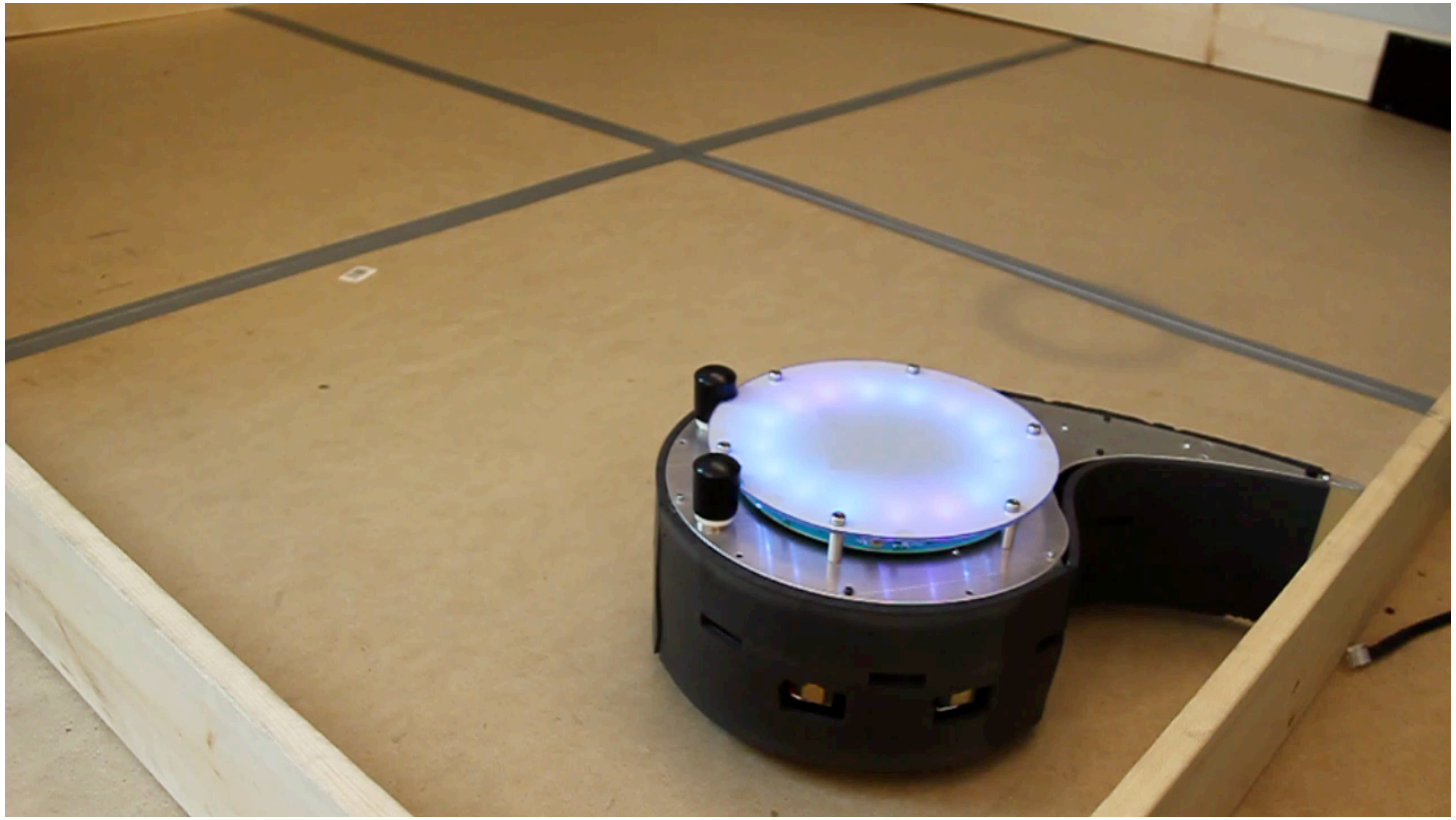
Markov Decision Processes

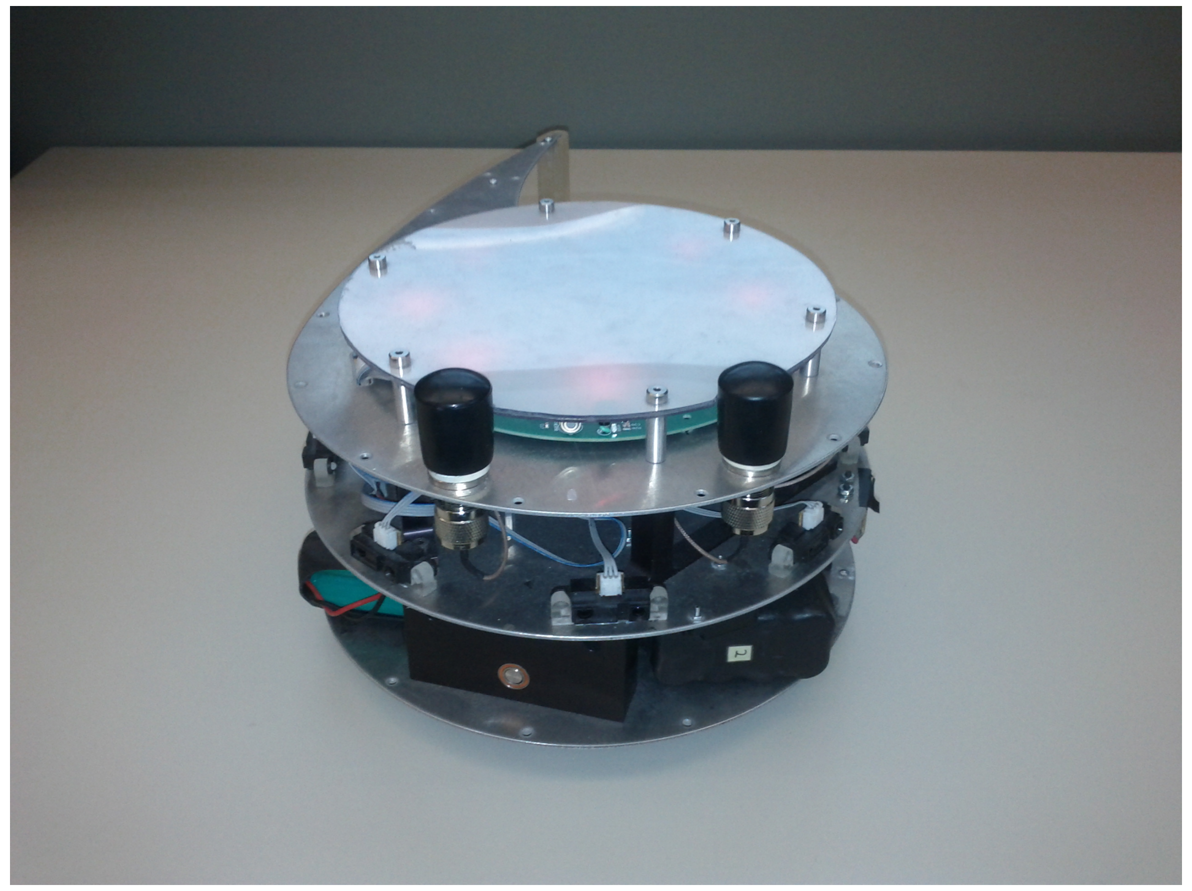
CMPUT 655
Fall 2022

Example MDPs

- Consider this robot called the critterbot

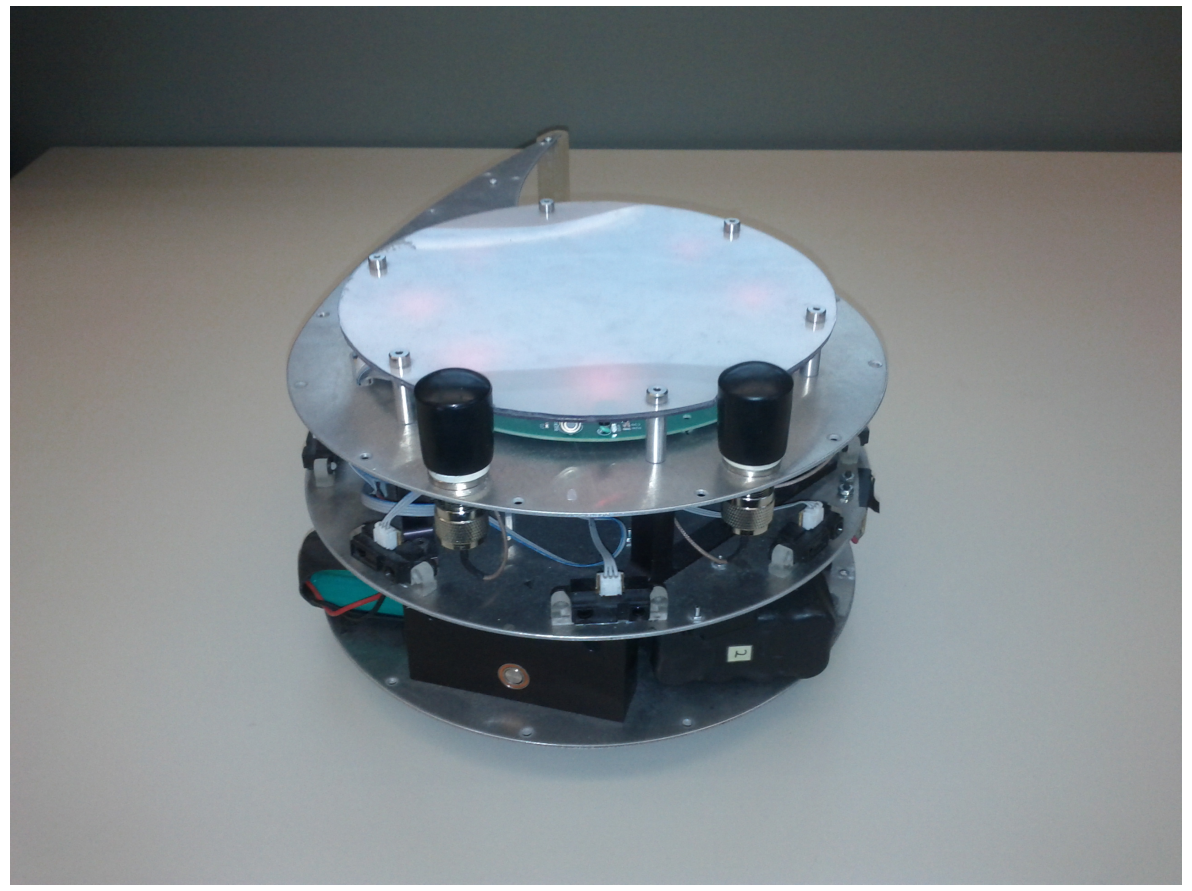


Robot sensors

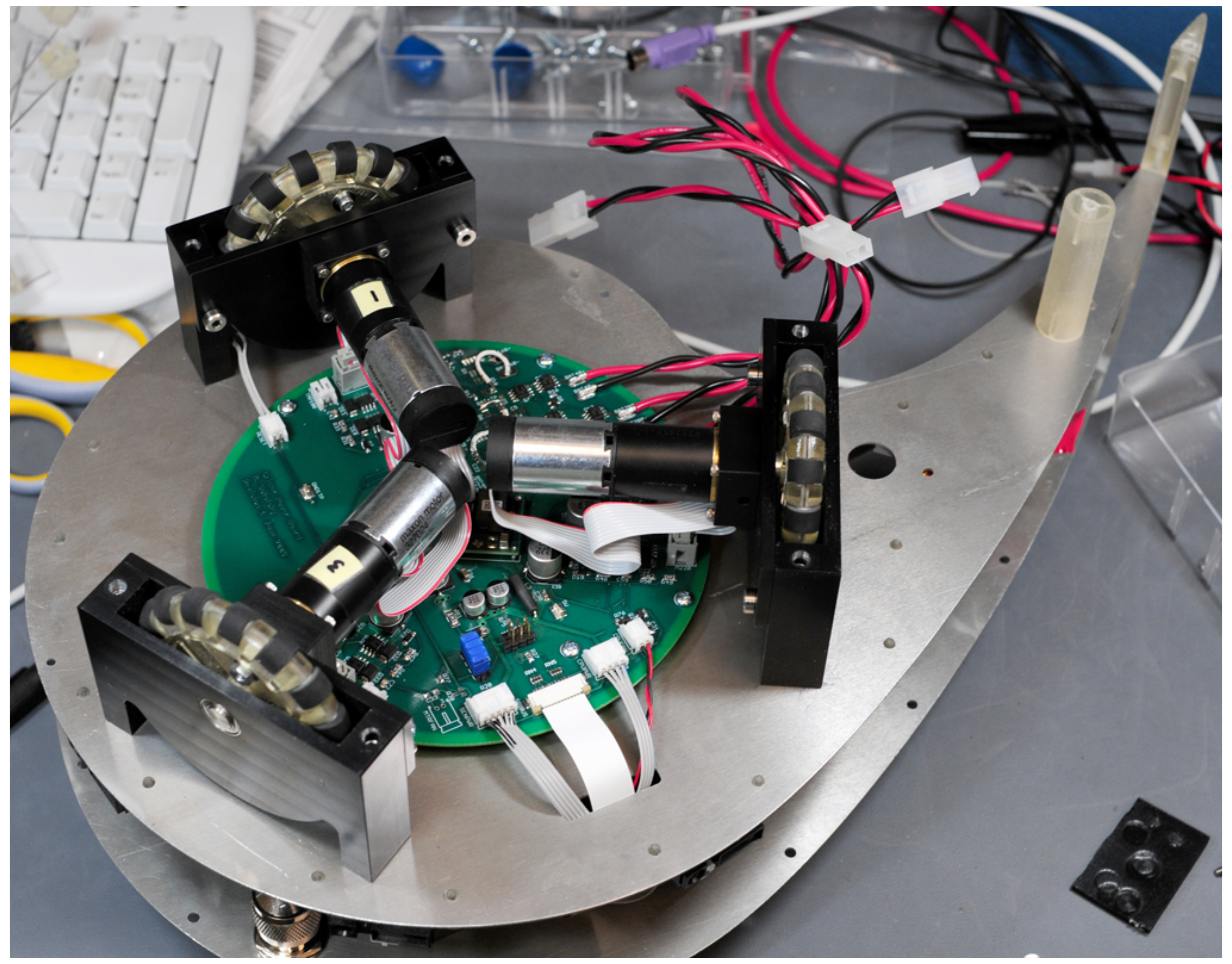


- This robot has **sensors** all over it:
  - Distance sensors
  - Light sensors
  - Thermal sensors
  - Motor information like speed, current, velocity, and temp
  - Accel XYZ
  - Rotational vel
  - etc

Robot actuation



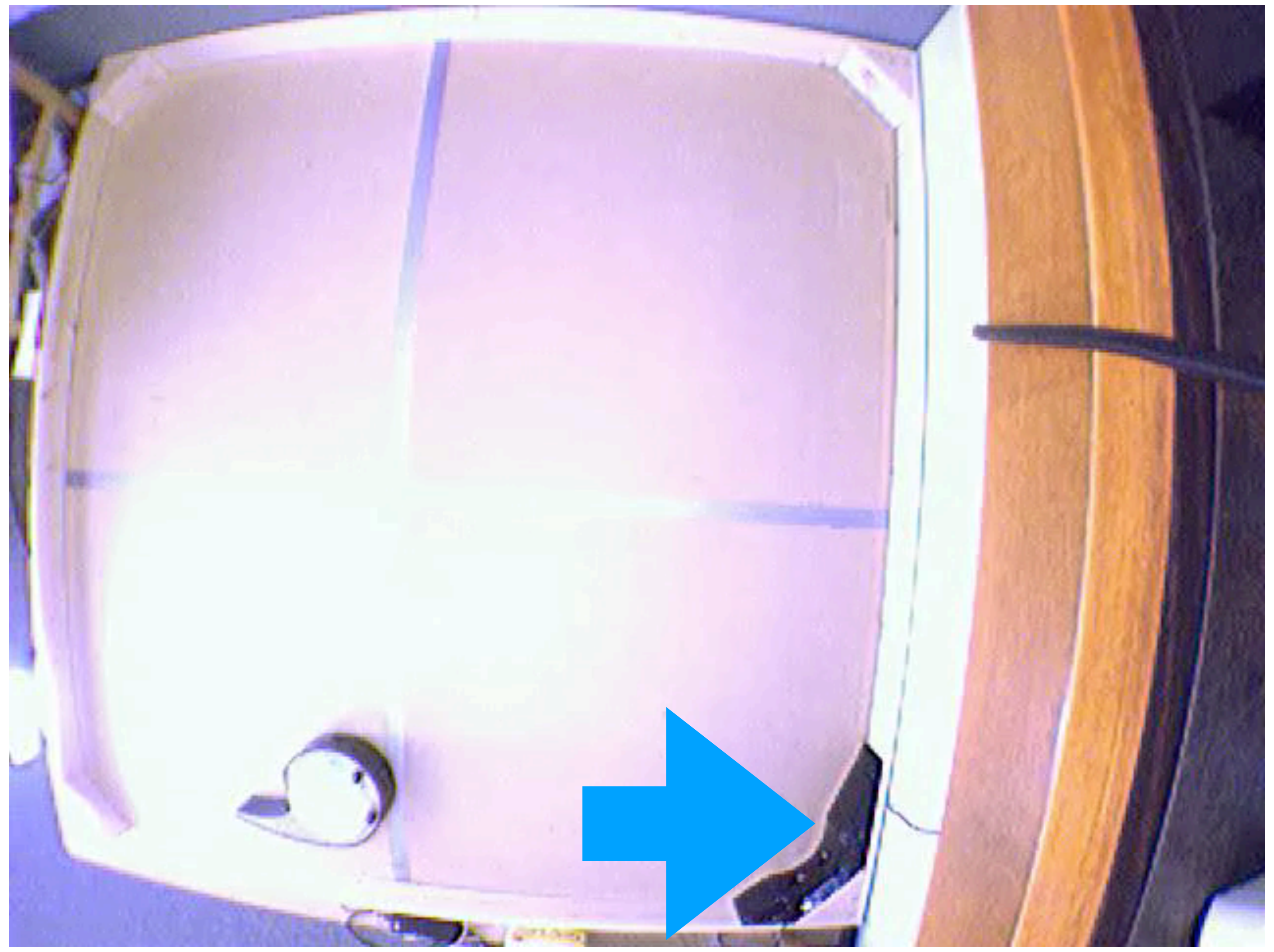
- This robot has an omni directional drive system:
  - Can move in any direction on a surface



An RL robot MDP

- Imagine we wanted this robot to **goto the light quickly**
  - What is the **state**?
  - What are the **actions**?
  - What is the **reward**?
- In other words how to formulate this as an **RL problem**?

Robot state



- Perhaps the sensors are so good that every situation in the pen looks **unique** to the agent:
