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Lab 1

MVMAMA001

Amandla Mvimbi

1. Introduction

The report comprises the determination of a model for the altitude control of a helicopter using both hands-methods and the system identification tool. The input of the model is a voltage that is used to control the angle of attack of the rotor blades of a helicopter, this, in turn, increases the lift force which alters the vertical velocity of the craft. The resultant model from the hand methods is then compared to that derived using the Matlab system identification toolbox.

2. Data

Table 1 below shows the data collected from the simulation of the step response of the craft. The columns are time, input voltage, DAC output, height and approximate velocity starting from the left. Velocity was calculated using an approximate derivative algorithm shown below.

$$v_i(t) = \frac{\Delta h_i}{\Delta t_i}$$

	Time(s)	Input(v)	voltage-to-Angle	y_t(m)	v_(t)
66	46.2	5.0	0.000	0.000	0.00
67	46.3	5.0	0.037	0.037	0.37
68	46.4	5.0	0.110	0.111	0.74
69	46.5	5.0	0.220	0.222	1.11
70	46.6	5.0	0.366	0.370	1.48
...	...	...	...	...	...
2002	239.8	5.0	10.000	9658.164	53.89
2003	239.9	5.0	10.000	9663.553	53.89
2004	240.0	5.0	10.000	9668.942	53.89
2005	240.1	5.0	10.000	9674.331	53.89
2006	240.2	5.0	10.000	9679.720	53.89

Table 1: Simulation Data Summary

3. Results

3.1 Eyeballing

This method involves decomposing the transfer function into a partial fraction in order to build a circuit that exhibits similar behaviour using electronic components.

$$Y(s) = AB\left(\frac{1}{s(Ts+1)}\right)$$

$$Y(s) = AB\left(\frac{m}{s} + \frac{n}{(Ts+1)}\right)$$

$$Y(s) = AB\left(\frac{1}{s} - \frac{T}{(Ts+1)}\right)$$

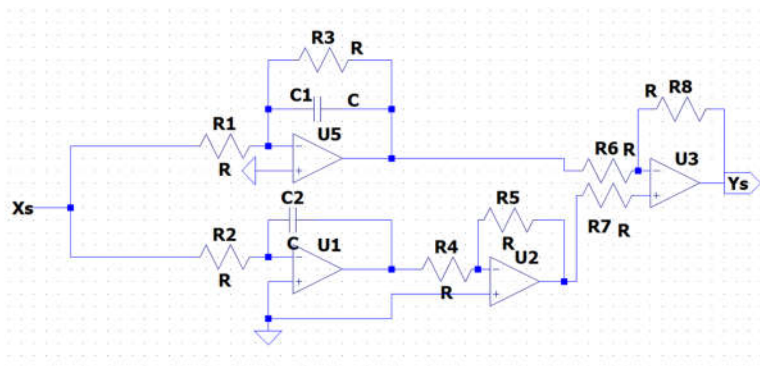

Figure 1: Electronic model of transfer function

Figure 1 shows the circuit that models the height of the helicopter where voltage (Xs) is the input. It comprises two integrators, one is passed through an inverting buffer to ensure that the effect of integrator U5 is subtracted from u1. U1 give the $\frac{1}{s}$ term in the Laplace domain. The results of the two integrators are added, which means adding ($\frac{1}{s}$) and ($\frac{-1}{Ts+1}$) in the Laplace domain. This is achieved using a summing amplifier. Resistor and capacitors are chosen in order to get the correct gain and time constant.

After iterative simulations in LTspice, the parameters are found to be;

C1 = 100 μF, C2 =10 μF,R1=143.4K,R2 = 1.86k, R3=2.66k R4 = R5=1k, R6=6.97k, R7 =R8 = 100k.

3.2 Method 2: Step Response

The step response of the vertical velocity of the helicopter is modelled as a first-order system with a generic Laplace domain function shown below, where T the time constant, A the gain of the system and B is the magnitude of the step input.

$$g(s) = \frac{AB}{s(1 + Ts)}$$

The parameters A and B can be derived from the time domain step response curve, whose equation is$g(t) = AB(1 - e^{-\frac{t}{T}})$. The graph of velocity begins as a transient (exponential) increasing function and then settles at the terminal velocity-AB- as time tends to infinity. The instantaneous velocity is calculated from the gradient of the height function, that is $v_i(t) = \frac{\Delta h_i}{\Delta t_i}$ as mentioned previously.

