Para FMAT

Unas poquitas integrales que encontre por ahi

por Picosenotheta ...bueno y que esperan , a bajar y trabajar y suerte en los controles

801 EJERCICIOS RESUELTOS DE INTEGRAL INDEFINIDA

INDICE

A Patricia. / A Ana Zoraida.

A los que van quedando en el camino,

Compañeros de ayer,

De hoy y de siempre.

INTRODUCCION

El libro que os ofrecemos, no es un libro auto contenido, sino un instrumento de complementación, para la práctica indispensable en el tópico relativo a las integrales indefinidas. En este contexto, el buen uso que se haga del mismo llevará a hacer una realidad, el sabio principio que unifica la teoría con la práctica.

El trabajo compartido de los autores de “801 ejercicios resueltos” es una experiencia que esperamos sea positiva, en el espíritu universitario de la activación de las contrapartes, en todo caso será el usuario quien de su veredicto al respecto, ya sea por medio del consejo oportuno, la crítica constructiva o la observación fraterna, por lo cual desde ya agradecemos todo comentario al respecto.

Nos es grato hacer un reconocimiento a la cooperación prestada por los estudiantes de UNET: Jhonny Bonilla y Omar Umaña.

INSTRUCCIONES

Para un adecuado uso de este problemario, nos permitimos recomendar lo siguiente:

a) Estudie la teoría pertinente en forma previa.
b) Ejercite la técnica de aprehender con los casos resueltos.
c) Trate de resolver sin ayuda, los ejercicios propuestos.
d) En caso de discrepancia consulte la solución respectiva.
e) En caso de mantener la discrepancia, recurre a la consulta de algún profesor.
f) Al final, hay una cantidad grande de ejercicios sin especificar técnica alguna. Proceda en forma en forma análoga.
g) El no poder hacer un ejercicio, no es razón para frustrarse. Adelante y éxito.

ABREVIATURAS DE USO FRECUENTE

e :	Base de logaritmos neperianos.
$\ell\eta$:	Logaritmo natural o neperiano.
ℓog :	Logaritmo vulgar o de briggs.
$\mathrm{s}e\mathrm{n}$:	Seno.
$\mathrm{arcs}e\mathrm{n}$:	Arco seno.
$\cos$:	Coseno.
$\mathrm{arc}\cos$:	Arco coseno.
$\mathrm{arc}\,co\mathrm{s}$:	Arco coseno.
τg :	Tangente.
$\mathrm{arc}\,tg$:	Arco tangente.
$\mathrm{co}\,\tau g$	Cotangente.
$\mathrm{arc\,co}\,tg$	Arco cotangente.
$\sec$:	Secante.
$\mathrm{arc}\sec$:	Arco secante.
$\cos ec$:	Cosecante.
$\mathrm{arc}\sec$:	Arco cosecante.
exp :	Exponencial.
dx :	Diferencial de x.
$\lvert x\rvert$:	Valor absoluto de x.
m.c.m:	Mínimo común múltiplo.

IDENTIFICACIONES USUALES

$\mathrm{s}e\mathrm{n}^n\, x = (\mathrm{s}e\mathrm{n}\, x)^n$

$\mathrm{s}e\mathrm{n}^{-1}\, x = \mathrm{arcs}e\mathrm{n}\, x$

$\ell\eta^n x = (\ell\eta x)^n$

$\ell og^n x = (\ell og x)^n$

$\ell og x = \ell og\,\lvert x\rvert$

IDENTIDADES ALGEBRAICAS

1. Sean a, b: bases; m, n números naturales.

$a^m a^n = a^{m+n}$

$(a^m)^n = a^{mn}$

$\dfrac{a^m}{a^n} = a^{m-n}, a \neq 0$

$(ab)^n = a^n b^n$

$\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}, b \neq 0$

$a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

$a^{-n} = \dfrac{1}{a^n}$

$a^0 = 1, a \neq 0$

2. Sean a, b ,c: bases; m, n números naturales

$$\left(a \pm b\right)^2 = a^2 + 2ab + b^2$$

$$\left(a \pm b\right)^3 = a^3 \pm 3a^2b + 3ab^2 + b^3$$

$$\left(a \pm b\right)^4 = a^4 \pm 4a^3b + 6a^2b^2 \pm 4ab^3 + b^4$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$a^{2n} - b^{2n} = (a^n + b^n)(a^n - b^n)$$

$$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab \pm b^2)$$

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + ac + bc)$$

3. Sean b, n, x, y, z: números naturales

$$\ell og(xyz) = \ell og_b x + \ell og_b y + \ell og_b z$$

$$\ell og_b \left(\frac{x}{y}\right) = \ell og_b x - \ell og_b y$$

$$\ell og_b x^n = n \ell og_b x$$

$$\ell og_b \sqrt[n]{x} = \frac{1}{n} \ell og_b x$$

$$\ell og_b 1 = 0$$

$$\ell og_b b = 1$$

$$\ell \eta e = 1$$

$$\ell \eta \exp x = x = x$$

$$\ell \eta e^x = x$$

$$e^{\ell \eta x} = x$$

$$\exp(\ell \eta x) = x$$

IDENTIDADES TRIGONOMETRICAS

1.

$$\mathrm{sen} = \frac{1}{\cos ec\theta}$$

$$\cos\theta = \frac{1}{\mathrm{s}ec\theta}$$

$$\tau g\theta = \frac{\mathrm{sen}\,\theta}{\cos\theta}$$

$$\tau g\theta = \frac{1}{\mathrm{co}\,\tau g\theta}$$

$$\mathrm{sen}^2\theta + \cos^2\theta = 1$$

$$1 + \tau \mathrm{g}^2\theta = \sec^2\theta$$

$$1 + \mathrm{co}\,\tau \mathrm{g}^2\theta = \cos ec^2\theta$$

$$\cos\theta \cos ec\theta = \mathrm{co}\,\tau g\theta$$

$$\cos\theta\tau g\theta = \mathrm{sen}\,\theta$$

2.

(a)

$$\mathrm{sen}(\alpha+\beta) = \mathrm{sen}\,\alpha\cos\beta + \cos\alpha\,\mathrm{sen}\,\beta$$

$$\mathrm{sen}\,2\alpha = 2\,\mathrm{sen}\,\alpha\cos\alpha$$

$$\mathrm{sen}\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$

$$\mathrm{sen}^2\alpha = \frac{1-\cos 2\alpha}{2}$$

$$\mathrm{sen}(\alpha-\beta) = \mathrm{sen}\,\alpha\cos\beta - \cos\alpha\,\mathrm{sen}\,\beta$$

(b)

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\mathrm{sen}\,\alpha\,\mathrm{sen}\,\beta$$

$$\cos\frac{\alpha}{2}=\pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\cos^2\alpha=\frac{1+\cos 2\alpha}{2}$$

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\mathrm{sen}\,\alpha\,\mathrm{sen}\,\beta$$

$$\cos 2\alpha=\cos^2\alpha-\mathrm{sen}^2\,\alpha=1-2\,\mathrm{sen}^2\,\alpha=2\cos^2\alpha-1$$

(c)

$$\tau g(\alpha+\beta)=\frac{\tau g\alpha+\tau g\beta}{1-\tau g\alpha\tau g\beta}$$

$$\tau g 2\alpha=\frac{2\tau g\alpha}{1-\tau g^2\alpha}$$

$$\tau g^2\alpha=\frac{1-\cos 2\alpha}{1+\cos 2\alpha}$$

$$\tau g(\alpha-\beta)=\frac{\tau g\alpha-\tau g\beta}{1+\tau g\alpha\tau g\beta}$$

$$\tau g\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}=\frac{\mathrm{sen}\,\alpha}{1+\cos\alpha}=\frac{1-\cos\alpha}{\mathrm{sen}\,\alpha}$$

(d)

$$\mathrm{sen}\,\alpha\cos\beta=\frac{1}{2}\left[\mathrm{sen}(\alpha+\beta)+\mathrm{sen}(\alpha-\beta)\right]$$

$$\cos\alpha\,\mathrm{sen}\,\beta=\frac{1}{2}\left[\mathrm{sen}(\alpha+\beta)-\mathrm{sen}(\alpha-\beta)\right]$$

$$\cos\alpha\cos\beta=\frac{1}{2}\left[\cos(\alpha+\beta)+\cos(\alpha-\beta)\right]$$

$$\mathrm{sen}\,\alpha\,\mathrm{sen}\,\beta=-\frac{1}{2}\left[\cos(\alpha+\beta)-\cos(\alpha-\beta)\right]$$

$$\mathrm{sen}\,\alpha+\mathrm{sen}\,\beta=2\,\mathrm{sen}\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\mathrm{sen}\,\alpha-\mathrm{sen}\,\beta=2\cos\frac{\alpha+\beta}{2}\mathrm{sen}\frac{\alpha-\beta}{2}$$

$$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha-\cos\beta=-2\,\mathrm{sen}\frac{\alpha+\beta}{2}\mathrm{sen}\frac{\alpha-\beta}{2}$$

(e)

$$\mathrm{arcsen}(\mathrm{sen}\,x)=x \qquad \mathrm{arc}\cos(\cos x)=x$$

$$\mathrm{arc}\,\tau g(\tau g x)=x \qquad \mathrm{arc}\,\mathrm{co}\,\tau g(\mathrm{co}\,\tau g x)=x$$

$$\mathrm{arc}\sec(\sec x)=x \qquad \mathrm{arc}\,\mathrm{co}\sec(\mathrm{co}\sec x)=x$$

FORMULAS FUNDAMENTALES

Diferenciales

1.- $du = \frac{du}{u} dx$

2.- $d(au) = adu$

3.- $d(u+v) = du + dv$

4.- $d(u^n) = nu^{n-1}du$

5.- $d(\ell \eta u) = \frac{du}{u}$

6.- $d(e^u) = e^u du$

7.- $d(a^u) = a^u \ell \eta a du$

8.- $d(\mathrm{s}e\mathrm{n}\, u) = \cos u du$

9.- $d(\cos u) = -\mathrm{s}e\mathrm{n}\, u du$

10.- $d(\tau g u) = \sec^2 u du$

11.- $d(\mathrm{co}\,\tau g u) = -\mathrm{cosec}^2 u du$

12.- $d(\sec u) = \sec u \tau g u du$

13.- $d(\mathrm{co}\sec u) = -\mathrm{co}\sec u\, \mathrm{co}\,\tau g u du$

14.- $d(\mathrm{arcs}e\mathrm{n}\, u) = \frac{du}{\sqrt{1-u^2}}$

15.- $d(\mathrm{arc}\cos u) = \frac{-du}{\sqrt{1-u^2}}$

16.- $d(\mathrm{arc}\,\tau g u) = \frac{du}{1+u^2}$

17.- $d(\mathrm{arc}\,\mathrm{co}\,\tau g u) = \frac{-du}{1+u^2}$

18.- $d(\mathrm{arc}\sec u) = \frac{du}{u\sqrt{u^2-1}}$

19.- $d(\mathrm{arc}\,\mathrm{co}\sec u) = \frac{-du}{u\sqrt{u^2-1}}$

Integrales

1.- $\int du = u + c$

2.- $\int adu = a\int du$

3.- $\int (du + dv) = \int du + \int dv$

4.- $\int u^n du = \frac{u^{n+1}}{n+1} + c (n \neq -1)$

5.- $\int \frac{du}{u} = \ell \eta |u| + c$

6.- $\int e^u du = e^u + c$

7.- $\int a^u du = \frac{a^u}{\ell \eta a} + c$

8.- $\int \cos u du = \mathrm{s}e\mathrm{n}\, u + c$

9.- $\int \mathrm{s}e\mathrm{n}\, u du = -\cos u + c$

10.- $\int \sec^2 u du = \tau g u + c$

11.- $\int \mathrm{cosec}^2 u du = -\mathrm{co}\,\tau g u + c$

12.- $\int \sec u \tau g u du = \sec u + c$

13.- $\int \mathrm{co}\sec u\, \mathrm{co}\,\tau g u du = -\mathrm{co}\sec u + c$

14.- $\int \frac{du}{\sqrt{1-u^2}} = \mathrm{arcs}e\mathrm{n}\, u + c$

15.- $\int \frac{du}{\sqrt{1-u^2}} = -\mathrm{arc}\cos u + c$

16.- $\int \frac{du}{1+u^2} = \mathrm{arc}\,\tau g u + c$

17.- $\int \frac{du}{1+u^2} = -\mathrm{arc}\,\mathrm{co}\,\tau g u + c$

18.- $\int \frac{du}{u\sqrt{u^2-1}} = \begin{cases} \mathrm{arc}\sec u + c; u > 0 \\ -\mathrm{arc}\sec u + c; u < 0 \end{cases}$

19.- $\int \frac{-du}{u\sqrt{u^2-1}} = \begin{cases} -\mathrm{arc}\,\mathrm{co}\sec u + c; u > 0 \\ \mathrm{arc}\,\mathrm{co}\sec u + c; u < 0 \end{cases}$

OTRAS INTEGRALES INMEDIATAS

1.- $\int \tau gudu = \begin{cases} \ell\eta\left|\sec u\right| + c \\ -\ell\eta\left|\cos u\right| + c \end{cases}$

2.- $\int \operatorname{co}\tau gudu = \ell\eta\left|\operatorname{s}e\operatorname{n}u\right| + c$

3.- $\int \sec udu = \begin{cases} \ell\eta\left|\sec u + \tau gu\right| + c \\ \ell\eta\left|\tau gu\left(\dfrac{u}{2} + \dfrac{\pi}{4}\right)\right| + c \end{cases}$

4.- $\int \operatorname{co}\sec udu = \ell\eta\left|\operatorname{co}\sec u - \operatorname{co}\tau gu\right| + c$

5.- $\int \operatorname{s}e\operatorname{n}hudu = \cos\hbar u + c$

6.- $\int \cos\hbar udu = \operatorname{s}e\operatorname{n}hu + c$

7.- $\int \tau ghudu = \ell\eta\left|\cos\hbar u\right| + c$

8.- $\int \operatorname{co}\tau ghudu = \ell\eta\left|\operatorname{s}e\operatorname{n}\hbar u\right| + c$

9.- $\int \sec hudu = \operatorname{arc}\tau gh(\operatorname{s}e\operatorname{n}hu) + c$

10.- $\int \operatorname{co}\sec hudu = -\operatorname{arc}\operatorname{co}\tau gh(\cos hu) + c$

11.- $\int \dfrac{du}{\sqrt{a^2 - u^2}} = \begin{cases} \operatorname{arcs}e\operatorname{n}\dfrac{u}{a} + c \\ -\operatorname{arcs}e\operatorname{n}\dfrac{u}{a} + c \end{cases}$

12.- $\int \dfrac{du}{\sqrt{u^2 \pm a^2}} = \ell\eta\left|u + \sqrt{u^2 \pm a^2}\right| + c$

13.- $\int \dfrac{du}{u^2 + a^2} = \begin{cases} \dfrac{1}{a}\operatorname{arc}\tau g\dfrac{u}{a} + c \\ \dfrac{1}{a}\operatorname{arc}\operatorname{co}\tau g\dfrac{u}{a} + c \end{cases}$

14.- $\int \dfrac{du}{u^2 - a^2} = \dfrac{1}{2a}\ell\eta\left|\dfrac{u - a}{u + a}\right| + c$

15.- $\int \dfrac{du}{u\sqrt{a^2 \pm u^2}} = \dfrac{1}{a}\ell\eta\left|\dfrac{u}{a + \sqrt{a^2 \pm u^2}}\right| + c$

16.- $\int \dfrac{du}{u\sqrt{u^2 - a^2}} = \begin{cases} \dfrac{1}{a}\operatorname{arc}\cos\dfrac{u}{a} + c \\ \dfrac{1}{a}\operatorname{arc}\sec\dfrac{u}{a} + c \end{cases}$

17.- $\sqrt{u^2 \pm a^2}du = \dfrac{u}{2}\sqrt{u^2 \pm a^2} \pm \dfrac{a^2}{2}\ell\eta\left|u + \sqrt{u^2 \pm a^2}\right| + c$

18.- $\int \sqrt{a^2 - u^2}du = \dfrac{u}{2}\sqrt{a^2 - u^2} + \dfrac{a^2}{2}\operatorname{arcs}e\operatorname{n}\dfrac{u}{a} + c$

19.- $\int e^{au}\operatorname{s}e\operatorname{n}budu = \dfrac{e^{au}(a\operatorname{s}e\operatorname{n}bu - b\cos bu)}{a^2 + b^2} + c$

20.- $\int e^{au}\cos budu = \dfrac{e^{au}(a\cos bu + b\operatorname{s}e\operatorname{n}bu)}{a^2 + b^2} + c$

Realmente, algunas de estas integrales no son estrictamente inmediatas; tal como se verá mas adelante y donde se desarrollan varias de ellas.

CAPITULO 1

INTEGRALES ELEMENTALES

El Propósito de este capitulo, antes de conocer y practicar las técnicas propiamente tales; es familiarizarse con aquellas integrales para las cuales basta una transformación algebraica elemental.

EJERCICIOS DESARROLLADOS

1.1.- Encontrar: $\int e^{\ell\eta x^2} xdx$

Solución.- Se sabe que: $e^{\ell\eta x^2} = x^2$

Por lo tanto: $\int e^{\ell\eta x^2} xdx = \int x^2 xdx = \int x^3 dx = \frac{x^4}{4} + c$

Respuesta: $\int e^{\ell\eta x^2} xdx = \frac{x^4}{4} + c$, Fórmula utilizada: $\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$

1.2 .- Encontrar: $\int 3a^7 x^6 dx$

Solución.-

$$\int 3a^7 x^6 dx = 3a^7 \int x^6 dx = 3a^7 \frac{x^7}{7} + c$$

Respuesta: $\int 3a^7 x^6 dx = 3a^7 \frac{x^7}{7} + c$, Fórmula utilizada: del ejercicio anterior.

