Lab 7

Amy Butler

11:59PM April 22, 2021

We will get some experience with speeding up R code using C++ via the `Rcpp` package.

First, clear the workspace and load the `Rcpp` package.

```
pacman::p_load(Rcpp)
```

Create a variable `n` to be 10 and a vaiable `Nvec` to be 100 initially. Create a random vector via `rnorm` `Nvec` times and load it into a `Nvec` x `n` dimensional matrix.

```
n=10
Nvec=100
X=matrix(data=rnorm(Nvec*n), nrow=Nvec, ncol=n)
head(X)
```

```
##            [,1]       [,2]       [,3]       [,4]       [,5]        [,6]
## [1,] -0.5244505  0.8784550  0.1032754 -0.1961418  1.3316479 -0.42355677
## [2,] -2.0396655  1.2125500  0.7139426 -0.3007843  0.3456104 -0.01736970
## [3,] -0.7203845 -1.6244293 -0.5252175 -0.6642094  1.7918216  0.76601917
## [4,] -0.2819080  0.6103835 -0.5815852 -0.6541447  1.7908089  0.05398963
## [5,]  0.5480315 -0.1137883 -1.8707033  0.1857361 -1.7532119 -1.07996065
## [6,] -0.6019071  1.2816607  0.1123712  0.4462137  1.2573843 -0.08834486
##            [,7]        [,8]        [,9]      [,10]
## [1,] -1.2616912 -1.52872003 -0.65200494 -0.1604301
## [2,]  0.7704068 -0.03932468 -0.43790830 -2.2488365
## [3,] -0.9727069 -1.58934492 -0.27026273  0.4789867
## [4,]  0.8050429  2.02661328 -0.52061150  0.6828799
## [5,]  0.3162035  1.00481594 -0.51855713  1.6057275
## [6,] -1.3723291 -1.27424375 -0.01504413  1.9213735
```

