IIITB COMET FOUNDATION

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Sridharacharya (C.E. 1025) derived a formula, now known as the quadratic formula, (as quoted by Bhaskara II) for solving a quadratic equation by the method of completing the square. An Arab mathematician Al-Khwarizmi (about C.E. 800) also studied quadratic equations of different types. Abraham bar Hiyya Ha-Nasi, in his book 'Libre embadorum' published in Europe in C.E. 1145 gave complete solutions of different quadratic equations.

In this chapter, you will study quadratic equations, and various ways of finding their roots. You will also see some applications of quadratic equations in daily life situations.

4.2 Quadratic Equations

A quadratic equation in the variable x is an equation of the form $ax^2+bx+c=0$, where a,b,c are real numbers, $a \neq 0$. For example, $2x^2+x-300=0$ is a quadratic equation. Similarly, $2x^2-3x+1=0$, $4x-3x^2+2=0$ and $1-x^2+300=0$ are also quadratic equations.

In fact, any equation of the form $p(x)=0$, where $p(x)$ is a polynomial of degree 2, is a quadratic equation. But when we write the terms of $p(x)$ in descending order of their degrees, then we get the standard form of the equation. That is, $ax^2+bx+c=0$, $a \neq 0$ is called the **standard form of a quadratic equation**.

Quadratic equations arise in several situations in the world around us and in different fields of mathematics. Let us consider a few examples.

Example 1: Represent the following situations mathematically:

(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.

(ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was 750. We would like to find out the number of toys produced on that day.

Solution:

(i) Let the number of marbles John had be x.
Then the number of marbles Jivanti had = $45-x$ (Why?).
The number of marbles left with John, when he lost 5 marbles = $x-5$
The number of marbles left with Jivanti, when she lost 5 marbles = $45-x-5$
$= 40-x$

Therefore, their product = (x – 5) (40 – x)

= 40x – x^2–200 + 5x

So, $-x^2+45x-200=124$ (Given that product = 124)

i.e., $-x^2+45x-324=0$

i.e., $x^2-45x+324=0$

Therefore, the number of marbles John had, satisfies the quadratic equation

$$x^2-45x+324=0$$

which is the required representation of the problem mathematically.

(ii) Let the number of toys produced on that day be x.

Therefore, the cost of production (in rupees) of each toy that day $=55-x$

So, the total cost of production (in rupees) that day $=x(55-x)$

Therefore, $x(55-x)=750$

i.e., $55x-x^2=750$

i.e., $-x^2+55x-750=0$

i.e., $x^2-55x+750=0$

Therefore, the number of toys produced that day satisfies the quadratic equation

$$x^2-55x+750=0$$

which is the required representation of the problem mathematically.

Example 2 : Check whether the following are quadratic equations:

(i) $(x-2)^2+1=2x-3$ (ii) $x(x+1)+8=(x+2)(x-2)$

(iii) $x(2x+3)=x^2+1$ (iv) $(x+2)^3=x^3-4$

Solution :

(i) LHS = $(x-2)^2+1=x^2-4x+4+1=x^2-4x+5$

Therefore, $(x-2)^2+1=2x-3$ can be rewritten as

$$x^2-4x+5=2x-3$$

i.e., $x^2-6x+8=0$

It is of the form $ax^2+bx+c=0$.

Therefore, the given equation is a quadratic equation.

(ii) Since $x(x+1)+8 = x^2+x+8$ and $(x+2)(x-2) = x^2-4$

Therefore, $x^2+x+8 = x^2-4$

i.e., $x+12 = 0$

It is not of the form $ax^2+bx+c = 0$.

Therefore, the given equation is not a quadratic equation.

(iii) Here, LHS $= x(2x+3) = 2x^2+3x$

So, $x(2x+3) = x^2+1$ can be rewritten as

$$2x^2+3x = x^2+1$$

Therefore, we get $x^2+3x-1 = 0$

It is of the form $ax^2+bx+c = 0$.

So, the given equation is a quadratic equation.

(iv) Here, LHS $= (x+2)^3 = x^3+6x^2+12x+8$

Therefore, $(x+2)^3 = x^3-4$ can be rewritten as

$$x^3+6x^2+12x+8 = x^3-4$$

i.e., $6x^2+10x+6 = 0$ or, $x^2+2x+2 = 0$

It is of the form $ax^2+bx+c = 0$.

So, the given equation is a quadratic equation.

Remark: Be careful! In (ii) above, the given equation appears to be a quadratic equation, but it is not a quadratic equation.

In (iv) above, the given equation appears to be a cubic equation (an equation of degree 3) and not a quadratic equation. But it turns out to be a quadratic equation. As you can see, often we need to simplify the given equation before deciding whether it is quadratic or not.

EXERCISE 4.1

1. Check whether the following are quadratic equations:

(i) $x^2+1 = 2x$

(ii) $x^2-2x = (-2)(3-x)$

(iii) $(x-2)(x+1) = x(x+5)$

(iv) $x^2+(2x-1)(2x+3) = x^2+2$

(v) $x^2+1 = (x+1)^2$

(vi) $x^3-3x^2+x+1 = (x-2)^2$

(vii) $x+2)^3 = 2x(x^2–1$

(viii)$x^3–4x^2–x+1 = (x–2)^3$

2. Represent the following situations in the form of quadratic equations:

   (i) The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.