

Lexicon:

p -- The user enters a valid password.

q -- The user has paid the subscription fee.

r -- Access is granted.

The user has paid the subscription fee, but does not enter a valid password.

$$q \wedge \neg p$$

Access is granted whenever the user has paid the subscription fee and enters a valid password.

$$p \wedge q \Rightarrow r$$

Access is denied if the user has not paid the subscription fee.

$$\neg q \Rightarrow \neg r$$

If the user has not entered a valid password but has paid the subscription fee, then access is granted.

$$\neg p \wedge q \Rightarrow r$$

p	q	r	$q \wedge \neg p$	$p \wedge q \Rightarrow r$	$\neg q \Rightarrow \neg r$	$\neg p \wedge q \Rightarrow r$
+	+	+				

CSC 520 / 507 / 1

Lexicon:

p -- The system software is being upgraded

q -- Users can access the file system.

r -- Users can save new files.

Whenever the system software is being upgraded, users cannot access the file system. If users can access the file system, then they can save new files. If users cannot save new files, then the system software is not being upgraded.

$$p \Rightarrow \neg q$$

$$q \Rightarrow r$$

$$\neg r \Rightarrow \neg p$$

CNF

$$1. \neg p \vee \neg q$$

$$2. q \vee r$$

$$3. r \vee \neg p$$

Consistent

p	q	r	$\neg p \vee \neg q$	$q \vee r$	$r \vee \neg p$
t	t	t	f	t	t
t	t	f	f	f	f
t	f	t	t	t	t
t	f	f	t	f	f
f	t	t	t	t	t
f	t	f	t	f	f
f	f	t	t	t	t
f	f	f	t	f	f

8 rows

← satisfiable

CSC 520/008/1

LEXICON:

p -- The file system is locked.

q -- New messages will be queued.

r -- The system is functioning normally.

s -- New messages will be sent to the message buffer.

If the file system is not locked, then new messages will be queued. If the file system is not locked, then the system is functioning normally, and conversely. If new messages are not queued, then they will be sent to the message buffer. If the file system is not locked, then new messages will be sent to the message buffer. New messages will not be sent to the message buffer.

Consistent

$\neg p \Rightarrow q$
 $\neg p \Leftrightarrow r$
 $\neg q \Rightarrow s$
 $\neg p \Rightarrow s$
 $\neg s$

$\frac{p}{t} \quad \frac{q}{t} \quad \frac{r}{f} \quad \frac{s}{f} \quad \frac{\neg p}{t} \quad \frac{\neg q}{t} \quad \frac{\neg r}{t} \quad \frac{\neg s}{t}$

CNF
 $\neg p \vee q$
 $\neg p \vee r$
 $\neg r \vee \neg p$
 $q \vee s$
 $p \vee s$
 $\neg s$
 $\neg p$
 $\neg r$
 p
 \vdots

$6 + 5$
 $7 + 3$
 $8 + 2$

CSC 520 / 008 / 2

Lexicon:

- m -- The system is in multiuser state.
- n -- The system is operating normally.
- k -- The kernel is functioning.
- i -- The system is in interrupt mode.

The system is in multiuser state if and only if it is operating normally. If the system is operating normally, the kernel is functioning. Either the kernel is not functioning or the system is in interrupt mode. If the system is not in multiuser state, then it is in interrupt mode. The system is not in interrupt mode.

Inconsistent

Handwritten logical expressions and a truth table:

$m \leftrightarrow n$
 $n \Rightarrow k$
 $\neg k \vee i$
 $\neg m \Rightarrow i$
 $\neg i$

These are grouped by a bracket pointing to a set of clauses under the heading CNF:

1	$\neg m \vee n$
2	$\neg n \vee m$
3	$\neg n \vee k$
4	$\neg k \vee i$
5	$m \vee i$
6	$\neg i$

7	$\neg k$
8	$\neg n$
9	$\neg m$
10	$\neg n$
11	$\neg i$
12	\square

Resolution

$6 + 4$
 $7 + 3$
 $8 + 1$
 $9 + 2$
 $9 + 5$
 $11 + 6$
 CSC 520/008/3

Lexicon:

Constants: none

Variables: X

Functions: none

Predicates:

student(X) -- X is a student in this class.

calc(X) -- X has studied calculus.

Every student in this class has studied calculus.

$$\forall X (\text{student}(X) \Rightarrow \text{calc}(X))$$

csc 520/009/1

Lexicon:

Constants: none

Variables: X

Functions: none

Predicates:

student(X) -- X is a student in this class.

mex(X) -- X has visited Mexico.

One or more students in this class have visited Mexico.

$$\exists X (\text{student}(X) \wedge \text{mex}(X))$$

csc 520/009/2

Lexicon:

Constants: none

Variables: X

Functions: none

Predicates:

lion(X) -- X is a lion.

fierce(X) -- X is fierce.

coffee(X) -- X drinks coffee.

All lions are fierce.

$$\forall X (\text{lion}(X) \Rightarrow \text{fierce}(X))$$

Some lions do not drink coffee.

$$\exists X (\text{lion}(X) \wedge \neg \text{coffee}(X))$$

Some fierce creatures do not drink coffee.

$$\exists X (\text{fierce}(X) \wedge \neg \text{coffee}(X))$$

Not $\exists X (\text{lion}(X) \Rightarrow \neg \text{coffee}(X))$
true for non-lions
even if all lions
drink coffee

csc 520/209/3

Lexicon:

Constants: none

Variables: X

Functions: none

Predicates:

bird(X) -- X is a hummingbird.

large(X) -- X is large.

honey(X) -- X lives on honey.

color(X) -- X is richly colored.

