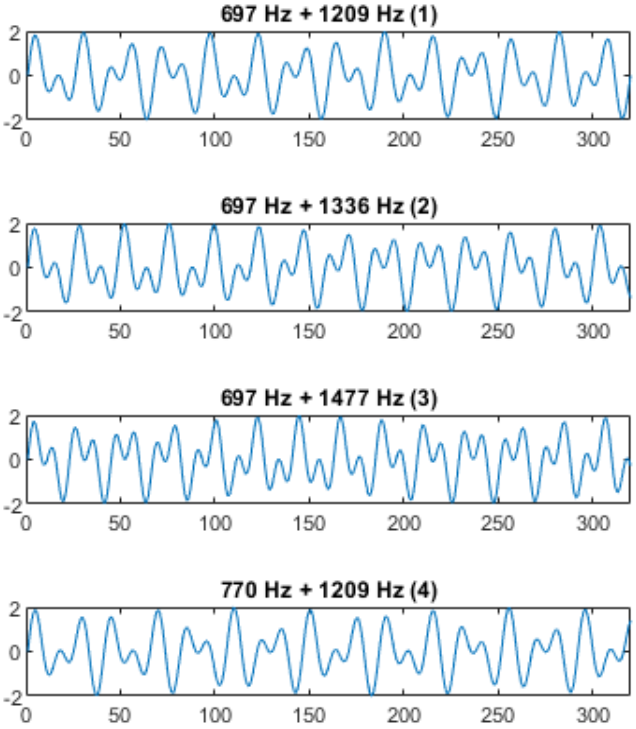
Simulating the Goertzel Algorithm

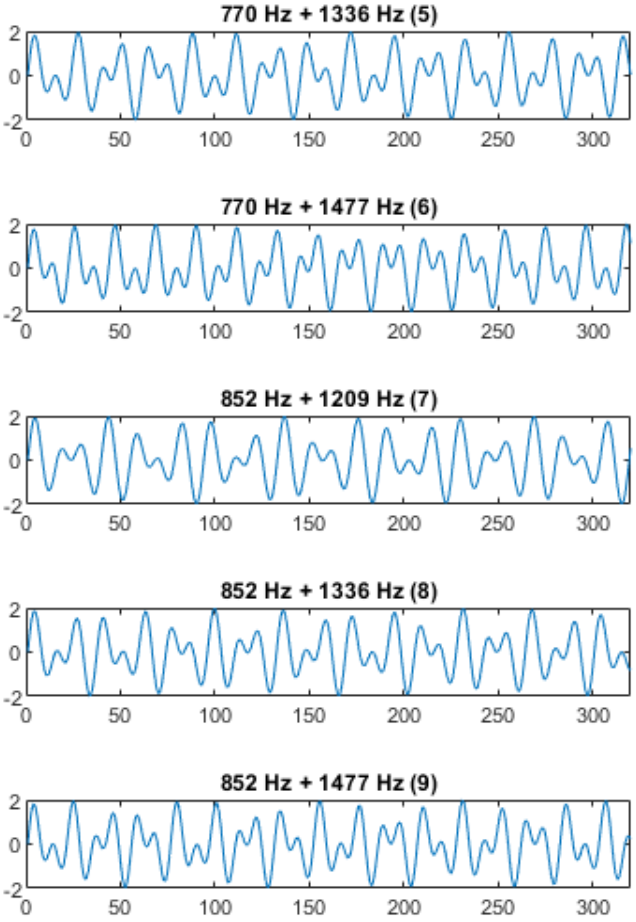
Detection Results with 12 Tones, Randomized Noise, Frame Shifting, + AWGN.

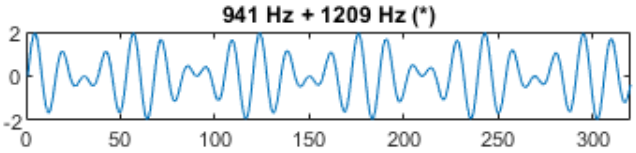
29 October 2024

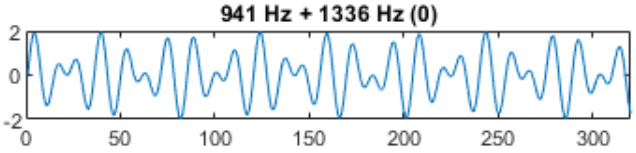
Configuration used: $F_s = 16000$, duration = 0.02 s, N = 320.

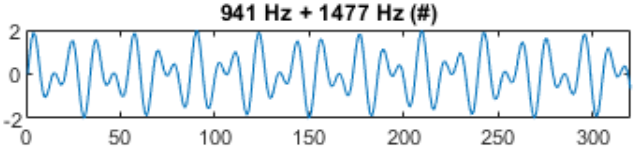
The 12 Tones


697 Hz + 1209 Hz (1)
697 Hz + 1336 Hz (2)
697 Hz + 1477 Hz (3)
770 Hz + 1209 Hz (4)
2
0
-2
0
50
100
150
200
250
300


770 Hz + 1336 Hz (5)
770 Hz + 1477 Hz (6)
852 Hz + 1209 Hz (7)
852 Hz + 1336 Hz (8)
852 Hz + 1477 Hz (9)
2
0
-2
0
50
100
150
200
250
300

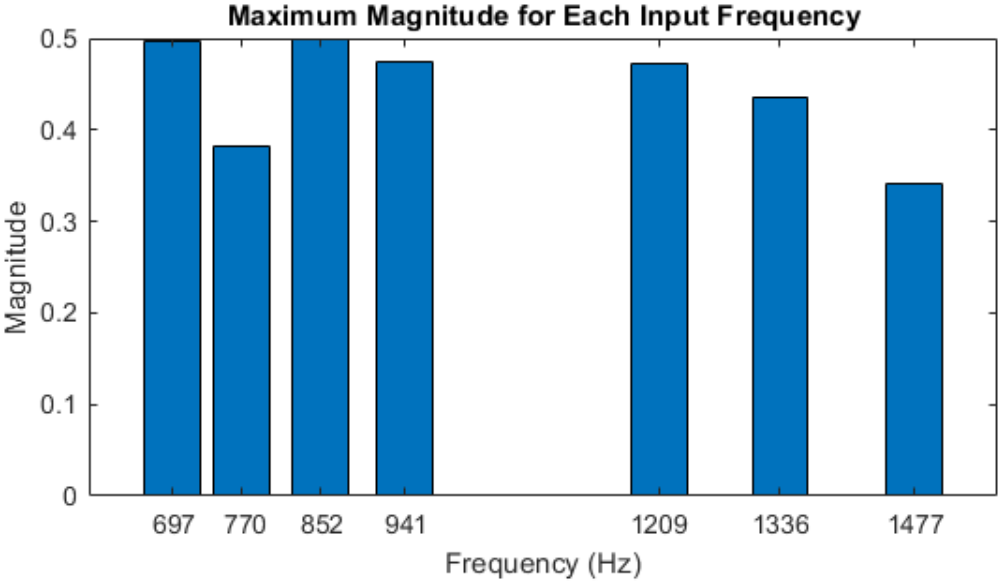

941 Hz + 1209 Hz (*)
2
0
-2
0
50
100
150
200
250
300


941 Hz + 1336 Hz (0)
2
0
-2
0
50
100
150
200
250
300

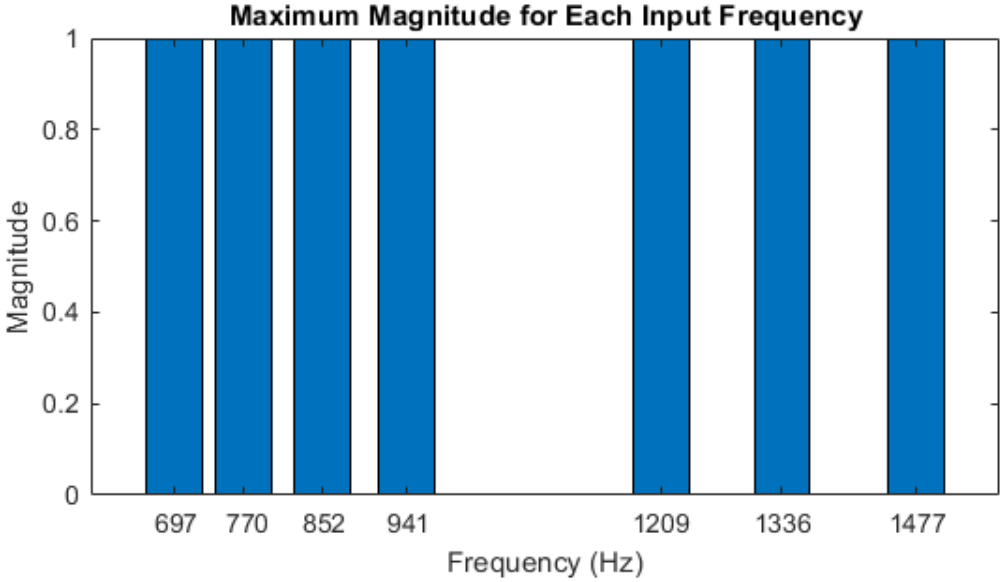

941 Hz + 1477 Hz (#)
2
0
-2
0
50
100
150
200
250
300

Detection using the Goertzel Algorithm

Before Normalization

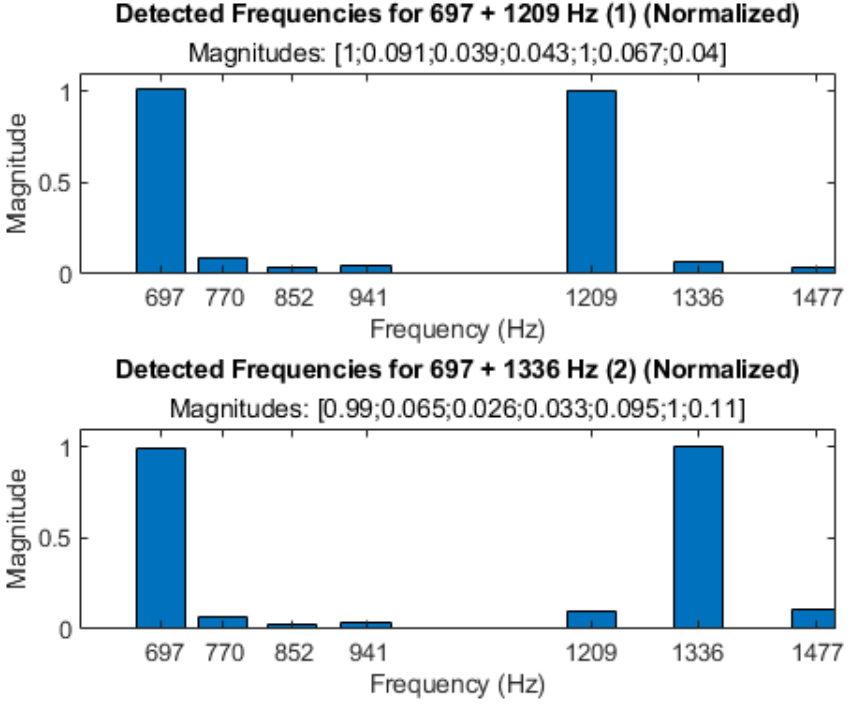


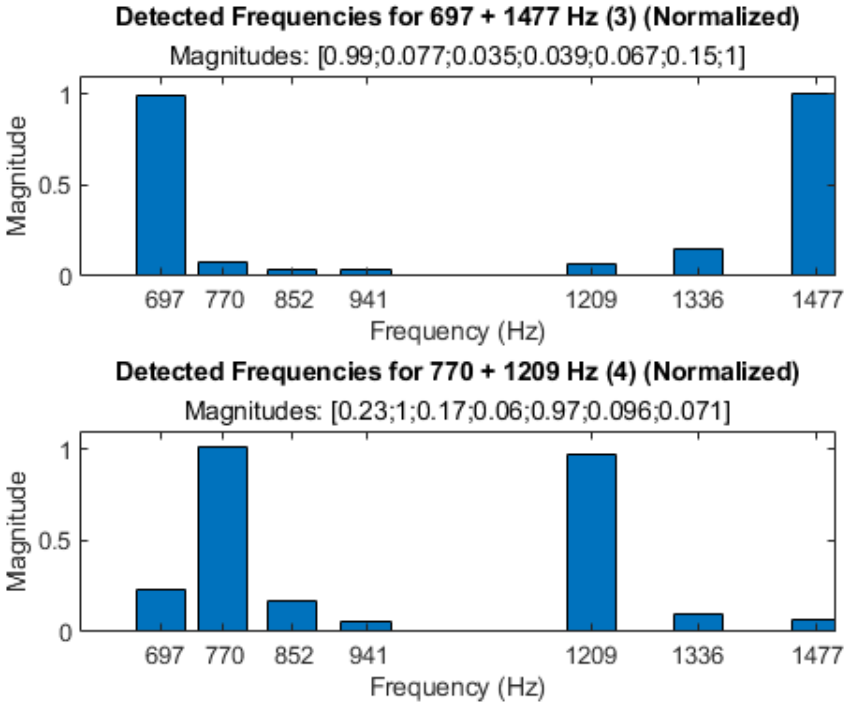
After Normalization

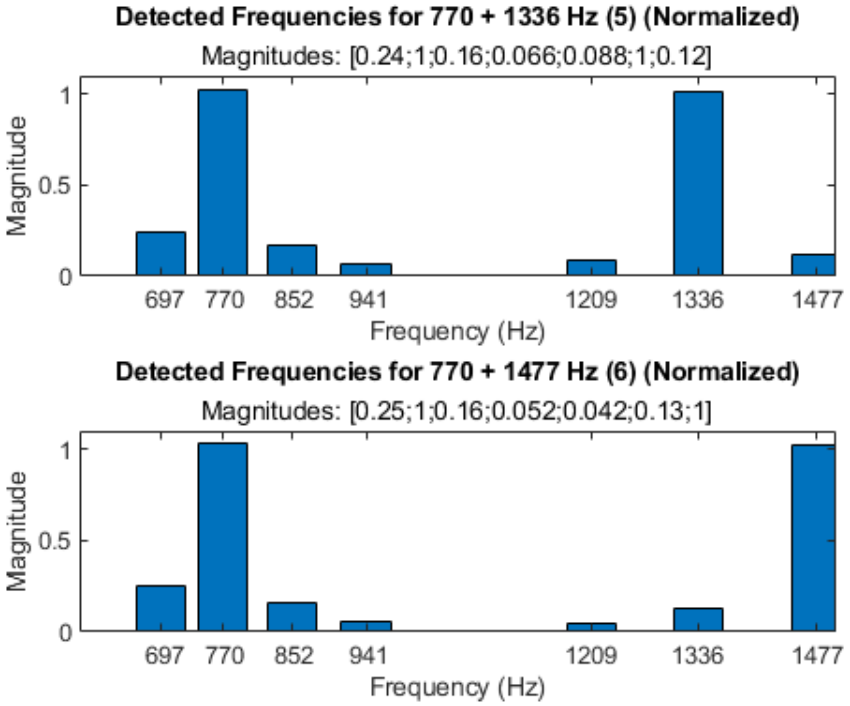


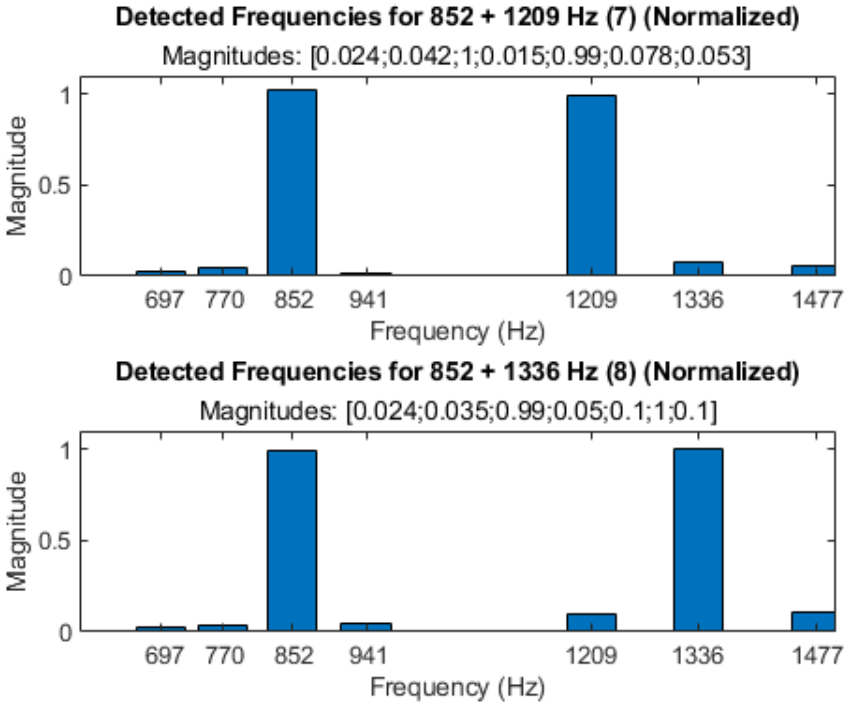
Frequency	Scaling Factor
697 Hz	2.0084
770 Hz	2.6215
852 Hz	2.0077
941 Hz	2.11
1209 Hz	2.1172
1336 Hz	2.2914
1477 Hz	2.936