Write a function `all_angles` that measures the angle between each of the pairs of vectors. You should measure the vector on a scale of 0 to 180 degrees with negative angles coerced to be positive.

```
angle = function(u,v){
  acos(sum(u*v)/sqrt(sum(u^2)*sum(v^2)))*(180/pi)
}

all_angles = function(X){
  A=matrix(NA, nrow=nrow(X), ncol=nrow(X))
  for(i in 1:(nrow(X)-1)){
    for(j in (i+1):nrow(X)){
      A[i,j]=angle(X[i,],X[j,])
    }
```

```
  }
  A
}
all_angles(X)

##        [,1]     [,2]     [,3]     [,4]      [,5]      [,6]      [,7]
[,8]
##   [1,]   NA 74.96096 58.30506 95.14488 117.37176  44.41146  77.16515
56.76138
##   [2,]   NA       NA 98.34774 85.05845 121.95566 100.73389  93.65988
60.09741
##   [3,]   NA       NA       NA 91.79928 111.56005  66.55170 111.26631
84.35589
##   [4,]   NA       NA       NA       NA  83.68704  87.39752  73.30535
83.46179
##   [5,]   NA       NA       NA       NA        NA  96.65098  89.75711
121.88241
##   [6,]   NA       NA       NA       NA        NA        NA  74.74980
55.19153
##   [7,]   NA       NA       NA       NA        NA        NA        NA
87.16600
##   [8,]   NA       NA       NA       NA        NA        NA        NA
NA
##   [9,]   NA       NA       NA       NA        NA        NA        NA
NA
##  [10,]   NA       NA       NA       NA        NA        NA        NA
NA
##  [11,]   NA       NA       NA       NA        NA        NA        NA
NA
##  [12,]   NA       NA       NA       NA        NA        NA        NA
NA
##  [13,]   NA       NA       NA       NA        NA        NA        NA
NA
##  [14,]   NA       NA       NA       NA        NA        NA        NA
NA
##  [15,]   NA       NA       NA       NA        NA        NA        NA
NA
##  [16,]   NA       NA       NA       NA        NA        NA        NA
NA
##  [17,]   NA       NA       NA       NA        NA        NA        NA
NA
##  [18,]   NA       NA       NA       NA        NA        NA        NA
NA
##  [19,]   NA       NA       NA       NA        NA        NA        NA
NA
##  [20,]   NA       NA       NA       NA        NA        NA        NA
NA
##  [21,]   NA       NA       NA       NA        NA        NA        NA
NA
##  [22,]   NA       NA       NA       NA        NA        NA        NA
```

```
NA
## [23,] NA NA NA NA NA NA NA
NA
## [24,] NA NA NA NA NA NA NA
NA
## [25,] NA NA NA NA NA NA NA
NA
## [26,] NA NA NA NA NA NA NA
NA
## [27,] NA NA NA NA NA NA NA
NA
## [28,] NA NA NA NA NA NA NA
NA
## [29,] NA NA NA NA NA NA NA
NA
## [30,] NA NA NA NA NA NA NA
NA
## [31,] NA NA NA NA NA NA NA
NA
## [32,] NA NA NA NA NA NA NA
NA
## [33,] NA NA NA NA NA NA NA
NA
## [34,] NA NA NA NA NA NA NA
NA
## [35,] NA NA NA NA NA NA NA
NA
## [36,] NA NA NA NA NA NA NA
NA
## [37,] NA NA NA NA NA NA NA
NA
## [38,] NA NA NA NA NA NA NA
NA
## [39,] NA NA NA NA NA NA NA
NA
## [40,] NA NA NA NA NA NA NA
NA
## [41,] NA NA NA NA NA NA NA
NA
## [42,] NA NA NA NA NA NA NA
NA
## [43,] NA NA NA NA NA NA NA
NA
## [44,] NA NA NA NA NA NA NA
NA
## [45,] NA NA NA NA NA NA NA
NA
## [46,] NA NA NA NA NA NA NA
NA
## [47,] NA NA NA NA NA NA NA
```

```
NA
## [48,]   NA      NA      NA      NA      NA      NA      NA
NA
## [49,]   NA      NA      NA      NA      NA      NA      NA
NA
## [50,]   NA      NA      NA      NA      NA      NA      NA
NA
## [51,]   NA      NA      NA      NA      NA      NA      NA
NA
## [52,]   NA      NA      NA      NA      NA      NA      NA
NA
## [53,]   NA      NA      NA      NA      NA      NA      NA
NA
## [54,]   NA      NA      NA      NA      NA      NA      NA
NA
## [55,]   NA      NA      NA      NA      NA      NA      NA
NA
## [56,]   NA      NA      NA      NA      NA      NA      NA
NA
## [57,]   NA      NA      NA      NA      NA      NA      NA
NA
## [58,]   NA      NA      NA      NA      NA      NA      NA
NA
## [59,]   NA      NA      NA      NA      NA      NA      NA
NA
## [60,]   NA      NA      NA      NA      NA      NA      NA
NA
## [61,]   NA      NA      NA      NA      NA      NA      NA
NA
## [62,]   NA      NA      NA      NA      NA      NA      NA
NA
## [63,]   NA      NA      NA      NA      NA      NA      NA
NA
## [64,]   NA      NA      NA      NA      NA      NA      NA
NA
## [65,]   NA      NA      NA      NA      NA      NA      NA
NA
## [66,]   NA      NA      NA      NA      NA      NA      NA
NA
## [67,]   NA      NA      NA      NA      NA      NA      NA
NA
## [68,]   NA      NA      NA      NA      NA      NA      NA
NA
## [69,]   NA      NA      NA      NA      NA      NA      NA
NA
## [70,]   NA      NA      NA      NA      NA      NA      NA
NA
## [71,]   NA      NA      NA      NA      NA      NA      NA
NA
## [72,]   NA      NA      NA      NA      NA      NA      NA
```

```
NA
## [73,]   NA      NA      NA      NA      NA      NA      NA
NA
## [74,]   NA      NA      NA      NA      NA      NA      NA
NA
## [75,]   NA      NA      NA      NA      NA      NA      NA
NA
## [76,]   NA      NA      NA      NA      NA      NA      NA
NA
## [77,]   NA      NA      NA      NA      NA      NA      NA
NA
## [78,]   NA      NA      NA      NA      NA      NA      NA
NA
## [79,]   NA      NA      NA      NA      NA      NA      NA
NA
## [80,]   NA      NA      NA      NA      NA      NA      NA
NA
## [81,]   NA      NA      NA      NA      NA      NA      NA
NA
## [82,]   NA      NA      NA      NA      NA      NA      NA
NA
## [83,]   NA      NA      NA      NA      NA      NA      NA
NA
## [84,]   NA      NA      NA      NA      NA      NA      NA
NA
## [85,]   NA      NA      NA      NA      NA      NA      NA
NA
## [86,]   NA      NA      NA      NA      NA      NA      NA
NA
## [87,]   NA      NA      NA      NA      NA      NA      NA
NA
## [88,]   NA      NA      NA      NA      NA      NA      NA
NA
## [89,]   NA      NA      NA      NA      NA      NA      NA
NA
## [90,]   NA      NA      NA      NA      NA      NA      NA
NA
## [91,]   NA      NA      NA      NA      NA      NA      NA
NA
## [92,]   NA      NA      NA      NA      NA      NA      NA
NA
## [93,]   NA      NA      NA      NA      NA      NA      NA
NA
## [94,]   NA      NA      NA      NA      NA      NA      NA
NA
## [95,]   NA      NA      NA      NA      NA      NA      NA
NA
## [96,]   NA      NA      NA      NA      NA      NA      NA
NA
## [97,]   NA      NA      NA      NA      NA      NA      NA
```

```
NA
##  [98,]   NA       NA       NA       NA        NA        NA        NA
NA
##  [99,]   NA       NA       NA       NA        NA        NA        NA
NA
## [100,]   NA       NA       NA       NA        NA        NA        NA
NA
##              [,9]     [,10]     [,11]     [,12]     [,13]     [,14]
[,15]
##   [1,]  60.59757  97.26438  93.56614  80.52468 105.27069 104.92753
104.30534
##   [2,]  62.55616 109.69861  84.23470  63.54389 100.99383  73.00741
87.68469
##   [3,]  78.94462  71.79650  80.85141  57.81922  98.40114 126.56114
112.29206
##   [4,]  83.30746  82.34693 122.69800  85.29530  80.29521  86.47545
59.51062
##   [5,] 131.59485 105.55147 103.27777 118.77607  67.66877  66.83092
65.58701
##   [6,]  64.09884  99.69495  99.71056 102.34458 109.28511  98.99705
90.31687
##   [7,]  76.86236  97.77261 108.48006 130.24591  81.16166  83.73414
77.38380
##   [8,]  30.30287  91.81374  83.89255  75.49393 134.35808  81.89048
81.02478
##   [9,]        NA  85.06199  73.10132  76.41457 128.57666  82.89797
80.02632
##  [10,]        NA        NA  79.86289  68.54413 110.93908 128.44481
104.91299
##  [11,]        NA        NA        NA  84.07293 112.53367  86.56154
90.69649
##  [12,]        NA        NA        NA        NA 109.65610 106.65114
108.43218
##  [13,]        NA        NA        NA        NA        NA  79.18247
76.16355
##  [14,]        NA        NA        NA        NA        NA        NA
39.29581
##  [15,]        NA        NA        NA        NA        NA        NA
NA
##  [16,]        NA        NA        NA        NA        NA        NA
NA
##  [17,]        NA        NA        NA        NA        NA        NA
NA
##  [18,]        NA        NA        NA        NA        NA        NA
NA
##  [19,]        NA        NA        NA        NA        NA        NA
NA
##  [20,]        NA        NA        NA        NA        NA        NA
NA
##  [21,]        NA        NA        NA        NA        NA        NA
```

```
NA
## [22,]     NA     NA     NA     NA     NA     NA
NA
## [23,]     NA     NA     NA     NA     NA     NA
NA
## [24,]     NA     NA     NA     NA     NA     NA
NA
## [25,]     NA     NA     NA     NA     NA     NA
NA
## [26,]     NA     NA     NA     NA     NA     NA
NA
## [27,]     NA     NA     NA     NA     NA     NA
NA
## [28,]     NA     NA     NA     NA     NA     NA
NA
## [29,]     NA     NA     NA     NA     NA     NA
NA
## [30,]     NA     NA     NA     NA     NA     NA
NA
## [31,]     NA     NA     NA     NA     NA     NA
NA
## [32,]     NA     NA     NA     NA     NA     NA
NA
## [33,]     NA     NA     NA     NA     NA     NA
NA
## [34,]     NA     NA     NA     NA     NA     NA
NA
## [35,]     NA     NA     NA     NA     NA     NA
NA
## [36,]     NA     NA     NA     NA     NA     NA
NA
## [37,]     NA     NA     NA     NA     NA     NA
NA
## [38,]     NA     NA     NA     NA     NA     NA
NA
## [39,]     NA     NA     NA     NA     NA     NA
NA
## [40,]     NA     NA     NA     NA     NA     NA
NA
## [41,]     NA     NA     NA     NA     NA     NA
NA
## [42,]     NA     NA     NA     NA     NA     NA
NA
## [43,]     NA     NA     NA     NA     NA     NA
NA
## [44,]     NA     NA     NA     NA     NA     NA
NA
## [45,]     NA     NA     NA     NA     NA     NA
NA
## [46,]     NA     NA     NA     NA     NA     NA
```

```
NA
##  [47,]        NA        NA        NA        NA        NA        NA
NA
##  [48,]        NA        NA        NA        NA        NA        NA
NA
##  [49,]        NA        NA        NA        NA        NA        NA
NA
##  [50,]        NA        NA        NA        NA        NA        NA
NA
##  [51,]        NA        NA        NA        NA        NA        NA
NA
##  [52,]        NA        NA        NA        NA        NA        NA
NA
##  [53,]        NA        NA        NA        NA        NA        NA
NA
##  [54,]        NA        NA        NA        NA        NA        NA
NA
##  [55,]        NA        NA        NA        NA        NA        NA
NA
##  [56,]        NA        NA        NA        NA        NA        NA
NA
##  [57,]        NA        NA        NA        NA        NA        NA
NA
##  [58,]        NA        NA        NA        NA        NA        NA
NA
##  [59,]        NA        NA        NA        NA        NA        NA
NA
##  [60,]        NA        NA        NA        NA        NA        NA
NA
##  [61,]        NA        NA        NA        NA        NA        NA
NA
##  [62,]        NA        NA        NA        NA        NA        NA
NA
##  [63,]        NA        NA        NA        NA        NA        NA
NA
##  [64,]        NA        NA        NA        NA        NA        NA
NA
##  [65,]        NA        NA        NA        NA        NA        NA
NA
##  [66,]        NA        NA        NA        NA        NA        NA
NA
##  [67,]        NA        NA        NA        NA        NA        NA
NA
##  [68,]        NA        NA        NA        NA        NA        NA
NA
##  [69,]        NA        NA        NA        NA        NA        NA
NA
##  [70,]        NA        NA        NA        NA        NA        NA
NA
##  [71,]        NA        NA        NA        NA        NA        NA
```

```
NA
## [72,]        NA        NA        NA        NA        NA        NA
NA
## [73,]        NA        NA        NA        NA        NA        NA
NA
## [74,]        NA        NA        NA        NA        NA        NA
NA
## [75,]        NA        NA        NA        NA        NA        NA
NA
## [76,]        NA        NA        NA        NA        NA        NA
NA
## [77,]        NA        NA        NA        NA        NA        NA
NA
## [78,]        NA        NA        NA        NA        NA        NA
NA
## [79,]        NA        NA        NA        NA        NA        NA
NA
## [80,]        NA        NA        NA        NA        NA        NA
NA
## [81,]        NA        NA        NA        NA        NA        NA
NA
## [82,]        NA        NA        NA        NA        NA        NA
NA
## [83,]        NA        NA        NA        NA        NA        NA
NA
## [84,]        NA        NA        NA        NA        NA        NA
NA
## [85,]        NA        NA        NA        NA        NA        NA
NA
## [86,]        NA        NA        NA        NA        NA        NA
NA
## [87,]        NA        NA        NA        NA        NA        NA
NA
## [88,]        NA        NA        NA        NA        NA        NA
NA
## [89,]        NA        NA        NA        NA        NA        NA
NA
## [90,]        NA        NA        NA        NA        NA        NA
NA
## [91,]        NA        NA        NA        NA        NA        NA
NA
## [92,]        NA        NA        NA        NA        NA        NA
NA
## [93,]        NA        NA        NA        NA        NA        NA
NA
## [94,]        NA        NA        NA        NA        NA        NA
NA
## [95,]        NA        NA        NA        NA        NA        NA
NA
## [96,]        NA        NA        NA        NA        NA        NA
```

```
NA
##  [97,]         NA         NA         NA         NA         NA         NA
NA
##  [98,]         NA         NA         NA         NA         NA         NA
NA
##  [99,]         NA         NA         NA         NA         NA         NA
NA
## [100,]         NA         NA         NA         NA         NA         NA
NA
##            [,16]     [,17]     [,18]     [,19]     [,20]     [,21]
[,22]
##   [1,]  91.98214  68.06754  91.54459  80.95780  84.84030 128.57494
90.45932
##   [2,] 103.06467 112.11123  90.99458  63.24730  80.45766 106.39780
72.83575
##   [3,]  85.01218  64.07856  80.06673  97.03381  78.06210 108.85076
110.37590
##   [4,] 108.91790  65.83452  48.32446  62.82481  98.43500  93.92910
44.18006
##   [5,]  89.33365  94.60526  88.39732  80.50524 129.33105  91.18200
74.60763
##   [6,]  77.15475  72.10205  77.92631 104.07996 102.53450 104.28836
100.10577
##   [7,]  80.01281  71.00167  82.38293  78.18361 102.66497  97.77852
61.21458
##   [8,]  91.36410  92.46143  87.56575 101.22362  77.18124  85.55245
93.92356
##   [9,]  77.61099  87.85596  90.16290 102.66485  72.41494  80.88375
94.96139
##  [10,]  99.73696  58.99458  83.05380 114.03912  53.35998  64.81252
96.16805
##  [11,]  80.00053  99.09818 116.30250 113.10306  66.27202  70.63742
117.26747
##  [12,] 100.66036  93.59760  83.50694  86.12970  67.97998  97.10962
94.82765
##  [13,] 106.19302  80.19818  98.29565  53.89205  98.51877 104.04790
77.35714
##  [14,]  88.25644 118.27950 112.68559  73.49474 106.35235  80.47863
78.98142
##  [15,] 103.78361  84.99745  96.47167  71.80581  95.78443  74.53930
68.54371
##  [16,]        NA 115.24882  83.29011 120.67432 119.51729  84.05717
110.38046
##  [17,]        NA        NA  81.78279  79.94363  69.21357  99.32962
76.37906
##  [18,]        NA        NA        NA  84.95671 116.24318 103.32634
62.39624
##  [19,]        NA        NA        NA        NA 100.32935 131.90807
41.13716
##  [20,]        NA        NA        NA        NA        NA  66.83336
```

```
108.94092
##  [21,]          NA          NA          NA          NA          NA          NA
114.96974
##  [22,]          NA          NA          NA          NA          NA          NA
NA
##  [23,]          NA          NA          NA          NA          NA          NA
NA
##  [24,]          NA          NA          NA          NA          NA          NA
NA
##  [25,]          NA          NA          NA          NA          NA          NA
NA
##  [26,]          NA          NA          NA          NA          NA          NA
NA
##  [27,]          NA          NA          NA          NA          NA          NA
NA
##  [28,]          NA          NA          NA          NA          NA          NA
NA
##  [29,]          NA          NA          NA          NA          NA          NA
NA
##  [30,]          NA          NA          NA          NA          NA          NA
NA
##  [31,]          NA          NA          NA          NA          NA          NA
NA
##  [32,]          NA          NA          NA          NA          NA          NA
NA
##  [33,]          NA          NA          NA          NA          NA          NA
NA
##  [34,]          NA          NA          NA          NA          NA          NA
NA
##  [35,]          NA          NA          NA          NA          NA          NA
NA
##  [36,]          NA          NA          NA          NA          NA          NA
NA
##  [37,]          NA          NA          NA          NA          NA          NA
NA
##  [38,]          NA          NA          NA          NA          NA          NA
NA
##  [39,]          NA          NA          NA          NA          NA          NA
NA
##  [40,]          NA          NA          NA          NA          NA          NA
NA
##  [41,]          NA          NA          NA          NA          NA          NA
NA
##  [42,]          NA          NA          NA          NA          NA          NA
NA
##  [43,]          NA          NA          NA          NA          NA          NA
NA
##  [44,]          NA          NA          NA          NA          NA          NA
NA
##  [45,]          NA          NA          NA          NA          NA          NA
```

```
NA
## [46,]        NA        NA        NA        NA        NA        NA
NA
## [47,]        NA        NA        NA        NA        NA        NA
NA
## [48,]        NA        NA        NA        NA        NA        NA
NA
## [49,]        NA        NA        NA        NA        NA        NA
NA
## [50,]        NA        NA        NA        NA        NA        NA
NA
## [51,]        NA        NA        NA        NA        NA        NA
NA
## [52,]        NA        NA        NA        NA        NA        NA
NA
## [53,]        NA        NA        NA        NA        NA        NA
NA
## [54,]        NA        NA        NA        NA        NA        NA
NA
## [55,]        NA        NA        NA        NA        NA        NA
NA
## [56,]        NA        NA        NA        NA        NA        NA
NA
## [57,]        NA        NA        NA        NA        NA        NA
NA
## [58,]        NA        NA        NA        NA        NA        NA
NA
## [59,]        NA        NA        NA        NA        NA        NA
NA
## [60,]        NA        NA        NA        NA        NA        NA
NA
## [61,]        NA        NA        NA        NA        NA        NA
NA
## [62,]        NA        NA        NA        NA        NA        NA
NA
## [63,]        NA        NA        NA        NA        NA        NA
NA
## [64,]        NA        NA        NA        NA        NA        NA
NA
## [65,]        NA        NA        NA        NA        NA        NA
NA
## [66,]        NA        NA        NA        NA        NA        NA
NA
## [67,]        NA        NA        NA        NA        NA        NA
NA
## [68,]        NA        NA        NA        NA        NA        NA
NA
## [69,]        NA        NA        NA        NA        NA        NA
NA
## [70,]        NA        NA        NA        NA        NA        NA
```

```
NA
## [71,]        NA        NA        NA        NA        NA        NA
NA
## [72,]        NA        NA        NA        NA        NA        NA
NA
## [73,]        NA        NA        NA        NA        NA        NA
NA
## [74,]        NA        NA        NA        NA        NA        NA
NA
## [75,]        NA        NA        NA        NA        NA        NA
NA
## [76,]        NA        NA        NA        NA        NA        NA
NA
## [77,]        NA        NA        NA        NA        NA        NA
NA
## [78,]        NA        NA        NA        NA        NA        NA
NA
## [79,]        NA        NA        NA        NA        NA        NA
NA
## [80,]        NA        NA        NA        NA        NA        NA
NA
## [81,]        NA        NA        NA        NA        NA        NA
NA
## [82,]        NA        NA        NA        NA        NA        NA
NA
## [83,]        NA        NA        NA        NA        NA        NA
NA
## [84,]        NA        NA        NA        NA        NA        NA
NA
## [85,]        NA        NA        NA        NA        NA        NA
NA
## [86,]        NA        NA        NA        NA        NA        NA
NA
## [87,]        NA        NA        NA        NA        NA        NA
NA
## [88,]        NA        NA        NA        NA        NA        NA
NA
## [89,]        NA        NA        NA        NA        NA        NA
NA
## [90,]        NA        NA        NA        NA        NA        NA
NA
## [91,]        NA        NA        NA        NA        NA        NA
NA
## [92,]        NA        NA        NA        NA        NA        NA
NA
## [93,]        NA        NA        NA        NA        NA        NA
NA
## [94,]        NA        NA        NA        NA        NA        NA
NA
## [95,]        NA        NA        NA        NA        NA        NA
```

```
NA
##  [96,]        NA        NA        NA        NA        NA        NA
NA
##  [97,]        NA        NA        NA        NA        NA        NA
NA
##  [98,]        NA        NA        NA        NA        NA        NA
NA
##  [99,]        NA        NA        NA        NA        NA        NA
NA
## [100,]        NA        NA        NA        NA        NA        NA
NA
##            [,23]     [,24]     [,25]     [,26]     [,27]     [,28]
[,29]
##   [1,] 121.55778  67.57681  81.75875  91.45731  88.58148 114.27518
105.27142
##   [2,]  97.75274  75.81881 113.35499  88.68689  64.34215  97.59511
84.85499
##   [3,]  80.47371  71.58851  90.91357 114.44428 112.69241  91.06972
106.84797
##   [4,]  81.84031  94.02179  75.60113 139.82884  75.62958  56.34639
31.99502
##   [5,]  76.97775  86.24645  66.45985  95.41582  76.97236  63.43646
80.00763
##   [6,] 107.40008  88.37305  57.93561  96.97067  87.80392  88.92652
92.65139
##   [7,] 105.35871 114.94031  90.38281  88.53410  93.94594 104.22665
58.61851
##   [8,] 114.23023  96.61969  75.27627  95.01237  59.21425  95.84037
93.38507
##   [9,] 101.11394 113.17747  93.40009 100.49108  75.26599 101.30014
86.36431
##  [10,]  91.76468 108.63249  92.17741 105.76015  98.80750 106.80888
99.53459
##  [11,]  71.81020 105.56276 107.92367  76.63521  92.20353 104.81925
120.16151
##  [12,]  87.38478  69.80631 107.11653 113.40302  80.09303  94.10321
104.22555
##  [13,]  82.99690  69.18425  86.71816  82.93624 120.41315  67.41222
70.75334
##  [14,]  90.02133  96.39598  80.05769  82.54200  62.01859  67.92931
75.79609
##  [15,]  85.55285 101.75218  61.61565 103.95548  68.60957  51.93287
59.32771
##  [16,]  66.62748 122.82064 114.01767  92.69522  99.19768  97.74341
89.23920
##  [17,] 103.71573  85.06835  67.52095 108.81949 108.38981  93.31938
82.85427
##  [18,]  65.70801  95.23543  93.71289 135.46158  81.95109  70.36459
51.27777
##  [19,]  94.54239  54.81827  92.79186 102.92295  80.39125  78.78969
```

```
65.13774
## [20,] 109.59847  91.11799  91.09322  75.12045 104.64089 109.25563
112.90875
## [21,]  83.05146 134.69377  82.89060  79.80845  92.27051  81.51257
91.40109
## [22,]  91.15508  84.54451  91.58731 117.92126  63.03390  82.70297
47.11483
## [23,]        NA 104.62525 114.03440 115.81360 100.88039  60.45532
70.48229
## [24,]        NA        NA  79.11552  86.34290  88.85533  85.33582
107.50622
## [25,]        NA        NA        NA  91.33120  77.99220  64.89548
90.49910
## [26,]        NA        NA        NA        NA 103.71543 116.10841
123.80683
## [27,]        NA        NA        NA        NA        NA  86.17393
84.93536
## [28,]        NA        NA        NA        NA        NA        NA
56.98281
## [29,]        NA        NA        NA        NA        NA        NA
NA
## [30,]        NA        NA        NA        NA        NA        NA
NA
## [31,]        NA        NA        NA        NA        NA        NA
NA
## [32,]        NA        NA        NA        NA        NA        NA
NA
## [33,]        NA        NA        NA        NA        NA        NA
NA
## [34,]        NA        NA        NA        NA        NA        NA
NA
## [35,]        NA        NA        NA        NA        NA        NA
NA
## [36,]        NA        NA        NA        NA        NA        NA
NA
## [37,]        NA        NA        NA        NA        NA        NA
NA
## [38,]        NA        NA        NA        NA        NA        NA
NA
## [39,]        NA        NA        NA        NA        NA        NA
NA
## [40,]        NA        NA        NA        NA        NA        NA
NA
## [41,]        NA        NA        NA        NA        NA        NA
NA
## [42,]        NA        NA        NA        NA        NA        NA
NA
## [43,]        NA        NA        NA        NA        NA        NA
NA
## [44,]        NA        NA        NA        NA        NA        NA
```

```
NA
## [45,]     NA     NA     NA     NA     NA     NA
NA
## [46,]     NA     NA     NA     NA     NA     NA
NA
## [47,]     NA     NA     NA     NA     NA     NA
NA
## [48,]     NA     NA     NA     NA     NA     NA
NA
## [49,]     NA     NA     NA     NA     NA     NA
NA
## [50,]     NA     NA     NA     NA     NA     NA
NA
## [51,]     NA     NA     NA     NA     NA     NA
NA
## [52,]     NA     NA     NA     NA     NA     NA
NA
## [53,]     NA     NA     NA     NA     NA     NA
NA
## [54,]     NA     NA     NA     NA     NA     NA
NA
## [55,]     NA     NA     NA     NA     NA     NA
NA
## [56,]     NA     NA     NA     NA     NA     NA
NA
## [57,]     NA     NA     NA     NA     NA     NA
NA
## [58,]     NA     NA     NA     NA     NA     NA
NA
## [59,]     NA     NA     NA     NA     NA     NA
NA
## [60,]     NA     NA     NA     NA     NA     NA
NA
## [61,]     NA     NA     NA     NA     NA     NA
NA
## [62,]     NA     NA     NA     NA     NA     NA
NA
## [63,]     NA     NA     NA     NA     NA     NA
NA
## [64,]     NA     NA     NA     NA     NA     NA
NA
## [65,]     NA     NA     NA     NA     NA     NA
NA
## [66,]     NA     NA     NA     NA     NA     NA
NA
## [67,]     NA     NA     NA     NA     NA     NA
NA
## [68,]     NA     NA     NA     NA     NA     NA
NA
## [69,]     NA     NA     NA     NA     NA     NA
```

```
NA
## [70,]      NA     NA     NA     NA     NA     NA
NA
## [71,]      NA     NA     NA     NA     NA     NA
NA
## [72,]      NA     NA     NA     NA     NA     NA
NA
## [73,]      NA     NA     NA     NA     NA     NA
NA
## [74,]      NA     NA     NA     NA     NA     NA
NA
## [75,]      NA     NA     NA     NA     NA     NA
NA
## [76,]      NA     NA     NA     NA     NA     NA
NA
## [77,]      NA     NA     NA     NA     NA     NA
NA
## [78,]      NA     NA     NA     NA     NA     NA
NA
## [79,]      NA     NA     NA     NA     NA     NA
NA
## [80,]      NA     NA     NA     NA     NA     NA
NA
## [81,]      NA     NA     NA     NA     NA     NA
NA
## [82,]      NA     NA     NA     NA     NA     NA
NA
## [83,]      NA     NA     NA     NA     NA     NA
NA
## [84,]      NA     NA     NA     NA     NA     NA
NA
## [85,]      NA     NA     NA     NA     NA     NA
NA
## [86,]      NA     NA     NA     NA     NA     NA
NA
## [87,]      NA     NA     NA     NA     NA     NA
NA
## [88,]      NA     NA     NA     NA     NA     NA
NA
## [89,]      NA     NA     NA     NA     NA     NA
NA
## [90,]      NA     NA     NA     NA     NA     NA
NA
## [91,]      NA     NA     NA     NA     NA     NA
NA
## [92,]      NA     NA     NA     NA     NA     NA
NA
## [93,]      NA     NA     NA     NA     NA     NA
NA
## [94,]      NA     NA     NA     NA     NA     NA
```

```
NA
##  [95,]        NA        NA        NA        NA        NA        NA
NA
##  [96,]        NA        NA        NA        NA        NA        NA
NA
##  [97,]        NA        NA        NA        NA        NA        NA
NA
##  [98,]        NA        NA        NA        NA        NA        NA
NA
##  [99,]        NA        NA        NA        NA        NA        NA
NA
## [100,]        NA        NA        NA        NA        NA        NA
NA
##            [,30]     [,31]     [,32]     [,33]     [,34]     [,35]
[,36]
##   [1,]  66.45258  61.00755  72.28247  83.92081  78.73531 105.22744
101.20947
##   [2,] 110.59404  63.34378  94.68981  61.67224  77.94747 120.37691
88.09326
##   [3,]  63.27947  68.46336  57.76947  81.85014  82.26801 106.58461
70.99485
##   [4,] 109.27015  95.09481  73.79018 102.61759  65.42123  66.78330
58.29385
##   [5,]  95.62230  90.45621  72.14685 116.25523 103.73521  51.18826
84.94778
##   [6,]  63.49576  73.93083  82.79854  94.41585  78.11812  79.14295
101.69693
##   [7,] 112.51852 112.40139 100.06256 134.85151  80.90126  61.37836
109.21268
##   [8,]  79.16941  72.15233  98.68701  64.38608  84.26171 110.80054
93.14568
##   [9,]  93.61569  83.81215  98.32163  81.10123  89.12864 109.43339
87.80077
##  [10,]  77.01126 112.41848  87.21323  88.07758  95.68108 108.92964
76.84915
##  [11,]  98.27467  77.58391  85.63609  85.98625 134.97065 130.77897
106.02315
##  [12,]  77.78575  68.98070  75.35225  49.69975  76.49242 126.00515
50.98562
##  [13,]  97.88879 105.36315  82.96131 106.42626  91.45579  62.57177
89.70968
##  [14,] 110.91902  88.04959  95.60681  92.49347 112.31252  75.68191
87.95174
##  [15,] 108.98991  95.06190  80.27425 103.19237 114.05539  71.06724
81.40542
##  [16,] 101.47258  95.45959 103.98254 112.69542  80.87442  70.54986
84.86807
##  [17,]  75.04833  98.92064  62.01826 108.93434  94.01513  88.57647
90.94712
##  [18,] 107.52915  81.08545  78.94664 103.05910  41.10887  70.06719
```

```
67.72580
## [19,] 110.29696  71.99552  61.79357  96.57670  79.99284  83.43363
79.94954
## [20,]  72.20853 104.97418  99.78679  61.99451 112.56049 136.09320
98.90906
## [21,]  89.94819 129.96487 124.75552  83.26382 114.36762  92.17383
90.90544
## [22,] 124.70058  83.03699  68.92784 113.10504  72.14966  73.28982
77.89930
## [23,] 123.83731  83.54367  76.53131 108.16391  88.85409  80.64400
69.28752
## [24,]  65.78111  53.50670  61.86398  63.81811  81.58631 102.60192
87.82940
## [25,]  56.03038  90.35624  81.10662  84.60187 101.87898  73.63362
95.94481
## [26,]  76.18118 102.28519 126.03004  76.41613 111.92591 103.40957
142.00474
## [27,]  99.93913  63.97309  82.01260  77.96772  89.84906  95.96978
80.11997
## [28,]  98.05141  87.11495  78.56316  89.91734  86.10347  62.24510
64.89347
## [29,] 130.12601 104.16107  89.42991 118.00195  63.72501  51.60234
69.95368
## [30,]        NA  88.48314  89.61583  60.34967  93.33033  99.65159
93.76709
## [31,]        NA        NA  52.65593  74.51096  86.19389 112.25851
88.12692
## [32,]        NA        NA        NA 103.66416  97.71130  93.38619
69.86286
## [33,]        NA        NA        NA        NA  86.22660 127.75580
85.04837
## [34,]        NA        NA        NA        NA        NA  71.73549
70.95016
## [35,]        NA        NA        NA        NA        NA        NA
78.95323
## [36,]        NA        NA        NA        NA        NA        NA
NA
## [37,]        NA        NA        NA        NA        NA        NA
NA
## [38,]        NA        NA        NA        NA        NA        NA
NA
## [39,]        NA        NA        NA        NA        NA        NA
NA
## [40,]        NA        NA        NA        NA        NA        NA
NA
## [41,]        NA        NA        NA        NA        NA        NA
NA
## [42,]        NA        NA        NA        NA        NA        NA
NA
## [43,]        NA        NA        NA        NA        NA        NA
```

```
NA
## [44,]        NA        NA        NA        NA        NA        NA
NA
## [45,]        NA        NA        NA        NA        NA        NA
NA
## [46,]        NA        NA        NA        NA        NA        NA
NA
## [47,]        NA        NA        NA        NA        NA        NA
NA
## [48,]        NA        NA        NA        NA        NA        NA
NA
## [49,]        NA        NA        NA        NA        NA        NA
NA
## [50,]        NA        NA        NA        NA        NA        NA
NA
## [51,]        NA        NA        NA        NA        NA        NA
NA
## [52,]        NA        NA        NA        NA        NA        NA
NA
## [53,]        NA        NA        NA        NA        NA        NA
NA
## [54,]        NA        NA        NA        NA        NA        NA
NA
## [55,]        NA        NA        NA        NA        NA        NA
NA
## [56,]        NA        NA        NA        NA        NA        NA
NA
## [57,]        NA        NA        NA        NA        NA        NA
NA
## [58,]        NA        NA        NA        NA        NA        NA
NA
## [59,]        NA        NA        NA        NA        NA        NA
NA
## [60,]        NA        NA        NA        NA        NA        NA
NA
## [61,]        NA        NA        NA        NA        NA        NA
NA
## [62,]        NA        NA        NA        NA        NA        NA
NA
## [63,]        NA        NA        NA        NA        NA        NA
NA
## [64,]        NA        NA        NA        NA        NA        NA
NA
## [65,]        NA        NA        NA        NA        NA        NA
NA
## [66,]        NA        NA        NA        NA        NA        NA
NA
## [67,]        NA        NA        NA        NA        NA        NA
NA
## [68,]        NA        NA        NA        NA        NA        NA
```

```
NA
## [69,]     NA     NA     NA     NA     NA     NA
NA
## [70,]     NA     NA     NA     NA     NA     NA
NA
## [71,]     NA     NA     NA     NA     NA     NA
NA
## [72,]     NA     NA     NA     NA     NA     NA
NA
## [73,]     NA     NA     NA     NA     NA     NA
NA
## [74,]     NA     NA     NA     NA     NA     NA
NA
## [75,]     NA     NA     NA     NA     NA     NA
NA
## [76,]     NA     NA     NA     NA     NA     NA
NA
## [77,]     NA     NA     NA     NA     NA     NA
NA
## [78,]     NA     NA     NA     NA     NA     NA
NA
## [79,]     NA     NA     NA     NA     NA     NA
NA
## [80,]     NA     NA     NA     NA     NA     NA
NA
## [81,]     NA     NA     NA     NA     NA     NA
NA
## [82,]     NA     NA     NA     NA     NA     NA
NA
## [83,]     NA     NA     NA     NA     NA     NA
NA
## [84,]     NA     NA     NA     NA     NA     NA
NA
## [85,]     NA     NA     NA     NA     NA     NA
NA
## [86,]     NA     NA     NA     NA     NA     NA
NA
## [87,]     NA     NA     NA     NA     NA     NA
NA
## [88,]     NA     NA     NA     NA     NA     NA
NA
## [89,]     NA     NA     NA     NA     NA     NA
NA
## [90,]     NA     NA     NA     NA     NA     NA
NA
## [91,]     NA     NA     NA     NA     NA     NA
NA
## [92,]     NA     NA     NA     NA     NA     NA
NA
## [93,]     NA     NA     NA     NA     NA     NA
```

```
NA
##  [94,]        NA        NA        NA        NA        NA        NA
NA
##  [95,]        NA        NA        NA        NA        NA        NA
NA
##  [96,]        NA        NA        NA        NA        NA        NA
NA
##  [97,]        NA        NA        NA        NA        NA        NA
NA
##  [98,]        NA        NA        NA        NA        NA        NA
NA
##  [99,]        NA        NA        NA        NA        NA        NA
NA
## [100,]        NA        NA        NA        NA        NA        NA
NA
##            [,37]     [,38]     [,39]     [,40]     [,41]     [,42]
[,43]
##   [1,]  81.01843  43.48128 131.31129  88.91146  85.53458  62.47276
109.50676
##   [2,]  93.93589  74.79411  97.59905 109.83133 121.66784  74.17696
96.08316
##   [3,]  60.29523  68.09765 122.15826  88.66658  78.72131 109.54205
123.08527
##   [4,] 100.98923 128.26318  67.00187  89.39084  93.56333  94.51010
97.36976
##   [5,]  90.95608 107.99931  69.28346  67.56626  88.65379 105.23388
62.28681
##   [6,]  89.46139  66.33324 122.81936  85.14346  80.00183  62.24319
112.30152
##   [7,] 104.47406  92.74131  68.58593  62.88354  60.66927  38.63330
64.40068
##   [8,]  97.18560  70.20069 121.05868 119.50946 118.13521  64.97801
126.02355
##   [9,]  89.18780  70.41984 111.04913 103.47865 100.93759  62.35632
119.67428
##  [10,]  79.32879 103.11834  83.41436 116.74602  93.21403 114.83563
105.54618
##  [11,]  43.98957  59.45992  90.42745 103.67887  99.87016  93.96991
97.34884
##  [12,]  83.83072  84.77156 115.71687 118.44789 120.70599 123.53916
119.41309
##  [13,]  96.33664 111.71010  69.18841  50.31228  53.71066  98.79952
74.58933
##  [14,] 103.61508  94.96715  84.29269  74.89258 102.54841  79.89009
83.52490
##  [15,]  92.10576 109.51012  71.81344  78.88951  94.66786  87.76875
98.06009
##  [16,]  92.97262  79.62880  97.21409  63.81162  73.65251  75.00221
72.12481
##  [17,]  72.32115  91.97276  84.84005  85.83282  65.90386  92.02421
```

```
106.22619
##  [18,]  92.63375 111.71633  73.17231  95.14474  94.88786  90.67152
83.49133
##  [19,]  89.94953  93.24636  76.74270  73.31738  90.58528  88.10197
77.04341
##  [20,]  79.78984  85.90076  97.43531 121.06269  93.51387 100.51761
124.84452
##  [21,] 100.27558 117.47105  76.75795 108.03660  94.73081 101.17130
103.57906
##  [22,]  95.49181 106.51266  57.59406  84.47389  99.93179  79.76279
64.91598
##  [23,]  63.83894 104.18616  63.06465  77.22315  85.15709 111.63905
77.73723
##  [24,]  83.61328  73.57297 113.56246  92.45608  98.69113 106.35774
99.61429
##  [25,]  98.66066  98.61816 102.87885  95.24107  92.16361  93.02892
116.56428
##  [26,]  99.65222  72.36423  97.08397  92.78334  81.82554  70.59420
80.08432
##  [27,]  97.85478  86.71299  92.70955 116.39697 148.53369  85.35498
88.20016
##  [28,]  93.68077 124.88658  77.77474  80.46691  90.93467 112.05148
104.41191
##  [29,] 106.89920 129.95950  53.44047  71.35172  79.52876  78.93260
77.29665
##  [30,]  97.36759  77.05540 138.74556 102.66247  88.37983 104.60753
125.49485
##  [31,]  60.18029  50.97737 112.15588 101.33565 119.48529  91.09890
96.87073
##  [32,]  48.97806  74.40563  92.15504  80.13894  93.79962 111.03249
94.14227
##  [33,] 101.20130  84.13752 124.81034 133.20158 125.64444 108.25189
128.26770
##  [34,] 118.68868 104.58545  94.75625  89.53754  89.07418  81.24987
84.24714
##  [35,] 117.87745 121.74767  72.82023  46.92658  61.28197  82.04262
66.62508
##  [36,]  98.12845 115.30800  94.91161  85.44813 101.74127 127.96443
102.95590
##  [37,]        NA  59.79530  84.07441  92.81595  89.81568 101.53192
93.80950
##  [38,]        NA        NA 123.93651  91.11726  94.22280  68.57532
93.84399
##  [39,]        NA        NA        NA  83.11288  81.82963  89.65133
53.90469
##  [40,]        NA        NA        NA        NA  40.38024  79.03516
65.87765
##  [41,]        NA        NA        NA        NA        NA  77.14334
80.05251
##  [42,]        NA        NA        NA        NA        NA        NA
```

```
74.59312
##  [43,]         NA         NA         NA         NA         NA         NA
NA
##  [44,]         NA         NA         NA         NA         NA         NA
NA
##  [45,]         NA         NA         NA         NA         NA         NA
NA
##  [46,]         NA         NA         NA         NA         NA         NA
NA
##  [47,]         NA         NA         NA         NA         NA         NA
NA
##  [48,]         NA         NA         NA         NA         NA         NA
NA
##  [49,]         NA         NA         NA         NA         NA         NA
NA
##  [50,]         NA         NA         NA         NA         NA         NA
NA
##  [51,]         NA         NA         NA         NA         NA         NA
NA
##  [52,]         NA         NA         NA         NA         NA         NA
NA
##  [53,]         NA         NA         NA         NA         NA         NA
NA
##  [54,]         NA         NA         NA         NA         NA         NA
NA
##  [55,]         NA         NA         NA         NA         NA         NA
NA
##  [56,]         NA         NA         NA         NA         NA         NA
NA
##  [57,]         NA         NA         NA         NA         NA         NA
NA
##  [58,]         NA         NA         NA         NA         NA         NA
NA
##  [59,]         NA         NA         NA         NA         NA         NA
NA
##  [60,]         NA         NA         NA         NA         NA         NA
NA
##  [61,]         NA         NA         NA         NA         NA         NA
NA
##  [62,]         NA         NA         NA         NA         NA         NA
NA
##  [63,]         NA         NA         NA         NA         NA         NA
NA
##  [64,]         NA         NA         NA         NA         NA         NA
NA
##  [65,]         NA         NA         NA         NA         NA         NA
NA
##  [66,]         NA         NA         NA         NA         NA         NA
NA
##  [67,]         NA         NA         NA         NA         NA         NA
```

```
NA
## [68,] NA NA NA NA NA NA
NA
## [69,] NA NA NA NA NA NA
NA
## [70,] NA NA NA NA NA NA
NA
## [71,] NA NA NA NA NA NA
NA
## [72,] NA NA NA NA NA NA
NA
## [73,] NA NA NA NA NA NA
NA
## [74,] NA NA NA NA NA NA
NA
## [75,] NA NA NA NA NA NA
NA
## [76,] NA NA NA NA NA NA
NA
## [77,] NA NA NA NA NA NA
NA
## [78,] NA NA NA NA NA NA
NA
## [79,] NA NA NA NA NA NA
NA
## [80,] NA NA NA NA NA NA
NA
## [81,] NA NA NA NA NA NA
NA
## [82,] NA NA NA NA NA NA
NA
## [83,] NA NA NA NA NA NA
NA
## [84,] NA NA NA NA NA NA
NA
## [85,] NA NA NA NA NA NA
NA
## [86,] NA NA NA NA NA NA
NA
## [87,] NA NA NA NA NA NA
NA
## [88,] NA NA NA NA NA NA
NA
## [89,] NA NA NA NA NA NA
NA
## [90,] NA NA NA NA NA NA
NA
## [91,] NA NA NA NA NA NA
NA
## [92,] NA NA NA NA NA NA
```

```
NA
##  [93,]        NA        NA        NA        NA        NA        NA
NA
##  [94,]        NA        NA        NA        NA        NA        NA
NA
##  [95,]        NA        NA        NA        NA        NA        NA
NA
##  [96,]        NA        NA        NA        NA        NA        NA
NA
##  [97,]        NA        NA        NA        NA        NA        NA
NA
##  [98,]        NA        NA        NA        NA        NA        NA
NA
##  [99,]        NA        NA        NA        NA        NA        NA
NA
## [100,]        NA        NA        NA        NA        NA        NA
NA
##            [,44]     [,45]     [,46]     [,47]     [,48]     [,49]
[,50]
##   [1,]  83.38404  47.67418  84.45444  79.93174 108.72775  76.78554
99.35494
##   [2,]  56.20972  77.58560  91.74809  72.73757 101.04558 103.28315
111.28830
##   [3,]  89.56006  76.30905 100.68281 120.83053  93.79352  88.18093
89.81630
##   [4,]  89.06578  80.47255  96.56134 118.64537  73.47587 113.57977
108.75695
##   [5,]  87.84419 106.53575  66.18108  84.34189  79.73972  95.07873
75.79102
##   [6,] 106.36367  58.00728  77.26595  85.19988 107.76993  74.28958
88.05669
##   [7,] 115.34144  67.40776  76.49268  91.88544  92.80802  63.84161
74.55619
##   [8,]  81.93394  55.72418  80.66175  68.27104  94.83185  96.85092
112.20161
##   [9,]  88.69304  54.61227  74.96754  86.26361  92.19678  83.82972
92.67578
##  [10,] 108.09545  83.54024 116.76095 111.90870  46.10800 107.95058
110.11621
##  [11,]  94.65709  73.67983  82.71830  84.98840  70.30240  97.71356
78.26211
##  [12,]  54.94647  93.97104 105.20031  99.88353  84.61065 110.77421
116.79527
##  [13,]  84.73221 113.77774  95.43065 105.90266 107.94727  81.22837
73.59054
##  [14,]  61.76630  95.59771  44.70555  63.59264 100.53607  87.42993
74.10282
##  [15,]  78.70737  79.67830  61.03023  86.88048  79.10902 104.83573
85.43438
##  [16,] 106.79132 101.34957  60.27277  99.48276 107.91911  44.15018
```

```
39.00299
##  [17,] 108.93815  57.70684 106.93376 115.04759  67.72164  99.74435
103.91527
##  [18,] 108.24416  85.99165 107.47937 123.21113  79.77276 104.06265
102.31160
##  [19,]  58.30950  82.90177  88.63802  92.21433  95.42278 102.77781
101.63003
##  [20,]  89.67498  78.37115 119.05656  89.61597  73.77051 106.74016
114.62749
##  [21,] 108.46075 106.00150  92.14985  89.71116  70.96098  95.65936
85.71823
##  [22,]  80.13563  73.81598  87.37528  95.38979  71.16464 109.64449
107.47508
##  [23,] 101.35256 105.87041  89.69299 130.03900  79.14550  96.84011
64.43204
##  [24,]  55.38157  91.77823 103.40817  79.11712 108.23253 104.68632
113.47540
##  [25,]  91.99335  78.38038  83.83077  74.96502  87.66177 104.14563
106.69824
##  [26,]  95.56907 100.42967  94.16897  49.77379 110.73053  72.01946
83.69977
##  [27,]  62.90371  74.31622  67.69554  56.81723  72.43468 117.63838
116.59664
##  [28,]  84.09305 106.68643  86.25538 110.22514  93.57851 108.58854
86.74877
##  [29,]  98.95585  90.79142  87.70943 119.01475  87.15935  93.42506
85.04214
##  [30,]  85.05449  92.45826  99.29233  75.33394 103.59718  85.62920
105.48952
##  [31,]  64.48559  66.30250  82.02051  77.62914  93.67563 109.44501
104.21680
##  [32,]  73.43213  65.68743  81.00004 107.33401  70.47962 111.07304
95.30227
##  [33,]  59.84705 101.19224 110.67394  67.54494 102.59507 110.80520
125.99159
##  [34,]  92.72726 100.54984 108.62926 107.38715 111.25401  81.66868
101.13912
##  [35,] 104.11322 109.64783  69.84760 104.15711 104.78180  65.52221
61.79776
##  [36,]  62.23328 108.81210  82.07696 120.01695  84.19232  98.34839
91.16756
##  [37,]  98.03687  60.46478  92.59946 105.54157  63.14766 109.01870
85.73116
##  [38,]  82.07041  56.55021  74.42863  66.42867 101.42762  77.08743
84.42195
##  [39,] 113.65495  94.89343 101.18545 109.56295  58.65273 106.29472
85.53579
##  [40,]  93.09510 100.98264  59.76250 107.94872 116.21221  45.77534
35.14688
##  [41,] 120.09609  95.93265  90.89988 118.57460 110.45371  48.53230
```

```
50.45613
##  [42,] 109.05030  60.82745  75.62968  69.89120 109.22630  62.80151
79.45526
##  [43,] 100.86010 102.41727  79.70270  85.88309  86.32499  74.50173
68.46981
##  [44,]        NA  99.44495  71.64778  69.89708 102.38353 100.52128
102.80150
##  [45,]        NA        NA  83.95338  84.10376  70.46113 102.48037
105.34749
##  [46,]        NA        NA        NA  72.09732 100.16182  65.01871
54.95879
##  [47,]        NA        NA        NA        NA  99.86611  90.35550
103.98103
##  [48,]        NA        NA        NA        NA        NA 131.57271
111.65881
##  [49,]        NA        NA        NA        NA        NA        NA
40.06714
##  [50,]        NA        NA        NA        NA        NA        NA
NA
##  [51,]        NA        NA        NA        NA        NA        NA
NA
##  [52,]        NA        NA        NA        NA        NA        NA
NA
##  [53,]        NA        NA        NA        NA        NA        NA
NA
##  [54,]        NA        NA        NA        NA        NA        NA
NA
##  [55,]        NA        NA        NA        NA        NA        NA
NA
##  [56,]        NA        NA        NA        NA        NA        NA
NA
##  [57,]        NA        NA        NA        NA        NA        NA
NA
##  [58,]        NA        NA        NA        NA        NA        NA
NA
##  [59,]        NA        NA        NA        NA        NA        NA
NA
##  [60,]        NA        NA        NA        NA        NA        NA
NA
##  [61,]        NA        NA        NA        NA        NA        NA
NA
##  [62,]        NA        NA        NA        NA        NA        NA
NA
##  [63,]        NA        NA        NA        NA        NA        NA
NA
##  [64,]        NA        NA        NA        NA        NA        NA
NA
##  [65,]        NA        NA        NA        NA        NA        NA
NA
##  [66,]        NA        NA        NA        NA        NA        NA
```

```
NA
##  [67,]          NA          NA          NA          NA          NA          NA
NA
##  [68,]          NA          NA          NA          NA          NA          NA
NA
##  [69,]          NA          NA          NA          NA          NA          NA
NA
##  [70,]          NA          NA          NA          NA          NA          NA
NA
##  [71,]          NA          NA          NA          NA          NA          NA
NA
##  [72,]          NA          NA          NA          NA          NA          NA
NA
##  [73,]          NA          NA          NA          NA          NA          NA
NA
##  [74,]          NA          NA          NA          NA          NA          NA
NA
##  [75,]          NA          NA          NA          NA          NA          NA
NA
##  [76,]          NA          NA          NA          NA          NA          NA
NA
##  [77,]          NA          NA          NA          NA          NA          NA
NA
##  [78,]          NA          NA          NA          NA          NA          NA
NA
##  [79,]          NA          NA          NA          NA          NA          NA
NA
##  [80,]          NA          NA          NA          NA          NA          NA
NA
##  [81,]          NA          NA          NA          NA          NA          NA
NA
##  [82,]          NA          NA          NA          NA          NA          NA
NA
##  [83,]          NA          NA          NA          NA          NA          NA
NA
##  [84,]          NA          NA          NA          NA          NA          NA
NA
##  [85,]          NA          NA          NA          NA          NA          NA
NA
##  [86,]          NA          NA          NA          NA          NA          NA
NA
##  [87,]          NA          NA          NA          NA          NA          NA
NA
##  [88,]          NA          NA          NA          NA          NA          NA
NA
##  [89,]          NA          NA          NA          NA          NA          NA
NA
##  [90,]          NA          NA          NA          NA          NA          NA
NA
##  [91,]          NA          NA          NA          NA          NA          NA
```

```
NA
##  [92,]        NA        NA        NA        NA        NA        NA
NA
##  [93,]        NA        NA        NA        NA        NA        NA
NA
##  [94,]        NA        NA        NA        NA        NA        NA
NA
##  [95,]        NA        NA        NA        NA        NA        NA
NA
##  [96,]        NA        NA        NA        NA        NA        NA
NA
##  [97,]        NA        NA        NA        NA        NA        NA
NA
##  [98,]        NA        NA        NA        NA        NA        NA
NA
##  [99,]        NA        NA        NA        NA        NA        NA
NA
## [100,]        NA        NA        NA        NA        NA        NA
NA
##            [,51]     [,52]     [,53]     [,54]     [,55]     [,56]
[,57]
##   [1,] 119.80951 110.70560 114.90205 105.73833  77.67257  51.56149
96.82209
##   [2,] 115.81036  73.43549 110.64805  90.81839  78.77331  41.83958
88.50464
##   [3,]  84.20155  90.14501  79.93246  88.73752  84.30845  80.19117
56.74699
##   [4,] 106.03679  60.75391  51.65040  71.41562  76.91555  78.94693
93.66596
##   [5,]  69.17160 105.99110  73.38754  82.26636  92.13893 128.47715
111.63918
##   [6,]  94.93676 104.87488  98.94998 105.98239 104.22256  76.39993
103.33101
##   [7,] 116.18347 106.07092  99.14834  70.08373  66.86679  93.76132
111.62138
##   [8,] 113.44989  67.82999 115.49166 116.06136 115.76099  47.74030
103.38424
##   [9,] 111.45581  63.46258 117.76124  92.17311 103.18762  61.06878
89.89693
##  [10,] 100.96878  63.91793  73.77261 100.38984  98.99043  97.01360
71.24286
##  [11,]  88.25645  84.77060 109.16412  85.91515 113.29674 102.59617
78.88911
##  [12,]  92.58895  56.50692  88.38229 100.80984  85.57952  59.62932
56.51695
##  [13,]  82.19736 115.17809  68.18143  63.76325  58.67945 101.33171
99.13154
##  [14,]  86.48326  84.52958 110.69664  79.76837  98.16791  91.41693
123.54466
##  [15,]  97.90549  74.49033  84.19472  74.52780 101.65971  93.48649
```

```
126.16901
## [16,]  53.62489  91.87697 107.62680  68.14675  90.67051 119.01008
64.25750
## [17,] 121.10285  97.83830  68.22653  88.55218  77.67822  85.60651
98.38414
## [18,]  86.28922  71.78096  49.32985  74.78188  79.86050  89.82434
65.80009
## [19,] 116.28489 100.74824  80.87249  70.92152  44.77469  67.87368
103.41586
## [20,] 115.55491  74.17040  96.20299 109.36582 102.96161  68.89585
89.48034
## [21,]  74.26235  57.70782  87.87854  96.91237 134.10898 111.44171
93.00591
## [22,] 122.49406  84.91384  73.80214  70.81241  56.46500  80.25795
103.86414
