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Artificial Intelligence – Assignment 4

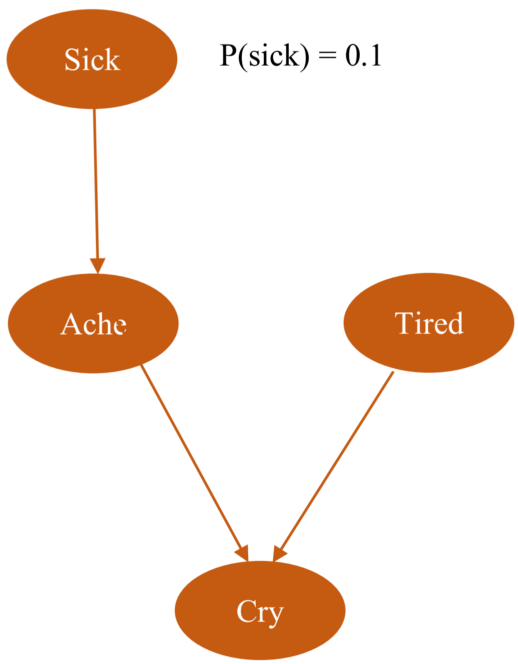
1. Babies have few and simple needs, but they have even fewer ways to communicate those needs to their parents. Parents need some way to figure out what is the matter with their child. Assume we know the following:

P(sick) = .1	P(ache | sick) = .75	P(ache | ¬sick) = .1	P(tired) = .6
P(cry | ache, tired) = .95	P(cry | ache, ¬tired) = .7	P(cry | ¬ache, tired) = .75	P(cry | ¬ache, ¬tired) = .05

   a. Assuming that the above conditional probabilities are sufficient to specify the values for a Bayes net for this problem, draw a picture of such a Bayesian network.

⇨

P(sick) = 0.1



P(tired) = 0.6

S	P(A)
0	0.1
1	0.75

A	T	P(C)
0	0	0.05
0	1	0.75
1	0	0.7
1	1	0.95

b. Use exact inference to compute the full joint probability table for this Bayesian network. Please model your table on the following template:

⇨ P(ache, tired, sick, cry) = P(sick) * P(tired) * P(ache|sick) * P(cry|ache,tired)
= 0.1 * 0.6 * 0.75 * 0.95
= 0.04275

P(ache, ¬tired, sick, cry) = P(sick) * P(¬tired) * P(ache|sick) * P(cry|ache, ¬tired)
= 0.1 * 0.4 * 0.75 * 0.7
= 0.021

P(ache, tired, ¬sick, cry) = P(¬sick) * P(tired) * P(ache|¬sick) * P(cry|ache, tired)
= 0.9 * 0.6 * 0.1 * 0.95
= 0.0513

P(ache, tired, ¬sick, cry) = P(¬sick) * P(¬tired) * P(ache|¬sick) * P(cry|ache, ¬tired)
= 0.9 * 0.4 * 0.1 * 0.7
= 0.0252

P(ache, tired, sick, ¬cry) = P(sick) * P(tired) * P(ache|sick) * P(¬cry|ache,tired)
= 0.1 * 0.6 * 0.75 * 0.05
= 0.0025

P(ache, ¬tired, sick, ¬cry) = P(sick) * P(¬tired) * P(ache|sick) * P(¬cry|ache, ¬tired)
= 0.1 * 0.4 * 0.75 * 0.3
= 0.009

P(ache, tired, ¬sick, ¬cry) = P(¬sick) * P(tired) * P(ache|¬sick) * P(¬cry|ache, tired)
= 0.9 * 0.6 * 0.1 * 0.05
= 0.0027

P(ache, tired, ¬sick,¬cry) = P(¬sick)*P(¬tired)*P(ache|¬sick)*P(¬cry|ache,¬tired)
= 0.9 * 0.4 * 0.1 * 0.3
= 0.0108

P(¬ache, tired, sick, cry) = P(sick) * P(tired) * P(¬ache|sick) * P(cry|¬ache,tired)
= 0.1 * 0.6 * 0.25 * 0.75
= 0.01125