All hummingbirds are richly colored.

$$\forall X (b(X) \Rightarrow c(X))$$

No large birds live on honey.

$$\neg (\exists X (l(X) \wedge h(X))) \quad \left| \begin{array}{l} \text{CNF} \\ \forall X \neg l(X) \vee \neg h(X) \\ \forall X l(X) \Rightarrow \neg h(X) \end{array} \right.$$

Birds that do not live on honey are dull in color.

$$\forall X (\neg h(X) \Rightarrow \neg c(X))$$

Hummingbirds are small.

$$\forall X (b(X) \Rightarrow s(X))$$

csc 520 / 009 / 4

Lexicon:

Constants: 0

Variables: X, Y, z ←

Functions:

succ(X) -- the successor of X

pred(X) -- the predecessor of X

Predicates:

e(X, Y) -- X = Y.

Universe: natural numbers

For every number, there is one and only one successor.

$$\forall X \exists Y (e(Y, \text{succ}(X)) \wedge \forall Z e(Z, \text{succ}(X)) \Rightarrow e(Z, Y))$$

There is no number for which 0 is the immediate successor.

$$\neg \exists X e(0, \text{succ}(X)) \Leftrightarrow \forall X \neg e(0, \text{succ}(X))$$

For every number other than 0, there is one and only one immediate predecessor.

$$\forall X (\neg e(X, 0) \Rightarrow \exists Y (e(Y, \text{pred}(X))))$$

$$\forall Z (\hat{\neg} (e(Z, \text{pred}(X)) \Rightarrow e(Y, Z)))$$

csc 520 / 009 / 5

Lexicon:

Constants: none

Variables: X, Y

Functions: none

Predicates:

patient(X) -- X is a patient.

quack(X) -- X is a quack.

likes(X, Y) -- X likes Y.

doctor(X) -- X is a doctor

Some patients like all doctors.

$$\exists X p(X) \wedge \forall Y d(Y) \Rightarrow l(X, Y)$$

No patient likes any quack.

$$\forall X p(X) \Rightarrow \forall Y q(Y) \Rightarrow \neg l(X, Y)$$

No doctor is a quack.

$$\forall X (d(X) \Rightarrow \neg q(X))$$

$$\forall X \forall Y (p(X) \wedge q(Y)) \Rightarrow \neg l(X, Y)$$

csc 520/209/6

Lexicon:

Constants: none

Variables: X, Y

Functions: none

Predicates:

patient(X) -- X is a patient.

quack(X) -- X is a quack.

likes(X, Y) -- X likes Y.

Some patients like all doctors.

$$\exists X (p(X) \wedge \forall Y (d(Y) \Rightarrow l(X, Y)))$$

No patient likes any quack.

$$\forall X p(X) \Rightarrow \forall Y (q(Y) \Rightarrow \neg l(X, Y))$$

No doctor is a quack.

$$\forall X (d(X) \Rightarrow \neg q(X))$$

csc 520/06/1

Lexicon:

Constants: none

Variables: X, Y

Functions: none

Predicates:

patient(X) -- X is a patient.

quack(X) -- X is a quack.

likes(X, Y) -- X likes Y.

Some patients like all doctors.

No patient likes any quack.

THEREFORE, no doctor is a quack.

CNF

$$\begin{array}{l} 1 \exists X (p(X) \wedge \forall Y (d(Y) \Rightarrow l(X, Y))) \longrightarrow \left\{ \begin{array}{l} 1 \quad p(a) \\ 2 \quad \neg d(Y_2) \vee l(a, Y_2) \end{array} \right. \\ 2 \forall X (p(X) \Rightarrow \forall Y (q(Y) \Rightarrow \neg l(X, Y))) \longrightarrow \left\{ \begin{array}{l} 3 \quad \neg p(X) \vee \neg q(Y_3) \vee \neg l(X, Y_3) \end{array} \right. \\ \hline 3 \forall X (d(X) \Rightarrow \neg q(X)) \longrightarrow \left\{ \begin{array}{l} 4 \quad d(b) \\ 5 \quad q(b) \end{array} \right. \\ \hline \neg [\forall X (d(X) \Rightarrow \neg q(X))] \\ \neg [\neg \exists X (\neg d(X) \vee \neg \neg q(X))] \\ \neg [\neg \exists X (\neg d(X) \vee \neg q(X))] \\ \neg [\neg \exists X (\neg d(X) \wedge \neg \neg q(X))] \\ \neg [\neg \exists X (\neg d(X) \wedge q(X))] \\ \neg [\neg \exists X (d(X) \wedge q(X))] \\ \neg [\neg \exists X (d(X) \wedge q(X))] \\ d(b) \wedge q(b) \end{array}$$
$$\begin{array}{l} \hline 6 \neg p(X) \vee \neg l(X, b) \\ \quad \quad \quad 5+3 \{b/Y_3\} \\ 7 \neg l(a, b) \quad 6+1 \{a/X\} \\ 8 \neg d(b) \quad 7+2 \{b/Y_2\} \\ 9 \quad \square \quad 8+4 \{\} \end{array}$$

csc 520/010/2

Lexicon:

Constants: none

Variables: X, H

Functions: none

Predicates:

dog(X) -- X is a dog.

animal(X) -- X is an animal.

headOf(H, X) -- H is the head of X.

The head of a dog is the head of an animal.

$$\forall H (\exists Y (dog(Y) \wedge headOf(H, Y)))$$

$$\Rightarrow \exists Z (animal(Z) \wedge headOf(H, Z))$$

$$\downarrow \text{Asmt \#2}$$
$$\neg [\quad] = ?$$

CSC 520 / 010 / 3