  - The agent would not do better by **remembering** a history of the sensors
  - This is true because of the **IR beacon on the charger**, and **magnetic** sensors + all the other sensors
- We could also use an overhead camera + a localization algorithm to extract X,Y, theta position —Markov state
- It would be easy to imagine that if the robot only had distance sensors and the pen was square, that the state would not be fully observable

Robot actions

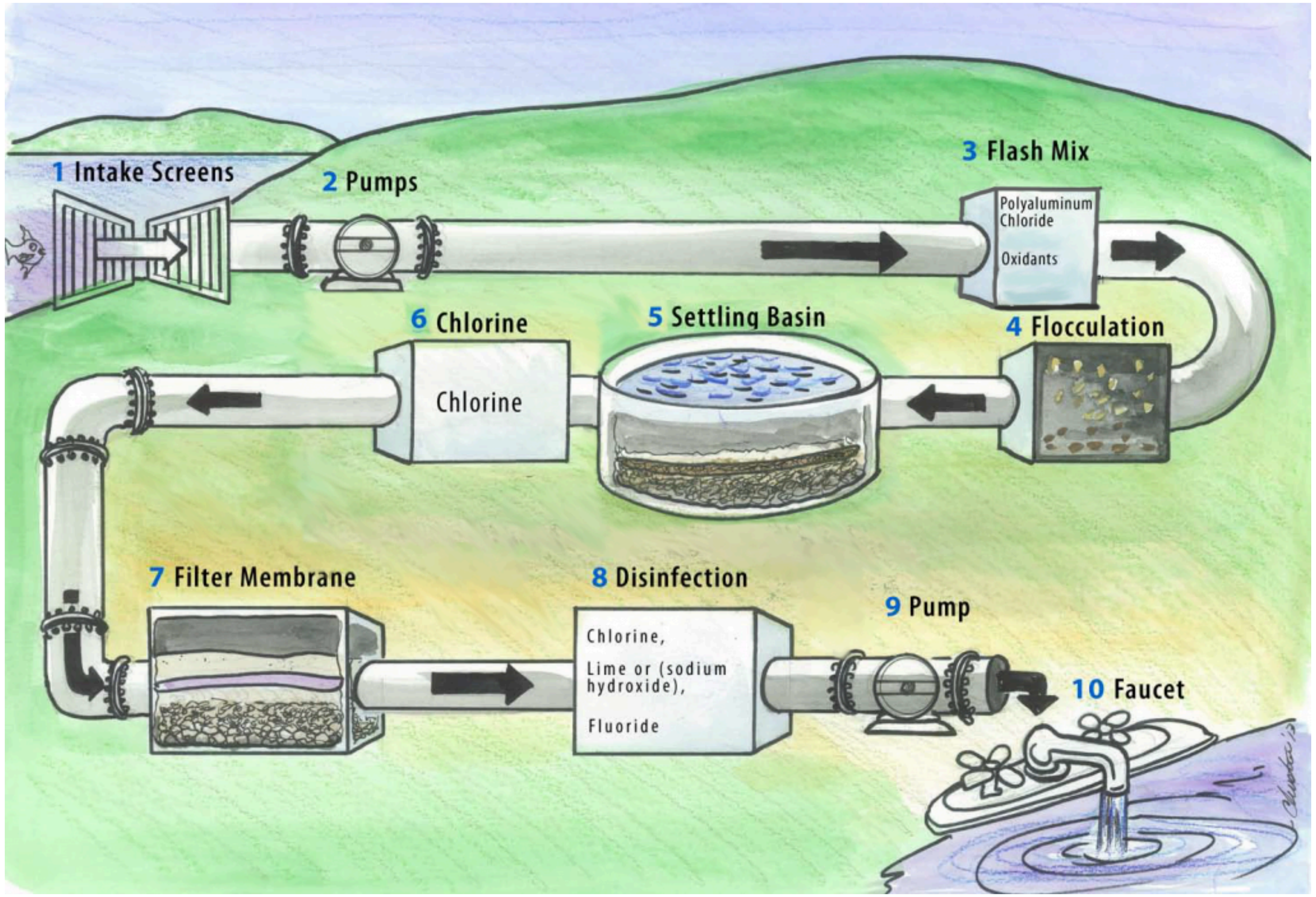
- This robot can be controlled in two basic ways:
  - X,Y,theta mode -> specify amount of translation in X and Y, and rotation
  - Voltage mode -> how much voltage to send to each of the three motors

Robot Reward

- To encourage light seeking
  - -1 per step, until front light sensor > threshold
  - OR
  - Reward = front_light_sensor_reading
- In both cases terminate when light sensor > threshold; **Episodic problem**
  - What is gamma?
- How do we reset to the state state? ME :(

