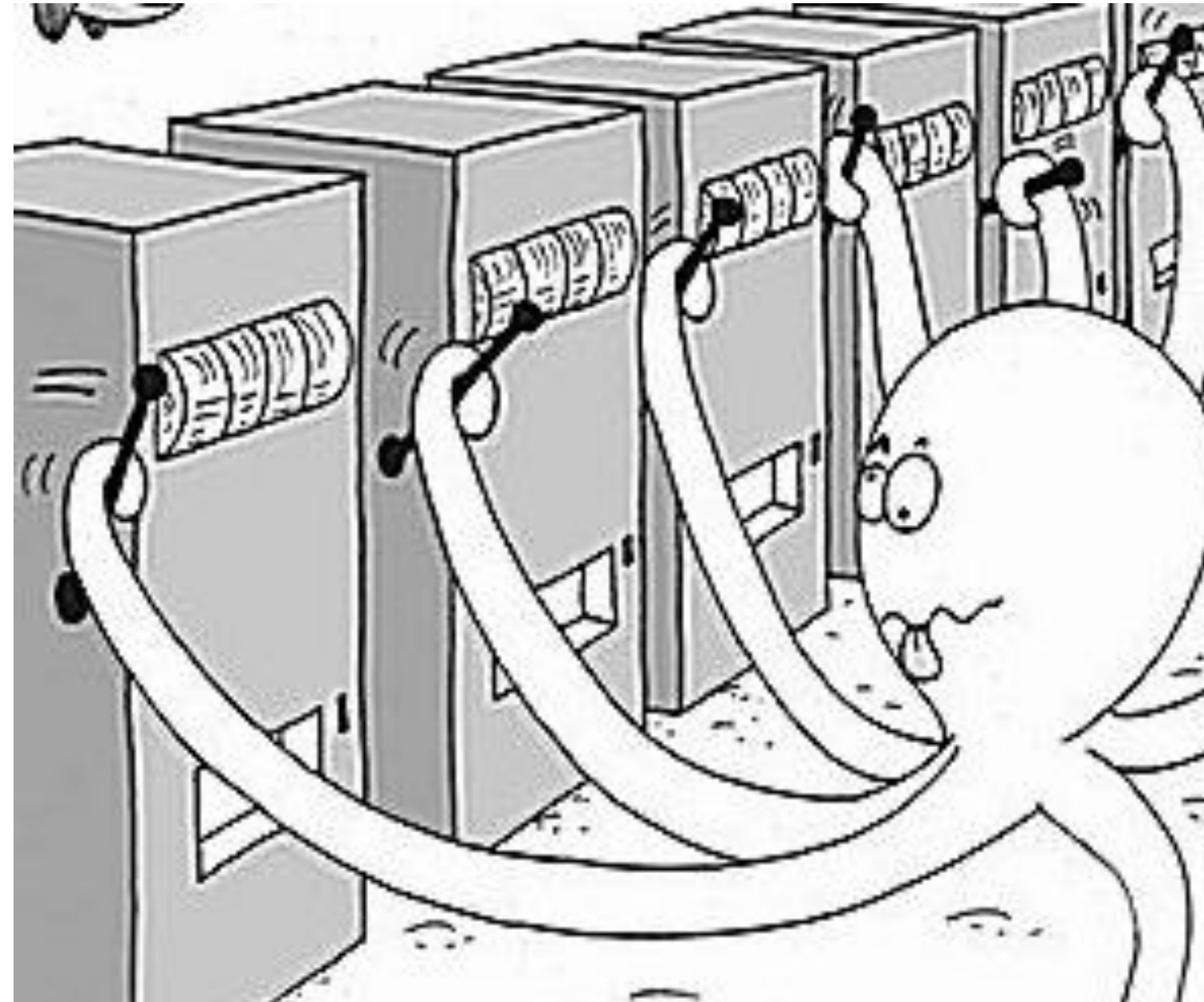


# Work Session: seeing through the eyes of the agent

Course 1, Module 1  
Sequential Decision Making

# Reminders: Sept 8, 2021

- Schedule with deadlines on github pages ([https://docs.google.com/spreadsheets/d/1ooFqttGCklw7rsst9xwL77\\_SA84LszLvZwpWo06Ltas](https://docs.google.com/spreadsheets/d/1ooFqttGCklw7rsst9xwL77_SA84LszLvZwpWo06Ltas))
- Next practice Quiz **due Sunday**, for Course 1, Module 2 (MDPs)
- *You all are reading along right??*
- TAs have posted office hours (likely different times next week). Over zoom/meet for now
- Any questions about admin?



**Microsoft Research: <http://slivkins.com/work/bandits-svc/>**

# Demo of Bandits

- <https://www.coursera.org/learn/fundamentals-of-reinforcement-learning/ungradedWidget/44Z9R/lets-play-a-game>
- <https://www.coursera.org/learn/fundamentals-of-reinforcement-learning/ungradedWidget/jEYTO/whats-underneath>