## [23,]  53.69860  74.46225  58.86798  45.10609  88.80312 124.40858
51.00880
## [24,]  97.72349 114.13108  85.07896 107.78399  72.06203  60.85251
95.14754
## [25,]  95.36233  96.47761  79.96216 119.62620 119.15417  84.80419
134.18037
## [26,]  90.35955 122.45158 122.30318 113.70548 102.36215  90.64533
111.87798
## [27,] 107.08560  68.62183 103.51576 108.70263 104.79910  72.66743
112.03214
## [28,]  64.61750  72.36736  50.67507  73.17858 102.50343  98.31986
93.63728
## [29,]  93.67661  70.91216  59.55552  51.50813  70.62998  93.14739
91.11505
## [30,]  85.61083 103.30857  97.59424 139.19942 110.06263  75.80102
99.13640
## [31,]  96.73330  98.37005  96.62041  97.12726  88.40284  67.41935
85.37398
## [32,] 100.28789  97.61416  72.96031  76.61366  71.71914  90.13352
85.84682
## [33,]  90.22141  71.49328  99.63712 132.23686 113.15706  49.85852
87.00102
## [34,]  86.27272  82.87284  73.03683  86.59249  66.15916  70.54375
66.71350
## [35,]  67.26811 100.82850  71.54056  67.34272  77.61895 117.74905
101.19587
## [36,]  73.28283  50.81379  69.65882  73.35717  77.47716  88.50974
62.94214
## [37,]  95.77503  98.54042  82.50252  75.42602  91.92056 102.19389
74.96722
## [38,] 104.41471 118.46763 131.75604 102.46587  88.94190  74.62204
91.06640
## [39,]  94.60555  83.80306  59.33789  56.23286  78.60449 119.99561
91.02569
## [40,]  71.87092 119.31791  91.46620  48.18242  56.80839 115.65776
```

```
91.05063
## [41,]  79.43513 120.79538  82.04294  61.65527  67.04576 112.69159
85.76041
## [42,] 114.37363 110.69335 120.42452  87.58475  83.60914  77.18195
112.42099
## [43,]  85.21488 114.09999  92.06380  66.06589  61.49556 121.19739
89.58506
## [44,]  94.88507  80.07688 107.77224  98.00774  77.86987  60.87331
96.14032
## [45,] 137.87180  95.09378 102.20045  94.47107  91.78733  69.15241
107.17308
## [46,]  80.54316  95.08252 122.96828  77.02590  91.50976 103.38777
108.96780
## [47,] 104.28244 105.72274 134.45016 132.03802 110.05282  72.87416
131.22854
## [48,] 109.28704  67.02253  70.19369  89.48832 101.40166 106.29128
89.66357
## [49,]  71.13640 115.76367 118.96724  75.86894  73.06792 103.37725
84.78078
## [50,]  55.02619 107.79839 102.37649  50.98188  80.37641 130.13493
79.15949
## [51,]        NA  88.28991  78.18179  78.25428 104.02862 126.22959
64.21665
## [52,]        NA        NA  77.06485  86.96027 108.00067  80.56475
72.09577
## [53,]        NA        NA        NA  73.25904  86.12334 104.72155
73.68231
## [54,]        NA        NA        NA        NA  57.66424 115.12693
67.82311
## [55,]        NA        NA        NA        NA        NA  84.08742
76.36413
## [56,]        NA        NA        NA        NA        NA        NA
99.14085
## [57,]        NA        NA        NA        NA        NA        NA
NA
## [58,]        NA        NA        NA        NA        NA        NA
NA
## [59,]        NA        NA        NA        NA        NA        NA
NA
## [60,]        NA        NA        NA        NA        NA        NA
NA
## [61,]        NA        NA        NA        NA        NA        NA
NA
## [62,]        NA        NA        NA        NA        NA        NA
NA
## [63,]        NA        NA        NA        NA        NA        NA
NA
## [64,]        NA        NA        NA        NA        NA        NA
NA
## [65,]        NA        NA        NA        NA        NA        NA
```

```
NA
## [66,]      NA      NA      NA      NA      NA      NA
NA
## [67,]      NA      NA      NA      NA      NA      NA
NA
## [68,]      NA      NA      NA      NA      NA      NA
NA
## [69,]      NA      NA      NA      NA      NA      NA
NA
## [70,]      NA      NA      NA      NA      NA      NA
NA
## [71,]      NA      NA      NA      NA      NA      NA
NA
## [72,]      NA      NA      NA      NA      NA      NA
NA
## [73,]      NA      NA      NA      NA      NA      NA
NA
## [74,]      NA      NA      NA      NA      NA      NA
NA
## [75,]      NA      NA      NA      NA      NA      NA
NA
## [76,]      NA      NA      NA      NA      NA      NA
NA
## [77,]      NA      NA      NA      NA      NA      NA
NA
## [78,]      NA      NA      NA      NA      NA      NA
NA
## [79,]      NA      NA      NA      NA      NA      NA
NA
## [80,]      NA      NA      NA      NA      NA      NA
NA
## [81,]      NA      NA      NA      NA      NA      NA
NA
## [82,]      NA      NA      NA      NA      NA      NA
NA
## [83,]      NA      NA      NA      NA      NA      NA
NA
## [84,]      NA      NA      NA      NA      NA      NA
NA
## [85,]      NA      NA      NA      NA      NA      NA
NA
## [86,]      NA      NA      NA      NA      NA      NA
NA
## [87,]      NA      NA      NA      NA      NA      NA
NA
## [88,]      NA      NA      NA      NA      NA      NA
NA
## [89,]      NA      NA      NA      NA      NA      NA
NA
## [90,]      NA      NA      NA      NA      NA      NA
```

```
NA
##  [91,]        NA        NA        NA        NA        NA        NA
NA
##  [92,]        NA        NA        NA        NA        NA        NA
NA
##  [93,]        NA        NA        NA        NA        NA        NA
NA
##  [94,]        NA        NA        NA        NA        NA        NA
NA
##  [95,]        NA        NA        NA        NA        NA        NA
NA
##  [96,]        NA        NA        NA        NA        NA        NA
NA
##  [97,]        NA        NA        NA        NA        NA        NA
NA
##  [98,]        NA        NA        NA        NA        NA        NA
NA
##  [99,]        NA        NA        NA        NA        NA        NA
NA
## [100,]        NA        NA        NA        NA        NA        NA
NA
##             [,58]     [,59]     [,60]     [,61]     [,62]     [,63]
[,64]
##   [1,]  81.59287  68.47282  75.39687  80.66601  80.17749  73.27594
81.47100
##   [2,]  94.65121  96.43896  73.60668  94.33425  64.45937  99.93101
91.00106
##   [3,]  98.23845  82.39410  67.52648  86.58718  96.08290  55.41903
104.97795
##   [4,]  92.11667  99.07505 128.12568  99.12300  97.60041 105.48499
57.13321
##   [5,]  91.75598  71.59258 102.31217 108.62963  96.16744 103.28973
79.68669
##   [6,]  82.28115  46.12920  87.88133  62.94911  90.98335  92.06835
61.48902
##   [7,]  89.39408  91.13199 132.14733  96.84200  90.69626 101.46098
62.38342
##   [8,]  62.37867  73.80810  84.56893  71.23884  62.73696  98.55211
78.06821
##   [9,]  70.13283  87.80706  90.79824  86.89950  57.98043  89.90525
92.00763
##  [10,]  63.58595 120.16730 111.48861  94.20503  94.51941  61.97598
108.71376
##  [11,]  84.28347  94.39183  76.61454 105.56023  75.71976  89.15116
125.84591
##  [12,]  84.06143 103.82702  61.96290  95.72028  69.98998  56.84780
119.32811
##  [13,] 118.03334 108.38220  94.32634  98.07291 125.71332 103.34222
80.43612
##  [14,]  81.81384  85.88209  87.93494 109.01847  65.37156 121.35932
```

```
88.79461
##  [15,]  76.18784  94.10765 113.58454 110.03526  84.71767 128.71337
74.07723
##  [16,] 106.51980  66.29059  79.79417  92.20570  67.08002  73.12642
105.08713
##  [17,]  76.73609 104.13367 121.06056  94.91166 116.24443  81.43334
75.98565
##  [18,] 114.37569  73.48881 111.75477  82.94927  99.46819  92.79418
57.79819
##  [19,] 105.51681  99.54045  95.21212 115.73704  93.41709  99.84174
75.30025
##  [20,]  63.43276 134.11478  89.51717  82.93825  98.04718  82.98349
110.85905
##  [21,]  66.59147 112.16769 104.79805  80.39443  90.36120 103.40081
102.12499
##  [22,]  93.87433  94.93630 126.31453 118.68820  85.08949 102.41725
63.17515
##  [23,] 131.37533  86.43278  90.31646 107.27957  96.13169  95.70845
99.13724
##  [24,] 100.22725  80.83695  57.77181  84.80540 103.16559  83.58777
85.69775
##  [25,]  61.51212  72.37268 100.52230  70.25057 106.86267 111.38270
59.33438
##  [26,]  84.12638  94.22272  73.05608  68.11830  99.16953 101.35053
96.62016
##  [27,]  63.39594  70.68424  94.31882 103.06111  50.27560 101.84900
81.03012
##  [28,] 107.71384  82.10363  94.31691  86.34204 108.45043 122.41016
67.43589
##  [29,] 110.28796  96.19660 129.16687  99.55591  97.85829 115.59024
55.41557
##  [30,]  63.78318  80.33999  65.17107  58.11559 100.49631  68.75385
94.75053
##  [31,] 101.05784  53.17117  60.20146  96.60979  74.45549  89.23926
87.85936
##  [32,]  94.37433  77.94482  89.15539 124.16708  88.80287  76.58957
93.18703
##  [33,]  76.10053  95.28515  51.37977  61.58769  84.67825  87.87305
100.75592
##  [34,] 115.14536  78.11874  90.32905  68.53605  94.18777  80.50753
64.42597
##  [35,] 104.91042  74.95237 112.65698  90.07388 101.67253 101.35775
61.92264
##  [36,]  91.56769  99.21516  85.69429 111.33793  71.75186  65.83503
105.84824
##  [37,]  99.89881  84.73633  85.95154 113.24553  95.04938  85.56752
106.27656
##  [38,]  86.07943  64.67301  59.32277  93.38145  69.57923  74.10148
105.81775
##  [39,] 106.82101 111.65941 137.26923 111.31011 108.68715 116.06227
```

```
76.12114
## [40,] 115.74504  84.12779  92.13509 109.36585  95.08666  85.70240
89.78253
## [41,] 112.43399  98.44691 100.85969  86.19852 121.42600  85.19760
84.93301
## [42,]  91.26170  72.90970 103.76824  78.33627  81.71295 108.98080
62.81434
## [43,] 111.66214  85.20626 106.26702 112.65661  86.86027  91.15750
87.39529
## [44,]  79.95666  94.70949  57.46596 110.65268  59.23252  79.27932
110.73405
## [45,]  72.08240  77.68162 107.87267  98.60377  81.02153  96.06640
74.79882
## [46,]  77.41253  66.80072  84.13416 118.20354  47.99359  89.06949
100.91144
## [47,]  59.78625  73.94577  73.80387  80.29244  67.72719 102.14611
88.96222
## [48,]  67.12627 106.87844 126.27988 116.97149  87.83787  89.05971
96.65006
## [49,] 101.09524  80.98186  78.25390  84.50793  81.51402  71.53866
99.38005
## [50,] 112.99946  81.21165  83.66192 104.71522  84.26120  84.81103
106.34080
## [51,] 115.12668  74.13533  65.81337  79.99217  95.04751  84.83924
104.22815
## [52,]  75.52995 110.36084  99.94862  95.56057  69.72262  90.54650
99.55980
## [53,] 113.78837  97.97361 113.41088  86.02216 132.30345 101.81664
67.27485
## [54,] 129.14086 100.59063 105.77570 124.03252  92.00630  94.02490
93.85384
## [55,] 116.42976 103.99510  94.61348 116.87917  91.02705  69.12221
91.83594
## [56,]  78.22282  93.06903  74.29783  73.24392  80.30066  91.14814
78.29838
## [57,] 121.86178  96.46296  73.30430  93.23146  88.41882  57.32708
116.29092
## [58,]        NA  99.20160 100.52069  92.93152  66.95146  81.57481
99.86062
## [59,]        NA        NA  74.16128  75.22358  80.83417  96.47601
68.27695
## [60,]        NA        NA        NA  78.05176  76.79638  71.44705
118.06640
## [61,]        NA        NA        NA        NA 116.80406 104.04579
63.14727
## [62,]        NA        NA        NA        NA        NA  70.40424
117.17329
## [63,]        NA        NA        NA        NA        NA        NA
130.49514
## [64,]        NA        NA        NA        NA        NA        NA
```

```
NA
##  [65,]         NA         NA         NA         NA         NA         NA
NA
##  [66,]         NA         NA         NA         NA         NA         NA
NA
##  [67,]         NA         NA         NA         NA         NA         NA
NA
##  [68,]         NA         NA         NA         NA         NA         NA
NA
##  [69,]         NA         NA         NA         NA         NA         NA
NA
##  [70,]         NA         NA         NA         NA         NA         NA
NA
##  [71,]         NA         NA         NA         NA         NA         NA
NA
##  [72,]         NA         NA         NA         NA         NA         NA
NA
##  [73,]         NA         NA         NA         NA         NA         NA
NA
##  [74,]         NA         NA         NA         NA         NA         NA
NA
##  [75,]         NA         NA         NA         NA         NA         NA
NA
##  [76,]         NA         NA         NA         NA         NA         NA
NA
##  [77,]         NA         NA         NA         NA         NA         NA
NA
##  [78,]         NA         NA         NA         NA         NA         NA
NA
##  [79,]         NA         NA         NA         NA         NA         NA
NA
##  [80,]         NA         NA         NA         NA         NA         NA
NA
##  [81,]         NA         NA         NA         NA         NA         NA
NA
##  [82,]         NA         NA         NA         NA         NA         NA
NA
##  [83,]         NA         NA         NA         NA         NA         NA
NA
##  [84,]         NA         NA         NA         NA         NA         NA
NA
##  [85,]         NA         NA         NA         NA         NA         NA
NA
##  [86,]         NA         NA         NA         NA         NA         NA
NA
##  [87,]         NA         NA         NA         NA         NA         NA
NA
##  [88,]         NA         NA         NA         NA         NA         NA
NA
##  [89,]         NA         NA         NA         NA         NA         NA
```

```
NA
##  [90,]        NA        NA        NA        NA        NA        NA
NA
##  [91,]        NA        NA        NA        NA        NA        NA
NA
##  [92,]        NA        NA        NA        NA        NA        NA
NA
##  [93,]        NA        NA        NA        NA        NA        NA
NA
##  [94,]        NA        NA        NA        NA        NA        NA
NA
##  [95,]        NA        NA        NA        NA        NA        NA
NA
##  [96,]        NA        NA        NA        NA        NA        NA
NA
##  [97,]        NA        NA        NA        NA        NA        NA
NA
##  [98,]        NA        NA        NA        NA        NA        NA
NA
##  [99,]        NA        NA        NA        NA        NA        NA
NA
## [100,]        NA        NA        NA        NA        NA        NA
NA
##            [,65]     [,66]     [,67]     [,68]     [,69]     [,70]
[,71]
##   [1,] 139.13765 105.51659  81.31865  73.31840  84.32306 112.45749
82.42218
##   [2,] 110.17543 104.56588  87.22973  53.85043 128.10830  88.50649
71.11615
##   [3,]  99.44843  70.25662  52.49140  81.20948  84.87318  95.23564
109.28701
##   [4,]  99.66851 113.61927 105.43522  78.47483  80.13400 103.73514
109.85828
##   [5,]  73.49020  97.98017 117.29835 130.87093  71.09160  81.00485
93.37391
##   [6,] 118.90775 102.26838  77.02430  96.50694  58.31420 115.85249
81.75326
##   [7,] 100.93381 140.25217 117.23551  71.90187  83.28813 116.68080
85.26705
##   [8,] 128.49853 103.44720  82.49098  82.12462  91.25313  95.91825
59.56247
##   [9,] 117.22261 109.22042  76.97290  66.03533 104.93663  97.60152
62.17034
##  [10,]  84.63856  75.10667  96.55986  96.54766  84.11541  67.56638
103.79813
##  [11,]  72.58548  80.82011  79.96209 100.02511 105.67502  50.75929
48.65430
##  [12,] 102.38345  68.87077  68.33565  70.66120 118.90402  76.22256
105.11969
##  [13,]  81.87544  83.69215  92.17158  85.10180  80.12055 117.97446
```

```
116.13875
## [14,]  97.00612 108.77355 102.56323  96.23081 103.71258  94.72498
59.80483
## [15,]  99.44607 111.43032 109.15571 103.85115  80.36233  96.43764
68.35088
## [16,]  66.73100  98.17040  67.25124  78.57832 109.63684  92.48538
89.51164
## [17,] 110.04720  97.13815 103.51712  93.80547  55.87549 102.65746
104.13785
## [18,]  77.65881 106.33578  89.69172  77.88548  82.97338  90.38084
116.43726
## [19,] 109.60933 114.07308 110.43750  69.78378  99.42177 104.85827
99.85783
## [20,] 103.22920  64.60169  83.88517  85.84749  89.93757  82.78812
81.42614
## [21,]  68.38699  69.80108  87.36539 109.51927  85.25303  75.04053
76.77403
## [22,] 100.81494 138.05294 134.37681  76.62877  95.24744  90.78271
96.05416
## [23,]  36.71417  84.90306  71.35404  87.73301 102.87663  69.27918
98.47971
## [24,] 113.43951  74.46264  84.57888  91.67963  83.48607  98.79885
104.35768
## [25,] 116.19686  85.75650 101.17732 132.05283  34.34129 105.12149
84.05031
## [26,]  88.63871  73.29153  90.62788 100.23404  86.52592  92.74671
68.12462
## [27,] 109.06053 117.46736 119.69921 102.59360 100.93261  67.25993
62.59186
## [28,]  78.11289  81.66803  78.30901 103.46957  72.12555 100.28811
101.21648
## [29,]  83.00046 121.92255 101.82700  69.12464  89.29804 110.02215
105.86214
## [30,] 117.02460  53.97778  73.05775 112.64302  60.99336 103.25252
100.55571
## [31,] 104.12929  98.19620  83.22172  91.98567  97.50092  74.19576
73.21832
## [32,] 100.38681 101.68211  98.49714  99.03011  83.99836  78.36530
95.12378
## [33,] 107.96184  52.26341  66.89668  90.07775  97.49883  85.38066
86.64401
## [34,]  93.88073  97.79672  77.87859  56.75816  95.49122 113.81080
128.73195
## [35,]  80.12514 107.44964  98.60153  94.73979  72.03117 120.19050
114.15971
## [36,]  90.28571  85.36012  77.96497  76.20613 110.62410  90.73190
122.66233
## [37,]  76.59305  89.32065  86.49955 101.58424  87.94481  57.51777
71.36178
## [38,] 110.11198  97.71765  80.98307  87.10539  99.47913  82.53930
```

```
59.01735
## [39,]  53.16337 109.07306 122.04213  92.11970  88.76637  73.35957
91.91754
## [40,]  83.39756 105.65662  85.01652  77.79207  91.66035 122.58609
105.89249
## [41,]  80.41808  88.51008  76.91070  78.14036  73.35430 127.08489
112.26382
## [42,] 107.02015 130.75317 100.67556  71.18887  88.98056 112.37376
63.53415
## [43,]  61.99671 117.45344 120.57589  86.80043 104.66231  78.83021
92.92388
## [44,] 117.17617  86.95215  89.07054  80.82016 120.17259  90.11702
88.77425
## [45,] 120.18432 124.62436 107.07013  89.07630  78.14159  85.40843
57.69380
## [46,] 100.84676 118.18027  97.22309  96.15745 104.45676  94.58383
65.65251
## [47,] 114.08185  95.49412 109.92777 109.55662  91.30721  82.82708
51.90515
## [48,]  77.13263  99.46917 125.17070 112.77998  84.26757  42.59141
79.47046
## [49,]  90.31416  95.70716  71.08361  69.30522 100.10806 124.68004
98.48344
## [50,]  65.18026  95.48251  72.22684  83.32143 101.53135 103.20404
89.75883
## [51,]  50.52022  59.16220  55.38938 102.13064  92.51367  88.96203
108.10956
## [52,]  85.66815  85.96184  83.48060  79.97040 109.50710  74.20453
91.31813
## [53,]  64.93130  78.00793  89.60527  98.84744  64.75917  87.45584
125.58664
## [54,]  62.26110 111.15721  85.33943  63.29534 110.07345  96.07692
100.93284
## [55,]  95.64874 112.06001  98.67359  49.28961 111.88347 109.46743
120.26066
## [56,] 140.63831  93.26533  83.28865  64.08266  97.51457 108.14866
85.31855
## [57,]  59.22652  71.04887  54.06676  63.20302 117.53870  77.14451
118.51802
## [58,] 123.19806  92.46774 114.52652 113.38899  81.00825  80.26354
68.36850
## [59,]  94.28286 103.22587  83.17913 108.18382  74.51128  91.16256
77.06132
## [60,]  95.79133  57.22898  48.34938  85.17946 109.73768  91.08573
89.06376
## [61,]  94.90991  62.91511  64.28799  95.26171  60.81502 111.63864
96.45294
## [62,] 105.94568 113.39752  96.00112  78.11397 133.78892  74.80042
66.29452
## [63,]  96.49106  78.80926  78.36156  76.07676 111.02752  85.37266
```

```
116.37225
## [64,] 100.71466 112.35338 104.90171  94.89745  51.51203 114.43322
92.38837
## [65,]        NA  75.34214  82.11002  98.48772  96.62225  63.17507
98.21560
## [66,]        NA        NA  53.43783 104.77057  81.37236  85.79538
108.22661
## [67,]        NA        NA        NA  72.19272  97.68648 106.96688
103.54430
## [68,]        NA        NA        NA        NA 128.86746 114.37033
106.12299
## [69,]        NA        NA        NA        NA        NA 103.52820
93.95704
## [70,]        NA        NA        NA        NA        NA        NA
68.00715
## [71,]        NA        NA        NA        NA        NA        NA
NA
## [72,]        NA        NA        NA        NA        NA        NA
NA
## [73,]        NA        NA        NA        NA        NA        NA
NA
## [74,]        NA        NA        NA        NA        NA        NA
NA
## [75,]        NA        NA        NA        NA        NA        NA
NA
## [76,]        NA        NA        NA        NA        NA        NA
NA
## [77,]        NA        NA        NA        NA        NA        NA
NA
## [78,]        NA        NA        NA        NA        NA        NA
NA
## [79,]        NA        NA        NA        NA        NA        NA
NA
## [80,]        NA        NA        NA        NA        NA        NA
NA
## [81,]        NA        NA        NA        NA        NA        NA
NA
## [82,]        NA        NA        NA        NA        NA        NA
NA
## [83,]        NA        NA        NA        NA        NA        NA
NA
## [84,]        NA        NA        NA        NA        NA        NA
NA
## [85,]        NA        NA        NA        NA        NA        NA
NA
## [86,]        NA        NA        NA        NA        NA        NA
NA
## [87,]        NA        NA        NA        NA        NA        NA
NA
## [88,]        NA        NA        NA        NA        NA        NA
```

```
NA
##  [89,]         NA         NA         NA         NA         NA         NA
NA
##  [90,]         NA         NA         NA         NA         NA         NA
NA
##  [91,]         NA         NA         NA         NA         NA         NA
NA
##  [92,]         NA         NA         NA         NA         NA         NA
NA
##  [93,]         NA         NA         NA         NA         NA         NA
NA
##  [94,]         NA         NA         NA         NA         NA         NA
NA
##  [95,]         NA         NA         NA         NA         NA         NA
NA
##  [96,]         NA         NA         NA         NA         NA         NA
NA
##  [97,]         NA         NA         NA         NA         NA         NA
NA
##  [98,]         NA         NA         NA         NA         NA         NA
NA
##  [99,]         NA         NA         NA         NA         NA         NA
NA
## [100,]         NA         NA         NA         NA         NA         NA
NA
##            [,72]      [,73]      [,74]      [,75]      [,76]      [,77]
[,78]
##   [1,]  80.04521  79.81874  68.23240  76.78142  60.26028 107.12595
105.22992
##   [2,] 102.71865  64.76905  74.73275  75.46626  85.89388 117.89022
87.26148
##   [3,]  69.69995 101.06493 104.02283  73.40820  89.29689 116.53470
86.89906
##   [4,] 127.62994  63.58751  92.66263 110.72560 111.32586  87.92237
115.45926
##   [5,] 124.16616  90.84033  77.96324 132.81590  75.08305  48.71182
116.99311
##   [6,]  88.66801  95.34575  84.23667  95.10450  63.86625 100.40833
113.84263
##   [7,]  90.03179  80.72452  68.11741  98.63284  95.30036  77.98271
104.34272
##   [8,]  95.48575  89.98478  87.73914  79.78585  73.88961 109.00684
91.77035
##   [9,]  83.42452 103.51398  94.56955  72.04818  88.89991 112.20906
75.69821
##  [10,]  64.36328  99.79565 106.16172  73.77062 122.36370  86.34815
73.34526
##  [11,]  71.07136 134.33121  88.95687  89.35030  98.75180  95.57518
59.52008
##  [12,]  84.76856  79.11302 104.64553  56.12189  92.38725 118.32802
```

```
76.40847
##  [13,] 103.77715  80.40256  95.50297 110.35657  94.83899  68.63855
91.47307
##  [14,] 129.74986  98.88793  86.19181 112.99601  68.26596  65.44250
86.14954
##  [15,] 139.54535 100.82347  91.75481 133.72604  87.90866  57.28992
94.57098
##  [16,]  67.35201 115.70000  99.71591  69.67515  86.28972 109.50847
79.92443
##  [17,]  86.13697  87.90127  85.40723 103.94260 103.50598  74.28882
101.72126
##  [18,] 103.12844  61.15368  86.34648  95.78501 113.28002 118.58534
127.28565
##  [19,] 126.19594  49.79848  62.98419 108.35185  83.45598  81.96041
114.12277
##  [20,]  65.03217 103.81068 108.07939  72.19487 109.40721  88.50041
50.52261
##  [21,]  80.16259 126.72635 127.87254  88.91095 114.32456  75.56963
52.75883
##  [22,] 128.94826  45.78997  54.59848 112.91032  97.60745  79.56510
128.58767
##  [23,]  95.25781 106.45923 103.49343 102.39052 121.00869 108.65785
87.29922
##  [24,] 104.41627  59.89763  74.94374  93.90587  61.67648  92.91606
110.37212
##  [25,] 116.73659  93.38564  93.47102 123.91946  66.79347  57.78214
111.24923
##  [26,]  66.62398 100.59448  85.39781  82.42469  75.03009  78.25639
68.88053
##  [27,] 129.28806  71.32640  63.24069 102.54677  68.24143  83.28192
115.12178
##  [28,] 133.21176  92.06708 113.46917 125.47959  96.74984  85.07974
102.24816
##  [29,] 121.45620  69.86616  92.15435 110.54881 115.73680  91.53001
109.65057
##  [30,]  72.99643  97.08721 106.34345  75.73001  60.74268  82.91876
85.77590
##  [31,] 108.98182  76.32368  59.04741  99.14599  63.10957 112.67047
118.50922
##  [32,] 112.44103  84.63422  67.79866 113.98281  80.67657  83.64743
117.39104
##  [33,]  87.58701  81.86939 108.14428  66.78796  77.36773 108.17835
73.49182
##  [34,]  89.78661  46.05238  91.63977  68.38695  95.57134 127.46872
114.66434
##  [35,] 109.76950  84.34537  95.78702 110.14045  86.11387  71.40863
112.55938
##  [36,] 107.20850  83.16162 114.74405  78.18822  96.04307  97.48877
88.10452
##  [37,]  84.15052 112.08233  73.42030 107.78667 100.54972  95.99417
```

```
91.20891
##  [38,]  71.44576 100.66300  60.40864  79.01086  58.11920 104.51735
91.04792
##  [39,] 103.92481  83.33929  79.88970 119.26565 134.12195  74.49867
97.50966
##  [40,]  92.31671  99.65976  90.47966  98.99124  80.54387  76.81836
92.26511
##  [41,]  66.60234 105.61415 103.19246  90.67846 100.81699  83.05627
80.52808
##  [42,]  84.29725  83.50363  62.72613  90.07555  79.71721  97.54693
101.54810
##  [43,]  91.02255  74.71243  57.65541  97.93657  94.04781  79.75701
107.03822
##  [44,] 116.15329  74.06950  86.53274  81.37095  58.59447  86.01373
86.56061
##  [45,]  94.27056  90.18533  55.66438 104.81648  85.37403  92.26984
109.01543
##  [46,] 108.68441 111.05688  80.03195  99.53805  54.30210  70.62905
91.60198
##  [47,]  98.18176  84.82426  65.52134  91.09538  46.77407  72.58179
93.75117
##  [48,]  97.02640  95.00180  77.76090 108.97446 118.68457  71.74492
95.25275
##  [49,]  60.71259 105.65293  98.82146  64.51778  73.73426  94.08795
74.68025
##  [50,]  76.27705 125.52990 101.42960  88.98871  87.01775  86.75147
72.78628
##  [51,]  83.25708 114.13585 124.70354  85.04504  89.08376  98.34746
77.53444
##  [52,] 100.75763  93.16592 118.55585  78.54040 116.82575 103.38647
73.00644
##  [53,] 106.29068  76.78885 104.49531 114.56624 126.64874  91.76592
109.06934
##  [54,]  96.34111  97.57866  92.36507 100.46212 117.49418  96.53969
86.54259
##  [55,]  89.16099  58.19107  70.65243  79.57737  93.72706  95.36581
101.37437
##  [56,]  97.81049  58.93250  82.61038  72.76456  74.68200 111.28967
95.03089
##  [57,]  61.11631  93.87782 110.59236  56.49938 118.31586 135.76424
74.54446
##  [58,]  91.50907 100.48931  89.21393  86.59814  72.71020  58.30082
80.67869
##  [59,] 102.48093  88.69972  69.09106 104.09131  53.86714 102.36135
128.57436
##  [60,]  77.72843  97.38281 100.72225  63.91454  57.77528 113.26470
73.80912
##  [61,]  73.93140  85.36175 109.87628  78.95597  86.56342 109.99592
90.93954
##  [62,]  95.06130  94.82125  78.46297  68.79388  66.61374  94.83779
```

```
82.44611
##  [63,]  56.15670  91.78614  95.52712  45.44634  82.52603  99.89438
77.28437
##  [64,] 117.50462  60.99704  74.80542 122.03522  90.72917  89.77058
135.41324
##  [65,]  78.32171 107.53251 103.22908  96.35102 125.85195  97.33960
79.39433
##  [66,]  67.57870 107.49872 132.97945  73.06202  94.93559  96.13466
59.31859
##  [67,]  66.32631 111.01435 131.29322  62.74613  91.05235 130.59678
63.94575
##  [68,]  76.24559  70.57233  93.33927  54.60408 104.41878 127.06464
77.80280
##  [69,]  97.53618  93.45285  91.61965 124.34802  86.98165  68.18443
113.87042
##  [70,]  89.12301 100.22882  74.92182  96.44683 103.38257  88.61630
87.82649
##  [71,]  97.77437 112.74506  67.30058 106.82501  75.76363  81.64253
84.42154
##  [72,]        NA 110.41237 104.67248  45.38294 102.94167 109.50685
56.52843
##  [73,]        NA        NA  63.37721  88.18041  86.68045 100.15199
128.21581
##  [74,]        NA        NA        NA 108.98560  72.03238  83.02142
130.20123
##  [75,]        NA        NA        NA        NA  90.17585 121.39221
58.78398
##  [76,]        NA        NA        NA        NA        NA  77.37631
105.31607
##  [77,]        NA        NA        NA        NA        NA        NA
91.71315
##  [78,]        NA        NA        NA        NA        NA        NA
NA
##  [79,]        NA        NA        NA        NA        NA        NA
NA
##  [80,]        NA        NA        NA        NA        NA        NA
NA
##  [81,]        NA        NA        NA        NA        NA        NA
NA
##  [82,]        NA        NA        NA        NA        NA        NA
NA
##  [83,]        NA        NA        NA        NA        NA        NA
NA
##  [84,]        NA        NA        NA        NA        NA        NA
NA
##  [85,]        NA        NA        NA        NA        NA        NA
NA
##  [86,]        NA        NA        NA        NA        NA        NA
NA
##  [87,]        NA        NA        NA        NA        NA        NA
```

```
NA
##  [88,]        NA        NA        NA        NA        NA        NA
NA
##  [89,]        NA        NA        NA        NA        NA        NA
NA
##  [90,]        NA        NA        NA        NA        NA        NA
NA
##  [91,]        NA        NA        NA        NA        NA        NA
NA
##  [92,]        NA        NA        NA        NA        NA        NA
NA
##  [93,]        NA        NA        NA        NA        NA        NA
NA
##  [94,]        NA        NA        NA        NA        NA        NA
NA
##  [95,]        NA        NA        NA        NA        NA        NA
NA
##  [96,]        NA        NA        NA        NA        NA        NA
NA
##  [97,]        NA        NA        NA        NA        NA        NA
NA
##  [98,]        NA        NA        NA        NA        NA        NA
NA
##  [99,]        NA        NA        NA        NA        NA        NA
NA
## [100,]        NA        NA        NA        NA        NA        NA
NA
##             [,79]     [,80]     [,81]     [,82]     [,83]     [,84]
[,85]
##   [1,]  96.14200  61.22534 111.39858  91.48159  96.68347  75.95129
74.50134
##   [2,]  94.56847  61.73100 100.75887  70.04074  95.15753  90.87506
111.16859
##   [3,]  69.64072  94.36382  96.11323 111.27130 103.68920  65.21612
96.20614
##   [4,]  86.24309 105.71291 119.48695  97.96377  78.65832  98.22227
64.23631
##   [5,]  74.18650 120.12478  78.18083  83.75269  62.42741 138.67419
82.63346
##   [6,]  90.57894  82.31150  92.31396  89.57232  84.84649  71.72996
51.63961
##   [7,]  99.27346  65.34645 121.14775  80.88267  65.14856 100.58250
57.85625
##   [8,] 111.47869  59.25019 101.90003  68.72580 106.05705  62.83477
72.32060
##   [9,] 102.43528  45.41739 115.33573  64.55060  97.29532  62.87795
81.16583
##  [10,] 106.17036  89.69578 106.82246 105.78718 135.89327  59.19239
87.12342
##  [11,]  78.33858  66.86833  77.01590  68.86608 105.33970  79.50394
```

```
121.60423
## [12,]  89.48003  86.89040 101.84571  93.63809 122.34579  72.71318
118.67982
## [13,]  81.06815 121.80609  87.17630 123.16362  53.63471 117.42595
85.86246
## [14,]  93.42040  82.60751  91.65792  57.66958  53.33032 125.78300
89.86661
## [15,]  92.78564  95.29999 103.11892  76.80719  59.71979 114.96244
68.35749
## [16,]  65.32321  73.16425  84.72456  59.40979  68.78075  90.18154
103.90700
## [17,]  98.09517  95.07579 119.59952 123.64757 100.07868  78.30830
55.97348
## [18,]  63.67135 106.10167  94.63016  92.01537  88.84381  87.22490
81.78272
## [19,]  81.69052  93.56744 113.77796  97.02610  67.67793 128.53038
91.73753
## [20,] 124.25099  79.03360  97.94143 112.41238 130.97456  50.45627
90.76657
## [21,] 111.71538  95.02176  76.81947  85.77103 102.66930  67.19972
84.53091
## [22,]  84.72123  86.41900 124.06960  80.66527  76.27731 125.71240
80.12195
## [23,]  31.52321 109.45392  72.33342  81.92954  71.98001 100.97442
120.82608
## [24,]  85.91007 108.96488  82.07366 115.60032  94.21042 101.62561
97.09764
## [25,] 110.77862 114.03519  84.70589 106.47834  88.32134  87.96817
38.33065
## [26,] 116.57630  80.43196  61.58355  93.97417  95.32778  82.80108
93.12658
## [27,]  98.28716  74.65310 100.12022  48.39263  97.47291 109.79326
89.00673
## [28,]  68.85171 135.99886  74.56118 100.37925  61.45003 103.04830
79.03505
## [29,]  76.48532 100.76710 110.08201  87.84562  56.73202 107.10324
71.56100
## [30,] 116.47449 103.22297  79.63897 119.02410 115.04770  62.29665
70.35938
## [31,]  62.06207  83.27966  82.41405  73.78395  93.55596 102.08815
109.49080
## [32,]  58.72527  97.62244 107.62167  93.03531  85.49716 114.35723
95.65441
## [33,] 113.57902  95.26336  71.47585 100.35466 123.95743  62.10025
101.91162
## [34,]  82.13028  97.90542  96.48473  98.74523  87.71658  81.79326
84.09233
## [35,]  78.88058 113.51249  93.18288  92.07192  42.76738 116.44701
62.21831
## [36,]  75.48717 104.66333 112.50013  88.63777  85.16014  98.66100
```

```
101.59503
## [37,]  55.97747  84.79712  85.05160  87.67985  99.15745  91.41685
111.93040
## [38,]  83.10681  52.39671  89.59633  71.97971  98.68228  86.07672
104.70600
## [39,]  76.97828 101.34083  91.29004  88.62070  77.32642 111.70677
92.01706
## [40,]  66.57529  93.73420 100.95662  91.19253  30.43410 121.59633
85.78579
## [41,]  81.51440  97.99495  94.29865 117.58418  60.78404  87.80852
73.63579
## [42,] 100.38602  51.41365 100.24136  69.06459  73.75343  88.22032
68.57157
## [43,]  72.58731  84.01604  88.78413  70.64760  68.87072 130.35487
105.23384
## [44,]  95.24175  85.66158 103.00937  79.24039  86.85919 113.39982
111.15397
## [45,]  92.42429  56.33394 115.64719  77.41515 100.61991  82.50860
70.89768
## [46,]  81.87546  67.57680 104.85449  45.13879  53.97829 123.50851
89.25213
## [47,] 122.12426  69.55374  78.69242  66.94041  99.09001  98.48029
86.37935
## [48,]  90.35879  88.76079 103.29221  82.91424 118.19799  92.03926
91.76845
## [49,]  89.29361  72.09169  95.24193  83.67588  61.11197  90.38812
86.66137
## [50,]  63.11286  84.29249  85.40858  75.12907  45.13889 106.63280
100.56918
## [51,]  62.52572 120.80858  52.70261  89.98718  71.24423  92.74356
108.33852
## [52,]  95.92367  89.10509 103.18136  77.34900 106.97828  73.13945
96.88682
## [53,]  67.95083 142.62805  80.83480 123.81071  88.88470  89.83374
81.60130
## [54,]  47.98630  89.84329 104.42395  81.94005  48.42157 114.64168
105.92702
## [55,]  74.95767  82.62378 121.99545  98.57525  69.96122 116.20991
101.20045
## [56,] 114.43782  73.21945 106.64309  96.43336 108.10443  71.81569
82.13943
## [57,]  56.61805  93.70122  85.24755  94.01918 101.15013  72.78890
129.15449
## [58,] 139.50390  70.95757 111.14402  80.15369 118.36760  79.08290
67.29487
## [59,]  66.73004  90.84081  69.85668  68.83423  76.26239  99.29895
81.80847
## [60,]  81.16635  90.06316  65.30530  89.07738  93.89531  83.97840
122.55249
## [61,] 109.32522 106.49502  56.81035 114.79064 104.60383  47.42327
```

```
66.62168
##  [62,]  93.44619  45.98221 114.29331  32.39310  91.25739 102.64898
109.44916
##  [63,]  88.89861  76.74247 111.62978  93.09059 110.27251  81.67725
109.31599
##  [64,]  92.33669 106.65784  85.48910 101.29583  75.21351  91.73347
44.87649
##  [65,]  56.80038 111.09781  57.34086  87.44856  83.23017  95.19661
118.55254
##  [66,]  96.30483 122.91868  53.58839 125.29609 112.74537  61.04726
104.24412
##  [67,]  72.96503  99.37969  68.38247 103.71985  87.28857  58.59956
105.78382
##  [68,]  84.92293  64.94920 117.84267  87.91622  80.65000  82.84268
104.47371
##  [69,]  99.62606 121.68867  74.30172 122.84604  92.02190  78.26359
39.93722
##  [70,]  77.77918  84.64293  76.67308  66.69425 120.77665  95.04229
123.96171
##  [71,]  98.15994  55.60520  83.40684  51.25846  92.95849  92.30168
92.10952
##  [72,]  98.07264  72.28286  84.33809 101.68700 114.64560  51.23228
102.72413
##  [73,]  95.45544  96.07288 102.82978  99.28640  94.49724 103.64801
84.67335
##  [74,]  83.00161  65.88716 101.44933  67.82256  89.09066 120.18776
91.25538
##  [75,] 104.87693  66.46913  99.83574  89.34617 113.16567  60.21060
110.92239
##  [76,]  99.40953  80.33065  88.23659  75.17565  79.27102 107.98820
82.58306
##  [77,] 112.65577  99.82376  99.36281  93.49694  79.43440 118.12134
67.03676
##  [78,] 106.38334  77.69625  87.50419  92.89673  99.80811  65.62420
110.99629
##  [79,]        NA 102.77348  77.17211  80.39415  64.81974 111.16198
118.02625
##  [80,]        NA        NA 119.93873  50.57417  97.30007  85.53739
98.30535
##  [81,]        NA        NA        NA  98.44188  92.71808  81.24681
103.18605
##  [82,]        NA        NA        NA        NA  79.84348 110.90574
107.39522
##  [83,]        NA        NA        NA        NA        NA 128.53089
83.16552
##  [84,]        NA        NA        NA        NA        NA        NA
79.71420
##  [85,]        NA        NA        NA        NA        NA        NA
NA
##  [86,]        NA        NA        NA        NA        NA        NA
```

```
NA
##  [87,]        NA        NA        NA        NA        NA        NA
NA
##  [88,]        NA        NA        NA        NA        NA        NA
NA
##  [89,]        NA        NA        NA        NA        NA        NA
NA
##  [90,]        NA        NA        NA        NA        NA        NA
NA
##  [91,]        NA        NA        NA        NA        NA        NA
NA
##  [92,]        NA        NA        NA        NA        NA        NA
NA
##  [93,]        NA        NA        NA        NA        NA        NA
NA
##  [94,]        NA        NA        NA        NA        NA        NA
NA
##  [95,]        NA        NA        NA        NA        NA        NA
NA
##  [96,]        NA        NA        NA        NA        NA        NA
NA
##  [97,]        NA        NA        NA        NA        NA        NA
NA
##  [98,]        NA        NA        NA        NA        NA        NA
NA
##  [99,]        NA        NA        NA        NA        NA        NA
NA
## [100,]        NA        NA        NA        NA        NA        NA
NA
##            [,86]     [,87]     [,88]     [,89]     [,90]     [,91]
[,92]
##   [1,]  82.68244  87.72568 112.59423  70.36652  97.44296  79.32844
72.56626
##   [2,]  88.76440  68.16211  98.57995  93.11144  90.58892  78.78812
57.67163
##   [3,]  87.43470 119.81784 126.80934  62.38513  77.34059 106.78177
75.65806
##   [4,] 139.68376  69.75151 113.74620  95.01457 106.55242  84.60602
101.38625
##   [5,] 120.78396  85.02202  60.65752 111.15713  73.23425 103.29735
114.88845
##   [6,]  90.07862  72.92566 121.31626  71.87074 106.02473  71.60822
89.61563
##   [7,]  94.58603  73.99241  77.13614  95.61148 141.54659  70.25361
88.15992
##   [8,]  87.28975  67.53153 119.91286  84.21909 109.81807  58.08271
77.29948
##   [9,]  82.72816  84.03213 112.24336  80.06686 124.26108  66.64920
60.58379
##  [10,]  89.74963 128.93624 100.00232  99.00162 101.79180  86.03601
```

```
108.85313
## [11,]  77.79648 116.33715  80.11651  96.09698  95.51064 108.56713
50.93304
## [12,]  88.31936 111.16320 114.09702  85.12454  64.47496  90.20123
82.83938
## [13,]  98.76006  85.59750  74.22462  67.52505  72.37100 130.36889
94.32347
## [14,] 102.91683  66.25613  62.15966  92.23675  87.37721  96.16855
77.61917
## [15,] 128.26812  69.43760  81.49763  88.63233 100.99921 105.02352
84.99118
## [16,]  67.53935  99.84896  81.39066  93.63772 103.73024  71.62997
76.16270
## [17,] 110.49900 105.91752 106.95412  76.78823 107.34847 104.57481
98.93531
## [18,] 118.90927  72.25464 125.02906 108.92100 104.66116  67.28144
100.04225
## [19,] 121.42438  74.73335  83.61934  89.82206  79.10966 108.07971
79.73914
## [20,]  71.62202 111.93239  99.64226  70.76621  94.05395 103.15286
84.30706
## [21,]  78.05573  96.44945  83.83580  91.50538 103.79287  86.47068
103.23762
## [22,] 136.84804  70.35431  83.83305 119.52769 105.43001  80.78145
93.82270
## [23,] 106.44546  99.14880  94.34034 100.78992  87.62676 110.17528
69.86847
## [24,]  96.89153  84.97162 100.16495  74.46540  42.41022 111.47289
92.03217
## [25,] 110.85711  70.44109 102.96302  76.42138  84.50765  93.52838
120.28779
## [26,]  45.52525  84.51714  63.58846  80.68028  84.15085  92.17564
90.40916
## [27,] 117.34961  67.40347  85.81520 127.62512  90.71688  61.69881
94.93130
## [28,] 125.04726  66.11878 108.33765  78.17714  75.89507 110.88712
96.07827
## [29,] 125.00081  61.44620  99.35193  96.45809 116.23079  82.80182
91.37656
## [30,]  69.38215 101.75042 106.42071  60.71030  63.67863  91.22254
116.84925
## [31,] 107.19082  81.25581 106.50153  98.59513  69.87689  94.14834
63.90364
## [32,] 132.39012 107.95867  96.69453  95.14304  74.98820 118.02146
77.98540
## [33,]  70.97868  82.85336 112.38685  74.02703  57.98469  86.26639
93.96359
## [34,]  90.09346  68.00825 120.43136  90.47319  92.53413  57.10340
103.19430
## [35,] 106.16166  71.69255  80.40730  89.26808  97.13695  84.06544
```

```
114.56333
## [36,] 111.96459 105.05845 102.53732  88.62489  72.05219  92.54734
96.97175
## [37,] 104.82048 116.93325  94.03405  96.67163  90.26380 122.99626
55.48637
## [38,]  71.36261 101.22014  88.08244  86.03220  88.49738  90.64359
54.93742
## [39,] 115.95331  84.04785  70.86662 120.70003 113.07283  97.93385
93.15572
## [40,]  91.14727  95.41241  67.91629  72.08698  91.44738 109.75173
79.92280
## [41,]  75.47544 101.57790  84.46396  58.15047 103.36459 110.08718
87.09939
## [42,]  77.16293  59.48174  86.39969  91.58508 133.56650  60.45292
72.71457
## [43,]  92.57289  88.03615  47.03804 128.28523  98.42414  81.46142
93.03104
## [44,]  97.49940  90.57942  81.08788  84.56830  50.69155 100.04358
82.14719
## [45,] 108.21956  86.06634 105.07926  96.43649 123.88452  83.06059
66.38804
## [46,]  96.87129  89.86818  61.65810  94.58535  93.70398  86.80441
75.63700
## [47,]  74.91984  70.44797  67.02615 101.50155  81.08756  71.53436
96.31288
## [48,] 121.28605 110.68833  84.31321 129.37218 105.99342  92.21730
98.56861
## [49,]  52.00432  97.17748  73.31090  68.06178  99.50858  80.71766
83.03575
## [50,]  74.30388 103.70366  65.02521  78.08180  96.87943 103.70507
70.00413
## [51,]  76.28680  97.26968  87.26571  81.60773  64.55115  99.42995
97.33379
## [52,] 103.37680  91.10858 109.80732  99.08410  98.54890  76.22827
91.00376
## [53,] 124.33332  86.29723 114.53173  92.72180  84.72420 107.21963
109.91119
## [54,] 106.67179  97.21163  80.19854  90.78037 106.03914 110.51362
60.63554
## [55,]  96.17160  98.91970  78.27900  88.91760  88.92959  97.72055
79.82040
## [56,]  87.15960  68.58972 119.80440  72.62893  86.15704  76.20032
80.54811
## [57,]  77.21409 118.14250 108.31536  89.95640  82.89511  91.80895
75.22500
## [58,]  89.68007 100.18784  80.92917  96.26494  97.10096  74.16258
110.95091
## [59,]  96.70248  67.59970 101.56086  98.81857  85.89725  72.00532
88.49894
## [60,]  59.98560  98.10224  94.87583  66.82842  44.11471 101.10456
```

```
73.15256
## [61,]  62.77190  62.43902 123.74430  66.70411  88.48853  70.96076
109.33258
## [62,]  86.84978  97.89265  73.57453 110.08742  94.12641  64.50177
73.26986
## [63,]  70.76374 141.13835  86.76946  87.80381  75.64140  85.13461
94.70500
## [64,] 114.57774  34.59840 113.41339  91.74278 109.90016  73.20209
106.17324
## [65,]  86.80464 101.12897  77.42122 109.40754  89.65778 101.42600
88.49797
## [66,]  63.15714 108.87591 102.38596  62.33038  50.39762 112.82311
103.79844
## [67,]  60.95330  96.91802 121.39020  47.28734  73.27452 101.09512
70.22162
## [68,]  75.95046  88.03555 101.17083  76.21346 104.70006  79.42658
62.00049
## [69,] 104.06292  75.64904 107.99681  77.73492  94.03422  97.18307
119.99417
## [70,] 100.35385 110.93370  76.37271 136.59629  85.20746  91.38953
85.51724
## [71,]  88.90258  78.04255  76.11646 103.86807 108.78028  85.74781
61.75842
## [72,]  35.29111 123.91320  90.30008  77.09533  99.02580  83.80220
84.84959
## [73,] 106.99647  62.85815  99.90847 105.63102  83.26613  69.51791
107.08763
## [74,] 108.62181  78.47389  72.47040 125.54128 100.62844  79.19976
80.02654
## [75,]  42.51116 112.11089  95.81347  78.24581  85.87381  67.43714
87.01552
## [76,]  83.29893  80.45948  78.65300  81.54251  65.00018  83.59720
93.57694
## [77,] 106.53054  96.66547  50.67184  96.22786  86.52401 106.89792
116.16294
## [78,]  50.66320 119.31651  78.62475  65.90834  88.08399 105.02929
74.83267
## [79,] 109.57884  95.92829  96.02983  98.84021  82.79180 109.56470
62.91768
## [80,]  72.23404  97.23879  76.64590 100.87363 121.91146  65.35438
58.16800
## [81,]  73.04985  76.50944  93.75960  87.78688  66.23530  99.68321
95.09686
## [82,]  94.12629  83.64422  68.27791 125.03712 107.29026  63.82549
68.09373
## [83,] 102.95324  70.83450  73.57993  77.86909  93.33930 104.31365
76.17950
## [84,]  59.27047  96.04990 130.00006  66.57989 103.02866  76.29614
91.96438
## [85,] 103.14193  65.11058 105.46716  75.82092 114.02847  75.86929
```

```
118.74105
##  [86,]        NA 102.72374  82.72290  68.45041  86.63828  79.49663
85.55583
##  [87,]        NA        NA 105.01502  91.71979 100.21690  68.69020
94.36230
##  [88,]        NA        NA        NA 111.23332  86.33775  98.09620
92.21621
##  [89,]        NA        NA        NA        NA  77.56707 114.73571
80.85964
##  [90,]        NA        NA        NA        NA        NA 115.96321
101.56855
##  [91,]        NA        NA        NA        NA        NA        NA
107.14176
##  [92,]        NA        NA        NA        NA        NA        NA
NA
##  [93,]        NA        NA        NA        NA        NA        NA
NA
##  [94,]        NA        NA        NA        NA        NA        NA
NA
##  [95,]        NA        NA        NA        NA        NA        NA
NA
##  [96,]        NA        NA        NA        NA        NA        NA
NA
##  [97,]        NA        NA        NA        NA        NA        NA
NA
##  [98,]        NA        NA        NA        NA        NA        NA
NA
##  [99,]        NA        NA        NA        NA        NA        NA
NA
## [100,]        NA        NA        NA        NA        NA        NA
NA
##            [,93]     [,94]     [,95]     [,96]     [,97]     [,98]
[,99]
##   [1,] 103.57173  78.78443 107.22455  82.43087 103.45284 104.36192
69.84669
##   [2,] 105.38845  78.09306  96.47844 105.13064  92.08202  56.25765
74.36341
##   [3,]  93.40629  88.33399 122.40077 106.45231 126.00863  92.11876
58.39533
##   [4,]  87.95386 136.84051  95.15822  94.58935  81.89280  94.74131
70.51631
##   [5,]  88.20363  97.11468  76.61479  90.28982  92.97781 103.20421
102.71718
##   [6,]  97.27606  83.70745  95.43438  78.97334  79.46207 116.63029
73.70622
##   [7,]  60.40531  96.18287  77.11173  41.78492  65.79475 133.17032
94.89771
##   [8,] 118.23858  82.53853  82.04137 104.66728  77.32935  81.61896
84.12310
##   [9,]  98.69186  75.98537  84.76683  98.39482  85.30853  87.64230
```

```
82.46612
## [10,]  86.93861 121.14315  74.42804 104.53835  99.60467  99.49726
104.53519
## [11,]  87.86489  63.72574  77.91875 108.81842 109.46980  87.68043
111.91479
## [12,] 113.93423  92.28176 104.97463 138.50564 123.33857  48.86721
64.98559
## [13,]  80.95138 100.84547 126.72914  59.49047  91.50083  90.73120
89.65427
## [14,] 107.41568  70.10995  84.84195  92.80432  84.03468  72.39307
104.68287
## [15,] 104.96672 101.09994  88.88841  91.94943  83.39934  90.08979
106.32216
## [16,]  56.82724  48.62332  76.21603  86.04418  87.79695  99.44461
72.74968
## [17,]  87.97403 129.85270 103.32494  74.45666 100.45037 127.07449
94.40128
## [18,]  65.64735 118.56681  84.45595  98.29989  78.80685 101.81157
49.13681
## [19,]  95.78903 104.80354 114.42966  84.41729 106.22847  83.99427
74.46488
## [20,] 109.78776 102.70003  98.88449  93.84839  94.81181  78.13618
116.12066
## [21,]  92.35844  94.64001  66.76582  95.65139  65.51709  81.81828
126.39157
## [22,]  81.00755 120.90543  80.25615  87.33546  90.43672 102.19133
80.06572
## [23,]  56.96105  85.94031  92.20708 107.66150 101.02869  86.48559
70.24439
## [24,] 119.82739  93.54846 126.82242  98.16425 114.57272  71.98189
72.53338
## [25,] 124.77427 112.00339  93.49456  86.48209  76.36323 102.09441
106.39544
## [26,]  96.31604  65.05278  86.32509  61.68546  70.03228  87.04145
125.80405
## [27,] 116.64435  92.09053  56.79532 123.39032  89.63779  77.48750
92.98881
## [28,]  98.09440 106.77642 109.70266 101.69438  82.11474  76.20329
76.00023
## [29,]  66.47547 116.26615  91.85846  77.52616  68.92331  98.79542
70.23797
## [30,] 126.79622  90.62154 105.99191  92.70849  95.64681  86.09961
95.61250
## [31,] 107.19057  75.06484  98.41709 120.51580 118.29304  80.50387
63.67999
## [32,]  96.13082 101.89406 104.74290 112.12485 140.59506 102.05753
72.91610
## [33,] 137.65859  85.80559 104.03420 118.63470  90.12664  38.00206
87.91091
## [34,]  75.47238 101.76686  99.17879  85.59935  74.36483  85.44289
```

```
38.71123
## [35,]  68.85620  97.84254  89.92085  63.74213  69.65186 110.68949
79.56139
## [36,]  97.30987 101.14912 101.92299 126.57201 115.92507  64.27120
58.45605
## [37,]  80.53026  86.25072  93.43234 106.45733 126.43668 106.21932
91.99749
## [38,]  97.56984  51.34990  92.22693  92.80539 114.64666  98.53310
87.03383
## [39,]  53.28117 117.53787  70.15151  76.29146  74.55976 108.19145
105.76334
## [40,]  63.45413  72.35934 110.74254  57.89437  98.66658 109.35112
77.53351
## [41,]  56.81082  87.45870 115.22180  38.42900  84.62009 121.24783
87.44733
## [42,]  73.63936  73.12777  77.28219  51.20657  59.14696 116.93783
91.46393
## [43,]  49.37537  82.52498  64.27676  69.54136  84.99753 110.13821
94.39903
## [44,] 131.77493  77.16083 107.52643 120.98847 121.23383  43.52796
79.63216
## [45,]  93.67454  96.73872  81.20922  88.43093  97.27362 121.54823
93.16887
## [46,]  95.15676  54.69740  78.00532  94.11011 102.46595  94.28977
92.77418
## [47,] 123.50072  68.47026  68.62532  90.24290  78.40857  79.59036
118.39605
## [48,]  85.35699 123.88375  55.46556 112.76396 101.42967 106.49514
114.84297
## [49,]  67.46203  51.69389  97.91639  54.60777  83.70809 102.93369
81.14120
## [50,]  57.15745  51.76954  94.95299  69.32689  94.45158 103.45385
86.29426
## [51,]  78.22349  67.65206  96.38606  99.06828  89.09646  74.76637
77.08085
## [52,]  99.88523 106.13982  78.30843 128.71589  87.20868  61.34029
83.76247
## [53,]  75.37468 138.55411 102.34287  94.59665  85.76129  96.31281
75.55378
## [54,]  45.63709  85.80814 101.12799  79.51635 100.71015 100.46296
70.89604
## [55,]  66.26272  93.47358 110.82199  71.38860 110.82984  97.35759
63.52769
## [56,] 125.48123  94.85976 110.40751  97.89577  89.46523  66.18879
73.05600
## [57,]  62.34865  82.45337  99.86430 109.99384 109.79748  77.10888
54.06076
## [58,] 127.43210  98.88383  64.80201 102.27019  90.72671  91.25485
126.79884
## [59,]  91.43180  68.70068  81.37330  95.37337  86.15951 104.23187
```

```
69.32959
## [60,] 113.98546  47.32337 116.09658 110.22957 112.85742  51.34170
72.09082
## [61,]  99.98804  92.63401  99.43611  74.62596  48.55574  85.43076
85.54223
## [62,] 102.07744  58.87729  65.27215 119.19395 108.30466  73.56527
85.74116
## [63,]  87.27346  79.69442  93.40650 104.28039 128.99058  88.14169
74.33495
## [64,]  85.92449 118.32480  90.38689  65.21522  49.29952 112.63910
80.72939
## [65,]  48.79556  86.69042  76.13796  91.70224  83.46488  92.34797
93.26484
## [66,] 108.21934  88.39263 113.18081 102.77756  94.84889  60.41383
95.45100
## [67,]  90.89299  64.17535 126.79954  97.73993  95.19793  65.78411
59.39848
## [68,]  71.06672  80.53708 110.65685  79.68277  92.42543  77.62990
54.75741
## [69,]  97.67010 119.89033  94.54353  69.69373  69.00855 121.83012
104.55306
## [70,]  88.29560  90.94011  52.49363 128.85303 107.89183  85.74902
109.36998
## [71,] 103.20162  66.39263  64.24030  95.78866  84.10567  94.02072
120.58709
## [72,]  69.23491  72.21327  89.50653  74.69905  90.97684  99.28585
94.57247
## [73,]  93.36789 115.72370  93.78632  87.20805  82.87234  83.79221
66.37504
## [74,]  84.00533  88.97228  68.99787  83.75913  98.28569 115.46261
93.43223
## [75,]  88.66491  69.00767  94.24585  95.44903  96.56478  67.99515
73.80362
## [76,] 124.77481  60.34453  94.67627  93.34908 101.81492  83.77076
88.11627
## [77,] 104.42255 104.56946  77.43067  77.41794  91.01722 105.58717
136.84931
## [78,]  92.92078  68.96028  98.19370  91.66724  94.53801  65.43905
109.67466
## [79,]  58.72044  77.86250 100.13071 102.44131 112.57983  96.63402
55.69250
## [80,]  85.55887  61.98248  68.92580  87.08786  96.09187  98.05196
95.97340
## [81,]  90.68891  75.94590  91.06326  92.07899  71.93911  77.74586
96.88878
## [82,]  85.16279  57.72686  50.25191 108.81936  92.56148  88.02400
92.09922
## [83,]  71.97072  73.20452 106.30592  65.97261  84.93589  98.11936
76.23969
## [84,]  95.58013  95.43309  95.71419  90.18807  73.10690  86.36849
```