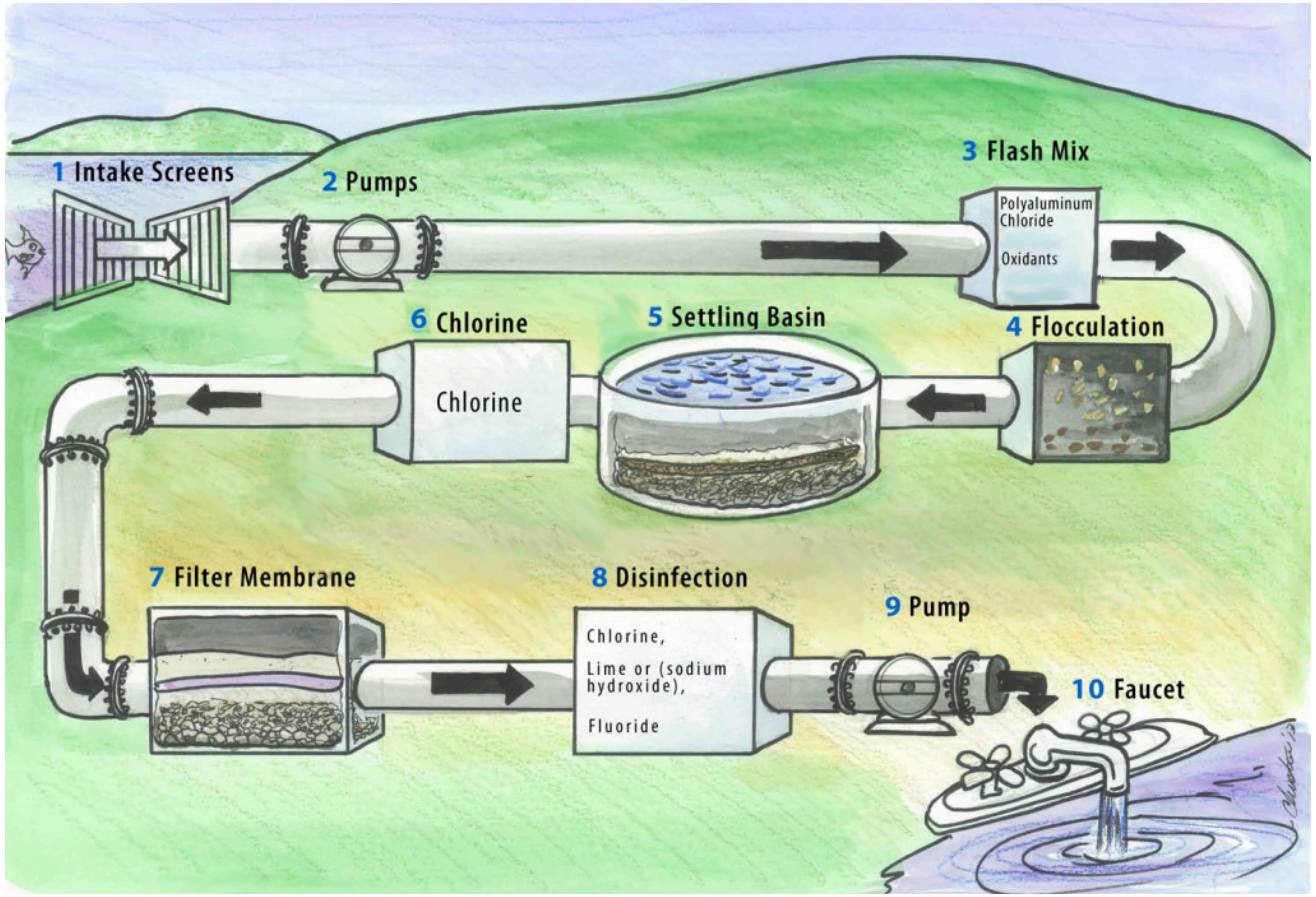
Fresh Water Treatment

- Water from the river
- Passes through many stages:
  - Chemical treatment
  - Settling
  - Mixing
  - Filters
  - Then to you to drink



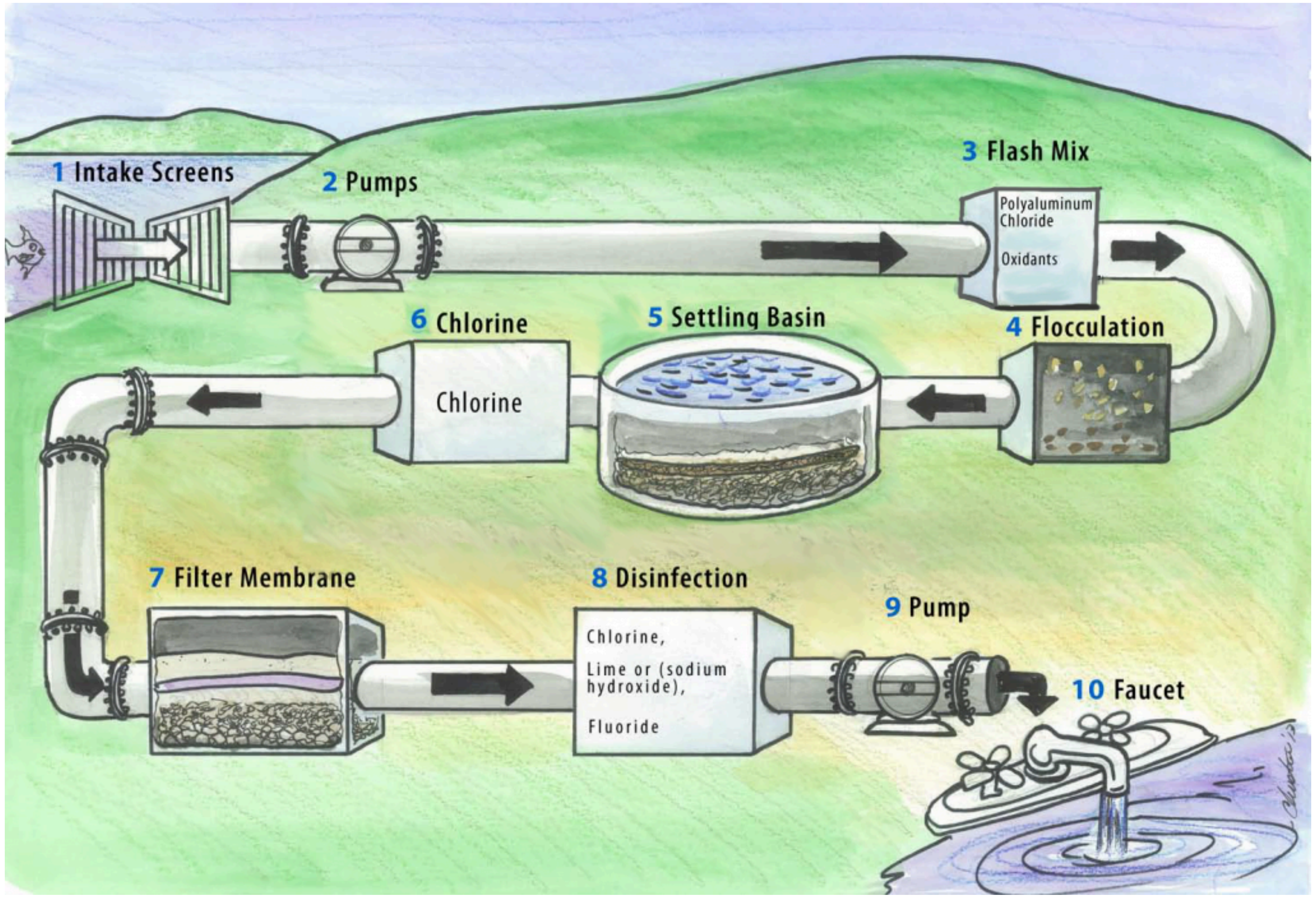
Fresh Water Treatment

- The reward is easy
  - R_t = Water flow - energy
- The system is **safe** by design:
  - Hard to break the components
  - Plant cannot produce unclean water
- Good for trial and error learning



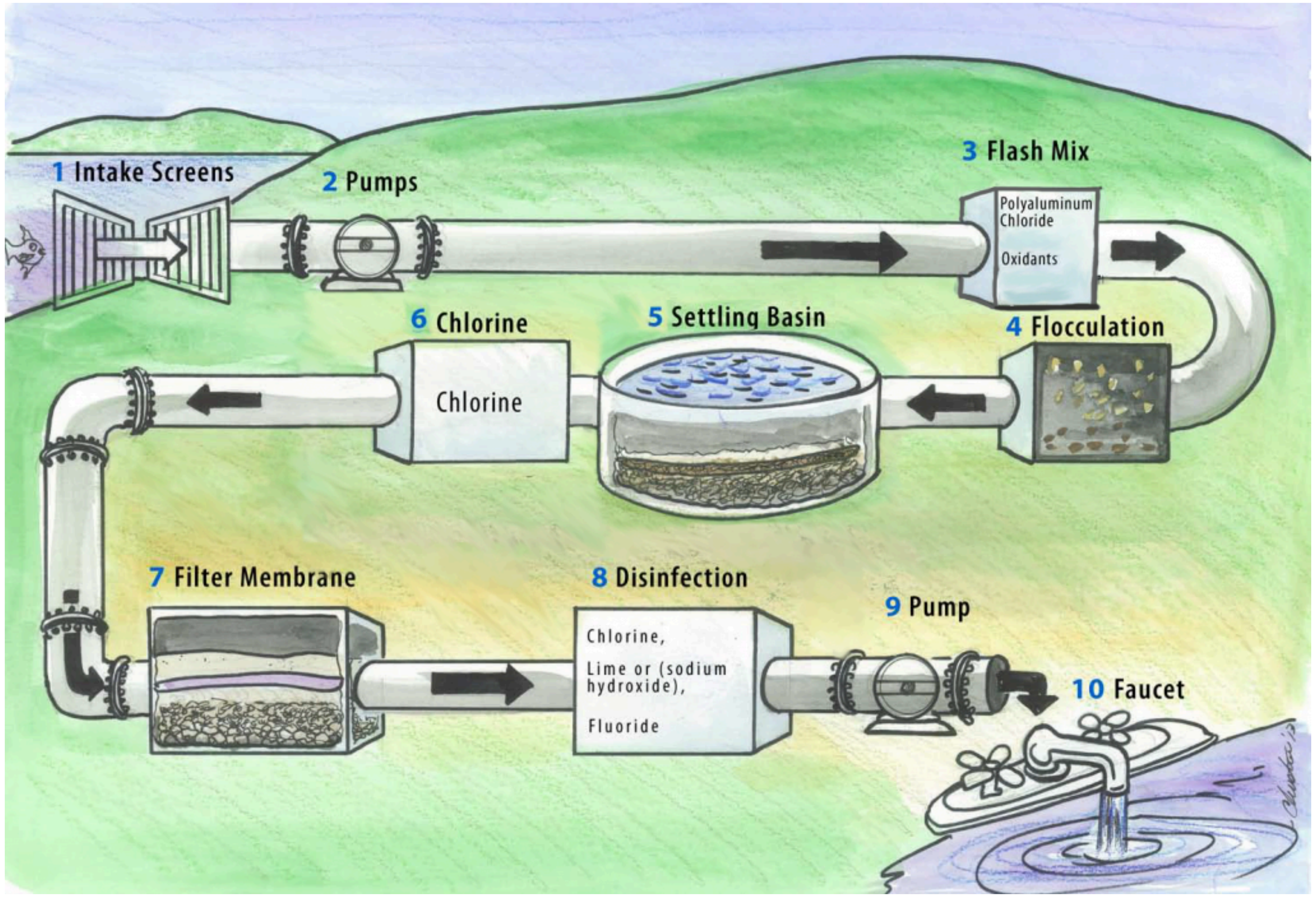
Fresh Water Treatment

- **Many sensors:**
  - Pump speeds and temp
  - TMP & other water data
  - Turbidity
  - Weather & future weather
  - River conditions
  - State of the filter
- Could be very close to **Markov state**



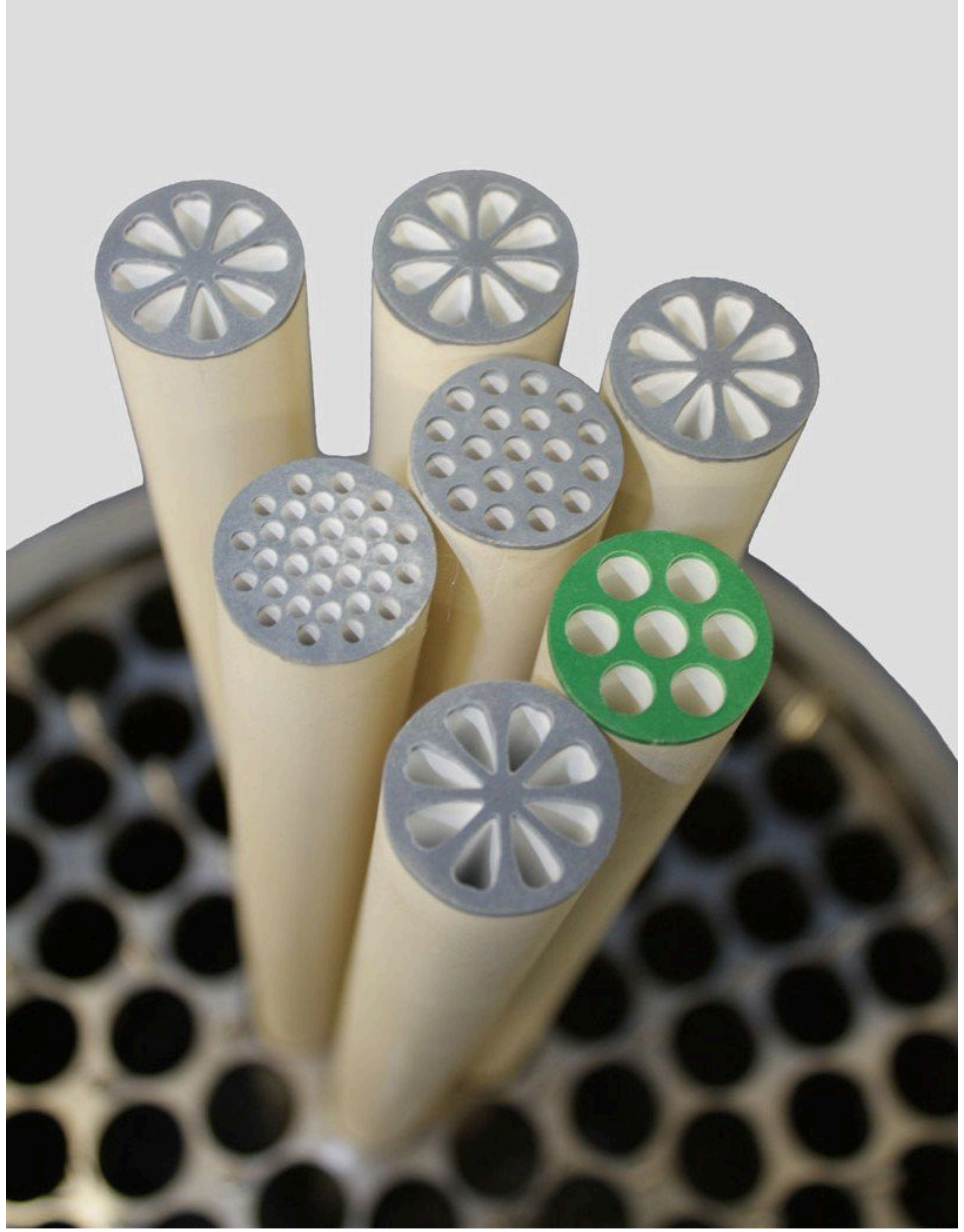
Fresh Water Treatment

- Many possible **actions**:
  - Pump speeds
  - Chemical treatments
  - **Backwashing the filter …**