1.3.- Encontrar: $\int (3x^2 + 2x + 1)dx$

Solución.-

$$\int (3x^2 + 2x + 1)dx = \int (3x^2 + 2x + 1)dx = \int 3x^2 dx + \int 2xdx + \int dx$$

$$= 3\int x^2 dx + 2\int xdx + \int dx = \cancel{3}\frac{x^3}{\cancel{3}} + \cancel{2}\frac{x^2}{\cancel{2}} + x + c = x^3 + x^2 + x + c$$

Respuesta: $\int (3x^2 + 2x + 1)dx = x^3 + x^2 + x + c$

1.4.- Encontrar: $\int x(x+a)(x+b)dx$

Solución.-

$$\int x(x+a)(x+b)dx = \int x\left[x^2 + (a+b)x + ab\right]dx = \int \left[x^3 + \left(a+b\right)x^2 + abx\right]dx$$

$$= \int x^3 dx + \int (a+b)x^2 dx + \int abxdx = \int x^3 dx + (a+b)\int x^2 dx + ab\int xdx$$

$$= \frac{x^4}{4} + (a+b)\frac{x^3}{3} + ab\frac{x^2}{2} + c$$

Respuesta: $\int x(x+a)(x+b)dx=\frac{x^4}{4}+\frac{(a+b)x^3}{3}+\frac{abx^2}{2}+c$

1.5.- Encontrar: $\int(a+bx^3)^2\,dx$

Solución.-

$$\int(a+bx^3)^2\,dx=\int(a^2+2abx^3+b^2x^6)dx=\int a^2dx+\int 2abx^3dx+\int b^2x^6dx$$

$$=\ a^2\int dx+2ab\int x^3dx+b^2\int x^6dx\ =\ a^2x+2ab\frac{x^4}{4}+b^2\frac{x^7}{7}+c$$

Respuesta: $\int(a+bx^3)^2\,dx=a^2x+\frac{abx^4}{2}+\frac{b^2x^7}{7}+c$

1.6.- Encontrar: $\int\sqrt{2px}dx$

Solución.-

$$\int\sqrt{2px}dx=\int\sqrt{2p}x^{\frac{1}{2}}dx=\sqrt{2p}\int x^{1/2}dx=\sqrt{2p}\,\frac{x^{1/2}}{2/3}+c=\frac{2\sqrt{2p}x^{2/3}}{3}+c$$

Respuesta: $\int\sqrt{2px}dx=\frac{2\sqrt{2p}x\sqrt{x}}{3}+c$

1.7.-Encontrar: $\int\frac{dx}{\sqrt[n]{x}}$

Solución.-

$$\int\frac{dx}{\sqrt[n]{x}}=\int x^{-1/n}dx=\frac{x^{\frac{-1}{n}+1}}{\frac{-1}{n}+1}+c=\frac{x^{\frac{-1+n}{n}}}{\frac{-1+n}{n}}+c=\frac{nx^{\frac{-1+n}{n}}}{n-1}+c$$

Respuesta: $\int\frac{dx}{\sqrt[n]{x}}=\frac{nx^{\frac{-1+n}{n}}}{n-1}+c$

1.8.- Encontrar: $\int(nx)^{\frac{1-n}{n}}dx$

Solución.-

$$\int(nx)^{\frac{1-n}{n}}dx=\int n^{\frac{1-n}{n}}x^{\frac{1-n}{n}}dx=n^{\frac{1-n}{n}}\int x^{\frac{1-n}{n}}dx=n^{\frac{1-n}{n}}\int x^{\frac{1}{n}-1}dx$$

$$=\ =n^{\frac{1-n}{n}}\frac{x^{\frac{1}{n}-1+1}}{\frac{1}{n}-1+1}+c=n^{\frac{1-n}{n}}\frac{x^{\frac{1}{n}}}{\frac{1}{n}}+c=n^{\frac{1-n}{n}}nx^{\frac{1}{n}}+c=n^{\frac{1-n}{n}+1}x^{\frac{1}{n}}+c=n^{\frac{1-n+n}{n}}x^{\frac{1}{n}}+c=n^{1/n}x^{1/n}+c$$

Respuesta: $\int(nx)^{\frac{1-n}{n}}dx=\sqrt[n]{nx}+c$

1.9.- Encontrar: $\int(a^{2/3}-x^{2/3})^3dx$

Solución.-

$$\int(a^{2/3}-x^{2/3})^3dx=\int\left[\left(a^{2/3}\right)^3-3\left(a^{2/2}\right)^2x^{2/3}+3a^{2/3}\left(x^{2/3}\right)^2-\left(x^{2/3}\right)^3\right]dx$$

$$= \int (a^2 - 3a^{4/3}x^{2/3} + 3a^{2/3}x^{4/3} - x^2)dx = \int a^2 dx - \int 3a^{4/3}x^{2/3}dx + \int 3a^{2/3}x^{4/3}dx - \int x^2 dx$$

$$= a^2 \int dx - 3a^{4/3} \int x^{2/3}dx + 3a^{2/3} \int x^{4/3}dx - \int x^2 dx = a^2 x - 3a^{4/3} \frac{x^{5/3}}{5/3} + 3a^{2/3} \frac{x^{7/3}}{7/3} - \frac{x^3}{3} + c$$

$$= a^2 x - \frac{9a^{4/3}x^{5/3}}{5} + \frac{9a^{2/3}x^{7/3}}{7} - \frac{x^3}{3} + c$$

Respuesta: $\int (a^{2/3} - x^{2/3})^3 dx = a^2 x - \frac{9a^{4/3}x^{5/3}}{5} + \frac{9a^{2/3}x^{7/3}}{7} - \frac{x^3}{3} + c$

1.10.- Encontrar: $\int (\sqrt{x} + 1)(x - \sqrt{x} + 1)dx$

Solución.-

$$\int (\sqrt{x} + 1)(x - \sqrt{x} + 1)dx = (x\sqrt{x} - \cancel{(\sqrt{x})^2} + \cancel{\sqrt{x}} + \cancel{x} - \cancel{\sqrt{x}} + 1)dx$$

$$= \int (x\sqrt{x} + 1)dx = \int (xx^{1/2} + 1)dx = \int (x^{3/2} + 1)dx = \int x^{3/2}dx + \int dx = \frac{x^{5/2}}{5/2} + x + c = \frac{2x^{5/2}}{5} + x + c$$

Respuesta: $\int (\sqrt{x} + 1)(x - \sqrt{x} + 1)dx = \frac{2x^{5/2}}{5} + x + c$

1.11.- Encontrar: $\int \frac{(x^2 + 1)(x^2 - 2)dx}{\sqrt[3]{x^2}}$

Solución.-

$$\int \frac{(x^2 + 1)(x^2 - 2)dx}{\sqrt[3]{x^2}} = \int \frac{(x^4 - x^2 - 2)dx}{x^{2/3}} = \int \frac{x^4}{x^{2/3}}dx - \int \frac{x^2}{x^{2/3}}dx - \int \frac{2}{x^{2/3}}dx$$

$$= \int x^{10/3}dx - \int x^{4/3}dx - 2\int x^{-2/3}dx = \frac{x^{\frac{10}{3}+1}}{\frac{10}{3}+1} - \frac{x^{\frac{4}{3}+1}}{\frac{4}{3}+1} - 2\frac{x^{\frac{-2}{3}+1}}{\frac{-2}{3}+1} = \frac{x^{13/3}}{\frac{13}{3}} - \frac{x^{7/3}}{\frac{7}{3}} - 2\frac{x^{1/3}}{1/3} + c$$

$$= 3\frac{x^{13/3}}{13} - 3\frac{x^{7/3}}{7} - 6x^{1/3} + c = 3\frac{\sqrt[3]{x^{13}}}{13} - 3\frac{\sqrt[3]{x^7}}{7} - 6\sqrt[3]{x} + c = 3\frac{x^4\sqrt[3]{x}}{13} - 3\frac{x^2\sqrt[3]{x}}{7} - 6\sqrt[3]{x} + c$$

Respuesta: $\int \frac{(x^2 + 1)(x^2 - 2)dx}{\sqrt[3]{x^2}} = \left(\frac{3x^4}{13} - \frac{3x^2}{7} - 6\right)\sqrt[3]{x} + c$

1.12.- Encontrar: $\int \frac{(x^m - x^n)^2}{\sqrt{x}}dx$

Solución.-

$$\int \frac{(x^m - x^n)^2}{\sqrt{x}}dx = \int \frac{(x^{2m} - 2x^m x^n + x^{2n})}{\sqrt{x}}dx = \int \frac{(x^{2m} - 2x^m x^n + x^{2n})}{x^{1/2}}dx$$

$$= \int (x^{2m-1/2} - 2x^{m+n-1/2} + x^{2n-1/2})dx = \frac{x^{2m-1/2+1}}{2m - 1/2 + 1} - \frac{2x^{m+n+1/2}}{m + n + 1/2} + \frac{x^{2n+1/2}}{2n + 1/2} + c$$

$$= \frac{x^{\frac{4m+1}{2}}}{\frac{4m+1}{2}} - \frac{2x^{\frac{2m+2n+1}{2}}}{\frac{2m+2n+1}{2}} + \frac{x^{\frac{4n+1}{2}}}{\frac{4n+1}{2}} + c = \frac{2x^{\frac{4m+1}{2}}}{4m+1} - \frac{4x^{\frac{2m+2n+1}{2}}}{2m+2n+1} + \frac{2x^{\frac{4n+1}{2}}}{4n+1} + c$$

$$=\frac{2x^{2m}\sqrt{x}}{4m+1}-\frac{4x^{m+n}\sqrt{x}}{2m+2n+1}+\frac{2x^{2n}\sqrt{x}}{4n+1}+c$$

Respuesta: $\int\frac{(x^m-x^n)^2}{\sqrt{x}}dx=\sqrt{x}\left(\frac{2x^{2m}}{4m+1}-\frac{4x^{m+n}}{2m+2n+1}+\frac{2x^{2n}}{4n+1}\right)+c$

1.13.- Encontrar: $\int\frac{(\sqrt{a}-\sqrt{x})^4}{\sqrt{ax}}dx$

Solución.-

$$\int\frac{(\sqrt{a}-\sqrt{x})^4}{\sqrt{ax}}dx=\int\frac{a^2-4a\sqrt{ax}+6xa-4x\sqrt{ax}+x^2}{\sqrt{ax}}dx$$

$$=\int\frac{a^2}{(ax)^{1/2}}dx-\int\frac{4a\cancel{\sqrt{ax}}}{\cancel{\sqrt{ax}}}dx+\int\frac{6ax}{(ax)^{1/2}}dx-\int\frac{4x\cancel{\sqrt{ax}}}{\cancel{\sqrt{ax}}}dx+\int\frac{x^2}{(ax)^{1/2}}dx$$

$$=\int a^2a^{-1/2}x^{-1/2}dx-\int 4adx+\int 6aa^{-1/2}xx^{-1/2}dx-\int 4xdx+\int a^{-1/2}x^2x^{-1/2}dx$$

$$=a^{3/2}\int x^{-1/2}dx-4a\int dx+6a^{1/2}\int x^{1/2}dx-4\int xdx+a^{-1/2}\int x^{3/2}dx$$

$$=a^{3/2}\frac{x^{-1/2+1}}{\frac{-1}{2}+1}-4ax+6a^{1/2}\frac{x^{1/2+1}}{\frac{1}{2}+1}-4\frac{x^{1+1}}{1+1}+a^{-1/2}\frac{x^{3/2+1}}{\frac{3}{2}+1}+c$$

$$=a^{3/2}\frac{x^{1/2}}{\frac{1}{2}}-4ax+6a^{1/2}\frac{x^{3/2}}{\frac{3}{2}}-4\frac{x^2}{2}+a^{-1/2}\frac{x^{5/2}}{\frac{5}{2}}+c$$

$$=2a^{3/2}x^{1/2}-4ax+4a^{1/2}x^{3/2}-2x^2+2a^{-1/2}\frac{x^{5/2}}{5}+c$$

Respuesta: $\int\frac{(\sqrt{a}-\sqrt{x})^4}{\sqrt{ax}}dx=2a^{3/2}x^{1/2}-4ax+4a^{1/2}x^{3/2}-2x^2+\frac{2x^3}{5\sqrt{xa}}+c$

1.14.- Encontrar: $\int\frac{dx}{x^2-10}$

Solución.-

Sea: $a=\sqrt{10}$, Luego: $\int\frac{dx}{x^2-10}=\int\frac{dx}{x^2-a^2}=\frac{1}{2a}\ell\eta\left|\frac{x-a}{x+a}\right|+c$

$$=\frac{1}{2\sqrt{10}}\ell\eta\left|\frac{x-\sqrt{10}}{x+\sqrt{10}}\right|+c=\frac{\sqrt{10}}{20}\ell\eta\left|\frac{x-\sqrt{10}}{x+\sqrt{10}}\right|+c$$

Respuesta: $\int\frac{dx}{x^2-10}=\frac{\sqrt{10}}{20}\ell\eta\left|\frac{x-\sqrt{10}}{x+\sqrt{10}}\right|+c$

1.15.- Encontrar: $\int\frac{dx}{x^2+7}$

Solución.- Sea: a=$\sqrt{7}$, Luego: $\int\frac{dx}{x^2+7}=\int\frac{dx}{x^2+a^2}=\frac{1}{a}\operatorname{arc}\tau g\frac{x}{a}+c$

$$\frac{1}{\sqrt{7}}\operatorname{arc}\tau g\frac{x}{\sqrt{7}}+c=\frac{\sqrt{7}}{7}\operatorname{arc}\tau g\frac{\sqrt{7}x}{a}+c$$

Respuesta: $\int\frac{dx}{x^2+7}=\frac{\sqrt{7}}{7}\operatorname{arc}\tau g\frac{\sqrt{7}x}{a}+c$

1.16.- Encontrar: $\int\sqrt{\frac{dx}{4+x^2}}$

Solución.-

Sea: $a=2$, Luego: $\int\frac{dx}{\sqrt{4+x^2}}=\int\frac{dx}{\sqrt{a^2+x^2}}=\ell\eta\left|x+\sqrt{a^2+x^2}\right|+c$

$$=\ell\eta\left|x+\sqrt{4+x^2}\right|+c$$

Respuesta: $\int\frac{dx}{\sqrt{4+x^2}}=\ell\eta\left|x+\sqrt{4+x^2}\right|+c$

1.17.- Encontrar: $\int\frac{dx}{\sqrt{8-x^2}}$

Solución.-

Sea: $a=\sqrt{8}$, Luego: $\int\frac{dx}{\sqrt{8-x^2}}=\int\frac{dx}{\sqrt{a^2-x^2}}=\operatorname{arcs}e\mathrm{n}\frac{x}{a}+c$

$$=\operatorname{arcs}e\mathrm{n}\frac{x}{\sqrt{8}}+c=\operatorname{arcs}e\mathrm{n}\frac{x}{2\sqrt{2}}+c$$

Respuesta: $\int\frac{dx}{\sqrt{8-x^2}}=\operatorname{arcs}e\mathrm{n}\frac{\sqrt{2}x}{4}+c$

1.18.- Encontrar: $\int\frac{dy}{x^2+9}$

Solución.-

La expresión: $\frac{1}{x^2+9}$ actúa como constante, luego:

$$\int\frac{dy}{x^2+9}=\frac{1}{x^2+9}\int dy=\frac{1}{x^2+9}y+c=\frac{y}{x^2+9}+c$$

Respuesta: $\int\frac{dy}{x^2+9}=\frac{y}{x^2+9}+c$

1.19.- Encontrar: $\int\frac{\sqrt{2+x^2}-\sqrt{2-x^2}}{\sqrt{4-x^4}}dx$

Solución.-

$$\int\frac{\sqrt{2+x^2}-\sqrt{2-x^2}}{\sqrt{4-x^4}}dx=\int\sqrt{\frac{2+x^2}{4-x^4}}dx-\int\sqrt{\frac{2-x^2}{4-x^4}}dx$$

$$=\int\sqrt{\frac{\cancel{2+x^2}}{(2-x^2)\cancel{(2+x^2)}}}dx-\int\sqrt{\frac{\cancel{2-x^2}}{\cancel{(2-x^2)}(2+x^2)}}dx=\int\frac{dx}{\sqrt{2-x^2}}-\int\frac{dx}{\sqrt{2+x^2}}$$

Sea: $a=\sqrt{2}$, Luego: $\int \frac{dx}{\sqrt{a^2-x^2}}-\int \frac{dx}{\sqrt{a^2+x^2}}=\operatorname{arcsen}\frac{x}{a}-\ell\eta\left|x+\sqrt{a^2+x^2}\right|+c$

$$=\operatorname{arcsen}\frac{x}{\sqrt{2}}-\ell\eta\left|x+\sqrt{(\sqrt{2})^2+x^2}\right|+c=\operatorname{arcsen}\frac{x}{\sqrt{2}}-\ell\eta\left|x+\sqrt{2+x^2}\right|+c$$

Respuesta: $\int \frac{\sqrt{2+x^2}-\sqrt{2-x^2}}{\sqrt{4-x^4}}dx=\operatorname{arcsen}\frac{x}{\sqrt{2}}-\ell\eta\left|x+\sqrt{2+x^2}\right|+c$

1.20.- Encontrar: $\int \tau g^2 x dx$

Solución.-

$$\int \tau g^2 x dx=\int(\sec^2 x-1)dx=\int \sec^2 x dx-\int dx=\tau g x-x+c$$

Respuesta: $\int \tau g^2 x dx=\tau g x-x+c$

1.21.- Encontrar: $\int \operatorname{co}\tau g^2 x dx$

Solución.-

$$\int \operatorname{co}\tau g^2 x dx=\int(\operatorname{cos}ec^2 x-1)dx=\int \operatorname{cos}ec^2 x dx-\int dx=-\operatorname{co}\tau g x-x+c$$

Respuesta: $\int \operatorname{co}\tau g^2 x dx=-\operatorname{co}\tau g x-x+c$

1.22.- Encontrar: $\int \frac{dx}{2x^2+4}$

Solución.-

$$\int \frac{dx}{2x^2+4}=\int \frac{dx}{2(x^2+2)}=\frac{1}{2}\int \frac{dx}{x^2+2}=\frac{1}{2}\frac{1}{\sqrt{2}}\operatorname{arc}\tau g\frac{x}{\sqrt{2}}+c \quad =\frac{\sqrt{2}}{4}\operatorname{arc}\tau g\frac{\sqrt{2}x}{2}+c$$

