ASEN 3728 Aircraft Dynamics
Written Homework 3

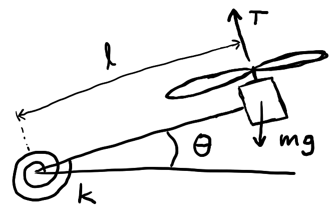
Due date listed on Gradescope.

Question 1. Consider a quadrotor with $I_y = 3$ kg m^2. Perform a closed-loop modal analysis of the quadrotor's longitudinal $\Delta\theta$ and Δq motion using the information below. Subscripts on the natural frequencies and damping ratios correspond to the eigenvalues of the state space model for the motion.

$$\begin{aligned} \Delta M_c &= -k_1 \Delta q - k_2 \Delta\theta \\ \omega_{n,1,2} &= 1.8 \quad \text{[rad/s]} \\ \zeta_{1,2} &= 0.5 \end{aligned}$$

1. Calculate the values of k_1 and k_2, as well as the eigenvalues λ_1 and λ_2 of the size 2×2 state space model $\mathbf{A}$ matrix.
2. At $t = 0$, the quadrotor's state is $\mathbf{x}(0) = 5\mathbf{v}_1 + \mathbf{v}_2$. The vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ are the unit-length eigenvectors of the $\mathbf{A}$ matrix which correspond to the eigenvalues λ_1 and λ_2. Write the solution $\mathbf{x}(t) = (\Delta\theta(t), \Delta q(t))^T$ in terms of t, λ_1, λ_2, $\mathbf{v}_1$, and $\mathbf{v}_2$.
3. Calculate the eigenvectors $\mathbf{v}_1$ and $\mathbf{v}_2$.
4. Describe the behavior of $\Delta\theta$ over time using the eigenvalues you calculated in Part (1).

Question 2. Recall the test rig described in Homework W2, Problem 3. The test rig in the diagram below is used to measure the thrust of a rotor, T. The rotor, which has mass m, is mounted at the end of a massless rod of length l, which has a torsional spring with stiffness k at its base.



The equation of motion is

$$\ddot{\theta} = \frac{-mgl\cos(\theta) - k\theta + Tl}{ml^2}. \tag{1}$$

If k is known, the equilibrium thrust T_0 can be measured with $T_0 = mg\cos(\theta_0) + \frac{k\theta_0}{l}$.

Describe the motion of the rig according to the linear model (e.g. "growing oscillations, exponential decay", etc.) after a perturbation if $k > mgl\sin(\theta_0)$ and T stays constant. Why does this make the rig unsuitable for its intended purpose?

Question 3. Suppose that the control forces and moments for a quadrotor with arm length $d = 10cm$ and rotor moment coefficient $k_m = 0.003$ are given by

$$\begin{bmatrix} Z_c \\ L_c \\ M_c \\ N_c \end{bmatrix} = \begin{bmatrix} -5N \\ 0Nm \\ 0.2Nm \\ 0.01Nm \end{bmatrix}.$$

If the quadrotor has the standard rotor configuration described in class, what are the thrust forces generated by each rotor?

Question 4. Consider the longitudinal dynamics of the linearized quadrotor EOM:

$$\begin{pmatrix} \Delta \dot{x}_E \\ \Delta \dot{u} \\ \Delta \dot{\theta} \\ \Delta \dot{q} \end{pmatrix} = \begin{pmatrix} \Delta u \\ -g\Delta\theta \\ \Delta q \\ \frac{1}{I_u}\Delta M_c \end{pmatrix}$$

where ΔM_c is defined in terms of k_1 and k_2 as in question 1.

1. Suppose a closed-loop modal analysis of the system was performed and you were given only the following values:

$$\lambda_1 = -1.5 + 4.2i \quad \lambda_2 = -0.0023 + 0.037i$$

$$\mathbf{v}_1 = \begin{pmatrix} 0.005 + 0.0021i \\ 0.075 + 0.0019i \\ 0.0085 + 0.0065i \\ 0.05 \end{pmatrix} \quad \mathbf{v}_2 = \begin{pmatrix} 0.006 + 0.0089i \\ 0.095 + 0.0030i \\ 0.0075 + 0.0015i \\ 0.0120 + 0.0030i \end{pmatrix}$$

   True or False: It is possible to determine λ_3, λ_4, $\mathbf{v}_3$, and $\mathbf{v}_4$ if k_1 or k_2 are unknown. Explain your answer.

   ☐ TRUE ☐ FALSE

2. Consider the following eigenvalues for another quadrotor system with the same dynamics described above:

$$\lambda_1 = -1.5 \quad \lambda_2 = -0.0023 + 0.037i$$

   True or False: It is possible to determine λ_3, λ_4 if k_1 or k_2 are unknown. Explain your answer.

   ☐ TRUE ☐ FALSE