Math 100 Test 1 Tuesday

Results

- Name: Felicia, Jessica
- ID: 49715998
- Test number: 148

question	version	mark	out of
Q1	2	8	8
Q2	2	3	6
Q3	3	2	6
total		13	20

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MATH 100 — TEST 1 — 45 minutes

Tuesday, October 10 2023

- The test consists of 6 pages and 3 questions worth a total of 20 marks.
- This is a closed-book examination. **None of the following are allowed**: documents, cheat sheets or electronic devices of any kind (including calculators, cell phones, etc.)
- No work on this page will be marked.
- Fill in the information below before turning to the questions. Your "Section" is your small class discussion section.

Student number	4	9	7	1	5	9	9	8
Section	A	1	2					
Name	Jessica Felicia							
Signature								

Q1: 8 out of 8

1. 8 marks ★★☆☆ Use the definition of the derivative to compute the derivative of the function

$$f(x) = \frac{x}{x^2+1}.$$

~~f'(x)~~

$u = x \quad v = (x^2+1) \quad , u' = 1 \quad v' = 2x$

$$f'(x) = \frac{u'v - v'u}{v^2}$$

$$f'(x) = \frac{1.(x^2+1) - 2x(x)}{(x^2+1)^2}$$

$$= \frac{x^2+1-2x^2}{(x^2+1)^2}$$

$$= \frac{1-x^2}{(x^2+1)^2}$$

$$= \frac{1-x^2}{x^4+2x^2+1}$$

8 of 8 full marks

Q2: 3 out of 6

2. 6 marks ★★★☆ Consider the function

$$f(x) = \frac{e^{x+1}}{e^{2x} + e^x}.$$

(a) Give the domain of $f(x)$. Justify your answer.

~~because it's known that this function can't be have a value of negative number so $x \geq 0$, based on the horizontal asymptote the graph~~

x can be any real number. $x \in \mathbb{R}$. Since $e^{2x} + e^x$ can never be value 0, hence it has no vertical asimtot, it will be continuos for all $x \in \mathbb{R}$.

+2 a) Correct domain with correct justification

(b) Find all horizontal asymptotes of $f(x)$, if any. Justify your answer.

horizontal asimtot $= \lim_{x\to\infty} \frac{e^{x+1}}{e^{2x}+e^x}$

$= \lim_{x\to\infty} \frac{e^x+1}{e^{2x}+e^x} \times \frac{e^{2x}}{e^{2x}}$

$= \lim_{x\to\infty} \frac{\frac{e^x}{e^{2x}} + \frac{1}{e^{2x}}}{\frac{e^{2x}}{e^{2x}} + \frac{e^x}{e^{2x}}}$

$= \lim_{x\to\infty} \frac{e^x \cdot e^{-2x} + e^{-2x}}{1 + e^x \cdot e^{-2x}}$

$= \lim_{x\to\infty} \frac{e^{-x} + e^{-2x}}{1 + e^{-x}}$

$= \frac{\frac{1}{\infty} + \frac{1}{\infty}}{1 + \frac{1}{\infty}}$

$\approx \frac{0}{1} = 0$

$y = 0$

$\lim_{x\to-\infty} \frac{e^x+1}{e^{2x}+e^x} = \lim_{x\to-\infty} \frac{e^x+1}{e^{2x}+e^x} \cdot \frac{e^{2x}}{e^{2x}}$

$= \lim_{x\to-\infty} \frac{\frac{e^x}{e^{2x}} + \frac{1}{e^{2x}}}{\frac{e^{2x}}{e^{2x}} + \frac{e^x}{e^{2x}}}$

$= \lim_{x\to-\infty} \frac{\frac{1}{e^x} + \frac{1}{e^{2x}}}{1 + \frac{1}{e^x}}$

$\approx \frac{\frac{1}{-\infty} + \frac{1}{-\infty}}{1 + \frac{1}{-\infty}}$

$\approx \frac{0}{1} = 0$

b) Need to find both asymptotes at 0 and e with correct justification

(c) Find all vertical asymptotes of $f(x)$, if any. Justify your answer.

vertical $\Rightarrow$ $e^{2x} + e^x > 0$

$e^{2x} = -e^x$

$\log(e^{2x}) = -\log(e^x)$ cannot bring a negative sign outside a log like this

$2x = -x$

$2 \neq 1$

~~there exist~~ doesn't exist, there is no vertical asymtote.

+1 c) Correct that there are no vertical asymptotes but missing correct justification

can just repeat this argument here

Q3: 2 out of 6

3. 6 marks ★★★☆ Let

$$f(x) = \begin{cases} a(e^x + 2) & x \le b \\ 1 & x > b \end{cases}$$

where a is a constant and b is an x value, both to be determined.

(a) If $b = 2$ find the value or values of a that make the function continuous.

If continuous

$\lim_{x \to b^-} a(e^x + 2) = \lim_{x \to b^+} 1$

$\lim_{x \to 2} a(e^x + 2) = 1$

$a(e^2 + 2) = 1$

$a = \frac{1}{e^2 + 2}$

+2 correct

(b) Determine the pair or pairs of values a and b such that the function is (continuous and) differentiable or show that such a pair of values does not exist.

If continuous

$\lim_{x \to b^-} a(e^x + 2) = \lim_{x \to b^+} 1$

If differentiable

$f'(x) = f'(x)$

- $\lim_{x \to b^-} a(e^x + 2) = \lim_{x \to b^+} 1$

  $a(e^b + 2) = 1$ ... (1)

- ~~f(x)~~ (b) $a(e^b + 2)$

  $f'(b) = ae^b + 2a$

  $f'(b) = ae^b$

  $f(b) = 1$

  $f'(b) = 0$

  $ae^b = 0$ ... (2)

- Sub 1 & 2

  $ae^b + a2 = 1$

  $0 + a2 = 1$

  $a = \frac{1}{2}$ ... (3)

- Sub 3 & 2

  $\frac{1}{2} e^b = 0$

This equation can't be ~~defined~~ solved, as e^b will never be 0, so the function is not differentiable and the value of b is not exist.

3 points out of 4 - derivative matching should have used the definition of the derivative.