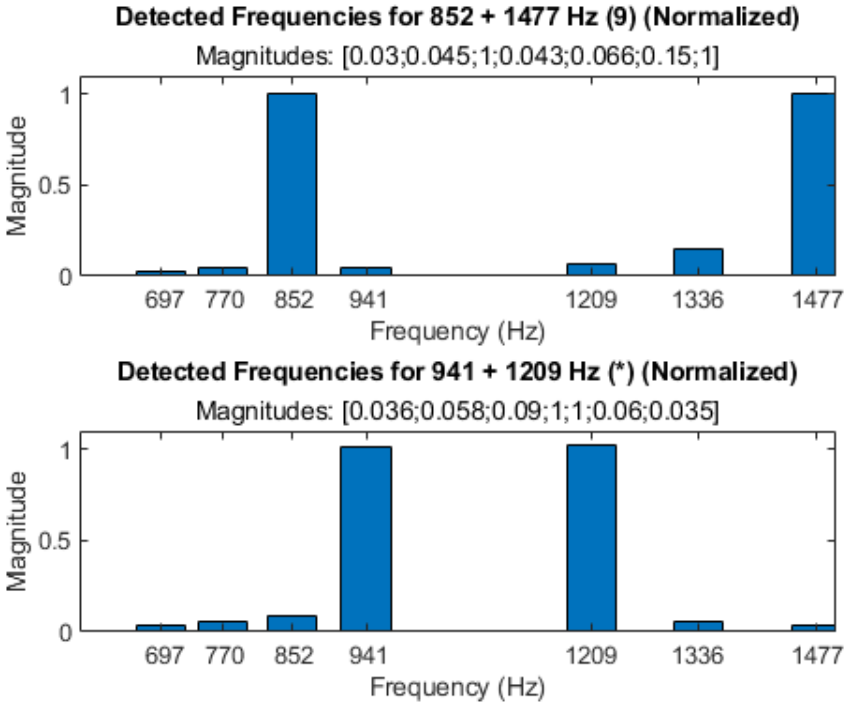
Detecting 12 DTMF (Pure) Tones

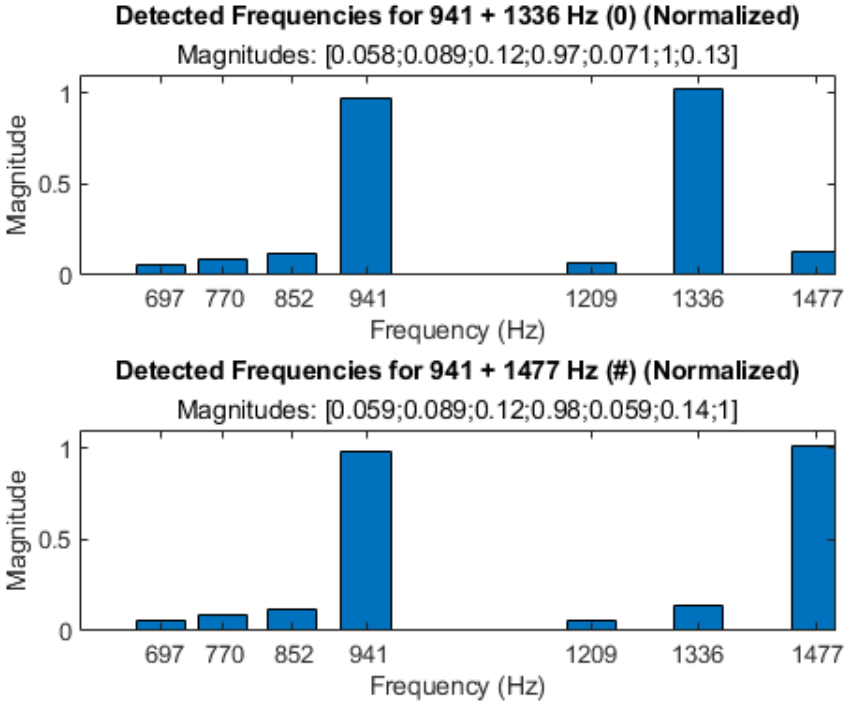



Detected Frequencies for 697 + 1477 Hz (3) (Normalized)
Magnitudes: [0.99;0.077;0.035;0.039;0.067;0.15;1]
Magnitude
1
0.5
0
697
770
852
941
1209
1336
1477
Frequency (Hz)
Detected Frequencies for 770 + 1209 Hz (4) (Normalized)
Magnitudes: [0.23;1;0.17;0.06;0.97;0.096;0.071]
Magnitude
1
0.5
0
697
770
852
941
1209
1336
1477
Frequency (Hz)


Detected Frequencies for 770 + 1336 Hz (5) (Normalized)
Magnitudes: [0.24;1;0.16;0.066;0.088;1;0.12]
Magnitude
1
0.5
0
697
770
852
941
1209
1336
1477
Frequency (Hz)
Detected Frequencies for 770 + 1477 Hz (6) (Normalized)
Magnitudes: [0.25;1;0.16;0.052;0.042;0.13;1]
Magnitude
1
0.5
0
697
770
852
941
1209
1336
1477
Frequency (Hz)


Detected Frequencies for 852 + 1209 Hz (7) (Normalized)
Magnitudes: [0.024;0.042;1;0.015;0.99;0.078;0.053]
Magnitude
1
0.5
0
697
770
852
941
1209
1336
1477
Frequency (Hz)
Detected Frequencies for 852 + 1336 Hz (8) (Normalized)
Magnitudes: [0.024;0.035;0.99;0.05;0.1;1;0.1]
Magnitude
1
0.5
0
697
770
852
941
1209
1336
1477
Frequency (Hz)


Detected Frequencies for 852 + 1477 Hz (9) (Normalized)
Magnitudes: [0.03;0.045;1;0.043;0.066;0.15;1]
Magnitude
1
0.5
0
697
770
852
941
1209
1336
1477
Frequency (Hz)
Detected Frequencies for 941 + 1209 Hz (*) (Normalized)
Magnitudes: [0.036;0.058;0.09;1;1;0.06;0.035]
Magnitude
1
0.5
0
697
770
852
941
1209
1336
1477
Frequency (Hz)


Detected Frequencies for 941 + 1336 Hz (0) (Normalized)
Magnitudes: [0.058;0.089;0.12;0.97;0.071;1;0.13]
Magnitude
1
0.5
0
697
770
852
941
1209
1336
1477
Frequency (Hz)
Detected Frequencies for 941 + 1477 Hz (#) (Normalized)
Magnitudes: [0.059;0.089;0.12;0.98;0.059;0.14;1]
Magnitude
1
0.5
0
697
770
852
941
1209
1336
1477
Frequency (Hz)

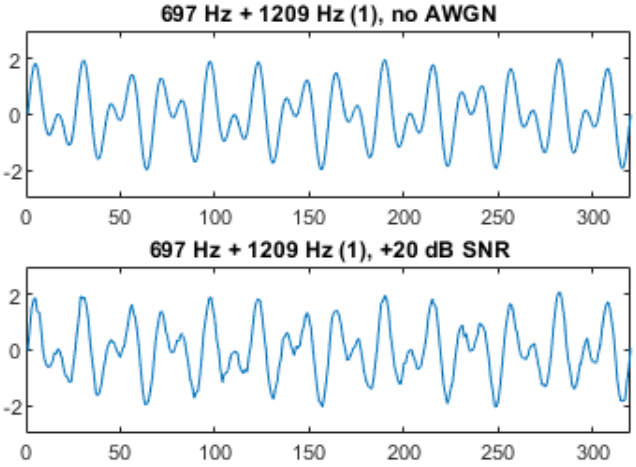
Summary of Resulting Detection Magnitudes for Pure Tone Inputs

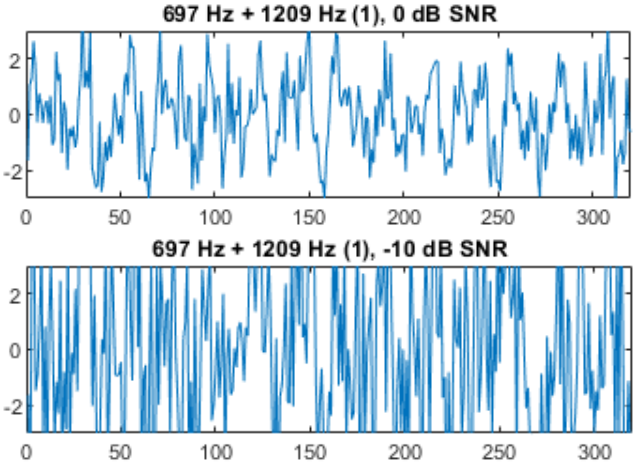
Input Signal	g(697)	g(770)	g(852)	g(941)	g(1209)	g(1336)	g(1477)
697 + 1209	1	0.091	0.039	0.043	1	0.067	0.04
697 + 1336	0.99	0.065	0.026	0.033	0.095	1	0.11
697 + 1477	0.99	0.077	0.035	0.039	0.067	0.15	1
770 + 1209	0.23	1	0.17	0.06	0.97	0.096	0.071
770 + 1336	0.24	1	0.16	0.066	0.088	1	0.12
770 + 1477	0.25	1	0.16	0.052	0.042	0.13	1
852 + 1209	0.024	0.042	1	0.015	0.99	0.078	0.053
852 + 1336	0.024	0.035	0.99	0.05	0.1	1	0.1
852 + 1477	0.03	0.045	1	0.043	0.066	0.15	1
941 + 1209	0.036	0.058	0.09	1	1	0.06	0.035
941 + 1336	0.058	0.089	0.12	0.97	0.071	1	0.13

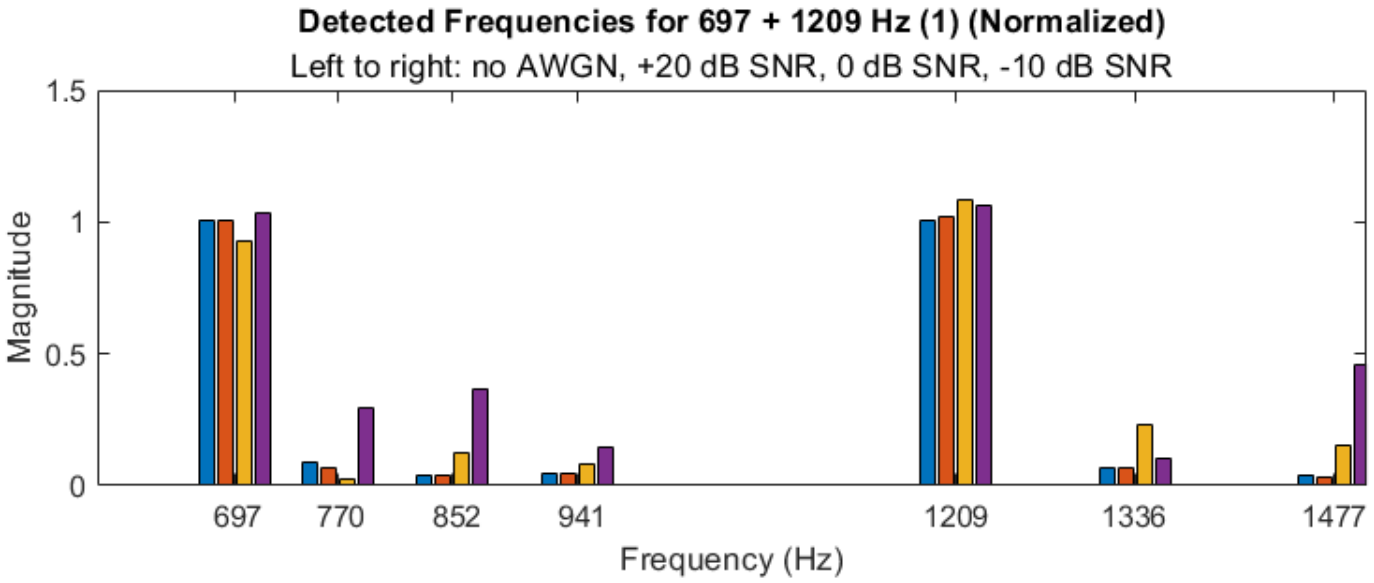
941 + 1477	0.059	0.089	0.12	0.98	0.059	0.14	1

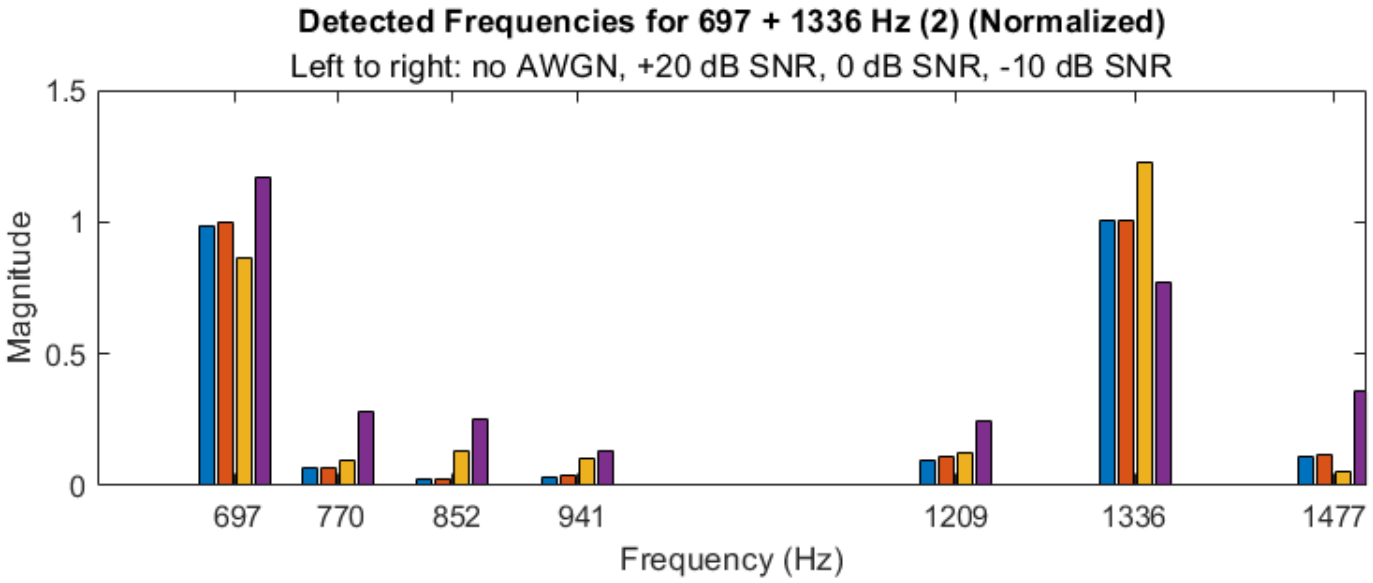
Input Signals with Additive White Gaussian Noise (AWGN)

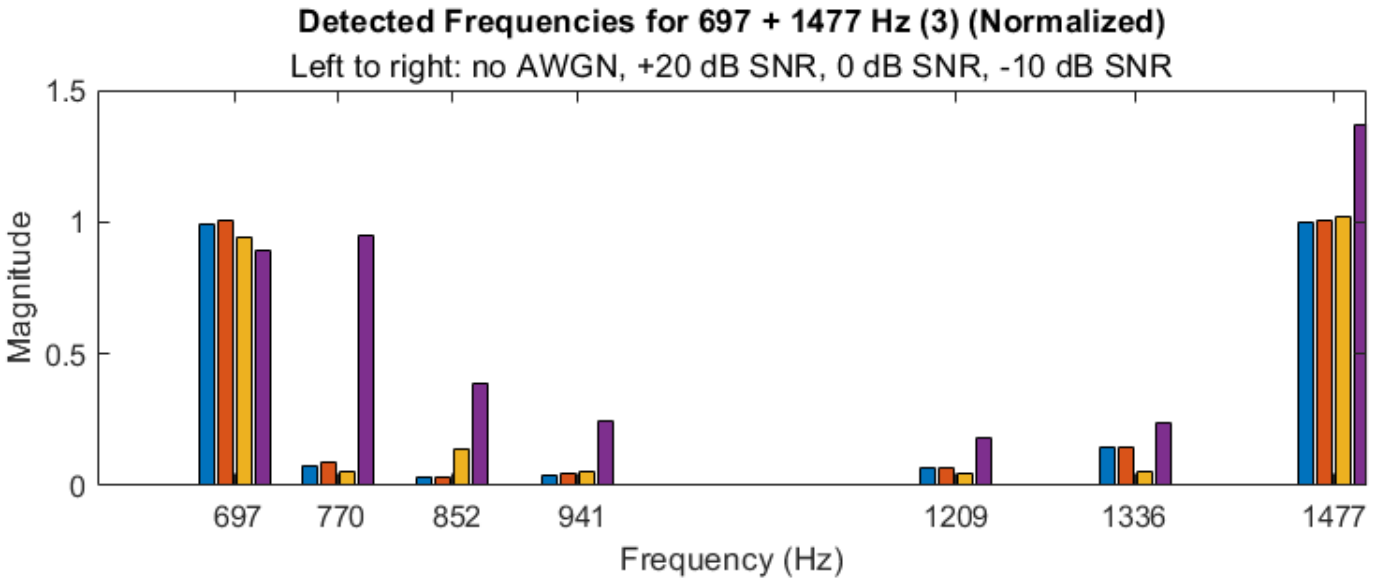
Configuration: awgn(pure tone, SNR in dB, measured)

