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Power Electronics

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Part One

$$V_m \sin(\omega t) = Ri(t) + L\frac{di(t)}{dt}$$

$$i(t) = i_f(t) + i_n(t)$$

$$i_f(t) = \frac{V_m}{Z}\sin(\omega t - \theta)$$

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$Ri(t) + L\frac{di(t)}{dt} = 0$$

$$i_n(t) = A^{-t/\tau}$$

$$i(t) = \frac{V_m}{Z}\sin(\omega t - \theta) + Ae^{-t/\tau}$$

$$i(0) = \frac{V_m}{Z}\sin(0 - \theta) + Ae^0 = 0$$

$$\Rightarrow A = -\frac{V_m}{Z}\ sin(\theta)$$

$$i(t) = \frac{V_m}{Z}\left[\sin(\omega t - \theta) + \sin(\theta)\, e^{\frac{-t}{\tau}}\right]$$

$$i(t) = \frac{35.35}{80.2952}[\sin(100\pi\ t - 0.8986) + \sin\left(0.8986\right)e^{-250t}]$$

b)

$$I_0 = \frac{1}{2\pi} \int_0^\beta i(\omega t)\, d(\omega t)$$

$$I_0 = \frac{1}{2\pi} \int_0^\beta \frac{V_m}{Z} \left[\sin(\beta - \theta) + \sin(\theta)\, e^{-\beta/\omega\tau}\right] d(\omega t)$$

$$I_0 = \frac{35.35}{2\pi(80.2952)} \int_0^{4.0709} \left[\sin(\beta - 0 \cdot 8986) + \sin(0.8986)\, e^{-\beta/1.2566}\right] d\beta$$

$$I_0 = 179.9mA$$

c)RMS Current

$$I_{rms} = \sqrt{\frac{1}{2\pi} \int_0^\beta i^2(\omega t)\, d(\omega t)}$$

$$I_{rms} = \sqrt{\frac{1}{2\pi} \int_0^\beta \left(\frac{V_m}{Z}\right)^2 \left[\sin(\beta - \theta) + \sin(\theta)\, e^{-\beta/\omega\tau}\right]^2 d(\omega t)}$$

$$I_{rms} = \sqrt{\frac{1}{2\pi} \int_0^{4.0907} \left(\frac{0.35}{80.2952}\right)^2 [\sin(\beta - 0.8986) + \sin(0.8986)\, e^{-\beta/(\omega\tau)}]^2 d(\omega t)}$$

$$I_{rms} = 240mA$$

d) Power Dissipated by Load

$$P_L = Irms^2 R$$
$$P_L = \left(240 \times 10^{-3}\right)^2 \times 50$$
$$P_L = 2.88W$$

e) Power Factor

$$PF = \cos(tan^{-1}\left(\frac{\omega L}{R}\right))$$

$$PF = \cos(tan^{-1}\left(\frac{100\pi \times 0.2}{50}\right))$$

$$PF = 0.623$$

Part Two

1. Waveforms

a)

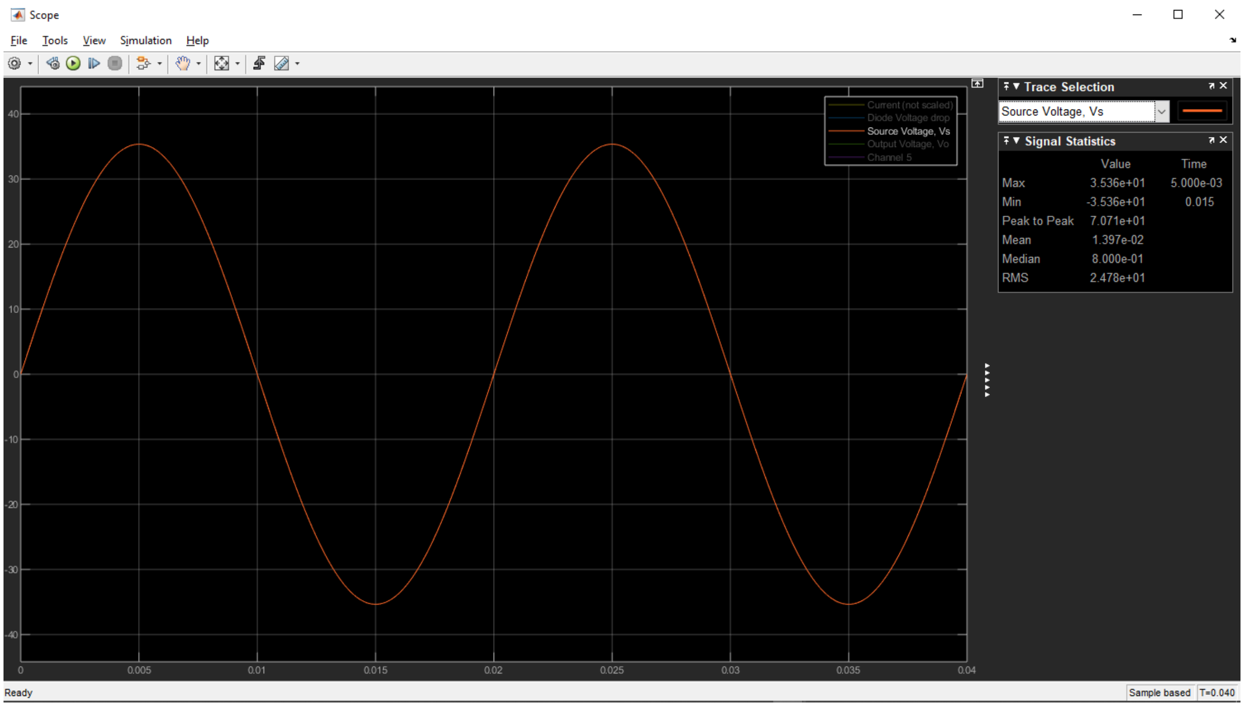

Figure 1: Source Simulation Voltage

b)