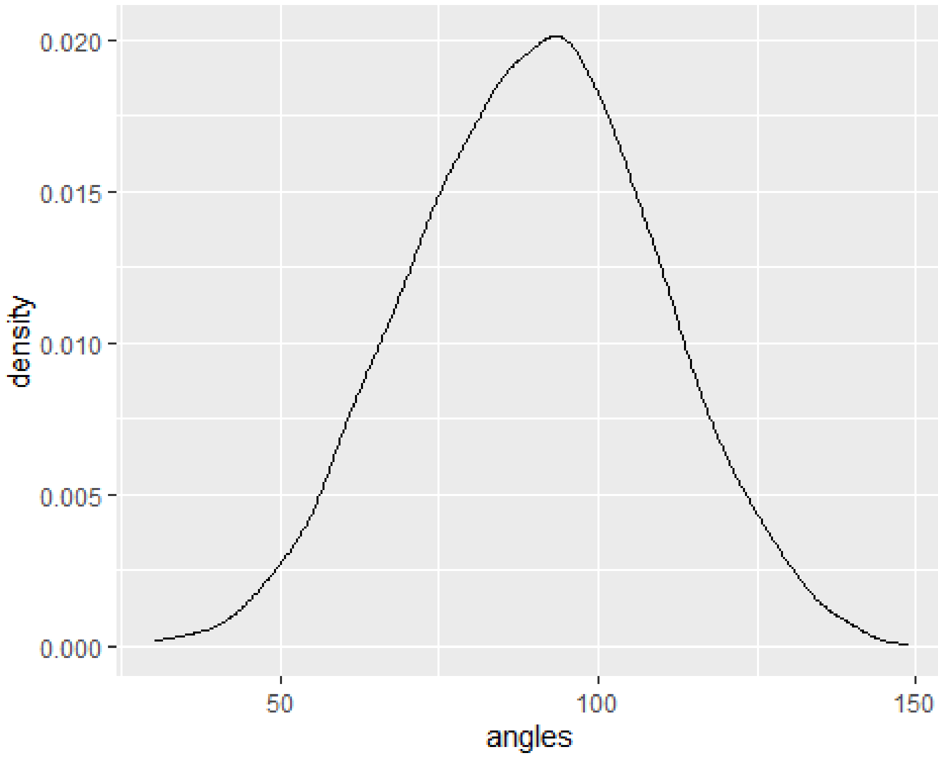
```
89.59081
##  [85,]  99.70814 118.32458  89.51160  61.12351  58.34224 120.40553
100.25243
##  [86,]  86.66734  54.09985  94.00177  72.56217  79.17417  78.66814
97.48062
##  [87,]  97.62856  96.15837  89.05921  76.03617  45.04439  85.95717
81.43220
##  [88,]  81.06330  70.97157  67.40958  77.37317  97.67913  95.33099
124.83723
##  [89,] 105.80571  80.87372 147.34293  72.28800  91.07790  78.41434
79.57484
##  [90,] 120.58093  81.54759 116.28947 114.65775 117.53309  52.08846
81.37616
##  [91,]  85.55563  88.51478  52.84130  89.75253  59.85505  95.38498
81.64073
##  [92,]  78.88945  57.16492 104.33002  94.46037 109.83024  85.00096
76.78426
##  [93,]        NA  88.13980  78.65018  64.15720  80.66644 122.19613
78.80605
##  [94,]        NA        NA  91.10465  90.72320 100.21057  77.22834
85.14996
##  [95,]        NA        NA        NA  98.57986  73.80565 107.14678
116.86905
##  [96,]        NA        NA        NA        NA  59.50439 123.68963
101.10620
##  [97,]        NA        NA        NA        NA        NA  98.09311
102.13299
##  [98,]        NA        NA        NA        NA        NA        NA
79.80459
##  [99,]        NA        NA        NA        NA        NA        NA
NA
## [100,]        NA        NA        NA        NA        NA        NA
NA
##          [,100]
##   [1,]  94.69071
##   [2,]  76.34965
##   [3,] 100.25397
##   [4,] 105.38488
##   [5,] 100.00734
##   [6,]  96.25055
##   [7,]  96.01720
##   [8,]  68.24094
##   [9,]  60.84084
##  [10,]  91.05705
##  [11,]  77.12034
##  [12,]  74.61991
##  [13,] 115.54373
##  [14,]  62.29437
##  [15,]  83.68285
##  [16,]  62.29929
```

```
##  [17,] 122.97892
##  [18,] 113.26563
##  [19,] 110.03772
##  [20,]  86.72242
##  [21,]  66.24609
##  [22,] 107.34260
##  [23,]  97.57348
##  [24,] 113.71230
##  [25,] 102.43296
##  [26,]  85.23125
##  [27,]  71.10244
##  [28,] 101.40703
##  [29,] 101.00925
##  [30,]  89.62595
##  [31,]  99.54130
##  [32,] 112.50655
##  [33,]  76.07837
##  [34,]  99.28723
##  [35,]  95.74952
##  [36,]  72.18115
##  [37,] 111.38691
##  [38,]  85.14902
##  [39,] 112.20261
##  [40,]  91.66197
##  [41,] 107.23352
##  [42,]  88.22824
##  [43,]  97.30500
##  [44,]  65.70615
##  [45,] 102.70678
##  [46,]  53.87234
##  [47,]  70.40643
##  [48,]  99.94271
##  [49,]  69.19927
##  [50,]  74.21085
##  [51,]  78.06731
##  [52,]  64.41276
##  [53,] 131.80545
##  [54,]  95.36775
##  [55,] 103.41732
##  [56,]  87.41893
##  [57,]  88.03632
##  [58,]  64.35135
##  [59,]  94.42467
##  [60,]  71.94910
##  [61,] 100.17212
##  [62,]  35.19286
##  [63,]  73.42548
##  [64,] 124.50755
##  [65,]  98.34227
##  [66,]  90.42567
```

```
##  [67,]  80.25367
##  [68,]  80.25802
##  [69,] 127.66852
##  [70,]  87.24110
##  [71,]  74.01766
##  [72,]  83.22953
##  [73,] 111.26480
##  [74,] 108.27765
##  [75,]  58.76574
##  [76,]  72.93956
##  [77,]  89.86206
##  [78,]  56.41208
##  [79,] 107.05446
##  [80,]  60.12879
##  [81,] 102.16184
##  [82,]  52.27128
##  [83,]  89.05798
##  [84,]  88.15354
##  [85,] 104.25729
##  [86,]  67.06879
##  [87,] 100.26802
##  [88,]  72.26598
##  [89,]  89.75313
##  [90,]  89.86119
##  [91,]  70.38860
##  [92,]  82.94953
##  [93,] 105.15911
##  [94,]  54.82662
##  [95,]  74.16380
##  [96,] 108.88545
##  [97,]  94.04320
##  [98,]  62.22942
##  [99,]  96.06411
## [100,]        NA
```

