

Math 100 Test 1 Thursday

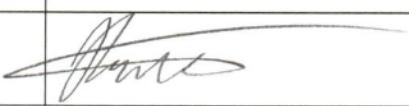
Results

- Name: Johal, Raavin
- ID: 31618176
- Test number: 116

question	version	mark	out of
Q1	3	8	8
Q2	1	3	6
Q3	4	6	6
total		17	20

**MATH 100 — TEST 1 — 45 minutes****Thursday, October 5, 2023**

- The test consists of 6 pages and 3 questions worth a total of 20 marks.
- This is a closed-book examination. **None of the following are allowed:** documents, cheat sheets or electronic devices of any kind (including calculators, cell phones, etc.)
- No work on this page will be marked.
- Fill in the information below before turning to the questions. Your “Section” is your small class discussion section.

Student number	3	1	6	1	8	1	7	6
Section	A	1	5					
Name	Raavin Johal							
Signature								





1. 8 marks ★★☆☆ A function $f(t)$ has instantaneous rate of change $f'(t) = \sqrt{t} + \sqrt[3]{t-1}$. When $t = 9$, it is observed that $f(9) = 3$.

Use a linear approximation to approximate $f(8.8)$.

Remember to simplify your final answers for all questions of this exam, unless instructed otherwise.

$$F(x) = f(a) + f'(a)(x-a)$$

$$F(8.8) = f(a) + f'(a)(8.8-a)$$

$$F(8.8) = 3 + 5(8.8-9)$$

$$F(8.8) = 3 + 5\left(-\frac{1}{5}\right)$$

$$F(8.8) = 3 - 1 = 2$$

$$f(a) = 3 \quad f'(a) = \sqrt{9} + \sqrt[3]{9-1} = 3 + 2 = 5$$

$$x = 8.8 \quad a = 9$$

$$f(8.8) \approx 2$$

8 of 8 full marks



2. [6 marks] ★★☆☆ Consider the function

$$f(x) = \begin{cases} \frac{1}{x} & \text{if } x < b, \\ 1 - \frac{x}{4} & \text{if } b \leq x. \end{cases}$$

(a) Determine a value of b so that $f(x)$ is continuous everywhere.to be continuous everywhere $\frac{1}{x} = 1 - \frac{x}{4}$ at b

$$\frac{1}{b} = 1 - \frac{b}{4} \quad \text{both sides } \times 4b$$

$$4 = 4b - b^2$$

$$0 = -b^2 + 4b - 4$$

$$0 = b^2 - 4b + 4$$

$$0 = (b - 2)^2$$

$$\boxed{b = 2}$$

$f(x) = 1 - b/4$ at $x=b$. You want to equate the two functions in the limit of $x \rightarrow b$ from left and right. Technically, your answer is not quite correct - you need limit expression in first line.



+3 correct

- (b) For the value of
- b
- you found in part (a), decide (with justification) whether
- $f(x)$
- is differentiable at
- $x = b$
- or not.

in order to be continuous $\lim_{x \rightarrow b^+} f(x) = a$ and $\lim_{x \rightarrow b^-} f(x) = a$, and $f(b) = a$ This is continuity.

lets say $b=2$, as this is what I found for my answer

$$\lim_{x \rightarrow 2^+} 1 - \frac{x}{4} = \lim_{x \rightarrow 2^+} 1 - \frac{x}{4} = \lim_{x \rightarrow 2^+} 1 - \frac{1}{2} = \boxed{\frac{1}{2}}$$

$$\lim_{x \rightarrow 2^-} \frac{1}{x} = \lim_{x \rightarrow 2^-} \frac{1}{x} = \boxed{\frac{1}{2}}$$

Ok, this calculation fixes what was missing in part (a). But you haven't said anything about differentiability.

$$\text{and } f(2) = 1 - \frac{2}{4} = \boxed{\frac{1}{2}}$$

therefore, the function is continuous if $b=2$

differentiable \neq continuous

3. [6 marks]

★★★ Consider the function given by $f(x) = \frac{e^x}{\sqrt{e^{2x} + 1}}$.

