1 Ray-Sphere Intersection

Equation of a Line: $x = o + dl$

- x: a point along the line
- o: the origin point of the line
- d: the distance along the line
- l: the direction vector of the line (normalized)

Equation of a Sphere: $\|x - c\|^2 = r^2$

- x: a point on the sphere
- c: the center point on the sphere
- r: the radius of the sphere

Math Occurs

$$d = -(l \cdot (o - c)) \pm \sqrt{(l \cdot (o - c))^2 - (\|o - c\|^2 - r^2)}$$

Notes:

- For rays, all intersections must be $d \geq 0$ since anything outside of that is outside of the ray.
- If the term inside the square root is negative, the line does not intersect the sphere.
- If the term is zero, there is exactly a single intersection. Otherwise, there are two.

2 Ray-Plane Intersection

Equation of a Line: $x = o + dl$

- x: a point along the line
- o: the origin point of the line
- d: the distance along the line
- l: the direction vector of the line (normalized)

Equation of a Plane: $(p - p_0) \cdot n = 0$

- p: a point on the plane
- p_0: the origin point of the plane
- n: the surface normal of the plane

Math Occurs

$$d = \frac{(p_0 - o) \cdot n}{l \cdot n}$$

Notes:

- For rays, all intersections must be $d \geq 0$ since anything outside of that is outside of the ray.
- If $l \cdot n = 0$, the line is parallel to the plane.

3 Lambertian Light

$$I_d = l \cdot nCI_l$$

- I_d: the intensity of diffuse light
- l: the vector toward the light from the point of intersection (normalized)
- n: the surface normal at the point of intersection
- C: the color of the surface
- I_l: the intensity of the light

4 Whitted Model

$$I = I_a + k_d \sum_{j=0}^{lights} (n \cdot l_j) + k_s S + k_t T$$

- I: the intensity of light in the Whitted model
- I_a: the intensity of ambient light (constant in the scene)
- k_d: the diffuse coefficient
- n: the surface normal at the point of intersection
- l_j: the vector from the point of intersection to the light (normalized)

- k_s: the specular coefficient
- S: the intensity of reflected light
- k_t: the transmission coefficient
- T: the intensity of refracted light

5 Reflection

Equation for a vector that is the component of the surface normal that is perpendicular to the normal direction: $perp_n l = l - (n \cdot l)n$

$$R = 2(n \cdot l)n - l$$

This is much easier to explain via a diagram. Mathematics for 3D Game Programming & Computer Graphics contains an easy-to-follow diagram and explanation on page 152-153.