Plot the density of these angles.

```
pacman::p_load(ggplot2) #load ggplot

ggplot(data.frame(angles=c(all_angles(X)))) + aes(x=angles) +geom_density()

## Warning: Removed 5050 rows containing non-finite values (stat_density).
```



Write an Rcpp function `all_angles_cpp` that does the same thing. Use an IDE if you want, but write it below in-line.

```
writeLines('PATH="${RTOOLS40_HOME}\\usr\\bin;${PATH}"', con = "~/.Renviron")

cppFunction('
  NumericMatrix all_angles_cpp(NumericMatrix X) {
    int n = X.nrow();
    int p = X.ncol();
    NumericMatrix A(n, n);
    std::fill(A.begin(), A.end(), NA_REAL);
    for (int i_1 = 0; i_1 < (n - 1); i_1++){
      //Rcout << "computing for row #: " << (i_1 + 1) << "\\n";
      for (int i_2 = i_1 + 1; i_2 < n; i_2++){
        double sum_sqd_u = 0;
        double sum_sqd_v = 0;
        double sum_u_v = 0;
        for (int j = 0; j < p; j++){
          sum_sqd_u += pow(X(i_1, j), 2);
          sum_sqd_v += pow(X(i_2, j), 2);
          sum_u_v = X(i_1, j) * X(i_2, j);
          //acos(sum(u * v)/sqrt(sum(u^2)*sum(v^2)))
          acos(sum_u_v/sqrt(sum_sqd_u * sum_sqd_v)) * (180/M_PI);
        }
        A(i_1, i_2) = acos(sum_u_v/sqrt(sum_sqd_u * sum_sqd_v)) * (180/M_PI);
      }
    }
```

```
    return A;
  }
  ')
#all_angles_cpp(X)
```