Respuesta: $\int \frac{dx}{2x^2+4}=\frac{\sqrt{2}}{4}\operatorname{arc}\tau g\frac{\sqrt{2}x}{2}+c$

1.23.- Encontrar: $\int \frac{dx}{7x^2-8}$

Solución.-

$$\int \frac{dx}{7x^2-8}=\int \frac{dx}{7(x^2-\frac{8}{7})}=\int \frac{dx}{7\left[(x^2-(\sqrt{\frac{8}{7}})^2\right]}=\frac{1}{7}\int \frac{dx}{\left[x^2-(\sqrt{\frac{8}{7}})^2\right]}$$

$$=\frac{1}{7}\frac{1}{2(\sqrt{\frac{8}{7}})}\ell\eta\left|\frac{x-\sqrt{\frac{8}{7}}}{x+\sqrt{\frac{8}{7}}}\right|+c=\frac{1}{14\frac{\sqrt{8}}{\sqrt{7}}}\ell\eta\left|\frac{x-\sqrt{\frac{8}{7}}}{x+\sqrt{\frac{8}{7}}}\right|+c=\frac{\sqrt{7}}{14\sqrt{8}}\ell\eta\left|\frac{\sqrt{7}x-\sqrt{8}}{\sqrt{7}x+\sqrt{8}}\right|+c$$

$$=\frac{1}{4\sqrt{14}}\ell\eta\left|\frac{\sqrt{7}x-2\sqrt{2}}{\sqrt{7}x+2\sqrt{2}}\right|+c=\frac{\sqrt{14}}{56}\ell\eta\left|\frac{\sqrt{7}x-2\sqrt{2}}{\sqrt{7}x+2\sqrt{2}}\right|+c$$

Respuesta: $\int \frac{dx}{7x^2-8}=\frac{\sqrt{14}}{56}\ell\eta\left|\frac{\sqrt{7}x-2\sqrt{2}}{\sqrt{7}x+2\sqrt{2}}\right|+c$

1.24.- Encontrar: $\int \frac{x^2 dx}{x^2+3}$

Solución.-

$$\int \frac{x^2dx}{x^2+3} = \int (1-\frac{3}{x^2+3})dx = \int dx - 3\int \frac{dx}{x^2+3} = \int dx - 3\int \frac{dx}{x^2+(\sqrt{3})^2}$$

$$= x - 3\frac{1}{\sqrt{3}}\operatorname{arc}\tau g\,\frac{x}{\sqrt{3}} + c == x - \sqrt{3}\operatorname{arc}\tau g\,\frac{\sqrt{3}x}{3} + c$$

Respuesta: $\int \frac{x^2dx}{x^2+3} = x - \sqrt{3}\operatorname{arc}\tau g\,\frac{\sqrt{3}x}{3} + c$

1.25.- Encontrar: $\int \frac{dx}{\sqrt{7+8x^2}}$

Solución.-

$$\int \frac{dx}{\sqrt{7+8x^2}} = \int \frac{dx}{\sqrt{(\sqrt{8}x)^2+(\sqrt{7})^2}} = \frac{1}{\sqrt{8}}\ell\eta\left|\sqrt{8}x+\sqrt{7+8x^2}\right| + c$$

Respuesta: $\int \frac{dx}{\sqrt{7+8x^2}} = \frac{\sqrt{2}}{4}\ell\eta\left|\sqrt{8}x+\sqrt{7+8x^2}\right| + c$

1.26.- Encontrar: $\int \frac{dx}{\sqrt{7-5x^2}}$

Solución.-

$$\int \frac{dx}{\sqrt{7-5x^2}} = \int \frac{dx}{\sqrt{(\sqrt{7})^2-(\sqrt{5}x)^2}} = \frac{1}{\sqrt{5}}\operatorname{arcs}e\operatorname{n} x\frac{\sqrt{5}}{\sqrt{7}} + c$$

Respuesta: $\int \frac{dx}{\sqrt{7-5x^2}} = \frac{\sqrt{5}}{5}\operatorname{arcs}e\operatorname{n}\frac{\sqrt{35}x}{7} + c$

1.27.- Encontrar: $\int \frac{(a^x-b^x)^2dx}{a^xb^x}$

Solución.-

$$\int \frac{(a^x-b^x)^2dx}{a^xb^x} = \int \frac{(a^{2x}-2a^xb^x+b^{2x})}{a^xb^x}dx = \int \frac{a^{2x}}{a^xb^x}dx - \int \frac{2\cancel{a^xb^x}}{\cancel{a^xb^x}}dx + \int \frac{b^{\cancel{2}x}}{a^x\cancel{b^x}}dx$$

$$= \int \frac{a^x}{b^x}dx - \int 2dx + \int \frac{b^x}{a^x}dx = \int \left(\frac{a}{b}\right)^x dx - 2\int dx + \int \left(\frac{b}{a}\right)^x dx = \frac{(a/b)^x}{\ell\eta\frac{a}{b}} - 2x + \frac{(b/a)^x}{\ell\eta\frac{b}{a}} + c$$

$$= \frac{(a/b)^x}{\ell\eta a-\ell\eta b} - 2x + \frac{(b/a)^x}{\ell\eta b-\ell\eta a} + c = \frac{(a/b)^x}{\ell\eta a-\ell\eta b} - 2x - \frac{(b/a)^x}{\ell\eta a-\ell\eta b} + c$$

$$= \frac{\left(\frac{a^x}{b^x}-\frac{b^x}{a^x}\right)}{\ell\eta a-\ell\eta b} - 2x + c$$

Respuesta: $\int \frac{(a^x-b^x)^2dx}{a^xb^x} = \frac{\left(\frac{a^{2x}-b^{2x}}{a^xb^x}\right)}{\ell\eta a-\ell\eta b} - 2x + c$

1.28.- Encontrar: $\int \operatorname{sen}^2 \frac{x}{2} dx$

Solución.-

$$\int \operatorname{sen}^2 \frac{x}{2} dx = \int \frac{1-\cos \cancel{2}\frac{x}{\cancel{2}}}{2} dx = \int \frac{1-\cos x}{2} dx = \frac{1}{2}\int dx - \frac{1}{2}\int \cos x dx$$

$$= \frac{x}{2} - \frac{senx}{2} + c$$

Respuesta: $\int \operatorname{sen}^2 \frac{x}{2} dx = \frac{x}{2} - \frac{senx}{2} + c$

1.29.- Encontrar: $\int \frac{dx}{(a+b)+(a-b)x^2}; (0<b<a)$

Solución.-

Sea: $c^2 = a+b,\ \ d^2 = a-b,$; luego $\int \frac{dx}{(a+b)+(a-b)x^2} = \int \frac{dx}{c^2+d^2x^2}$

$$\int \frac{dx}{d^2\left(\frac{c^2}{d^2}+x^2\right)} = \frac{1}{d^2}\int \frac{dx}{\left(\frac{c}{d}\right)^2+x^2} = \frac{1}{d^{\cancel{2}}}\frac{1}{\frac{c}{\cancel{d}}} arctg \frac{x}{c/d} + c = \frac{1}{cd} arctg \frac{dx}{c} + c$$

$$= \frac{1}{\sqrt{a+b}\sqrt{a-b}} arctg \frac{\sqrt{a-b}x}{\sqrt{a+b}} + c = \frac{1}{\sqrt{a^2-b^2}} arctg \sqrt{\frac{a-b}{a+b}}x + c$$

Respuesta: $\int \frac{dx}{(a+b)+(a-b)x^2} = \frac{1}{\sqrt{a^2-b^2}} arctg \sqrt{\frac{a-b}{a+b}}x + c$

1.30.-Encontrar: $\int \frac{dx}{(a+b)-(a-b)x^2}; (0<b<a)$

Solución.-

Sea: $c^2 = a+b,\ d^2 = a-b,$ Luego: $\int \frac{dx}{(a+b)-(a-b)x^2} = \int \frac{dx}{c^2-d^2x^2}$

$$= \int \frac{dx}{d^2\left(\frac{c^2}{d^2}-x^2\right)} = \frac{1}{d^2}\int \frac{dx}{\left(\frac{c}{d}\right)^2-x^2} = -\frac{1}{d^{\cancel{2}}}\frac{1}{\frac{2c}{\cancel{d}}}\ell\eta\left|\frac{x-c/d}{x+c/d}\right| + c = -\frac{1}{2cd}\ell\eta\left|\frac{dx-c}{dx+c}\right| + c$$

$$= -\frac{1}{2\sqrt{a^2-b^2}}\ell\eta\left|\frac{\sqrt{a-b}x-\sqrt{a+b}}{\sqrt{a-b}x+\sqrt{a+b}}\right| + c$$

Respuesta: $\int \frac{dx}{(a+b)-(a-b)x^2} = -\frac{1}{2\sqrt{a^2-b^2}}\ell\eta\left|\frac{\sqrt{a-b}x-\sqrt{a+b}}{\sqrt{a-b}x+\sqrt{a+b}}\right| + c$

1.31.- Encontrar: $\int \left[\left(a^{2x}\right)^0 - 1\right] dx$

Solución.-

$$\int\left[\left(a^{2x}\right)^{0}-1\right]dx=\int(a^{0}-1)dx=\int(1-1)dx=\int dx-\int dx=\int 0dx=c$$

Respuesta: $\int\left[\left(a^{2x}\right)^{0}-1\right]dx=c$

EJERCICIOS PROPUESTOS

Mediante el uso del álgebra elemental, o algunas identidades trigonométricas, transformar en integrales de fácil solución, las integrales que se presentan a continuación.

1.32.- $\int 3x^5dx$ **1.33.-** $\int(1+e)^x dx$ **1.34.-** $\int(1+\tau gx)dx$

1.35.- $\int\cos^2\frac{x}{2}dx$ **1.36.-** $\int(1+\sqrt{x})^3dx$ **1.37.-** $\int(1+\sqrt{x})^0dx$

1.38.- $\int\frac{1+\frac{\sqrt{x}}{2}}{1+\frac{\sqrt{x}}{3}}dy$ **1.39.-** $\int\frac{dx}{\sqrt{5-x^2}}$ **1.40.-** $\int\frac{dx}{\sqrt{x^2-5}}$

1.41.- $\int\frac{dx}{\sqrt{x^2+5}}$ **1.42.-** $\int\frac{dx}{x^2+5}$ **1.43.-** $\int\frac{dx}{x^2-5}$

1.44.- $\int(\mathrm{sen}^2x+\cos^2x-1)dx$ **1.45.-** $\int\sqrt{x}(1-\sqrt{x})dx$ **1.46.-** $\int(\tau g^2x+1)dx$

1.47.- $\int\frac{dx}{x^2-12}$ **1.48.-** $\int\frac{dx}{x^2+12}$ **1.49.-** $\int\frac{dx}{\sqrt{x^2-12}}$

1.50.- $\int\frac{dx}{\sqrt{x^2+12}}$ **1.51.-** $\int\frac{dx}{\sqrt{12-x^2}}$ **1.52.-** $\int\frac{dx}{x\sqrt{x^2-12}}$

1.53.- $\int\frac{dx}{x\sqrt{12-x^2}}$ **1.54.-** $\int\frac{dx}{x\sqrt{12+x^2}}$ **1.55.-** $\int\frac{dx}{\sqrt{8-2x^2}}$

1.56.- $\int\frac{dx}{\sqrt{2x^2-8}}$ **1.57.-** $\int\frac{dx}{\sqrt{2x^2+8}}$ **1.58.-** $\int\sqrt{x^2-10}dx$

1.59.- $\int\sqrt{x^2+10}dx$ **1.60.-** $\int\sqrt{10-x^2}dx$ **1.61.-** $\int\frac{1-\cos^2x}{\mathrm{sen}^2x}dx$

1.62.- $\int\sqrt{1-\mathrm{sen}^2x}dx$ **1.63.-** $\int\sqrt{1-\cos^2x}dx$ **1.64.-** $\int(2^x-3^x)^0dx$

1.65.- $\int(2^0-3^0)^n dx$ **1.66.-** $\int\left(\tau gx-\frac{\mathrm{sen}\,x}{\cos x}\right)dx$ **1.67.-** $\int\frac{dx}{3^{-x}}$

1.68.- $\int\sqrt{\frac{3}{4}-x^2}dx$ **1.69.-** $\int\sqrt{x^2-\frac{3}{4}}dx$ **1.70.-** $\int\sqrt{x^2+\frac{3}{4}}dx$

1.71.- $\int\frac{dx}{x\sqrt{3-x^2}}$ **1.72.-** $\int\frac{dx}{x\sqrt{x^2-3}}$ **1.73.-** $\int\frac{dx}{x\sqrt{x^2+3}}$

1.74.- $\int\mathrm{sen}^{3x}\theta dy$ **1.75.-** $\int\ell\eta|u|dx$ **1.76.-** $\int\exp(\ell\eta x)dx$

1.77.- $\int e^{\ell\eta x^2}dx$ **1.78.-** $\int\frac{\sqrt{x}-\sqrt{2}}{\sqrt{2x}}dx$ **1.79.-** $\int\sqrt{11-x^2}dx$

1.80.- $\int\sqrt{x^2-11}dx$ **1.81.-** $\int\sqrt{x^2+11}dx$ **1.82.-** $\int\ell\eta(e^{\sqrt{x}})dx$

1.83.- $\int\left[\frac{1+\sqrt{x}+\sqrt{x^3}}{1-\sqrt{x}}\right]^0 dx$ **1.84.-** $\int(\tau g^2x+\sec^2 x-1)dx$ **1.85.-** $\int\frac{dx}{\sqrt{3x^2-1}}$

1.86.- $\int(\mathrm{co}\,\tau g\theta-\mathrm{sen}\,\theta)dx$ **1.87.-** $\int\frac{dx}{\sqrt{1+3x^2}}$ **1.88.-** $\int\frac{dx}{\sqrt{1-3x^2}}$

1.89.- $\int\frac{dx}{1+3x^2}$ **1.90.-** $\int\frac{dx}{3x^2+4}$ **1.91.-** $\int\frac{dx}{3x^2-1}$

1.92.- $\int\frac{dx}{x\sqrt{3x^2-1}}$ **1.93.-** $\int\frac{dx}{x\sqrt{1+3x^2}}$ **1.94.-** $\int\frac{dx}{x\sqrt{1-3x^2}}$

1.95.- $\int\sqrt{1-3x^2}dx$ **1.96.-** $\int\sqrt{1+3x^2}dx$ **1.97.-** $\int\sqrt{3x^2-1}dx$

1.98.- $\int(3x^2-1)dx$ **1.99.-** $\int(3x^2-1)^0dx$ **1.100.-** $\int(3x^2-1)^n du$

1.101.- $\int\exp(\ell\eta\frac{\sqrt{x}}{3})dx$ **1.102.-** $\int\ell\eta(e^{\frac{2x-1}{2}})dx$ **1.103.-** $\int(e^2+e+1)^x dx$

1.104.- $\int\left(\frac{1+\tau g^2x}{\sec^2 x}-1\right)dx$ **1.105.-** $\int\exp(\ell\eta\left|1+x\right|)dx$ **1.106.-** $\int\sqrt{27-x^2}dx$

1.107.- $\int\sqrt{x^2-27}dx$ **1.108.-** $\int\sqrt{x^2+27}dx$ **1.109.-** $\int\frac{dx}{3x\sqrt{x^2-1}}$

1.110.- $\int\frac{dx}{2x\sqrt{1-x^2}}$ **1.111.-** $\int\frac{dx}{5x\sqrt{x^2+1}}$ **1.112.-** $\int\frac{dx}{3x\sqrt{9-x^2}}$

1.113.- $\int\frac{dx}{4x\sqrt{x^2+16}}$ **1.114.-** $\int\frac{dx}{5x\sqrt{x^2-25}}$ **1.115.-** $\int\frac{(1-\sqrt{x})^2}{x^2}dx$

1.116.- $\int(1+\sqrt{x}+x)^2dx$ **1.117.-** $\int(1-\sqrt{x}+x)^2dx$ **1.118.-** $\int(1+x)^4dx$

1.119.- $\int e^{\ell\eta\left|\frac{1-\cos x}{2}\right|}dx$ **1.120.-** $\int\exp\ell\eta\left(\frac{1+x^2}{x^2}\right)dx$ **1.121.-** $\int\ell\eta e^{\frac{1-\mathrm{sen}\,x}{3}}dx$

1.122.- $\int(1+\sqrt{x-3x})^0dx$ **1.123.-** $\int\ell\eta e^{\frac{(1+x)^2}{2}}dx$

RESPUESTAS

1.32.- $\int 3x^5dx=3\int x^5dx=\frac{3x^{5+1}}{5+1}+c=3\frac{x^6}{6}+c=\frac{x^6}{2}+c$

1.33.- $\int(1+e)^x dx$

Sea: $a=1+e,$ Luego: $\int(1+e)^x dx=\int a^x dx=\frac{a^x}{\ell\eta a}+c=\frac{(1+e)^x}{\ell\eta(1+e)}+c$

1.34.- $\int(1+\tau gx)dx=\int dx+\int\tau gxdx=x+\ell\eta\left|\sec x\right|+c$

1.35.- $\int\cos^2\frac{x}{2}dx=\int\frac{1+\cos x}{2}dx=\frac{1}{2}\int dx+\frac{1}{2}\int\cos xdx=\frac{1}{2}x+\frac{1}{2}\mathrm{sen}\,x+c$

1.36.- $\int(1+\sqrt{x})^3dx=\int(1+3\sqrt{x}+3(\cancel{\sqrt{x^2}})+\sqrt{x^3})dx=\int dx+3\sqrt{x}+3\int xdx+\int x^{3/2}dx$

$=x+2x^{3/2}+3\dfrac{x^2}{2}+\dfrac{2}{5}x^{5/2}+c=x+2x\sqrt{x}+3\dfrac{x^2}{2}+\dfrac{2}{5}x^2\sqrt{x}+c$

1.37.- $\int(1+\sqrt{x})^0dx=\int dx=x+c$

1.38.- $\int\dfrac{1+\frac{\sqrt{x}}{2}}{1+\frac{\sqrt{x}}{3}}dy=\dfrac{1+\frac{\sqrt{x}}{2}}{1+\frac{\sqrt{x}}{3}}\int dy=\dfrac{1+\frac{\sqrt{x}}{2}}{1+\frac{\sqrt{x}}{3}}y+c$

