Math 100 Test 1 Friday

Results

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- ID: 40552606
- Test number: 112

| question | version | mark | out of |
|---|---|---|---|
| Q1 | 1 | 2 | 8 |
| Q2 | 2 | 1 | 6 |
| Q3 | 1 | 0 | 6 |
| total | | 3 | 20 |

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# MATH 100 — TEST 1 — 45 minutes

## Friday, October 6, 2023

- The test consists of 6 pages and 3 questions worth a total of 20 marks.
- This is a closed-book examination. **None of the following are allowed**: documents, cheat sheets or electronic devices of any kind (including calculators, cell phones, etc.)
- No work on this page will be marked.
- Fill in the information below before turning to the questions. Your "Section" is your small class discussion section.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Student number | 4 | 0 | 5 | 5 | 2 | 6 | 0 | 6 |
| Section | A | 2 | 3 | | | | | |
| Name | Anna-Liisa Volohhonski | | | | | | | |
| Signature | | | | | | | | |

Q1: 2 out of 8



1. 8 marks ★★☆☆ Let $f$ be a piecewise defined function with parameters $a$ and $b$:

$$f(x) = \begin{cases} x & \text{for } x \leq b, \\ \sqrt{x-a} & \text{for } x > b. \end{cases}$$

Determine the values of $a$ and $b$ that make $f$ continuous and differentiable. Hint: it is much easier to impose the differentiability condition before the continuity condition.

$x = \sqrt{x-a}$

$f(x) = \sqrt{x-a} - x =$

$f'(x) = (\sqrt{x-a})' - (x)'$

+1 Correct $f'$ for $x \leq b$

$(\sqrt{x-a})' = \frac{1}{2\sqrt{x-a}} \cdot (x-a)' = \frac{1}{2\sqrt{x-a}}$

+1 Correct $f'$ for $x > b$

$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$

Q2: 1 out of 6



2. 6 marks ★★★☆ Let $f(x) = \dfrac{e^x - e^{-x}}{e^{2x} + e^{-2x}}$.

(a) What is the asymptotic behavior of $f(x)$ for large positive values of $x$? What about when $x$ is large in the negative direction?

b) $f(x) = \dfrac{e^x - e^{-x}}{e^{2x} + e^{-2x}}$

a) The larger positive values of x the closer the asymptote to 0 it's domain, when the values are larger in the negative then it's ~~closer~~ further to

$f_1'(x) = (e^x - e^{-x})' = (e^x)' - (e^{-x})' = e^x + e^{-x}$

$f_2'(x) = (e^{2x} + e^{-2x})' = (e^{2x})' + (e^{-2x})' = 2e^{2x} - e^{-2x}$

$f'(x) = \dfrac{(e^x - e^{-x})'(e^{2x} + e^{-2x}) - (e^x - e^{-x})(e^{2x} + e^{-2x})'}{(e^{2x} + e^{-2x})^2} =$

$= \dfrac{(e^x + e^{-x})(e^{2x} + e^{-2x}) - (e^x - e^{-x})(2e^{2x} - e^{-2x})}{(e^{2x} + e^{-2x})^2} =$

no answer

(b) What is the slope of $f(x)$ at $x = 0$? You can either take the derivative of $f(x)$ (harder) or you can replace each exponential by its linear approximation near $x = 0$ and find the slope of the resulting function (easier).

$= \dfrac{e^{3x} + e^{-x} + e^{x} + \cancel{e^{-3x}} - 2e^{3x} + e^{-x} + 2e^{x} - \cancel{e^{-3x}}}{(e^{2x} + e^{-2x})^2} = \dfrac{-e^{3x} + 2e^{-x} + 3e^{x}}{(e^{2x} + e^{-2x})^2} =$

$= \dfrac{e^x(-1^{2x} + 2^{-1} + 3)}{(e^{2x} + e^{-2x})^2}$ ?

$f'(0) = \dfrac{e^0(-1^{2\cdot 0} + 2^{-1} + 3)}{(e^{2\cdot 0} + e^{-2\cdot 0})^2} = \dfrac{1(-1 + \frac{1}{2} + 3)}{(e^1 + e^1)^2} = \dfrac{\frac{3}{2}}{4e^2} = \dfrac{3}{8e^2}$

$-\frac{1}{2} + 2 = \frac{-1+4}{2} = \frac{3}{2}$

$f'(0) = \frac{3}{8e^2}$

incorrect derivative compuitattion

$(e^{2x} + e^{-2x})^2 = (e^{2x})^2 + 2e^{2x} \cdot e^{-2x} + (e^{-2x})^2 = e^{4x^2} + 2e + e^{4x^2}$



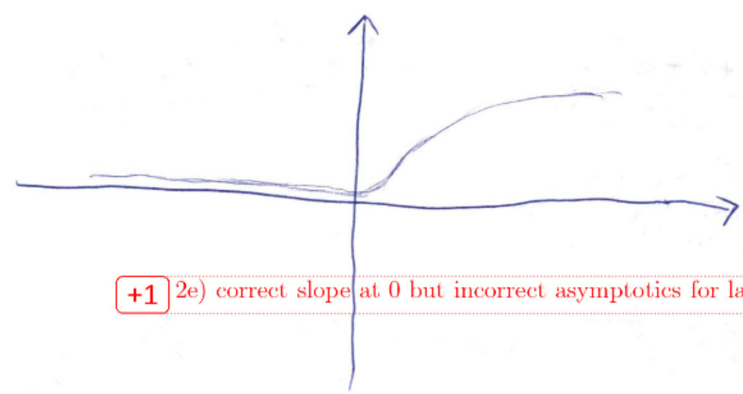


(c) Evaluate $f(-x)$ and compare to $f(x)$. What does this tell you about the symmetry of $f(x)$?

$f(-x) = \dfrac{e^{-x} - e^{+x}}{e^{2\cdot(-x)} + e^{-2\cdot(-x)}} = \dfrac{e^{-x} - e^{x}}{e^{-2x} + e^{2x}}$

~~They are the same~~ They are assymetrical

(d) Use the information above to sketch the graph of $f(x)$.



+1 2e) correct slope at 0 but incorrect asymptotics for large $x$

Q3: 0 out of 6



3. $\boxed{\text{6 marks}}$ ★★★☆ There are two distinct straight lines that pass through the point $(1, -3)$ and are tangent to the curve $y = x^2$. Find their equations.

$y = x^2$

$(1; -3)$ $x_1 = 1$ ; $y = -3$

$kx + c$

$mx + b$

$f(x)(x - x_0) + f(x_0)$

This is not the equation of a tangent line for a couple reasons. There should be a ' on the first f and an x0 in its ( ).

$\cancel{k = \frac{f(x)}{x} = \frac{x^2}{x} = x}$ $k = \lim_{x \to \infty} \frac{f(x)}{x} = \frac{x^2}{x} = 0$

incorrect formula for the derivative

$b = \lim_{x \to \infty} = f(x) - (kx) = x^2 - 0 = 0$

$f(h)(h - a) + f'(h) - mx + b$

$y = 0$

$y' = 2x$

Finding y' evaluated at the as-yet unknown point of tangency was worth a point but not just for taking the derivative of x^2.

$y = 2x + 5$

$y = 2 \cdot 1 = 2$

$-3 = 2 \cdot 1 = b$

$b = 5$

$\boxed{0 \text{ of } 6}$ no marks

$f(x) = 1^2 + 3 = 4$

$\cancel{f'(x) = 2x} =$





*This blank page is for your solution to **Question 3**, if you need more space.*