Birthday Protocols for Low Energy Deployment and Flexible Neighbor Discovery in Ad Hoc Wireless Networks

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ABSTRACT
In this paper, we address two problems associated with static ad hoc wireless networks: methods of saving energy during a deployment of the nodes, and efficient methods of performing adjacent neighbor discovery. To meet these goals we introduce a family of "birthday protocols" which use random independent transmissions to discover adjacent nodes. Various modes of the birthday protocol are used to solve the two problems.

We provide a mathematical model and analysis of two modes of the protocol and are led to a third mode which is the probabilistic analog of the deterministic round robin scheduling algorithm.

We show by analysis and simulation that the birthday protocols are a promising tool for saving energy during the deployment of an ad hoc network as well as an efficient and flexible means of having the nodes discover their neighbors.

Categories and Subject Descriptors
C.2.1 [**Computer Communication Networks**]: Wireless communications; G.3 [**Probability and Statistics**]: Probabilistic algorithms

General Terms
Algorithms, Design

Keywords
birthday, ad hoc, wireless, sensor, deployment, discovery

1. INTRODUCTION
In ad hoc wireless networking, one considers the problem of organizing nodes whose connectivity is unpredictable. Such networks can be static, like a sensor network whose nodes are dropped by a plane into a forest, or mobile, like the headsets of a squadron of soldiers. In static ad hoc wireless networks, energy conservation can be an especially important concern, since nodes may be deployed with small batteries. When the battery fails, the node essentially disappears with perhaps dire consequences for the data path it helped to support.

In this paper we consider two aspects of static ad hoc networks. First, we consider the problem of energy conservation during the deployment of nodes over an extended period. Second, we examine the problem of neighbor discovery following deployment.

To motivate our problems, consider the following scenario. A large number of wireless battery-powered sensors are released from an airplane into a forest. Several drops are required before all the sensors are deployed. After a week, all the sensors are in place, and we would like to initiate network discovery through some trigger mechanism.

During the week-long deployment phase, the nodes initially deployed must wait for up to a week before discovery begins. If they listen constantly during this period, their batteries will be exhausted long before they can do anything useful. Thus, saving energy during deployment is one goal. The discovery phase itself might only last a couple of minutes. The nodes must participate vigorously during discovery to maximize their chance of hearing and being heard by neighbors. Thus, maximizing the probability of discovery is another goal. After discovery is complete, data transmission among the nodes would begin. We do not concern ourselves here with data transmission.

We present a family of probabilistic protocols that we call "birthday protocols," which allow us to save a great deal of energy during deployment, while also allowing us a high probability of discovering our neighbors during a discovery phase. We present three different birthday protocols which differ only in their parameters, but which can be used to solve the aforementioned problems. Through analysis and simulation we show that the protocols perform well, and also robustly, in the sense that if conditions are not quite what was expected during deployment, reasonable results may still be obtained.

The inspiration for our protocol is the *birthday paradox*, in which we compute the probability that at least two people in a room have the same birthday. When there are as few as

23 people, the probability that at least two have the same birthday already exceeds 1/2. We apply a similar idea to channel access. Over a period of n slots, two wireless nodes independently and randomly select k slots. The first node transmits a message on its k slots, and the second node listens on its k slots. For the remaining $n-k$ slots, each node is idle. The probability that the second node hears the first is:

$$Q(n,k) = 1 - \frac{\binom{n-k}{k}}{\binom{n}{k}}$$

which is nearly 1 when the ratio k/n is relatively small. For example, $Q(1000, 70) \approx 0.995$. With these parameters, each node is idle 93% of the time, yet there is a very high probability that the second node will hear the first. This is the crux of the birthday protocol. Although saving significant energy, the protocol discovers links with high probability. If we use this method on a network containing many links, our protocol may fail to discover some of the links. However, by trading energy for a minimal discovery loss, we show that the protocol is highly effective, even when a node has many neighbors.

1.1 Related Work

Many ad hoc networks do not explicitly perform neighbor discovery. In mobile networks, it makes little sense to discover the local topology, because soon it will change. In these cases, wireless broadcasts are used by upper layers in the protocol stack, and nodes communicate with a whole set of neighbors without caring who they are. Ad hoc routing protocols such as AODV, DSR, and ZRP [1, 2, 3] assume that neighboring nodes will always listen for transmissions when not transmitting. Such methods of constant listening, which avoid explicit discovery of neighbors, work well but are not energy efficient. Energy inefficiency represents a real impediment to the deployment of ad hoc networks whose devices are battery operated.

In a static network, where the topology changes only when nodes run out of energy, it makes more sense to determine the fixed topology. Neighbor discovery enables clever scheduling schemes like rendezvous[4], cluster-based routing [5, 6, 7, 8, 9] , and source routing strategies, which might be well suited to sensor networks, although they have been little explored. Discovery of neighbors is also required before running some distributed algorithms, such as the distributed minimum weight spanning tree algorithm of [10].

