

A dark blue vertical bar runs down the left side of the page. A blue arrow points to the right from this bar, containing the date.

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# Inductor Design

## Planar Finite Element Analysis

Several thin, curved lines in dark blue and light grey originate from the bottom left corner and sweep upwards and to the right.

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# Numerical Analyses

## Method 1 (Theoretical Modelling)

1.  $K = 0.245$

2. 
$$N = \frac{A_{window} * K_{fill}}{\pi * (R_{wire})^2}$$

$$= \frac{(28 * 10^{-3} * 9.5 * 10^{-3}) * 0.245}{\pi (0.225 * 10^{-3})^2}$$

$$= 410 \text{ Turns}$$

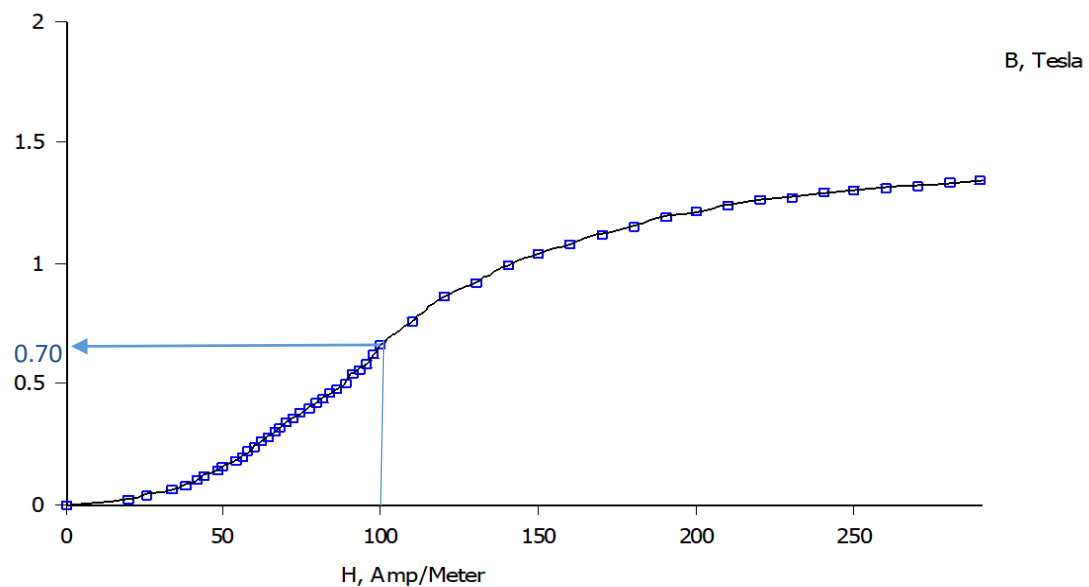
3.  $\Phi = \frac{Li}{n}, \quad IL = \frac{n^2}{R}$

R can be approximated to  $\frac{2lg}{\mu Ag}$  since the reluctance of the gap is much larger than that of the silicon steel (i.e. approximately 4000 times more).

$$\Phi = \frac{410 * 5}{2.44 * 10^6}$$

$$= 8.40 \mu W$$

4.



5. 
$$A_{ff} = \frac{\Phi}{B}$$

$$= \frac{840 \mu W}{0.7 T}$$

$$= 1.2 * 10^{-3} m^2$$

6. 
$$A_{act} = \frac{A_{ff}}{K_{stack}}$$

$$= \frac{1.2 * 10^{-3}}{0.97}$$

$$= 1.24e-3 \text{ m}^2$$

$$\begin{aligned} \text{Depth} &= \frac{A_{act}}{\text{Width of core center}} \\ &= \frac{1.24e-3}{19e-3} \\ &= 65\text{mm} \end{aligned}$$

$$\begin{aligned} 7. \text{ Number of laminations} &= \frac{\text{Depth}}{\text{Thickness of sheet}} \\ &= \frac{65\text{mm}}{0.5\text{mm}} \\ &= 130\text{mm} \end{aligned}$$

$$\begin{aligned} 8. \quad 0.5\Phi_{tot} &= \frac{2Ni}{R} \\ &= \frac{2(410*5)}{840\mu W} \\ &= 4.90*10^6 \end{aligned}$$

$$R_L = R_c + R_g$$

$$\begin{aligned} 4.90*10^6 &= \frac{lc}{\mu_g Ag} + \frac{lc}{\mu_c Ac} \\ &= 3098 + 3.46 * 10^9 lg \\ &= 1.42\text{mm} \end{aligned}$$

While so far the determined parameters are fairly reasonable, the depth of the inductor is concluded to be suboptimal as it will need more lamina hence cost more, consequently, the analytic method is used for the optimized result.