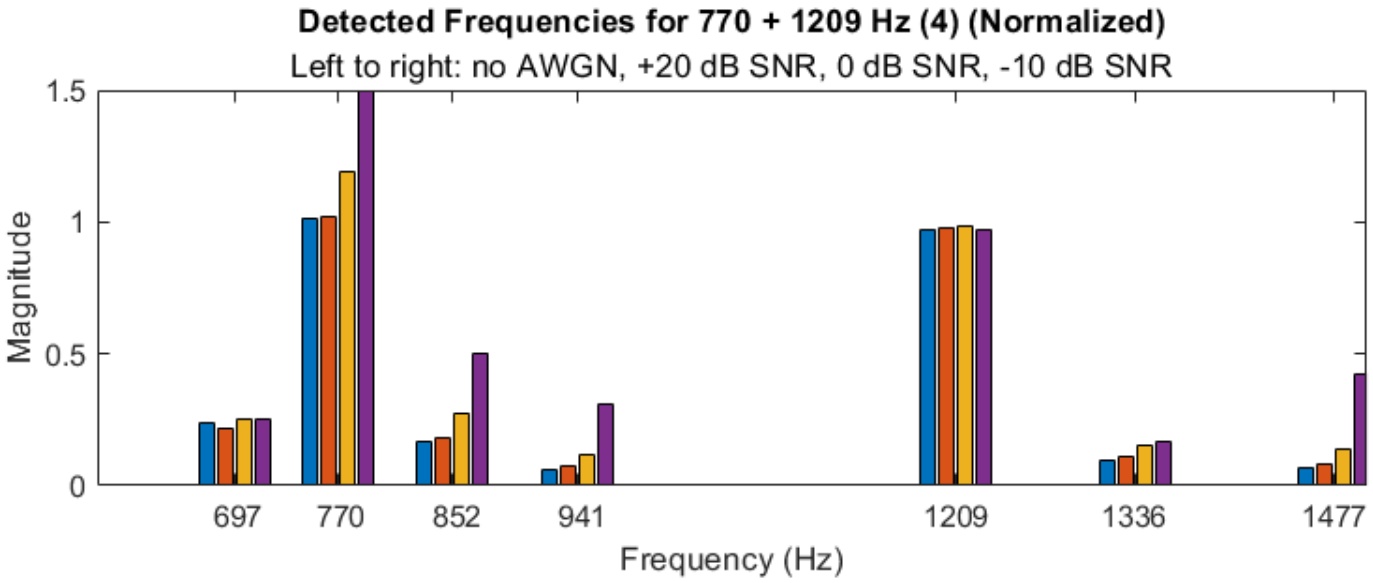


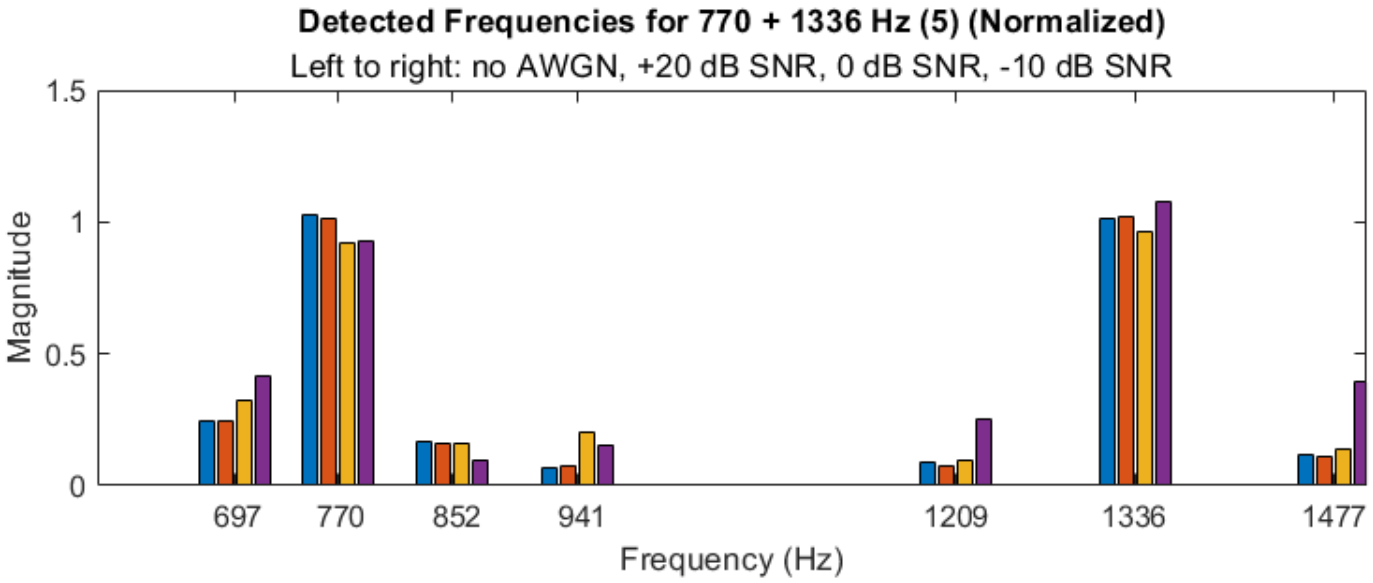

697 Hz + 1209 Hz (1), 0 dB SNR
2
0
-2
0
50
100
150
200
250
300
697 Hz + 1209 Hz (1), -10 dB SNR
2
0
-2
0
50
100
150
200
250
300

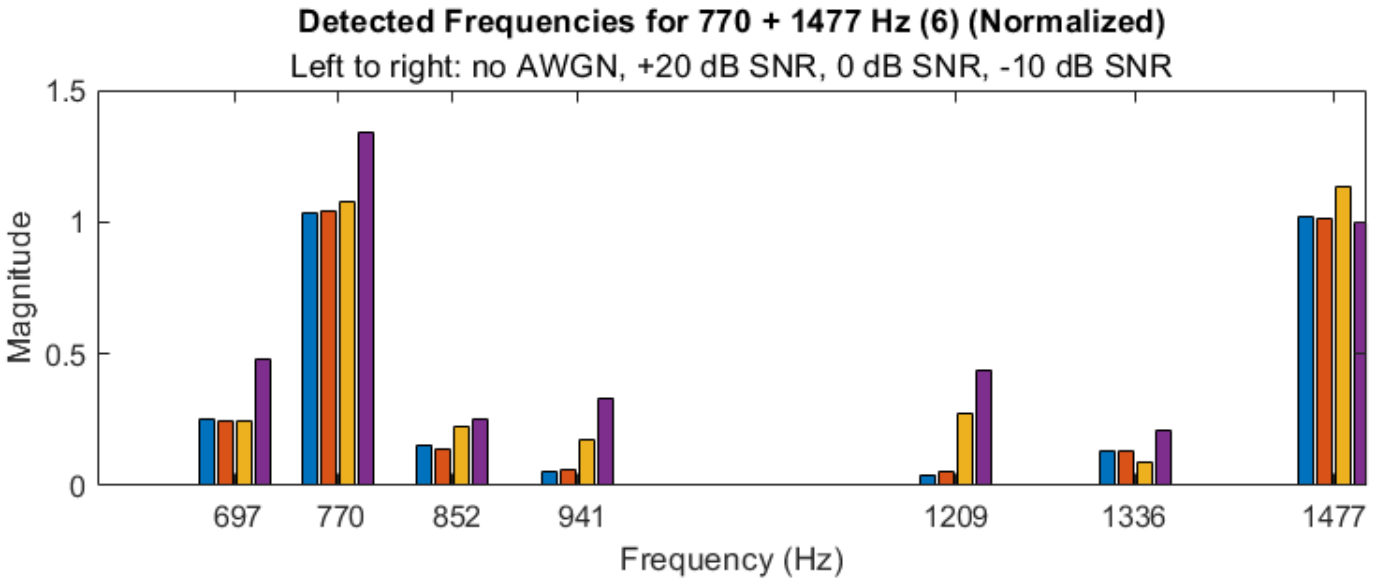

Detected Frequencies for 697 + 1209 Hz (1) (Normalized)
Left to right: no AWGN, +20 dB SNR, 0 dB SNR, -10 dB SNR
Magnitude
1.5
1
0.5
0
697
770
852
941
1209
1336
1477
Frequency (Hz)

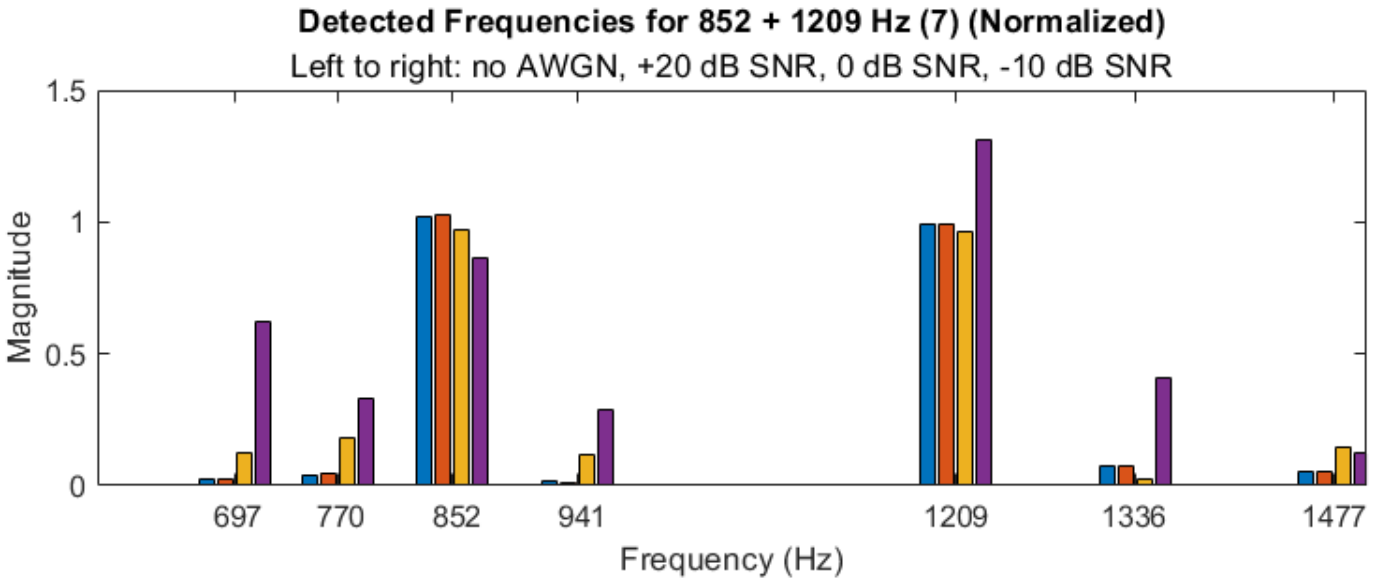

Detected Frequencies for 697 + 1336 Hz (2) (Normalized)
Left to right: no AWGN, +20 dB SNR, 0 dB SNR, -10 dB SNR
Magnitude
1.5
1
0.5
0
697
770
852
941
1209
1336
1477
Frequency (Hz)

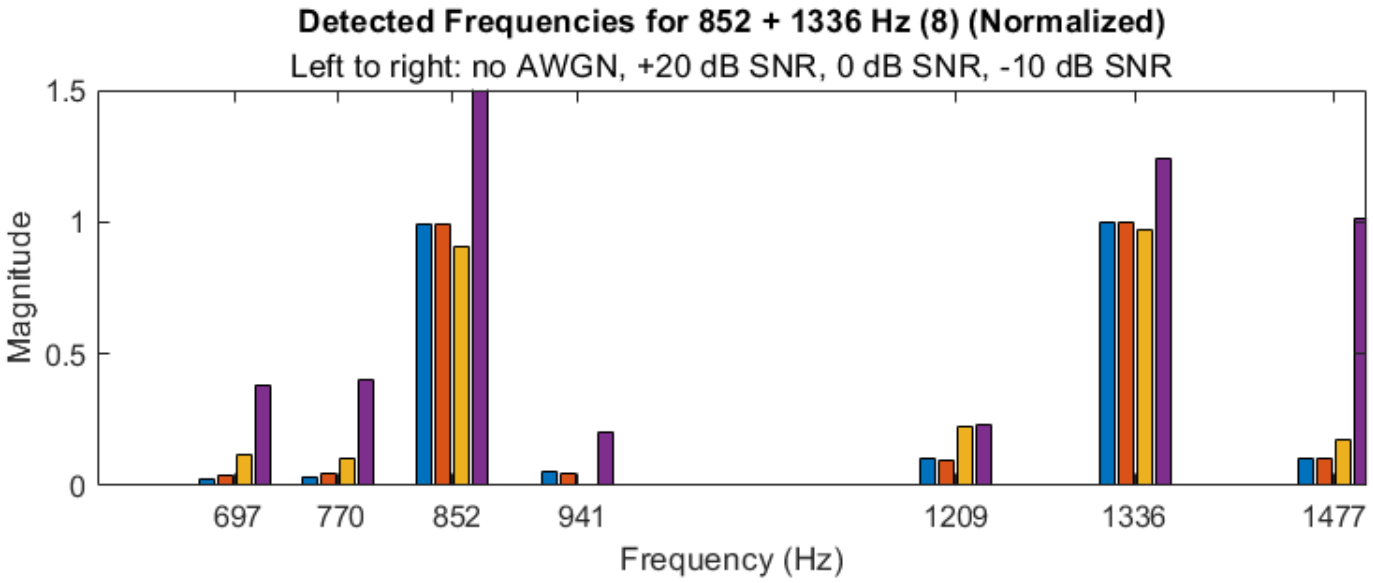

Detected Frequencies for 697 + 1477 Hz (3) (Normalized)
Left to right: no AWGN, +20 dB SNR, 0 dB SNR, -10 dB SNR
Magnitude
1.5
1
0.5
0
697
770
852
941
1209
1336
1477
Frequency (Hz)

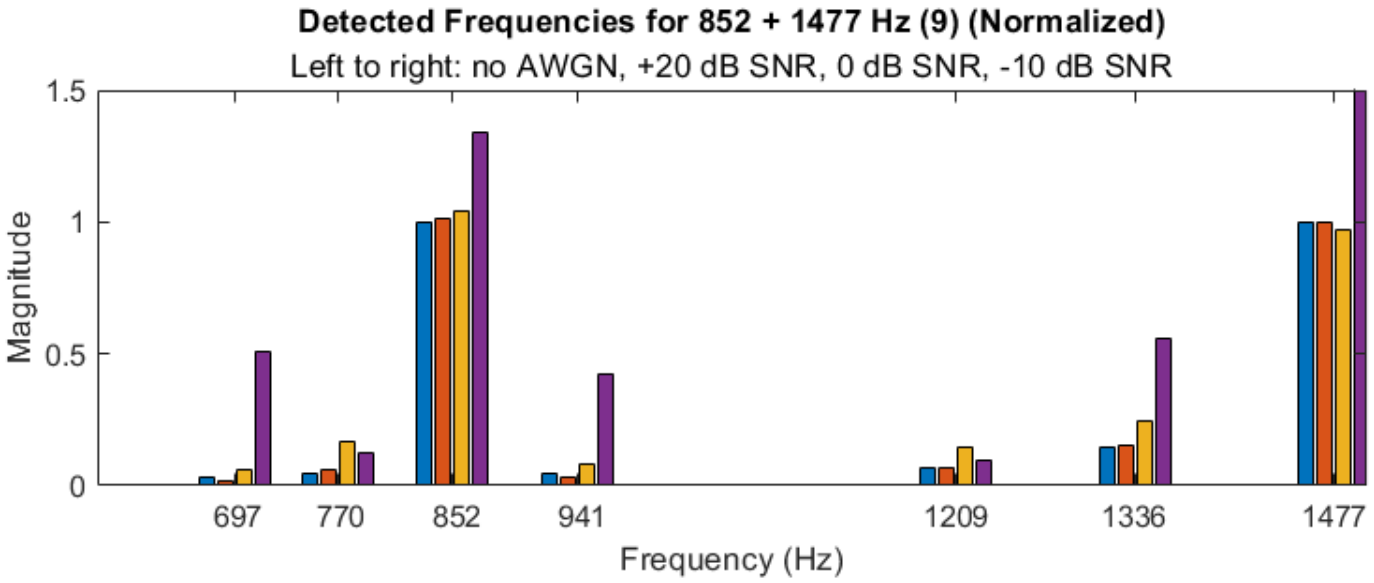

Detected Frequencies for 770 + 1209 Hz (4) (Normalized)
Left to right: no AWGN, +20 dB SNR, 0 dB SNR, -10 dB SNR
Magnitude
1.5
1
0.5
0
697
770
852
941
1209
1336
1477
Frequency (Hz)