P(¬ache, ¬tired, sick, cry) = P(sick)*P(¬tired)*P(¬ache|sick)*P(cry|¬ache, ¬tired)
= 0.1 * 0.4 * 0.25 * 0.05
= 0.0005

P(¬ache, tired, ¬sick, cry) = P(¬sick)*P(tired)*P(¬ache|¬sick)*P(cry|¬ache, tired)
= 0.9 * 0.6 * 0.9 * 0.75

= 0.3645

P(¬ache, tired,¬sick,cry)=P(¬sick)*P(¬tired)*P(¬ache|¬sick)*P(cry|¬ache, ¬tired)
= 0.9 * 0.4 * 0.9 * 0.05
= 0.0162

P(ache, tired, sick, ¬cry) = P(sick) * P(tired) * P(ache|sick) * P(¬cry|ache,tired)
= 0.1 * 0.6 * 0.25 * 0.25
= 0.00375

P(ache, ¬tired, sick, ¬cry) = P(sick) * P(¬tired) * P(ache|sick) * P(¬cry|ache, ¬tired)
= 0.1 * 0.4 * 0.25 * 0.95
= 0.0095

P(¬ache, tired,¬sick,¬cry)=P(¬sick)*P(tired)*P(¬ache|¬sick)*P(¬cry|¬ache, tired)
= 0.9 * 0.6 * 0.9 * 0.25
= 0.0125

P(¬ache,tired,¬sick,¬cry)=P(¬sick)*P(¬tired)*P(¬ache|¬sick)*P(¬cry|¬ache,¬tired)
= 0.9 * 0.4 * 0.9 * 0.95
= 0.3078

		sick		¬sick	
		tired	¬tired	tired	¬tired
ache	cry	0.04275	0.021	0.0513	0.0252
	¬cry	0.00225	0.009	0.0027	0.0108
¬ache	cry	0.01125	0.0005	0.3645	0.0162
	¬cry	0..00375	0.0095	0.0125	0.3078

2. Here's some domain knowledge you can use to construct an intelligent agent for simple medical diagnosis.

   Lexicon

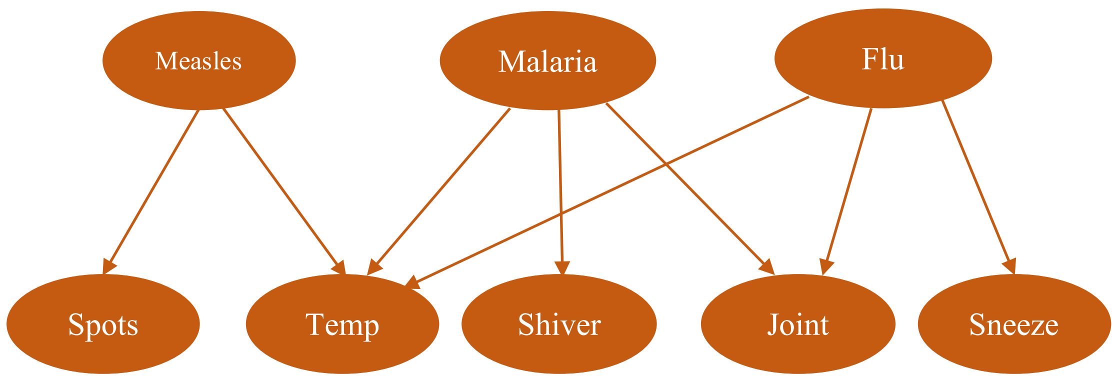
   measles -- the patient has measles;
   malaria -- the patient has malaria;
   flu -- the patient has flu;
   spots -- the patient is covered with spots;
   temp -- the patient has a high temperature;
   shiver -- the patient is shivering violently;

joint -- the patient has pain in his/her joints;
sneeze -- the patient is sneezing.

Symptom/Diagnosis Rules

If (spots and temp) then measles
If (temp and shiver and joint) then malaria
If (temp and joint and sneeze) then flu

1. Using the knowledge base, show the corresponding Bayes Net. (Caution: the complexity of the rest of the assignment depends on the directionality of the edges in the net, so think carefully: do symptoms cause diseases or do diseases cause symptoms?)