Test the time difference between these functions for `n = 1000` and `Nvec = 100, 500, 1000, 5000`. Store the results in a matrix.

#Practice Lecture 18 Notes

```
pacman::p_load(microbenchmark)
n=1000
Nvec=100
X=matrix(data=rnorm(Nvec*n), nrow=Nvec, ncol=n)

microbenchmark(all_angles(X),all_angles_cpp(X), times=10)
```

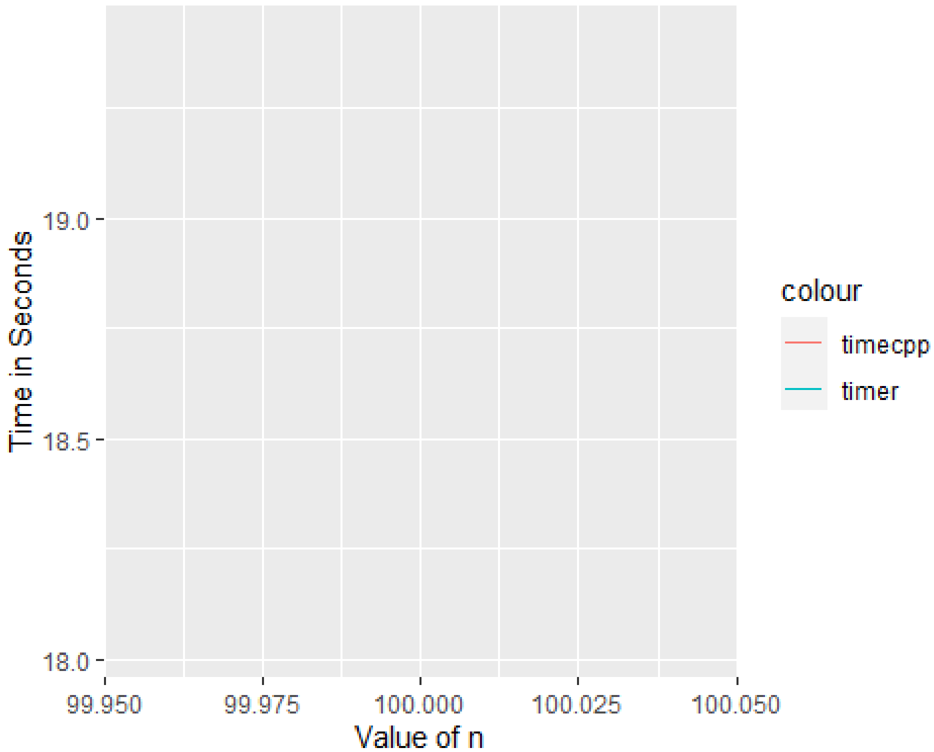
```
## Unit: milliseconds
##               expr     min       lq      mean   median       uq      max
neval
##      all_angles(X) 99.2188 146.1557 154.45308 154.7281 169.0726 216.7984
10
##  all_angles_cpp(X) 38.5164  43.4165  54.45628  55.1737  60.7115  70.5362
10
```

```
#C++ is faster by about 20x
time_r = c()
time_cpp = c()
for (i in 1:length(Nvec)){
  X = c()
  for (j in 1:n){
    x = rnorm(Nvec[i])
    X = cbind(X, x)
  }
  time_r = c(time_r, mean(microbenchmark(angles_r = all_angles(X), times = 3,
unit = "s")$time))
  time_cpp = c(time_cpp, mean(microbenchmark(angles_cpp = all_angles_cpp(X),
times = 3, unit = "s")$time))
}
```

Plot the divergence of performance (in log seconds) over n using a line geometry. Use two different colors for the R and CPP functions. Make sure there's a color legend on your plot.

```
pacman::p_load(ggplot2)

ggplot() +
  geom_line(aes(x = Nvec, y = log(time_r), col = "timer")) +
  geom_line(aes(x = Nvec, y = log(time_cpp), col = "timecpp")) +
  xlab("Value of n") +
  ylab("Time in Seconds")
```

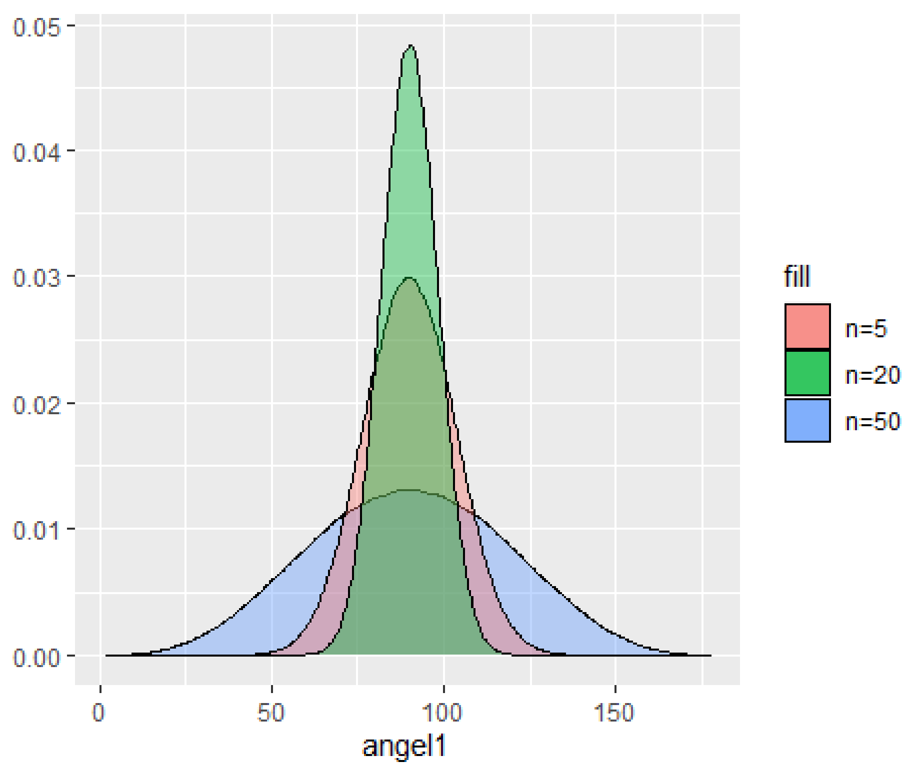
```
## geom_path: Each group consists of only one observation. Do you need to
adjust
## the group aesthetic?
## geom_path: Each group consists of only one observation. Do you need to
adjust
## the group aesthetic?
```



Let `Nvec = 10000` and vary `n` to be 10, 100, 1000. Plot the density of angles for all three values of `n` on one plot using color to signify `n`. Make sure you have a color legend. This is not easy.

```
Nvec = 1000
X = c()
for (i in 1:5){
  x = rnorm(Nvec)
  X = cbind(X, x)
}
angel1 = all_angles(X)
X = c()
for (i in 1:20){
  x = rnorm(Nvec)
  X = cbind(X, x)
}
angel2 = all_angles(X)
X = c()
for (i in 1:50){
  x = rnorm(Nvec)
```

```
  X = cbind(X, x)
}
angel3 = all_angles(X)
ggplot() +
  geom_density(aes(x = angel1, fill = "red"), alpha = .4) +
  geom_density(aes(x = angel2, fill = "blue"), alpha = .4) +
  geom_density(aes(x = angel3, fill = "green"), alpha = .4) +
  scale_fill_discrete(labels = c("n=5", "n=20", "n=50")) +
  ylab("Density") +
  ylab("")

## Warning: Removed 500500 rows containing non-finite values (stat_density).

## Warning: Removed 500500 rows containing non-finite values (stat_density).

## Warning: Removed 500500 rows containing non-finite values (stat_density).
```



Write an R function `nth_fibonnaci` that finds the nth Fibonnaci number via recursion but allows you to specify the starting number. For instance, if the sequency started at 1, you get the familiar 1, 1, 2, 3, 5, etc. But if it started at 0.01, you would get 0.01, 0.01, 0.02, 0.03, 0.05, etc.

```
nth_fibonacci = function(n, start){
  if (n == 1 | n == 2) return(start)
  else return(nth_fibonacci(n-1, start) + nth_fibonacci(n-2, start))
}
nth_fibonacci(5, 0.01)
```

```
## [1] 0.05
```

Write an Rcpp function `nth_fibonnaci_cpp` that does the same thing. Use an IDE if ou want, but write it below in-line.