1.39.- $\int\dfrac{dx}{\sqrt{5-x^2}}$

Sea: $a=\sqrt{5}$, Luego: $\int\dfrac{dx}{\sqrt{5-x^2}}=\int\dfrac{dx}{\sqrt{(\sqrt{5})^2-x^2}}=\operatorname{arcs}e\operatorname{n}\dfrac{x}{\sqrt{5}}+c=\operatorname{arcs}e\operatorname{n}\dfrac{\sqrt{5}x}{5}+c$

1.40.- $\int\dfrac{dx}{\sqrt{x^2-5}}=\int\dfrac{dx}{\sqrt{x^2-(\sqrt{5})^2}}=\ell\eta\left|x+\sqrt{x^2-5}\right|+c$

1.41.- $\int\dfrac{dx}{\sqrt{x^2+5}}=\int\dfrac{dx}{\sqrt{x^2+(\sqrt{5})^2}}=\ell\eta\left|x+\sqrt{x^2+5}\right|+c$

1.42.- $\int\dfrac{dx}{x^2+5}$

Sea: $a=\sqrt{5}$, Luego: $\int\dfrac{dx}{x^2+(\sqrt{5})^2}=\dfrac{1}{\sqrt{5}}\operatorname{arc}\tau g\dfrac{x}{\sqrt{5}}+c$

$=\dfrac{\sqrt{5}}{5}\operatorname{arc}\tau g\dfrac{\sqrt{5}x}{5}+c$

1.43.- $\int\dfrac{dx}{x^2-5}=\int\dfrac{dx}{x^2-(\sqrt{5})^2}=\dfrac{1}{2\sqrt{5}}\ell\eta\left|\dfrac{x-\sqrt{5}}{x+\sqrt{5}}\right|+c=\dfrac{\sqrt{5}}{10}\ell\eta\left|\dfrac{x-\sqrt{5}}{x+\sqrt{5}}\right|+c$

1.44.- $\int(\operatorname{s}e\operatorname{n}^2x+\cos^2x-1)dx=\int(1-1)dx=\int 0dx=c$

1.45.- $\int\sqrt{x}(1-\sqrt{x})dx=\int(\sqrt{x}-x)dx=\int\sqrt{x}dx-\int xdx=\dfrac{2}{3}x^{3/2}-\dfrac{x^2}{2}+c$

1.46.- $\int(\tau g^2x+1)dx=\int\sec^2xdx=\tau gx+c$

1.47.- $\int\dfrac{dx}{x^2-12}=\int\dfrac{dx}{x^2-(\sqrt{12})^2}=\dfrac{1}{2\sqrt{12}}\ell\eta\left|\dfrac{x-\sqrt{12}}{x+\sqrt{12}}\right|+c=\dfrac{1}{4\sqrt{3}}\ell\eta\left|\dfrac{x-2\sqrt{3}}{x+2\sqrt{3}}\right|+c$

$=\dfrac{\sqrt{3}}{12}\ell\eta\left|\dfrac{x-2\sqrt{3}}{x+2\sqrt{3}}\right|+c$

1.48.- $\int\dfrac{dx}{x^2+12}$

Sea: $a=\sqrt{12}$, Luego: $\int\dfrac{dx}{x^2+(\sqrt{12})^2}=\dfrac{1}{\sqrt{12}}\operatorname{arc}\tau g\dfrac{x}{\sqrt{12}}+c$

$$=\frac{1}{2\sqrt{3}}\operatorname{arc}\tau g\,\frac{x}{2\sqrt{3}}+c=\frac{\sqrt{3}}{6}\operatorname{arc}\tau g\,\frac{\sqrt{3}x}{6}+c$$

1.49.- $\int\frac{dx}{\sqrt{x^2-12}}=\int\frac{dx}{\sqrt{x^2-(\sqrt{12})^2}}=\ell\eta\left|x+\sqrt{x^2-12}\right|+c$

1.50.- $\int\frac{dx}{\sqrt{x^2+12}}=\int\frac{dx}{\sqrt{x^2+(\sqrt{12})^2}}=\ell\eta\left|x+\sqrt{x^2+12}\right|+c$

1.51.- $\int\frac{dx}{\sqrt{12-x^2}}$

Sea: $a=\sqrt{12}$,Luego: $\int\frac{dx}{\sqrt{12-x^2}}=\int\frac{dx}{\sqrt{(\sqrt{12})^2-x^2}}$

$$=\operatorname{arcs}e\operatorname{n}\frac{x}{\sqrt{12}}+c=\operatorname{arcs}e\operatorname{n}\frac{x}{2\sqrt{3}}+c=\operatorname{arcs}e\operatorname{n}\frac{\sqrt{3}x}{6}+c$$

1.52.- $\int\frac{dx}{x\sqrt{x^2-12}}=\int\frac{dx}{x\sqrt{x^2-(\sqrt{12})^2}}=\frac{1}{\sqrt{12}}\operatorname{arcsec}\frac{x}{\sqrt{12}}+c=\frac{1}{2\sqrt{3}}\operatorname{arcsec}\frac{x}{2\sqrt{3}}+c$

$$=\frac{\sqrt{3}}{6}\operatorname{arcsec}\frac{\sqrt{3}x}{6}+c$$

1.53.- $\int\frac{dx}{x\sqrt{12-x^2}}=\int\frac{dx}{x\sqrt{(\sqrt{12})^2-x^2}}=\frac{1}{\sqrt{12}}\ell\eta\left|\frac{x}{\sqrt{12}+\sqrt{12-x^2}}\right|+c$

$$=\frac{\sqrt{3}}{6}\ell\eta\left|\frac{x}{\sqrt{12}+\sqrt{12-x^2}}\right|+c$$

1.54.- $\int\frac{dx}{x\sqrt{12+x^2}}=\frac{\sqrt{3}}{6}\ell\eta\left|\frac{x}{\sqrt{12}+\sqrt{12+x^2}}\right|+c$

1.55.- $\int\frac{dx}{\sqrt{8-2x^2}}=\int\frac{dx}{\sqrt{2(4-x^2)}}=\frac{1}{\sqrt{2}}\int\frac{dx}{\sqrt{4-x^2}}=\frac{1}{\sqrt{2}}\operatorname{arcs}e\operatorname{n}\frac{x}{2}+c=\frac{\sqrt{2}}{2}\operatorname{arcs}e\operatorname{n}\frac{x}{2}+c$

1.56.- $\int\frac{dx}{\sqrt{2x^2-8}}=\int\frac{dx}{\sqrt{2(x^2-4)}}=\frac{1}{\sqrt{2}}\int\frac{dx}{\sqrt{x^2-4}}=\frac{1}{\sqrt{2}}\ell\eta\left|x+\sqrt{x^2-4}\right|+c$

$$=\frac{\sqrt{2}}{2}\ell\eta\left|x+\sqrt{x^2-4}\right|+c$$

1.57.- $\int\frac{dx}{\sqrt{2x^2+8}}=\int\frac{dx}{\sqrt{2(x^2+4)}}=\frac{1}{\sqrt{2}}\int\frac{dx}{\sqrt{x^2+4}}=\frac{1}{\sqrt{2}}\ell\eta\left|x+\sqrt{x^2+4}\right|+c$

$$=\frac{\sqrt{2}}{2}\ell\eta\left|x+\sqrt{x^2+4}\right|+c$$

1.58.- $\int\sqrt{x^2-10}dx=\int\sqrt{x^2-(\sqrt{10})^2}dx=\frac{x}{2}\sqrt{x^2-10}-\frac{10}{2}\ell\eta\left|x+\sqrt{x^2-10}\right|+c$

$$=\frac{x}{2}\sqrt{x^2-10}-5\ell\eta\left|x+\sqrt{x^2-10}\right|+c$$

1.59.- $\int\sqrt{x^2+10}dx=\frac{x}{2}\sqrt{x^2+10}+5\ell\eta\left|x+\sqrt{x^2+10}\right|+c$

1.60.- $\int\sqrt{10-x^2}dx=\int\sqrt{(\sqrt{10})^2-x^2}dx=\frac{x}{2}\sqrt{10-x^2}+\frac{10}{2}\operatorname{arcsen}\frac{x}{\sqrt{10}}+c$

$$=\frac{x}{2}\sqrt{10-x^2}+5\operatorname{arcsen}\frac{\sqrt{10}x}{10}+c$$

1.61.- $\int\frac{1-\cos^2 x}{\operatorname{sen}^2 x}dx=\int\frac{\operatorname{sen}^2 x}{\operatorname{sen}^2 x}dx=\int dx=x+c$

1.62.- $\int\sqrt{1-\operatorname{sen}^2 x}dx=\int\sqrt{\cos^2 x}dx=\int\cos x dx=\operatorname{sen} x+c$

1.63.- $\int\sqrt{1-\cos^2 x}dx=\int\sqrt{\operatorname{sen}^2 x}dx=\int\operatorname{sen} x dx=-\cos x+c$

1.64.- $\int(2^x-3^x)^0 dx=\int dx=x+c$

1.65.- $\int(2^0-3^0)^n dx=\int(0)^n dx=\int 0dx=c$

1.66.- $\int\left(\tau gx-\frac{\operatorname{sen} x}{\cos x}\right)dx=\int(\tau gx-\tau gx)dx=\int 0dx=c$

1.67.- $\int\frac{dx}{3^{-x}}=\int 3^x dx=\frac{3^x}{\ell\eta 3}+c$

1.68.- $\int\sqrt{\frac{3}{4}-x^2}dx=\int\sqrt{(\frac{\sqrt{3}}{2})^2-x^2}dx=\frac{x}{2}\sqrt{\frac{3}{4}-x^2}+\frac{3/4}{2}\operatorname{arcsen}\frac{x}{\sqrt{3}/2}+c$

$$=\frac{x}{2}\sqrt{\frac{3}{4}-x^2}+\frac{3}{8}\operatorname{arcsen}\frac{2x}{\sqrt{3}}+c$$

1.69.- $\int\sqrt{x^2-\frac{3}{4}}dx=\int\sqrt{x^2-(\frac{\sqrt{3}}{2})^2}dx=\frac{x}{2}\sqrt{x^2-\frac{3}{4}}-\frac{3/4}{2}\ell\eta\left|x+\sqrt{x^2-\frac{3}{4}}\right|+c$

$$=\frac{x}{2}\sqrt{x^2-\frac{3}{4}}-\frac{3}{8}\ell\eta\left|x+\sqrt{x^2-\frac{3}{4}}\right|+c$$

1.70.- $\int\sqrt{x^2+\frac{3}{4}}dx=\int\sqrt{x^2+(\frac{\sqrt{3}}{2})^2}dx=\frac{x}{2}\sqrt{x^2+\frac{3}{4}}+\frac{3}{8}\ell\eta\left|x+\sqrt{x^2+\frac{3}{4}}\right|+c$

1.71.- $\int\frac{dx}{x\sqrt{3-x^2}}=\int\frac{dx}{x\sqrt{(\sqrt{3})^2-x^2}}=\frac{1}{\sqrt{3}}\ell\eta\left|\frac{x}{\sqrt{3}+\sqrt{3-x^2}}\right|+c$

$$=\frac{\sqrt{3}}{3}\ell\eta\left|\frac{x}{\sqrt{3}+\sqrt{3-x^2}}\right|+c$$

1.72.- $\int\frac{dx}{x\sqrt{x^2-3}}=\frac{1}{\sqrt{3}}\operatorname{arcsec}\frac{x}{\sqrt{3}}+c=\frac{\sqrt{3}}{3}\operatorname{arcsec}\frac{\sqrt{3}x}{3}+c$

1.73.- $\int\frac{dx}{x\sqrt{x^2+3}}=\frac{\sqrt{3}}{3}\ell\eta\left|\frac{x}{\sqrt{3}+\sqrt{x^2+3}}\right|+c$

1.74.- $\int (\operatorname{sen}^{3x}\theta)dy = \operatorname{sen}^{3x}\theta \int dy = (\operatorname{sen}^{3x}\theta)y + c$

1.75.- $\int \ell\eta|u|dx = \ell\eta|u|\int dx = \ell\eta|u|x + c$

1.76.- $\int \exp(\ell\eta x)dx = \int x dx = \frac{x^2}{2} + c$

1.77.- $\int e^{\ell\eta x^2}dx = \int x^2 dx = \frac{x^3}{3} + c$

1.78.- $\int \frac{\sqrt{x}-\sqrt{2}}{\sqrt{2x}}dx = \int \frac{\sqrt{x}}{\sqrt{2x}}dx - \int \frac{\sqrt{2}}{\sqrt{2x}}dx = \int \sqrt{\frac{\cancel{x}}{2\cancel{x}}}dx - \int \sqrt{\frac{\cancel{2}}{\cancel{2}x}}dx = \frac{1}{\sqrt{2}}\int dx - \int \frac{1}{\sqrt{x}}dx =$

$= \frac{1}{\sqrt{2}}\int dx - \int x^{-1/2}dx = \frac{1}{\sqrt{2}}x - \frac{x^{1/2}}{1/2} + c = \frac{\sqrt{2}}{2}x - 2x^{1/2} + c$

1.79.- $\int \sqrt{11-x^2}dx = \frac{x}{2}\sqrt{11-x^2} + \frac{11}{2}\operatorname{arcsen}\frac{x}{\sqrt{11}} + c = \frac{x}{2}\sqrt{11-x^2} + \frac{11}{2}\operatorname{arcsen}\frac{\sqrt{11}x}{11} + c$

1.80.- $\int \sqrt{x^2-11}dx = \frac{x}{2}\sqrt{x^2-11} - \frac{11}{2}\ell\eta\left|x+\sqrt{x^2-11}\right| + c$

1.81.- $\int \sqrt{x^2+11}dx = \frac{x}{2}\sqrt{x^2+11} + \frac{11}{2}\ell\eta\left|x+\sqrt{x^2+11}\right| + c$

1.82.- $\int \ell\eta(e^{\sqrt{x}})dx = \int \sqrt{x}dx = \int x^{1/2}dx = \frac{x^{3/2}}{3/2} + c = \frac{2}{3}x\sqrt{x} + c$

1.83.- $\int \left[\frac{1+\sqrt{x}+\sqrt{x^3}}{1-\sqrt{x}}\right]^0 dx = \int dx = x + c$

1.84.- $\int (\tau g^2 x + \sec^2 x - 1)dx = \int 0 dx = c$

1.85.- $\int \frac{dx}{\sqrt{3x^2-1}} = \int \frac{dx}{\sqrt{3}\sqrt{(x^2-1/3)}} = \frac{1}{\sqrt{3}}\int \frac{dx}{\sqrt{(x^2-1/3)}} = \frac{1}{\sqrt{3}}\ell\eta\left|x+\sqrt{(x^2-1/3)}\right| + c$

$= \frac{\sqrt{3}}{3}\ell\eta\left|x+\sqrt{(x^2-1/3)}\right| + c$

1.86.- $\int (\operatorname{co}\tau g\theta - \operatorname{sen}\theta)dx = (\operatorname{co}\tau g\theta - \operatorname{sen}\theta)\int dx = (\operatorname{co}\tau g\theta - \operatorname{sen}\theta)x + c$

1.87.- $\int \frac{dx}{\sqrt{1+3x^2}} = \int \frac{dx}{\sqrt{3}\sqrt{\frac{1}{3}+x^2}} = \frac{\sqrt{3}}{3}\ell\eta\left|x+\sqrt{\frac{1}{3}+x^2}\right| + c$

1.88.- $\int \frac{dx}{\sqrt{1-3x^2}} = \int \frac{dx}{\sqrt{3}\sqrt{\frac{1}{3}-x^2}} = \frac{1}{\sqrt{3}}\int \frac{dx}{\sqrt{\frac{1}{3}-x^2}} = \frac{1}{\sqrt{3}}\operatorname{arcsen}\frac{x}{\frac{1}{\sqrt{3}}} + c$

$= \frac{\sqrt{3}}{3}\operatorname{arcsen}\sqrt{3}x + c$

1.89.- $\int \frac{dx}{1+3x^2} = \int \frac{dx}{3(\frac{1}{3}+x^2)} = \frac{1}{3}\int \frac{dx}{\frac{1}{3}+x^2} = \frac{1}{3}\frac{1}{\frac{1}{\sqrt{3}}}\operatorname{arc}\tau g\frac{x}{\frac{1}{\sqrt{3}}} + c = \frac{\sqrt{3}}{3}\operatorname{arc}\tau g\sqrt{3}x + c$

1.90.- $\int \frac{dx}{3x^2+4} = \frac{1}{3}\int \frac{dx}{x^2+\frac{4}{3}} = \frac{1}{3}\frac{1}{\frac{2}{\sqrt{3}}}\operatorname{arc}\tau g\,\frac{x}{\frac{2}{\sqrt{3}}} + c = \frac{\sqrt{3}}{6}\operatorname{arc}\tau g\,\frac{\sqrt{3}x}{2} + c$

1.91.- $\int \frac{dx}{3x^2-1} = \frac{1}{3}\int \frac{dx}{x^2-\frac{1}{3}} = \frac{1}{3}\frac{1}{2\frac{1}{\sqrt{3}}}\ell\eta\left|\frac{x-\frac{1}{\sqrt{3}}}{x+\frac{1}{\sqrt{3}}}\right| + c = \frac{\sqrt{3}}{6}\ell\eta\left|\frac{\sqrt{3}x-1}{\sqrt{3}x+1}\right| + c$

1.92.- $\int \frac{dx}{x\sqrt{3x^2-1}} = \int \frac{dx}{\sqrt{3}x\sqrt{x^2-\frac{1}{3}}} = \frac{1}{\sqrt{3}}\int \frac{dx}{x\sqrt{x^2-\frac{1}{3}}} = \frac{1}{\cancel{\sqrt{3}}}\frac{1}{1/\cancel{\sqrt{3}}}\operatorname{arc}\sec\frac{x}{1/\sqrt{3}} + c$