Bayes Net

2. Using the ideas of conditional independence discussed in class and in Ch. 14 of R&N, identify which variables in this network are conditionally independent of which others given what conditions. For example, is measles conditionally independent of malaria given high temperature? Is joint pain conditionally independent of shivering given malaria (etc. etc.)? Where appropriate, it's sufficient to make shorthand statements such as "X is conditionally independent of the rest of the network given Y", to save having to enumerate all the nodes in the rest of the network.

⇨ The following are the conditional relations between various elements:
   a. **Malaria** is conditionally independent of the rest of the network given **Temp**, **Shiver**, **Joint**, **Flu** and **Measles**.(A node is independent of the rest of the network given its children and its spouse)

b. **Measles** is conditionally independent of the rest of the network given **Spots**, **Temp**, **Flu** and **Malaria**. (A node is independent of the rest of the network given its children and its spouse)
c. **Flu** is conditionally independent of the rest of the network given **Temp**, **Joint**, **Sneeze**, **Measles** and **Malaria.** (A node is independent of the rest of the network given its children and its spouse)
d. **Shiver** is conditionally independent of the rest of the network given **Malaria**. (A node is independent of the rest of the network given its parents)
e. **Spots** is conditionally independent of the rest of the network given **Measles**. (A node is independent of the rest of the network given its parents)
f. **Sneeze** is conditionally independent of the rest of the network given **Flu**. (A node is independent of the rest of the network given its parents)
g. **Temp** is conditionally independent of the rest of the network given **Measles**, **Malaria** and **Flu**. (A node is independent of the rest of the network given its parents)
h. **Joint** is conditionally independent of the rest of the network given **Malaria** and **Flu**. (A node is independent of the rest of the network given its parents)
i. **Temp**, **Joint** and **sneeze** are conditionally independent of each other given **Flu**.
j. **Spots** and **temp** are conditionally independent of each other given **Measles**.
k. **Temp**, **shiver** and **joint** are conditionally independent of each other given **Malaria**.

3. Use the prior and conditional probabilities provided to construct probability tables for each node of this network. Then use exact inference to answer the following questions. What is the probability that –

⇨ The probability tables are constructed below:

Measles	Spots	P(Spots, Measles)
T	T	0.9*0.02 = 0.018
F	T	0.03*0.98 = 0.0294
P(Spots = True)		0.0474

Malaria	Shiver	P(Shiver, Malaria)
T	T	0.6*0.01 = 0.006
F	T	0.02*0.99 = 0.0198
P(Spots = True)		0.0258

Flue	Sneeze	P(Sneeze, Flu)
T	T	0.7*0.04 = 0.028
F	T	0.4*0.96 = 0.0384

P(Sneeze = True)		0.0664

Malaria	flu	joint	P (joint, malaria, flu)
T	T	T	0.95*0.01*0.04 = 0.00038
T	F	T	0.9*0.01*0.96 = 0.00864
F	T	T	0.75*0.99*0.04 = 0.0297
F	F	T	0.1*0.99*0.96 = 0.09504
P(Joint = True)			0.13376

Measles	Malaria	Flue	Temp	P (Temp, Flue, malaria, flu)
T	T	T	T	0.95*0.02*0.01*0.04 = 7.6*10-6
T	F	T	T	0.5*0.02*0.99*0.04 = 3.96*10-4
F	T	T	T	0.8*0.98*0.01*0.04 = 3.136*10-4
F	F	T	T	0.9*0.98*0.99*0.04 = 0.0349272
T	T	F	T	0.75*0.02*0.01*0.96 = 1.44*10-4
T	F	F	T	0.5*0.02*0.99*0.96 = 9.504*10-3
F	T	F	T	0.7*0.98*0.01*0.96 = 6.5856*10-3
F	F	F	T	0.05*0.98*0.99*0.96 = 0.0465696
P(Measles)				0.0984476

a. the patient has joint pain?