Fresh Water Treatment

- The filter is comprised of hundreds of filter tubes
- They get full of dirt and other stuff
- To clean them we just run the system backwards (**backwashing**)
    - But backwashing uses a **lot of energy** and **stops water production**
- **Action**: backwash or not on every step



More example MDPs

- White in 1969 (not me!!)
  - Salmon Harvesting
  - Agriculture: how much to plant based on weather and soil state.
  - Water resources: keep the correct water level at reservoirs.
  - Inspection, maintenance and repair: when to replace/inspect based on age, condition, etc.
  - Purchase and production: how much to produce based on demand.
  - Queues

More example MDPs

- Flying REAL (small) Helicopters ~Andrew Ng et al

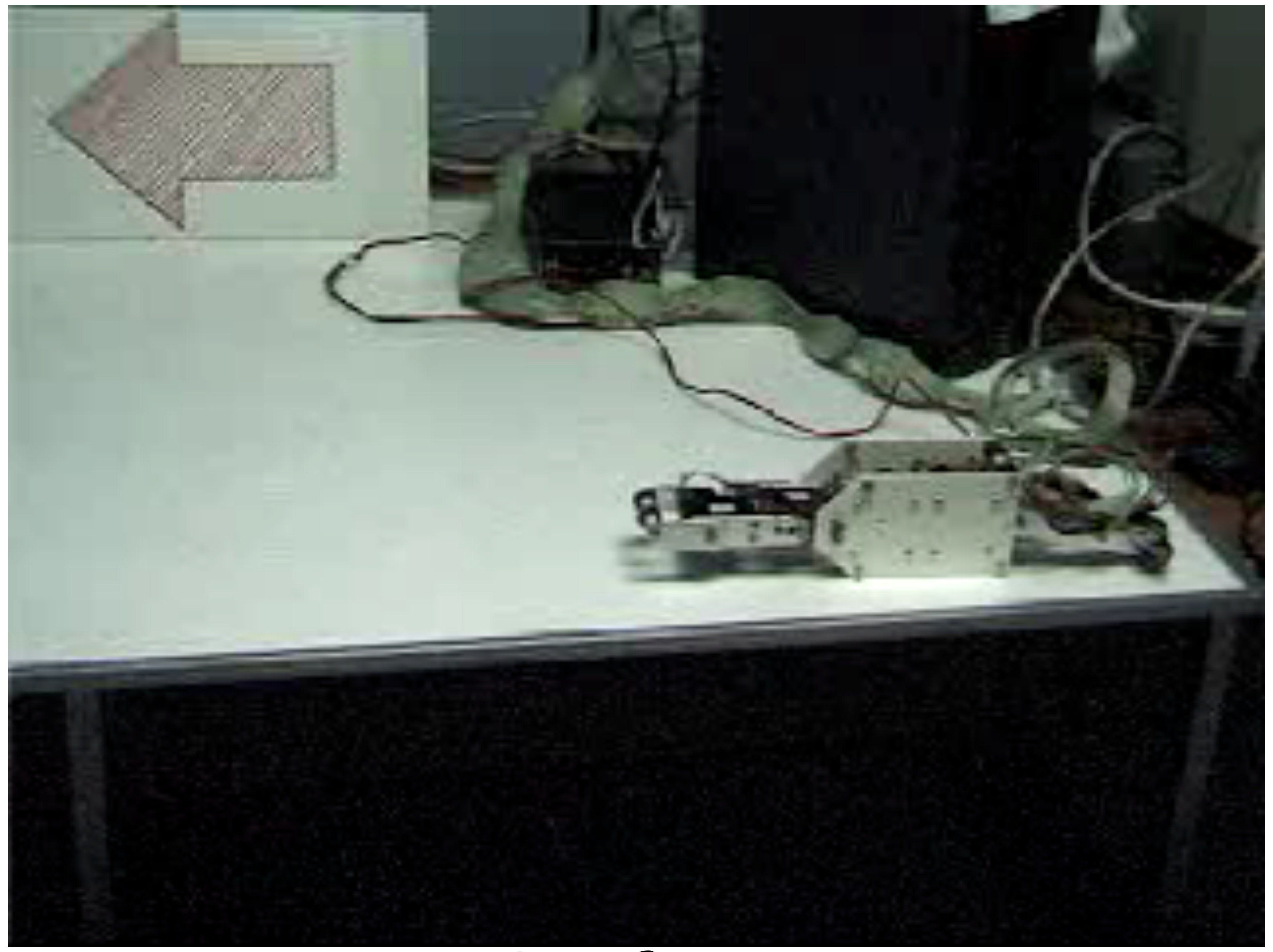
Heli MDP

- **Actions (multi-dimensional & continuous!!):**
  - Longitudinal and latitudinal Pitch controls
  - Main rotor pitch
  - Tail rotor pitch
  - Throttle

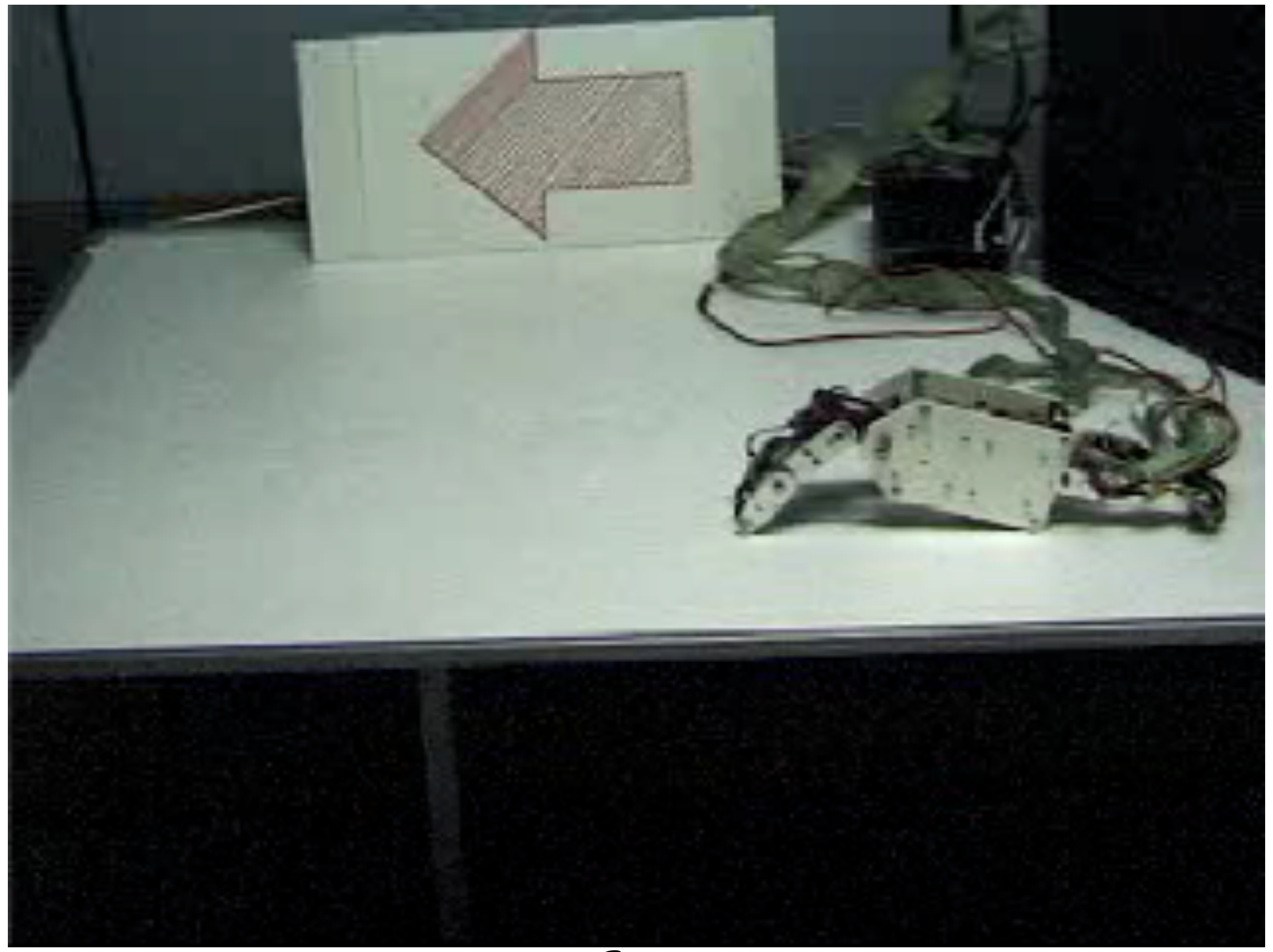
$$R(s^s) = -(\alpha_x(x - x^*)^2 + \alpha_y(y - y^*)^2 + \alpha_z(z - z^*)^2 \\ + \alpha_{\dot{x}}\dot{x}^2 + \alpha_{\dot{y}}\dot{y}^2 + \alpha_{\dot{z}}\dot{z}^2 + \alpha_\omega(\omega - \omega^*)^2),$$

- **State (8 dim):**
  - helicopter's position (x, y, z), orientation (roll ϕ, pitch θ, yaw ω), velocity ($\dot{x}$,$\dot{y}$,$\dot{z}$) and angular velocities ($\dot{\phi}$,$\dot{\theta}$,$\dot{\omega}$

