Problem 1.6

According to the given information:

	Clock Rate	CPI class A	CPI class B	CPI class C	CPI class D
P1	2.5GHz	1	2	3	3
P2	3 GHz	2	2	2	2

From the question, we get the number of instructions $I = 1.0E6 = 10^6$,
and class A=0.1, class B=0.2, class C=0.5, class D=0.2

The CPU time for processor P1:

$$\text{CPU Time} = \sum \frac{I \times CPI}{ClockRate}$$

$$= \frac{10^6 \times [(0.1\times1)+(0.2\times2)+(0.5\times3)+(0.2\times3)]}{2.5\times10^9}$$

$$= \frac{10^{-3}\times(0.1+0.4+1.5+0.6)}{2.5}$$

$$= \frac{2.6}{2.5} \times 10^{-3}$$

$$= 1.04 \text{ milliseconds}$$

The CPU time for processor P2:

$$\text{CPU Time} = \sum \frac{I \times CPI}{ClockRate}$$

$$= \frac{10^6 \times [(0.1\times2)+(0.2\times2)+(0.5\times2)+(0.2\times2)]}{3\times10^9}$$

$$= \frac{10^{-3}\times(0.2+0.4+1.0+0.4)}{3}$$

$$= \frac{2}{3} \times 10^{-3}$$

$$\approx 0.67 \text{ milliseconds}$$

The CPU time of processor P1 is 1.04 milliseconds while the CPU time of processor P2 is 0.67 milliseconds. Thus, P2 is faster than P1.

a. What is the global CPI for each implementation?

Global CPI for P1:

$$\text{Global CPI} = \frac{CPItime \times ClockRate}{I}$$

$$= \frac{1.04\times10^{-3}\times2.5\times10^9}{10^6}$$

$$= 2.6$$

Thus, the global CPI of processor P1 is 2.6.

Global CPI for P2:

Global CPI $= \frac{CPItime \times ClockRate}{I}$

$= \frac{0.67 \times 10^{-3} \times 3 \times 10^{9}}{10^{6}}$

$= 2.01$

Thus, the global CPI of processor P2 is 2.01.

b. Find the clock cycles required in both cases.

The number of clock cycles for P1:

Clock Cycles = Global CPI $\times$ Number of Instructions

$= 2.6 \times 10^{6}$

Thus, the number of clock cycles of P1 is 2.6×10^{6}

The number of clock cycles for P2:

Clock Cycles = Global CPI $\times$ Number of Instructions

$= 2.01 \times 10^{6}$

Thus, the number of clock cycles of P2 is 2.01×10^{6}

Problem 1.9

1.9.1 Find the total execution time for this program on 1, 2, 4, 8 processors, and show the relative speedup of the 2, 4, and 8 processor result relative to the single processor result.

CPI for Arithmetic instruction is 1.
CPI for Load/store instruction is 12.
CPI for Branch instruction is 5.
Number of Arithmetic Instruction per processor is 2.56×10^9.
Number of Load/store Instruction per processor is 1.28×10^9.
Number of Branch Instruction per processor is 256×10^6.
Each processor has clock rate 2GHz.

Execution time for a processor $= \frac{ClockCycle}{ClockRate}$

Execution time for 1 processor:

$$\begin{aligned} ClockCycles &= CPI_{ai} \times \text{ number of Ar instruction} \\ &+ CPI_{l/s} \times \text{ number of L/S instruction} \\ &+ CPI_{branch} \times \text{ number of Branch instruction} \\ &= 2.56 \times 10^9 \times 1 + 1.28 \times 10^9 \times 12 + 256 \times 10^6 \times 5 = 19.2 \times 10^9 \end{aligned}$$

Hence, execution time for 1 processor $= \frac{19.2 \times 10^9}{2 \times 10^9} = 9.6 sec$

Execution time for 2 processors:

$$\begin{aligned} ClockCycles &= \frac{CPI_{ai} \times NumberOfArInstruction}{0.7 \times p} \\ &+ \frac{CPI_{l/s} \times NumberOfL/SInstruction}{0.7 \times p} \\ &+ CPI_{branch} \times \text{ number of Branch instruction} \\ &= \frac{2.56 \times 10^9 \times 1}{0.7 \times 2} + \frac{1.28 \times 10^9 \times 12}{0.7 \times 2} + 256 \times 10^6 \times 5 = 14.08 \times 10^9 \end{aligned}$$

Hence, execution time for 2 processors $= \frac{14.08 \times 10^9}{2 \times 10^9} = 7.04 sec$

Execution time for 4 processors:

$ClockCycles = \frac{2.56 \times 10^9 \times 1}{0.7 \times 4} + \frac{1.28 \times 10^9 \times 12}{0.7 \times 4} + 256 \times 10^6 \times 5 = 7.68 \times 10^9$

Hence, execution time for 4 processors $= \frac{7.68 \times 10^9}{2 \times 10^9} = 3.84 sec$

Execution time for 8 processors:

$ClockCycles = \frac{2.56 \times 10^9 \times 1}{0.7 \times 8} + \frac{1.28 \times 10^9 \times 12}{0.7 \times 8} + 256 \times 10^6 \times 5 = 4.48 \times 10^9$

Hence, execution time for 8 processors $= \frac{4.48 \times 10^9}{2 \times 10^9} = 2.24 sec$

Total execution time for processor 1, 2, 4, and 8 is

Total execution time = 9.6 + 7.04 + 3.84 + 2.24 = 22.72 sec

Relative speedup of the 2 processor result:
Relative speedup = $\frac{9.6}{7.04} = 1.36$

Relative speedup of the 4 processor result:
Relative speedup = $\frac{9.6}{3.84} = 2.5$

Relative speedup of the 8 processor result:
Relative speedup = $\frac{9.6}{2.24} = 4.29$

1.9.2 If the CPI of the arithmetic instructions was doubled, what would the impact be on the execution time of the program on 1, 2, 4, or 8 processor?

Execution time for 1 processor:
Clock Cycles $= 2.56 \times 10^9 \times 2 + 1.28 \times 10^9 \times 12 + 256 \times 10^6 \times 5 = 21.76 \times 10^9$

Execution time $= \frac{21.76 \times 10^9}{2 \times 10^9} = 10.88 sec$

Execution time for 2 processors:
Clock Cycles $= \frac{2.56 \times 10^9 \times 2}{0.7 \times 2} + \frac{1.28 \times 10^9 \times 12}{0.7 \times 2} + 256 \times 10^6 \times 5 = 15.908 \times 10^9$

Execution time $= \frac{21.76 \times 10^9}{2 \times 10^9} = 7.954 sec$

Execution time for 4 processors:
Clock Cycles $= \frac{2.56 \times 10^9 \times 2}{0.7 \times 4} + \frac{1.28 \times 10^9 \times 12}{0.7 \times 4} + 256 \times 10^6 \times 5 = 8.594 \times 10^9$

Execution time $= \frac{21.76 \times 10^9}{2 \times 10^9} = 4.297 sec$

Execution time for 8 processors:
Clock Cycles $= \frac{2.56 \times 10^9 \times 2}{0.7 \times 8} + \frac{1.28 \times 10^9 \times 12}{0.7 \times 8} + 256 \times 10^6 \times 5 = 4.937 \times 10^9$

Execution time $= \frac{21.76 \times 10^9}{2 \times 10^9} = 2.468 sec$

1.9.3 To what should the CPI of load/store instructions be reduced in order for a single processor to match the performance of four processors using the original CPI values?

The original execution time for 4 processors we got in 1.9.1 is 3.86 sec.

Now, reduce CPI of a single processor to match the performance of 4 processors.

Then, Clock Cycles $= 2.56 \times 10^9 \times 1 + 1.28 \times 10^9 \times x + 256 \times 10^6 \times 5 = 3.84 \times 10^9 + 1.28 \times 10^9 \times x$

Execution time $= \frac{3.84 \times 10^9 + 1.28 \times 10^9 \times x}{2 \times 10^9} = 1.92 + 0.64 \times x$
$3.86 = 1.92 + 0.64 \times x$
$x = 3.03$

Hence, the reduced CPI of load/store instructions is $\frac{3.03}{12} = 25\%$.

Problem 1.12
1.12.1
There are two processors, P1 and P2, with different clock rates, CPIs, and number of instructions.

	Clock Rate	CPI	# of Instructions
P1	4×10^9 Hz	0.9	5.0×10^9
P2	3×10^9 Hz	0.75	1.0×10^9

CPU time = $\frac{CPI \times \#of instructions}{Clock Rate}$

CPU time for P1: $P1_{CPU} = \frac{0.9 \times 5 \times 10^9}{4 \times 10^9} = \frac{4.5}{4} = 1.125 sec$

CPU time for P2: $P2_{CPU} = \frac{0.75 \times 1 \times 10^9}{3 \times 10^9} = \frac{0.75}{3} = 0.25 sec$

Since CPU time for P1 is longer than the one for P2, processor P2 performs better than P1.

However, processor P1 has larger clock rate than P2 does, thus the statement "the computer with the largest clock rate as having the largest performance" is false.

