Math 100 Test 1 Wednesday

Results

- Name: Ha, Thao Linh
- ID: 30304992
- Test number: 731

question	version	mark	out of
Q1	4	8	8
Q2	3	4	6
Q3	4	1	6
total		13	20

Cover page produced on 2023-10-26 at 10:22 PDT

MATH 100 — TEST 1 — 45 minutes

Wednesday, October 11 2023

- The test consists of 6 pages and 3 questions worth a total of 20 marks.
- This is a closed-book examination. **None of the following are allowed**: documents, cheat sheets or electronic devices of any kind (including calculators, cell phones, etc.)
- No work on this page will be marked.
- Fill in the information below before turning to the questions. Your "Section" is your small class discussion section.

Student number	3	0	3	0	4	9	9	2
Section	C	3	6					
Name	Ha Thao Linh							
Signature	Halue							

1. $\boxed{\text{8 marks}}$ ★★☆☆ Suppose the tangent line to the graph of $f(x)$ at $x = 3$ is $y = 5x - 2$, and suppose $y = 4x + 1$ is the tangent line to the graph of $g(x)$ at $x = 3$.

(a) Find the equation of the tangent line to $y = f(x)g(x)$ at $x = 3$.

For $f(x)$ the tangent line is $\left.\begin{array}{l} y = 5x-2 = 5(x-3)+13 \\ y = f'(x_0)(x-3) + y_0 \end{array}\right\}$

$\rightarrow \begin{cases} f'(x_0) = 5 \\ y_0 = 13 \text{ at } x = 3. \end{cases}$

For $g(x)$ the tangent line is $y = 4x+1 = 4(x-3)+13 = g'(x_0)(x-3)+y_0$.

$\rightarrow \begin{cases} g'(x_0) = 4 \\ y_0 = 13 \text{ at } x = 3. \end{cases}$

$\Rightarrow (g(x).f(x))' = f'(x)(g(x) + g'(x).f(x) = 5.13 + 13.4 = 13.9 = 117$ at $x=3$.

$g(x).f(x) = g(3).f(3) = 13.13$.

$\Rightarrow$ The tangent line to $y = f(x)\,g(x)$ at $x=3$ is $y = (f(x_0)\,g(x_0))'(x-3) + f(x_0)g(x_0)$

$\rightarrow y = 117(x-3) + 169 = 117x - 182$

(b) Find the equation of the tangent line to $y = \dfrac{f(x)}{g(x)}$ at $x = 3$.

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - g'(x)f(x)}{(g(x))^2} = \frac{5.13 - 4.13}{(13)^2} = \frac{13}{(13)^2} = \frac{1}{13} \text{ at } x=3$$

$\rightarrow$ The tangent line to $y = \dfrac{f(x)}{g(x)}$ at $x=3$ is

$$y = \left(\frac{f(x_0)}{g(x_0)}\right)'(x-3) + \frac{f(x_0)}{g(x_0)} = \frac{1}{13}(x-3) + \frac{13}{13} = \frac{1}{13}(x-3)+1$$

$$\Rightarrow y = \frac{1}{13}(x-3) + 1 = \frac{1}{13}x + \frac{10}{13}$$

8 of 8 full marks

Q2: 4 out of 6

2. 6 marks ★★★☆ Consider the function $f(x)$ below

$$f(x) = \begin{cases} mx + b & \text{for } x < 0, \\ 1 & \text{for } x = 0 \\ g(x) & \text{for } x > 0. \end{cases}$$

where m and b are constants and g satisfies the differential equation

$$g' = g$$

(a) What is $g(x)$?

$g(x) = e^x$

+1 for e^x

One point off for not including a constant: g(x)=ce^x

(b) If f is continuous, what is the value of b?

If f is continuous $\rightarrow \begin{cases} mx + b = 1 & \text{at } x = 0 \\ g(0) = 1 \end{cases}$

$\Leftrightarrow \begin{cases} 0 + b = 1 & \Rightarrow b = 1 \\ e^0 = 1 & \text{(~~it is~~ always right).} \end{cases}$

Therefore, when $b = 1 \Rightarrow f$ is continuous

+1 for b=1

(c) If f is differentiable, what are the possible values of m and b? Justify briefly.