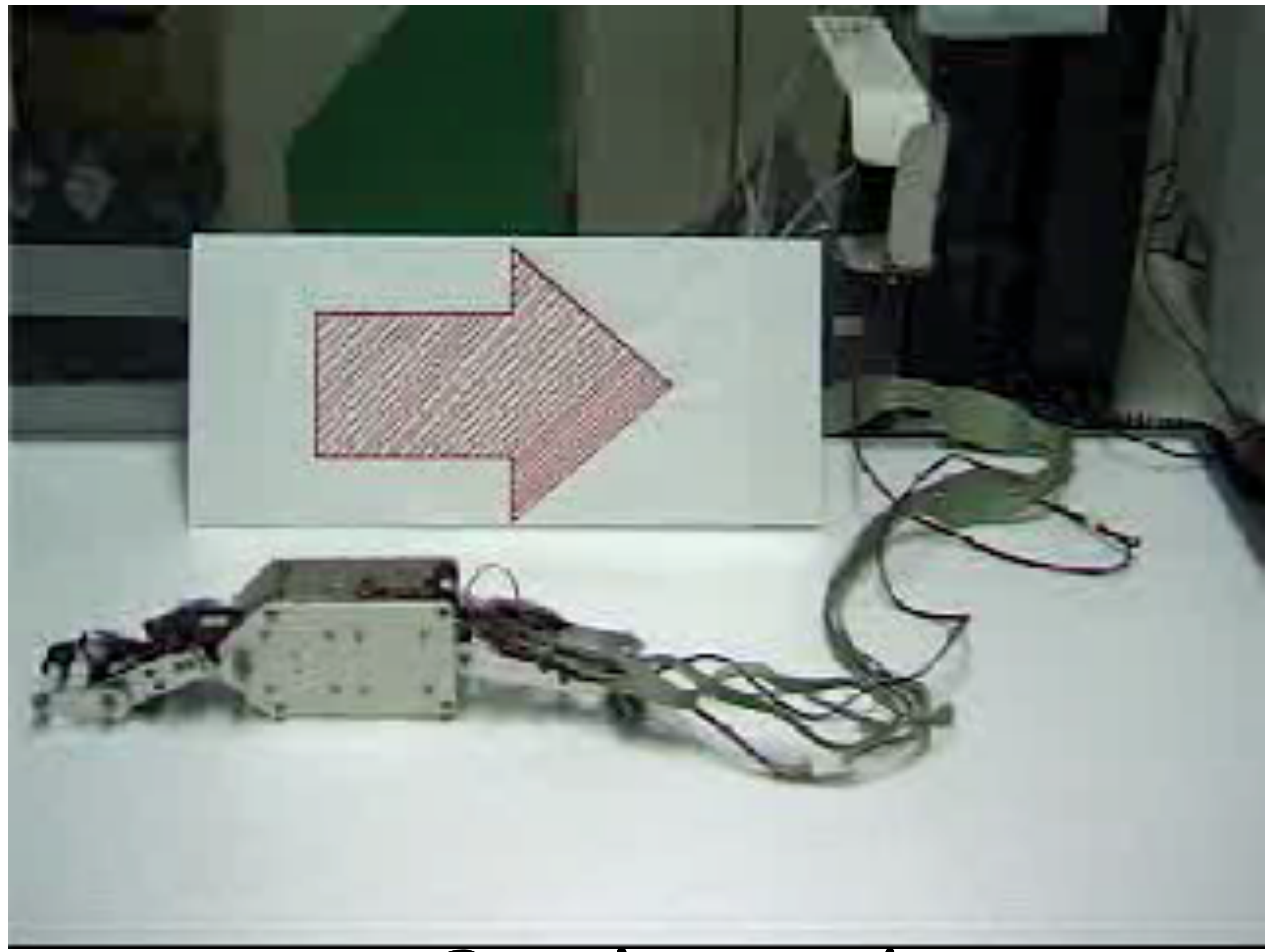
Hajime Kimura's RL Robots



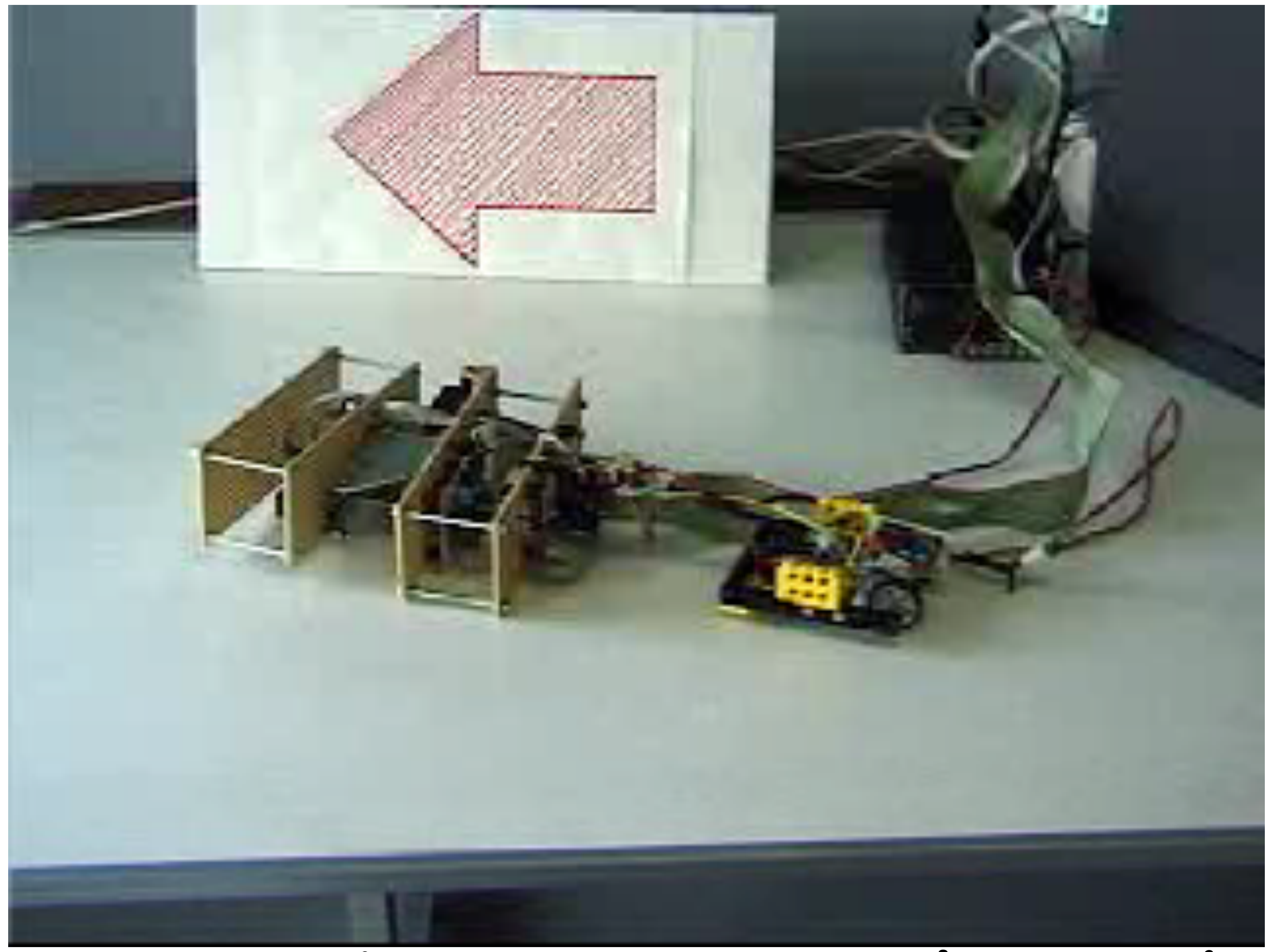
Before



After

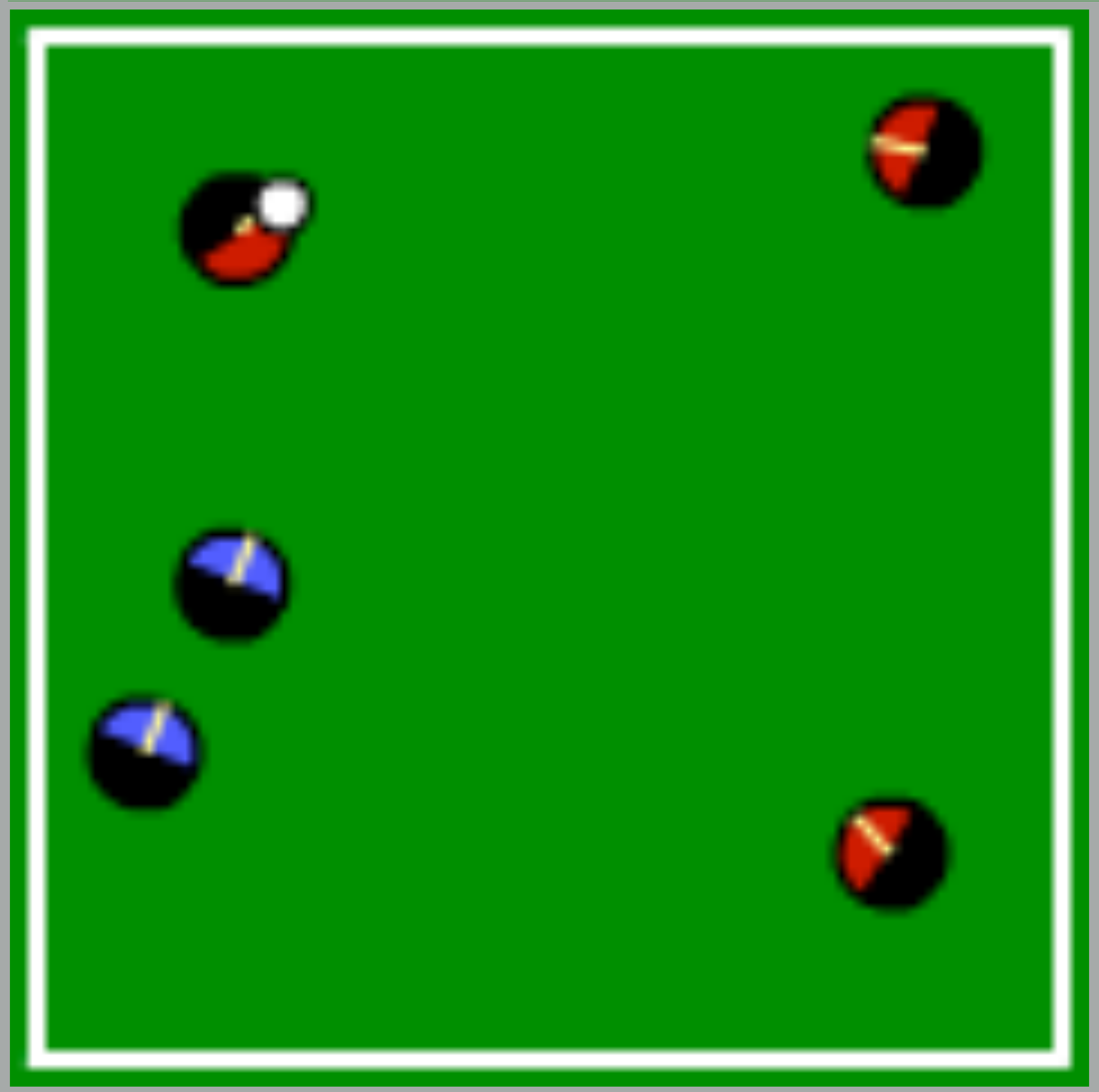


Backward



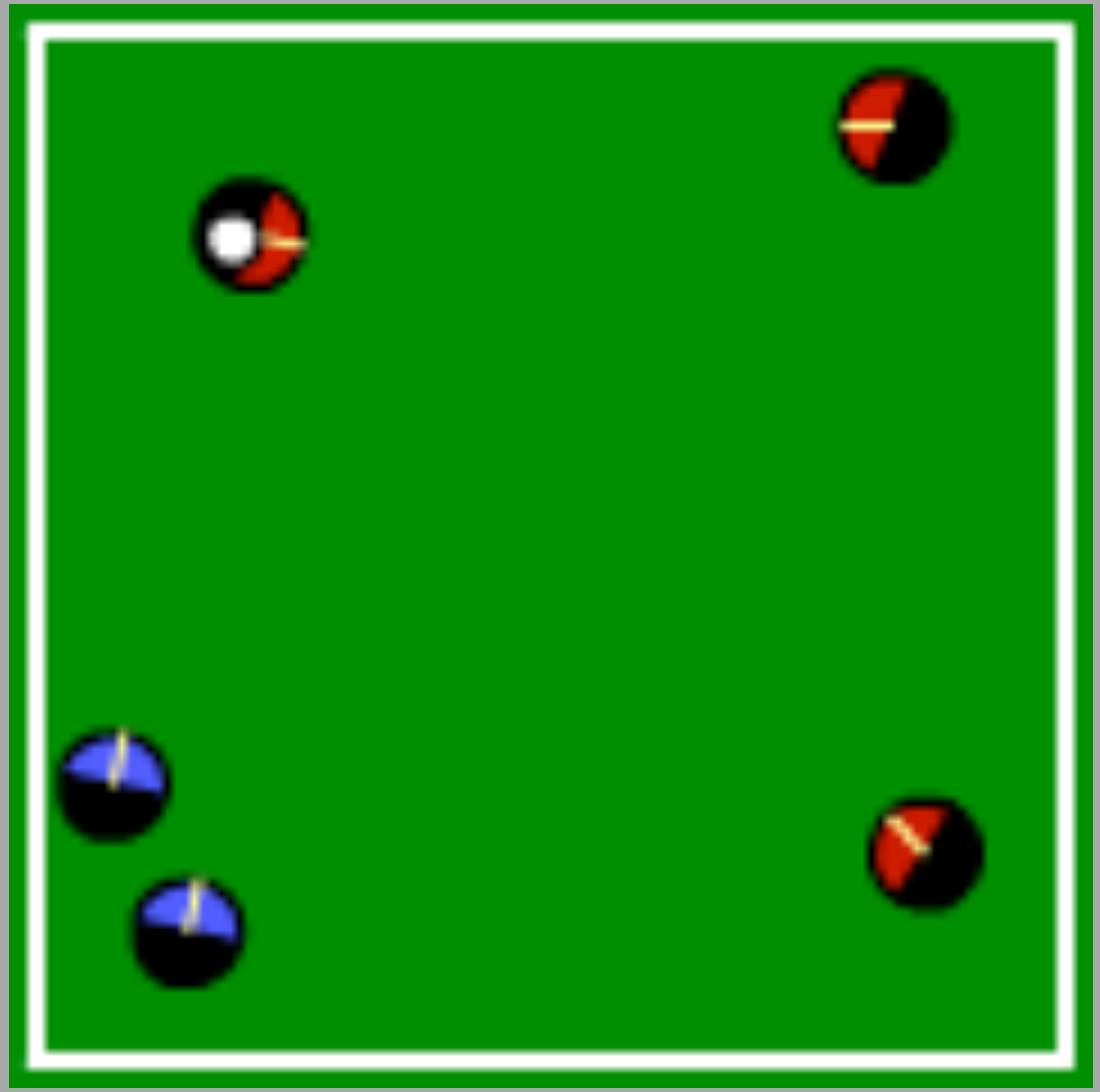
New Robot, Same algorithm

