Statistical Inference Course Project - Part 1

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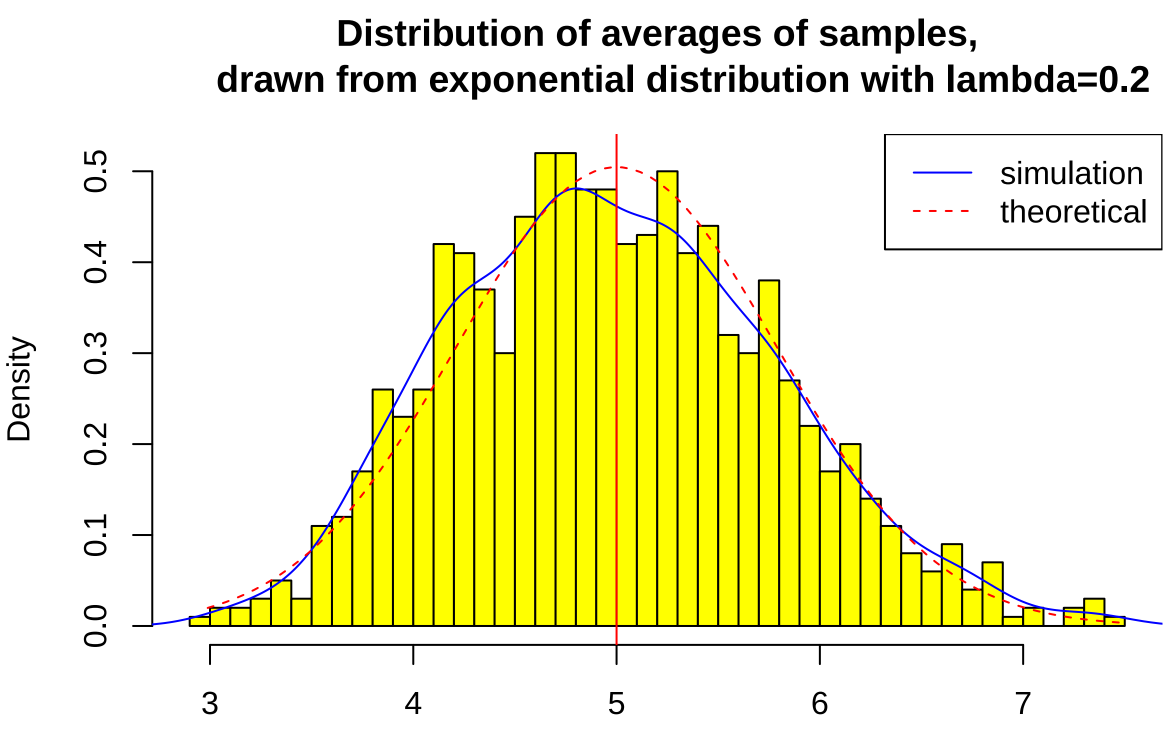
Simulation Exercises

In this exercise the exponential distribution is simulated in R with rexp(n, lambda) where lambda λ is the rate parameter. The mean of exponential distribution is $1/\lambda$ and the standard deviation is also $1/\lambda$. In this simulation, we investigate the distribution of averages of 40 numbers sampled from exponential distribution with $\lambda = 0.2$.

Here is the code for generating 1000 simulated averages of 40 exponentials.

```
set.seed(3)
lambda <- 0.2
num_sim <- 1000
sample_size <- 40
sim <- matrix(rexp(num_sim*sample_size, rate=lambda), num_sim, sample_size)
row_means <- rowMeans(sim)
```

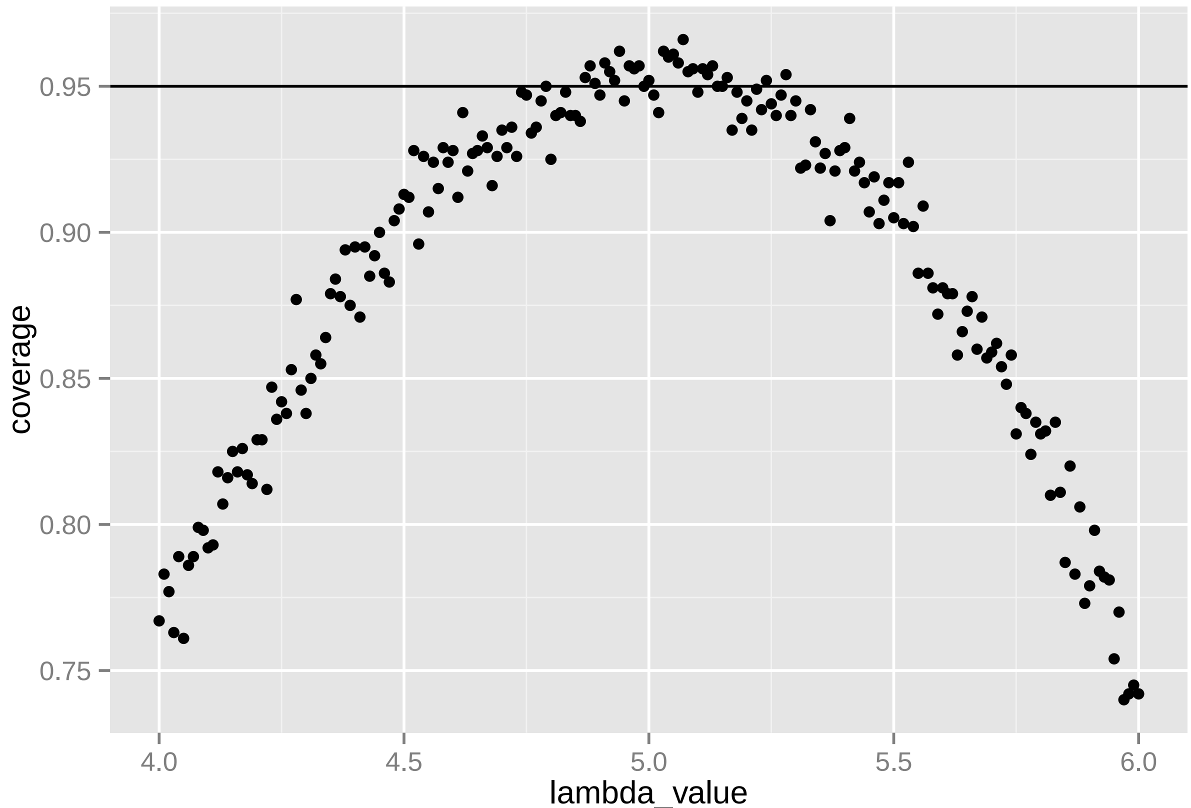
The distribution of sample means are shown as follows:



The distribution of sample means is centered at 4.9866197 and the theoretical center of the distribution is $\lambda^{-1} = 5$. The variance of sample means is 0.6257575 where the theoretical variance of the distribution is $\sigma^2/n = 1/(\lambda^2 n) = 1/(0.04 \times 40) = 0.625$.

The above figure has shown that the distribution is about normal, according to Central Limit Theorem.

The following figure illustrates the coverage of the confidence interval for $1/\lambda = \bar{X} \pm 1.96\frac{S}{\sqrt{n}}$



The 95% confidence intervals for the rate parameter (λ) to be estimated ($\hat{\lambda}$) are $\hat{\lambda}low = \hat{\lambda}(1 - \frac{1.96}{\sqrt{n}})$ and $\hat{\lambda}upp = \hat{\lambda}(1 + \frac{1.96}{\sqrt{n}})$. As shown in the above figure, the average of the sample mean falls within the confidence interval at least 95% of the time.