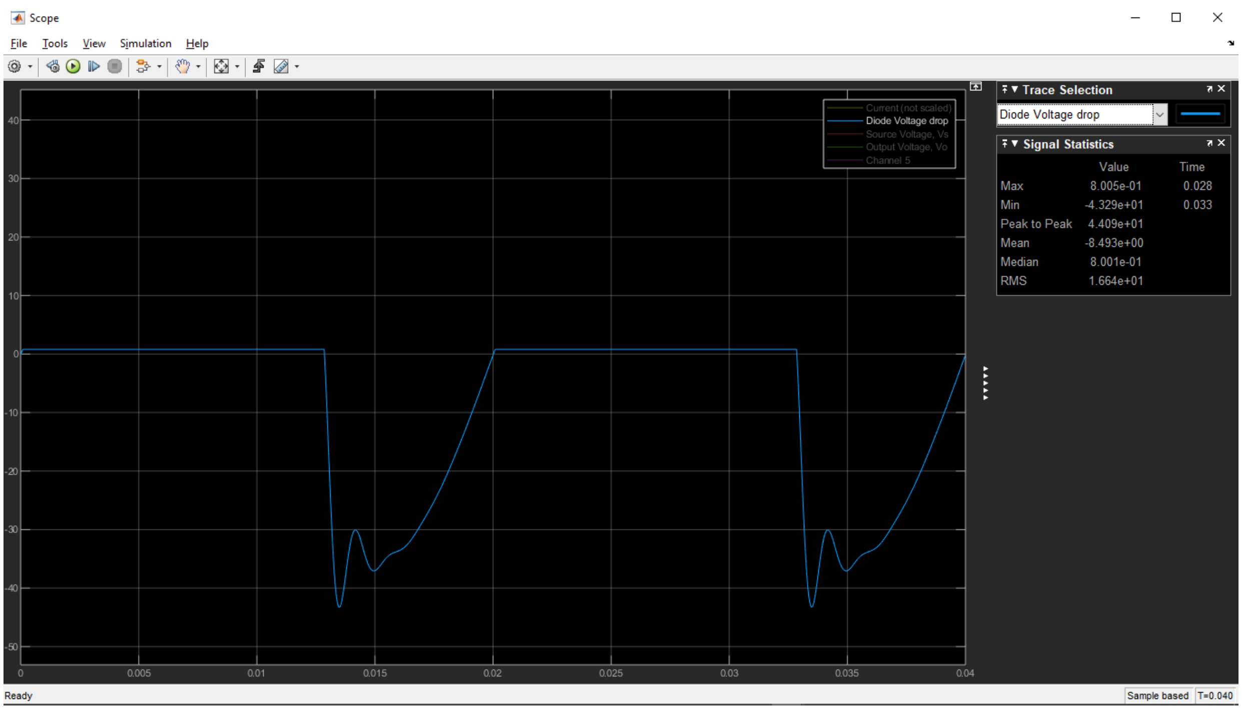

Figure 2: Diode Drop Simulation

c)