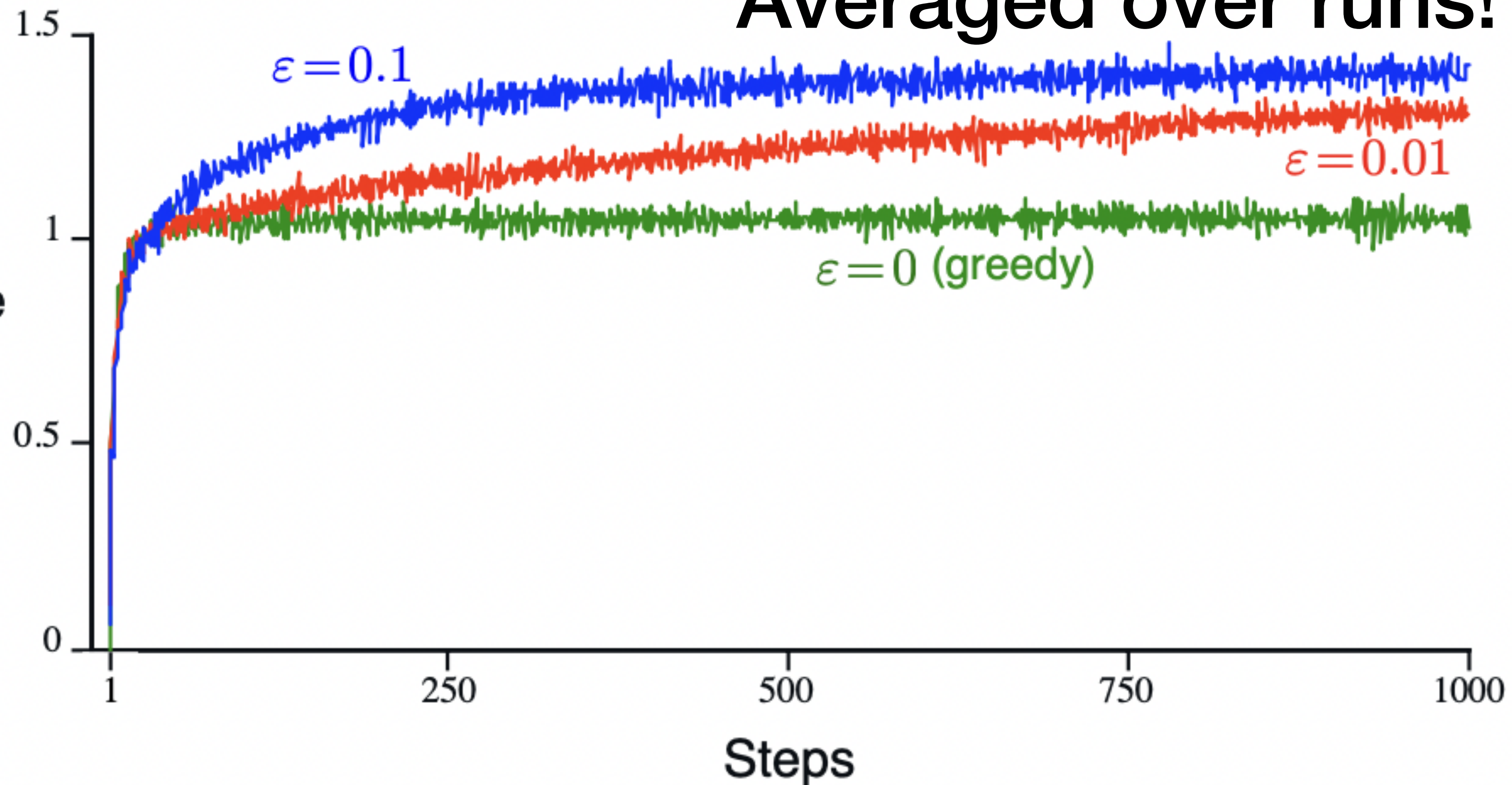
# Digging deeper into learning curves

- There were some questions about impact of epsilon
- I noticed the curves in the notebook don't match the book! Sup with that!?!
- Learning curves are super critical and we will be looking a lot at them, so comfort with them is key

# Anatomy of a learning curve

Performance  
or  
Error  
(Some go up,  
some go down)