In short, while mobile ad hoc networks will not use discovery, discovery can be useful in static ad hoc networks.

Current commercial radio systems have no way to transmit or receive a special wakeup broadcast. To remain in touch with neighbors nodes have to spend time listening, which costs energy. What's worse is that the power of listening is on the order of the power for transmitting, for the low power radios (e.g. Bluetooth[11]) that would likely be used in applications where battery size is an issue. Without the ability to be woken up, there would appear to be only two ways to save transmit power: transmit at lower power, or not transmit. Research in the first area is exemplified by [12] in which transmit power is controlled to give a desirable graph topology among the communicating nodes. Research in the second area is exemplified by [4, 13, 14] in which the nodes in the network arrange a schedule for communication, saving energy in the periods when they are not communicating. Our paper describes a method which saves energy in a method similar to the second branch, except that instead of a schedule we use a probabilistic method.

A probabilistic protocol which solves a different problem, electing a leader from a clique of N nodes, is given in [15]. Although it may be possible to extend leader election to a method of neighbor discovery, the hidden node problem makes their work inapplicable to the multi-hop environments we are considering. There is no obvious way to extend their analysis of a complete graph to the far more likely case that some nodes will be in more than one clique, which is the hidden node problem. It is a goal of this paper to show how our probabilistic methods can succeed where deterministic methods and schedules fail.

The paper is organized as follows. We provide a formal model of our wireless network and define our protocol in its terms. We immediately follow with a definition of "birthday-listen" and "birthday-listen-and-transmit" modes. We then provide analysis of its performance with complete graphs, introduce "probabilistic round-robin" mode, and show how its features suit it to arbitrary graphs where one cannot be sure how many neighbors a node will have. We then give a detailed example of the deployment and discovery situation where we think the birthday protocols can be of benefit, and provide a simulation to give flesh to our arguments.

2. MODEL OF THE WIRELESS NETWORK

In this analysis we will assume that

- time is slotted (as in [15].) We have n timeslots which taken together comprise an interval of length I. Thus each timeslot has length I/n.
- n is large.
- the total number of nodes is known.
- the nodes do not coordinate their actions in any way.
- the nodes are placed randomly in some area.
- nodes are distinguishable by an ID such as a MAC address.
- each node has some internal memory to record local topology cd /home/pro ad information.

A node can be in one of three states: transmit (T), listen (L), or energy-saving (S). During the *transmission state*, a node is broadcasting a discovery message advertising itself. Such a message would consist of, at a minimum, the address of the broadcaster. During the *listen state*, a node is listening for discovery messages. If such a message is heard, the node records the source address in its local topology table.

During the *energy-saving state*, a node spends zero energy on its radio subsystem. We assume that only a small but positive amount of time is required to transit between listening, transmitting and energy saving.

Each node in our system will choose randomly to enter one of the three states at the beginning of each timeslot. Its choice will depend on the mode the node is in, and π_t and π_s, to be defined below. The choice of state for one timeslot will be independent of choices made for other timeslots.

We will describe two modes of operation: birthday-listen-and-transmit (BLT) and birthday-listen. Later, a third mode will be described.

A node in BLT mode can be in any of the three states, L, T and S. In each timeslot it chooses state T with probability p_t, L with probability p_l and S with probability $p_s = 1 - p_l - p_t$.

A node in birthday-listen mode (a.k.a. BL mode or listen-only mode) does not transmit. It alternates between states L and S. On each timeslot it picks randomly between the two states, choosing L with probability p_l and S with probability $p_s = 1 - p_l$.

To enable all the modes, the nodes store two globally fixed values π_t and π_l for their lifetimes. The first quantity π_t represents p_t when in BLT mode. The quantity π_l represents p_l in both BLT and birthday-listen modes. A node which transits into BLT mode sets

$$\begin{aligned} p_t &\leftarrow \pi_t \\ p_l &\leftarrow \pi_l \\ p_s &\leftarrow 1 - \pi_t - \pi_l \end{aligned} \tag{1}$$

A node which transits into birthday-listen mode sets

$$\begin{aligned} p_t &\leftarrow 0 \\ p_l &\leftarrow \pi_l \\ p_s &\leftarrow 1 - \pi_l \end{aligned} \tag{2}$$

We now define neighbor discovery. We will distinguish between node X "hearing Y" and "discovering Y". We say that X hears Y on a timeslot if X is in state L, Y is in state T, and no other node in range of X is in state T. X may hear Y on many timeslots. We say that "X discovers Y" on the first timeslot he hears Y. We distinguish between the events "X hears Y" and "Y hears X." In other words we do not presume that links are necessarily bi-directional. In this paper, we use "link" interchangeably with "unidirectional link" and treat the discovery of each direction as a separate achievement.