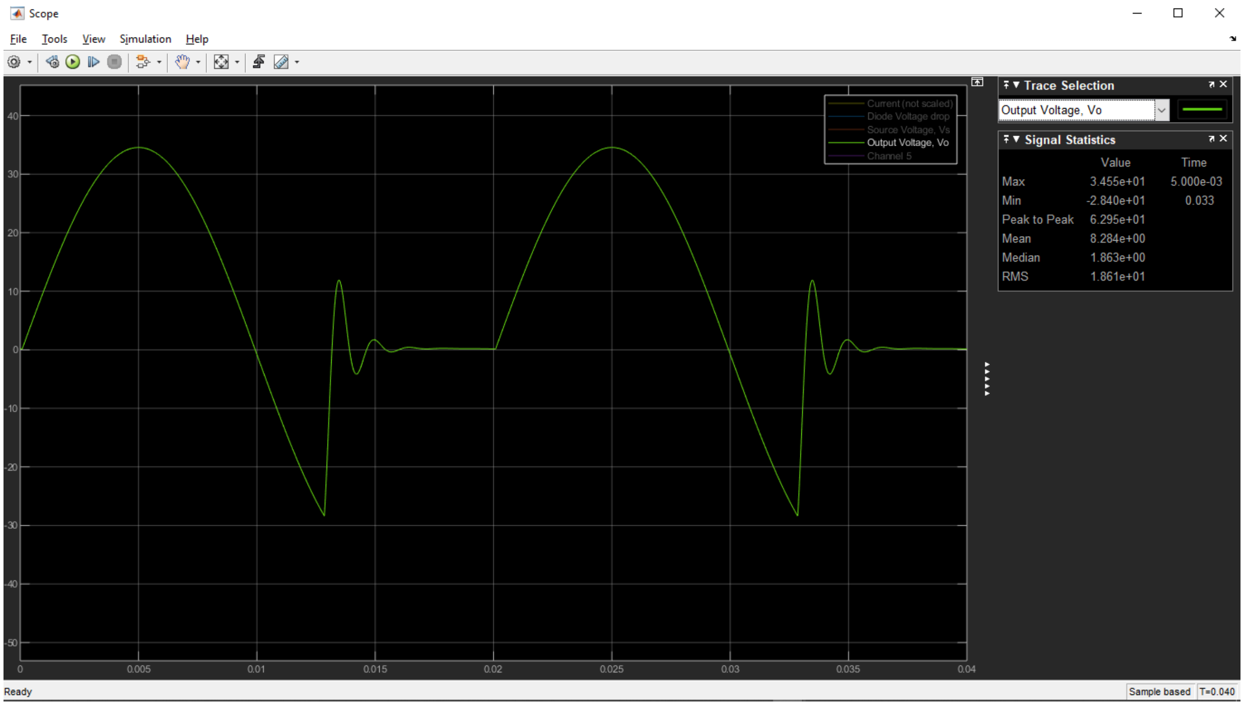

Figure 3: Output Voltage Simulation

d)

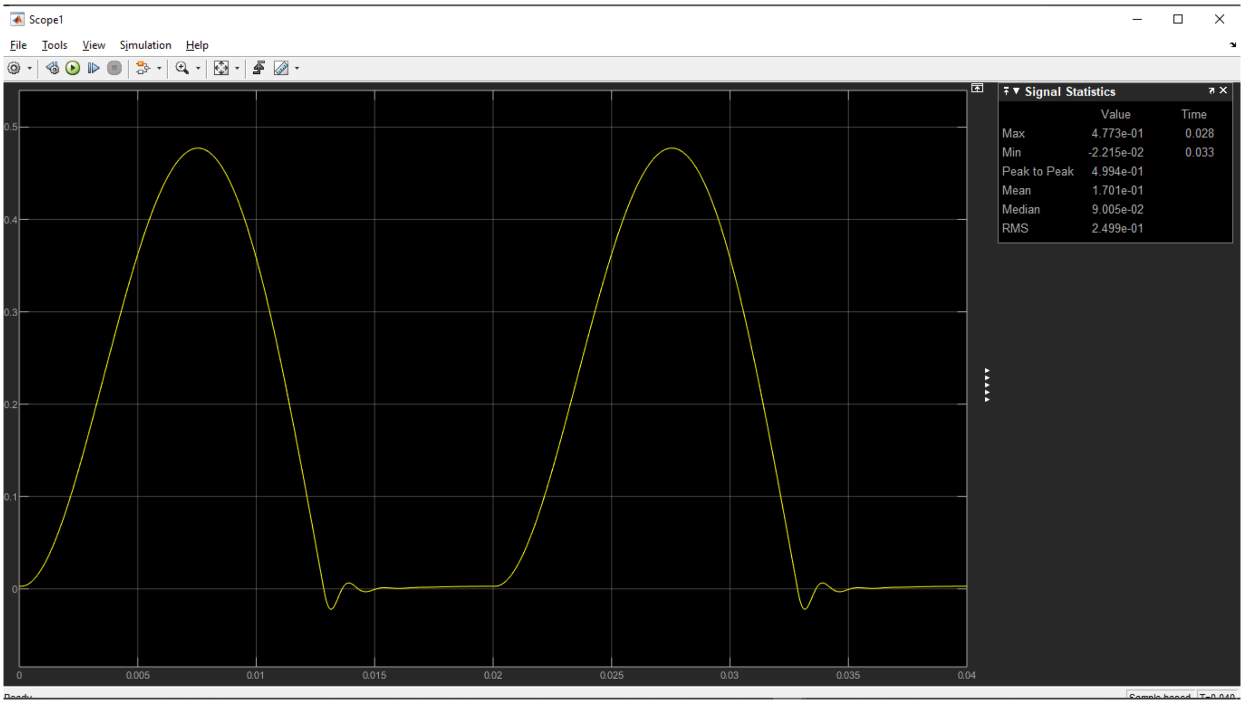

Figure 4: Load Current Simulation

Determining B from the simulations:

Looking at the voltage waveform:
T(+) = 0.033s – 0.02s
= 0.013s

Therefore β = * 2 4.0841 radians
= 4.08 rad

2. Performance Characteristics

a) Extinction Angle

$$\tau = \frac{200 \times 10^{-3}}{50} \qquad \omega = 100\pi$$

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$
$$\theta = \tan^{-1}\left(\frac{2\pi \times 50 \times 200 \times 10^{-3}}{50}\right)$$
$$\theta = 0.898637\ rads$$

$$f(\beta) = \sin(\beta - \theta) + \sin(\theta)\, e^{-\beta/\omega\tau}$$

<u>Newtons' Approximation Algorithm</u>

$$\beta_{n+1} = \beta_n - \frac{f(\beta_n)}{f'(\beta_n)}$$

$$\beta_1 = 4$$
$$\beta_2 \approx 4 - \frac{f(4)}{f'(4)}$$
$$\beta_2 \approx 4 - \frac{0.07036}{-0.67946}$$
$$\beta_2 \approx 4.107$$

Since f(β) = 0, the value of β can be found using Newton's approximation above. By increasing the number of iterations of the algorithm, the result more accurately approximates β. In this case, the number of iterations is chosen so that the result is accurate to within 0.0001% of the actual value