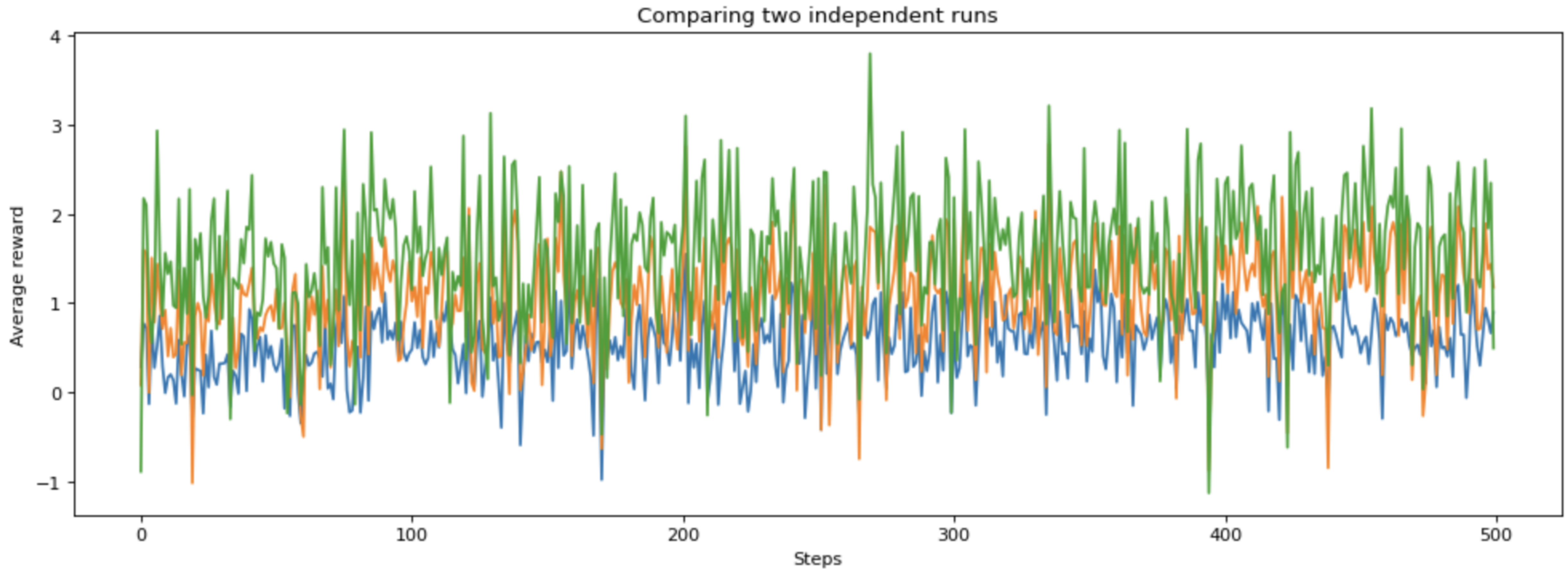
Average  
reward



<-Experiment duration->



# Individual runs of the system can be different due to randomness in rewards and the exploration

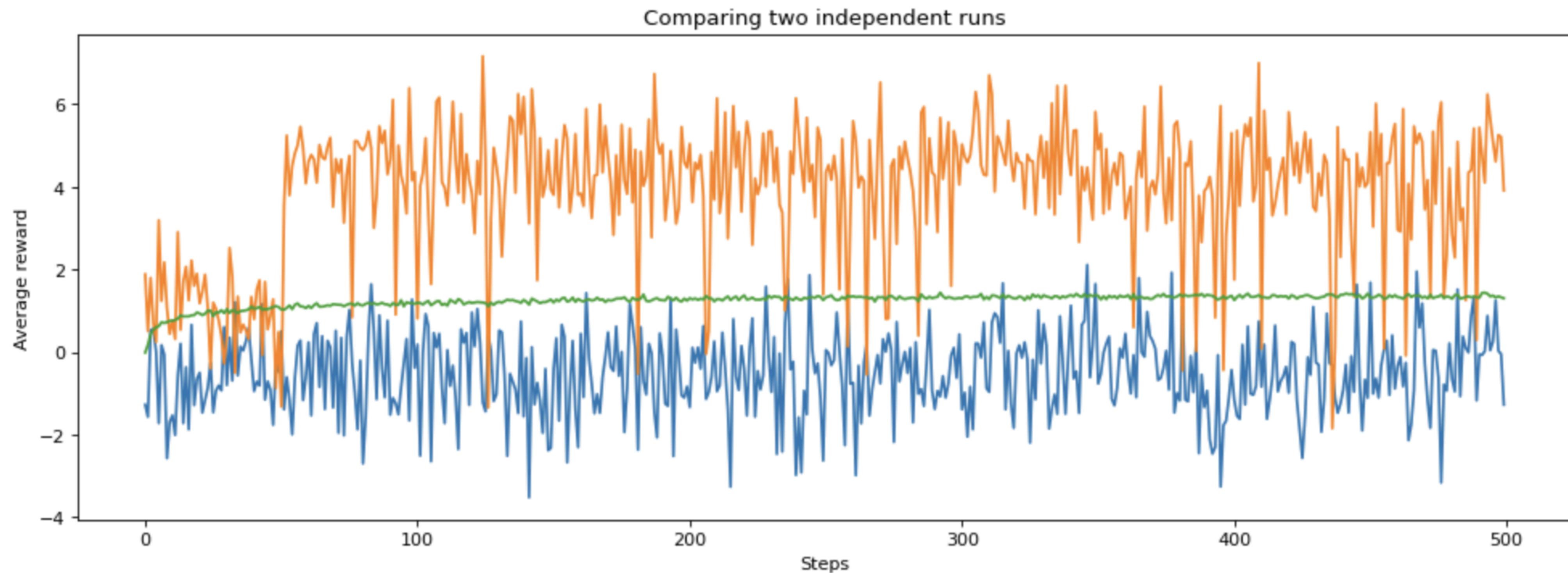


# Averaging helps

- Imagine you wanted to empirically (using experiments and data analysis) convince me that the probability of rolling 2 on a fair six sided dice was  $1/6$ ?
- If you rolled it once that won't help
- If you rolled it 6 times you might never see a '2' (**unlucky**)
- If you rolled it 6 times, maybe you observe '2' five times (**lucky**)
- Using a large number of rolls and counting the '2's you observe, eventually will give you a good answer