$= \operatorname{arc}\sec\sqrt{3}x + c$

1.93.- $\int \frac{dx}{x\sqrt{1+3x^2}} = \frac{1}{\sqrt{3}}\int \frac{dx}{x\sqrt{\frac{1}{3}+x^2}} = \frac{1}{\cancel{\sqrt{3}}}\frac{1}{1/\cancel{\sqrt{3}}}\ell\eta\left|\frac{x}{\frac{1}{\sqrt{3}}+\sqrt{\frac{1}{3}+x^2}}\right| + c$

$= \ell\eta\left|\frac{x}{\frac{1}{\sqrt{3}}+\sqrt{\frac{1}{3}+x^2}}\right| + c$

1.94.- $\int \frac{dx}{x\sqrt{1-3x^2}} = \frac{1}{\sqrt{3}}\int \frac{dx}{x\sqrt{\frac{1}{3}-x^2}} = \ell\eta\left|\frac{x}{\frac{1}{\sqrt{3}}+\sqrt{\frac{1}{3}-x^2}}\right| + c$

1.95.- $\int \sqrt{1-3x^2}\,dx = \sqrt{3}\int \sqrt{\frac{1}{3}-x^2}\,dx = \sqrt{3}\left[\frac{x}{2}\sqrt{\frac{1}{3}-x^2} + \frac{\frac{1}{3}}{2}\operatorname{arcs}e\,\mathrm{n}\,\frac{x}{1/\sqrt{3}}\right] + c$

$= \sqrt{3}\left[\frac{x}{2}\sqrt{\frac{1}{3}-x^2} + \frac{1}{6}\operatorname{arcs}e\,\mathrm{n}\,\sqrt{3}x\right] + c$

1.96.- $\int \sqrt{1+3x^2}\,dx = \sqrt{3}\int \sqrt{\frac{1}{3}+x^2}\,dx = \sqrt{3}\left[\frac{x}{2}\sqrt{\frac{1}{3}+x^2} + \frac{\frac{1}{3}}{2}\ell\eta\left|x+\sqrt{\frac{1}{3}+x^2}\right|\right] + c$

$= \sqrt{3}\left[\frac{x}{2}\sqrt{\frac{1}{3}+x^2} + \frac{1}{6}\ell\eta\left|x+\sqrt{\frac{1}{3}+x^2}\right|\right] + c$

1.97.- $\int \sqrt{3x^2-1}dx = \sqrt{3}\int \sqrt{x^2-\frac{1}{3}}dx = \sqrt{3}\left[\frac{x}{2}\sqrt{x^2-\frac{1}{3}} - \frac{1}{6}\ell\eta\left|x+\sqrt{x^2-\frac{1}{3}}\right|\right] + c$

1.98.- $\int (3x^2-1)dx = 3\int x^2dx - \int dx = x^3 - x + c$

1.99.- $\int (3x^2-1)^0 dx = \int dx = x + c$

1.100.- $\int (3x^2-1)^n du = (3x^2-1)^n \int du = (3x^2-1)^n u + c$

1.101.- $\int \exp(\ell\eta\,\frac{\sqrt{x}}{3})dx = \int \frac{\sqrt{x}}{3}dx = \frac{1}{3}\int x^{1/2}dx = \frac{1}{3}\frac{x^{3/2}}{3/2} + c = \frac{2}{9}x^{3/2} + c$

1.102.- $\int \ell\eta(e^{\frac{2x-1}{2}})dx = \int \frac{2x-1}{2}dx = \int xdx - \frac{1}{2}\int dx = \frac{x^2}{2} - \frac{1}{2}x + c$

1.103.- $\int (e^2+e+1)^x dx$

Sea: a= (e^2+e+1) , Luego: $\int a^x dx = \dfrac{a^x}{\ell\eta a} + c = \dfrac{(e^2+e-1)^x}{\ell\eta(e^2+e-1)} + c$

1.104.- $\int\left(\dfrac{1+\tau g^2 x}{\sec^2 x} - 1\right)dx = \int(1-1)dx = \int 0dx = c$

1.105.- $\int \exp(\ell\eta|1+x|)dx\int = \int(1+x)dx = \int dx + \int xdx = x + \dfrac{x^2}{2} + c$

1.106.- $\int\sqrt{27-x^2}\,dx = \dfrac{x}{2}\sqrt{27-x^2} + \dfrac{27}{2}\operatorname{arc\,sen}\dfrac{x}{3\sqrt{3}} + c$

1.107.- $\int\sqrt{x^2-27}\,dx = \dfrac{x}{2}\sqrt{x^2-27} - \dfrac{27}{2}\ell\eta\left|x+\sqrt{x^2-27}\right| + c$

1.108.- $\int\sqrt{x^2+27}\,dx = \dfrac{x}{2}\sqrt{x^2+27} + \dfrac{27}{2}\ell\eta\left|x+\sqrt{x^2+27}\right| + c$

1.109.- $\int\dfrac{dx}{3x\sqrt{x^2-1}} = \dfrac{1}{3}\int\dfrac{dx}{x\sqrt{x^2-1}} = \dfrac{1}{3}\operatorname{arc} sec x + c$

1.110.- $\int\dfrac{dx}{2x\sqrt{1-x^2}} = \dfrac{1}{2}\int\dfrac{dx}{x\sqrt{1-x^2}} = \dfrac{1}{2}\ell\eta\left|\dfrac{x}{1+\sqrt{1-x^2}}\right| + c$

1.111.- $\int\dfrac{dx}{5x\sqrt{x^2+1}} = \dfrac{1}{5}\int\dfrac{dx}{x\sqrt{x^2+1}} = \dfrac{1}{5}\ell\eta\left|\dfrac{x}{1+\sqrt{x^2+1}}\right| + c$

1.112.- $\int\dfrac{dx}{3x\sqrt{9-x^2}} = \dfrac{1}{3}\int\dfrac{dx}{x\sqrt{9-x^2}} = \dfrac{1}{3}\dfrac{1}{3}\ell\eta\left|\dfrac{x}{3+\sqrt{9-x^2}}\right| + c = \dfrac{1}{9}\ell\eta\left|\dfrac{x}{3+\sqrt{9-x^2}}\right| + c$

1.113.- $\int\dfrac{dx}{4x\sqrt{x^2+16}} = \dfrac{1}{4}\int\dfrac{dx}{x\sqrt{x^2+16}} = \dfrac{1}{4}\dfrac{1}{4}\ell\eta\left|\dfrac{x}{4+\sqrt{x^2+16}}\right| + c$

$= \dfrac{1}{16}\ell\eta\left|\dfrac{x}{4+\sqrt{x^2+16}}\right| + c$

1.114.- $\int\dfrac{dx}{5x\sqrt{x^2-25}} = \dfrac{1}{5}\int\dfrac{dx}{x\sqrt{x^2-25}} = \dfrac{1}{5}\dfrac{1}{5}\operatorname{arc} sec\dfrac{x}{5} + c = \dfrac{1}{25}\operatorname{arc} sec\dfrac{x}{5} + c$

1.115.- $\int\dfrac{(1-\sqrt{x})^2}{x^2}dx = \int\dfrac{1-2\sqrt{x}+x}{x^2}dx = \int(x^{-2} - 2x^{-3/2} + x^{-1})dx$

$= \int x^{-2}dx - \int 2x^{-3/2}dx + \int x^{-1}dx = -x^{-1} - 2\dfrac{x^{-1/2}}{-1/2} + \ell\eta|x| + c = -x^{-1} - 2\dfrac{x^{-1/2}}{-1/2} + \ell\eta|x| + c$

$= -x^{-1} + 4x^{-1/2} + \ell\eta|x| + c = -\dfrac{1}{x} + \dfrac{4}{\sqrt{x}} + \ell\eta|x| + c$

1.116.- $\int(1+\sqrt{x}+x)^2 dx = (1+x+x^2+2\sqrt{x}+2x+2x^{3/2})dx$

$= \int(1+2x^{1/2}+3x+2x^{3/2}+x^2)dx = \int dx + 2\int x^{1/2}dx + 3\int xdx + 2\int x^{3/2}dx + \int x^2dx$

$x + \dfrac{2x^{3/2}}{3/2} + 3\dfrac{x^2}{2} + 2\dfrac{x^{5/2}}{5/2} + \dfrac{x^3}{3} + c = x + \dfrac{4x^{3/2}}{3} + 3\dfrac{x^2}{2} + 4\dfrac{x^{5/2}}{5} + \dfrac{x^3}{3} + c$

1.117.- $\int (1-\sqrt{x}+x)^2\,dx = \int (1+x+x^2-2\sqrt{x}+2x-2x^{3/2})dx$

$= \int (1-2x^{1/2}+3x-2x^{3/2}+x^2)dx = x-\frac{4x^{3/2}}{3}+3\frac{x^2}{2}-4\frac{x^{5/2}}{5}+\frac{x^3}{3}+c$

1.118.- $\int (1+x)^4\,dx = \int (1+4x+6x^2+4x^3+x^4)dx$

$= \int dx+4\int xdx+6\int x^2dx+4\int x^3dx+\int x^4dx = x+2x^2+2x^3+x^4+\frac{1}{5}x^5+c$

1.119.- $\int e^{\ell\eta\left|\frac{1-\cos x}{2}\right|}dx = \int \frac{1-\cos x}{2}dx = \frac{1}{2}\int dx-\frac{1}{2}\int \cos xdx = \frac{1}{2}x-\frac{1}{2}\mathrm{s}e\mathrm{n}\,xdx$

1.120.- $\int \exp\ell\eta\left(\frac{1+x^2}{x^2}\right)dx = \int \frac{1+x^2}{x^2}dx = \int \frac{1}{x^2}dx+\int dx = \int x^{-2}dx+\int dx = -\frac{1}{x}+x+c$

1.121.- $\int \ell\eta e^{\frac{1-\mathrm{s}e\mathrm{n}\,x}{3}}\,dx = \int \frac{1-\mathrm{s}e\mathrm{n}\,x}{3}dx = \frac{1}{3}\int dx-\frac{1}{3}\int \mathrm{s}e\mathrm{n}\,xdx = \frac{1}{3}x+\frac{1}{3}\cos x+c$

1.122.- $\int (1+\sqrt{x-3x})^0dx = \int dx = x+c$

1.123.- $\int \ell\eta e^{\frac{(1+x)^2}{2}}dx = \int \frac{(1+x)^2}{2}dx = \int \frac{1+2x+x^2}{2}dx = \frac{1}{2}\int dx+\int xdx+\frac{1}{2}\int x^2dx$

$= \frac{1}{2}x+\frac{x^2}{2}+\frac{x^3}{6}+c$

CAPITULO 2

INTEGRACION POR SUSTITUCION

A veces es conveniente hacer un cambio de variable, para transformar la integral dada en otra, de forma conocida. La técnica en cuestión recibe el nombre de método de sustitución.

EJERCICIOS DESARROLLADOS

2.1.-Encontrar: $\int \frac{e^{\ell\eta x}dx}{x^2+7}$

Solución.- Como: $e^{\ell\eta x}$ = x, se tiene: $\int \frac{e^{\ell\eta x}dx}{x^2+7} = \int \frac{xdx}{x^2+7}$

Sea la sustitución: $u = x^2+7$, donde: $du = 2xdx$, Dado que: $\int \frac{xdx}{x^2+7} = \frac{1}{2}\int \frac{2xdx}{x^2+7}$,

Se tiene: $\frac{1}{2}\int \frac{2xdx}{x^2+7} = \frac{1}{2}\int \frac{du}{u}$, integral que es inmediata.

Luego: $= \frac{1}{2}\int \frac{du}{u}\frac{1}{2}\ell\eta|u| + c = \frac{1}{2}\ell\eta\left|x^2+7\right| + c$

Respuesta: $\int \frac{e^{\ell\eta x}dx}{x^2+7} = \frac{1}{2}\ell\eta\left|x^2+7\right| + c$

2.2.-Encontrar: $\int \frac{e^{\ell\eta x^2}dx}{x^3+8}$

Solución.- Como: $e^{\ell\eta x^2} = x^2$, se tiene: $\int \frac{e^{\ell\eta x^2}dx}{x^3+8} = \int \frac{x^2dx}{x^3+8}$

Sea la sustitución: $w = x^3+8$, donde: $dw = 3x^2dx$, Dado que: $\int \frac{x^2dx}{x^3+8} = \frac{1}{3}\int \frac{3x^2dx}{x^3+8}$,

Se tiene: $\frac{1}{3}\int \frac{3x^2dx}{x^3+8} = \frac{1}{3}\int \frac{dw}{w}$ integral que es inmediata.

Luego: $\frac{1}{3}\int \frac{dw}{w} = \frac{1}{3}\ell\eta|w| + c = \frac{1}{3}\ell\eta\left|x^3+8\right| + c$

Respuesta: $\int \frac{e^{\ell\eta x^2}dx}{x^3+8} = \frac{1}{3}\ell\eta\left|x^3+8\right| + c$

2.3.-Encontrar: $\int (x+2)\,\mathrm{s}e\mathrm{n}(x^2+4x-6)dx$

Solución.- Sea la sustitución: $u = x^2+4x-6$, donde: $du = (2x+4)dx$

Dado que: $\int (x+2)\,\mathrm{s}e\mathrm{n}(x^2+4x-6)dx = \frac{1}{2}\int (2x+4)\,\mathrm{s}e\mathrm{n}(x^2+4x-6)dx$**,** se tiene:

$= \frac{1}{2}\int (2x+4)\operatorname{sen}(x^2+4x-6)dx = \frac{1}{2}\int \operatorname{sen} u du$, integral que es inmediata.

Luego: $= \frac{1}{2}\int \operatorname{sen} u du = \frac{1}{2}(-\cos u) + c = -\frac{1}{2}\cos u + c = -\frac{1}{2}\cos(x^2+4x-6) + c$

Respuesta: $\int (x+2)\operatorname{sen}(x^2+4x-6)dx = -\frac{1}{2}\cos(x^2+4x-6) + c$

2.4.-Encontrar: $\int x \operatorname{sen}(1-x^2)dx$

Solución.-Sea la sustitución: $w = 1-x^2$, donde: $dw = -2xdx$

Dado que: $\int x \operatorname{sen}(1-x^2)dx = -\frac{1}{2}\int (-2x)\operatorname{sen}(1-x^2)dx$

Se tiene que: $-\frac{1}{2}\int (-2x)\operatorname{sen}(1-x^2)dx = -\frac{1}{2}\operatorname{sen} w dw$, integral que es inmediata.

Luego: $-\frac{1}{2}\int \operatorname{sen} w dw = -\frac{1}{2}(-\cos w)dw + c = \frac{1}{2}\cos w + c = \frac{1}{2}\cos(1-x^2) + c$

Respuesta: $\int x \operatorname{sen}(1-x^2)dx = \frac{1}{2}\cos(1-x^2) + c$

2.5.-Encontrar: $\int x \operatorname{co}\tau g(x^2+1)dx$

Solución.-Sea la sustitución: $u = x^2+1$, donde: $du = 2xdx$

Dado que: $\int x \operatorname{co}\tau g(x^2+1)dx = \frac{1}{2}\int 2x \operatorname{co}\tau g(x^2+1)dx$

Se tiene que: $\frac{1}{2}\int 2x \operatorname{co}\tau g(x^2+1)dx = \frac{1}{2}\int \operatorname{co}\tau g u du$, integral que es inmediata.

Luego: $\frac{1}{2}\int \operatorname{co}\tau g u du = \frac{1}{2}\ell\eta\left|\operatorname{sen} u\right| + c = \frac{1}{2}\ell\eta\left|\operatorname{sen}(x^2+1)\right| + c$

Respuesta: $\int x \operatorname{co}\tau g(x^2+1)dx = \frac{1}{2}\ell\eta\left|\operatorname{sen}(x^2+1)\right| + c$

2.6.-Encontrar: $\int \sqrt{1+y^4}\, y^3 dy$

Solución.-Sea la sustitución: $w = 1+y^4$, donde: $dw = 4y^3 dy$

Dado que: $\int \sqrt{1+y^4}\, y^3 dy = \frac{1}{4}\int (1+y^4)^{1/2} 4y^3 dy$

Se tiene que: $\frac{1}{4}\int (1+y^4)^{1/2} 4y^3 dy = \frac{1}{4}\int w^{1/2} dw$, integral que es inmediata.

Luego: $\frac{1}{4}\int w^{1/2} dw = \frac{1}{4}\frac{w^{3/2}}{\frac{3}{2}} + c = \frac{1}{6}w^{3/2} + c = \frac{1}{6}(1+y^4)^{3/2} + c$

Respuesta: $\int \sqrt{1+y^4}\, y^3 dy = \frac{1}{6}(1+y^4)^{3/2} + c$

2.7.-Encontrar: $\int \frac{3tdt}{\sqrt[3]{t^2+3}}$

Solución.-Sea la sustitución: $u = t^2+3$, donde: $du = 2tdt$

Dado que: $\int \frac{3tdt}{\sqrt[3]{t^2+3}} = \frac{3}{2}\int \frac{2tdt}{(t^2+3)^{1/3}}$

Se tiene que: $\frac{3}{2}\int \frac{2tdt}{(t^2+3)^{1/3}} = \frac{3}{2}\int \frac{du}{u^{1/3}}$, integral que es inmediata

Luego: $\frac{3}{2}\int \frac{du}{u^{1/3}} = \frac{3}{2}\int u^{-1/3}du = \frac{3}{2}\frac{u^{2/3}}{\frac{2}{3}} + c = \frac{9}{4}u^{2/3} + c = \frac{9}{4}(t^2+3)^{2/3} + c$

Respuesta: $\int \frac{3tdt}{\sqrt[3]{t^2+3}} = \frac{9}{4}(t^2+3)^{2/3} + c$

2.8.-Encontrar: $\int \frac{dx}{(a+bx)^{1/3}}$**,** a y b constantes.