Now we define some random variables of interest. Let C be a set of nodes which are all within radio range of each other. In graph terms C is a clique. Suppose we have N nodes in a clique which includes nodes X, Y. Let $H(X, Y, I)$ equal the number of timeslots in an interval I in which X hears Y. Let $H = \sum_{X,Y \in C} H(X, Y, I)$ be the total number of events in which any node hears any other node during a whole interval I. We will sometimes denote by h the number of nodes which hear another node during a single timeslot. Thus h refers to a single timeslot whereas H refers to a whole interval.

Let $U \leq H$ be the number of discoveries made during I, which we can think of as de-duping the events counted in H. There are $N(N-1)$ discoveries which could be made. Clearly $U \leq N(N-1)$, and the fraction of links which are discovered is $F = \frac{U}{N(N-1)} \leq 1$.

To model energy expenditure, we use two levels: zero energy, for when the radio system is turned off, and a single level of energy for when the radio system is either listening or transmitting. This follows the trend observed in low power radios [11], such as those expected to be used in sensor networks. The model would have to be modified for longer range radio systems such as cellular telephone or satellite systems where transmissions cost more than reception. Communication between nodes is asynchronous. Each node slots its time independently of other nodes, but at the same frequency.

Some research in low-power circuitry suggests that as the years pass Moore's law will allow the power draw of circuitry to decline arbitrarily far, while the power draw of wireless transmission is subject to Maxwell's Laws and cannot therefore be reduced beyond a certain point [16]. Thus we account only for the energy cost of transmission in this paper.

Under this two-level assumption of energy expenditure, nodes in both birthday-listen and BLT modes save energy a fraction p_s of the time. The birthday protocol would still be useful given a more complicated energy model, but we wish to expose its advantages in this simpler setting.

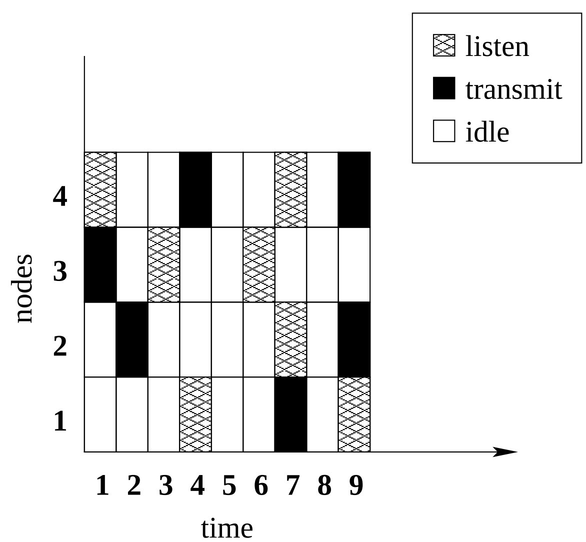

Figure 1: Discovery example. A clique of $N = 4$ nodes, all in BLT mode, for $n = 9$ slots.

A simple example is provided in Figure 1. Four nodes, all within range of each other, have just entered BLT mode, performing discovery over 9 slots. Link discovery occurs when a single node transmits, and one or more nodes are listening. In this example, $N = 4$, $n = 9$, $p_t = p_l = \frac{1}{6}$, and $p_s = \frac{2}{3}$.

In slots 1, 4, and 7, a total of four links are discovered ($3 \to 4, 4 \to 1, 1 \to 2, 1 \to 4$). In the remaining slots, no links are discovered due to: no listeners (slots 2), no transmitter (slots 3 and 6), transmission collisions (slot 9), or no activity (slots 5 and 8).

Thus the fraction of links discovered is $F = 4/12$. The nodes spent 23 of the 36 slots sleeping and 13 transmitting or listening, so the actual energy gain is 36/13. The expected energy gain was $1/(p_t + p_l) = 3$.

3. ANALYSIS

In this section we analyze the tradeoffs between energy savings, probability of neighbor discovery, and delay.

We will compare our protocol to another which could be employed to discover neighbors. This protocol is in fact widely used in situations where power is not a concern. The protocol transmits data when it is necessary to do so, and at all other times listens for other nodes who transmit. In terms of the model of Section 2, we will allow that for this protocol

- a clique of size N will discover all $N(N-1)$ of its links.
- each node will have $p_l + p_t = 1$ and $p_s = 0$.

That is, it is completely successful at discovering neighbors, but is constantly using energy. We use the term "energy gain" to mean the quotient of energy used by this protocol divided by the energy used by the birthday protocol.