# Why we average

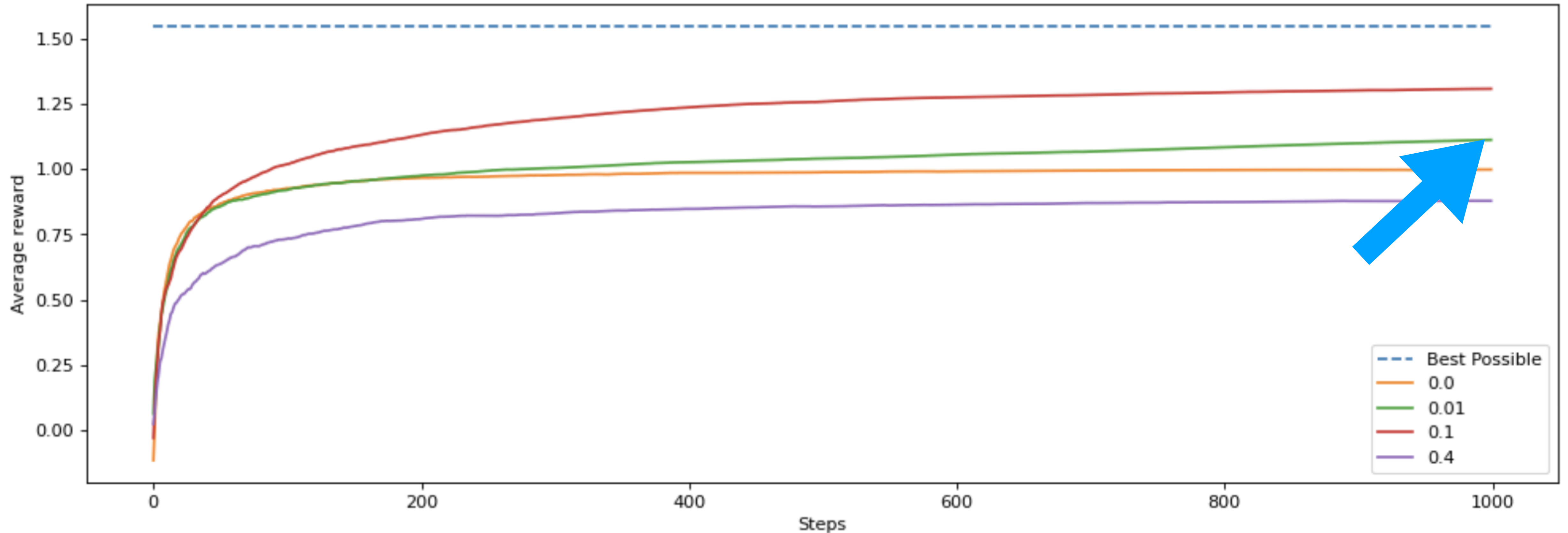


**Agent might be lucky, or unlucky!**

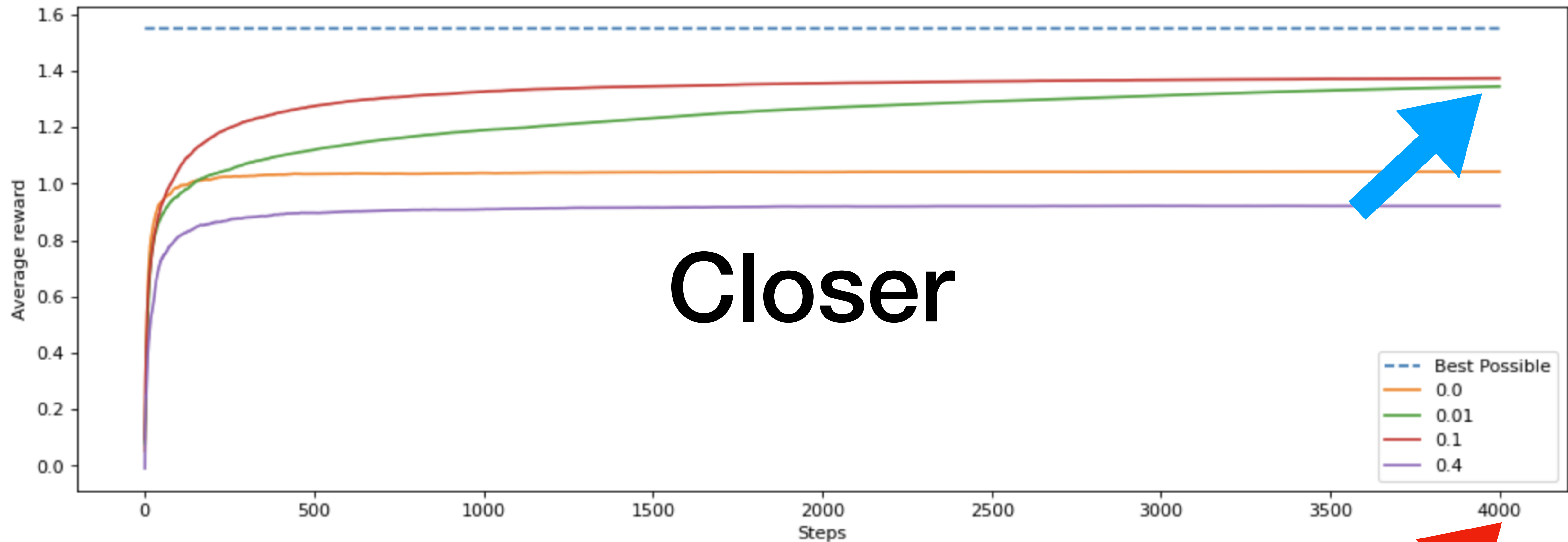
# What else might we be interested in?

- Average or mean performance is useful in comparing algorithms
  - We can make confidence intervals around the mean, conduct hypothesis tests
  - Makes for clear and precise comparisons
- But in deployment we rarely care about average performance!
  - Average success in folding your laundry
- We could also report Best Performance
- We could also report Worst Case Performance

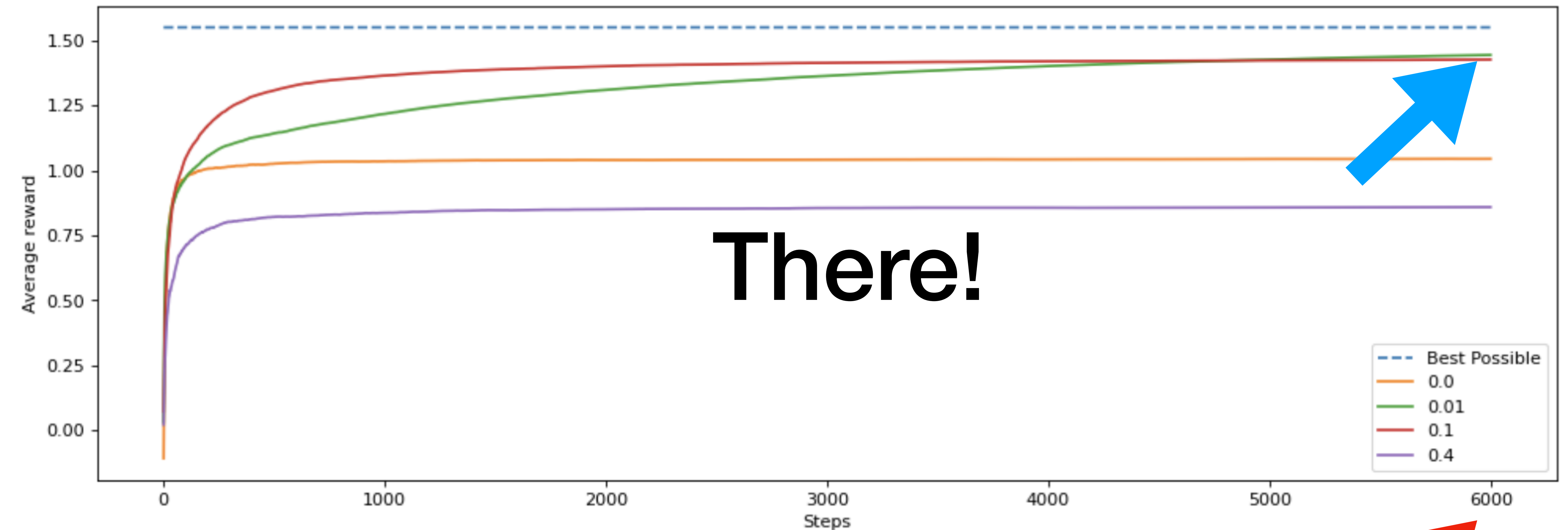
# Will small epsilon do better?



