Math 100 Test 1 Thursday

Results

- Name: Saleem, Athif
- ID: 15079189
- Test number: 43

question	version	mark	out of
Q1	1	8	8
Q2	2	2	6
Q3	2	5	6
total		15	20

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MATH 100 — TEST 1 — 45 minutes

Thursday, October 5, 2023

- The test consists of 6 pages and 3 questions worth a total of 20 marks.
- This is a closed-book examination. **None of the following are allowed**: documents, cheat sheets or electronic devices of any kind (including calculators, cell phones, etc.)
- No work on this page will be marked.
- Fill in the information below before turning to the questions. Your "Section" is your small class discussion section.

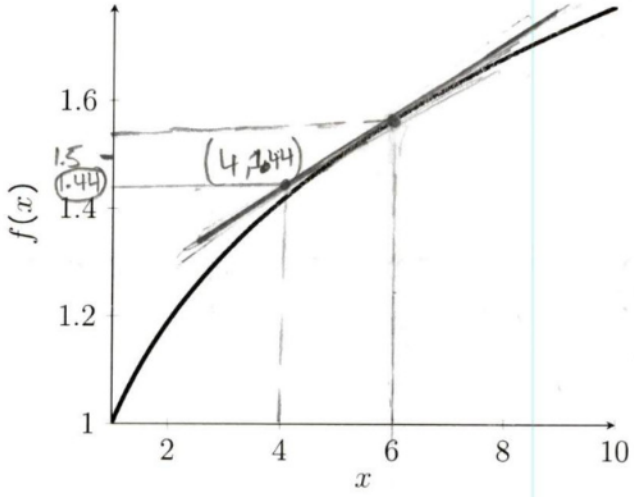
Student number	1	5	0	7	9	1	8	9
Section	A	1	4					
Name	Athif Saleem							
Signature								

Q1: 8 out of 8

1. 8 marks ★★☆☆ Consider the graph of $f(x)$ below.

   (a) On the graph draw the tangent line at the point with $x = 6$.

   (b) Indicate on the tangent line the value of the linear approximation to the function at $x = 4$.



8 of 8 full marks

Q2: 2 out of 6

2. [6 marks] ★★★☆ Consider the function

$$f(x) = \begin{cases} \frac{1}{x} & \text{if } x < b, \\ 1 - \frac{x}{4} & \text{if } b \le x. \end{cases}$$

$1-\frac{x}{4}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	0	$-\frac{1}{4}$
x	1	2	3	4	5

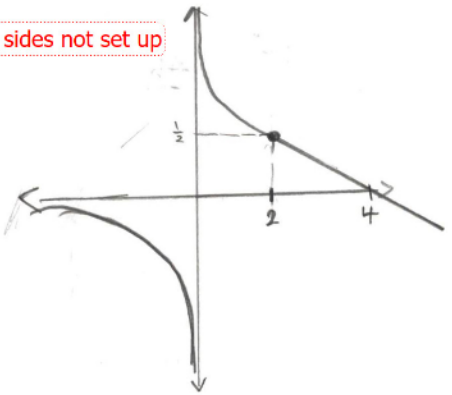
(a) Determine a value of b so that $f(x)$ is continuous everywhere.

$\left(\frac{1}{x} = 1 - \frac{x}{4}\right) \times$

Limit on both sides not set up

$1 = x - \frac{x^2}{4}$

$\left[\frac{x^2}{4} - x + 1 = 0\right] 4$



$x^2 - 4x + 4 = 0$

$(x-2)(x-2) = 0$

$x = 2$

+1 ~~Correct value of b~~

You need to equate the two pieces of the function in the limit as x-->b. You end up with an equivalent equation but no demonstration of understanding continuity in terms of limits.

(b) For the value of b you found in part (a), decide (with justification) whether $f(x)$ is differentiable at $x = b$ or not.

at $x = 2$, $f(x) = 1 - \frac{x}{4}$

$f'(2) \Rightarrow \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0} \frac{\left[1 - \frac{(2+h)}{4}\right] - \left[1 - \frac{2}{4}\right]}{h}$

$\lim_{h \to 0} \frac{-\frac{2+h-2}{4}}{h} \Rightarrow \lim_{h \to 0} \frac{-\frac{h}{4}}{h} \Rightarrow \frac{-1}{4}$

+1 No gluing; expected a definition of the derivative

ANSWER $f'(2) = -\frac{1}{4}$, so $f(x)$ is differentiable at $x = b$.

You just found the “right-side derivative”. You need to also find the “left-side derivative”, show that they are equal and then conclude differentiability.

Q3: 5 out of 6

3. [6 marks] ★★★☆ Consider the function

$$f(x) = \frac{\sqrt{2x^2+1}}{x-2}$$

(a) Find all horizontal asymptotes and all vertical asymptotes.

+1 Correct vertical asymptote

vertical asymptote at $x=2$

as $f(2) = \frac{\sqrt{2(2)^2+1}}{2-2} \Rightarrow \frac{\sqrt{9}}{0} \Rightarrow$ undefined

horizontal asymptote is $y=0$

(b) Find all x-intercepts and all y-intercepts.

x-intercept is when $y=0$

$f(x)=0$ so $0 = \frac{\sqrt{2x^2+1}}{x-2}$

$0 = \sqrt{2x^2+1} \Rightarrow 2x^2+1=0$

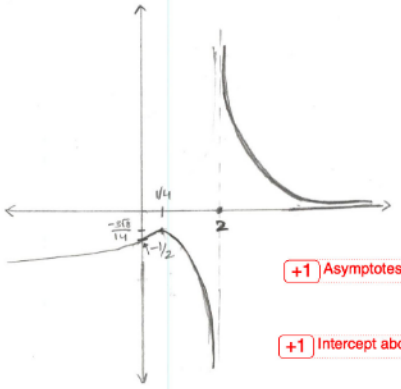
$\frac{2x^2}{2} = \frac{-1}{2}$

$x^2 = -\frac{1}{2}$ as $\sqrt{-\frac{1}{2}}$ does not exist.

There is no x-intercept.

+1 Correct x-intercept (there is none)

(c) Sketch the graph of $f(x)$ showing all intercepts and asymptotes.



+1 Asymptotes match (a)

+1 Intercept above asymptote

Only 1 question per page!

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y-intercept is when $x=0$, so

$f(0) = \frac{\sqrt{2(0)^2+1}}{0-2}$

$f(0) = \frac{\sqrt{1}}{-2}$

$f(0) = \frac{1}{-2}$

$f(0) = -\frac{1}{2}$

y-intercept $= -\frac{1}{2}$

+1 Correct y-intercept

If you need more space, use back or another sheet.