## Method 2 (Analytic Solution)

This method involves using pre-calculated parameters (these are fixed because they constitute the required specifications) and varying – in theoretically reasoned manner- the other parameters to achieve a circuit which operates stably with the given specifications.

The fill factor and number of turns are fixed at the previous values.

$$1. \quad K = 0.245$$

$$\begin{aligned} 2. \quad N &= \frac{A_{window} * K_{fill}}{\pi * (R_{wire})^2} \\ &= \frac{(28*10e-3 * 9.5e-3) * 0.245}{\pi (0.225e-3)^2} \\ &= 410 \text{ Turns} \end{aligned}$$

On the other hand, the length of the gap is chosen to be small enough to ensure minimal fringing, while still providing the needed reluctance for a stable operation at desired specifications. It is worth noting that for each trial of the length, the corresponding reluctance is calculated and assessed whether it pushes the circuit closer to the desired functionality or not.

$$3. \quad \text{After a few permutations, a gap length of } 0.35\text{mm} \text{ is chosen, and the same approach was used to get to an optimal thickness of } 12.1\text{mm}. \text{ The operating reluctance is given by:}$$

$$\begin{aligned} R &= \frac{0.35e-e}{4\pi e-7 * (12.1e-3 * 19e-3)} \\ &= 2.44e6 \end{aligned}$$

4. In this scenario the operating flux becomes

$$\begin{aligned}\Phi_{\text{tot}} &= \frac{ni}{R} \\ &= \frac{410 \times 5}{2.44e6} \\ &= 8.40e-4\end{aligned}$$

$$\begin{aligned}B &= \frac{\Phi_{\text{tot}}}{A} \\ &= \frac{8.40e-4}{(12.1e-3 \times 19e-3)} \\ &= 3.65T\end{aligned}$$

5.  $A_{\text{ff}} = 12.1e-3 \times 19e-3$   
 $= 2.3e-4\text{m}$

6.  $A_{\text{act}} = \frac{2.3e-4}{0.97}$   
 $= 2.37e-4\text{m}^2$

$$\begin{aligned}\text{Depth} &= \frac{A_{\text{act}}}{\text{Width of center}} \\ &= \frac{2.37}{19e-3} \\ &= 12.5\text{mm}\end{aligned}$$

7. Number of laminations =  $\frac{\text{Depth}}{\text{Thickness of sheet}}$   
 $= \frac{12.5\text{mm}}{0.5\text{mm}}$   
 $= 25$

8. Length of gap = 0.35mm (predetermined by trials)

9. A.  $L_{\text{wire}} = 2\text{Width} + 2\text{Depth}$   
 $= 2 \times 19e-3 + 2 \times 12.5e-3$   
 $= 0.063$

B.  $N_T = \frac{28e-3}{0.45e-3}$   
 $= 63 \text{ Turns}$

C.  $N_L = \frac{410}{63}$   
 $= 7 \text{ Layers}$

D. By arithmetic progression

$$L_n = L_0 + (n-1) \times (4d): L_n \Rightarrow \text{Length of } n \text{ layers}$$