# Let the data guide you

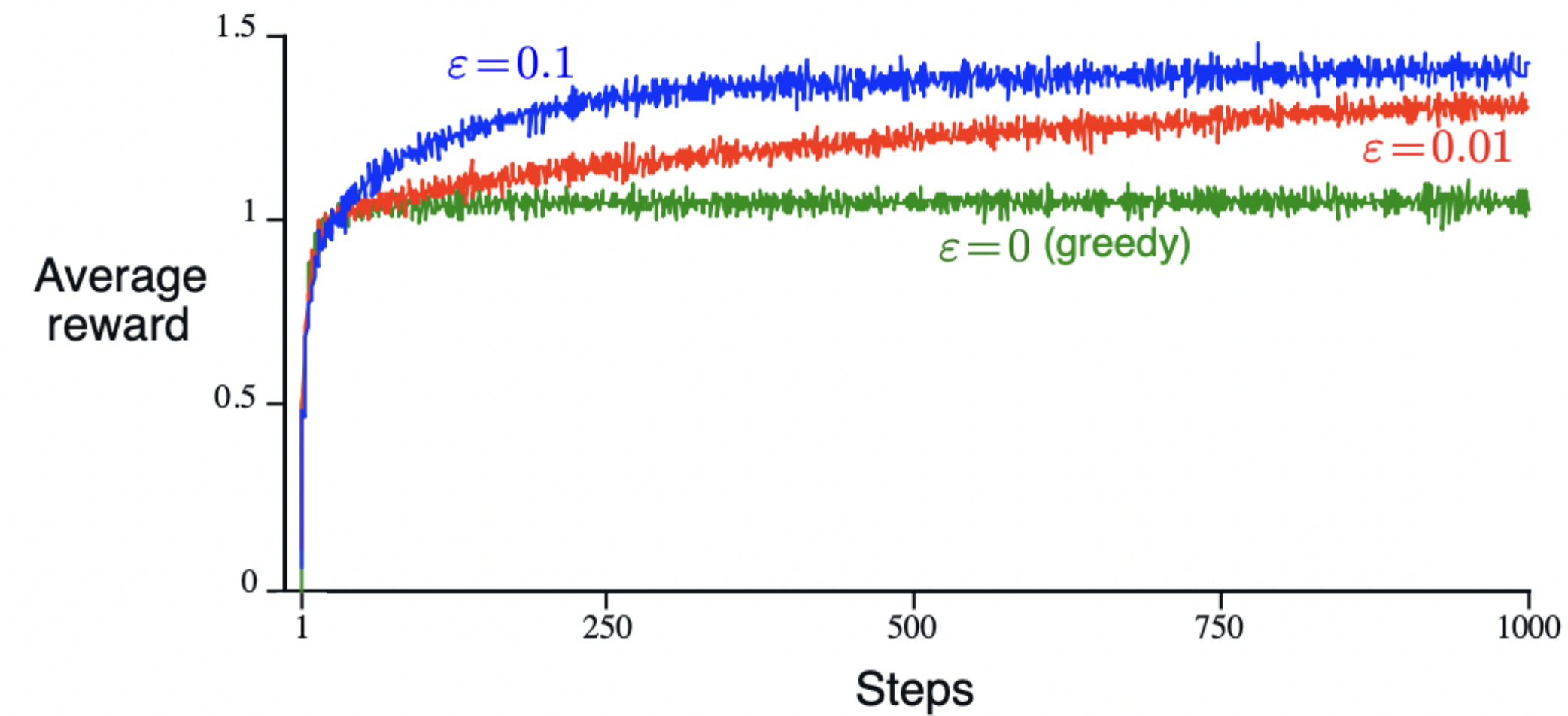


# Let the data guide you

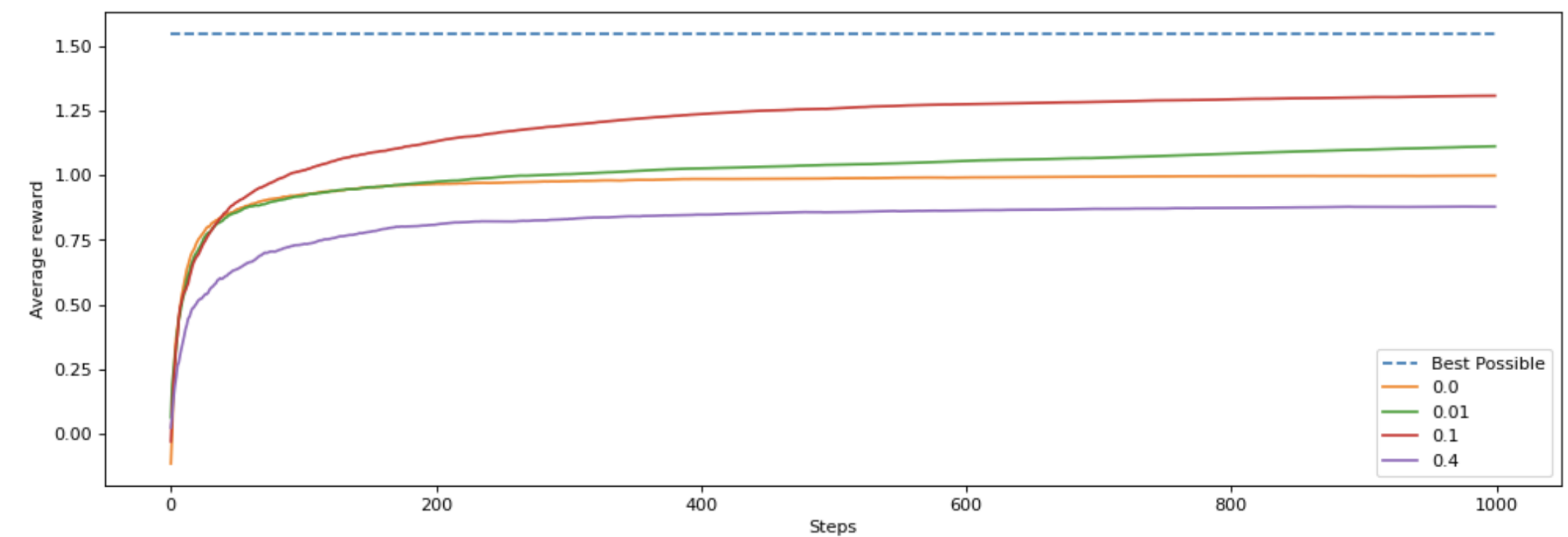




# You might have noticed, the textbook curves are not as smooth!

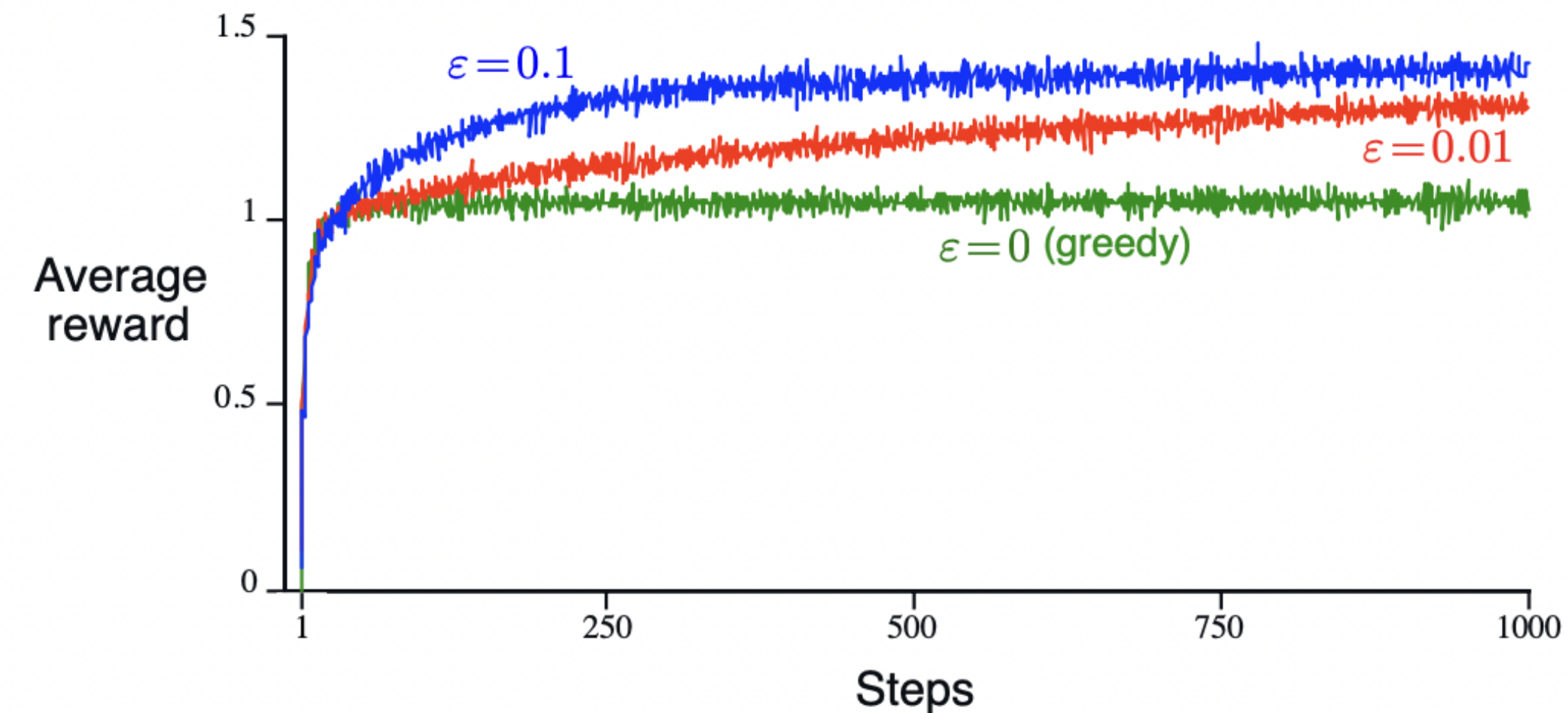


VS



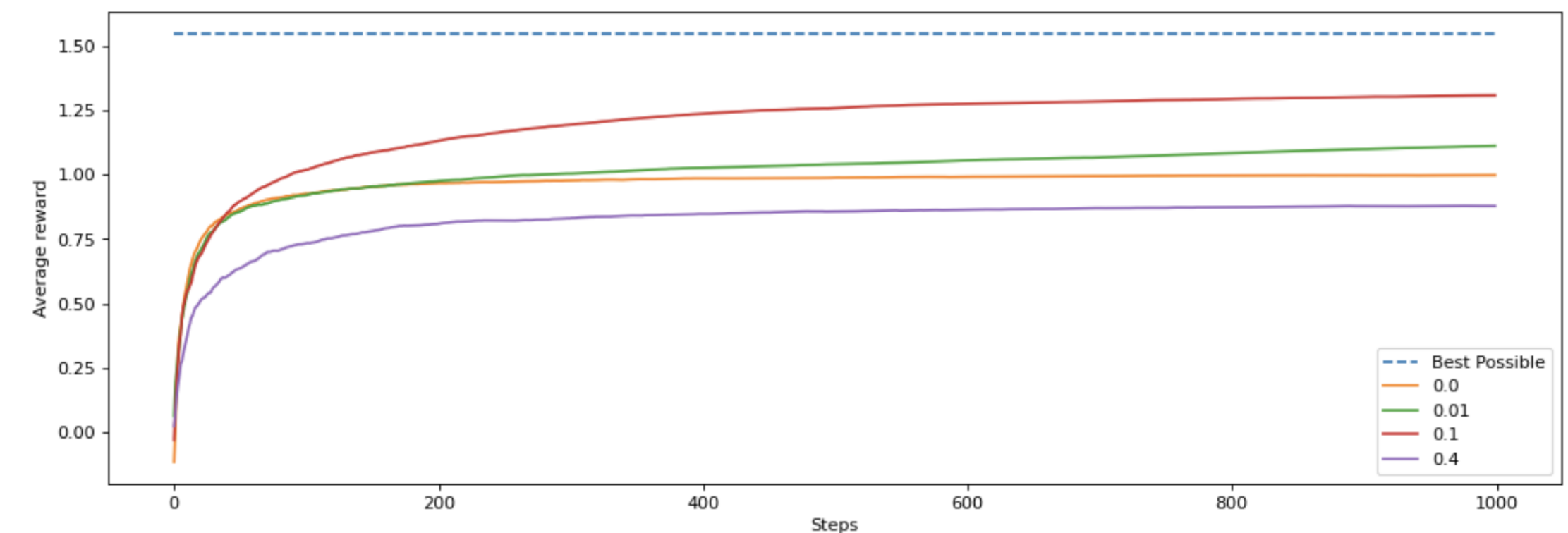


# The textbook is only averaging overruns!

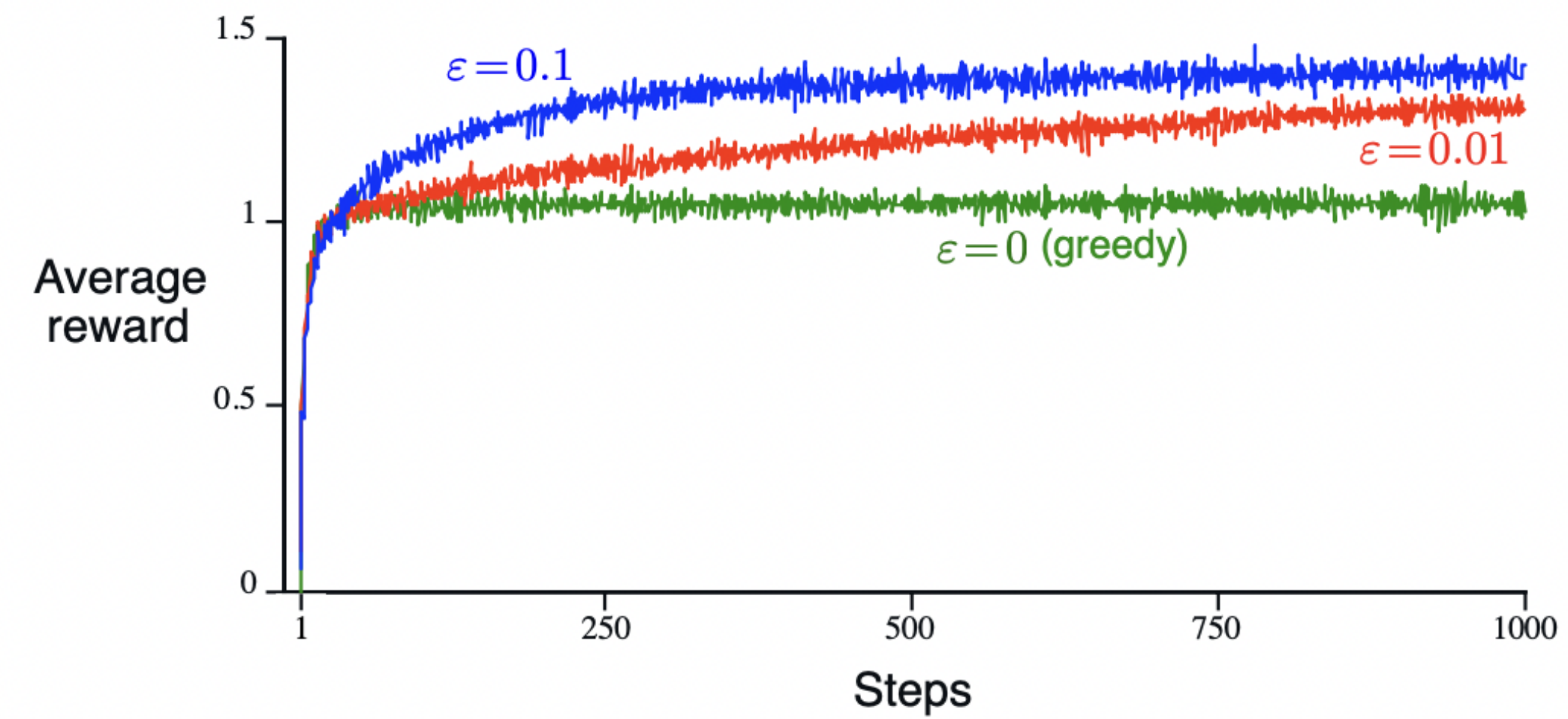


VS

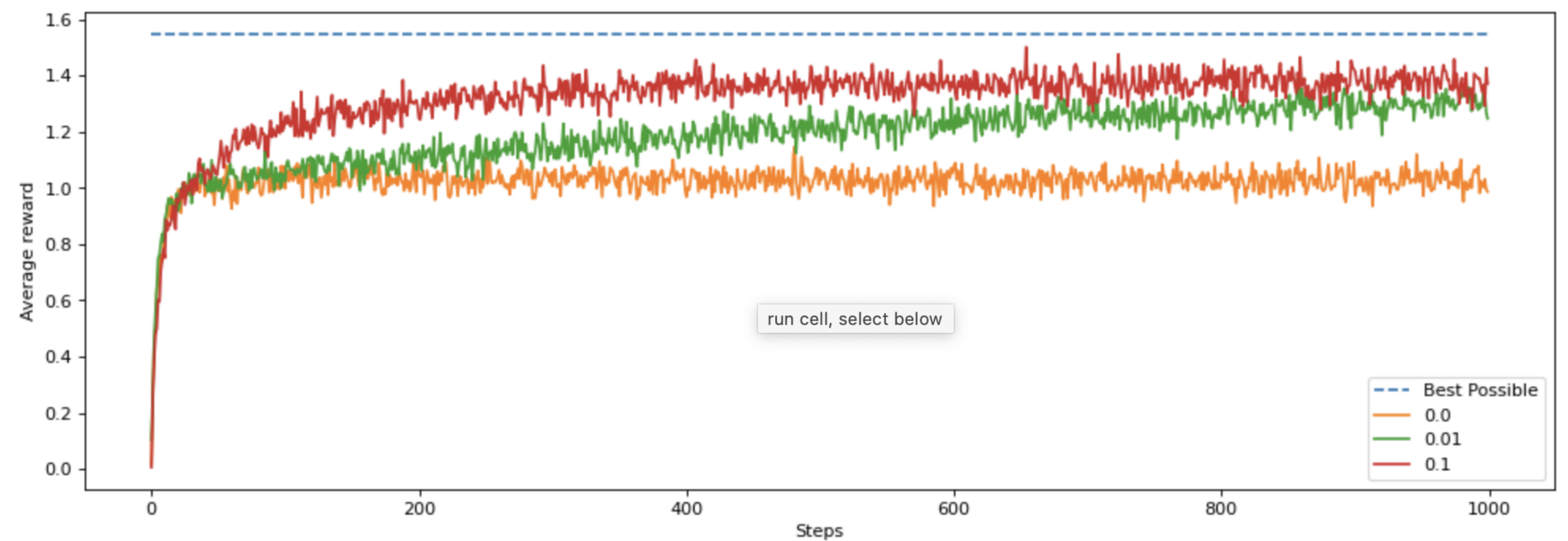
The notebook averaging the reward  
within the run first  
&  
using less runs (because compute :( )



# Easy to fix



# VS



# Let's review the quiz

- <https://www.coursera.org/learn/fundamentals-of-reinforcement-learning/quiz/IMCZf/sequential-decision-making/attempt>

# Worksheet question

3. (Exercise 2.2 from S&B 2nd edition) Consider a  $k$ -armed bandit problem with  $k = 4$  actions, denoted 1, 2, 3, and 4. Consider applying to this problem a bandit algorithm using  $\epsilon$ -greedy action selection, sample-average action-value estimates, and initial estimates of  $Q_1(a) = 0$ , for all  $a$ . Suppose the initial sequence of actions and rewards is  $A_1 = 1, R_1 = -1, A_2 = 2, R_2 = 1, A_3 = 2, R_3 = -2, A_4 = 2, R_4 = 2, A_5 = 3, R_5 = 0$ . On some