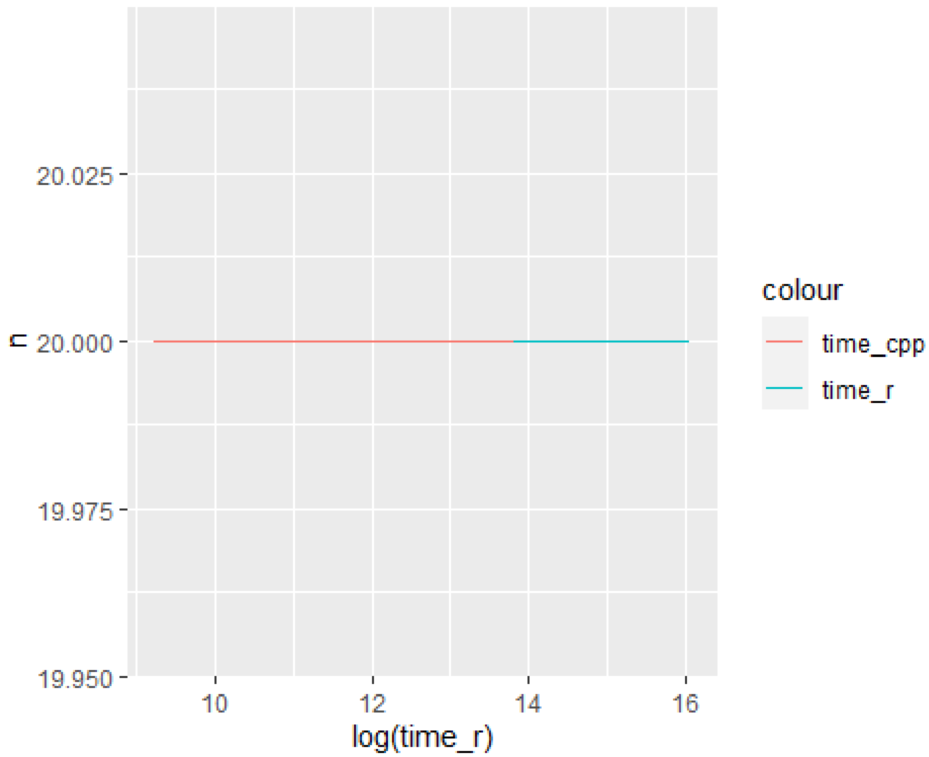
```
cppFunction('
  double nth_fibonacci_cpp(int n, double start) {
    if(n == 1 || n == 2)
      return start;
    return ( nth_fibonacci_cpp(n-1, start) + nth_fibonacci_cpp(n-2, start) );
  }
')
nth_fibonacci_cpp(5,1)

## [1] 5
```

Time the difference in these functions for n = 100, 200, …., 1500 while starting the sequence at the smallest possible floating point value in R. Store the results in a matrix.

```
n = 20
time_r = c()
time_cpp = c()
for (i in 1:n){
  time_r = c(time_r, mean(microbenchmark(fib_r = nth_fibonacci(i,
.Machine$double.xmin), times = 3, unit = "s")$time))
  time_cpp = c(time_cpp, mean(microbenchmark(fib_cpp = nth_fibonacci_cpp(i,
.Machine$double.xmin), times = 3, unit = "s")$time))
}
```

Plot the divergence of performance (in log seconds) over n using a line geometry. Use two different colors for the R and CPP functions. Make sure there’s a color legend on your plot.

```
ggplot() +
  geom_line(aes(y = n, x = log(time_r), col = "time_r")) +
  geom_line(aes(y = n, x = log(time_cpp), col = "time_cpp"))
```



Data Wrangling / Munging / Carpentry

Throughout this assignment you can use either the `tidyverse` package suite or `data.table` to answer but not base R. You can mix `data.table` with `magrittr` piping if you wish but don't go back and forth between `tbl_df`'s and `data.table` objects.

```
pacman::p_load(tidyverse, magrittr, data.table)
```

Load the `storms` dataset from the `dplyr` package and investigate it using `str` and `summary` and `head`. Which two columns should be converted to type factor? Do so below.

```
data(storms)
str(storms)

## tibble [10,010 x 13] (S3: tbl_df/tbl/data.frame)
##  $ name        : chr [1:10010] "Amy" "Amy" "Amy" "Amy" ...
##  $ year        : num [1:10010] 1975 1975 1975 1975 1975 ...
##  $ month       : num [1:10010] 6 6 6 6 6 6 6 6 6 6 ...
##  $ day         : int [1:10010] 27 27 27 27 28 28 28 28 29 29 ...
##  $ hour        : num [1:10010] 0 6 12 18 0 6 12 18 0 6 ...
##  $ lat         : num [1:10010] 27.5 28.5 29.5 30.5 31.5 32.4 33.3 34 34.4
34 ...
##  $ long        : num [1:10010] -79 -79 -79 -79 -78.8 -78.7 -78 -77 -75.8 -
74.8 ...
##  $ status      : chr [1:10010] "tropical depression" "tropical depression"
"tropical depression" "tropical depression" ...
```

```
##  $ category   : Ord.factor w/ 7 levels "-1"<"0"<"1"<"2"<..: 1 1 1 1 1 1 1
1 2 2 ...
##  $ wind       : int [1:10010] 25 25 25 25 25 25 25 30 35 40 ...
##  $ pressure   : int [1:10010] 1013 1013 1013 1013 1012 1012 1011 1006 1004
1002 ...
##  $ ts_diameter: num [1:10010] NA NA NA NA NA NA NA NA NA NA ...
##  $ hu_diameter: num [1:10010] NA NA NA NA NA NA NA NA NA NA ...
```

```
head(storms)
```

```
## # A tibble: 6 x 13
##   name   year month   day  hour   lat  long status       category  wind
pressure
##   <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>        <ord>    <int>
<int>
## 1 Amy    1975     6    27     0  27.5 -79   tropical de~ -1          25
1013
## 2 Amy    1975     6    27     6  28.5 -79   tropical de~ -1          25
1013
## 3 Amy    1975     6    27    12  29.5 -79   tropical de~ -1          25
1013
## 4 Amy    1975     6    27    18  30.5 -79   tropical de~ -1          25
1013
## 5 Amy    1975     6    28     0  31.5 -78.8 tropical de~ -1          25
1012
## 6 Amy    1975     6    28     6  32.4 -78.7 tropical de~ -1          25
1012
## # ... with 2 more variables: ts_diameter <dbl>, hu_diameter <dbl>
```

Reorder the columns so name is first, status is second, category is third and the rest are the same.

```
storms %>%
  select(name, status, category, everything())
```

```
## # A tibble: 10,010 x 13
##    name  status       category  year month   day  hour   lat  long  wind
pressure
##    <chr> <chr>        <ord>    <dbl> <dbl> <int> <dbl> <dbl> <dbl> <int>
<int>
##  1 Amy   tropical d~ -1        1975     6    27     0  27.5 -79      25
1013
##  2 Amy   tropical d~ -1        1975     6    27     6  28.5 -79      25
1013
##  3 Amy   tropical d~ -1        1975     6    27    12  29.5 -79      25
1013
##  4 Amy   tropical d~ -1        1975     6    27    18  30.5 -79      25
1013
##  5 Amy   tropical d~ -1        1975     6    28     0  31.5 -78.8    25
1012
##  6 Amy   tropical d~ -1        1975     6    28     6  32.4 -78.7    25
```

```
1012
##   7 Amy   tropical d~ -1        1975     6    28    12  33.3 -78      25
1011
##   8 Amy   tropical d~ -1        1975     6    28    18  34   -77      30
1006
##   9 Amy   tropical s~ 0         1975     6    29     0  34.4 -75.8    35
1004
## 10 Amy   tropical s~ 0         1975     6    29     6  34   -74.8    40
1002
## # ... with 10,000 more rows, and 2 more variables: ts_diameter <dbl>,
## #   hu_diameter <dbl>
```

Find a subset of the data of storms only in the 1970’s.

```
storms %>%
  filter(year>=1970 & year<=1979)
```

```
## # A tibble: 546 x 13
##    name   year month   day  hour   lat  long status      category  wind
pressure
##    <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>       <ord>    <int>
<int>
##   1 Amy    1975     6    27     0  27.5 -79   tropical d~ -1          25
1013
##   2 Amy    1975     6    27     6  28.5 -79   tropical d~ -1          25
1013
##   3 Amy    1975     6    27    12  29.5 -79   tropical d~ -1          25
1013
##   4 Amy    1975     6    27    18  30.5 -79   tropical d~ -1          25
1013
##   5 Amy    1975     6    28     0  31.5 -78.8 tropical d~ -1          25
1012
##   6 Amy    1975     6    28     6  32.4 -78.7 tropical d~ -1          25
1012
##   7 Amy    1975     6    28    12  33.3 -78   tropical d~ -1          25
1011
##   8 Amy    1975     6    28    18  34   -77   tropical d~ -1          30
1006
##   9 Amy    1975     6    29     0  34.4 -75.8 tropical s~ 0           35
1004
## 10 Amy    1975     6    29     6  34   -74.8 tropical s~ 0           40
1002
## # ... with 536 more rows, and 2 more variables: ts_diameter <dbl>,
## #   hu_diameter <dbl>
```

Find a subset of the data of storm observations only with category 4 and above and wind speed 100MPH and above.

```
storms %>%
  filter(category>=4 & wind>=100)
```

```
## # A tibble: 416 x 13
##    name   year month   day  hour   lat  long status    category  wind
pressure
##    <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>     <ord>    <int>
<int>
##  1 Anita  1977     9     2     0  24.6 -96.2 hurricane 5          140
931
##  2 Anita  1977     9     2     6  24.2 -97.1 hurricane 5          150
926
##  3 Anita  1977     9     2    12  23.7 -98   hurricane 4          120
940
##  4 David  1979     8    28     0  12.2 -52.9 hurricane 4          115
947
##  5 David  1979     8    28     6  12.5 -54.4 hurricane 4          125
941
##  6 David  1979     8    28    12  12.8 -55.7 hurricane 4          130
938
##  7 David  1979     8    28    18  13.2 -56.9 hurricane 4          125
941
##  8 David  1979     8    29     0  13.7 -58   hurricane 4          120
944
##  9 David  1979     8    29     6  14.2 -59.2 hurricane 4          120
942
## 10 David  1979     8    29    12  14.8 -60.3 hurricane 4          125
938
## # ... with 406 more rows, and 2 more variables: ts_diameter <dbl>,
## #   hu_diameter <dbl>
```

Create a new feature `wind_speed_per_unit_pressure`.

```
storms %>%
  mutate(wind_speed_per_unit_pressure = wind/pressure)
```

```
## # A tibble: 10,010 x 14
##    name   year month   day  hour   lat  long status      category  wind
pressure
##    <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>       <ord>    <int>
<int>
##  1 Amy    1975     6    27     0  27.5 -79   tropical d~ -1          25
1013
##  2 Amy    1975     6    27     6  28.5 -79   tropical d~ -1          25
1013
##  3 Amy    1975     6    27    12  29.5 -79   tropical d~ -1          25
1013
##  4 Amy    1975     6    27    18  30.5 -79   tropical d~ -1          25
1013
##  5 Amy    1975     6    28     0  31.5 -78.8 tropical d~ -1          25
1012
##  6 Amy    1975     6    28     6  32.4 -78.7 tropical d~ -1          25
1012
```

```
##   7 Amy     1975     6     28     12  33.3 -78   tropical d~ -1          25
1011
##   8 Amy     1975     6     28     18  34   -77   tropical d~ -1          30
1006
##   9 Amy     1975     6     29      0  34.4 -75.8 tropical s~ 0           35
1004
## 10 Amy     1975     6     29      6  34   -74.8 tropical s~ 0           40
1002
## # ... with 10,000 more rows, and 3 more variables: ts_diameter <dbl>,
## #   hu_diameter <dbl>, wind_speed_per_unit_pressure <dbl>
```

Create a new feature: average_diameter which averages the two diameter metrics. If one is missing, then use the value of the one that is present. If both are missing, leave missing.

```
storms %>%
  rowwise() %>%
  arrange(desc(year)) %>%
  mutate(average_diameter = mean(c(ts_diameter, hu_diameter), na.rm=TRUE))

## # A tibble: 10,010 x 14
## # Rowwise:
##    name    year month   day  hour   lat  long status      category  wind
pressure
##    <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>       <ord>    <int>
<int>
##   1 Ana     2015     5      9      6  32.2 -77.5 tropical s~ 0           50
998
##   2 Ana     2015     5      9     12  32.5 -77.8 tropical s~ 0           50
1001
##   3 Ana     2015     5      9     18  32.7 -78   tropical s~ 0           45
1001
##   4 Ana     2015     5     10      0  33.1 -78.3 tropical s~ 0           45
1001
##   5 Ana     2015     5     10      6  33.5 -78.6 tropical s~ 0           40
1002
##   6 Ana     2015     5     10     10  33.8 -78.8 tropical s~ 0           40
1002
##   7 Ana     2015     5     10     12  33.9 -78.8 tropical s~ 0           35
1002
##   8 Ana     2015     5     10     18  34.3 -78.7 tropical d~ -1          30
1006
##   9 Ana     2015     5     11      0  34.7 -78.5 tropical d~ -1          30
1009
## 10 Ana     2015     5     11      6  35.5 -78   tropical d~ -1          30
1010
## # ... with 10,000 more rows, and 3 more variables: ts_diameter <dbl>,
## #   hu_diameter <dbl>, average_diameter <dbl>

#Replace zeros with NA in category
```

For each storm, summarize the maximum wind speed. “Summarize” means create a new dataframe with only the summary metrics you care about.

```
storms %>%
  group_by(name) %>%
  summarize(max_wind_speed = max(wind, na.rm=TRUE))
```

```
## # A tibble: 198 x 2
##    name     max_wind_speed
##    <chr>             <int>
##  1 AL011993             30
##  2 AL012000             25
##  3 AL021992             30
##  4 AL021994             30
##  5 AL021999             30
##  6 AL022000             30
##  7 AL022001             25
##  8 AL022003             30
##  9 AL022006             45
## 10 AL031987             40
## # ... with 188 more rows
```

Order your dataset by maximum wind speed storm but within the rows of storm show the observations in time order from early to late.

```
storms %>%
  group_by(name) %>%
  mutate(max_wind_by_storm = max(wind, na.rm=TRUE)) %>%
  select(name, max_wind_by_storm, everything()) %>% #Make max_wind_by_storm
the first column
  arrange(desc(max_wind_by_storm), year, month, day, hour)
```

```
## # A tibble: 10,010 x 14
## # Groups:   name [198]
##    name   max_wind_by_sto~  year month   day  hour   lat  long status
category
##    <chr>             <int> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>
<ord>
##  1 Gilbe~              160  1988     9     8    18  12   -54   tropica~ -1
##  2 Gilbe~              160  1988     9     9     0  12.7 -55.6 tropica~ -1
##  3 Gilbe~              160  1988     9     9     6  13.3 -57.1 tropica~ -1
##  4 Gilbe~              160  1988     9     9    12  14   -58.6 tropica~ -1
##  5 Gilbe~              160  1988     9     9    18  14.5 -60.1 tropica~ 0
##  6 Gilbe~              160  1988     9    10     0  14.8 -61.5 tropica~ 0
##  7 Gilbe~              160  1988     9    10     6  15   -62.8 tropica~ 0
##  8 Gilbe~              160  1988     9    10    12  15.3 -64.1 tropica~ 0
##  9 Gilbe~              160  1988     9    10    18  15.7 -65.4 tropica~ 0
## 10 Gilbe~              160  1988     9    11     0  15.9 -66.8 hurrica~ 1
## # ... with 10,000 more rows, and 4 more variables: wind <int>, pressure
<int>,
## #   ts_diameter <dbl>, hu_diameter <dbl>
```

Find the strongest storm by wind speed per year.

```
storms %>%
  group_by(year) %>%
  arrange(year, desc(wind)) %>%
  slice(1) %>% #gives the first row for every storm
  select(name, year)
```

```
## # A tibble: 41 x 2
## # Groups:   year [41]
##    name      year
##    <chr>    <dbl>
##  1 Caroline  1975
##  2 Belle     1976
##  3 Anita     1977
##  4 Cora      1978
##  5 David     1979
##  6 Ivan      1980
##  7 Harvey    1981
##  8 Debby     1982
##  9 Alicia    1983
## 10 Diana     1984
## # ... with 31 more rows
```

For each named storm, find its maximum category, wind speed, pressure and diameters. Do not allow the max to be NA (unless all the measurements for that storm were NA).

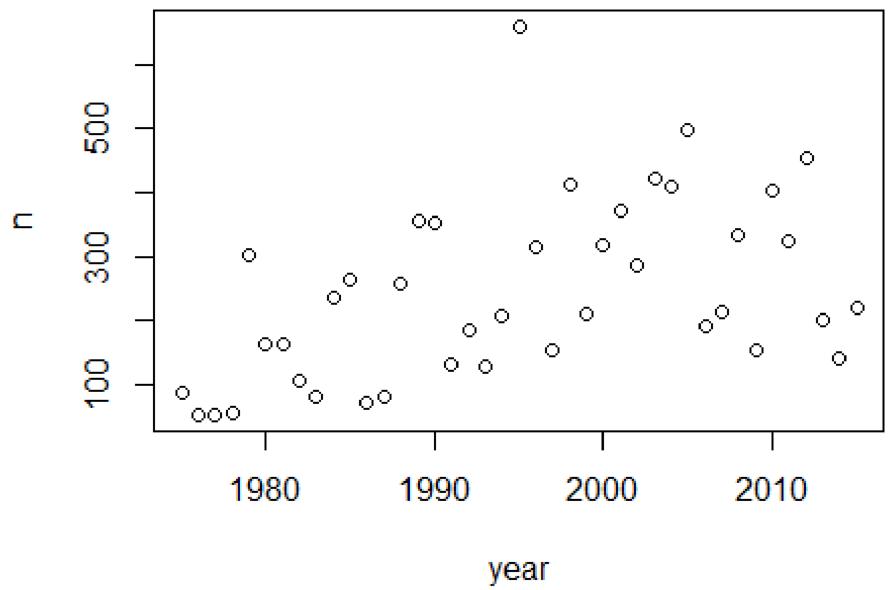
```
storms %>%
  group_by(name) %>%
  mutate(maximum_wind_by_storm = max(wind, na.rm = TRUE)) %>%
  select(name, maximum_wind_by_storm, everything()) %>%
  arrange(maximum_wind_by_storm, year, month, day, hour)
```

```
## # A tibble: 10,010 x 14
## # Groups:   name [198]
##    name   maximum_wind_by_~  year month   day  hour   lat  long status
category
##    <chr>              <int> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>
<ord>
##  1 AL101~                25  1991    10    24    12  13.4 -42.3 tropic~ -1
##  2 AL101~                25  1991    10    24    18  13.7 -43.6 tropic~ -1
##  3 AL101~                25  1991    10    25     0  13.8 -44.9 tropic~ -1
##  4 AL101~                25  1991    10    25     6  14   -46.4 tropic~ -1
##  5 AL101~                25  1991    10    25    12  14.1 -47.7 tropic~ -1
##  6 AL012~                25  2000     6     7    18  21   -93   tropic~ -1
##  7 AL012~                25  2000     6     8     0  20.9 -92.8 tropic~ -1
##  8 AL012~                25  2000     6     8     6  20.7 -93.1 tropic~ -1
##  9 AL012~                25  2000     6     8    12  20.8 -93.5 tropic~ -1
## 10 AL022~                25  2001     7    11    18  10.9 -42.1 tropic~ -1
## # ... with 10,000 more rows, and 4 more variables: wind <int>, pressure
```

```
<int>,
## #   ts_diameter <dbl>, hu_diameter <dbl>
```

For each year in the dataset, tally the number of storms. “Tally” is a fancy word for “count the number of”. Plot the number of storms by year. Any pattern?

```
storms %>%
  group_by(year) %>%
  summarize(num_storms = n_distinct(name))

## # A tibble: 41 x 2
##     year num_storms
##    <dbl>      <int>
##  1  1975          3
##  2  1976          2
##  3  1977          3
##  4  1978          4
##  5  1979          7
##  6  1980          8
##  7  1981          5
##  8  1982          5
##  9  1983          4
## 10  1984         10
## # ... with 31 more rows

storms_per_year =
storms %>%
  count(year, sort = TRUE)
plot(storms_per_year)
```




```
#As the years increase the number of stroms increases
```

For each year in the dataset, tally the storms by category.

```
storms_per_year_and_category =
 storms %>%
  group_by(year, category) %>%
  count(category, sort = TRUE)
storms_per_year_and_category

## # A tibble: 233 x 3
## # Groups:   year, category [233]
##      year category     n
##     <dbl> <ord>    <int>
##  1  2012 0          276
##  2  1995 0          247
##  3  2005 0          221
##  4  2011 0          203
##  5  2010 0          193
##  6  2003 0          186
##  7  2008 0          183
##  8  2004 0          166
##  9  1995 1          164
## 10  1995 -1         158
## # ... with 223 more rows
```

For each year in the dataset, find the maximum wind speed per status level.

```
storms %>%
  group_by(status) %>%
  summarize(max_wind_speed = max(wind))
```

```
## # A tibble: 3 x 2
##   status              max_wind_speed
##   <chr>                        <int>
## 1 hurricane                      160
## 2 tropical depression             30
## 3 tropical storm                  70
```

For each storm, summarize its average location in latitude / longitude coordinates.

```
storms %>%
  group_by(name) %>%
  summarize(avg_lat = mean(lat), avg_long = mean(long))
```

```
## # A tibble: 198 x 3
##    name     avg_lat avg_long
##    <chr>      <dbl>    <dbl>
##  1 AL011993   24.7     -78.0
##  2 AL012000   20.8     -93.1
##  3 AL021992   26.7     -84.5
##  4 AL021994   33.6     -79.7
##  5 AL021999   20.4     -96.4
##  6 AL022000    9.9     -28.5
##  7 AL022001   11.9     -45.3
##  8 AL022003    9.62    -43.4
##  9 AL022006   41.3     -63.5
## 10 AL031987   30.8     -88.7
## # ... with 188 more rows
```

For each storm, summarize its duration in number of hours (to the nearest 6hr increment).

```
storms %>%
  group_by(name) %>%
  summarise(duration = ifelse(sum(hour) %% 6 == 0, sum(hour),
                       ifelse(sum(hour) %% 6 < 3, sum(hour) - (sum(hour) %%
6),
                       sum(hour) + (6-(sum(hour) %% 6)))))
```

```
## # A tibble: 198 x 2
##    name     duration
##    <chr>       <dbl>
##  1 AL011993       72
##  2 AL012000       36
##  3 AL021992       48
##  4 AL021994       54
##  5 AL021999       30
##  6 AL022000      108
##  7 AL022001       54
```

```
## 8 AL022003 36
## 9 AL022006 42
## 10 AL031987 288
## # ... with 188 more rows
```

For storm in a category, create a variable `storm_number` that enumerates the storms 1, 2, ... (in date order).

```
storms %>%
  group_by(category, name) %>%
  slice(1) %>%
  group_by(category) %>%
  mutate(storm_number = dense_rank(paste(year, as.numeric(month), day))) %>%
  select(category, storm_number, year, month, day, name) %>%
  distinct %>%
  arrange(category,storm_number)
```

```
## # A tibble: 687 x 6
## # Groups:   category [7]
##    category storm_number  year month   day name
##    <ord>           <int> <dbl> <dbl> <int> <chr>
##  1 -1                  1  1975     6    27 Amy
##  2 -1                  2  1975     8    24 Caroline
##  3 -1                  3  1976     8     6 Belle
##  4 -1                  4  1976     9    26 Gloria
##  5 -1                  5  1977    10    13 Evelyn
##  6 -1                  6  1977     8    29 Anita
##  7 -1                  7  1977     9     5 Clara
##  8 -1                  8  1978    10     7 Juliet
##  9 -1                  9  1978     7    30 Amelia
## 10 -1                 10  1978     8     5 Bess
## # ... with 677 more rows
```