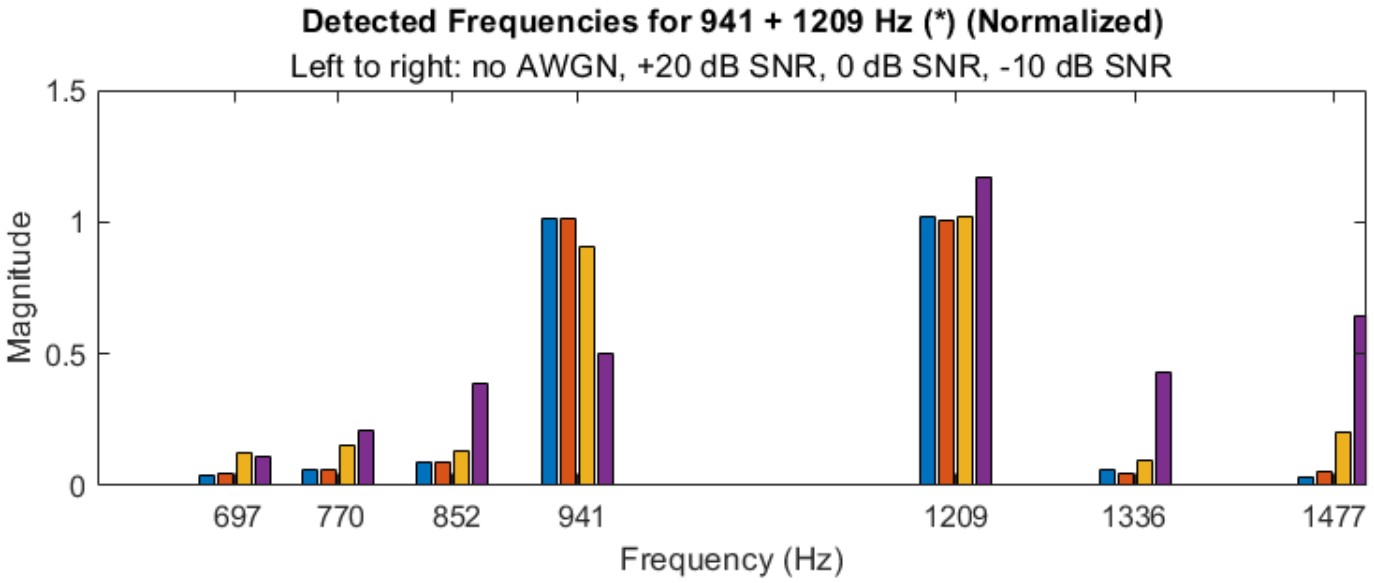

Detected Frequencies for 770 + 1336 Hz (5) (Normalized)
Left to right: no AWGN, +20 dB SNR, 0 dB SNR, -10 dB SNR
Magnitude
1.5
1
0.5
0
697
770
852
941
1209
1336
1477
Frequency (Hz)

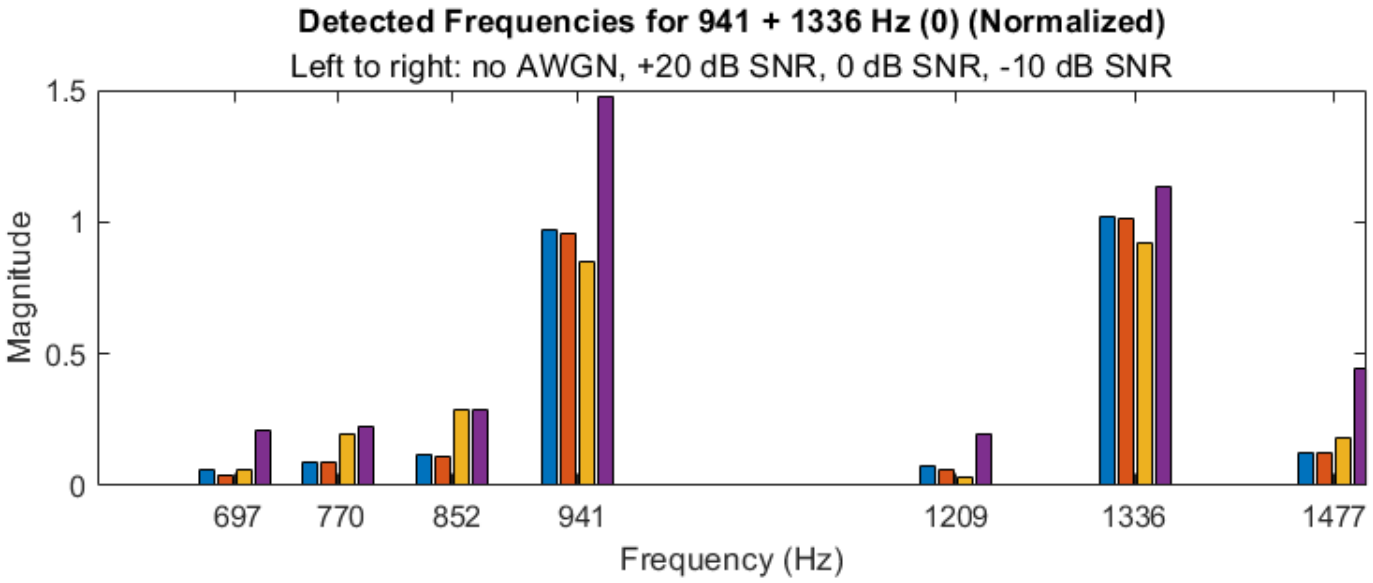

Detected Frequencies for 770 + 1477 Hz (6) (Normalized)
Left to right: no AWGN, +20 dB SNR, 0 dB SNR, -10 dB SNR
Magnitude
1.5
1
0.5
0
697
770
852
941
1209
1336
1477
Frequency (Hz)

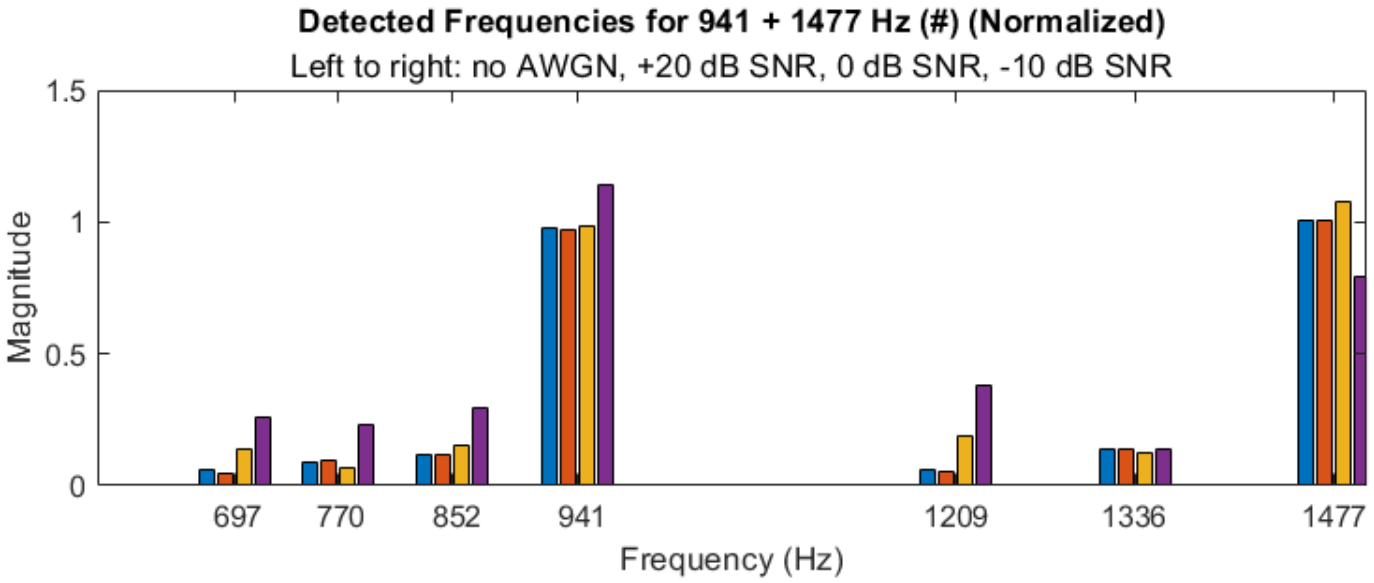

Detected Frequencies for 852 + 1209 Hz (7) (Normalized)
Left to right: no AWGN, +20 dB SNR, 0 dB SNR, -10 dB SNR
Magnitude
1.5
1
0.5
0
697
770
852
941
1209
1336
1477
Frequency (Hz)


Detected Frequencies for 852 + 1336 Hz (8) (Normalized)
Left to right: no AWGN, +20 dB SNR, 0 dB SNR, -10 dB SNR
Magnitude
1.5
1
0.5
0
697
770
852
941
1209
1336
1477
Frequency (Hz)


Detected Frequencies for 852 + 1477 Hz (9) (Normalized)
Left to right: no AWGN, +20 dB SNR, 0 dB SNR, -10 dB SNR
Magnitude
1.5
1
0.5
0
697
770
852
941
1209
1336
1477
Frequency (Hz)


Detected Frequencies for 941 + 1209 Hz (*) (Normalized)
Left to right: no AWGN, +20 dB SNR, 0 dB SNR, -10 dB SNR
Magnitude
1.5
1
0.5
0
697
770
852
941
1209
1336
1477
Frequency (Hz)


Detected Frequencies for 941 + 1336 Hz (0) (Normalized)
Left to right: no AWGN, +20 dB SNR, 0 dB SNR, -10 dB SNR
Magnitude
1.5
1
0.5
0
697
770
852
941
1209
1336
1477
Frequency (Hz)


Detected Frequencies for 941 + 1477 Hz (#) (Normalized)
Left to right: no AWGN, +20 dB SNR, 0 dB SNR, -10 dB SNR
Magnitude
1.5
1
0.5
0
697
770
852
941
1209
1336
1477
Frequency (Hz)

Summary of Resulting Detection Magnitudes for Inputs with AWGN

Input Signal	g(697)	g(770)	g(852)	g(941)	g(1209)	g(1336)	g(1477)
697 + 1209, no AWGN	1	0.091	0.039	0.043	1	0.067	0.04
697 + 1209, +20 dB	1	0.066	0.036	0.043	1	0.07	0.031
697 + 1209, 0 dB	0.93	0.023	0.12	0.084	1.1	0.23	0.15
697 + 1209, -10 dB	1	0.29	0.37	0.15	1.1	0.1	0.46

Input Signal	g(697)	g(770)	g(852)	g(941)	g(1209)	g(1336)	g(1477)
697 + 1336, no AWGN	0.99	0.065	0.026	0.033	0.095	1	0.11
697 + 1336, +20 dB	1	0.065	0.028	0.037	0.11	1	0.12
697 + 1336, 0 dB	0.86	0.094	0.13	0.1	0.12	1.2	0.052

697 + 1336, -10 dB	1.2	0.28	0.25	0.13	0.25	0.77	0.36

Input Signal	g(697)	g(770)	g(852)	g(941)	g(1209)	g(1336)	g(1477)
697 + 1477, no AWGN	0.99	0.077	0.035	0.039	0.067	0.15	1
697 + 1477, +20 dB	1	0.088	0.032	0.049	0.065	0.15	1
697 + 1477, 0 dB	0.94	0.054	0.14	0.054	0.045	0.052	1
697 + 1477, -10 dB	0.89	0.95	0.39	0.24	0.18	0.24	1.4

Input Signal	g(697)	g(770)	g(852)	g(941)	g(1209)	g(1336)	g(1477)
770 + 1209, no AWGN	0.23	1	0.17	0.06	0.97	0.096	0.071
770 + 1209, +20 dB	0.22	1	0.18	0.072	0.98	0.11	0.083
770 + 1209, 0 dB	0.25	1.2	0.28	0.12	0.98	0.15	0.14
770 + 1209, -10 dB	0.25	1.6	0.5	0.31	0.97	0.17	0.43

Input Signal	g(697)	g(770)	g(852)	g(941)	g(1209)	g(1336)	g(1477)
770 + 1336, no AWGN	0.24	1	0.16	0.066	0.088	1	0.12
770 + 1336, +20 dB	0.24	1	0.16	0.077	0.075	1	0.11
770 + 1336, 0 dB	0.32	0.92	0.16	0.2	0.093	0.96	0.14
770 + 1336, -10 dB	0.41	0.93	0.099	0.15	0.25	1.1	0.39

Input Signal	g(697)	g(770)	g(852)	g(941)	g(1209)	g(1336)	g(1477)
770 + 1477, no AWGN	0.25	1	0.16	0.052	0.042	0.13	1
770 + 1477, +20 dB	0.24	1	0.14	0.063	0.052	0.13	1
770 + 1477, 0 dB	0.24	1.1	0.23	0.18	0.27	0.09	1.1
770 + 1477, -10 dB	0.48	1.3	0.25	0.33	0.44	0.21	1

Input Signal	g(697)	g(770)	g(852)	g(941)	g(1209)	g(1336)	g(1477)

852 + 1209, no AWGN	0.024	0.042	1	0.015	0.99	0.078	0.053
852 + 1209, +20 dB	0.023	0.049	1	0.012	0.99	0.077	0.051
852 + 1209, 0 dB	0.13	0.18	0.97	0.12	0.96	0.027	0.15
852 + 1209, -10 dB	0.62	0.33	0.86	0.29	1.3	0.41	0.12

Input Signal	g(697)	g(770)	g(852)	g(941)	g(1209)	g(1336)	g(1477)
852 + 1336, no AWGN	0.024	0.035	0.99	0.05	0.1	1	0.1
852 + 1336, +20 dB	0.036	0.044	0.99	0.048	0.096	1	0.1
852 + 1336, 0 dB	0.12	0.1	0.91	0.0046	0.22	0.97	0.17
852 + 1336, -10 dB	0.38	0.4	1.6	0.21	0.23	1.2	1

Input Signal	g(697)	g(770)	g(852)	g(941)	g(1209)	g(1336)	g(1477)
852 + 1477, no AWGN	0.03	0.045	1	0.043	0.066	0.15	1

852 + 1477, +20 dB	0.02	0.058	1	0.032	0.071	0.15	1
852 + 1477, 0 dB	0.06	0.17	1	0.078	0.15	0.25	0.97
852 + 1477, -10 dB	0.51	0.12	1.3	0.42	0.098	0.56	1.6

Input Signal	g(697)	g(770)	g(852)	g(941)	g(1209)	g(1336)	g(1477)
941 + 1209, no AWGN	0.036	0.058	0.09	1	1	0.06	0.035
941 + 1209, +20 dB	0.047	0.059	0.092	1	1	0.049	0.053
941 + 1209, 0 dB	0.12	0.16	0.13	0.91	1	0.094	0.2
941 + 1209, -10 dB	0.11	0.21	0.39	0.5	1.2	0.43	0.64

Input Signal	g(697)	g(770)	g(852)	g(941)	g(1209)	g(1336)	g(1477)
941 + 1336, no AWGN	0.058	0.089	0.12	0.97	0.071	1	0.13
941 + 1336, +20 dB	0.042	0.09	0.11	0.96	0.064	1	0.13

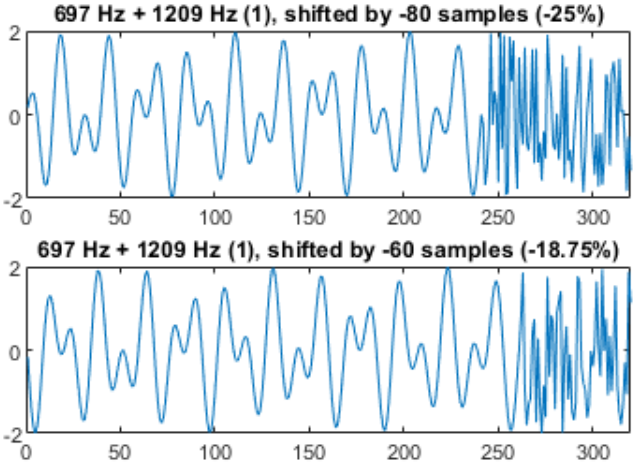
941 + 1336, 0 dB	0.061	0.19	0.29	0.85	0.03	0.92	0.18
941 + 1336, -10 dB	0.21	0.23	0.28	1.5	0.2	1.1	0.44

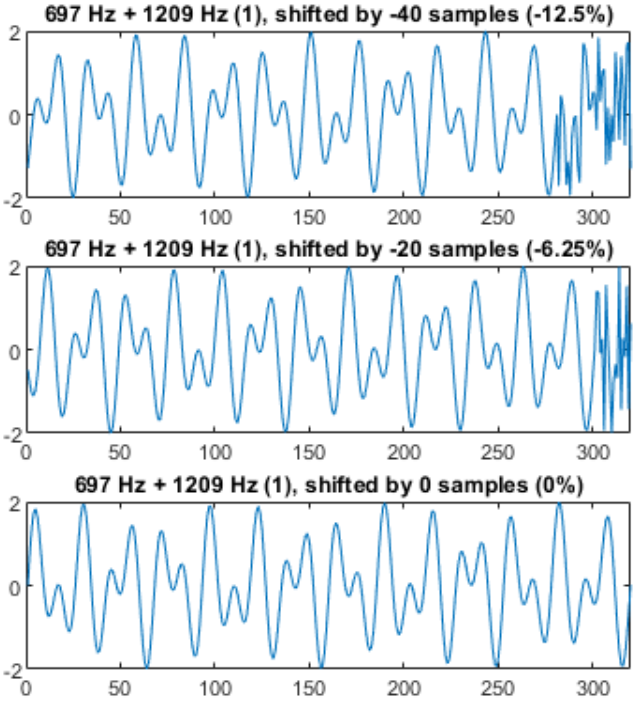
Input Signal	g(697)	g(770)	g(852)	g(941)	g(1209)	g(1336)	g(1477)
941 + 1477, no AWGN	0.059	0.089	0.12	0.98	0.059	0.14	1
941 + 1477, +20 dB	0.047	0.093	0.12	0.97	0.053	0.14	1
941 + 1477, 0 dB	0.14	0.069	0.15	0.99	0.19	0.13	1.1
941 + 1477, -10 dB	0.26	0.23	0.29	1.1	0.38	0.14	0.79

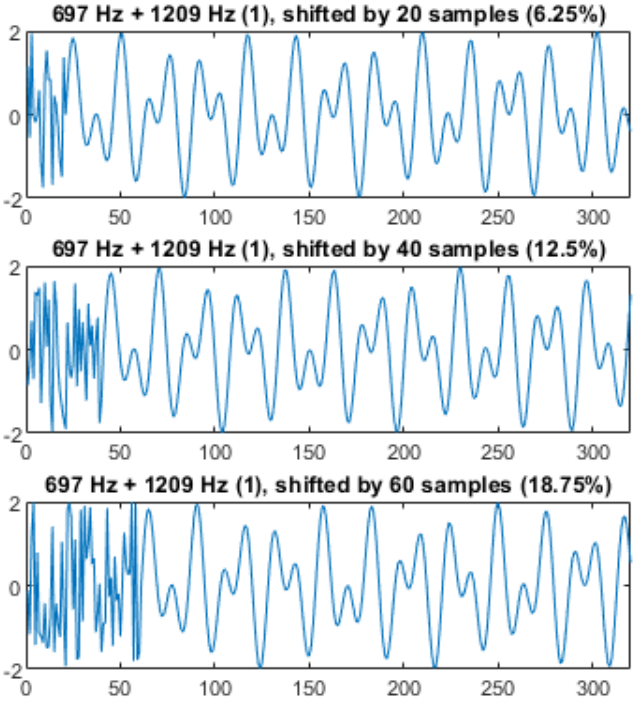
Frame Shifting

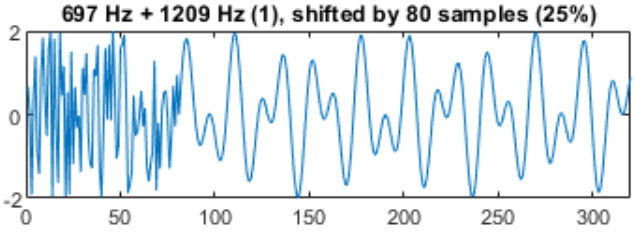
Simulating what happens when frame synchronization is not perfect.