$f(x) = \begin{cases} mx + b & \text{for } x < 0 \\ 1 & \text{for } x = 0 \\ e^x & \text{for } x > 0 \end{cases}$

If f is differentiable ~~in~~ all in its domain

$\Rightarrow \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = \lim_{x \to 0} f(x)$

This is part of the definition of continuity, not differentiability.

which means it should satisfy this:

+1 $\lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^+} \frac{f(x) - f(0)}{x \to 0} = 1$ ✓ ×

$\Leftrightarrow \begin{cases} \lim_{x \to 0^-} \frac{mx + b - b}{x} = 1 \\ \lim_{x \to 0^+} \frac{e^x - e^0}{x} = 1 \end{cases}$ $\Leftrightarrow \begin{cases} \lim_{x \to 0^-} m = 1 \\ \lim_{x \to 0^+} \frac{e^x - 1}{x} = 1 \end{cases}$ (this one is correct)

$\Rightarrow m = 1$

Moreover since f is differentiable, it should be continuous in its all domain

$\Rightarrow b = 1$

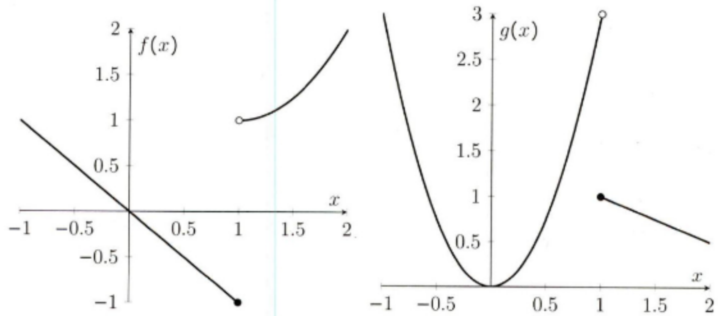
+1 for b=1 (independent of part (b))

This 1 appears to come from f(0) but that's not where it should be coming from. It should come from the slope of e^x at x=0, which you confirm below (so clearly not the source of the 1 here).

Q3: 1 out of 6

3. 6 marks ★★★☆

The functions $f(x)$ and $g(x)$ are defined by their graphs below. Answer each of the following questions.



(a) If $h(x) = f(x) + g(x)$, what is $\lim_{x\to 1} h(x)$? Is $h(x)$ a continuous function for the values of x shown?

$\lim_{x\to 1} h(x) = \lim_{x\to 1} f(x) + \lim_{x\to 1} g(x)$ = doesnot exist. Because:

(v4) not continuous, limit exists but not equal to value

$\lim_{x\to 1^+} f(x) = 1 \neq \lim_{x\to 1^-} f(x) = -1$

and $\lim_{x\to 1^+} f(x) = 1 \neq \lim_{x\to 1^-} g(x) = 3$

$\Rightarrow \lim_{x\to 1} f(x)$, $\lim_{x\to 1} g(x)$ doesnot exist

$\to$ They are not continuous at x=1

$\to h(x)$ is not continuous at $x=1$

(b) If $h(x) = f(x) - g(x)$, what is $\lim_{x\to 1} h(x)$? Is $h(x)$ a continuous function for the values of x shown?

$\lim_{x\to 1} (h(x) = \lim_{x\to 1} f(x) - \lim_{x\to 1} g(x)$ does not exist

The same explanation is justified above in (a)

$\to h(x)$ is not continuous at $x=1$

(v4) right and left limits of h do not match

(c) If $h(x) = f(x)/g(x)$, what is $\lim_{x\to 0} h(x)$? Note that $g(x)$ for $x < 1$ is quadratic.

$\lim_{x\to 0} h(x) = \lim_{x\to 0} \frac{f(x)}{g(x)} = \frac{\lim_{x\to 0} f(x)}{\lim_{x\to 0} g(x)} = \frac{0}{0}$ is indeterminate

We have $f(x)$: $f(x)$ has the slope: -1 and passes $(0,0)$

$\to f(x) = -1.x = -x$.

(v4) limit behaves like Ax/(Bx^2) so does not exist

$g(x)$ is quadratic. $\to g(x) = ax^2 + bx + c$ (a,b,c are constants and $a>0$).

Since $(0,0)$, $(1,3)$; $(-1,3)$ are on $g(x)$

$\Rightarrow \begin{cases} c = 0 \\ a + b + c = 3 \\ a - b + c = 3 \end{cases} \Leftrightarrow \begin{cases} a+b = 3 \\ a-b = 3 \\ c=0 \end{cases} \Leftrightarrow \begin{cases} a = 3 \\ b = 0 \\ c = 0 \end{cases} \Rightarrow g(x) = 3x^2$.

$+)$ $= \lim \frac{f(x) - f(0)}{x}$ =

(d) If $h(x) = g(x)/f(x)$, what is $\lim_{x\to 0} h(x)$?

$\lim_{x\to 0} h(x) = \lim_{x\to 0} \frac{g(x)}{f(x)} = \cancel{\frac{\lim_{x\to 0} g(x)}{\lim_{x\to 0} f(x)}} = \frac{0}{0}$ is indeterminate

+1

$= \lim_{x\to 0} \frac{3x^2}{-x} = \lim_{x\to 0} -3x = 0$

1 of 6 one mark