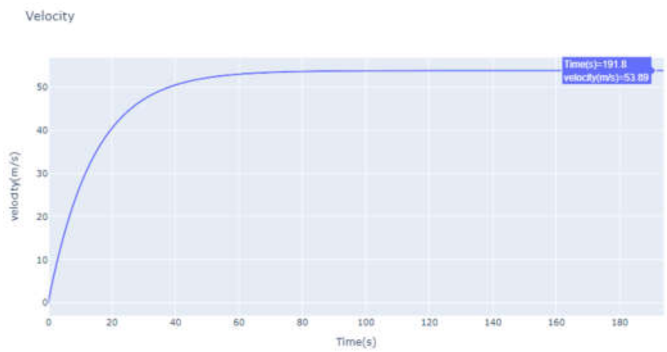

Figure 2: Height (Step Response)

A is calculated by dividing the terminal velocity by the magnitude of the step input.

$A = \frac{53.89}{2.5}$ (From the steady-state magnitude of figure 2)

$\boldsymbol{A = 21.556\ ms^{-1}v^{-1}}$

An exponentially increasing function gets to 63.21% of its final value after one time constant, therefore, the time constant T can be found graphically and or from the data by finding the time it takes for velocity to get to 63.21% of AB - 34.06. This time is 14.35 seconds, and the transfer function is now fully defined. Since five trials of the experiment were done, the same procedure is repeated for the five data sets and the average of the results is taken as summarized in the table below.

Trial 1	$g(s) = \frac{53.89}{s(1 + 14.35s)}$
Trial 2	$g(s) = \frac{53.89}{s(1 + 14.33s)}$
Trial 3	$g(s) = \frac{53.89}{s(1 + 14.33s)}$

Trial 4	$g(s) = \frac{53.89}{s(1+14.34s)}$
Trial 5	$g(s) = \frac{53.89}{s(1+14.35s)}$
Average	$\boldsymbol{g(s) = \frac{53.89}{s(1+14.34s)}}$

The Laplace step response function that maps the input voltage to the output velocity is the one with averaged parameters which is that in the last row;

$$\boldsymbol{g(s) = \frac{53.89}{s(1+14.34s)}}$$

$\Rightarrow \boldsymbol{V(s) = \frac{21.56}{1+14.34s}}$ is the transfer function for velocity.

By dividing the Laplace domain transfer function with $\frac{1}{s}$, the ramp response function-which is the step response for height is derived as shown below;

$$h(s) = AB\frac{1}{(sT+1)\,s^2}$$

$$h(s) = AB\left[\frac{m}{s^2} + \frac{n}{s} + \frac{q}{(Ts+1)}\right]$$

$$n = -T,\ q = T^2,\ m = 1$$

$$h(s) = AB\left[\frac{1}{s^2} - \frac{T}{s} + \frac{T^2}{sT+1}\right] \xrightarrow{\mathscr{L}^{-1}} h(t) = AB\left(t - T + Te^{-\frac{t}{T}}\right)$$