No AWGN, for now.



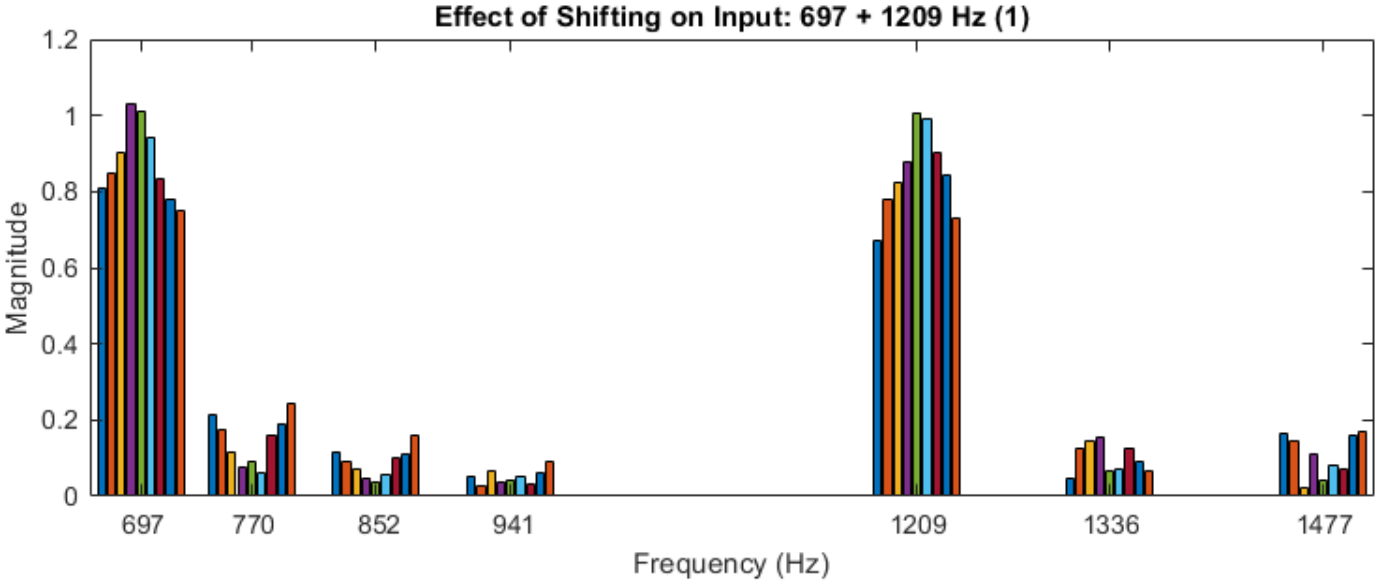

697 Hz + 1209 Hz (1), shifted by -40 samples (-12.5%)
697 Hz + 1209 Hz (1), shifted by -20 samples (-6.25%)
697 Hz + 1209 Hz (1), shifted by 0 samples (0%)
2
0
-2
0
50
100
150
200
250
300

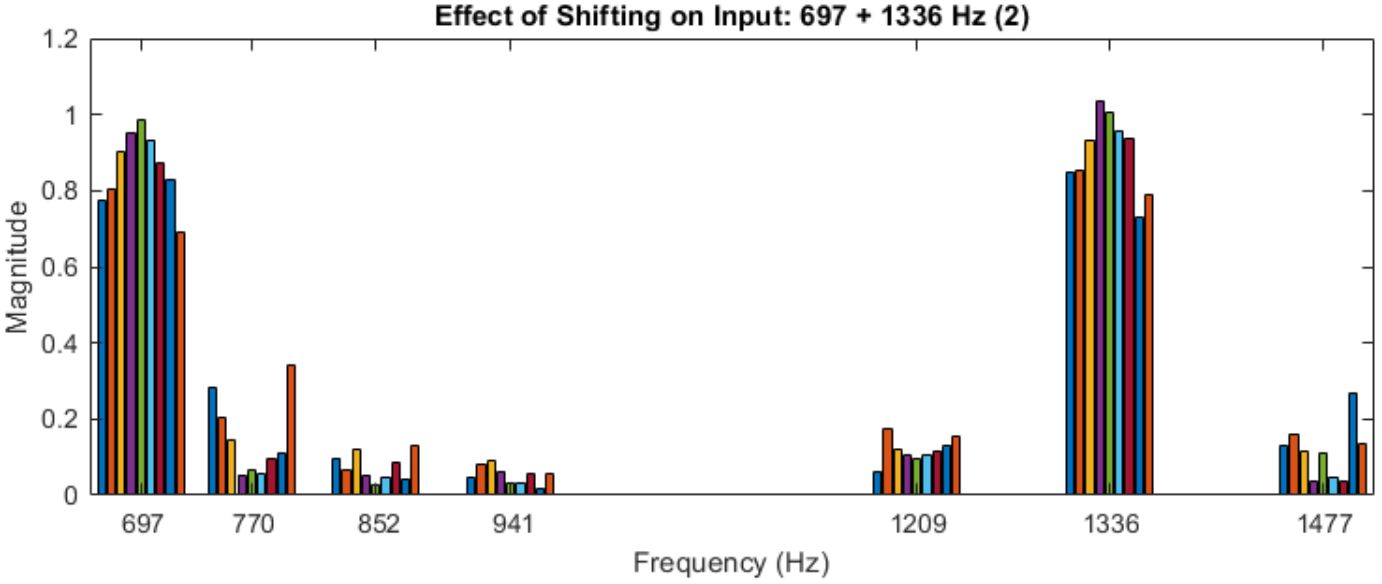

697 Hz + 1209 Hz (1), shifted by 20 samples (6.25%)
697 Hz + 1209 Hz (1), shifted by 40 samples (12.5%)
697 Hz + 1209 Hz (1), shifted by 60 samples (18.75%)
2
0
-2
0
50
100
150
200
250
300

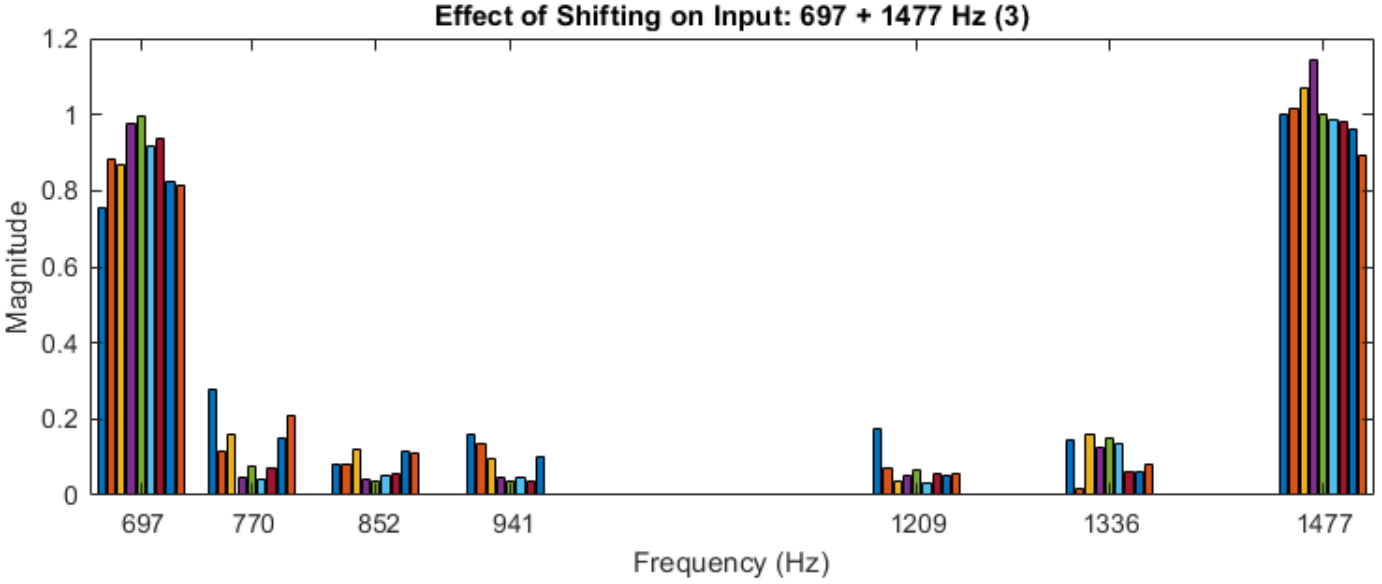

697 Hz + 1209 Hz (1), shifted by 80 samples (25%)
2
0
-2
0
50
100
150
200
250
300

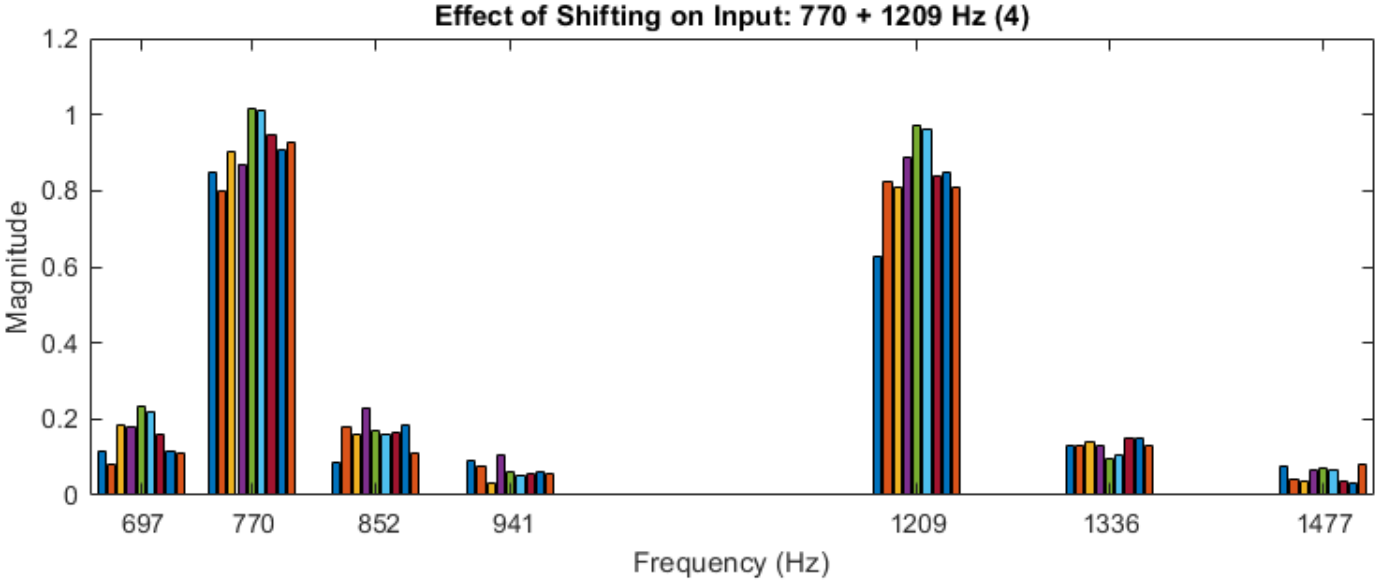
Left to right: Shifted -25 %, -18.75 %, -12.5 %, -6.25 %, 0 %, 6.25 %, 12.5 %, 18.75 %, 25%.

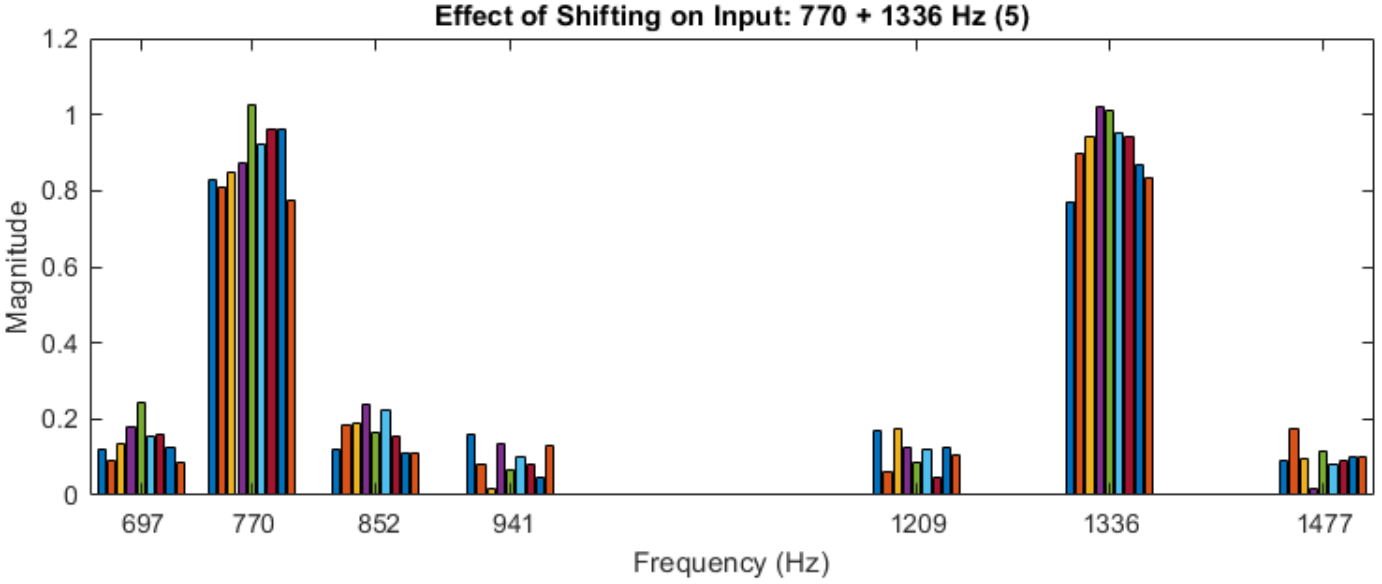
All magnitudes have been normalized.