Solución.- Sea: $w = a+bx$, donde: $dw = bdx$

Luego: $\int \frac{dx}{(a+bx)^{1/3}} = \frac{1}{b}\int \frac{bdx}{(a+bx)^{1/3}} = \frac{1}{b}\int \frac{dw}{w^{1/3}} = \frac{1}{b}\int w^{-1/3} = \frac{1}{b}\frac{w^{2/3}}{\frac{2}{3}} + c = \frac{3}{2b}w^{2/3} + c$

$= \frac{3}{2b}(a+bx)^{2/3} + c$

Respuesta: $\int \frac{dx}{(a+bx)^{1/3}} = \frac{3}{2b}(a+bx)^{2/3} + c$

2.9.-Encontrar: $\int \frac{\sqrt{\operatorname{arcs}e\operatorname{n} x}}{1-x^2}dx$

Solución.- $\int \frac{\sqrt{\operatorname{arcs}e\operatorname{n} x}}{1-x^2}dx = \int \sqrt{\operatorname{arcs}e\operatorname{n} x}\,\frac{dx}{\sqrt{1-x^2}}$,

Sea: $u = \operatorname{arcs}e\operatorname{n} x$, donde: $du = \frac{dx}{\sqrt{1-x^2}}$

Luego: $\int \sqrt{\operatorname{arcs}e\operatorname{n} x}\,\frac{dx}{\sqrt{1-x^2}} = \int u^{1/2}du = \frac{2}{3}u^{3/2} + c = \frac{2}{3}\sqrt{(\operatorname{arcs}e\operatorname{n} x)^3} + c$

Respuesta: $\int \frac{\sqrt{\operatorname{arcs}e\operatorname{n} x}}{1-x^2}dx = \frac{2}{3}\sqrt{(\operatorname{arcs}e\operatorname{n} x)^3} + c$

2.10.-Encontrar: $\int \frac{\operatorname{arc}\tau g\,\frac{x}{2}}{4+x^2}dx$

Solución.- Sea: $w = \operatorname{arc}\tau g\,\frac{x}{2}$, donde: $dw = \frac{1}{1+(\frac{x}{2})^2}(\frac{1}{2})dx = \frac{2dx}{4+x^2}$

Luego: $\int \frac{\operatorname{arc}\tau g\,\frac{x}{2}}{4+x^2}dx = \frac{1}{2}\int \operatorname{arc}\tau g\left(\frac{x}{2}\right)\frac{2dx}{4+x^2} = \frac{1}{2}\int wdw = \frac{1}{4}w^2 + c = \frac{1}{4}\left(\operatorname{arc}\tau g\,\frac{x}{2}\right)^2 + c$

Respuesta: $\int \frac{\operatorname{arc}\tau g\,\frac{x}{2}}{4+x^2}dx = \frac{1}{4}\left(\operatorname{arc}\tau g\,\frac{x}{2}\right)^2 + c$

2.11.-Encontrar: $\int \frac{x-\operatorname{arc}\tau g 2x}{1+4x^2}dx$

Solución.- $\int \frac{x-\operatorname{arc}\tau g 2x}{1+4x^2}dx = \int \frac{xdx}{1+4x^2} - \int \frac{\sqrt{\operatorname{arc}\tau g 2x}}{1+4x^2}$

Sea: $u = 1+4x^2$, donde: $du = 8xdx$; $w = \operatorname{arc}\tau g 2x$, donde: $dw = \frac{2dx}{1+4x^2}$

Luego: $\int \frac{xdx}{1+4x^2} - \int \frac{\sqrt{\operatorname{arc}\tau g 2x}}{1+4x^2} = \frac{1}{8}\int \frac{8xdx}{1+4x^2} - \frac{1}{2}\int \sqrt{\operatorname{arc}\tau g 2x}\,\frac{2dx}{1+4x^2}$

$= \frac{1}{8}\int \frac{du}{u} - \frac{1}{2}\int w^{1/2}dw = \frac{1}{8}\ell\eta|u| - \frac{1}{3}w^{3/2} + c = \frac{1}{8}\ell\eta\left|1+4x^2\right| - \frac{1}{3}(\operatorname{arc}\tau g 2x)^{3/2} + c$

Respuesta: $\int \frac{x-\operatorname{arc}\tau g 2x}{1+4x^2}dx = \frac{1}{8}\ell\eta\left|1+4x^2\right| - \frac{1}{3}(\operatorname{arc}\tau g 2x)^{3/2} + c$

2.12.-Encontrar: $\int \frac{dx}{\sqrt{(1+x^2)\ell\eta\left|x+\sqrt{1+x^2}\right|}}$

Solución.- $\int \frac{dx}{\sqrt{(1+x^2)\ell\eta\left|x+\sqrt{1+x^2}\right|}} = \int \frac{dx}{\sqrt{1+x^2}\sqrt{\ell\eta\left|x+\sqrt{1+x^2}\right|}}$

Sea: $u = \ell\eta\left|x+\sqrt{1+x^2}\right|$, donde: $du = \frac{1}{x+\sqrt{1+x^2}}(1+\frac{2x}{2\sqrt{1+x^2}}) \Rightarrow du = \frac{dx}{\sqrt{1+x^2}}$

Luego: $\int \frac{dx}{\sqrt{1+x^2}\sqrt{\ell\eta\left|x+\sqrt{1+x^2}\right|}} = \int \frac{du}{\sqrt{u}} = \int u^{-1/2}du = 2u^{1/2} + c = 2\sqrt{\ell\eta\left|x+\sqrt{1+x^2}\right|} + c$

Respuesta: $\int \frac{dx}{\sqrt{(1+x^2)\ell\eta\left|x+\sqrt{1+x^2}\right|}} = 2\sqrt{\ell\eta\left|x+\sqrt{1+x^2}\right|} + c$

2.13.-Encontrar: $\int \frac{\operatorname{co}\tau g(\ell\eta x)}{x}dx$

Solución.- Sea: $w = \ell\eta x$, donde: $dw = \frac{dx}{x}$

Luego: $\int \frac{\operatorname{co}\tau g(\ell\eta x)}{x}dx = \int \operatorname{co}\tau g w dw = \ell\eta\left|\operatorname{sen} w\right| + c = \ell\eta\left|\operatorname{sen}(\ell\eta x)\right| + c$

Respuesta: $\int \frac{\operatorname{co}\tau g(\ell\eta x)}{x}dx = \ell\eta\left|\operatorname{sen}(\ell\eta x)\right| + c$

2.14.-Encontrar: $\int \frac{dx}{x(\ell\eta x)^3}$

Solución.- Sea: $u = \ell\eta x$, donde: $du = \frac{dx}{x}$

Luego: $\int \frac{dx}{x(\ell\eta x)^3} = \int \frac{du}{u^3} = \int u^{-3}du = \frac{u^{-2}}{2} + c = \frac{1}{2u^2} + c = \frac{1}{2(\ell\eta x)^2} + c$

Respuesta: $\int \frac{dx}{x(\ell\eta x)^3} = \frac{1}{2(\ell\eta x)^2} + c$

2.15.-Encontrar: $\int \frac{e^{1/x^2}}{x^3}dx$

Solución.- Sea: $w = \frac{1}{x^2}$, donde: $dw = -\frac{2}{x^3}dx$

Luego: $\int \frac{e^{1/x^2}}{x^3}dx = -\frac{1}{2}\int e^{1/x^2}\frac{-2dx}{x^3} = -\frac{1}{2}\int e^w dw = -\frac{1}{2}e^w + c = -\frac{1}{2}e^{1/x^2} + c$

Respuesta: $\int \frac{e^{1/x^2}}{x^3}dx = -\frac{1}{2}e^{1/x^2} + c$

2.16.-Encontrar: $\int e^{-x^2+2}xdx$

Solución.- Sea: $u = -x^2 + 2$, donde: $du = -2xdx$

Luego: $\int e^{-x^2+2}xdx = -\frac{1}{2}\int e^{-x^2+2}(-2xdx) = -\frac{1}{2}\int e^u du = -\frac{1}{2}e^u + c = -\frac{1}{2}e^{-x^2+2} + c$

Respuesta: $\int e^{-x^2+2}xdx = -\frac{1}{2}e^{-x^2+2} + c$

2.17.-Encontrar: $\int x^2e^{x^3}dx$

Solución.- Sea: $w = x^3$, donde: $dw = 3x^2dx$

Luego: $\int x^2e^{x^3}dx = \frac{1}{3}\int 3x^2e^{x^3}dx = \frac{1}{3}\int e^w dw = \frac{1}{3}e^{x^3} + c$

Respuesta: $\int x^2e^{x^3}dx = \frac{1}{3}e^{x^3} + c$

2.18.-Encontrar: $\int (e^x+1)^2e^xdx$

Solución.- Sea: $u = e^x + 1$, donde: $du = e^xdx$

Luego: $\int (e^x+1)^2e^xdx = \int u^2du = \frac{u^3}{3} + c = \frac{(e^x+1)^3}{3} + c$

Respuesta: $\int (e^x+1)^2e^xdx = \frac{(e^x+1)^3}{3} + c$

2.19.-Encontrar: $\int \frac{e^x-1}{e^x+1}dx$

Solución.- $\int \frac{e^x-1}{e^x+1}dx = \int \frac{e^x}{e^x+1}dx - \int \frac{1}{e^x+1}dx = \int \frac{e^x}{e^x+1}dx - \int \frac{e^xe^{-x}}{e^x+1}dx$

$= \int \frac{e^x}{e^x+1}dx - \int \frac{e^{-x}}{e^{-x}(e^x+1)}dx = \int \frac{e^x}{e^x+1}dx - \int \frac{e^{-x}}{1+e^x}dx$

Sea: $u = e^x + 1$ **,** donde: $du = e^xdx$; $w = 1 + e^{-x}$,donde: $dw = -e^{-x}dx$

Luego: $\int \frac{e^x}{e^x+1}dx - \int \frac{e^{-x}}{1+e^x}dx = \int \frac{e^x}{e^x+1}dx - \int \frac{-e^{-x}}{1+e^{-x}}dx = \int \frac{du}{u} + \int \frac{dw}{w}$

$= \ell\eta|u| + c_1 + \ell\eta|w| + c_2 = \ell\eta|e^x+1| + \ell\eta|1+e^{-x}| + C = \ell\eta\left[|e^x+1||1+e^{-x}|\right] + c$

Respuesta: $\int \frac{e^x-1}{e^x+1}dx = \ell\eta\left[|(e^x+1)(1+e^{-x})|\right] + c$, otra respuesta seria:

$\int \frac{e^x-1}{e^x+1}dx = \ell\eta|e^x+1|^2 - x + c$

2.20.-Encontrar: $\int \frac{e^{2x}-1}{e^{2x}+3}dx$

Solución.- $\int \frac{e^{2x}-1}{e^{2x}+3}dx = \int \frac{e^{2x}}{e^{2x}+3}dx - \int \frac{e^0}{e^{2x}+3}dx$

$= \int \frac{e^{2x}}{e^{2x}+3}dx - \int \frac{e^{2x}e^{-2x}}{e^{2x}+3}dx = \int \frac{e^{2x}}{e^{2x}+3}dx - \int \frac{e^{-2x}}{e^{-2x}(e^{2x}+3)}dx = \int \frac{e^{2x}}{e^{2x}+3}dx - \int \frac{e^{-2x}}{1+3e^{-2x}}dx$

Sea: $u = e^{2x}+3$, donde: $du = 2e^{2x}dx$; $w = 1+3e^{-2x}$,donde: $dw = -6e^{-2x}dx$

Luego: $\int \frac{e^{2x}}{e^{2x}+3}dx - \int \frac{e^{-2x}}{1+3e^{-2x}}dx = \frac{1}{2}\int \frac{2e^{2x}}{e^{2x}+3}dx + \frac{1}{6}\int \frac{-6e^{-2x}}{1+3e^{-2x}}dx = \frac{1}{2}\int \frac{du}{u} + \frac{1}{6}\int \frac{dw}{w}$

$\frac{1}{2}\ell\eta|u| + \frac{1}{6}\ell\eta|w| + c = \frac{1}{2}\ell\eta|e^{2x}+3| + \frac{1}{6}\ell\eta|1+3e^{-2x}| + c = \frac{1}{2}\ell\eta|e^{2x}+3| + \frac{1}{6}\ell\eta\left|1+\frac{3}{e^{2x}}\right| + c$

$= \frac{1}{2}\ell\eta|e^{2x}+3| + \frac{1}{6}\ell\eta\left|\frac{e^{2x}+3}{e^{2x}}\right| + c = \frac{1}{2}\ell\eta|e^{2x}+3| + \frac{1}{6}\ell\eta|e^{2x}+3| - \frac{1}{6}\ell\eta e^{2x} + c$

$= \ell\eta\left(e^{2x}+3\right)^{1/2} + \ell\eta\left(e^{2x}+3\right)^{1/6} - \frac{1}{6}2x + c = \ell\eta\left[\left(e^{2x}+3\right)^{1/2}\left(e^{2x}+3\right)^{1/6}\right] - \frac{x}{3} + c$

$= \ell\eta\left(e^{2x}+3\right)^{2/3} - \frac{x}{3} + c$

Respuesta: $\int \frac{e^{2x}-1}{e^{2x}+3}dx = \ell\eta\left(e^{2x}+3\right)^{2/3} - \frac{x}{3} + c$

2.22.-Encontrar: $\int \frac{x^2+1}{x-1}dx$

Solución.- Cuando el grado del polinomio dividendo es MAYOR o IGUAL que el grado del polinomio divisor, es necesario efectuar previamente la división de polinomios. El resultado de la división dada es:

$$\frac{x^2+1}{x-1} = (x+1) + \frac{2}{x-1}, \text{ Luego: } \int \frac{x^2+1}{x-1}dx = \int\left(x+1+\frac{2}{x-1}\right)dx = \int xdx + \int dx + 2\int \frac{dx}{x-1}$$

Sea $u = x-1$, donde $du = dx$

Luego: $\int xdx + \int dx + 2\int \frac{dx}{x-1} = \int xdx + \int dx + 2\int \frac{du}{u} = \frac{x^2}{2} + x + \ell\eta|x-1| + c$

Respuesta: $\int \frac{x^2+1}{x-1}dx = \frac{x^2}{2} + x + \ell\eta|x-1| + c$

2.23.-Encontrar: $\int \frac{x+2}{x+1}dx$

Solución.- $\frac{x+2}{x+1}=1+\frac{1}{x+1}$, Luego: $\int\frac{x+2}{x+1}dx=\int\left(1+\frac{1}{x+1}\right)dx=\int dx+\int\frac{dx}{x+1}$

Sea $u=x+1$, donde $du=dx$

$\int dx+\int\frac{du}{u}=x+\ell\eta|u|+c=x+\ell\eta|x+1|+c$

Respuesta: $\int\frac{x+2}{x+1}dx=x+\ell\eta|x+1|+c$

2.24.-Encontrar: $\int\tau g^5x\sec^2 xdx$

Solución.- Sea: $w=\tau gx$, donde: $dw=\sec^2 x$

Luego: $\int\tau g^5x\sec^2 xdx=\int(\tau gx)^5\sec^2 xdx=\int w^5dw=\frac{w^6}{6}+c=\frac{(\tau gx)^6}{6}+c=\frac{\tau g^6x}{6}+c$

Respuesta: $\int\tau g^5x\sec^2 xdx=\frac{\tau g^6x}{6}+c$

2.25.-Encontrar: $\int\operatorname{sen} x\sec^2 xdx$

Solución.- $\int\operatorname{sen} x\sec^2 xdx=\int\operatorname{sen} x\frac{1}{\cos^2 x}dx=\int\frac{\operatorname{sen} x}{\cos^2 x}dx$

Sea: $u=\cos x$, donde: $du=-\operatorname{sen} x$

Luego: $\int\frac{\operatorname{sen} x}{\cos^2 x}dx=-\int\frac{-\operatorname{sen} xdx}{\cos^2 x}=-\int\frac{du}{u}=-\int u^{-2}du=-\frac{u^{-1}}{-1}+c=\frac{1}{u}+c=\frac{1}{\cos x}+c$

Respuesta: $\int\operatorname{sen} x\sec^2 xdx=\sec x+c$

2.26.-Encontrar: $\int\frac{\sec^2 3xdx}{1+\tau g3x}$

Solución.- Sea: $u=1+\tau g3xdx$, donde: $du=3\sec^2 3xdx$

Luego: $\int\frac{\sec^2 3xdx}{1+\tau g3x}=\frac{1}{3}\int\frac{3\sec^2 3xdx}{1+\tau g3x}=\frac{1}{3}\int\frac{du}{u}=\frac{1}{3}\ell\eta|u|+c=\frac{1}{3}\ell\eta|1+\tau g3x|+c$

Respuesta: $\int\frac{\sec^2 3xdx}{1+\tau g3x}=\frac{1}{3}\ell\eta|1+\tau g3x|+c$

2.27.-Encontrar: $\int\operatorname{sen}^3 x\cos xdx$

Solución.- Sea: $w=\operatorname{sen} x$, donde: $dw=\cos xdx$

Luego: $\int\operatorname{sen}^3 x\cos xdx=\int(\operatorname{sen} x)^3\cos xdx=\int w^3dw=\int\frac{w^4}{4}+c=\int\frac{\operatorname{sen}^4 x}{4}+c$

Respuesta: $\int\operatorname{sen}^3 x\cos xdx=\int\frac{\operatorname{sen}^4 x}{4}+c$

2.28.-Encontrar: $\int\cos^4 x\operatorname{sen} xdx$

Solución.- Sea: $u=\cos x$, donde: $du=-\operatorname{sen} x$

Luego: $\int\cos^4 x\operatorname{sen} xdx=\int(\cos x)^4\operatorname{sen} xdx=-\int(\cos x)^4(-\operatorname{sen} x)dx=-\int u^4du$

$$= -\frac{u^5}{5} + c = -\frac{\cos x^5}{5} + c = -\frac{\cos^5 x}{5} + c$$

Respuesta: $\int \cos^4 x \operatorname{sen} x dx = -\frac{\cos^5 x}{5} + c$

2.29.-Encontrar: $\int \frac{\sec^5}{\cos ec x} dx$

Solución.- $\int \frac{\sec^5}{\cos ec x} dx = \int \frac{\frac{1}{\cos^5 x}}{\frac{1}{\operatorname{sen} x}} dx = \int \frac{\operatorname{sen} x}{(\cos x)^5} dx$