We pose our problem as follows. A customer has an ad hoc network to deploy. The devices will not be individually placed, but dropped from a plane or otherwise dispersed. The number of nodes to be deployed is known, but their exact placement is not. The deployment may be done in several stages, so that the total time for deployment is long, say, a week. When the deployment is complete the customer would like to initiate the discovery, perhaps from a location near the edge of the area of deployment. As we shall see, discovery may be extremely short compared to the period of deployment. After discovery, data exchange among the nodes occurs, but we are concerned only with the phases of deployment and discovery.

We tentatively propose to deploy the nodes in birthday-listen mode, so that they can save energy during this longish period, and upon initiation of discovery, change into BLT mode when the goal is to maximize the fraction of neighbors each node discovers.

The customer specifies two of the following.

- The discovery period shall last no longer than some period I. That is, once the waiting period is over, and the discovery period begins, after I elapses all links have either been discovered or never will be. Because technology puts an upper bound on the rate at which our nodes can switch between states (timeslots must include time for both physically switching the radio hardware between L, T and S, plus time to transmit the message when the state is T), this is equivalent to specifying n.
- The expected fraction of links to be recovered is at least F.
- The expected energy gain is at least G. That is, if the devices would deplete their batteries in a period P of constant radio usage, the customer might desire the devices last a period PG.

Given any two of the requirements we can optimize the third.

In this analysis section, we examine first the smallest possible collection of nodes, $N = 2$ of them, to gain some intuition. Then we consider an arbitrarily large clique of N nodes all within radio range of each other. We compute the expected performance of the birthday protocols in such a clique. As a result we are led to a new, third, mode of the birthday protocols, which we call "probabilistic round robin." Finally we consider the performance of the birthday protocols in arbitrary graphs, and under conditions likely to occur in a real deployment.

3.1 Intuition from the two-node case, N = 2

Suppose our network contains two nodes within radio range of each other. Suppose also that both nodes are in BLT mode for some interval of length I. There are several questions of interest. What is the total energy the two nodes use, and what is the fraction F (0, $\frac{1}{2}$ or 1) of the two links which are discovered?

For X to discover Y, X must be listening while Y is transmitting. In one timeslot the probability of this is $p_l p_t$. The probability that X discovers Y over a whole interval of n slots is $1 - (1 - p_l p_t)^n$. By symmetry the probability for Y to discover X is the same. Treating these events as independent the expected number of links discovered is

$$E(U) = 2(1 - (1 - p_l p_t)^n) \tag{3}$$

We can use this formula to gain intuition about the design of our system. The expression increases as n increases. Since interval length is fixed, increasing n decreases the length I/n of a timeslot. However, there is a limit to how quickly the hardware can switch between the listening, transmitting and energy-saving states, and this puts a lower bound on the length of a timeslot, hence an upper bound on n. Nevertheless we clearly want to make n as large as possible and our timeslots as short as possible. Intuitively, the larger n is, the more timeslots we have to make discoveries within one interval.

If X is in birthday-listen mode and Y is in BLT mode, $Pr(X \text{ discovers } Y) = 1 - (1 - (p_l p_t)^2)^n$, unchanged from the case when both are in BLT mode.

3.2 Analysis of cliques of size N > 2

The reader might wonder whether the birthday protocol holds up when more and more nodes are within hearing distance of each other. After all, each of the nodes has a fixed probability of transmission at the time of deployment so the more nodes there are the higher the probability that two transmissions collide, wasting the chance to make new connections in that slot. In this section we address this question.

Suppose we have a clique of N nodes all in BLT mode. According to our model each picks one of the three states (T, L, S) randomly in each timeslot with corresponding probabilities (p_t, p_l, p_s). Denoting the number of nodes in these states in a single timeslot by T, L and S, we have $T + L + S = N$.

The random variables T, L and S in any one timeslot have a trinomial distribution. This follows from the independence of the actions of the nodes, an assumption of the model. The actions of each of the N nodes is an independent trial having three possible outcomes, T L or S.

$$Pr(T = t, L = l, S = s) = \binom{N}{t,l} p_t^t p_l^l p_s^s \qquad (4)$$

The events we are concerned with are when one node hears another. A node X hears one of his $N-1$ neighbors if and only if exactly one of the neighbors transmits while X listens. Let us first determine the number of times we expect one node to hear another in one timeslot. Let us call an event where one node hears another a "link." A link can only be made when there is exactly one transmitter, so what we want to compute is $E(h) = E(L \mid T = 1)Pr(T = 1)$.