- Stone & Sutton

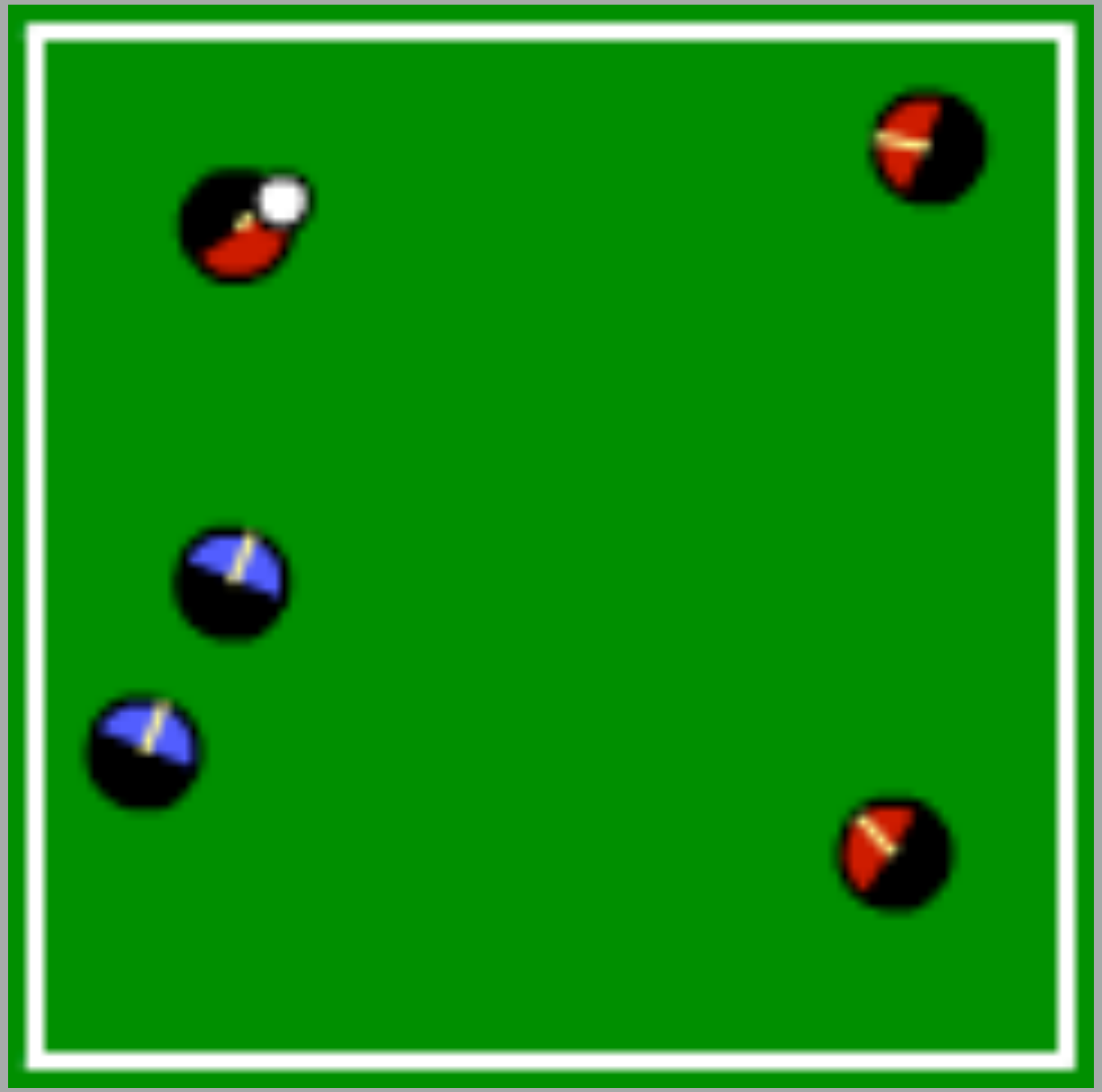


Hand-coded

- Stone & Sutton



Random



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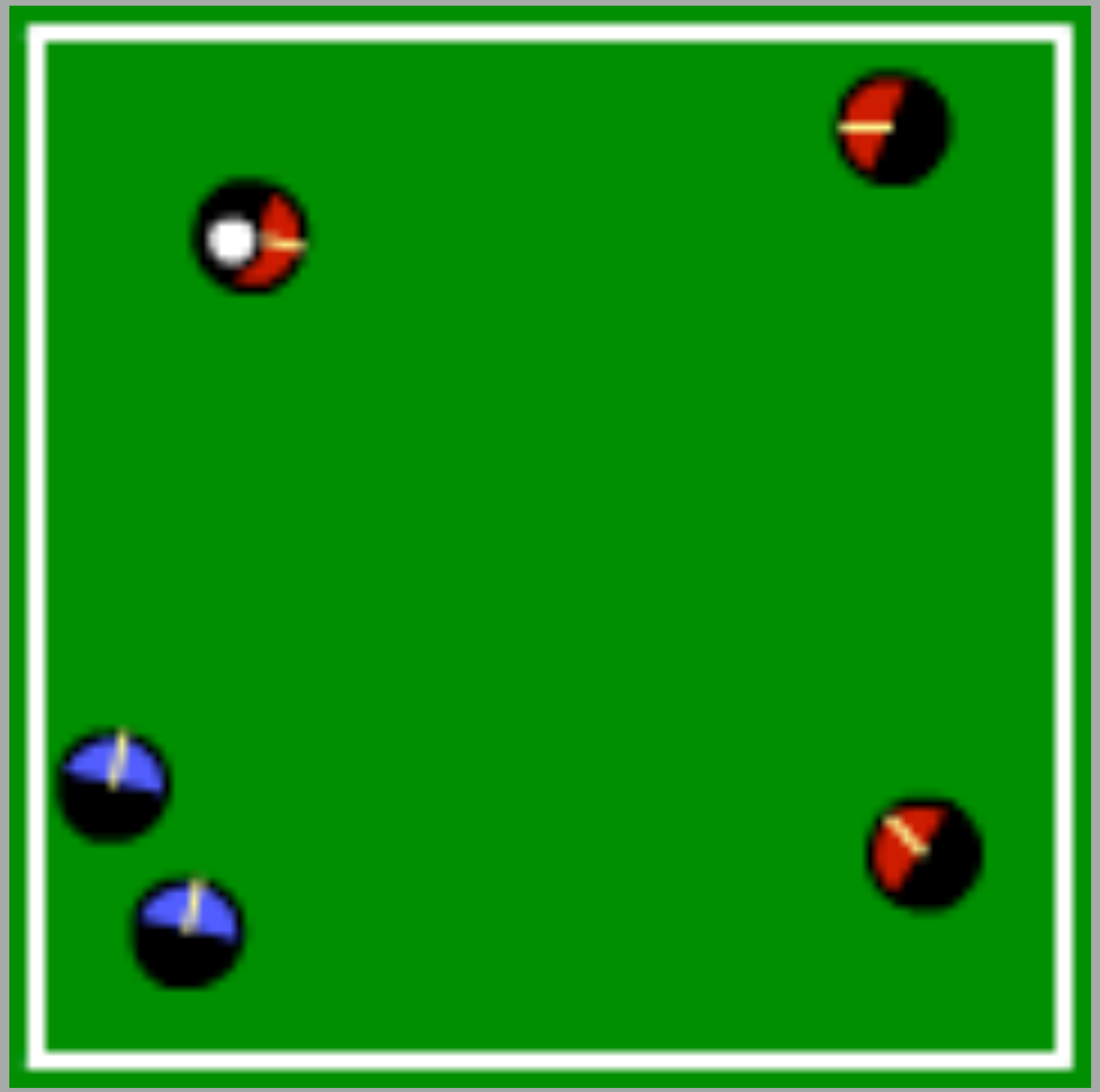
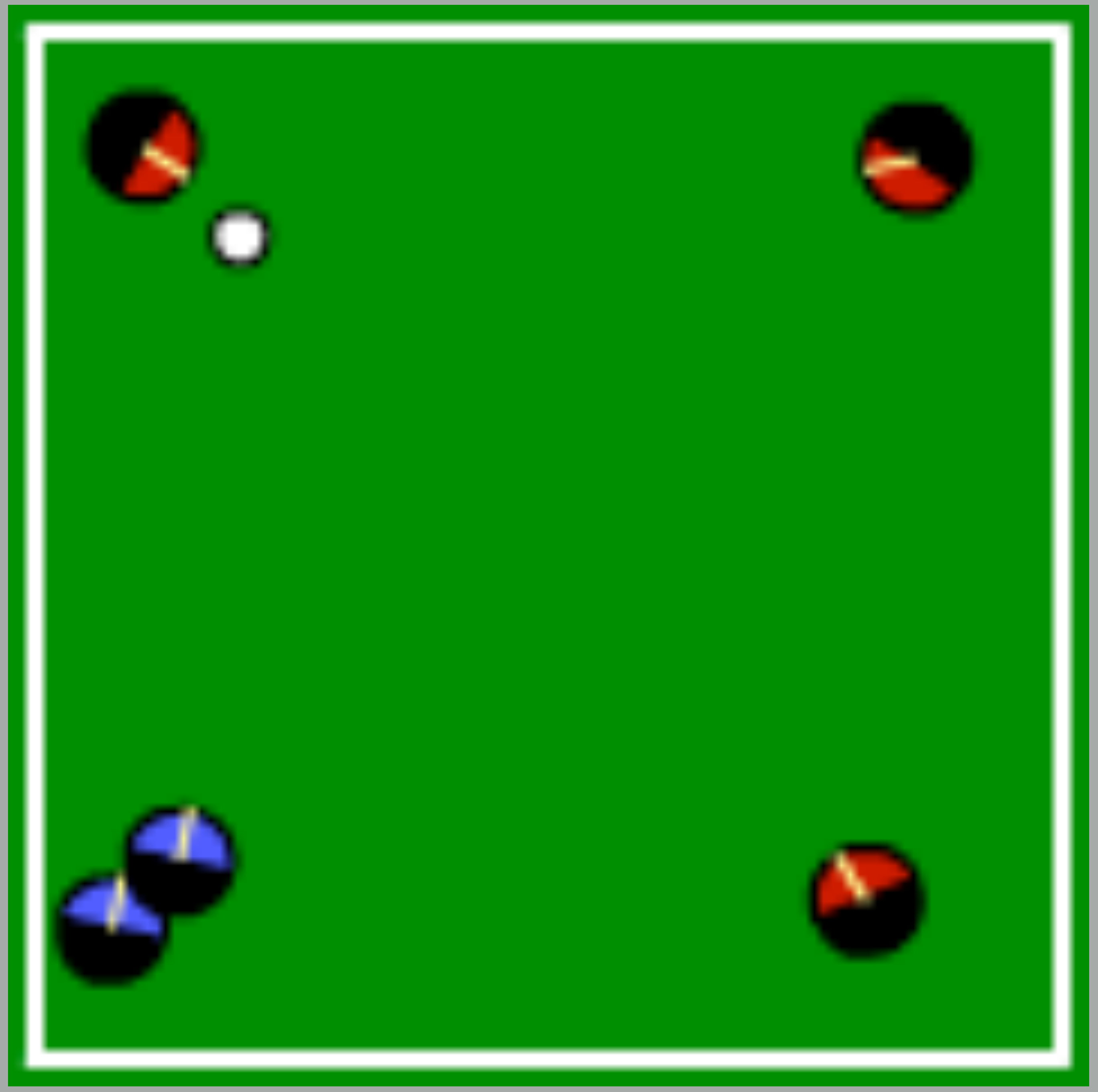
Hold

