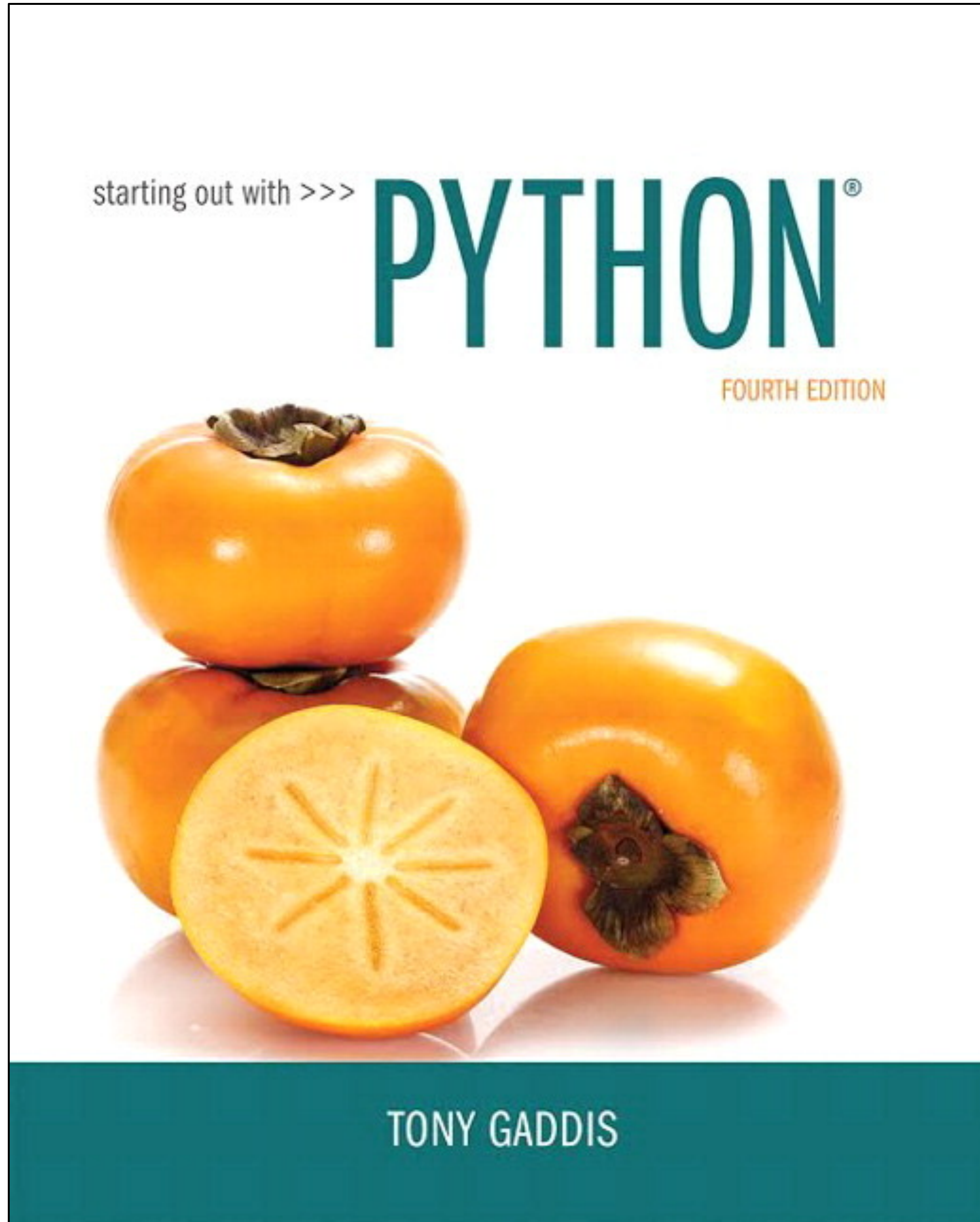
CHAPTER 12

# Recursion



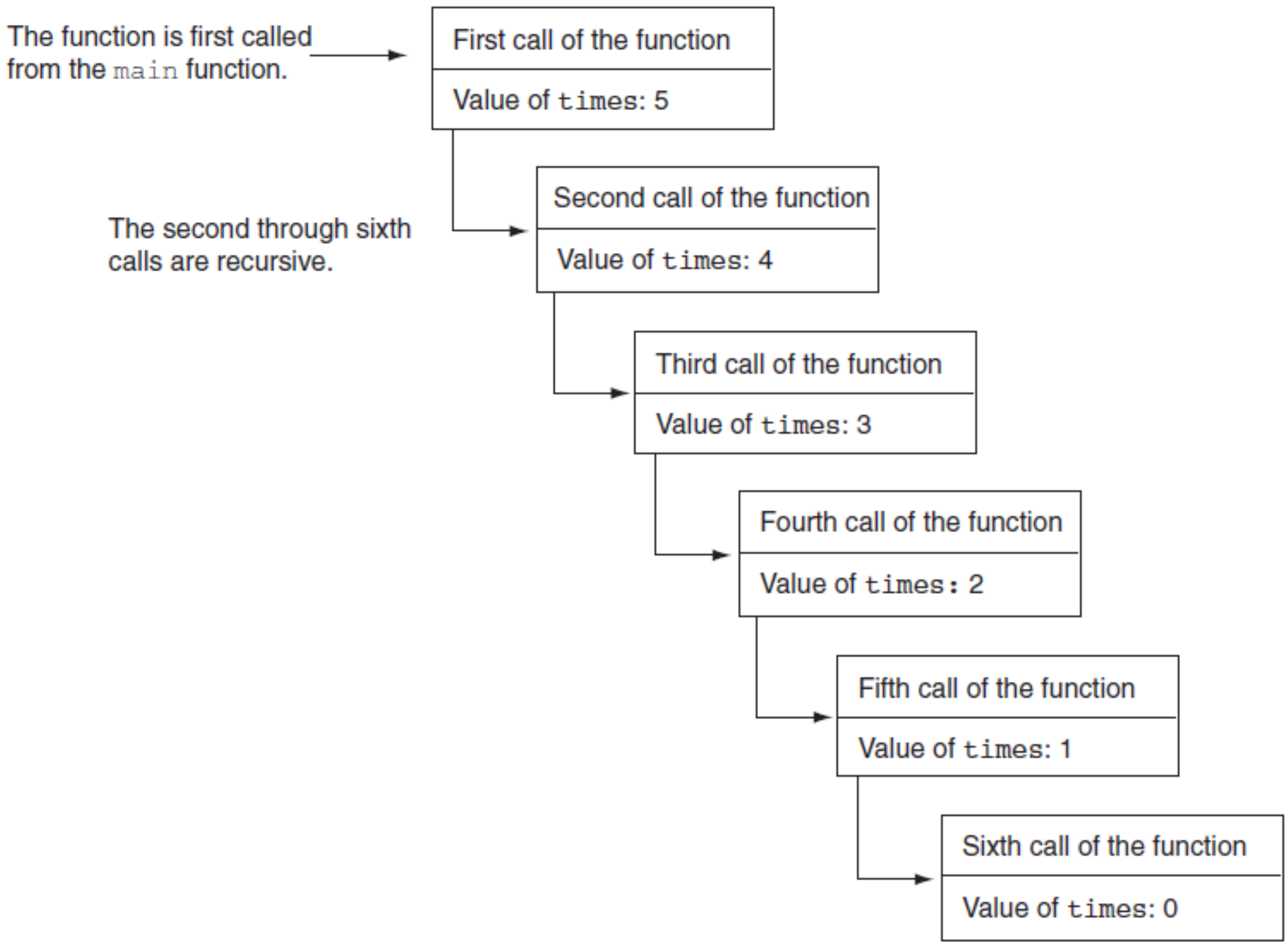
# Topics

- Introduction to Recursion
- Problem Solving with Recursion
- Examples of Recursive Algorithms

# Introduction to Recursion

- **<u>Recursive function</u>: a function that calls itself**
- **Recursive function must have a way to control the number of times it repeats**
  - Usually involves an `if-else` statement which defines when the function should return a value and when it should call itself
- **<u>Depth of recursion</u>: the number of times a function calls itself**

## Figure 12-2 Six calls to the message function



# Introduction to Recursion (cont'd.)

**Figure 12-3** Control returns to the point after the recursive function call

Recursive function call

```
def message(times):
    if times > 0:
        print('This is a recursive function.')
        message(times - 1)
```

Control returns here from the recursive call.
There are no more statements to execute
in this function, so the function returns.