Convert year, month, day, hour into the variable `timestamp` using the `lubridate` package. Although the new package `clock` just came out, `lubridate` still seems to be standard. Next year I'll probably switch the class to be using `clock`.

```
pacman::p_load(lubridate)
library(lubridate)
storms %<>%
  mutate(timestamp = ymd_h(paste(year, month, day, hour, sep = "-"))) %<>%
  select(-year, -month, -day, -hour)
storms
```

```
## # A tibble: 10,010 x 10
##    name    lat  long status       category  wind pressure ts_diameter
hu_diameter
##    <chr> <dbl> <dbl> <chr>        <ord>    <int>    <int>       <dbl>
<dbl>
##  1 Amy    27.5 -79   tropical d~ -1          25     1013          NA
NA
```

```
##   2 Amy     28.5 -79   tropical d~ -1          25      1013            NA
NA
##   3 Amy     29.5 -79   tropical d~ -1          25      1013            NA
NA
##   4 Amy     30.5 -79   tropical d~ -1          25      1013            NA
NA
##   5 Amy     31.5 -78.8 tropical d~ -1          25      1012            NA
NA
##   6 Amy     32.4 -78.7 tropical d~ -1          25      1012            NA
NA
##   7 Amy     33.3 -78   tropical d~ -1          25      1011            NA
NA
##   8 Amy     34   -77   tropical d~ -1          30      1006            NA
NA
##   9 Amy     34.4 -75.8 tropical s~ 0           35      1004            NA
NA
##  10 Amy     34   -74.8 tropical s~ 0           40      1002            NA
NA
## # ... with 10,000 more rows, and 1 more variable: timestamp <dttm>
```

Using the `lubridate` package, create new variables `day_of_week` which is a factor with levels "Sunday", "Monday", … "Saturday" and `week_of_year` which is integer 1, 2, …, 52.

```
storms %<>%
  mutate(day_of_week = weekdays(timestamp),
         week_of_year = week(timestamp))
```

For each storm, summarize the day in which is started in the following format "Friday, June 27, 1975".

```
storms %>%
  group_by(name) %>%
  summarize(start_date = min(timestamp)) %>%
  mutate(start_date = paste(weekdays(start_date),
                            paste(months(start_date), day(start_date), sep =
" "),
                            year(start_date), sep = ", "))

## # A tibble: 198 x 2
##    name      start_date
##    <chr>     <chr>
##  1 AL011993 Monday, May 31, 1993
##  2 AL012000 Wednesday, June 7, 2000
##  3 AL021992 Thursday, June 25, 1992
##  4 AL021994 Wednesday, July 20, 1994
##  5 AL021999 Friday, July 2, 1999
##  6 AL022000 Friday, June 23, 2000
##  7 AL022001 Wednesday, July 11, 2001
##  8 AL022003 Wednesday, June 11, 2003
##  9 AL022006 Monday, July 17, 2006
```

```
## 10 AL031987 Sunday, August 9, 1987
## # ... with 188 more rows
```

Create a new factor variable `decile_windspeed` by binning wind speed into 10 bins.

```
bins = 0:10
storms %<>%
  mutate(decile_windspeed = factor(cut(wind, breaks = quantile(wind,
bins/10), labels = FALSE)))
```

Create a new data frame `serious_storms` which are category 3 and above hurricanes.

```
serious_storms =
  storms %>%
    filter(category >= 3)
serious_storms
```

```
## # A tibble: 779 x 13
##    name       lat  long status   category  wind pressure ts_diameter
hu_diameter
##    <chr>    <dbl> <dbl> <chr>    <ord>    <int>    <int>       <dbl>
<dbl>
##  1 Caroline  24   -97   hurrica~ 3          100      973          NA
NA
##  2 Caroline  24.1 -97.5 hurrica~ 3          100      963          NA
NA
##  3 Belle     29.5 -75.3 hurrica~ 3          100      958          NA
NA
##  4 Belle     30.9 -75.3 hurrica~ 3          105      957          NA
NA
##  5 Belle     32.5 -75.2 hurrica~ 3          105      959          NA
NA
##  6 Anita     25.2 -95.5 hurrica~ 3          110      945          NA
NA
##  7 Anita     24.6 -96.2 hurrica~ 5          140      931          NA
NA
##  8 Anita     24.2 -97.1 hurrica~ 5          150      926          NA
NA
##  9 Anita     23.7 -98   hurrica~ 4          120      940          NA
NA
## 10 David     12.2 -52.9 hurrica~ 4          115      947          NA
NA
## # ... with 769 more rows, and 4 more variables: timestamp <dttm>,
## #   day_of_week <chr>, week_of_year <dbl>, decile_windspeed <fct>
```

In `serious_storms`, merge the variables lat and long together into `lat_long` with values `lat / long` as a string.

```
serious_storms %<>%
  unite(lat_long, lat, long, sep = " / ")
```

Let's return now to the original storms data frame. For each category, find the average wind speed, pressure and diameters (do not count the NA's in your averaging).

```
storms %>%
  group_by(category) %>%
  summarise(avg_wind_speed = mean(wind),
            avg_pressure = mean(pressure),
            avg_ts_diameter = mean(ts_diameter, na.rm = TRUE),
            avg_hu_diameter = mean(hu_diameter, na.rm = TRUE))
```

```
## # A tibble: 7 x 5
##   category avg_wind_speed avg_pressure avg_ts_diameter avg_hu_diameter
##   <ord>             <dbl>        <dbl>           <dbl>           <dbl>
## 1 -1                 27.3        1008.              0               0
## 2 0                  45.8         999.            160.              0
## 3 1                  70.9         982.            278.             57.3
## 4 2                  89.4         967.            282.             78.8
## 5 3                 105.          954.            307.             91.4
## 6 4                 122.          940.            315.            102.
## 7 5                 145.          916.            317.            120.
```

For each named storm, find its maximum category, wind speed, pressure and diameters (do not allow the max to be NA) and the number of readings (i.e. observations).

```
storms %>%
  group_by(name) %>%
  summarize(max_category = max(category),
            max_wind_speed = max(wind),
            max_pressure = max(pressure),
            max_hu_diameter = max(hu_diameter, na.rm = TRUE),
            max_ts_diameter = max(ts_diameter, na.rm = TRUE),
            readings = n())
```

```
## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
```

```
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
```

```
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf
```

```
## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf
```

```
## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
```

```
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
```

```
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf
```

```
## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(hu_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf
```

```
## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
```

```
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
```

```
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf
```

```
## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf
```

```
## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
```

```
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
```

```
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## Warning in max(ts_diameter, na.rm = TRUE): no non-missing arguments to
max;
## returning -Inf

## # A tibble: 198 x 7
##    name     max_category max_wind_speed max_pressure max_hu_diameter
##    <chr>    <ord>                 <int>        <int>           <dbl>
##  1 AL011993 -1                       30         1003            -Inf
##  2 AL012000 -1                       25         1010            -Inf
##  3 AL021992 -1                       30         1009            -Inf
##  4 AL021994 -1                       30         1017            -Inf
##  5 AL021999 -1                       30         1006            -Inf
##  6 AL022000 -1                       30         1010            -Inf
##  7 AL022001 -1                       25         1012            -Inf
##  8 AL022003 -1                       30         1010            -Inf
##  9 AL022006 0                        45         1008               0
## 10 AL031987 0                        40         1015            -Inf
## # ... with 188 more rows, and 2 more variables: max_ts_diameter <dbl>,
## #   readings <int>
```

Calculate the distance from each storm observation to Miami in a new variable `distance_to_miami`. This is very challenging. You will need a function that computes distances from two sets of latitude / longitude coordinates.

```
MIAMI_LAT_LONG_COORDS = c(25.7617, -80.1918)
distance = function(lat1, long1, lat2, long2){
  lat1 = lat1 * 180/pi
  lat2 = lat2 * 180/pi
  long1 = long1 * 180/pi
  long2 = long2 * 180/pi
  a = sin(lat2 - lat1 / 2)^2 + (cos(lat2) * cos(lat1)) * sin(long2 - long1 /
2)^2
  b = 2 * atan2(sqrt(a), sqrt(1 - a))
  distance = 6373.0 * b # Multiplying by radius of earth in KM
  return(distance)
}
```

```
storms %>%
  mutate(distance_to_miami = distance(lat, long, MIAMI_LAT_LONG_COORDS[1],
MIAMI_LAT_LONG_COORDS[2]))

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## # A tibble: 10,010 x 14
##    name    lat  long status      category  wind pressure ts_diameter
hu_diameter
##    <chr> <dbl> <dbl> <chr>       <ord>    <int>    <int>       <dbl>
<dbl>
##  1 Amy    27.5 -79   tropical d~ -1          25     1013          NA
NA
##  2 Amy    28.5 -79   tropical d~ -1          25     1013          NA
NA
##  3 Amy    29.5 -79   tropical d~ -1          25     1013          NA
NA
##  4 Amy    30.5 -79   tropical d~ -1          25     1013          NA
NA
##  5 Amy    31.5 -78.8 tropical d~ -1          25     1012          NA
NA
##  6 Amy    32.4 -78.7 tropical d~ -1          25     1012          NA
NA
##  7 Amy    33.3 -78   tropical d~ -1          25     1011          NA
NA
##  8 Amy    34   -77   tropical d~ -1          30     1006          NA
NA
##  9 Amy    34.4 -75.8 tropical s~ 0           35     1004          NA
NA
## 10 Amy    34   -74.8 tropical s~ 0           40     1002          NA
NA
## # ... with 10,000 more rows, and 5 more variables: timestamp <dttm>,
## #   day_of_week <chr>, week_of_year <dbl>, decile_windspeed <fct>,
## #   distance_to_miami <dbl>
```

For each storm observation, use the function from the previous question to calculate the distance it moved since the previous observation.

```
storms %<>%
  group_by(name) %>%
  mutate(dist_from_prev = ifelse(name != lag(name), 0, distance(lat, long,
lag(lat), lag(long)))) %>%
  mutate(dist_from_prev = ifelse(is.na(dist_from_prev), 0, dist_from_prev))

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced
```

```
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced
```

```
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
```

```
## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced
```

```
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
```

```
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
```

```
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
```

```
## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced
```

```
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
```

```
## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced
```

```
## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced
```

```
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced

## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
```

```
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
## Warning in sqrt(a): NaNs produced
## Warning in sqrt(1 - a): NaNs produced
```

```
## Warning in sqrt(a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced

## Warning in sqrt(a): NaNs produced

## Warning in sqrt(1 - a): NaNs produced
```

```
head(storms)
```

```
## # A tibble: 6 x 14
## # Groups:   name [1]
##   name    lat  long status       category  wind pressure ts_diameter
hu_diameter
##   <chr> <dbl> <dbl> <chr>        <ord>    <int>    <int>       <dbl>
<dbl>
## 1 Amy    27.5 -79   tropical de~ -1          25     1013          NA
NA
## 2 Amy    28.5 -79   tropical de~ -1          25     1013          NA
NA
## 3 Amy    29.5 -79   tropical de~ -1          25     1013          NA
NA
## 4 Amy    30.5 -79   tropical de~ -1          25     1013          NA
NA
## 5 Amy    31.5 -78.8 tropical de~ -1          25     1012          NA
NA
## 6 Amy    32.4 -78.7 tropical de~ -1          25     1012          NA
NA
## # ... with 5 more variables: timestamp <dttm>, day_of_week <chr>,
## #   week_of_year <dbl>, decile_windspeed <fct>, dist_from_prev <dbl>
```

For each storm, find the total distance it moved over its observations and its total displacement. “Distance” is a scalar quantity that refers to “how much ground an object has covered” during its motion. “Displacement” is a vector quantity that refers to “how far out of place an object is”; it is the object’s overall change in position.

```
storms %>%
  group_by(name) %>%
  summarise(Distance = sum(dist_from_prev),
            Displacement = paste(round(last(lat) - first(lat), 2),
round(last(long) - first(long), 2), sep = " / "))
```

```
## # A tibble: 198 x 3
##    name     Distance Displacement
##    <chr>       <dbl> <chr>
##  1 AL011993   48147. 6.3 / 12.2
##  2 AL012000   20759. -0.2 / -0.5
##  3 AL021992   46119. 4 / 2.6
##  4 AL021994   34482. 3 / -2.1
##  5 AL021999   23063. 0.2 / -2.3
##  6 AL022000   86454. 0.2 / -18.4
##  7 AL022001   23191. 2.2 / -6.4
##  8 AL022003   15264. 0.2 / -5.1
##  9 AL022006   24361. 4.6 / 6.3
## 10 AL031987  220656. 5.5 / 11.3
## # ... with 188 more rows
```

For each storm observation, calculate the average speed the storm moved in location.

```
storms %<>%
  mutate(speed = dist_from_prev / 6)
```

For each storm, calculate its average ground speed (how fast its eye is moving which is different from windspeed around the eye).

```
storms %>%
  group_by(name) %>%
  summarise(avg_ground_speed = mean(speed))
```

```
## # A tibble: 198 x 2
##    name     avg_ground_speed
##    <chr>               <dbl>
##  1 AL011993            1003.
##  2 AL012000             865.
##  3 AL021992            1537.
##  4 AL021994             958.
##  5 AL021999             961.
##  6 AL022000            1201.
##  7 AL022001             773.
##  8 AL022003             636.
##  9 AL022006             812.
## 10 AL031987            1149.
## # ... with 188 more rows
```

Is there a relationship between average ground speed and maximum category attained? Use a dataframe summary (not a regression).

```
speed_and_category = storms %>%
  group_by(name) %>%
  summarize(avg_ground_speed = mean(speed), maximum_category =
as.numeric(max(category)))
cor(speed_and_category[,2], speed_and_category[,3])
```

```
##                     maximum_category
## avg_ground_speed           0.2590877
```

Now we want to transition to building real design matrices for prediction. This is more in tune with what happens in the real world. Large data dump and you convert it into X and y how you see fit.

Suppose we wish to predict the following: given the first three readings of a storm, can you predict its maximum wind speed? Identify the `y` and identify which features you need $x_1, \ldots x_p$ and build that matrix with `dplyr` functions. This is not easy, but it is what it's all about. Feel free to "featurize" as creatively as you would like. You aren't going to overfit if you only build a few features relative to the total 198 storms.

```
storms_m = storms %>%
  group_by(name) %>%
  summarise(y = max(wind),
            avg_pressure = mean(pressure),
            avg_distance = mean(dist_from_prev),
            final_status = last(status)) %>%
  select(-name)
```

Fit your model. Validate it.

```
mod = lm(y ~ 0 + ., data = storms_m)
summary(mod)

##
## Call:
## lm(formula = y ~ 0 + ., data = storms_m)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -48.329 -14.249  -0.884  13.175  46.367
##
## Coefficients:
##                                Estimate Std. Error t value Pr(>|t|)
## avg_pressure                 -2.780e+00  1.314e-01 -21.159   <2e-16 ***
## avg_distance                  1.969e-03  9.110e-04   2.162   0.0319 *
## final_statushurricane         2.819e+03  1.304e+02  21.610   <2e-16 ***
## final_statustropical depression  2.835e+03  1.326e+02  21.382   <2e-16 ***
## final_statustropical storm    2.837e+03  1.317e+02  21.537   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 19.62 on 193 degrees of freedom
## Multiple R-squared:  0.9512, Adjusted R-squared:   0.95
## F-statistic: 753.1 on 5 and 193 DF,  p-value: < 2.2e-16

n = nrow(storms_m)
K = 5
test_indices = sample(1 : n, 1 / K * n)
```

```
train_indices = setdiff(1 : n, test_indices)
X = select(storms_m, -y)
y = storms_m$y
X_train = X[train_indices, ]
y_train = y[train_indices]
X_test = X[test_indices, ]
y_test = y[test_indices]
modv = lm(y_train ~ ., data.frame(X_train))
yhat_oos = predict(mod, data.frame(X_test))
oos_residuals = y_test - yhat_oos
sd(modv$residuals) - sd(oos_residuals)

## [1] -0.4585245

head(cbind(y_test, yhat_oos))

##   y_test yhat_oos
## 1     30 33.01424
## 2     30 66.75854
## 3     85 80.42200
## 4    100 98.19577
## 5     65 67.27280
## 6     30 22.55551
```

Assess your level of success at this endeavor.

#TO-DO

#The Forward Stepwise Procedure for Probability Estimation Models

Set a seed and load the `adult` dataset and remove missingness and randomize the order.

```
set.seed(1)
pacman::p_load_gh("coatless/ucidata")
data(adult)
adult = na.omit(adult)
adult = adult[sample(1 : nrow(adult)), ]
```

Copy from the previous lab all cleanups you did to this dataset.

```
adult$fnlwgt = NULL
adult$marital_status = as.character(adult$marital_status)
adult$marital_status = ifelse(adult$marital_status == "Married-AF-spouse" |
adult$marital_status == "Married-civ-spouse", "Married",
adult$marital_status)
adult$marital_status = as.factor(adult$marital_status)
adult$education = as.character(adult$education)
adult$education = ifelse(adult$education == "1st-4th" | adult$education ==
"Preschool", "<=4th", adult$education)
adult$education = as.factor(adult$education)
adult$education = NULL
tab = sort(table(adult$native_country))
```

```r
adult$native_country = as.character(adult$native_country)
adult$native_country= ifelse(adult$native_country %in% names(tab[tab<50]),
"Other", adult$native_country)
adult$native_country= as.factor(adult$native_country)
adult$worktype = paste(adult$occupation, adult$workclass, sep = ":")
tab_worktype = sort(table(adult$worktype))
adult$occupation = NULL
adult$workclass = NULL
adult$worktype = as.character(adult$worktype)
adult$worktype = ifelse(adult$worktype %in%
names(tab_worktype[tab_worktype<100]), "Other", adult$worktype)
adult$worktype = as.factor(adult$worktype)
adult$status = paste(as.character(adult$relationship),
as.character(adult$marital_status), sep = ":")
adult$status = as.character(adult$status)
tab_status = sort(table(adult$status))
adult$relationship = NULL
adult$marital_status = NULL
adult$status = as.factor(adult$status)
```

We will be doing model selection. We will split the dataset into 3 distinct subsets. Set the size of our splits here. For simplicitiy, all three splits will be identically sized. We are making it small so the stepwise algorithm can compute quickly. If you have a faster machine, feel free to increase this.

```r
Nsplitsize = 1000
```

Now create the following variables: `Xtrain`, `ytrain`, `Xselect`, `yselect`, `Xtest`, `ytest` with `Nsplitsize` observations. Binarize the y values.

```r
Xtrain = adult[1 : Nsplitsize, ]
Xtrain$income = NULL
ytrain = ifelse(adult[1 : Nsplitsize, "income"] == ">50K", 1, 0)
Xselect = adult[(Nsplitsize + 1) : (2 * Nsplitsize), ]
Xselect$income = NULL
yselect = ifelse(adult[(Nsplitsize + 1) : (2 * Nsplitsize), "income"]
==">50K", 1, 0)
Xtest = adult[(2 * Nsplitsize + 1) : (3 * Nsplitsize), ]
Xtest$income = NULL
ytest = ifelse(adult[(2 * Nsplitsize + 1) : (3 * Nsplitsize), "income"] ==
">50K", 1, 0)
```

Fit a vanilla logistic regression on the training set.

```r
#logistic_mod = glm(ytrain ~ ., Xtrain, family = binomial(link = logit))
```

and report the log scoring rule, the Brier scoring rule.

```r
#p_hat_train = predict(logistic_mod, Xtrain, type = 'response')
#mean(ytrain * log(p_hat_train) + (1 - ytrain) * log(1 - p_hat_train))
#mean(-(ytrain - p_hat_train)^2)
```

We will be doing model selection using a basis of linear features consisting of all first-order interactions of the 14 raw features (this will include square terms as squares are interactions with oneself).

Create a model matrix from the training data containing all these features. Make sure it has an intercept column too (the one vector is usually an important feature). Cast it as a data frame so we can use it more easily for modeling later on. We're going to need those model matrices (as data frames) for both the select and test sets. So make them here too (copy-paste). Make sure their dimensions are sensible.

```
Xmm_train = data.frame(model.matrix(~ . * ., Xtrain))
Xmm_select = data.frame(model.matrix(~ . * . , Xselect))
Xmm_test = data.frame(model.matrix(~ . * . , Xtest))
dim(Xmm_train)

## [1] 1000 3104

dim(Xmm_select)

## [1] 1000 3104

dim(Xmm_test)

## [1] 1000 3104
```

Write code that will fit a model stepwise. You can refer to the chunk in the practice lecture. Use the negative Brier score to do the selection. The negative of the Brier score is always positive and lower means better making this metric kind of like s_e so the picture will be the same as the canonical U-shape for oos performance.

Run the code and hit "stop" when you begin to the see the Brier score degrade appreciably oos. Be patient as it will wobble.

```
#pacman::p_load(Matrix)
#p_plus_one = ncol(Xmm_train)
#predictor_by_iteration = c() #keep a growing list of predictors by iteration
#in_sample_brier_by_iteration = c() #keep a growing list of briers by
iteration
#oos_brier_by_iteration = c() #keep a growing list of briers by iteration
#i = 1
#repeat {
  #all_briers = array(NA, p_plus_one)
  #for (j_try in 1 : p_plus_one){
    #if (j_try %in% predictor_by_iteration){
      #next
    #}
    #Xmm_sub = Xmm_train[, c(predictor_by_iteration, j_try), drop = FALSE]
    #logistic_mod = suppressWarnings(glm(ytrain ~ ., Xmm_sub, family =
"binomial"))
    #phat_train = suppressWarnings(predict(logistic_mod, Xmm_sub, type =
'response'))
```

```
    #all_briers[j_try] = -mean(-(ytrain - phat_train)^2)
  #}
  #j_star = which.max(all_briers)
  #predictor_by_iteration = c(predictor_by_iteration, j_star)
  #in_sample_brier_by_iteration = c(in_sample_brier_by_iteration,
all_briers[j_star])
  #Xmm_sub = Xmm_train[, predictor_by_iteration, drop = FALSE]
    #logistic_mod = suppressWarnings(glm(ytrain ~ ., Xmm_sub, family =
"binomial"))
    #phat_train = suppressWarnings(predict(logistic_mod, Xmm_sub, type =
'response'))
    #all_briers[j_try] = -mean(-(ytrain - phat_train)^2)
    #phat_select = suppressWarnings(predict(logistic_mod, Xmm_select[,
predictor_by_iteration, drop = FALSE], type = 'response'))
    #oos_brier = -mean(-(yselect - phat_select)^2)
    #oos_brier_by_iteration = c(oos_brier_by_iteration, oos_brier)
  #cat("i =", i, "in-sample_brier =", all_briers[j_star], "oos_brier =",
oos_brier, "\n   predictor added:", colnames(Xmm_train)[j_star],   "\n")
  #i = i + 1
  #if (i > Nsplitsize || i > p_plus_one){
    #break
  #}
#}
```

Plot the in-sample and oos (select set) Brier score by p. Does this look like what's expected?

```
#in_sample = -(ytrain - phat_train)**2
```