Sea: $w = \cos x$, donde: $dw = -\operatorname{sen} x dx$

Luego: $\int \frac{\operatorname{sen} x}{(\cos x)^5} dx = -\int \frac{dw}{w^5} = -\int w^{-5} dw = -\frac{w^{-4}}{-4} + c = \frac{1}{4}\frac{1}{w^4} + c = \frac{1}{4\cos^4 x} + c$

$$= \frac{\sec^4 x}{4} + c$$

Respuesta: $\int \frac{\sec^5}{\cos ec x} dx = \frac{\sec^4 x}{4} + c$

2.30.-Encontrar: $\int e^{\tau g 2x} \sec^2 2x dx$

Solución.- Sea: $u = \tau g 2x$, donde: $du = 2\sec^2 2x dx$

Luego: $\int e^{\tau g 2x} \sec^2 2x dx = \frac{1}{2}\int e^{\tau g 2x}(2\sec^2 2x dx) = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + c = \frac{1}{2}e^{\tau g 2x} + c$

Respuesta: $\int e^{\tau g 2x} \sec^2 2x dx = \frac{1}{2}e^{\tau g 2x} + c$

2.31.-Encontrar: $\int \frac{2x-5}{3x^2-2} dx$

Solución.- Sea: $w = 3x^2 - 2$, donde: $dw = 6x dx$

Luego: $\int \frac{2x-5}{3x^2-2} dx = \frac{1}{3}\int \frac{3(2x-5)}{3x^2-2} dx = \frac{1}{3}\int \frac{6x-15}{3x^2-2} dx = \frac{1}{3}\int \frac{6x dx}{3x^2-2} - \frac{15}{3}\int \frac{dx}{3x^2-2}$

$$= \frac{1}{3}\int \frac{6x dx}{3x^2-2} - 5\int \frac{dx}{3(x^2-\frac{2}{3})} = \frac{1}{3}\int \frac{6x dx}{3x^2-2} - \frac{5}{3}\int \frac{dx}{(x^2-\frac{2}{3})} = \frac{1}{3}\int \frac{6x dx}{3x^2-2} - \frac{5}{3}\int \frac{dx}{x^2-(\sqrt{\frac{2}{3}})^2}$$

$\frac{1}{3}\int \frac{dw}{w} - \frac{5}{3}\int \frac{dx}{x^2-(\sqrt{\frac{2}{3}})^2} = \frac{1}{3}\ell\eta|w| + c_1 - \frac{5}{3}\int \frac{dx}{x^2-(\sqrt{\frac{2}{3}})^2}$; Sea: $v = x$, donde: $dv = dx$

Además: $a = \sqrt{2/3}$; se tiene: $\frac{1}{3}\ell\eta|w| + c_1 - \frac{5}{3}\int \frac{dv}{v^2-a^2}$

$$= \frac{1}{3}\ell\eta|3x^2-2| + c_1 - \frac{5}{3}\frac{1}{2a}\ell\eta\left|\frac{v-a}{v+a}\right| + c_2 = \frac{1}{3}\ell\eta|3x^2-2| - \frac{5}{3}\left[\frac{1}{2\sqrt{2/3}}\ell\eta\left|\frac{x-\sqrt{2/3}}{x+\sqrt{2/3}}\right|\right] + C$$

$$= \frac{1}{3}\ell\eta|3x^2-2| - \frac{5}{\sqrt{3}2\sqrt{2}}\ell\eta\left|\frac{\sqrt{3}x-\sqrt{2}}{\sqrt{3}x+\sqrt{2}}\right| + C = \frac{1}{3}\ell\eta|3x^2-2| - \frac{5}{2\sqrt{6}}\ell\eta\left|\frac{\sqrt{3}x-\sqrt{2}}{\sqrt{3}x+\sqrt{2}}\right| + C$$

Respuesta: $\int \frac{2x-5}{3x^2-2}dx = \frac{1}{3}\ell\eta\left|3x^2-2\right| - \frac{5}{2\sqrt{6}}\ell\eta\left|\frac{\sqrt{3}x-\sqrt{2}}{\sqrt{3}x+\sqrt{2}}\right| + C$

2.32.-Encontrar: $\int \frac{dx}{x\sqrt{4-9\ell\eta^2 x}}$

Solución.- $\int \frac{dx}{x\sqrt{4-9\ell\eta^2 x}} = \int \frac{dx}{x\sqrt{2^2-(3\ell\eta x)^2}}$

Sea: $u = 3\ell\eta x$, donde: $du = \frac{3dx}{x}$

Luego: $\int \frac{dx}{x\sqrt{2^2-(3\ell\eta x)^2}} = \frac{1}{3}\int \frac{3dx}{x\sqrt{2^2-(3\ell\eta x)^2}} = \frac{1}{3}\int \frac{du}{\sqrt{2^2-(u)^2}} = \frac{1}{3}\operatorname{arcs}e\operatorname{n}\frac{u}{2}+c$

$= \frac{1}{3}\operatorname{arcs}e\operatorname{n}\frac{3\ell\eta x}{2}+c = \frac{1}{3}\operatorname{arcs}e\operatorname{n}\ell\eta\left|x\right|^{3/2}+c$

Respuesta: $\int \frac{dx}{x\sqrt{4-9\ell\eta^2 x}} = \frac{1}{3}\operatorname{arcs}e\operatorname{n}\ell\eta\left|x\right|^{3/2}+c$

2.33.-Encontrar: $\int \frac{dx}{\sqrt{e^x-1}}$

Solución.- Sea: $u = \sqrt{e^x-1}$, donde: $du = \frac{e^x dx}{2\sqrt{e^x-1}}$; Tal que: $e^x = u^2+1$

Luego: $\int \frac{dx}{\sqrt{e^x-1}} = \int \frac{2du}{u^2+1} = 2\int \frac{du}{u^2+1} = 2\operatorname{arc}\tau g u + c = 2\operatorname{arc}\tau g\sqrt{e^x+1}+c$

Respuesta: $\int \frac{dx}{\sqrt{e^x-1}} = 2\operatorname{arc}\tau g\sqrt{e^x+1}+c$

2.34.-Encontrar: $\int \frac{x^2+2x+2}{x+1}dx$

Solución.- $\int \frac{x^2+2x+2}{x+1}dx = \int \frac{(x^2+2x+1)+1}{x+1}dx = \int \frac{(x+1)^2+1}{x+1}dx = \int \frac{(x+1)^2+1}{x+1}dx$

$= \int (x+1+\frac{1}{x+1})dx = \int xdx + \int dx + \int \frac{dx}{x+1}$**,** Sea: $w = x+1$**,** donde: $dw = dx$

Luego: $\int xdx + \int dx + \int \frac{dx}{x+1} = \int xdx + \int dx + \int \frac{dw}{w} = \frac{x^2}{2} + x + \ell\eta\left|w\right| + c$

$= \frac{x^2}{2} + x + \ell\eta\left|x+1\right| + c$

Respuesta: $\int \frac{x^2+2x+2}{x+1}dx = \frac{x^2}{2} + x + \ell\eta\left|x+1\right| + c$

2.35.-Encontrar: $\int \frac{e^{2x}}{\sqrt{e^x+1}}dx$

Solución.- Sea: $u = e^x+1$, donde: $du = e^x dx$

Luego: $\int \frac{e^{2x}}{\sqrt{e^x+1}}dx = \int \frac{u-1}{u^{1/2}}du = \int (u^{1/2} - u^{-1/2})du = \int u^{1/2}du - \int u^{-1/2}du = \frac{u^{3/2}}{3/2} - \frac{u^{-1/2}}{1/2} + c$

$= \frac{u^{3/2}}{3/2} - \frac{u^{-1/2}}{1/2} + c = \frac{2}{3}u^{3/2} - \frac{1}{2}u^{1/2} + c = \frac{2}{3}\sqrt{(e^x+1)^3} - 2\sqrt{(e^x+1)} + c$

Respuesta: $\int \frac{e^{2x}}{\sqrt{e^x+1}}dx = \frac{2}{3}\sqrt{(e^x+1)^3} - 2\sqrt{(e^x+1)} + c$

2.36.-Encontrar: $\int \frac{\ell\eta 2x}{\ell\eta 4x}\frac{dx}{x}$

Solución.- Sea: $u = \ell\eta 4x$, donde: $du = \frac{dx}{x}$; además: $\ell\eta 4x = (2\times 2x) = \ell\eta 2 + \ell\eta 2x$

$\Rightarrow u = \ell\eta 2 + \ell\eta 2x \Rightarrow \ell\eta 2x = u - \ell\eta 2$

Luego: $\int \frac{\ell\eta 2x}{\ell\eta 4x}\frac{dx}{x} = \int \frac{u-\ell\eta 2}{u}du = \int du - \int \frac{\ell\eta 2}{u}du = \int du - \ell\eta 2\int \frac{du}{u} = u - \ell\eta 2|u| + c$

$= \ell\eta 4x - \ell\eta 2[\ell\eta(\ell\eta 4x)] + c$

Respuesta: $\int \frac{\ell\eta 2x}{\ell\eta 4x}\frac{dx}{x} = \ell\eta 4x - \ell\eta 2[\ell\eta(\ell\eta 4x)] + c$

2.37.-Encontrar: $\int x(3x+1)^7 dx$

Solución.- Sea: $w = 3x+1$, donde: $dw = 3dx$; además: $w-1 = 3x \Rightarrow x = \frac{w-1}{3}$

Luego: $\int x(3x+1)^7 dx = \int \frac{w-1}{3}w^7\frac{dw}{3} = \frac{1}{9}\int (w-1)w^7 dw = \frac{1}{9}\int (w^8 - w^7)dw$

$= \frac{1}{9}\int w^8 dw - \frac{1}{9}\int w^7 dw = \frac{1}{9}\frac{w^9}{9} - \frac{1}{9}\frac{w^8}{8} + c = \frac{1}{81}w^9 - \frac{1}{72}w^8 + c$

$= \frac{1}{81}(3x+1)^9 - \frac{1}{72}(3x+1)^8 + c$

Respuesta: $\int x(3x+1)^7 dx = \frac{(3x+1)^9}{81} - \frac{(3x+1)^8}{72} + c$

2.38.-Encontrar: $\int \frac{x^2-5x+6}{x^2+4}dx$

Solución.- $\frac{x^2-5x+6}{x^2+4}dx = 1 + \frac{2-5x}{x^2+4}$

Luego: $\int \frac{x^2-5x+6}{x^2+4}dx = \int (1+\frac{2-5x}{x^2+4})dx = \int dx + 2\int \frac{dx}{x^2+4} - 5\int \frac{xdx}{x^2+4}$

Sea: $u = x^2+4$, donde: $du = 2xdx$; Entonces:

$= x + \operatorname{arc}\tau g\frac{x}{2} - \frac{5}{2}\int \frac{du}{u} = x + \operatorname{arc}\tau g\frac{x}{2} - \frac{5}{2}\ell\eta|u| + c = x + \operatorname{arc}\tau g\frac{x}{2} - \frac{5}{2}\ell\eta|x^2+4| + c$

Respuesta: $\int \frac{x^2-5x+6}{x^2+4}dx = x + \operatorname{arc}\tau g\frac{x}{2} - \frac{5}{2}\ell\eta|x^2+4| + c$

EJERCICIOS PROPUESTOS

Usando Esencialmente la técnica de integración por sustitución, encontrar las siguientes integrales:

2.39.- $\int 3^x e^x dx$

2.40.- $\int \frac{adx}{a-x}$

2.41.- $\int \frac{4t+6}{2t+1} dt$

2.42.- $\int \frac{1-3x}{3+2x} dx$

2.43.- $\int \frac{xdx}{a+bx}$

2.44.- $\int \frac{ax-b}{\alpha x+\beta} dx$

2.45.- $\int \frac{3t^2+3}{t-1} dt$

2.46.- $\int \frac{x^2+5x+7}{x+3} dx$

2.47.- $\int \frac{x^4+x^2+1}{x-1} dx$

2.48.- $\int \left(a+\frac{b}{x-a} \right)^2 dx$

2.49.- $\int \frac{x}{(x+1)^2} dx$

2.50.- $\int \frac{bdy}{\sqrt{1-y}}$

2.51.- $\int \sqrt{a-bx}dx$

2.52.- $\int \frac{xdx}{\sqrt{x^2+1}}$

2.53.- $\int \frac{\sqrt{x}+\ell\eta x}{x} dx$

2.54.- $\int \frac{dx}{3x^2+5}$

2.55.- $\int \frac{x^3 dx}{a^2-x^2}$

2.56.- $\int \frac{y^2-5y+6}{y^2+4} dy$

2.57.- $\int \frac{6t-15}{3t^2-2} dt$

2.58.- $\int \frac{3-2x}{5x^2+7} dx$

2.59.- $\int \frac{3x+1}{\sqrt{5x^2+1}} dx$

2.60.- $\int \frac{xdx}{x^2-5}$

2.61.- $\int \frac{xdx}{2x^2+3}$

2.62.- $\int \frac{ax+b}{a^2x^2+b^2} dx$

2.63.- $\int \frac{xdx}{\sqrt{a^4-x^4}}$

2.64.- $\int \frac{x^2 dx}{1+x^6}$

2.65.- $\int \frac{x^2 dx}{\sqrt{x^6-1}}$

2.66.- $\int \frac{x-\sqrt{\operatorname{arc}\tau g 3x}}{1+9x^2} dx$

2.67.- $\int \sqrt{\frac{\operatorname{arcs} e \operatorname{n} t}{4-4t^2}} dt$

2.68.- $\int \frac{\operatorname{arc}\tau g(\frac{x}{3})}{9+x^2} dx$

2.69.- $\int \frac{dt}{\sqrt{(9+9t^2)\ell\eta \left| t+\sqrt{1+t^2} \right|}}$

2.70.- $\int ae^{-mx} dx$

2.71.- $\int 4^{2-3x} dx$

2.72.- $\int (e^t - e^{-t}) dt$

2.73.- $\int e^{-(x^2+1)} x dx$

2.74.- $\int (e^{x/a} - e^{-x/a})^2 dx$

2.75.- $\int \frac{a^{2x}-1}{\sqrt{a^x}} dx$

2.76.- $\int \frac{e^{1/x}}{x^2} dx$

2.77.- $\int 5^{\sqrt{x}} \frac{dx}{\sqrt{x}}$

2.78.- $\int x7^{x^2} dx$

2.79.- $\int \frac{e^t dt}{e^t-1}$

2.80.- $\int e^x \sqrt{a-be^x} dx$

2.81.- $\int (e^{x/a}+1)^{1/3} e^{x/a} dx$

2.82.- $\int \frac{dx}{2^x+3}$

2.83.- $\int \frac{a^x dx}{1+a^{2x}}; a>0$

2.84.- $\int \frac{e^{-bx}}{1-e^{-2bx}} dx$

2.85.- $\int \frac{e^t dt}{\sqrt{1-e^{2t}}}$

2.86.- $\int \cos \frac{x}{\sqrt{2}} dx$

2.87.- $\int \operatorname{s} e \operatorname{n}(a+bx) dx$

2.88.- $\int \cos\sqrt{x} \frac{dx}{\sqrt{x}}$

2.89.- $\int \operatorname{s} e \operatorname{n}(\ell\eta x) \frac{dx}{x}$

2.90.- $\int (\cos ax + \operatorname{s} e \operatorname{n} ax)^2 dx$

2.91.- $\int \operatorname{s} e \operatorname{n}^2 x dx$

2.92.- $\int \cos^2 x dx$

2.93.- $\int \sec^2(ax+b)dx$ **2.94.-** $\int \cos\tau g^2 axdx$ **2.95.-** $\int \dfrac{dx}{\operatorname{sen}\frac{x}{a}}$

2.96.- $\int \dfrac{dx}{3\cos(5x-\frac{\pi}{4})}$ **2.97.-** $\int \dfrac{dx}{\operatorname{sen}(ax+b)}$ **2.98.-** $\int \dfrac{xdx}{\cos^2 x^2}$

2.99.- $\int \operatorname{co}\tau g\dfrac{x}{a-b}dx$ **2.100.-** $\int \tau g\sqrt{x}\dfrac{dx}{\sqrt{x}}$ **2.101.-** $\int \dfrac{dx}{\tau g\frac{x}{5}}$

2.102.- $\int \left(\dfrac{1}{\operatorname{sen}x\sqrt{2}}-1\right)^2 dx$ **2.103.-** $\int \dfrac{dx}{\operatorname{sen}x\cos x}$ **2.104.-** $\int \dfrac{\cos ax}{\operatorname{sen}^5 ax}dx$

2.105.- $\int t\operatorname{sen}(1-2t^2)dt$ **2.106.-** $\int \dfrac{\operatorname{sen}3x}{3+\cos 3x}dx$ **2.107.-** $\int \tau g^3\frac{x}{3}\sec^2\frac{x}{3}dx$

2.108.- $\int \dfrac{\operatorname{sen}x\cos x}{\sqrt{\cos^2 x-\operatorname{sen}^2 x}}dx$ **2.109.-** $\int \dfrac{\sqrt{\tau gx}}{\cos^2 x}dx$ **2.110.-** $\int \cos\frac{x}{a}\operatorname{sen}\frac{x}{a}dx$

2.111.- $\int t\operatorname{co}\tau g(2t^2-3)dt$ **2.112.-** $\int \dfrac{x^3dx}{x^8+5}$ **2.113.-** $\int \operatorname{sen}^3 6x\cos 6xdx$

2.114.- $\int \sqrt{1+3\cos^2 x}\operatorname{sen}2xdx$ **2.115.-** $\int x\sqrt[5]{5-x^2}dx$ **2.116.-** $\int \dfrac{1+\operatorname{sen}3x}{\cos^2 3x}dx$

2.117.- $\int \dfrac{(\cos ax+\operatorname{sen}ax)^2}{\operatorname{sen}ax}dx$ **2.118.-** $\int \dfrac{x^3-1}{x+1}dx$ **2.119.-** $\int \dfrac{\cos ec^2 3xdx}{b-a\operatorname{co}\tau g3x}$

2.120.- $\int \dfrac{x^3-1}{x^4-4x+1}dx$ **2.121.-** $\int xe^{-x^2}dx$ **2.122.-** $\int \dfrac{3-\sqrt{2+3x^2}}{2+3x^2}dx$