# Problem Solving with Recursion

- **Recursion is a powerful tool for solving repetitive problems**
- **Recursion is never required to solve a problem**
  - Any problem that can be solved recursively can be solved with a loop
  - Recursive algorithms usually less efficient than iterative ones
    - Due to *overhead* of each function call

# Problem Solving with Recursion (cont'd.)

- **Some repetitive problems are more easily solved with recursion**
- **General outline of recursive function:**
  - If the problem can be solved now without recursion, solve and return
    - Known as the *base case*
  - Otherwise, reduce problem to smaller problem of the same structure and call the function again to solve the smaller problem
    - Known as the *recursive case*

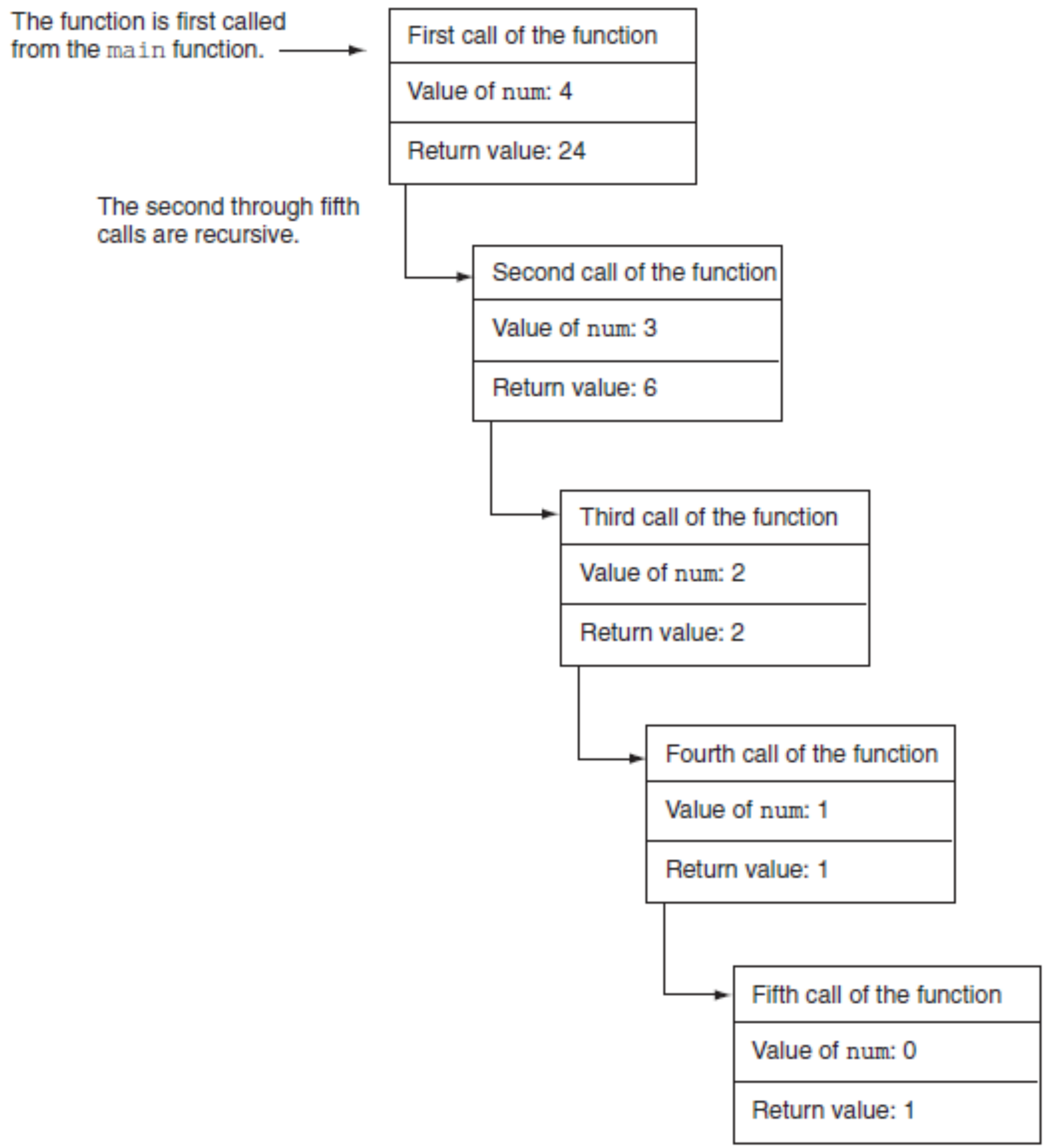
# Using Recursion to Calculate the Factorial of a Number