of these time steps the  $\epsilon$  case may have occurred, causing an action to be selected at random. On which time steps did this definitely occur? On which time steps could this possibly have occurred?

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T	Q1	Q2	Q3	Q4	{A*_t}	A_t	Explore?	R_1
1								
2								
3								
4								
5								

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T	Q1	Q2	Q3	Q4	{A*_t}	A_t	Explore?	R_t
1								-1
2								1
3								-2
4								2
5								0

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1								-1
2								1
3								-2
4								2
5								0

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T	Q1	Q2	Q3	Q4	{A*_t}	A_t	Explore?	R_1
1	0	0	0	0				-1
2								1
3								-2
4								2
5								0

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3								-2
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T	Q1	Q2	Q3	Q4	{A*_t}	A_t	Explore?	R_1
1	0	0	0	0	{1,2,3,4}	1	Maybe	-1
2								1
3								-2
4								2
5								0

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2	-1	0	0	0				1
3								-2
4								2
5								0

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1	0	0	0	0	{1,2,3,4}	1	Maybe	-1
2	-1	0	0	0	{2,3,4}			1
3								-2
4								2
5								0

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 $2, R_2 = 1, A_3 = 2, R_3 = -2, A_4 = 2, R_4 = 2, A_5 = 3, R_5 = 0$ . On some of these time steps the  $\epsilon$  case may have occurred, causing an action to be selected at random. On which time steps did this definitely occur? On which time steps could this possibly have occurred?

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2	-1	0	0	0	{2,3,4}	2	Maybe	1
3								-2
4								2
5								0

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1	0	0	0	0	{1,2,3,4}	1	Maybe	-1
2	-1	0	0	0	{2,3,4}	2	Maybe	1
3	?	?	?	?				-2
4								2
5								0

# Worksheet question

**Q3**

Suppose that in a lottery you have 0.01% chance of winning and the prize is \$1000. The ticket to enter the lottery costs you \$10. What is the expected amount you would earn, when buying a ticket for this lottery?

**What is the definition of expected value?**

**What is the random variable & what are the possible outcomes?**

# Worksheet question

## Q3

Suppose that in a lottery you have 0.01% chance of winning and the prize is \$1000. The ticket to enter the lottery costs you \$10. What is the expected amount you would earn, when buying a ticket for this lottery?

$$\mathbb{E}[X] \doteq \sum_{i=1}^k x_i p_i = x_1 p_1 + x_2 p_2 + \dots + x_k p_k$$



# Worksheet question

## Q3

Suppose that in a lottery you have 0.01% chance of winning and the prize is \$1000. The ticket to enter the lottery costs you \$10. What is the expected amount you would earn, when buying a ticket for this lottery?

$$\mathbb{E}[X] \doteq \sum_{i=1}^k x_i p_i = x_1 p_1 + x_2 p_2 + \dots + x_k p_k$$

Two outcomes: win or not

# Worksheet question

1. Suppose a game where you choose to flip one of two (possibly unfair) coins. You win \$1 if your chosen coin shows heads and lose \$1 if it shows tails.
  - (a) Model this as a K-armed bandit problem: define the action set.

**Can you formally define  $q^*$ ?**

- (b) Is the reward a deterministic or stochastic function of your action?
  - (c) You do not know the coin flip probabilities. Instead, you are able to view 6 sample flips for each coin respectively: (T,H,H,T,T,T) and (H,T,H,H,H,T). Use the sample average formula (equation 2.1 in the book) to compute the estimates of the value of each action.
  - (d) Decide on which coin to flip next! Assume it's an exploit step.