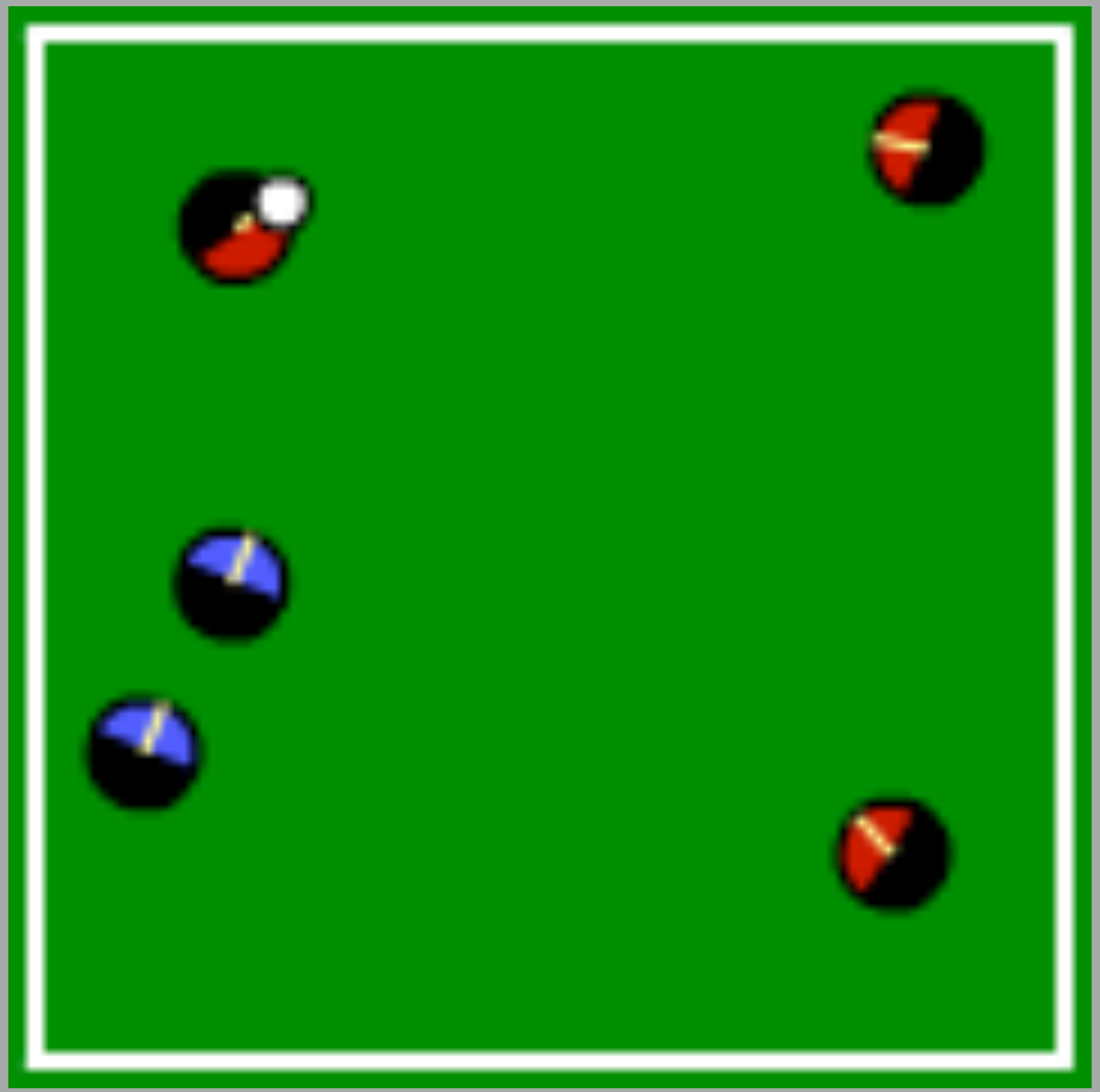
- Stone & Sutton



Random



Learned



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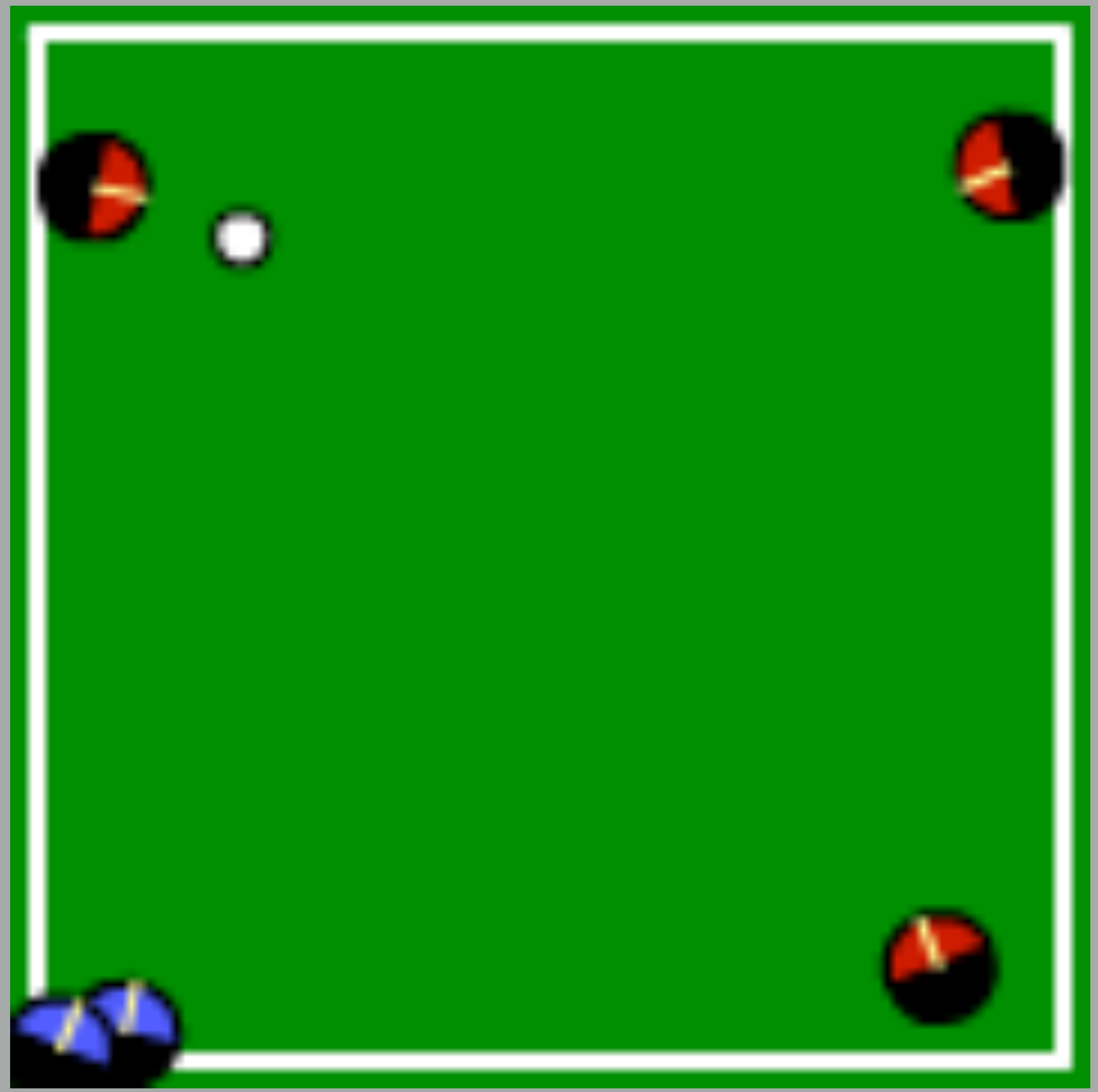
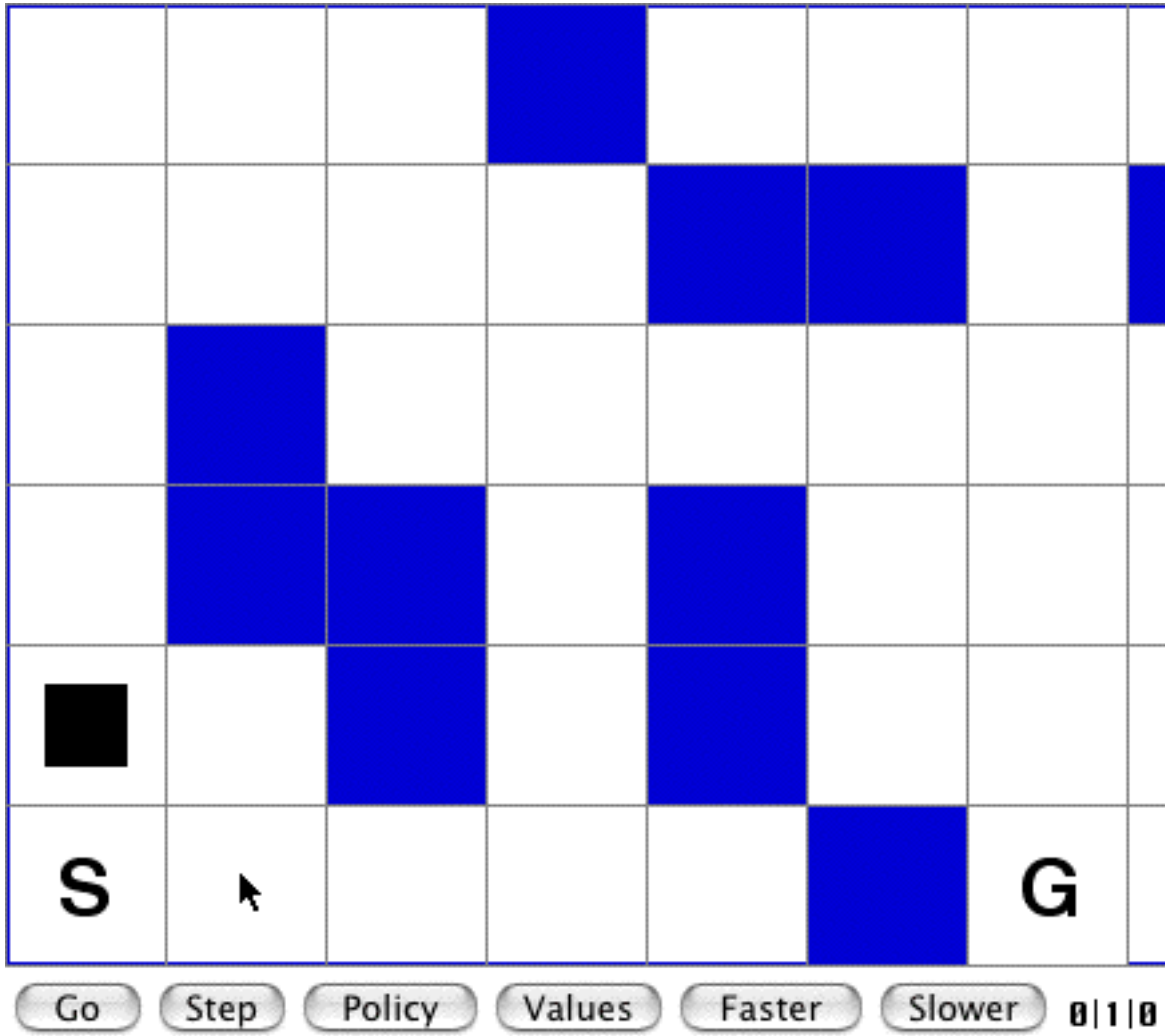


Hold

GridWorld Example



Discussion advice

- Don’t ask questions about the basics of the material e.g.
  - “I didn’t understand figure 3.x”
  - “Why is there a max in the bellman optimality equation?”
- This is something you should just ask in:
  - #ta-lounge, office hours, or in class
- Ask questions you would to start a discussion with a group of people
  - e.g., is the MDP formulation limited? What problems might **not** be well framed this way?

Discussion advice

- The goal of chapter 3 is to become comfortable with the formalizations and notations of RL
- Use the math when needed to ask precise questions, using regular language will often not be precise enough
  - E.g., if you are asking about the value of a state, just write `v_{\pi}`

How many learning methods are discussed in chapter 3?

- There **exists** an optimal for an MDP
- We can **compute** the value function for a given MDP and policy
    - e.g., by hand calculations
- We can **derive** the bellman equation from the **definition** of the value function and knowledge of MDPs
- **Estimating** the value function or optimal policy typically involves an iterative algorithm or data generated from an agent interacting with an MDP
- **Approximating** the value function or optimal policy typically involves data and a function approximator

Fancy MDPs

- What about infinite MDPs?
  - Example?
  - Do we care about them?
  - What about continuous time?

Optimal Policies

- Are all optimal policies deterministic?
  - In MDPs we assume things are stationary (ie the reward function and transitions don’t change with time)
  - There can be many optimal policies. How?
  - What changes if we consider 2-player zero sum games?
    - Do we care?