The initial value of β_1 was set to 4, $f'(\beta_n)$ refers to the nth derivative of f at β_n. The result is as follows:

β= 4.0709 radians

β = 233.25°

The python code used for this iterative process is attached in the appendix

The simulated β value differs from the one calculated in part one by a negligible margin.

b) Average Current

$$I_0 = \frac{1}{2\pi}\int_0^{\beta} i\left(\omega t\right) d\left(\omega t\right)$$

$$I_0 = \frac{1}{2\pi}\int_0^{\beta} \frac{V_m}{Z}\left[\sin\left(\beta-\theta\right)+\sin\left(\theta\right)e^{-\beta/\omega\tau}\right] d\left(\omega t\right)$$

$$I_0 = \frac{35.35}{2\pi\left(80.2952\right)}\int_0^{4.0709} \left[\sin\left(\beta-0\cdot 8986\right)+\sin\left(0.8986\right)e^{-\beta/1.2566}\right] d\beta$$

$$I_0 = 179.9mA$$

The load current in the simulations differs from the calculated load current by micro amperes because the calculated one assumes an ideal situation in which there are no attenuations in the signal caused by external factors. So due to factors such as other resistive circuit components other than the defined ones, the current signal waveform appears to have been attenuated a little bit.

c)

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\beta} i^2\left(\omega t\right) d\left(\omega t\right)}$$

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\beta} \left(\frac{V_m}{Z}\right)^2 \left[\sin\left(\beta-\theta\right)+\sin\left(\theta\right)e^{-\beta/\omega\tau}\right]^2 d\left(\omega t\right)}$$

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{4.0907} \left(\frac{0.35}{80.2952}\right)^2 [\sin\left(\beta-0.8986\right)+\sin\left(0.8986\right)e^{-\beta/(\omega\tau)}]^2 d\left(\omega t\right)}$$

$$I_{rms} = 240mA$$

The simulated rms current was found to differ from the calculated value by a very negligible amount that may be ignored.

3.

Diodes have a finite reverse recovery time, as such, if the operating frequency is too high, the diode will conduct momentarily in the reverse bias before it is reverse biased.

4. Waveforms with freewheeling diode

a) The source Voltage, Vs

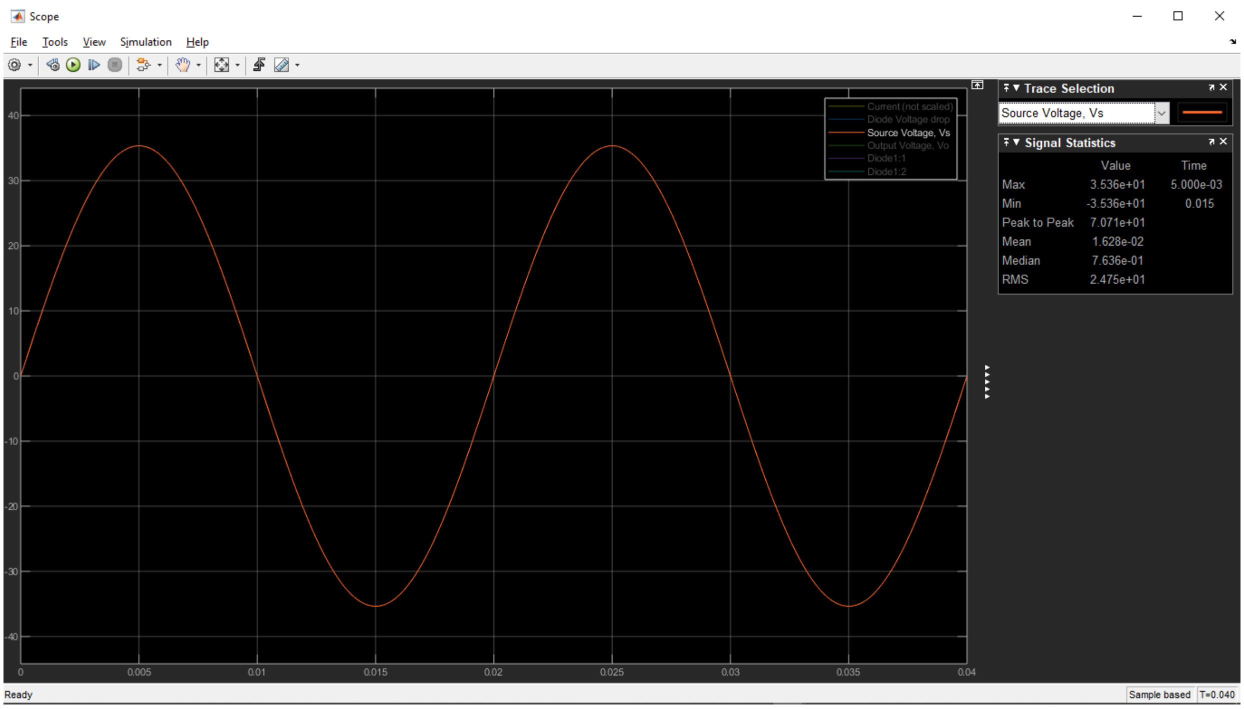

Figure 5

The source Voltage in the simulation varies very slightly from the calculated value,The value by which the two voltages vary is negligible to an accuracy of nanoVolts.