First we determine $E(L \mid T = 1)$. Given that there is exactly one transmitter, the other $N-1$ nodes are either listening or saving energy, with probabilities $\frac{p_l}{p_l+p_s}$ and $\frac{p_s}{p_l+p_s}$ respectively. The number of listeners therefore has a binomial($N-1$, $\frac{p_l}{p_l+p_s}$) distribution, whose expectation is

$$E(L \mid T = 1) = \frac{(N-1)p_l}{p_l + p_s}. \qquad (5)$$

Now we determine the probability that $T = 1$. The number of transmitters is also a random variable whose distribution is binomial(N, p_t). Therefore

$$Pr(T = 1) = \binom{N}{1} p_t (1 - p_t)^{N-1}. \qquad (6)$$

Thus within one timeslot we have

$$\begin{aligned} E(h) &= E(L \mid T = 1)Pr(T = 1) \\ &= N(N-1)\frac{p_l}{p_l + p_s} p_t (1-p_t)^{N-1} \\ &= N(N-1)\frac{p_l}{1 - p_t} p_t (1-p_t)^{N-1} \\ &= N(N-1) p_t p_l (1-p_t)^{N-2} \qquad (7) \end{aligned}$$

This is graphed in Figure 2 with the restriction $p_t = p_l$. The restriction is made only to reduce the dimensionality of the problem.

Now we calculate the expected fraction of links discovered, i.e. we try to get U by subtracting the duplicates counted in H. Over an entire interval (n timeslots), there are $E(H) = nE(h)$ links found. Since one of the assumptions from Section 2 is that n is large, $nE(h)$ is large. Each such "hearing" is on one of the $N(N-1)$ bidirectional pairings. If N is large, then the placing of hearings into bidirectional links is a Poisson process with mean $\lambda = \frac{nE(h)}{N(N-1)} = np_t p_l (1-p_t)^{N-2}$. The fraction of links discovered is therefore

$$F = 1 - e^{-\lambda} = 1 - e^{-np_t p_l (1-p_t)^{N-2}} \qquad (8)$$

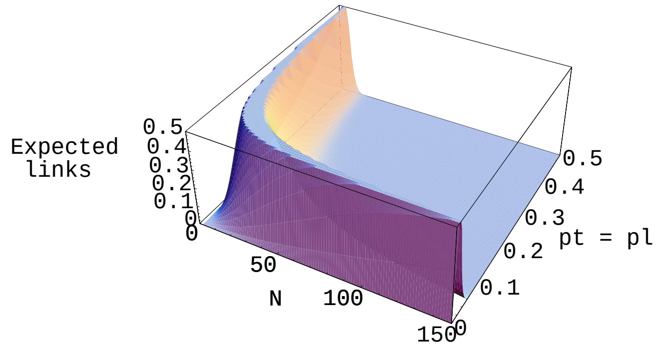

Figure 2: Expected number $E(h)$ of links heard in a single slot, assuming $p_t = p_l$.

Let us examine this expression for fixed n. Even as N becomes large, with the right choice of $p_t = p_l$ we can still manage to achieve a fraction $F = 1/e$ of all the links in the clique, corresponding to the ridge in the graph of Figure 2. This result may seem somewhat surprising, because the more transmitters there are, the more collisions there will be. On the other hand, in the case when there is exactly one transmitter, all the listeners will hear him. This is the effect which compensates for the noisier environment of larger N and makes it possible to discover links "in a crowd."

In the next section we will see what happens when we remove the restriction $p_t = p_l$.

3.3 Probabilistic Round Robin Mode

In this section we will exploit the conclusion of the previous section, namely, that by choosing the "right" transmission probability, we can maximize the fraction of links that we discover. Previously we chose to let $p_t = p_l$, in order to reduce Equation 7 to a function of only one variable.

For a particular value of N, we are interested in obtaining probabilities p_t and p_l that maximize the expected fraction of unique links discovered over n slots. To do this it is sufficient to maximize $E(h)$, the expected number links discovered in a single slot.

If we set $p_l = C - p_t$, for some C in the range $0 < p_t < C \leq 1$, then C is inversely related to energy savings. Substituting into Equation 7, the expected number of links discovered in a single slot is

$$E(h) = N(N-1)p_t(C - p_t)(1-p_t)^{N-2} \qquad (9)$$

This function of p_t is maximized at

$$p_t = \frac{1}{N} \quad \text{or} \quad p_t = \frac{1-(1-N)C}{N}$$

The second solution does not satisfy the constraint that p_t must be less than C. Thus, when $p_t = \frac{1}{N}$, we maximize the number of links discovered for some C.

To maximize discovery, we would disregard energy concerns and set $C = 1$. This implies that in each slot, a node is either transmitting or listening, never sleeping.

We can model this as a third birthday mode, in which we set

$$\begin{aligned} p_t &\leftarrow \frac{1}{N} \\ p_l &\leftarrow 1 - \frac{1}{N} \\ p_s &\leftarrow 0 \end{aligned} \tag{10}$$

Intuitively, since there are N nodes, and each is transmitting a fraction $\frac{1}{N}$ of the time and listening the rest of the time, it is the probabilistic analog of a round robin protocol. Therefore we call this mode of the birthday protocol "probabilistic round robin" (PRR). There is no energy conservation in PRR mode, and the number of links discovered is maximized.