- **In mathematics, the $n!$ notation represents the factorial of a number *n***
  - For $n = 0$, $n! = 1$
  - For $n > 0$, $n! = 1 \times 2 \times 3 \times \ldots \times n$
- **The above definition lends itself to recursive programming**
  - $n = 0$ is the base case
  - $n > 0$ is the recursive case
    - factorial(*n*) = $n$ x factorial($n$-1)

# Using Recursion (cont'd.)

```
# The factorial function uses recursion to
# calculate the factorial of its argument,
# which is assumed to be nonnegative.
def factorial(num):
    if num == 0:
        return 1
    else:
        return num * factorial(num - 1)
```

Figure 12-4 The value of `num` and the return value during each call of the function



# Using Recursion (cont'd.)

- **Since each call to the recursive function reduces the problem:**
  - Eventually, it will get to the base case which does not require recursion, and the recursion will stop
- **Usually the problem is reduced by making one or more parameters smaller at each function call**

# Direct and Indirect Recursion

- **Direct recursion: when a function directly calls itself**
  - All the examples shown so far were of direct recursion
- **Indirect recursion: when function A calls function B, which in turn calls function A**

# Examples of Recursive Algorithms

- **Summing a range of list elements with recursion**
  - Function receives a list containing range of elements to be summed, index of starting item in the range, and index of ending item in the range
  - Base case:
    - `if start index > end index return 0`
  - Recursive case:
    - `return current_number + sum(list, start+1, end)`

# Examples of Recursive Algorithms (cont’d.)

```
# The range_sum function returns the sum of a specified
# range of items in num_list. The start parameter
# specifies the index of the starting item. The end
# parameter specifies the index of the ending item.
def range_sum(num_list, start, end):
    if start > end:
        return 0
    else:
        return num_list[start] + range_sum(num_list, start + 1, end)
```

# The Fibonacci Series

- **Fibonacci series: has two base cases**
  - `if n = 0 then Fib(n) = 0`
  - `if n = 1 then Fib(n) = 1`
  - `if n > 1 then Fib(n) = Fib(n-1) + Fib(n-2)`
- **Corresponding function code:**

```
# The fib function returns the nth number
# in the Fibonacci series.
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n - 1) + fib(n - 2)
```

# Finding the Greatest Common Divisor

- **Calculation of the greatest common divisor (GCD) of two positive integers**
  - If x can be evenly divided by y, then
  - $\gcd(x,y) = y$
  - Otherwise, $\gcd(x,y) = \gcd(y, \text{remainder of } x/y)$
- **Corresponding function code:**

```
# The gcd function returns the greatest common
# divisor of two numbers.
def gcd(x, y):
    if x % y == 0:
        return y
    else:
        return gcd(x, x % y)
```

# The Towers of Hanoi

- **Mathematical game commonly used to illustrate the power of recursion**
  - Uses three pegs and a set of discs in decreasing sizes
  - <u>Goal of the game</u>: move the discs from leftmost peg to rightmost peg
    - Only one disc can be moved at a time
    - A disc cannot be placed on top of a smaller disc
    - All discs must be on a peg except while being moved