State

- The meaning of Markov State
    - The agent would never do better by keeping a history of prior states and actions
- Remember MDPs are an abstract mathematical concept
  - Its a set of assumptions which may not be true in practice
  - What is important is if MDPs can be used to prove useful results and develop algs
    - e.g., Temporal difference learning

General

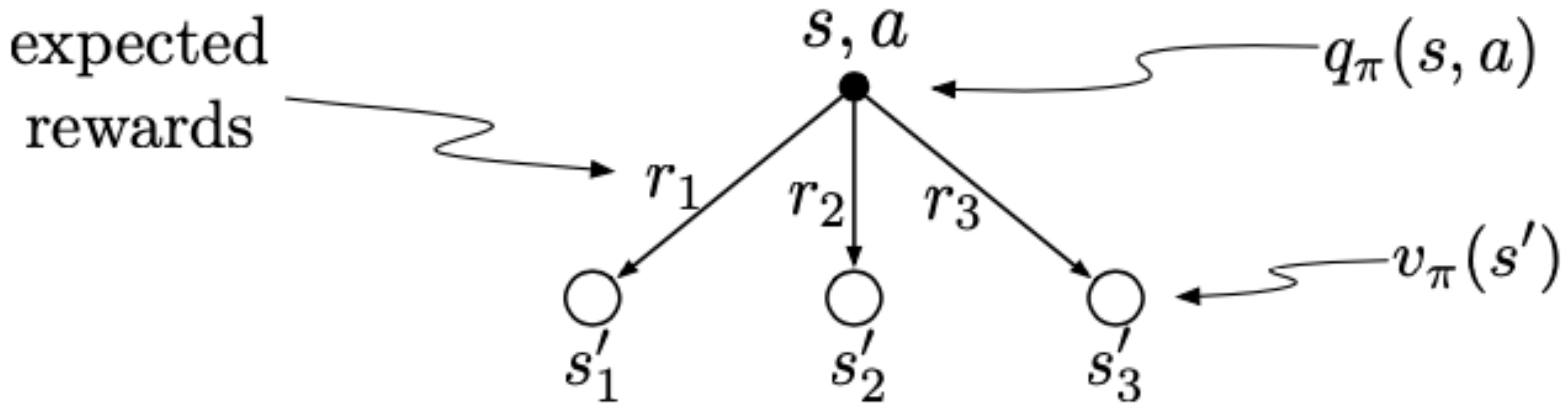
- Can we use A* or other search algorithms?
- Should such an agent still care about convergence if the environment is much bigger than the agent?
  - What do we mean here?
- How do we handle exploration in the optimal policy (Ch3 generally)?
- What is the difference between v_{\pi}(S) and q_{\pi}(S,A)?
  - Why would it be useful to ask q_{\pi} about A’s that \pi does not select?

Specifying the problem

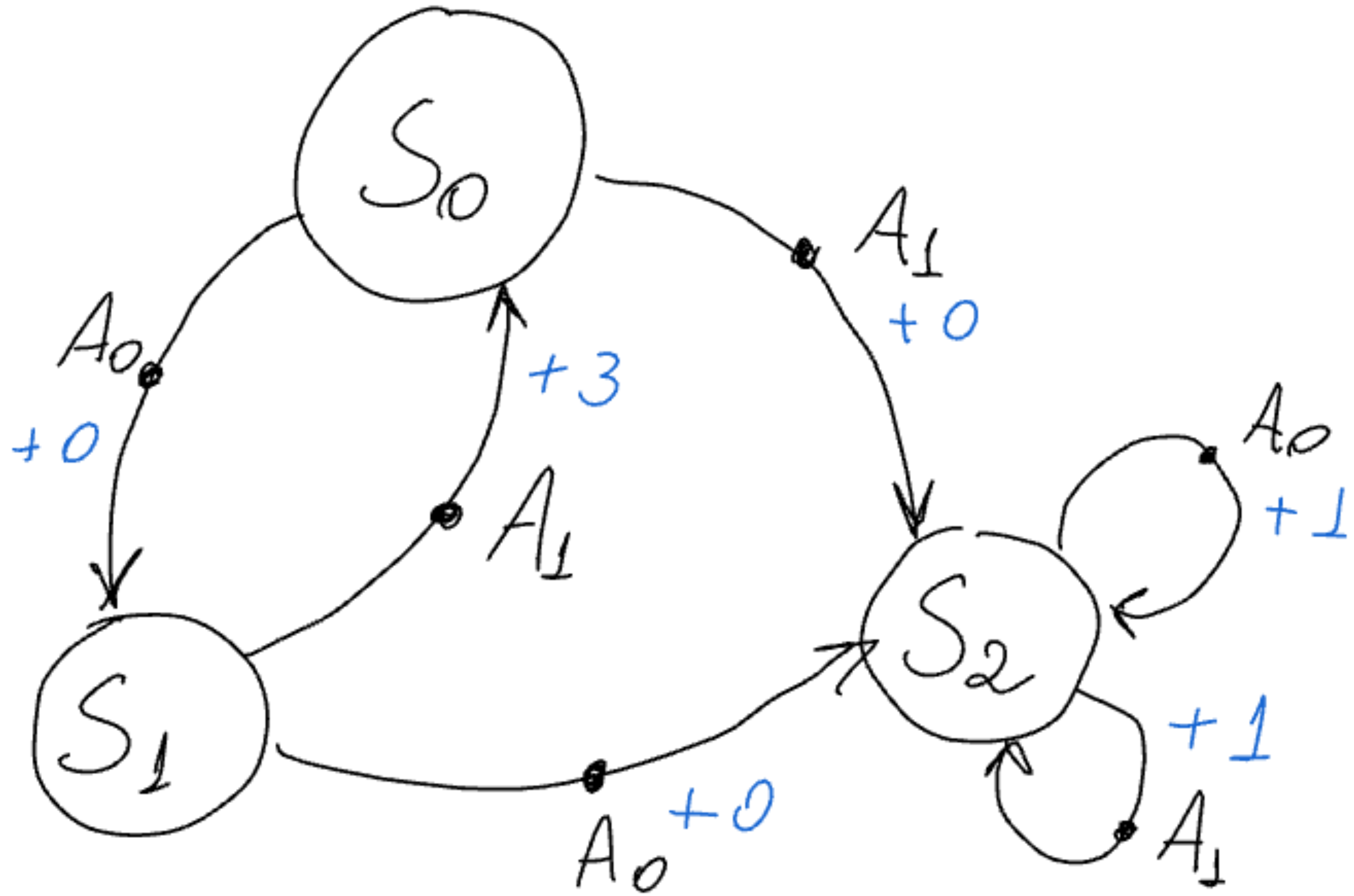
- Do we need to search for the best value of the **discount rate** also when finding the optimal policy?
  - Changing the problem is generally a bad idea
- This chapter is about problem formulations not solution methods
- Imagine learning about sorting list:
  - We know the best case performance depends on the length of the list
  - Would you ask: should we change the length of the list to get better performance?
    - NO!

Definitions vs other things

$$q_\pi(s,a) \doteq \mathbb{E}_\pi[G_t \mid S_t = s, A_t = a] = \mathbb{E}_\pi\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \,\middle|\, S_t = s, A_t = a\right]. \quad (3.13)$$



Solve for V*



Worksheet Review

3. (Exercise 2.2 from S&B 2nd edition) Consider a k-armed bandit problem with $k = 4$ actions, denoted 1, 2, 3, and 4. Consider applying to this problem a bandit algorithm using ϵ-greedy action selection, sample-average action-value estimates, and initial estimates of $Q_1(a) = 0$, for all a. Suppose the initial sequence of actions and rewards is $A_1 = 1, R_1 = -1, A_2 = 2, R_2 = 1, A_3 = 2, R_3 = -2, A_4 = 2, R_4 = 2, A_5 = 3, R_5 = 0$. On some of these time steps the ϵ case may have occurred, causing an action to be selected at random. On which time steps did this definitely occur? On which time steps could this possibly have occurred?

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T	Q1	Q2	Q3	Q4	{A*_t}	A_t	Explore?	R_1
1								
2								
3								
4								
5								

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T	Q1	Q2	Q3	Q4	{A*_t}	A_t	Explore?	R_1
1	0	0	0	0	{1,2,3,4}	1		
2								
3								
4								
5								

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T	Q1	Q2	Q3	Q4	{A*_t}	A_t	Explore?	R_1
1	0	0	0	0	{1,2,3,4}	1	Maybe	-1
2	-1	0	0	0	{2,3,4}	2	Maybe	1
3	-1	1	0	0		2		-2
4	-1	-0.5	0	0		2		2
5	-1	0.3333	0	0		3		0

Key learnings

T	Q1	Q2	Q3	Q4	{A*_t}	A_t	Explore?	R_1
1	0	0	0	0	{1,2,3,4}	1	Maybe	-1
2	-1	0	0	0	{2,3,4}	2	Maybe	1
3	-1	1	0	0		2		-2
4	-1	-0.5	0	0		2		2
5	-1	0.3333	0	0		3		0

- Initial Q-values (all zeros) do not impact the computation of the sample average
- We don't change the values of actions not taken
- The explore step or \epsilon step might choose the greedy action: can only know for sure when agent explores
- Try to imagine the agents life:
  - Look at the Q-values; pick an action; observe the reward; update one of the Q-values
  - Look at the Q-values; pick an action; observe the reward; update one of the Q-values
  - …. forever

Worksheet Review

1. Suppose $\gamma = 0.9$ and the reward sequence is $R_1 = 2$, $R_2 = -2$, $R_3 = 0$ followed by an infinite sequence of 7s. What are G_1 and G_0?

Worksheet Review

1. Suppose $\gamma = 0.9$ and the reward sequence is $R_1 = 2, R_2 = -2, R_3 = 0$ followed by an infinite sequence of 7s. What are G_1 and G_0?

- Work backwards
- Sequence of rewards is: R1 = 2, R2 = -2, R3 = 0, R4 = 7, R5 = 7, …..
- Let's start with R4=7, the first of the unending sequence of 7's
- Using our formula G_t = R_{t+1} + gamma R_{t+2} + gamma^2 R_{t+3} + …
  we write it as:
  - G_3 = R_4 + gamma R_5 + gamma^2 R_6 + gamma^3 R_7 + …
    = 7 + 0.9*7 + (0.9)^2*7 + + (0.9)^3*7…
  - Use our special formula to work out G_3: $\sum_{k=0}^{\infty} \gamma^k R = \frac{R}{1 - \gamma}$
-

Worksheet Review

1. Suppose $\gamma = 0.9$ and the reward sequence is $R_1 = 2, R_2 = -2, R_3 = 0$ followed by an infinite sequence of 7s. What are G_1 and G_0?

- Continue working backwards from G_3 using our other special formula
  - $G_t = R_{t+1} + \gamma G_{t+1}$

Answer:

$$G_3 = 7 + 0.9 \times 7 + 0.9^2 \times 7 + ... = 7 \times \frac{1}{1-0.9} = 70$$
$$G_2 = R_3 + 0.9 \times G_3 = 0 + 0.9 \times 70 = 63$$
$$G_1 = R_2 + 0.9 \times G_2 = -2 + 0.9 \times 63 = 54.7$$
$$G_0 = R_1 + 0.9 \times G_1 = 2 + 0.9 \times 54.7 = 51.23$$

Worksheet Question

4. Prove that the discounted sum of rewards is always finite, if the rewards are bounded: $|R_{t+1}| \leq R_{\max}$ for all t for some finite $R_{\max} > 0$.

$$\left| \sum_{i=0}^{\infty} \gamma^i R_{t+1+i} \right| < \infty \qquad \text{for } \gamma \in [0, 1)$$

Recall $\displaystyle\sum_{i=0}^{\infty} |a_i| < \infty$ then $\displaystyle\left| \sum_{i=0}^{\infty} a_i \right| < \infty$

4. Prove that the discounted sum of rewards is always finite, if the rewards are bounded: $|R_{t+1}| \leq R_{\max}$ for all t for some finite $R_{\max} > 0$.

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$$= \sum_{i=0}^{\infty} \gamma^i |R_{t+1+i}|$$

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$$\leq \sum_{i=0}^{\infty} \gamma^i R_{\max}$$

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$$= \sum_{i=0}^{\infty} \gamma^i |R_{t+1+i}|$$

$$\leq \sum_{i=0}^{\infty} \gamma^i R_{\max}$$

$$= R_{\max} \sum_{i=0}^{\infty} \gamma^i$$

4. Prove that the discounted sum of rewards is always finite, if the rewards are bounded: $|R_{t+1}| \leq R_{\max}$ for all t for some finite $R_{\max} > 0$.

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$$= \sum_{i=0}^{\infty} \gamma^i |R_{t+1+i}|$$

$$\leq \sum_{i=0}^{\infty} \gamma^i R_{\max}$$

$$= R_{\max} \sum_{i=0}^{\infty} \gamma^i = \frac{R_{\max}}{1 - \gamma}$$

$R_{\max}$ and $\frac{1}{1-\gamma}$ are finite.