Using chain rule:

P(Joint=true) =
$\sum_{measles,malaria,flu,joint,temp,sneeze,shiver,spot}$(P(measles)*P(malaria)*P(flu)*P(spot|measles)*P(shiver|malaria)*P(Sneeze|Flu)*P(Temp|Measles,Malaria,Flu)*P(Joint|Malaria,Flu)

Marginalizing and eliminating variables using exact inference:

=> $\sum_{malaria,flu}$ P(joint|malaria,flu) *P(malaria)*P(flu)* $\sum_{spots}\sum_{measles}$ P(spots|measles)* P(measles)* $\sum_{sneeze}$ P(sneeze|flu)* $\sum_{shiver}$P(shiver|malaria)* $\sum_{temp}$ P(temp|measles,malaria,flu)

Now, $\sum_{sneeze}$ P(sneeze|flu) = 1 upon marginalizing it over sneeze.
Similarly, $\sum_{shiver}$P(shiver|malaria) = 1 upon marginalizing it over shiver.
Similarly, $\sum_{temp}$ P(temp|measles,malaria,flu) upon marginalizing it over temp.

=> $\sum_{malaria,flu}$ P(joint|malaria,flu) *P(malaria)*P(flu)* $\sum_{spots}\sum_{measles}$ P(spots|measles)* P(measles)*1*1*1

Now, marginalizing P(spots|measles)*P(measles) = P(spots,measles) over spots and measles gives 1.

=> $\sum_{malaria,flu}$ P(joint|malaria,flu) *P(malaria)*P(flu)* 1*1*1*1

=> 0.99*0.04*0.75+0.01*0.96*0.9+0.01*0.04*0.95+0.99*0.96*0.1
=> 0.13376

b. the patient has malaria?

The probability that the patient has malaria is 0.01 as given earlier.

c. the patient has a high temperature?

Using chain rule:

P(Temp=true) =
$\sum_{measles,malaria,flu,joint,temp,sneeze,shiver,spot}$(P(measles)*P(malaria)*P(flu)*P(spot|measles)*P(shiver|malaria)*P(Sneeze|Flu)*P(Temp|Measles,Malaria,Flu)*P(Joint|Malaria,Flu))

We need to calculate the probability of temp, when its true, so our interest lies in the term P(Temp|Measles,Malaria,Flu)

=>$\sum_{measles,malaria,flu}$ P(Temp|Measles,Malaria,Flu)* P(measles)*P(malaria)*P(flu)* $\sum_{spot}$P(spot|measles)* $\sum_{shiver}$P(shiver|malaria)* $\sum_{sneeze}$P(sneeze|flu)* $\sum_{joint}$P(Joint|Malaria,Flu)

Now, $\sum_{spot}$P(spot|measles) = $\sum_{shiver}$P(shiver|malaria) = $\sum_{sneeze}$P(sneeze|flu) = $\sum_{joint}$P(Joint|Malaria,Flu) = 1

=>$\sum_{measles,malaria,flu}$ P(Temp|Measles,Malaria,Flu)* P(measles)*P(malaria)*P(flu) *1*1*1

=> 0.02*0.99*0.04*0.5+
0.02*0.01*0.04*0.95+
0.98*0.99*0.04*0.9+
0.98*0.01*0.04*0.8+
0.98*0.99*0.96*0.05+
0.02*0.01*0.96*0.75+
0.02*0.99*0.96*0.5+
0.98*0.01*0.96*0.7 = 0.0984476

BONUS:

1. Introduce the evidence that THE PATIENT DOES NOT HAVE A HIGH TEMPERATURE and HAS JOINT PAIN. Perform the needed inference to answer the following questions. What is the probability that –

a. The patient has the flu? Has this probability gone up from before the evidence, down, or stayed the same? Why?

- ⇨ P(FLU = TRUE| TEMP = FALSE, JOINT = TRUE)
  =>$\alpha\sum_{measles,malaria,sneeze,shiver,spot}$P(measles)*P(malaria)*P(flu)*P(spot|measles)*P(shiver|malaria)*P(Sneeze|Flu)*P(Temp|Measles,Malaria,Flu)*P(Joint|Malaria,Flu)

  Using elimination of variables, we get the following solution:

  For flu = TRUE

  =>αP(flu=true) $\sum_{malaria}$P(Malaria)*P(Joint=True|Malaria, flu=true)* $\sum_{measles}$P(Measles)*P(temp=false|malaria,measles,flu=true)

  =>α*0.04*[.99*.75*(.02*0.5+.98*0.1)+0.1*0.95*(0.02*0.25+0.98*0.5)]
  =>0. 0032825*α

  For flu = FALSE

  =>αP(flu=false) $\sum_{malaria}$P(Malaria)*P(Joint=True|Malaria, flu=false)* $\sum_{measles}$P(Measles)*P(temp=false|malaria,measles,flu=false)

  =>0.092023*α

  We know that, α(P(F=true))+ α(P(F=false)) = 1
  => α = 10.3576
  Hence, P(flu=true |temp=false and joint=T) = 0.03998.