1.12.2

	Clock Rate	CPI	# of Instructions
P1	4×10^9 Hz	0.9	1.0×10^9
P2	3×10^9 Hz	0.75	unknown

CPU time for P1: $P1_{CPU} = \frac{0.9 \times 1 \times 10^9}{4 \times 10^9} = \frac{0.9}{4} = 0.225 sec$

CPU time for P2: $P2_{CPU} = \frac{0.75 \times 1 \times 10^9}{3 \times 10^9} = \frac{0.75}{3} = 0.25 sec$

Find the number of instructions I for processor P2:

$0.225 = \frac{I \times 0.75}{3 \times 10^9}$

$I = \frac{0.225 \times 3 \times 10^9}{0.75} = \frac{0.675 \times 10^9}{0.75} = 9 \times 10^8$

Hence, the number of instructions that P2 can execute in the same time that P1 needs to execute 1.0E9 instructions is 9×10^9.

Problem 1.14

	Execution	CPI	Clock Rate
FP instructions	50×10^6	1	2×10^9
INT instructions	110×10^6	1	2×10^9
L/S instructions	80×10^6	4	2×10^9
branch instructions	16×10^6	2	2×10^9

1.14.1

$$\begin{aligned}\text{Clock Cycles} &= (CPI_{fp} \times I_{fp}) + (CPI_{int} \times I_{int}) + (CPI_{ls} \times I_{ls}) + (CPI_{br} \times I_{br}) \\ &= (50 \times 10^6 \times 1) + (110 \times 10^6 \times 1) + (80 \times 10^6 \times 4) + (16 \times 10^6 \times 2) \\ &= 512 \times 10^6\end{aligned}$$

Execution time = $\frac{ClockCycles}{ClockRates}$

If we want the program to run two times faster, then $\frac{ExecutionTime}{2} \implies \frac{ClockCycles}{2}$

$$\frac{ClockCycles}{2} = (CPI_{new-fp} \times I_{fp}) + (CPI_{int} \times I_{int}) + (CPI_{ls} \times I_{ls}) + (CPI_{br} \times I_{br})$$

$$CPI_{new-fp} = \frac{\frac{ClockCycles}{2} - \left[(CPI_{int} \times I_{int}) + (CPI_{ls} \times I_{ls}) + (CPI_{br} \times I_{br})\right]}{I_{fp}}$$

$$= \frac{\frac{512 \times 10^6}{2} - \left[(110 \times 10^6 \times 1) + (80 \times 10^6 \times 4) + (16 \times 10^6 \times 2)\right]}{50 \times 10^6}$$

$$= \frac{-206 \times 10^6}{50 \times 10^6}$$

$$= \text{-4.12} < 0$$

Thus, the CPI of FP instructions cannot be improved because the value might be negative when the program runs two times faster.

1.14.2

$$\frac{ClockCycles}{2} = (CPI_{fp} \times I_{fp}) + (CPI_{int} \times I_{int}) + (CPI_{new-ls} \times I_{ls}) + (CPI_{br} \times I_{br})$$

$$CPI_{new-ls} = \frac{\frac{ClockCycles}{2} - \left[(CPI_{fp} \times I_{fp}) + (CPI_{int} \times I_{int}) + (CPI_{br} \times I_{br})\right]}{I_{ls}}$$

$$= \frac{\frac{512 \times 10^6}{2} - \left[(50 \times 10^6 \times 1) + (110 \times 10^6 \times 1) + (16 \times 10^6 \times 2)\right]}{80 \times 10^6}$$

$$= \frac{64 \times 10^6}{80 \times 10^6}$$

$$= 0.8$$

Thus, the CPI of L/S instructions will be improved by 0.8 times if the program to be run two times faster.

1.14.3
Original Execution time $= \frac{ClockCycles}{ClockRates} = \frac{512\times10^6}{2\times10^9} = 0.256$ sec

Now calculate the CPI for FP, INT, L/S, and branch instructions:
$CPI_{fp} = 1 \times (1 - 0.4) = 0.6$
$CPI_{int} = 1 \times (1 - 0.4) = 0.6$
$CPI_{ls} = 4 \times (1 - 0.3) = 2.8$
$CPI_{br} = 2 \times (1 - 0.3) = 1.4$

Clock Cycles $= 50 \times 10^6 \times 0.6 + 110 \times 10^6 \times 0.6 + 80 \times 10^6 \times 2.8 + 16 \times 10^6 \times 1.4 = 342.4 \times 10^6$

New Execution Time $= \frac{342.4\times10^6}{2\times10^9} = 0.1712$ sec

$\frac{0.256}{0.1712} = 1.495$

Thus, it improves the execution time of the processor by 1.497 times.