(a) Find all horizontal asymptotes and all vertical asymptotes.

$$e^{2x} + 1 > 0$$

$$e^{2x} + 1 > 0$$

$$e^{2x} > -1$$

$$\ln e^{2x} = \ln(-1)$$

there are no V.A.

for this graph

as e^{2x} is always greater than -1

$$\lim_{x \rightarrow \infty} \frac{e^x}{\sqrt{e^{2x} + 1}} = \lim_{x \rightarrow \infty} \frac{e^x}{\sqrt{(e^x)^2 + 1}}$$

$$\lim_{x \rightarrow \infty} \frac{e^x}{\sqrt{(e^x)^2 (1 + \frac{1}{e^{2x}})}} = \lim_{x \rightarrow \infty} \frac{e^x}{e^x \sqrt{1 + \frac{1}{e^{2x}}}}$$

$$\lim_{x \rightarrow \infty} \frac{e^x}{e^x \sqrt{1 + \frac{1}{e^{2x}}}} = \lim_{x \rightarrow \infty} \frac{e^x}{e^x} = 1$$

+1 Correct asym

H.A at $y = 1$ for large x

$$\lim_{x \rightarrow -\infty} \frac{e^x}{\sqrt{e^{2x} + 1}} = \frac{\text{very small}}{\sqrt{\text{very small} + 1}} \approx 0$$

+1 Correct asym

H.A at $y = 0$ for very small x

(b) Find all x-intercepts and all y-intercepts.

 $0 = \frac{e^x}{\sqrt{e^{2x} + 1}}$ this value is always greater than 1,

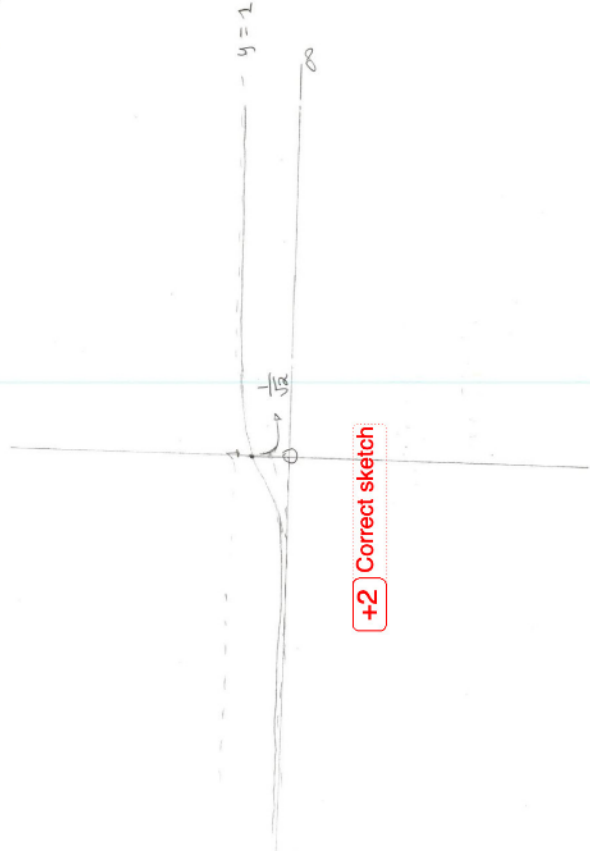
 $0 = \frac{e^x}{\sqrt{e^{2x} + 1}}$ this value is always greater than 1

This function can never be 0, so no x-intercept

+1 Correct x-intercept (there is none)

$$y = \frac{e^0}{\sqrt{e^{2(0)} + 1}} = \frac{1}{\sqrt{1 + 1}} = \frac{1}{\sqrt{2}} = y \text{ intercept}$$

+1 Correct y-intercept

(c) Sketch the graph of $f(x)$ showing all intercepts and asymptotes.

+2 Correct sketch