Using the parameters of PRR in Equation 7, the expected number of links discovered in a single slot is

$$\begin{aligned} E(h) &= N(N-1)p_t p_l (1-p_t)^{N-2} \\ &= (N-1)(1-\frac{1}{N})^{N-1} \end{aligned} \tag{11}$$

and for large N the Poisson approximation of Equation 8 holds with fraction of unique links discovered

$$\begin{aligned} F &= 1 - e^{-\lambda} \\ &= 1 - e^{-\frac{n}{N}(1-\frac{1}{N})^{N-1}} \end{aligned} \tag{12}$$

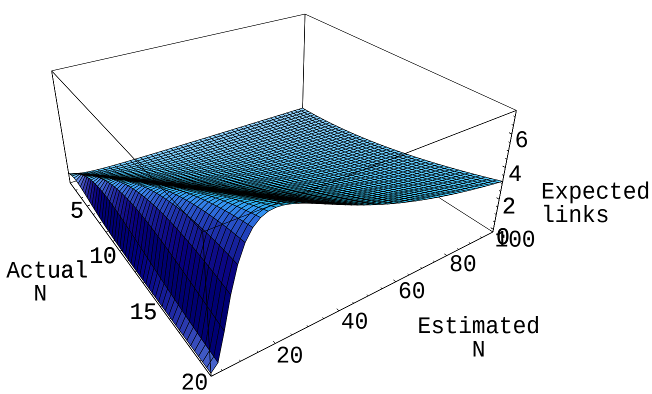

Figure 3: Robustness of birthday protocol when $\hat{N} \neq N$. The height of the graph is the expected number of discovered links given that the true number of neighbors is N but $p_t = \frac{1}{\hat{N}}$ and $p_l = 1 - \frac{1}{\hat{N}}$ are based on the estimate $\hat{N}$.

3.4 Analysis of Arbitrary Graph Topologies

We have referred to the "flexibility" of the birthday protocols. By flexibility we mean both that the protocol can be deployed with an arbitrary graph topology, and also that when conditions are not what was expected (such as when we incorrectly estimate a node's number of neighbors), the protocol degrades gracefully. Here we briefly analyze these issues.

Let $\hat{N}$ equal some estimate we make of the number of neighbors that a node will have after deployment. Given our assumptions of random placement of a known number of nodes within a known area, we will be able to produce such an estimate. What happens when a node has more or fewer neighbors than estimated, as will surely occur in a random deployment? In Figure 3, we show the expected number of discovered links per slot, $E(h)$, as a function of N and $\hat{N}$. The graph depicts Equation 7 with $p_t = \frac{1}{\hat{N}}$ and $p_l = 1 - \frac{1}{\hat{N}}$. When $N > \hat{N}$, transmission probabilities are set too high, causing collisions which reduce the number of links discovered below the optimum. When $N < \hat{N}$ transmission probabilities are set too low, missing opportunities to make links.

The graph shows the outcome is better when $\hat{N} > N$. This suggests the rule, "overestimate the number of neighbors." This rule could be formalized by assigning a cost to the difference between N and $\hat{N}$ but for simplicity we bypass this.

The fact that the scenario can be different than expected, and yet success still achieved is the primary difference between the probabilistic birthday protocols and rigid deterministic scheduling algorithms such as that proposed in [15].

Now let us consider the "mixed-mode" case where there is a clique of N nodes, some of them in BL mode, and the rest in PRR mode. (We will see in the next section why this case is important.) We want to calculate the probability that a certain node in BL mode hears a node in PRR mode. The nodes in birthday-listen mode are listening with probability p_l. In the worst case, only a single neighbor is in PRR mode. The probability that a node in BL mode hears this neighbor over the course of an interval is

$$\begin{aligned} Pr(\text{hear on interval}) &= 1 - (Pr(\text{don't hear on 1 slot}))^n \\ &= 1 - \left(1 - \frac{p_l}{N}\right)^n \end{aligned} \tag{13}$$

where n is the number of slots that the situation lasts. If more than one neighbor is in PRR mode, then the probability of a node in BL mode hearing a message increases.

4. AN EXAMPLE OF DEPLOYMENT AND DISCOVERY

In this section we connect our analysis from the previous section to our ideas for deployment, which we mentioned briefly in the introduction. We proceed to illustrate our results with a specific example and a simulation.

In the last section we analyzed the birthday protocol primarily in terms of a tradeoff between energy, success of neighbor discovery, and delay. We found that: (a) in birthday-listen mode, we could save a lot of energy, and (b) in PRR mode, we could maximize the probability of neighbor discovery while saving no energy. In each phase we are still using the birthday protocol; only its parameters change.