# The Towers of Hanoi (cont'd.)

**Figure 12-5** The pegs and discs in the Tower of Hanoi game

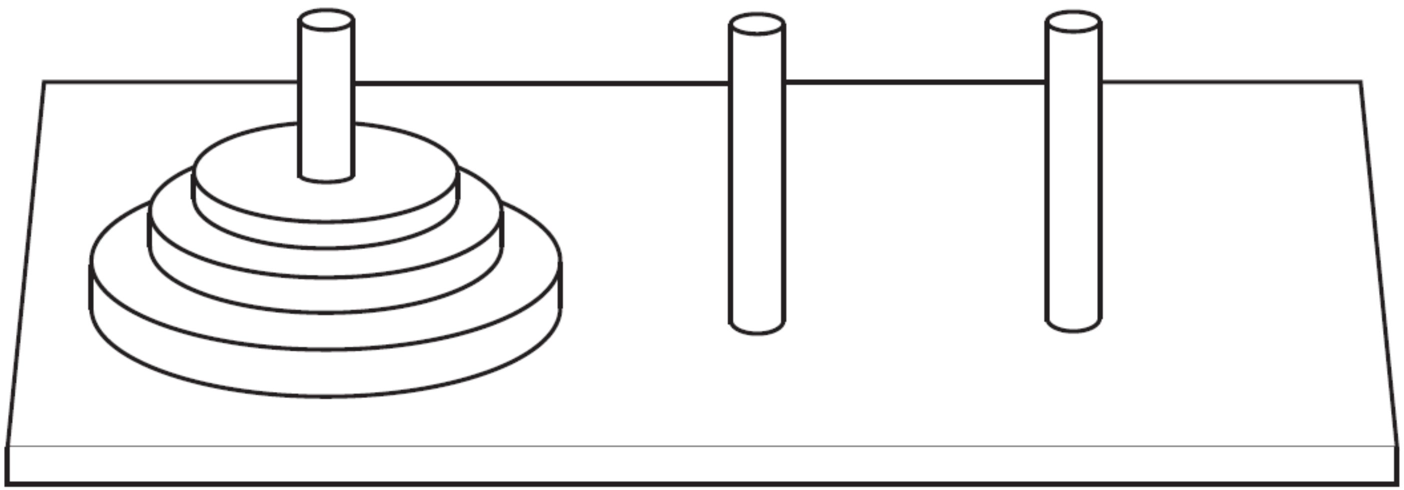
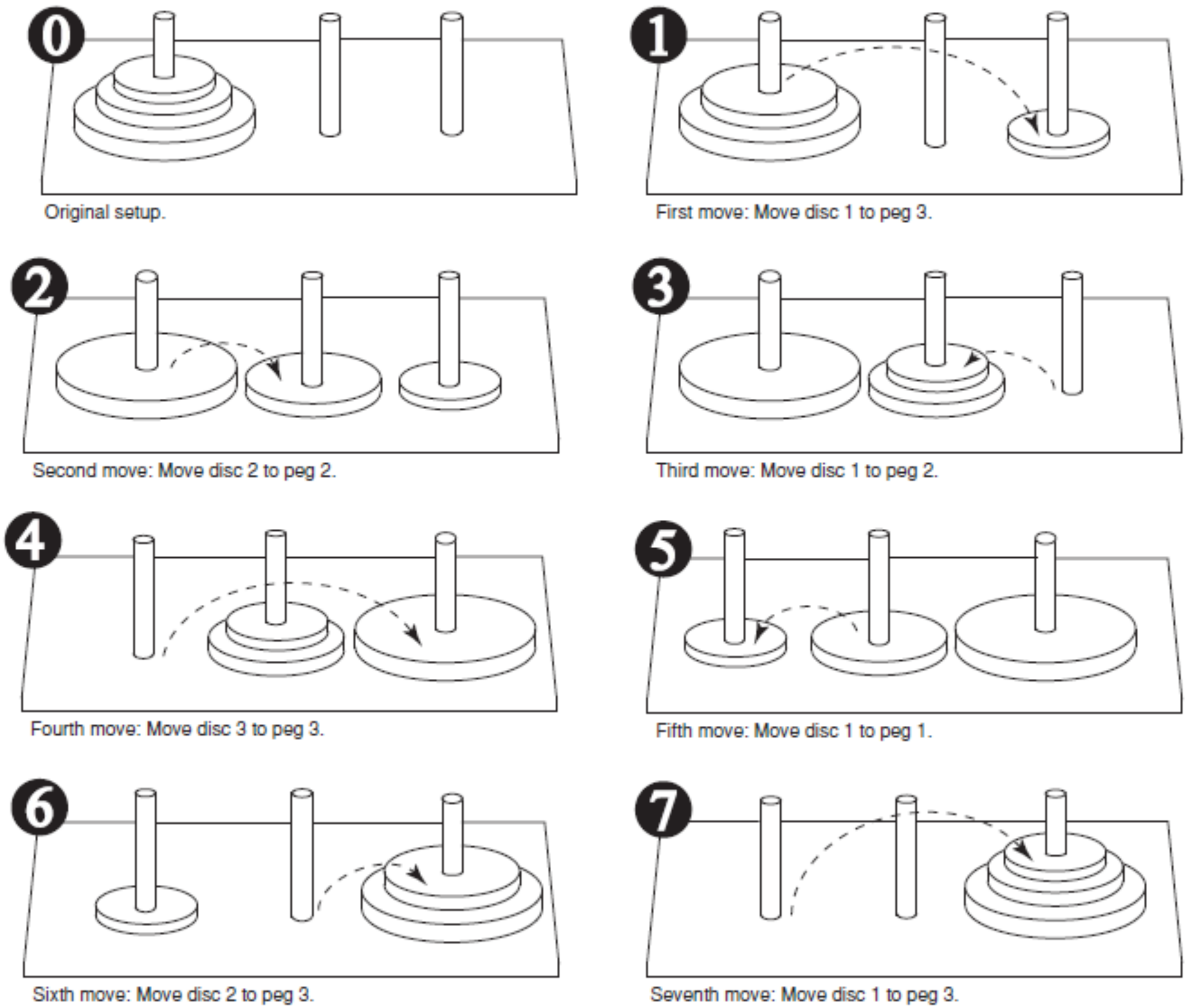