$d \Rightarrow \text{Diameter of wire}$

$L_0 \Rightarrow \text{Length of first layer}$

$$L_n = 0.063 + 1.8(n-1)$$

The total length is given by the sum of the n layers thus;

$$S_n = 0.5d*(L_0 + L_n)$$

$$= 0.5*7*(0.063+0.073)$$

$$= 0.476\text{m wire is needed}$$

$$IL (\text{Inductance}) = \frac{n^2}{R}$$

$$= \frac{410^2}{2.44\text{e}6}$$

$$= 0.0688\text{H}$$

## Simulation

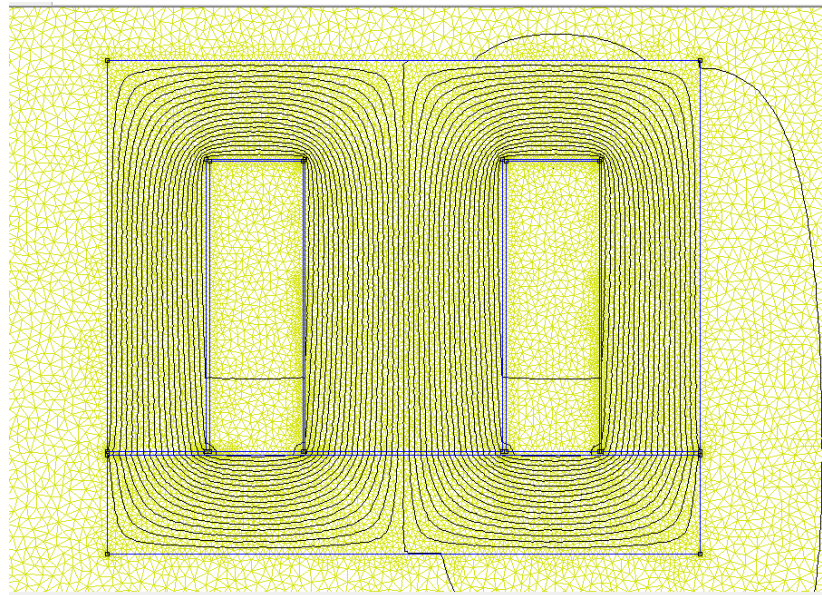


Figure 1: Simulation

Figure 1 above is a planar model of the inductor enclosed in a Dirichlet boundary. It shows that most of the flux flows through the core. However, a small portion of the flux leaks outside the core to air and the copper coil.

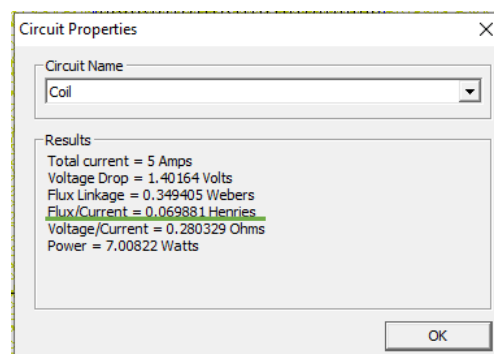


Figure 2: Simulated Inductance

Figure 2 above shows that the simulated inductance (Flux/current) =0.0698H, this is in agreement with the analytical solution to within 1.4% margin of error.

#### Finite Element Inductance

The inductance was also calculated from energy for confirmation:

$W = 0.5 \int B \cdot H dv$  which, from simulation, is 0.84497J as seen in figure 3 below.

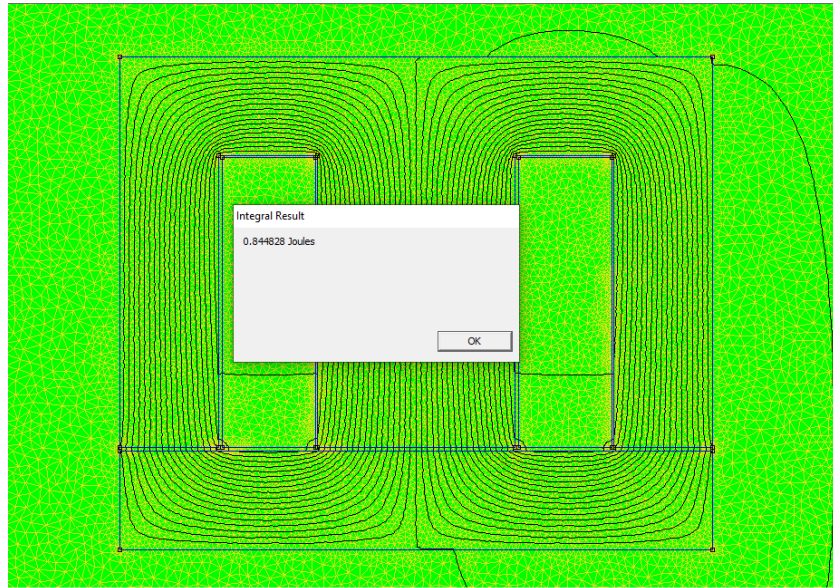


Figure 3: Energy Integral

$$\begin{aligned} L &= \frac{2W}{i^2} \\ &= \frac{2(0.8448)}{5^2} \\ &= 0.067585\text{H} \end{aligned}$$

This value also ties with the previously calculated values to within 3% margin of error.