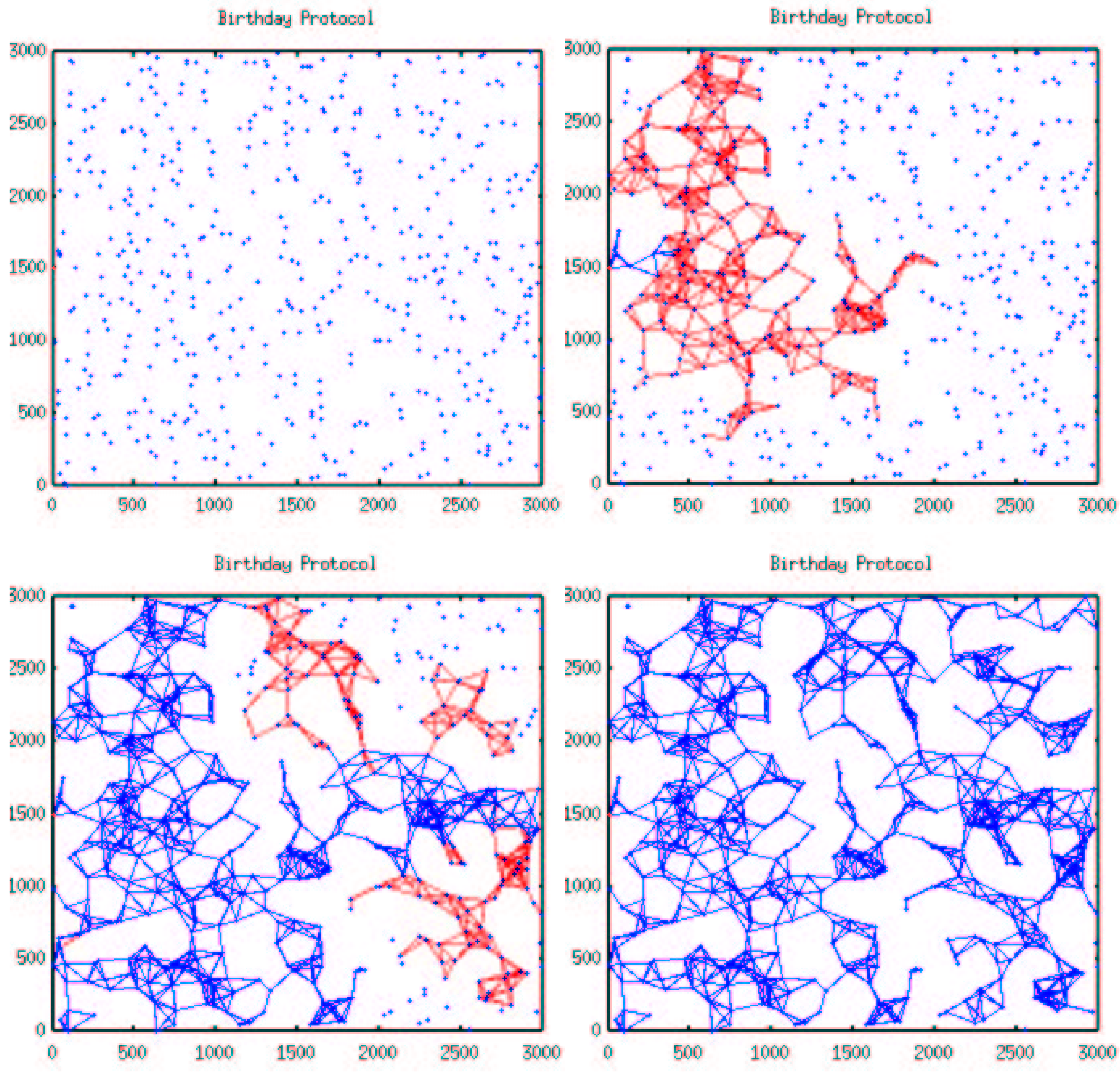

Figure 4: Simulation of the combined BL-PRR protocol at various times. $t = 0$ (upper left plot), $t = 5000$ (upper right), $t = 10000$ (lower left), $t = 19221$ (lower right). At $t = 0$, all nodes are in BL mode, except the trigger node at coordinates (1,1500) which is in PRR mode. At $t = 5000$, more than half the nodes are still in BL mode, including the original trigger node which reverted to BL mode at $t = 3000$. At $t = 10000$ most of the nodes are either in PRR mode or have completed PRR and reverted to BL mode. At $t = 19221$ all nodes are again in BL mode and discovery is finished. Three nodes were physically partitioned (two in the upper left corner and one on the bottom edge at (2554,11).) Discovery was almost completely successful, as only 12 links went undiscovered. Two undiscovered links are associated with the node at (2984,609) on the right side which was close enough to two other nodes to make links but did not.