b) The diode voltage, Vd

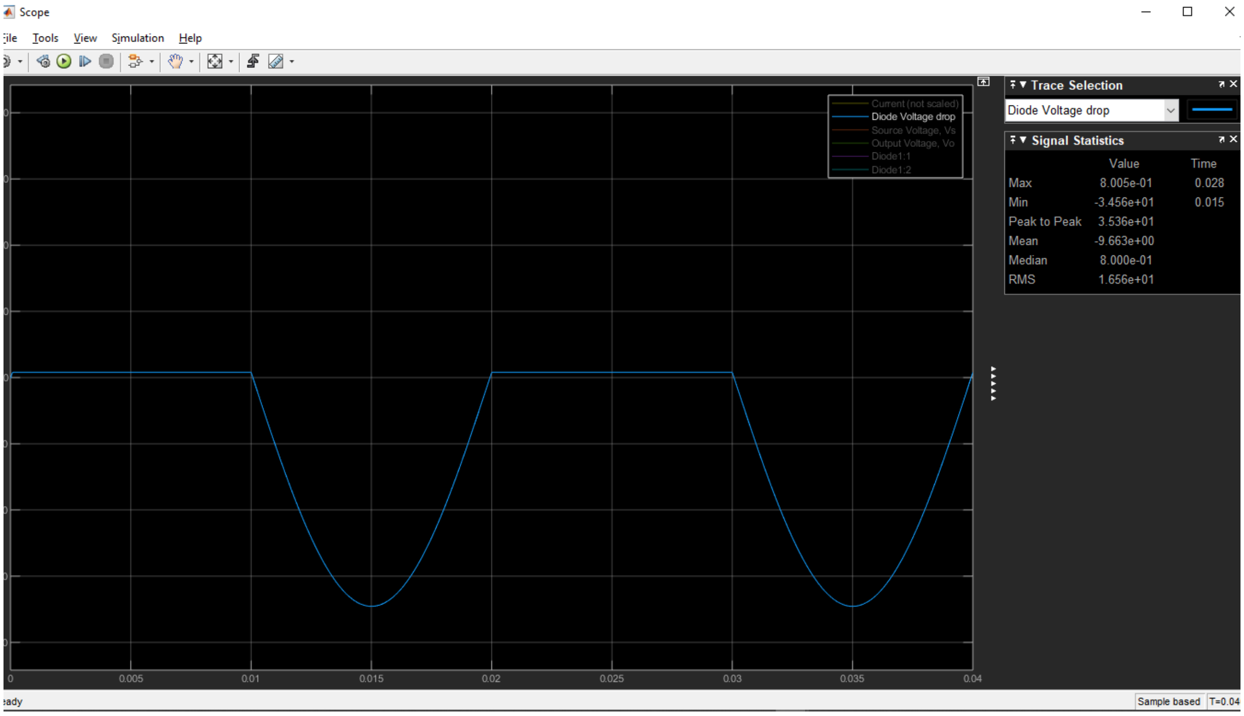

Figure 6:

The diode drop becomes smoother with the free wheeling diode being added.

c) Output Voltage, Vo

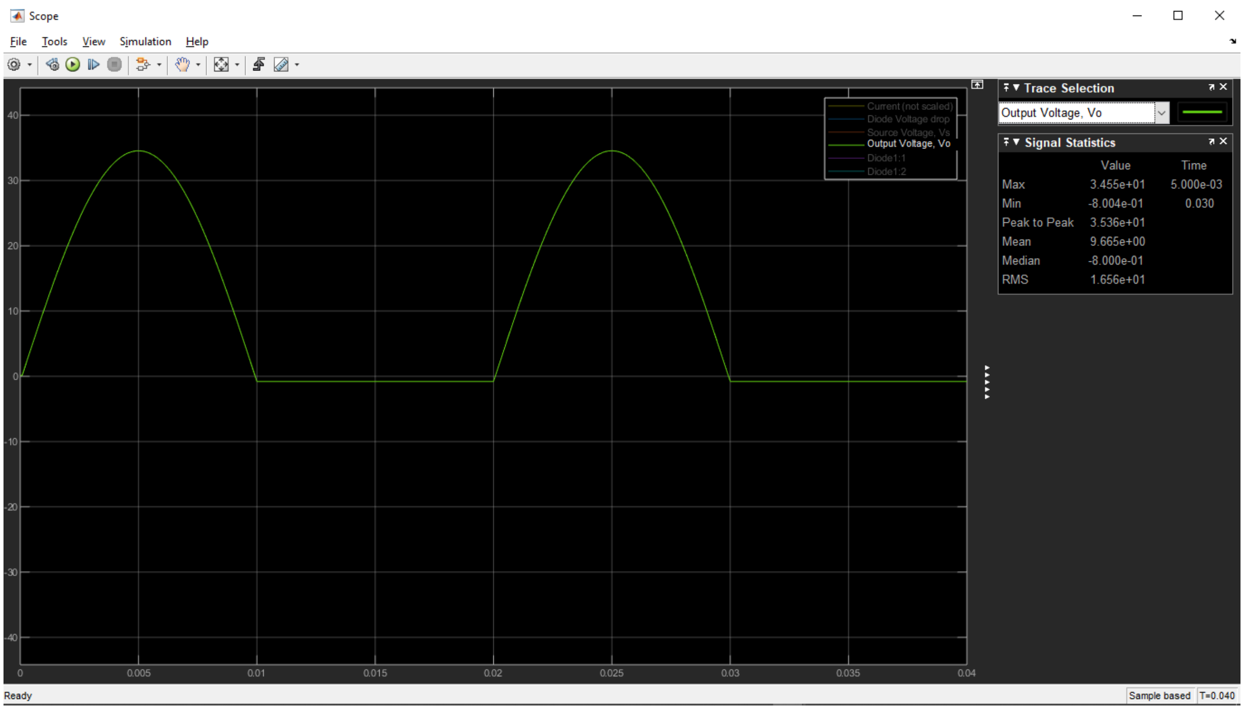

Figure 7

In the positive half cycle the inductor charge is the same as the polarity of the output dc voltage. The moment the negative half cycle starts, the inductor acts as like source, making the freewheeling diode forward bias. Thus, the output dc voltage becomes zero and the inductor current gradually decreases. So the freewheeling diode essentially blocks negative output voltage.