Table 1: Inductances vs Current

I (A)	$L_{\text{calculated}}$ (H)	$L_{\text{femm}}$ (H)
0.30	0.0688	0.0721
0.35	0.0688	0.0713
0.40	0.0688	0.0707
0.45	0.0688	0.0703
0.50	0.0688	0.0699
0.55	0.0688	0.0696
0.60	0.0688	0.0693
0.65	0.0688	0.0691
0.70	0.0688	0.0689

Table one shows the variation of inductance with changing current, the theoretically calculated inductance shows no change in response to current. This is the case because the equation used assumes an ideal situation in which inductance varies only with reluctance and the number of turns, alternatively, reactance. Also, an assumption that  $\mu$  is constant is inherent to the equation. This goes to prove that the analytical solution is much more accurate and should therefore be the method employed.

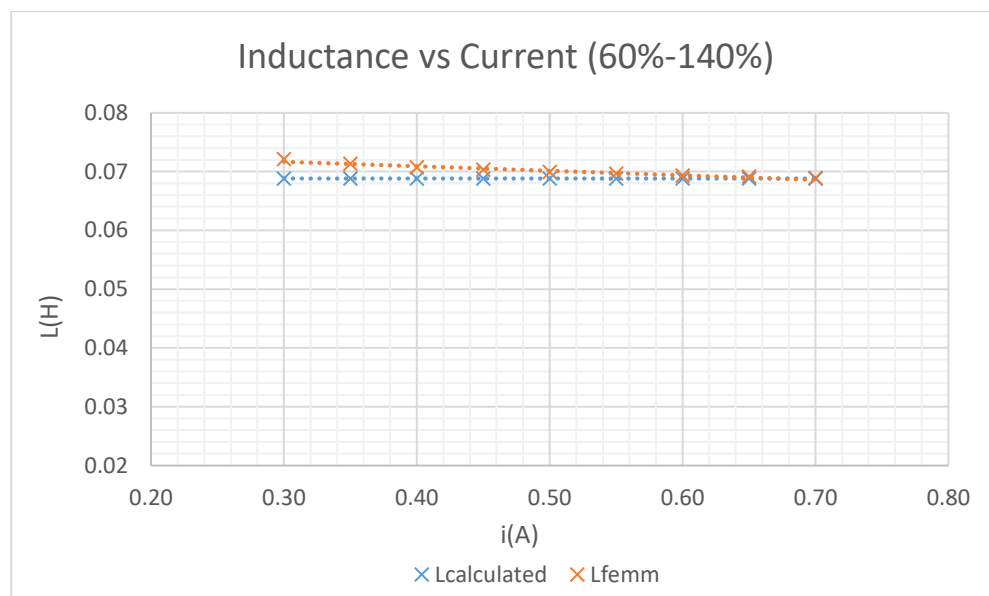


Figure 4: Inductance plots

Figure 4 is a plot of Inductance vs current for both the simulation and the theoretically determined inductances and the inductances simulated using femm. The plots summaries the trend of the data in table 1. They corroborate that the theoretically evaluated inductance is independent of the current, which is idealised to be constant in the equation. However, the simulation shows that in practice, the inductance of real material reduces with increase in current, while the rate of decrease is small, ways to mitigate this decrease are necessary, and those include altering the net permeability of the core ( by varying the gap length for instance).



## Conclusion

In conclusion, the resultant parameters from the analytical method were implemented in the design, they are more accurate because they accommodate physical factors that alter the fictional behaviours of the circuit, these include saturation, fringing, flux leakage etc. which the theoretical approach fails to take into account. With this chosen approach, the required specifications were achieved.

## Specification Summary

- $I = 5A$
- $X_L = 5\Omega$
- $IL = 0.689H$