Math 100 Test 1 Friday

Results

- Name: Weston, Sam
- ID: 56390362
- Test number: 549

question	version	mark	out of
Q1	3	7	8
Q2	3	6	6
Q3	4	1	6
total		14	20

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MATH 100 — TEST 1 — 45 minutes

Friday, October 6, 2023

- The test consists of 6 pages and 3 questions worth a total of 20 marks.
- This is a closed-book examination. **None of the following are allowed**: documents, cheat sheets or electronic devices of any kind (including calculators, cell phones, etc.)
- No work on this page will be marked.
- Fill in the information below before turning to the questions. Your "Section" is your small class discussion section.

Student number	5	6	3	9	0	3	6	2
Section	A	4	6					
Name	Samuel Weston							
Signature	Sam Weston							

Q1: 7 out of 8

1. 8 marks ★★☆☆ What is the domain of $g(x)$? Find all values of x where $g(x)$ is not continuous.

$$g(x) = \begin{cases} \dfrac{1}{(x+3)^2}, & \text{if } x \le -1, \\ 2-x, & \text{if } -1 < x \le 1, \\ \dfrac{3}{(x+2)}, & \text{if } x > 1. \end{cases}$$

Boundary Values

$-1, \ 1$

$\lim_{x\to -1^-} g(x) = \lim_{x\to -1}\left(\frac{1}{(x+3)^2}\right) = \frac{1}{2^2}$

$= \frac{1}{4}$

$\lim_{x\to 1^-} g(x) = \lim_{x\to 1}(2-x) = 2-1$

$= 1$

$\lim_{x\to -1^+} g(x) = \lim_{x\to -1}(2-x) = 2+1$

$= 3$

$\lim_{x\to 1^+} g(x) = \lim_{x\to 1}\left(\frac{3}{x+2}\right) = \frac{3}{3}$

$= 1$

+2 discontinuity at $x = -1$

$\boxed{\therefore g(x) \text{ is not continuous at } x = -1}$

$\therefore g(x)$ is continuous at $x = 1$

+2 continuous at $x = 1$

Domain:

if $x \le -1$, $(x+3) \neq 0$

$x \neq -3$

if $x > 1$ $\quad x+2 > 0$

$x > -2$

DNE

Domain =

$\boxed{\therefore x \mid x \neq -3}$

+2 exclude $x = -3$ from domain

+1 incorrect or incomplete treatment of $x = -2$

Q2: 6 out of 6

2. $\boxed{\text{6 marks}}$ ★★★☆ Let $f(x) = \frac{\cos(x)}{x^2+1}$.

(a) What is the asymptotic behaviour of $f(x)$ for x large in both positive and negative directions?

as $x \to +\infty$, $\frac{\cos(x)}{x^2+1} \to \frac{\pm 1}{\infty^2} \to 0$

as $x \to -\infty$, $\frac{\cos(x)}{x^2+1} \to \frac{\pm 1}{(-\infty)^2} \to 0$

notation don't use infinity

(b) What is the largest value for $\cos(x)$? What is the smallest value for x^2+1? Use this information to deduce the largest value for $f(x)$, and at which x value f attains this maximum.

the largest value of $\cos(x)$ is 1. →

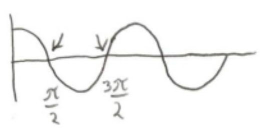
The smallest value of x^2+1 is 1. This occurs when $x=0$. At $x=0$, $f(x)$ is equal to $\frac{1}{1}$ or 1. Which is the global maximum of the function.

(c) Find the x-intercepts for $f(x)$.

X-int when $f(x) = 0$.

$f(x) = 0$ when $\cos(x) = 0$

$\cos(x) = 0$ @ $\frac{\pi}{2} + k\pi$, $k \in I$



$\therefore x = \frac{\pi}{2} + k\pi$, $k \in I$

6 of 6 full marks

(d) Compute $f(x) - f(-x)$ and deduce a symmetry for the graph of $f(x)$.

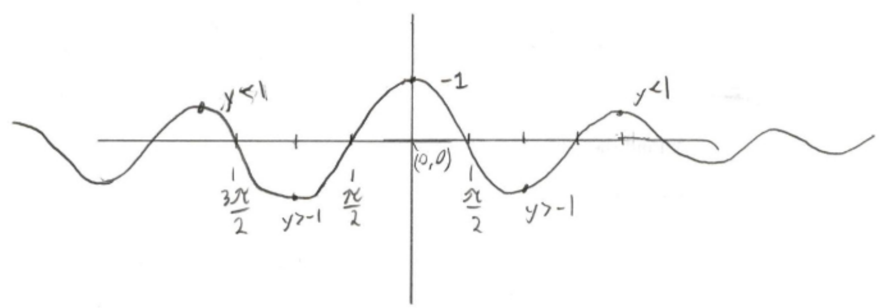
$$\frac{\cos(x)}{x^2+1} - \frac{\cos(-x)}{(-x)^2+1}$$

$$= \frac{\cos(x) - \cos(-x)}{x^2+1}$$

$$\frac{\cos(-x)}{x^2+1} = \frac{\cos(x)}{x^2+1}$$

$\therefore$ It is Symmetrical across the Y axis

(e) Sketch the graph of $f(x)$.



Q3: 1 out of 6

3. 6 marks ★★★☆ Consider the curve $y = \sqrt{2x+1}$. Find the equation of the line that is perpendicular to the tangent line of this curve at the point where $x = 4$. *Hint*: Two straight lines are perpendicular when the product of their slopes $m_1 m_2$ equals -1.

$y = (2x+1)^{\frac{1}{2}}$

$y' = \frac{1}{2}(2x+1) \cdot 2$ ×

$= 2x+1$

Incorrect derivative

$y = (2(4)+1)^{\frac{1}{2}}$

$y = (9)^{\frac{1}{2}}$

$y = \pm 3$ ×

$y = +3$

A function cannot be multi-valued. sqrt always returns the positive result.

$m = y'(4)$

$m = 2(4)+1$ ×

$m = 9$

$\perp m = -\frac{1}{9}$ +1

if $y = 3$

$-\frac{1}{9} = \frac{y-3}{x-4}$

$-\frac{1}{9}x + \frac{4}{9} = y - 3$

$-\frac{1}{9}x + \frac{31}{9} = y$

$y = -\frac{1}{9}x + \frac{31}{9}$

or

if $y = -3$

$-\frac{1}{9} = \frac{y+3}{x-4}$

$-\frac{1}{9}x + \frac{4}{9} = y + 3$

$-\frac{1}{9}x - \frac{23}{9} = y$

$y = -\frac{1}{9}x - \frac{23}{9}$

With the assumption that there are two points of interest (y=-3,3), you are really finding tangent lines to y^2=2x+1 so the two points should have different slopes, in this case m=9,-9.

Two possible equations that satisfy the solution are $y = -\frac{1}{9}x + \frac{31}{9}$ or $y = -\frac{1}{9}x - \frac{23}{9}$.

for negative y

∴ When y is positive, the equation is $\boxed{y = -\frac{1}{9}x + \frac{31}{9}}$

*This blank page is for your solution to **Question 3**, if you need more space.*