d) Load current, Io

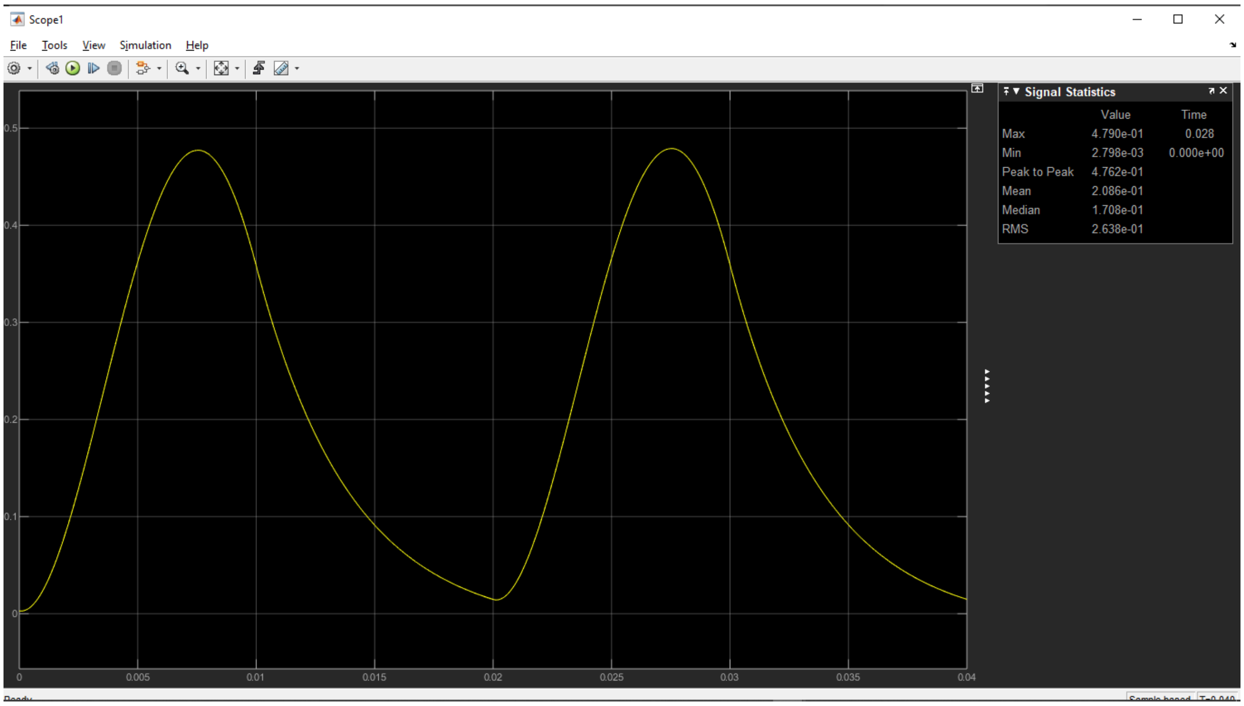

Figure 8

The load current when the free-wheeling diode has been added becomes smoother, hence improving the overall load performance.

5. In addition to adding a free-wheeling diode and an inductor, by connecting a smoothing capacitor in parallel with the loads, the load current can be made continuous.

Part 3 – Experimental Results

a) Half-wave rectifier

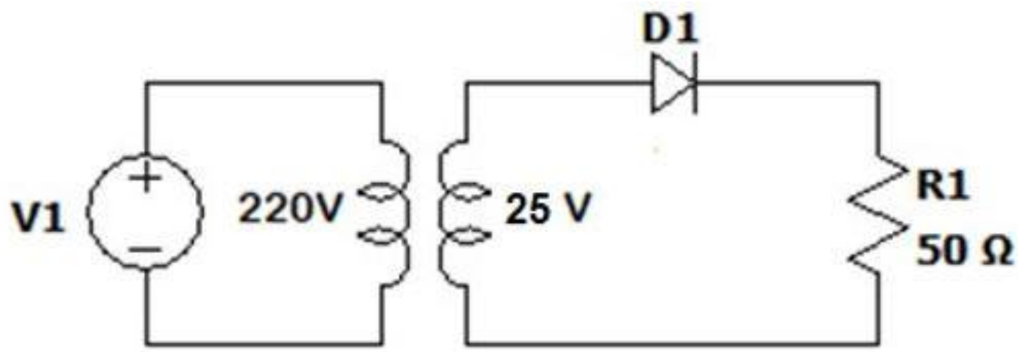

Figure 8

Load resistor (Ω)	V_{dc} (V)	I_{dc} (A)
20	9.71	0.496
50	9.87	0.158
90	8.83	0.0602

Table 1:

Diodes have a finite reverse recovery time and voltage drop that can alter the exact value of the average, hence causing differences.

b) RL Circuit

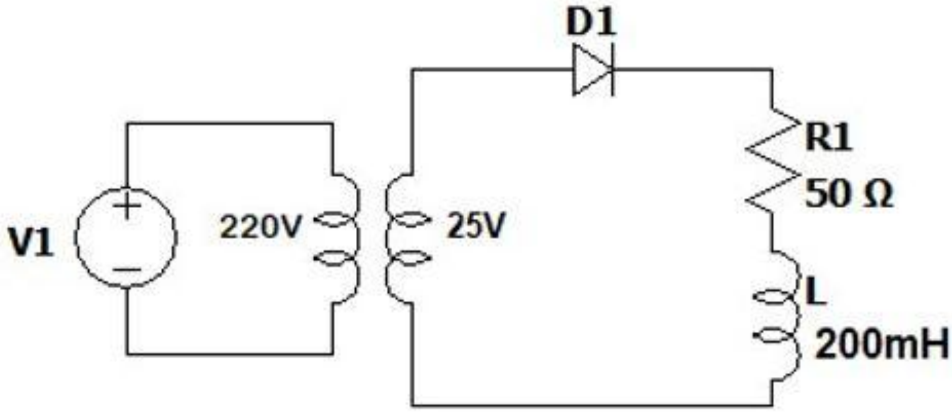

Figure 9

Inductor (mH)	V_{dc} (V)	I_{dc} (A)	V_{rms} (V)	I_{rms} (A)	β (°)
200	7.64	0.115	16.6	0.189	4.02
400	5.69	0.0765	19.0	0.137	4.40
600	4.54	0.0537	20.4	0.109	4.59

Table 2

Calculation for β:

$\beta =$ (t/20) *360° , where x is the time interval for the non zero voltage and is in milliseconds. A similar approach to Part 2 a) was used for the calculations of the above excitations values. These calculations can be found in the appendix.

Calculated values assume ideal conditions (e.g zero parasitic capacitances) which don't hold practically, and therefore leads to deviations.

c) R-L circuit with free wheeling diode

Impact of freewheeling diode across the load:

Impact on output voltage:

- Eliminates flyback spikes which can be higher than the Peak Inverse Voltage of the rectifying diode leading to damage. In essence, it prevents a negative output voltage across the load.

Impact on current:

- The current waveform becomes more smooth, improving the load performance.

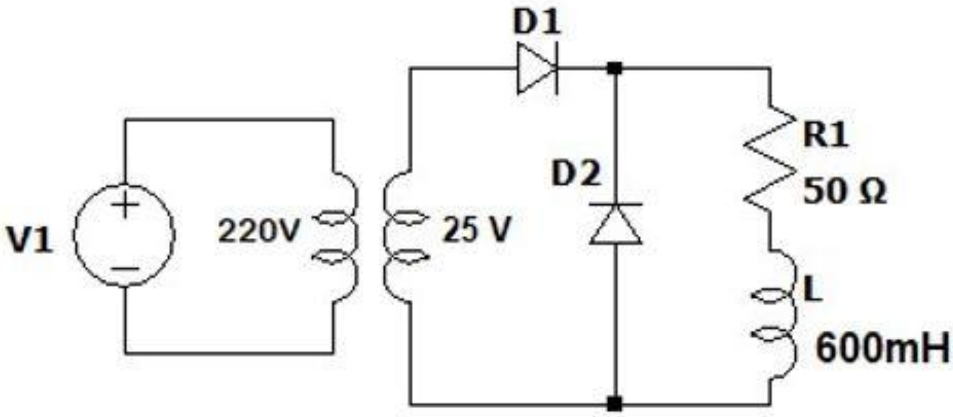

Figure 10.