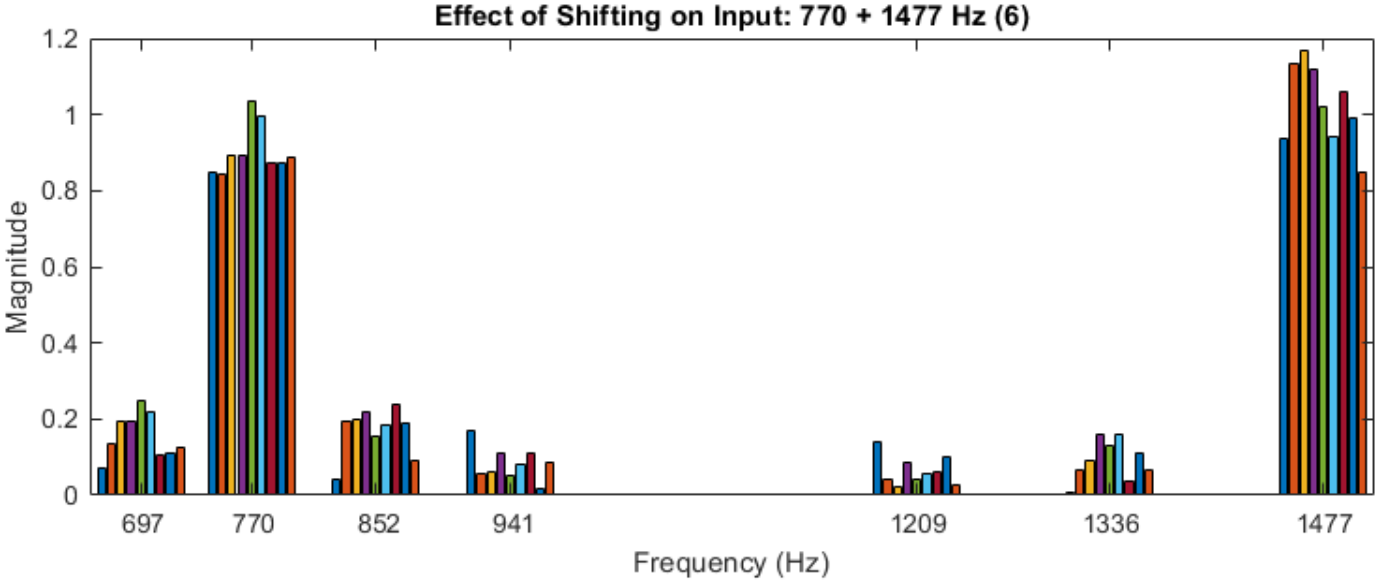


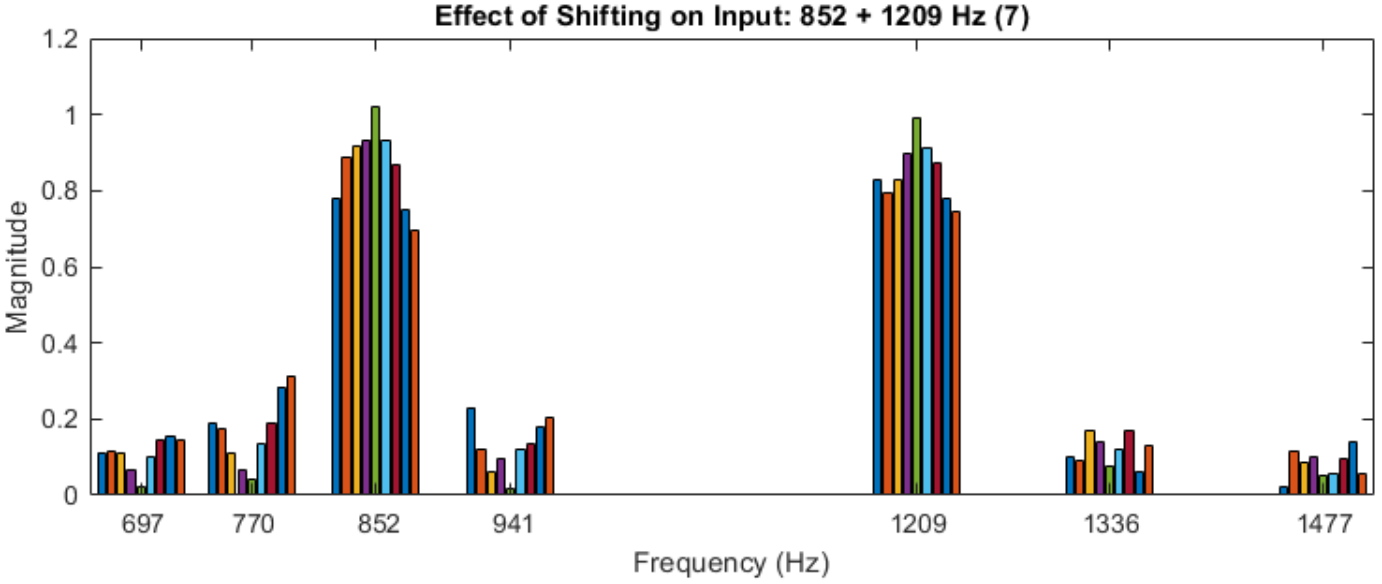

Effect of Shifting on Input: 697 + 1336 Hz (2)
Magnitude
Frequency (Hz)
0
0.2
0.4
0.6
0.8
1
1.2
697
770
852
941
1209
1336
1477

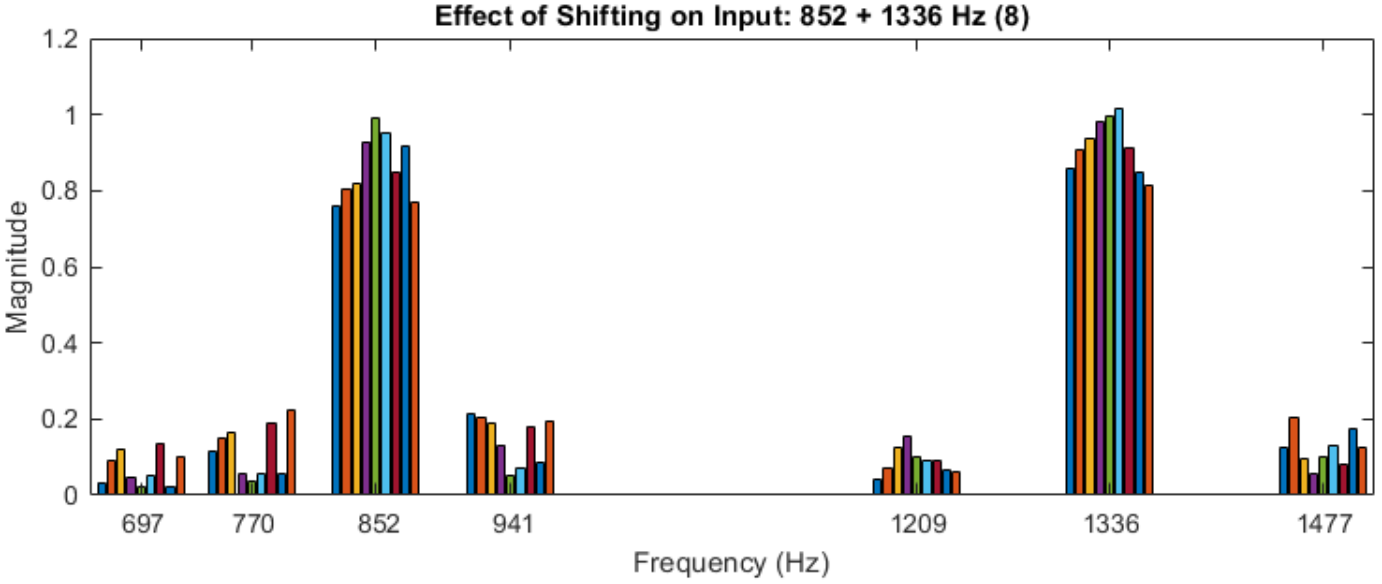

Effect of Shifting on Input: 697 + 1477 Hz (3)
Magnitude
1.2
1
0.8
0.6
0.4
0.2
0
697
770
852
941
1209
1336
1477
Frequency (Hz)

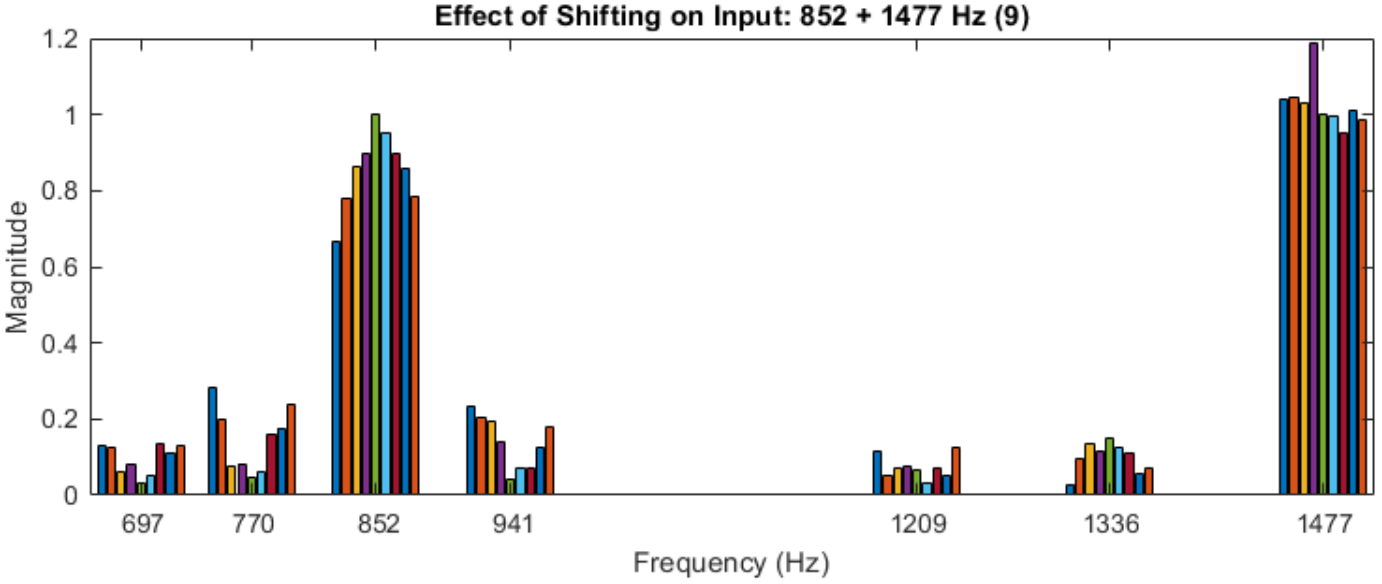

Effect of Shifting on Input: 770 + 1209 Hz (4)
Magnitude
1.2
1
0.8
0.6
0.4
0.2
0
697
770
852
941
1209
1336
1477
Frequency (Hz)

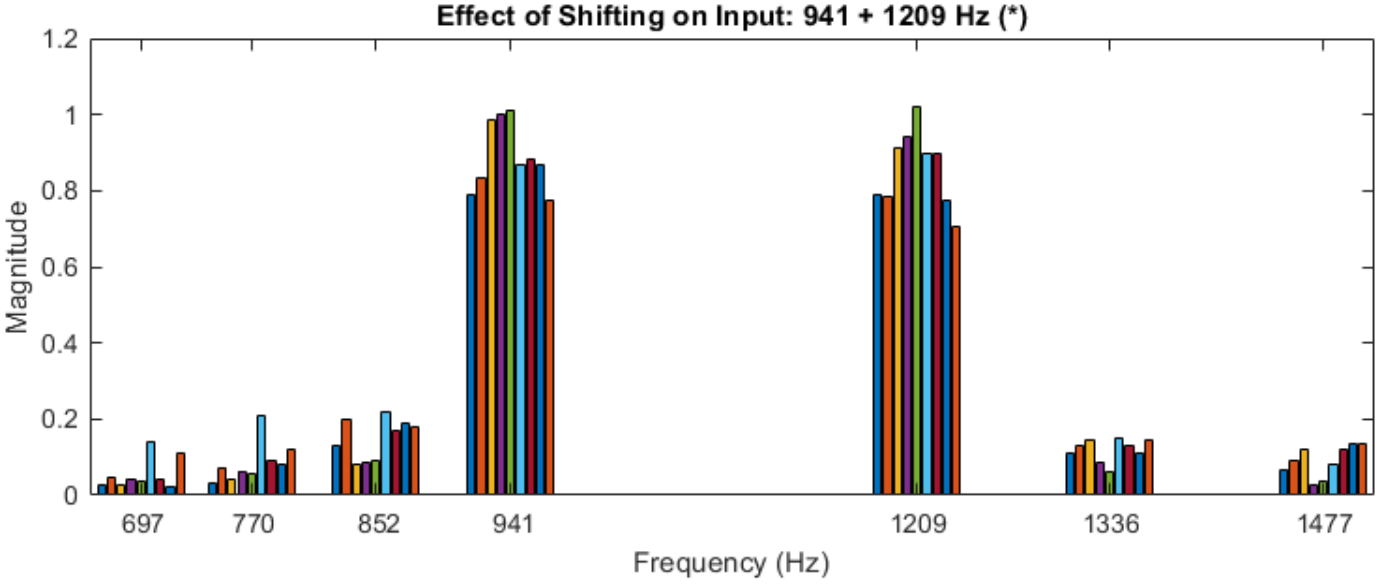

Effect of Shifting on Input: 770 + 1336 Hz (5)
Magnitude
Frequency (Hz)
0
0.2
0.4
0.6
0.8
1
1.2
697
770
852
941
1209
1336
1477

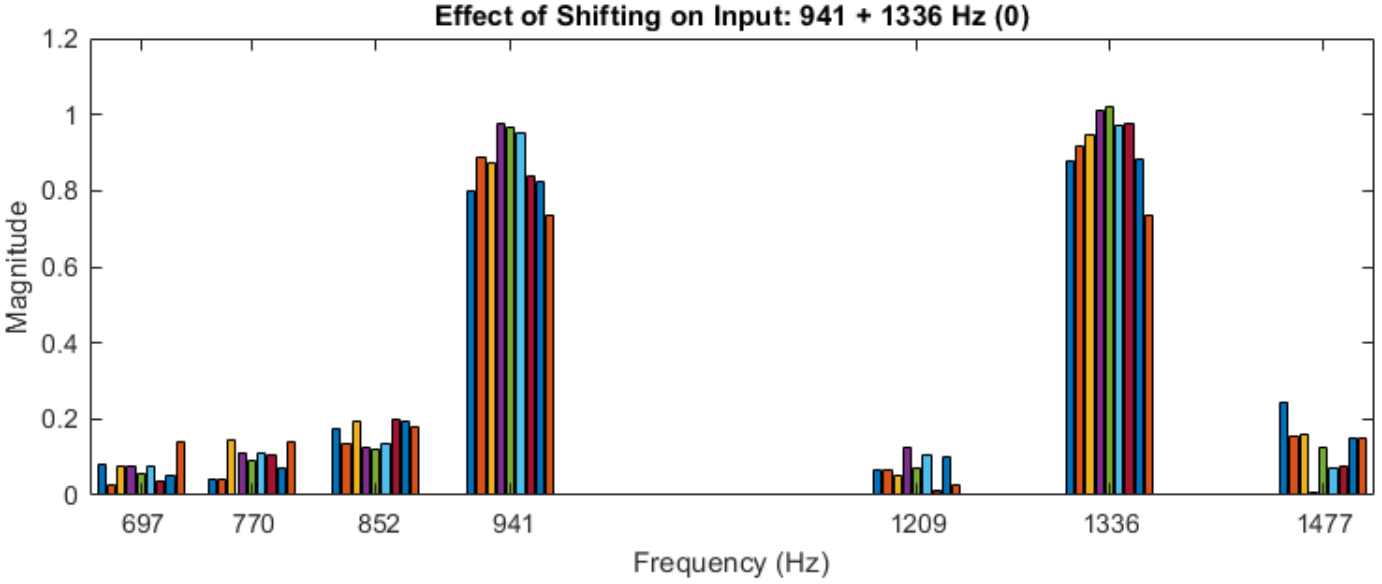

Effect of Shifting on Input: 770 + 1477 Hz (6)
Magnitude
1.2
1
0.8
0.6
0.4
0.2
0
697
770
852
941
1209
1336
1477
Frequency (Hz)

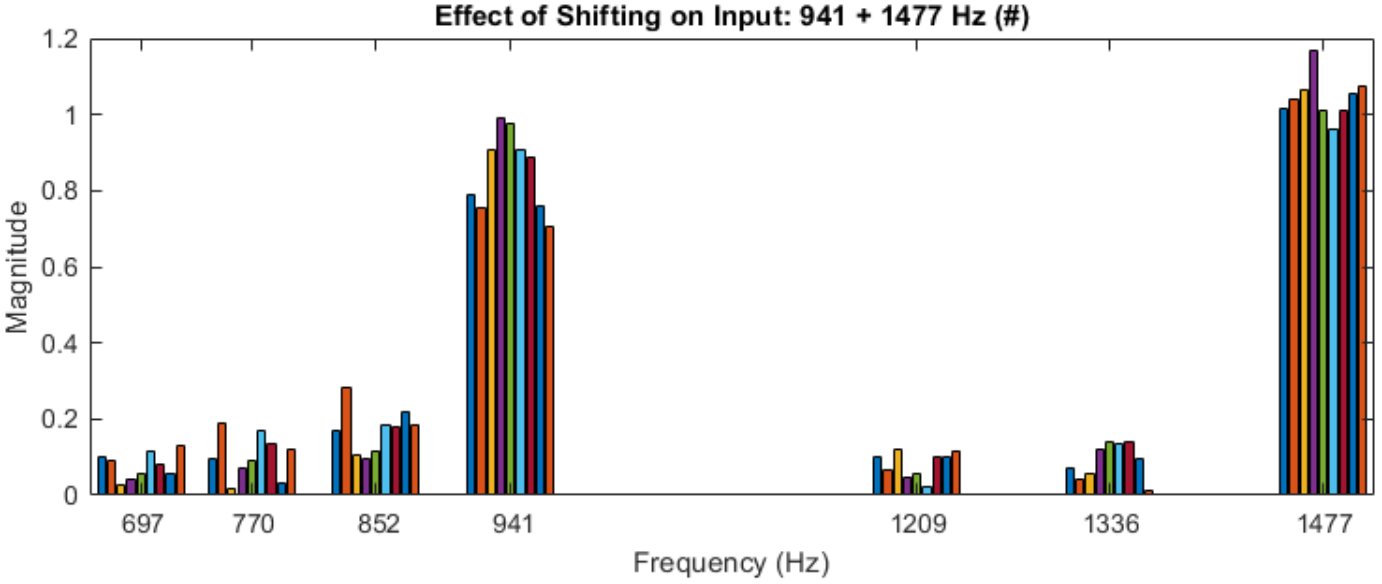

Effect of Shifting on Input: 852 + 1209 Hz (7)
Magnitude
Frequency (Hz)
0
0.2
0.4
0.6
0.8
1
1.2
697
770
852
941
1209
1336
1477


Effect of Shifting on Input: 852 + 1336 Hz (8)
Magnitude
Frequency (Hz)
0
0.2
0.4
0.6
0.8
1
1.2
697
770
852
941
1209
1336
1477