$$\Rightarrow H(s) = \frac{53.89}{s^2(14.34s+1)}$$

$$h(t) = 53.89(t + 14.34e^{\frac{-t}{14.34}} - 14.34)$$

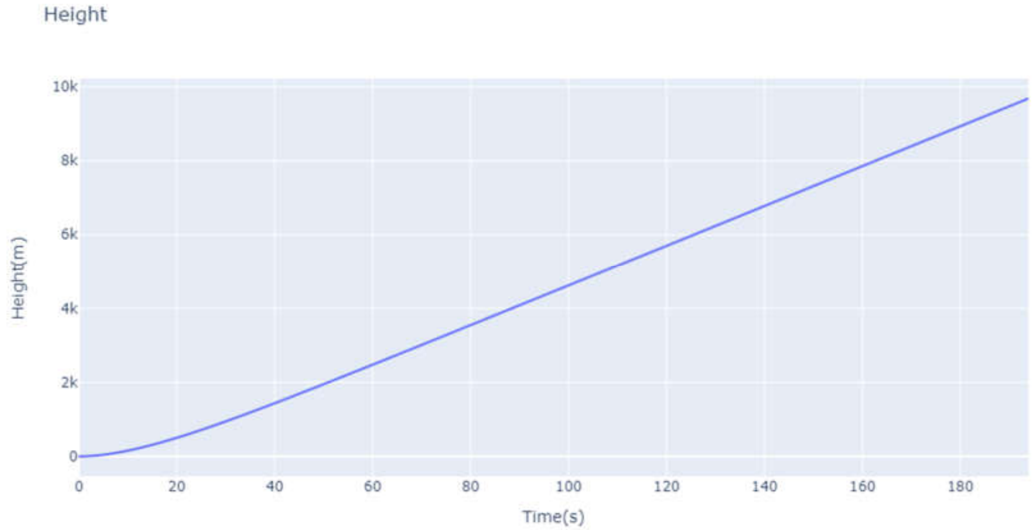

Figure 3:Height response to step

By dividing by $\frac{B}{s}$, the transfer function from input voltage to height is derived as shown below.

$$\boldsymbol{Y(s)} = \frac{\mathbf{21.56}}{\mathbf{s(14.34}\boldsymbol{s}\mathbf{ + 1)}}$$

Method 3: Least Squares System Identification

This method includes using known input data and output to derive a transfer function that characterizes the relationship between the input voltage which controls the angle of attack and the height.

$$T_{Y/V} = \frac{b_2 s^2 + b_1 s + b_0}{s^2 + a_1 s + a_0}$$

$b_1 = 0$, $b_2 = 0$, $b_0 = AB/T$

$a_0 = 0$, $a_1 = 1/T$

$$\theta = \begin{pmatrix} a_1 \\ a_0 \\ b_1 \\ b_0 \\ \frac{1}{T} \\ 0 \\ 0 \\ AB/T \end{pmatrix} y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ \cdot \\ \vdots \\ y_n \end{pmatrix}$$

$$M = \begin{vmatrix} -Iy_1 & -II_{y1} & I_{u1} & II_{u1} \\ -I_{y2} & -II_{y2} & I_{u2} & II_{u2} \\ \vdots & \vdots & \vdots & \vdots \\ -I_{yn} & -II_{yn} & I_{un} & II_{un} \end{vmatrix}$$

$$\theta = \left(M^T M\right)^{-1} \left(M^T y\right)$$

$$y = M\theta$$

```
Vsc =

      0.05833 s + 0.5984
  ---------------------------
  s^2 + 0.05322 s + 1.933e-05

Continuous-time transfer function.
```

The result from Matlab has errors as seen from the transfer function above. a_0 and b_0 are 0.0583 and 0.00001933 respectively instead of zero. These errors arise as a result of the least-squares algorithm and can be disregarded. Furthermore, the resultant gain is scaled up by a factor of two to get the true gain as Matlab reads the B as 5V instead of starting from the hovering voltage of 2.5V, then And that yields;

$$\boldsymbol{Y(s)} = \frac{\mathbf{22.49}}{\mathbf{s(18.79}\boldsymbol{s}\mathbf{+1)}}$$

The results above indicate that the least square method results in a greater level of inaccuracy in the time constant as it deviates from the true value by 31%. The time gain is however still within 3% of the values calculated in other methods.

The code snippet used to get the above is attached below.

```
>> N = length(Inputv);
>> Y = y_tm(3:N);

>> phi = [-y_tm(2:N-1) -y_tm(1:N-2) Inputv(2:N-1) Inputv(1:N-2)];
>> theta_hat = inv(phi'*phi)*phi'*Y

theta_hat =


  -1.9947

   0.9947

   0.0088

  -0.0028
```

```
>> Vsd = tf(theta_hat(3:4)',[1 theta_hat(1:2)'], 1);

>> Vsd = tf(theta_hat(3:4)',[1 theta_hat(1:2)'], 0.1);

>> Vsc = d2c(Vsd)
```

Method 4: System Identification Toolbox

This method used a Matlab system identification tool to find both the transfer functions that map voltage to velocity and height, using the input and output data collected from the lab.

```
  From input "u1" to output "y1":
    0.6782
  -----------
  s + 0.06276
Name: tf4
Continuous-time identified transfer function.
```

The snippet above shows the transfer function for velocity as calculated by Matlab. The function can be re-written as $\boldsymbol{V(s)} = \frac{\mathbf{21.62}}{\mathbf{14.86}\boldsymbol{s}+\mathbf{1}}$ after the gain is scaled to manually zero the data.