Screenshot of output voltage and current waveforms

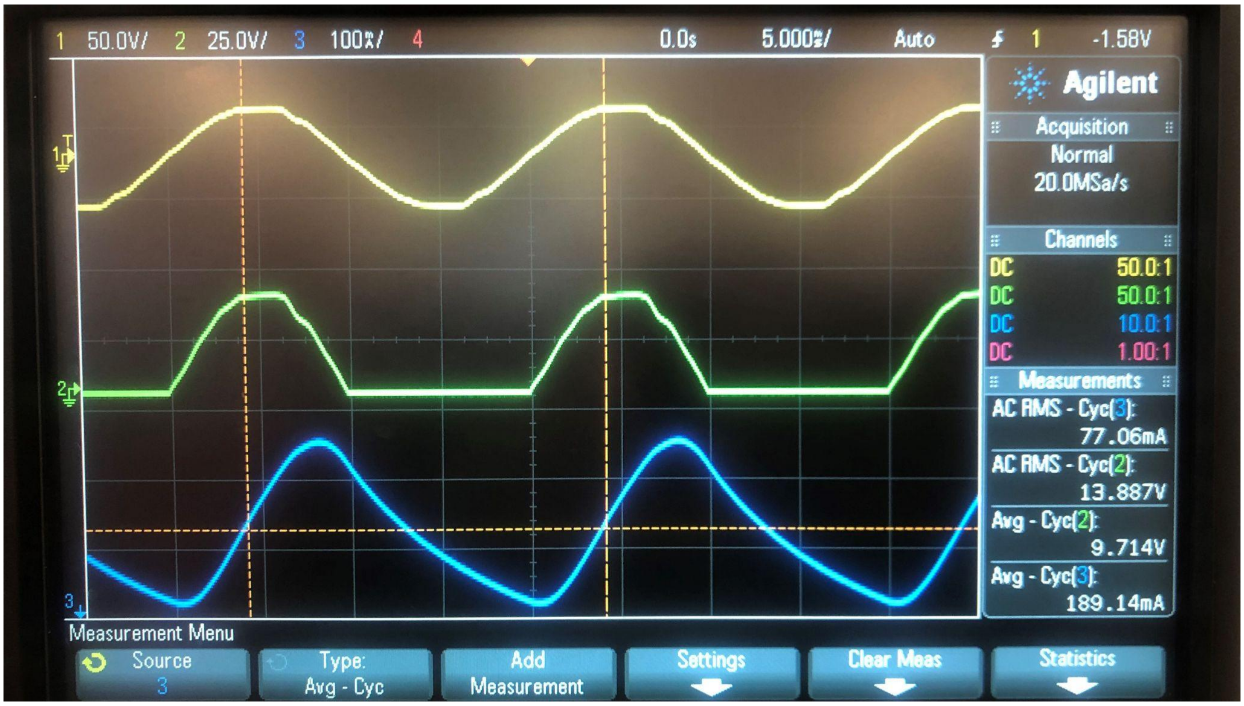

Figure 11

As seen in the above figure, the negative voltage has been clipped as required, the current is also further smoothed.

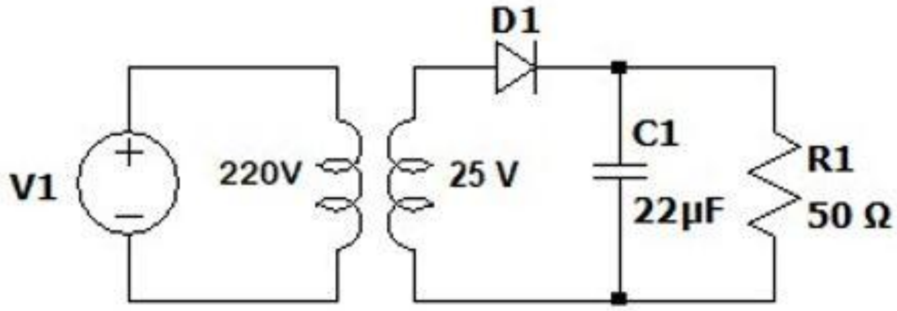

Figure 12

Capacitor (microFarad)	V_{dc} (V)
22	10.3
470	23.9

Table 3

In a practical rectifier the capacitor smooths the output which results in a waveform that more closely approximates DC voltage. The output capacitor limits the noise produced by the internal components.The capacitor (also called the reservoir capacitor) is used to smooth the output of the rectifier. It charges the moment the voltage from the rectifier goes above the capacitor voltage. When the voltage from the rectifier drops, the capacitor will provide the required current(charge). Therefore, the voltage varies less than in the absence of the capacitor

Appendix

14:30 Thu 17 Jun

Untitled - Jupyter Notebook

In [14]:

```python
from scipy import misc
import math

def NewtonsMethod(f, x, tolerance=0.000001):
    while True:
        x1 = x - f(x) / misc.derivative(f, x)
        t = abs(x1 - x)
        if t < tolerance:
            break
        x = x1
    return x

def f(x):
    return math.sin(x-0.89863709)+math.sin(0.89863709)*math.exp(-x/1.25663706)

x = 4

β = NewtonsMethod(f, x)

print('x: ', x)
print('β: ', x0)
print("f(β) = ", math.sin(x-0.89863709)+math.sin(0.89863709)*math.exp(-x/1.25663706))
```

```
x:  4
β:  4.070894771195649
f(β) =  0.0726581134159591
```

In [ ]:

Excitation Angle Calculations (β).

- $Z = \sqrt{R^2 + (L\omega)^2}$
- $\theta = \frac{L\omega}{R}$

$= \tan^{-1}\left(\frac{L\omega}{R}\right)$

- $\omega\tau = \omega \cdot \frac{L}{R}$
- $\frac{V_m}{Z}\left[\sin(\beta - \theta) + \sin(\theta)\, e^{\frac{-\beta}{\omega\tau}}\right] = 0$

200 mH :

- $Z = \sqrt{50^2 + (200\times10^{-3} \times 50Hz \times 2\pi)^2} = 80{,}29\Omega$
- $\theta = \frac{200\times10^{-3} \times 2\pi \times 50Hz}{50}$

$= \tan^{-1}(\theta)$

$= 0{,}898$ radians.

- $\omega\tau = 2\pi\,(50) \times \frac{200\times10^{-3}}{50}$

$= 1{,}2566$

$\therefore \sin(\beta - 0{,}898) + \sin(0{,}898)\, e^{\frac{-\beta}{1{,}2566}} = 0$

$\therefore \beta = 4{,}071$ rad / $233{,}25°$

400 mH :

$\sin(\beta - 1{,}192) + \sin(1{,}192) \cdot e^{\frac{-\beta}{2{,}51}} = 0$

$\therefore \beta = 4{,}49$ radians / $257{,}26°$

600 mH :

$\sin(\beta - 1{,}312) + \sin(1{,}312) \cdot e^{\frac{-\beta}{3{,}77}} = 0$

$\therefore \beta = 4{,}732$ radians / $271{,}12°$