**Figure 12-6** Steps for moving three pegs



# The Towers of Hanoi (cont'd)

- **<u>Problem statement</u>: move n discs from peg 1 to peg 3 using peg 2 as a temporary peg**
- **<u>Recursive solution</u>:**
  - If n == 1: Move disc from peg 1 to peg 3
  - Otherwise:
    - Move n-1 discs from peg 1 to peg 2, using peg 3
    - Move remaining disc from peg 1 to peg 3
    - Move n-1 discs from peg 2 to peg 3, using peg 1

# The Towers of Hanoi (cont'd.)

```
# The moveDiscs function displays a disc move in
# the Towers of Hanoi game.
# The parameters are:
#    num:         The number of discs to move.
#    from_peg:    The peg to move from.
#    to_peg:      The peg to move to.
#    temp_peg:    The temporary peg.
def move_discs(num, from_peg, to_peg, temp_peg):
    if num > 0:
        move_discs(num - 1, from_peg, temp_peg, to_peg)
        print('Move a disc from peg', from_peg, 'to peg', to_peg)
        move_discs(num - 1, temp_peg, to_peg, from_peg)
```

# Recursion versus Looping

- **Reasons not to use recursion:**
  - Less efficient: entails function calling overhead that is not necessary with a loop
  - Usually a solution using a loop is more evident than a recursive solution
- **Some problems are more easily solved with recursion than with a loop**
  - Example: Fibonacci, where the mathematical definition lends itself to recursion

# Summary

- **This chapter covered:**
  - Definition of recursion
  - The importance of the base case
  - The recursive case as reducing the problem size
  - Direct and indirect recursion
  - Examples of recursive algorithms
  - Recursion versus looping