Using the same process, the transfer function to height was found to be;

```
 From input "u1" to output "y1":
            0.7259
 --------------------------
 s^2 + 0.06721 s + 1.27e-06
ame: tf2
ontinuous-time identified transfer functior
```

$$Y(s) = \frac{0.7259}{s^2 + 0.06721s + 1.27e - 6}$$

which after disregarding errors can be re-written as;

$$Y(s) = \frac{10.8}{\mathrm{s}(14.88\mathrm{s} + 1)}$$

The resultant gain is scaled up by a factor of two to get the true gain as Matlab reads the B as 5 instead of 2.5 (This is because the software does not zero the values to account for hovering). By so doing, the step is zeroed to the magnitude of 2.5 then;

$$\boldsymbol{Y(s)} = \frac{\mathbf{21.60}}{\mathbf{s(14.88s + 1)}}$$

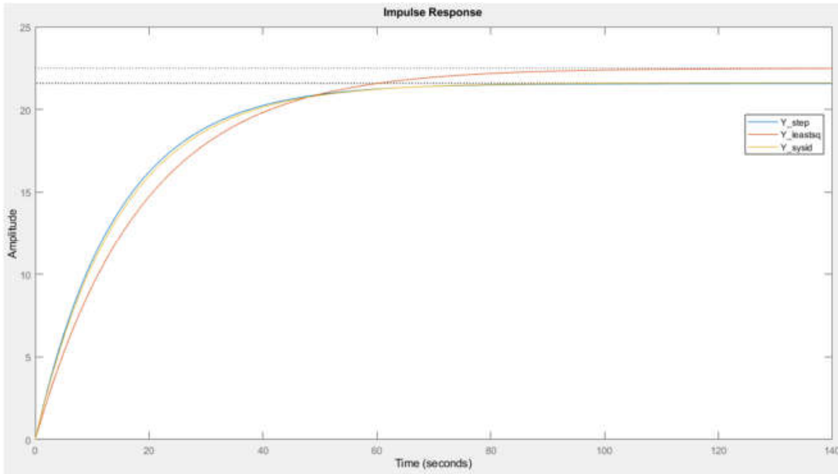

Figure 4: Impulse Responses of Height transfer functions

Figure 4 above is the impulse responses of the transfer functions for height calculated using different methods. Since the impulse response of a system characterises the system, it can be concluded that the results from various methods corroborate each other. However, the least-squares method deviates considerably in terms of the gain and time constant as seen from the graph above.

4. Conclusion

From the results above, it can be concluded that the transfer function for height of the helicopter is given by $\boldsymbol{Y}(\boldsymbol{s}) = \frac{\mathbf{21.56}}{\boldsymbol{s}(\mathbf{14.34}\boldsymbol{s}+\mathbf{1})}$, where 14.34 is the time constant in the transient region of the response and 21.56 is the gain of the system. It is worth noting that other methods such as the least-squares method and the system identification result in errors in the coefficients of the transfer function, however, after propagating these errors, the results validate each other. Furthermore, Matlab reads the magnitude of the step as 5V instead of 2.5V as the program does not account for dynamics such as hovering, as such, its resultant gain is scaled up by a factor of two to get the true gain, after accounting for this error the parameter A and T computed using different methods agree to within 0.001% with the gain from the least-squares method being the only exception by showing a 31% deviation in the time constant and a 3% deviation in the gain.

5. Appendix

	Time(s)	Input(v)	voltage-to-Angle	y_t(m)	v_(t)
66	46.2	5.0	0.000	0.000	0.00
67	46.3	5.0	0.037	0.037	0.37
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2006	240.2	5.0	10.000	9679.720	53.89

Figure 5: Table of raw Data

Bibliography

[1] M. Braae, Control Engineering, Cape Town: UCT Press, 2014.

[2] N. S. Nise, Control Systems Engineering, Pomona: Wiley, 2015.