  As we can see, the probability of flu=true has decreased after the introduction of the evidence because according to the probability distribution table, the dependency of flu on temperature is higher than the dependency of flu on joint pain.

b. the patient has joint pain? Has this probability gone up from before the evidence, down, or stayed the same? Why?

   P(Joint=TRUE) is 1 as that matches the evidence. And the probability that the evidence will happen will be 1. So, in this case it will increase to 1.

c. the patient has the malaria? Has this probability gone up from before the evidence, down, or stayed the same? Why?

   ⇨ P(MALARIA= TRUE| TEMP = FALSE, JOINT = TRUE)
   αP(malaria=TRUE)
   $\sum_{measles,flu}$P(measles)*P(flu)*P(joint=true|malaria=true,flu)*P(temp=true|measles,malaria=true,flu)
   => α*0.01*[(0.04*0.02*0.95*(1-0.95))+(0.04*0.98*0.95*(1-0.8))+(0.96*0.02*0.9*(1-0.75))+(0.96*0.98*0.9*(1-0.7))
   => 0.00265* α

   α P(malaria=FALSE)
   $\sum_{measles,flu}$P(measles)*P(flu)*P(joint=true|malaria=false,flu)*P(temp=true|measles,malaria=false,flu)
   => α*0.99*[(0.04*0.02*0.75*(1-0.5))+(0.04*0.98*0.75*(1-0.9))+(0.96*0.02*0.1*(1-0.5))+(0.96*0.98*0.1*(1-0.05))]
   => 0.09265* α
   We know that, α(P(F=true))+ α(P(F=false)) = 1
   => α = 10.4931
   Hence, P(malaria=true |temp=false and joint=T) = 0.02781.
   Here, we can see that the probability of malaria has increased with the introduction of evidence because according to the probability distribution table, the probability of Joint|malaria is pretty high as compared to the probability of Temp=false|Malaria.

d. the patient has the flu, given additional evidence that she is SHIVERING VIOLENTLY? Has this probability gone up from your answer to 1(a), down, or stayed the same? Why?

   ⇨ P(FLU=TRUE|TEMP=FALSE, JOINT=TRUE, SHIVER=TRUE)
   => αP(flu=TRUE) $\sum_{measles,malaria}$ P(measles)*P(malaria)*
   P(joint=True|malaria,flu=true)* P(temp=False|measles,malaria,flu=True)
   *P(shiver=True|malaria)

$=>\alpha*0.04*[(0.02 * 0.01 *0.95 * (0.05)*0.6)+(0.02*0.99*0.75*0.95*0.02)+(0.98*0.01*0.95*0.2*0.6)+(0.98*0.99*0.75*0.1*0.02)]$

$=> 0.000109* \alpha$

$=\alpha P(flu=False) \sum_{measles,malaria} P(measles)*P(malaria)* P(joint=True|malaria,flu=False)* P(temp=False|measles,malaria,flu=False)* P(shiver=True|malaria)$

$= \alpha*0.96*[(0.02*0.01*0.9*0.25*0.6) + (0.02*0.99*0.1*0.5*0.02) + (0.98*0.01*0.9*0.3*0.6) + (0.98*0.99*0.1*0.95*0.02)]$

$= 0.00338* \alpha$

We know that, $\alpha(P(F=true))+ \alpha(P(F=false)) = 1$

$\alpha =290.0453422$

P(flu=true |temp=false, joint=True, shiver=True)=0. 03998

Here we can see that when compared with 3.a, there is no change in probability because the introduction of shivering=true, doesn't change anything for flue=true as they are conditionally independent.