Effect of Shifting on Input: 852 + 1477 Hz (9)
Magnitude
Frequency (Hz)
0
0.2
0.4
0.6
0.8
1
1.2
697
770
852
941
1209
1336
1477


Effect of Shifting on Input: 941 + 1209 Hz (*)
Magnitude
Frequency (Hz)
0
0.2
0.4
0.6
0.8
1
1.2
697
770
852
941
1209
1336
1477

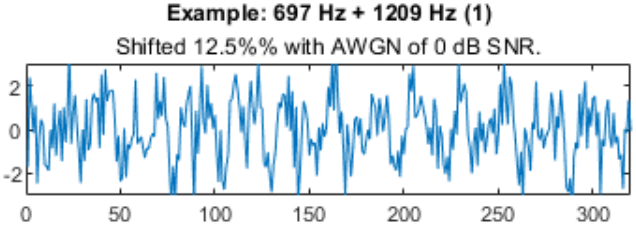

Effect of Shifting on Input: 941 + 1336 Hz (0)
Magnitude
Frequency (Hz)
0
0.2
0.4
0.6
0.8
1
1.2
697
770
852
941
1209
1336
1477


Effect of Shifting on Input: 941 + 1477 Hz (#)
Magnitude
1.2
1
0.8
0.6
0.4
0.2
0
697
770
852
941
1209
1336
1477
Frequency (Hz)

Frame Shifting + AWGN

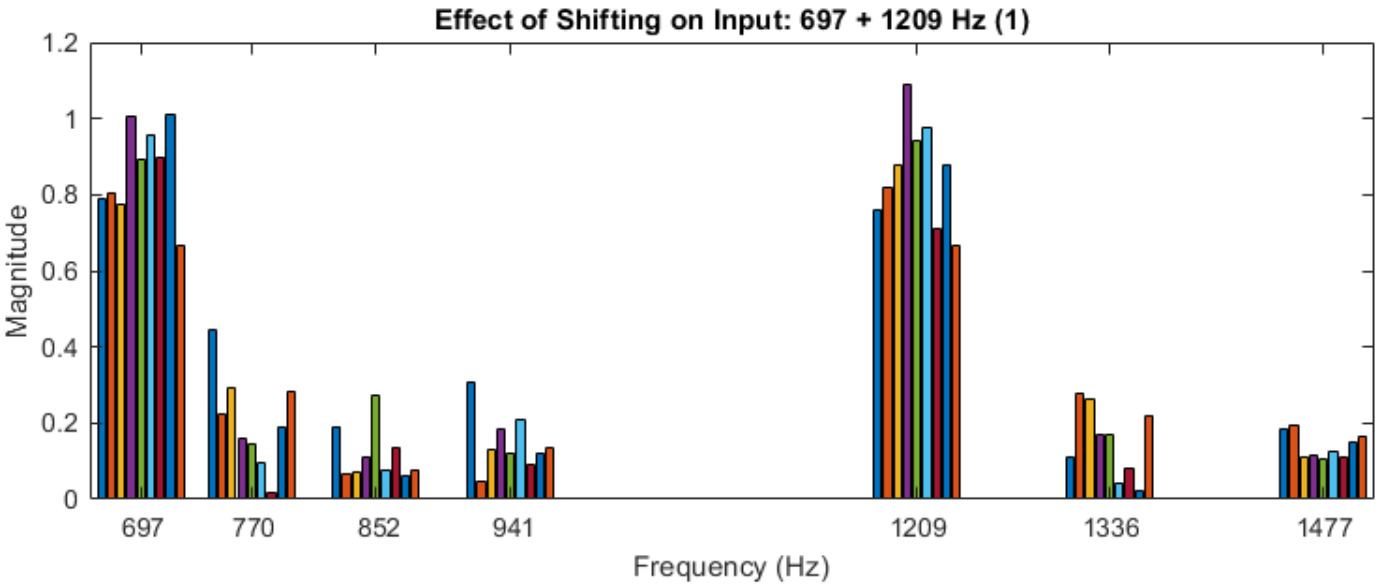
Combining both frame shifting and Additive White Gaussian Noise.

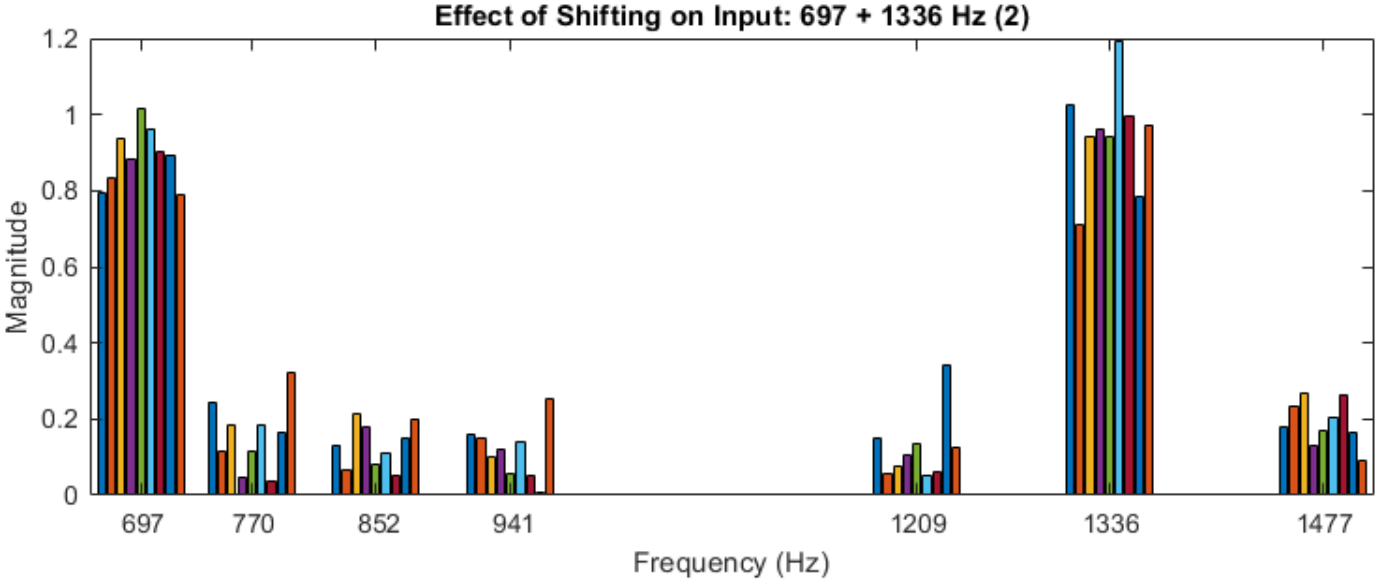
AWGN setting: 0 dB Signal-to-Noise ratio (SNR).

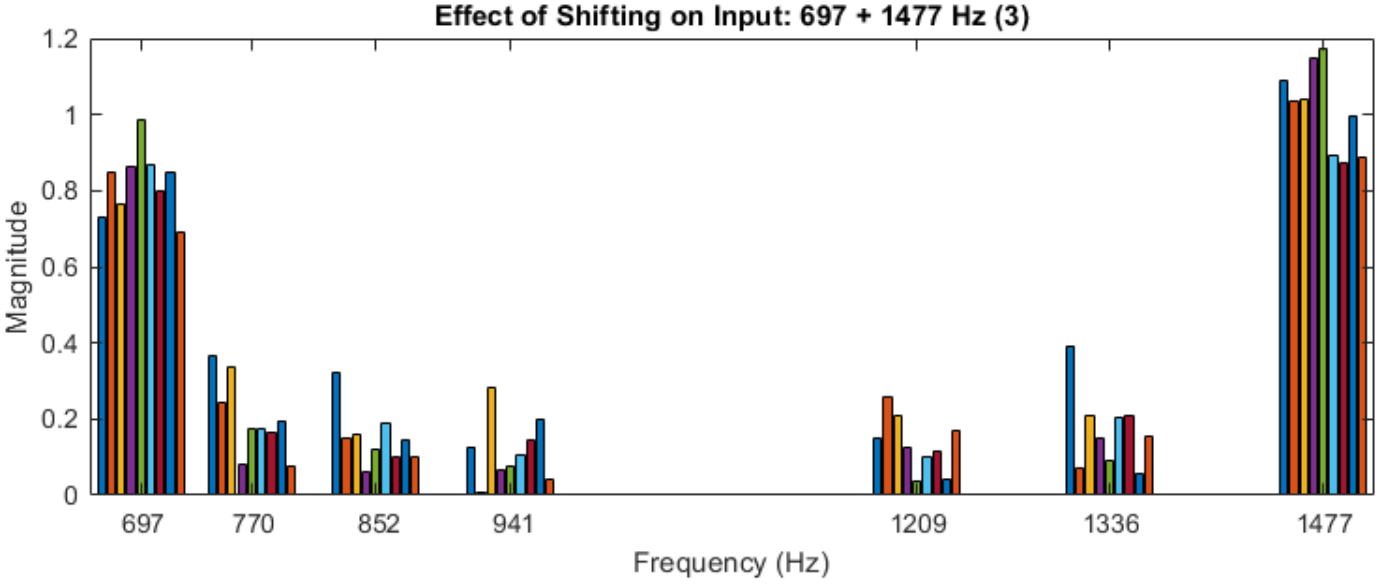


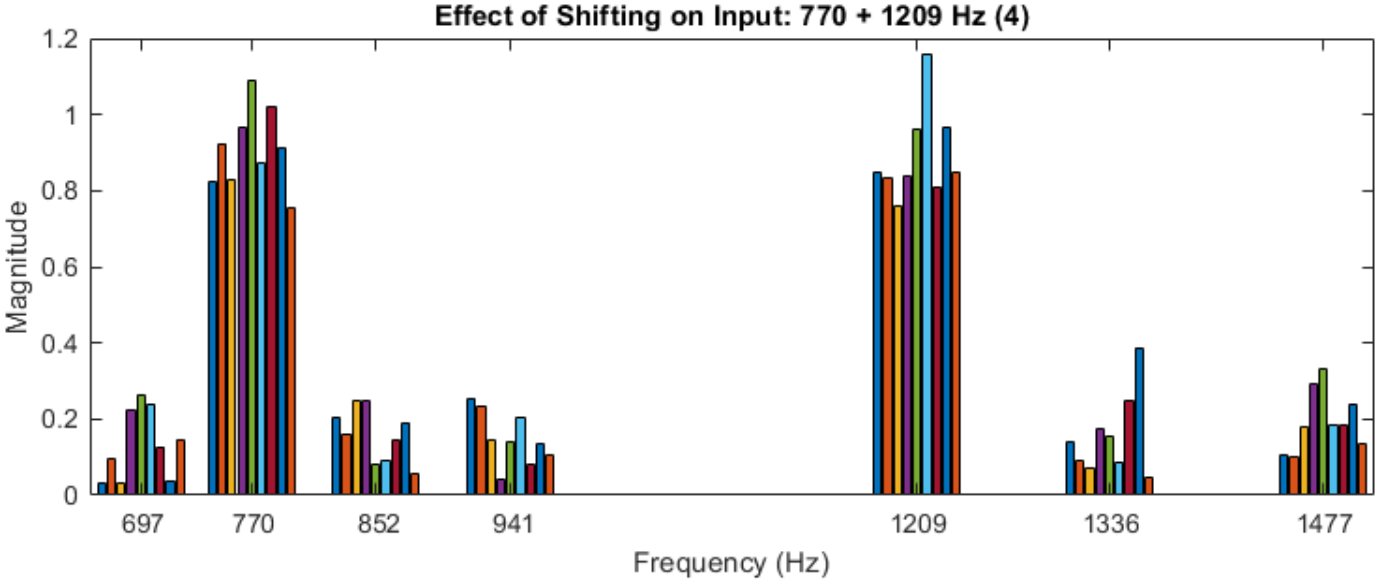
Left to right: Shifted -25 %, -18.75 %, -12.5 %, -6.25 %, 0 %, 6.25 %, 12.5 %, 18.75 %, 25%.

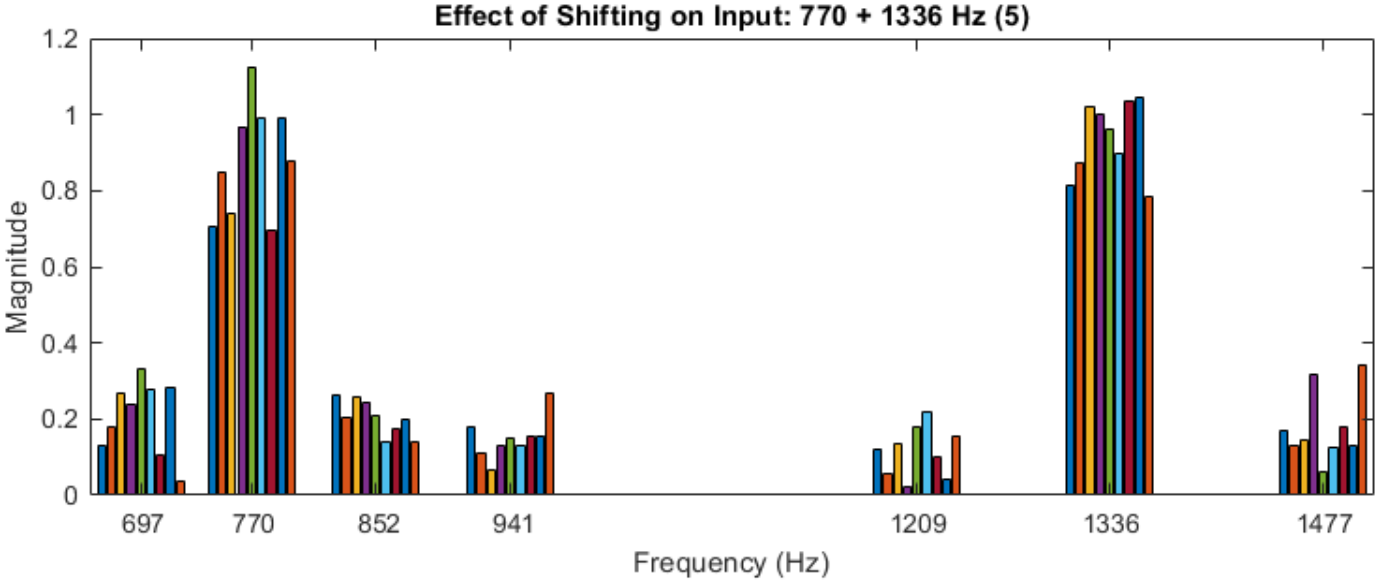
All magnitudes have been normalized using the scaling factors that were calculated using pure/ideal tone (no AWGN + shifting) inputs.

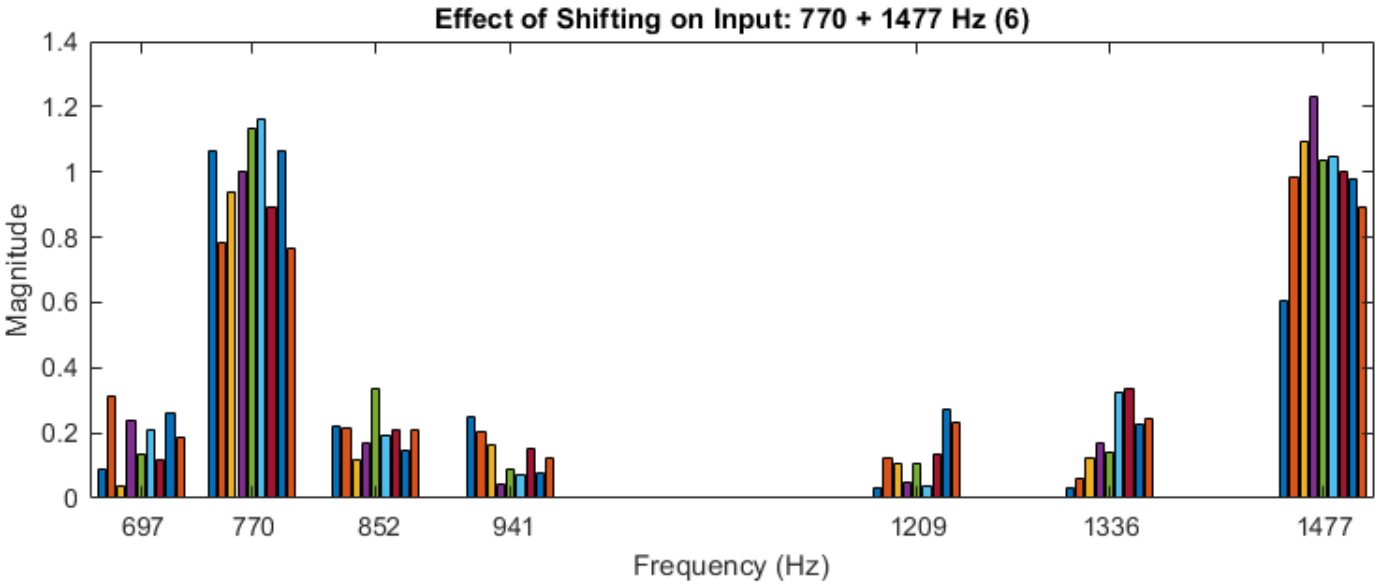

Effect of Shifting on Input: 697 + 1209 Hz (1)
Magnitude
Frequency (Hz)
0
0.2
0.4
0.6
0.8
1
1.2
697
770
852
941
1209
1336
1477