This leads us to believe that a successful strategy will be to deploy the nodes in birthday-listen mode, then upon the occurrence of some event, to transit to PRR mode for a period. As we mentioned earlier, this strategy will be most effective when the deployment phase lasts a long time. The time in PRR mode must be relatively short, since it uses full power. (The followup to neighbor discovery could be clustering, construction of a minimum spanning tree, or another algorithm that would allow for the data transmission phase to be energy-efficient. None of these things is considered here.)

To execute this strategy the nodes obey a few rules. The randomly placed nodes are deployed in BL mode, saving energy yet listening with some p_l to hear some neighbor. After the deployment phase is over, we put one node on an edge of the square in PRR mode. The specific PRR mode we use is $p_t = 1/\hat{N}$, $p_l = 1 - 1/\hat{N}$. The nodes are programmed as follows. When in BL mode, and a message is heard, it must transit to PRR mode for a certain fixed period, then transit back to BL mode. When there are no more nodes in PRR mode, our simulation is complete.

A simulator was developed to evaluate the birthday protocol. The simulator randomly places Θ wireless nodes into a rectangular grid of dimension $x \times y$, chosen by the user. For simplicity, each node has a transmission radius r and a node can hear all adjacent nodes within distance r. Finally, node clocks were slot synchronized.

For random graph simulations, each node is deployed in BL mode, and a node chosen by the user is triggered into PRR mode at time $t = 0$. At each time slot, each node generates a pseudorandom number which determines if the node should listen, transmit, or sleep with appropriate probabilities. After all nodes determine their state during the current slot, listening nodes determine if they heard a single neighbor transmission within distance r. If so, then a link has been discovered. The simulation ends when all modes have returned to BL mode.

Nearly all network events can be recorded for further analysis. Various network statistics also are computed including: number of links heard, number of unique links discovered out of the possible $\Theta(\Theta - 1)$, number of collisions heard, number of transmissions that were not heard, and total energy expended by the network.

To illustrate, we simulated the random placement of $\Theta = 500$ nodes in a 3000 ft x 3000 ft square ($x = y = 3000$). The nodes each have a range of $r = 200$ ft. Based on these numbers we roughly expect a node to have 7 neighbors. For safety, and in accordance with the theory from Figure 3, we will overestimate this and set $\hat{N} = 10$.

Let us say that we want the fraction of links discovered to be $F = 95\%$, we expect deployment to last 3 days, and that we want the energy gain during the 3-day period to be 100. From the energy gain we will need $p_l = 1/100$ in the BL mode. From this we calculate the necessary n with Equation 13:

$$\begin{aligned} 1 - \left(1 - \frac{p_l}{\hat{N}}\right)^n &\geq .95 \\ n &\geq \frac{log(.05)}{log(1 - p_l/\hat{N})} \\ n &\geq 2994 \qquad (14) \end{aligned}$$

We set $n = 3000$. Using the currently available Intersil HFA3683A chip from the Prism chipset, the time to transit between energy-saving and TX/RX modes is no more than 10 ms, which far exceeds the times to switch between TX and RX modes [17]. Using the conservative figure of 20 ms as a slot length, $n = 3000$ slots corresponds to a PRR mode which lasts 1 minute. So the nodes will be in BL mode for 3 days and PRR mode for 1 minute.

We display in Figure 4 the results of a simulation run with the above parameters. Out of 500 nodes deployed, 3 were out of radio range (physically partitioned) so that no protocol could have found them. Our BL-PRR protocol discovered 3068/3080 possible links, and managed to discover 494 of the 497 nodes.

Although each node was in PRR mode for 60 seconds the total length of the simulation was 19221 slots or 384 seconds. This corresponds to the ripple effect; the nodes closest to the trigger node will go into PRR mode before waking up their neighbors, who wake up theirs, and so on.

It seems likely that slot time can be reduced below our conservative estimate of 20 ms. Supposing that it could be reduced to 5 ms, with other parameters unchanged, what improvements could be made? During the 3-day deployment, the energy gain could be improved from 100 to 400; the fraction F of discovered links could be improved from $F = .95$ to $F = .999994$; or the length of PRR mode could be reduced from 60 seconds to 15 seconds.

5. CONCLUSION

We have presented the birthday protocols, a family of probabilistic neighbor discovery protocols. Their simplicity and flexibility make them attractive in static wireless ad hoc networks where energy conservation is critical.

The protocols provide a concrete way to trade off energy efficiency, success in discovering neighbors, and delay. A deployment and discovery scenario was detailed in Section 4. It was shown that the birthday-listen mode of the protocol was well suited to saving energy during a long deployment, while the probabilistic round-robin mode could quickly discover neighbors.

The flexibility of the birthday protocols springs from their probabilistic nature. Their robustness in unexpected settings, and their ability to easily handle deployments with arbitrary graph topology was predicted and illustrated. These make the birthday protocols a more realistic choice for ad hoc networks than deterministic or scheduling algorithms.

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