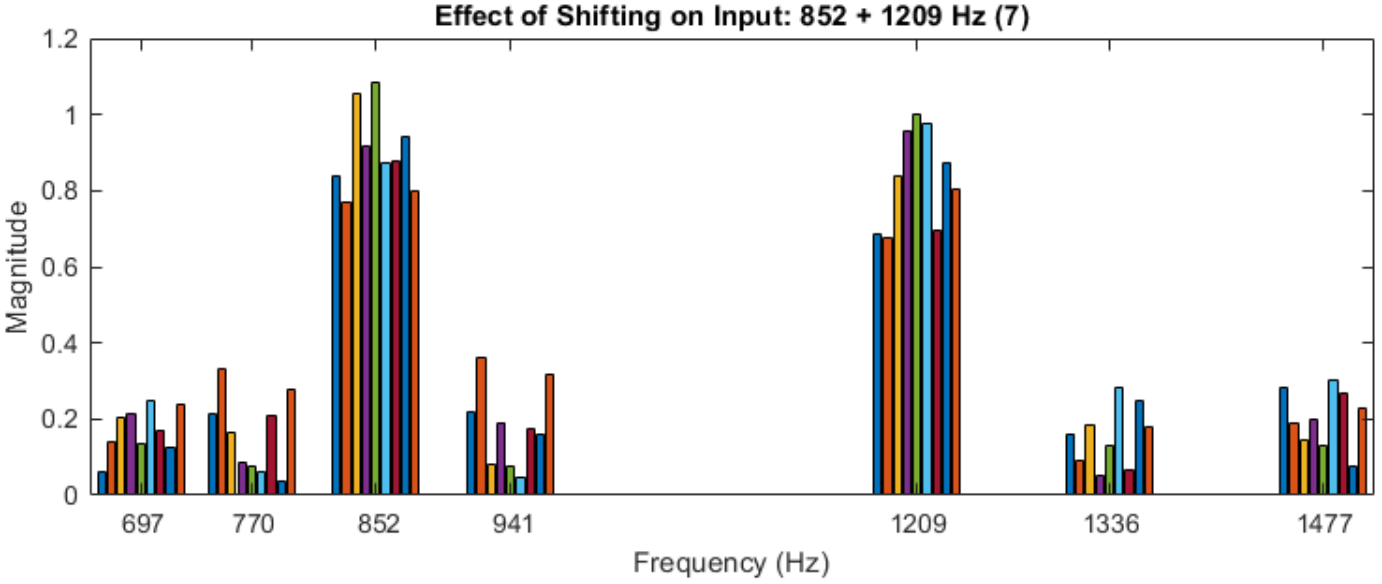

Effect of Shifting on Input: 697 + 1336 Hz (2)
Magnitude
1.2
1
0.8
0.6
0.4
0.2
0
697
770
852
941
1209
1336
1477
Frequency (Hz)

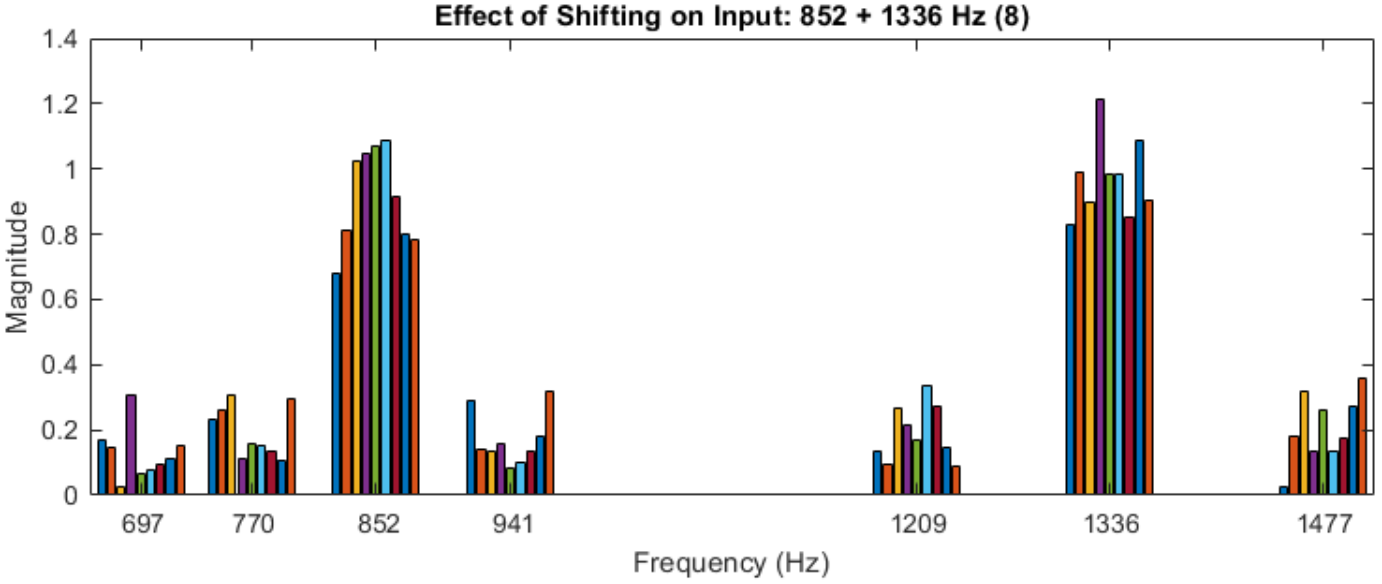

Effect of Shifting on Input: 697 + 1477 Hz (3)
Magnitude
Frequency (Hz)
0
0.2
0.4
0.6
0.8
1
1.2
697
770
852
941
1209
1336
1477

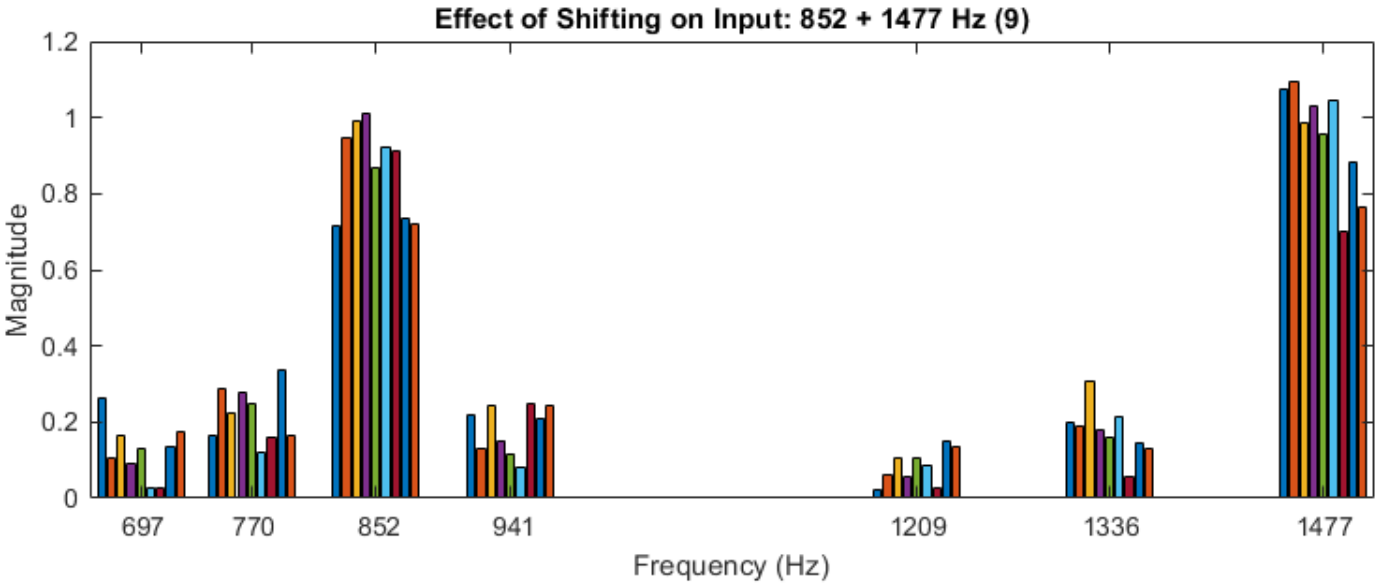

Effect of Shifting on Input: 770 + 1209 Hz (4)
Magnitude
Frequency (Hz)
0
0.2
0.4
0.6
0.8
1
1.2
697
770
852
941
1209
1336
1477

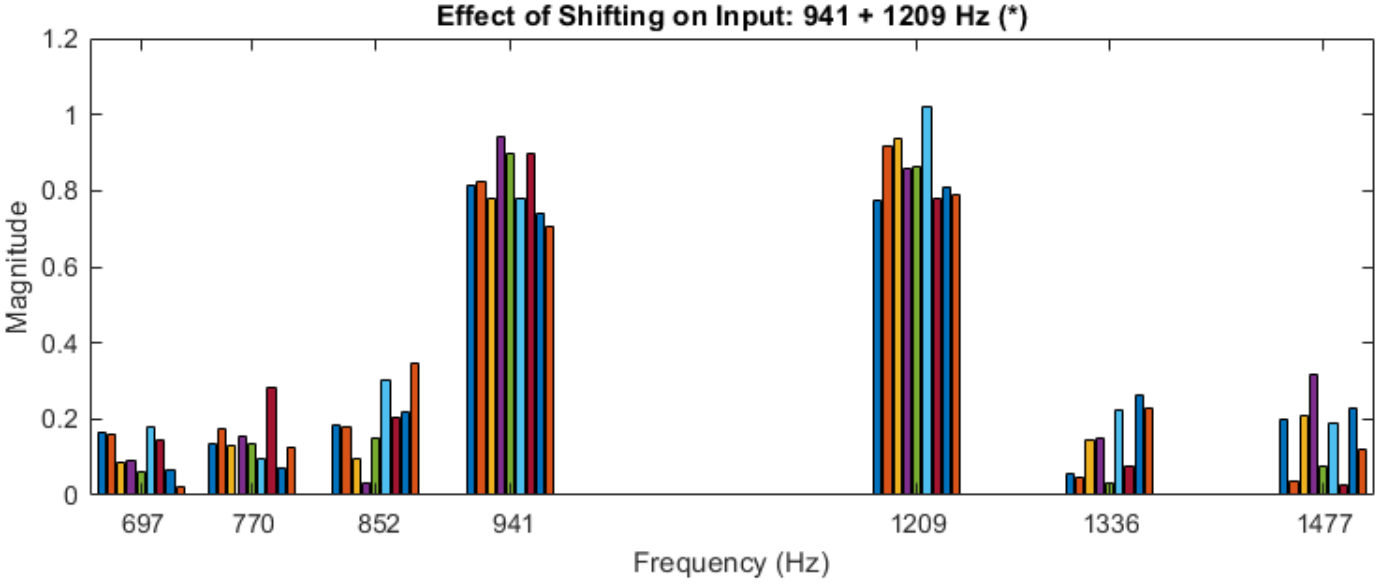

Effect of Shifting on Input: 770 + 1336 Hz (5)
Magnitude
Frequency (Hz)
0
0.2
0.4
0.6
0.8
1
1.2
697
770
852
941
1209
1336
1477

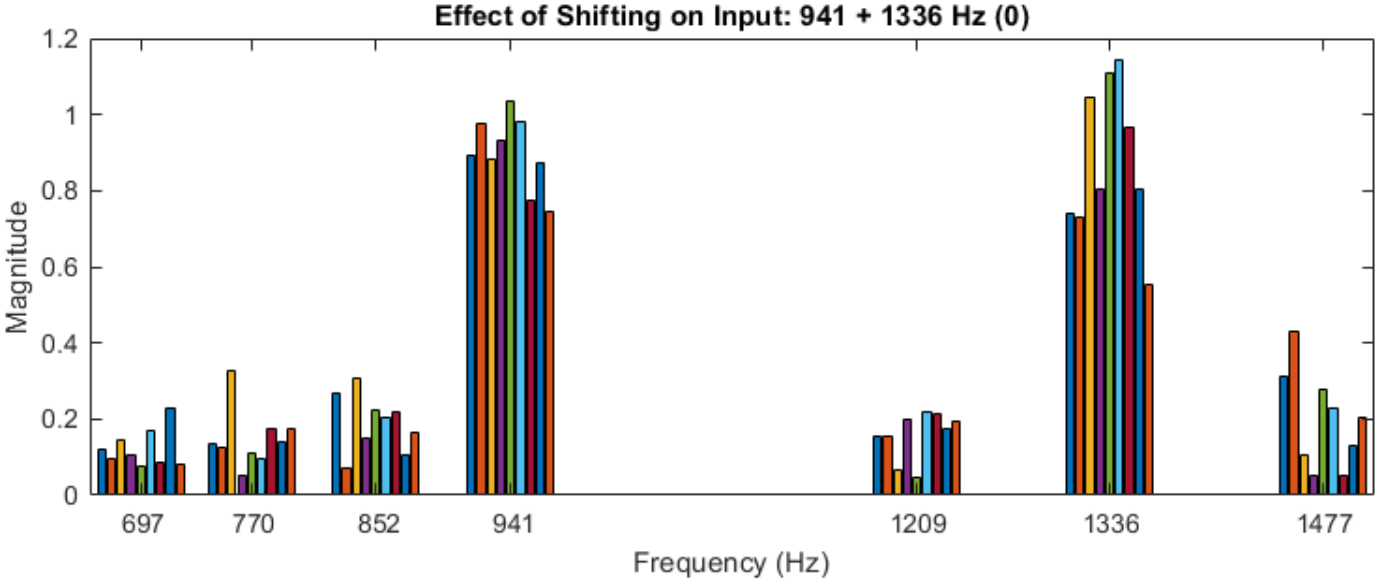

Effect of Shifting on Input: 770 + 1477 Hz (6)
Magnitude
Frequency (Hz)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
697
770
852
941
1209
1336
1477

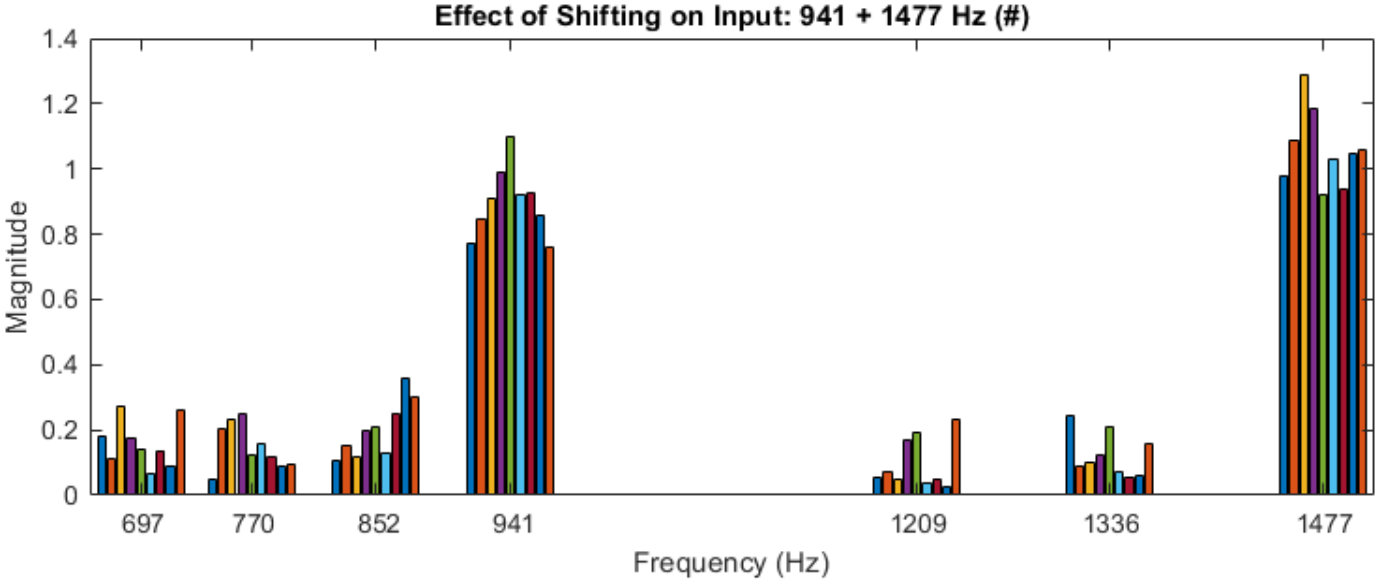

Effect of Shifting on Input: 852 + 1209 Hz (7)
Magnitude
1.2
1
0.8
0.6
0.4
0.2
0
697
770
852
941
1209
1336
1477
Frequency (Hz)


Effect of Shifting on Input: 852 + 1336 Hz (8)
Magnitude
Frequency (Hz)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
697
770
852
941
1209
1336
1477


Effect of Shifting on Input: 852 + 1477 Hz (9)
Magnitude
1.2
1
0.8
0.6
0.4
0.2
0
697
770
852
941
1209
1336
1477
Frequency (Hz)


Effect of Shifting on Input: 941 + 1209 Hz (*)
Magnitude
1.2
1
0.8
0.6
0.4
0.2
0
697
770
852
941
1209
1336
1477
Frequency (Hz)


Effect of Shifting on Input: 941 + 1336 Hz (0)
Magnitude
1.2
1
0.8
0.6
0.4
0.2
0
697
770
852
941
1209
1336
1477
Frequency (Hz)


Effect of Shifting on Input: 941 + 1477 Hz (#)
Magnitude
1.4
1.2
1
0.8
0.6
0.4
0.2
0
697
770
852
941
1209
1336
1477
Frequency (Hz)