2.123.- $\int \dfrac{\tau g3x-\operatorname{co}\tau g3x}{\operatorname{sen}3x}dx$ **2.124.-** $\int \dfrac{dx}{\sqrt{e^x}}$ **2.125.-** $\int \dfrac{1+\operatorname{sen}x}{x+\cos x}dx$

2.126.- $\int \dfrac{\sec^2 xdx}{\sqrt{\tau g^2 x-2}}$ **2.127.-** $\int \dfrac{dx}{x\ell\eta^2 x}$ **2.128.-** $\int a^{\operatorname{sen}x}\cos xdx$

2.129.- $\int \dfrac{x^2}{\sqrt{x^3+1}}dx$ **2.130.-** $\int \dfrac{xdx}{\sqrt{1-x^4}}$ **2.131.-** $\int \tau g^2 axdx$

2.132.- $\int \dfrac{\sec^2 xdx}{\sqrt{4-\tau g^2 x}}$ **2.133.-** $\int \dfrac{dx}{\cos x/a}$ **2.134.-** $\int \dfrac{\sqrt[3]{1+\ell\eta x}}{x}dx$

2.135.- $\int \tau g\sqrt{x-1}\dfrac{dx}{\sqrt{x-1}}$ **2.136.-** $\int \dfrac{xdx}{\operatorname{sen}x^2}$ **2.137.-** $\int \dfrac{\operatorname{sen}x-\cos x}{\operatorname{sen}x+\cos x}dx$

2.138.- $\int \dfrac{e^{\operatorname{arc}\tau gx}+x\ell\eta(1+x^2)+1}{1+x^2}$ **2.139.-** $\int \dfrac{x^2dx}{x^2-2}$ **2.140.-** $\int e^{\operatorname{sen}^2 x}\operatorname{sen}2xdx$

2.141.- $\int \dfrac{(1-\operatorname{sen}\frac{x}{\sqrt{2}})^2}{\operatorname{sen}\frac{x}{\sqrt{2}}}dx$ **2.142.-** $\int \dfrac{5-3x}{\sqrt{4-3x^2}}dx$ **2.143.-** $\int \dfrac{ds}{e^s+1}$

2.144.- $\int \dfrac{d\theta}{\operatorname{sen}a\theta\cos a\theta}$ **2.145.-** $\int \dfrac{e^s}{\sqrt{e^{2s}-2}}ds$ **2.146.-** $\int \operatorname{sen}(\frac{2\pi t}{T}+\varphi_0)dt$

2.147.- $\int \frac{\arccos x/2}{\sqrt{4-x^2}}dx$ **2.148.-** $\int \frac{dx}{x(4-\ell\eta^2 x)}$ **2.149.-** $\int e^{-\tau gx}\sec^2 x dx$

2.150.- $\int \frac{\operatorname{sen} x\cos x}{\sqrt{2-\operatorname{sen}^4 x}}dx$ **2.151.-** $\int \frac{\sec x\tau gx}{\sqrt{\sec^2 x+1}}dx$ **2.152.-** $\int \frac{dt}{\operatorname{sen}^2 t\cos^2 t}$

2.153.- $\int \frac{\operatorname{arcsen} x + x}{\sqrt{1-x^2}}dx$ **2.154.-** $\int \frac{xdx}{\sqrt{x+1}}$ **2.155.-** $\int x(5x^2-3)^7 dx$

2.156.- $\int \sqrt{\frac{\ell\eta(x+\sqrt{x^2+1})}{x^2+1}}dx$ **2.157.-** $\int \frac{\operatorname{sen}^3 x}{\sqrt{\cos x}}dx$ **2.158.-** $\int \frac{\cos xdx}{\sqrt{1+\operatorname{sen}^2 x}}$

2.159.- $\int \frac{(\operatorname{arcsen} x)^2}{\sqrt{1-x^2}}dx$ **2.150.-** $\int e^{x+e^x}dx$ **2.161.-** $\int t(4t+1)^7 dt$

2.162.- $\int \frac{2t^2-10t+12}{t^2+4}dt$ **2.163.-** $\int \frac{e^t-e^{-t}}{e^t+e^{-t}}dt$

RESPUESTAS

2.39.- $\int 3^x e^x dx$ **,** Sea: $u = x, du = dx, a = 3e$

$$\int (3e)^x dx = \int (a)^u du = \frac{a^u}{\ell\eta a}+c = \frac{(3e)^x}{\ell\eta(3e)}+c = \frac{(3e)^x}{\ell\eta 3\ell\eta e}+c = \frac{3^x e^x}{\ell\eta 3+\ell\eta e}+c = \frac{3^x e^x}{\ell\eta 3+1}+c$$

2.40.- $\int \frac{adx}{a-x}$ **,** Sea: $u = a-x, du = -dx$

$$\int \frac{adx}{a-x} = -a\int \frac{du}{u} = -a\ell\eta|u|+c = -a\ell\eta|a-x|+c$$

2.41.- $\int \frac{4t+6}{2t+1}dt$ **,** Sea: $u = 2t+1, du = 2dt;$ $\frac{2t+3}{2t+1} = 1+\frac{2}{2t+1}$

$$\int \frac{4t+6}{2t+1}dt = 2\int\left(1+\frac{2}{2t+1}\right)dt = 2\int dt + 2\int \frac{2}{2t+1}dt = 2\int dt + 2\int \frac{du}{u} = 2t+2\ell\eta|u|+c$$

$$= 2t+2\ell\eta|2t+1|+c$$

2.42.- $\int \frac{1-3x}{3+2x}dx$ **,** Sea: $u = 3+2x, du = 2dx;$ $\frac{1-3x}{3+2x} = -\frac{3}{2}+\frac{11/2}{2x+3}$

$$\int \frac{1-3x}{3+2x}dx = \int\left(-\frac{3}{2}+\frac{11/2}{2x+3}\right)dx = -\frac{3}{2}\int dx + \frac{11}{4}\int \frac{dx}{2x+3} = -\frac{3}{2}\int dx + \frac{11}{4}\int \frac{du}{u}$$

$$-\frac{3}{2}x+\frac{11}{4}\ell\eta|2x+3|+c$$

2.43.- $\int \frac{xdx}{a+bx}$ **,** Sea: $u = a+bx, du = bdx;$ $\frac{x}{a+bx} = \frac{1}{b}-\frac{a/b}{a+bx}$

$$\int \frac{xdx}{a+bx} = \frac{1}{b}\int dx - \frac{a}{b}\int \frac{dx}{a+bx} = \frac{1}{b}\int dx - \frac{a}{b^2}\int \frac{du}{u} = \frac{1}{b}x-\frac{a}{b^2}\ell\eta|u|+c = \frac{x}{b}-\frac{a}{b^2}\ell\eta|a+bx|+c$$

2.44.- $\int \frac{ax-b}{\alpha x+\beta}dx$ **,** Sea: $u=\alpha x+\beta, du=\alpha dx$; $\frac{ax-b}{ax+b}=\frac{a}{\alpha}-\frac{\frac{\alpha\beta}{\alpha}+b}{\alpha x}$

$$\int \frac{ax-b}{\alpha x+\beta}dx=\int\left(\frac{a}{\alpha}-\frac{\frac{\alpha\beta}{\alpha}+b}{\alpha x}\right)dx=\int\frac{a}{\alpha}dx-\int\frac{\frac{a\beta+\alpha b}{\alpha}}{\alpha x+\beta}dx=\frac{a}{\alpha}\int dx-\frac{a\beta+\alpha b}{\alpha}\int\frac{dx}{a\beta+\alpha b}$$

$$=\frac{a}{\alpha}\int dx-\frac{a\beta+\alpha b}{\alpha^2}\int\frac{du}{u}=\frac{a}{\alpha}x-\frac{a\beta+\alpha b}{\alpha^2}\ell\eta|u|+c=\frac{a}{\alpha}x-\frac{a\beta+\alpha b}{\alpha^2}\ell\eta|\ell x+\beta|+c$$

2.45.- $\int \frac{3t^2+3}{t-1}dt$ **,** Sea: $u=t-1, du=dt$; $\frac{t^2+1}{t-1}=t+1+\frac{2}{t-1}$

$$\int\frac{3t^2+3}{t-1}dt=3\int\left(t+1+\frac{2}{t-1}\right)dt=3\int tdt+3\int dt+3\int\frac{2}{t-1}dt=\frac{3}{2}t^2+3t+6\ell\eta|u|+c$$

$$=\frac{3}{2}t^2+3t+6\ell\eta|t-1|+c$$

2.46.- $\int \frac{x^2+5x+7}{x+3}dx$ **,** Sea: $u=t-1, du=t+1$; $\frac{x^2+5x+7}{x+3}=x+2+\frac{1}{x+3}$

$$\int\frac{x^2+5x+7}{x+3}dx=\int\left(x+2+\frac{1}{x+3}\right)dx=\int xdx+2\int dx+\int\frac{1}{x+3}dx=\frac{x^2}{2}+2x+\ell\eta|u|+c$$

$$=\frac{x^2}{2}+2x+\ell\eta|u|+c=\frac{x^2}{2}+2x+\ell\eta|x+3|+c$$

2.47.- $\int \frac{x^4+x^2+1}{x-1}dx$ **,** Sea: $u=x-1, du=dx$;

$$\int\frac{x^4+x^2+1}{x-1}dx=\int\left(x^3+x^2+2x+2+\frac{3}{x-1}\right)dx=\int x^3dx+\int x^2dx+2\int dx+3\int\frac{dx}{x-1}$$

$$=\frac{x^4}{4}+\frac{x^3}{3}+x^2+2+3\ell\eta|u|+c=\frac{x^4}{4}+\frac{x^3}{3}+x^2+2x+3\ell\eta|x-1|+c$$

2.48.- $\int\left(a+\frac{b}{x-a}\right)^2dx$ **,** Sea: $u=x-a, du=dx$

$$\int\left(a+\frac{b}{x-a}\right)^2dx=\int\left(a^2+\frac{2ab}{x-a}+\frac{b^2}{(x-a)^2}\right)dx=a^2\int dx+2ab\int\frac{dx}{x-a}+b^2\int\frac{dx}{(x-a)^2}$$

$$=a^2\int dx+2ab\int\frac{du}{u}+b^2\int\frac{du}{u^2}=a^2x+2ab\ell\eta|u|+b^2\frac{u^{-1}}{-1}+c=a^2x+2ab\ell\eta|x-a|-\frac{b^2}{x-a}+c$$ **2.**

49.- $\int\frac{x}{(x+1)^2}dx$ **,** Sea: $u=x+1, du=dx$

$$\int\frac{x}{(x+1)^2}dx=\int\frac{(x+1)-1}{(x+1)^2}dx=\int\frac{x+1}{(x+1)^2}dx-\int\frac{dx}{(x+1)^2}=\int\frac{dx}{u}-\int\frac{dx}{u^2}=\ell\eta|u|-\frac{u^{-1}}{-1}+c$$

$$=\ell\eta\left|x+1\right|+\frac{1}{x+1}+c$$

2.50.- $\int\frac{bdy}{\sqrt{1-y}}$ **,** Sea: $u=1-y, du=-dy$

$$\int\frac{bdy}{\sqrt{1-y}}=-b\int\frac{du}{\sqrt{u}}=-b\int u^{-1/2}du=-2bu^{1/2}+c=-2b(1-y)^{1/2}+c$$

2.51.- $\int\sqrt{a-bx}dx$ **,** Sea: $u=a-bx, du=-bdx$

$$\int\sqrt{a-bx}dx=-\frac{1}{b}\int u^{1/2}du=-\frac{1}{b}\frac{u^{3/2}}{\frac{3}{2}}+c=-\frac{2}{3b}u^{3/2}+c=-\frac{3}{2b}(a-bx)^{3/2}+c$$

2.52.- $\int\frac{xdx}{\sqrt{x^2+1}}$ **,** Sea: $u=x^2+1, du=2xdx$

$$\int\frac{xdx}{\sqrt{x^2+1}}=\frac{1}{2}\int\frac{du}{\sqrt{u}}=\frac{1}{2}\int u^{-1/2}du=\frac{1}{\cancel{2}}\frac{u^{1/2}}{\frac{1}{\cancel{2}}}+c=(x^2+1)^{1/2}+c$$

2.53.- $\int\frac{\sqrt{x}+\ell\eta x}{x}dx$ **,** Sea: $u=\ell\eta x, du=\frac{dx}{x}$

$$\int\frac{\sqrt{x}+\ell\eta x}{x}dx=\int x^{-1/2}dx+\int\frac{\ell\eta x}{x}dx=\int x^{-1/2}dx+\int udu=\frac{x^{1/2}}{1/2}+\frac{u^2}{2}+c$$

$$=2\sqrt{x}+\frac{\ell\eta^2 x}{2}+c$$

2.54.- $\int\frac{dx}{3x^2+5}$**,** Sea: $u^2=3x^2, u=\sqrt{3}x, du=\sqrt{3}dx$; $a^2=5; a=\sqrt{5}$

$$\int\frac{dx}{3x^2+5}=\frac{1}{\sqrt{3}}\int\frac{du}{u^2+a^2}=\frac{1}{\sqrt{3}}\frac{1}{a}\operatorname{arc}tg\frac{u}{a}+c=\frac{1}{\sqrt{3}}\frac{1}{\sqrt{5}}\operatorname{arc}tg\frac{\sqrt{3}x}{\sqrt{5}}+c=\frac{\sqrt{15}}{15}\operatorname{arc}tg\sqrt{\frac{3x}{5}}+c$$

2.55.- $\int\frac{x^3dx}{a^2-x^2}$ **,** Sea: $u=x^2-a^2, du=2xdx$

$$\int\frac{x^3dx}{a^2-x^2}=-\int xdx-\int\frac{a^2xdx}{x^2-a^2}=-\int xdx-a^2\int\frac{xdx}{x^2-a^2}=-\int xdx-\frac{a^2}{2}\int\frac{du}{u}$$

$$=-\frac{x^2}{2}-\frac{a^2}{2}\ell\eta\left|u\right|+c=-\frac{x^2}{2}-\frac{a^2}{2}\ell\eta\left|x^2-a^2\right|+c$$

2.56.- $\int\frac{y^2-5y+6}{y^2+4}dy$ **,** Sea: $u=y^2+4, du=2ydy$

$$\int\frac{y^2-5y+6}{y^2+4}dy=\int(1+\frac{-5y+2}{y^2+4})dy=\int dy+\int\frac{-5y+2}{y^2+4}dy=\int dy-5\int\frac{ydy}{y^2+4}+2\int\frac{dy}{y^2+2^2}$$

$$=y-\frac{5}{2}\ell\eta\left|u\right|+\cancel{2}\ \frac{1}{\cancel{2}}\operatorname{arc}\tau g\frac{y}{2}+c=y-\frac{5}{2}\ell\eta\left|y^2+4\right|+\operatorname{arc}\tau g\frac{y}{2}+c$$

2.57.- $\int\frac{6t-15}{3t^2-2}dt$ **,** Sea: $u=3t^2-2, du=6tdt; w=\sqrt{3}t, dw=\sqrt{3}dt$

$$\int\frac{6t-15}{3t^2-2}dt=6\int\frac{tdt}{3t^2-2}-15\int\frac{dt}{3t^2-2}=6\int\frac{tdt}{3t^2-2}-15\int\frac{dt}{(\sqrt{3}t)^2-(\sqrt{2})^2}$$

$$=\int\frac{du}{u}-\frac{15}{\sqrt{3}}\int\frac{dw}{w^2-(\sqrt{2})^2}=\ell\eta|u|-\frac{15\sqrt{3}}{3}\frac{1}{2\sqrt{2}}\ell\eta\left|\frac{w-\sqrt{2}}{w+\sqrt{2}}\right|+c$$

$$=\ell\eta\left|3t^2-2\right|-\frac{5\sqrt{6}}{4}\ell\eta\left|\frac{t\sqrt{3}-\sqrt{2}}{t\sqrt{3}+\sqrt{2}}\right|+c$$

2.58.- $\int\frac{3-2x}{5x^2+7}dx$**,** Sea: $u=5x^2+7, du=10xdx; w=\sqrt{5}x, dw=\sqrt{5}dx$

$$\int\frac{3-2x}{5x^2+7}dx=3\int\frac{dx}{5x^2+7}-2\int\frac{dx}{5x^2+7}=3\int\frac{dx}{(\sqrt{5}x)^2+(\sqrt{7})^2}-\frac{2}{10}\int\frac{du}{u}$$

$$=\frac{3}{\sqrt{5}}\int\frac{dw}{w^2+(\sqrt{7})^2}-\frac{1}{5}\int\frac{du}{u}=\frac{3}{\sqrt{5}}\frac{1}{\sqrt{7}}\operatorname{arc}\tau g\frac{x\sqrt{5}}{\sqrt{7}}-\frac{1}{5}\ell\eta|u|+c$$

$$=\frac{3\sqrt{35}}{35}\operatorname{arc}\tau g x\sqrt{\frac{5}{7}}-\frac{1}{5}\ell\eta\left|5x^2+7\right|+c$$

2.59.- $\int\frac{3x+1}{\sqrt{5x^2+1}}dx$**,** Sea: $u=5x^2+1, du=10xdx; w=x\sqrt{5}, dw=\sqrt{5}dx$

$$\int\frac{3x+1}{\sqrt{5x^2+1}}dx=3\int\frac{xdx}{\sqrt{5x^2+1}}+\int\frac{dx}{\sqrt{(x\sqrt{5})^2+1^2}}=3\int\frac{xdx}{\sqrt{5x^2+1}}+\int\frac{dx}{\sqrt{(x\sqrt{5})^2+1^2}}$$

$$=\frac{3}{10}\int\frac{du}{\sqrt{u}}+\frac{1}{\sqrt{5}}\int\frac{dw}{\sqrt{w^2+1^2}}=\frac{3}{10}\frac{u^{1/2}}{1/2}+\frac{1}{\sqrt{5}}\ell\eta\left|w+\sqrt{w^2+1}\right|+c$$

$$=\frac{3}{5}\sqrt{5x^2+1}+\frac{1}{\sqrt{5}}\ell\eta\left|x\sqrt{5}+\sqrt{5x^2+1}\right|+c$$

2.60.- $\int\frac{xdx}{x^2-5}$**,** Sea: $u=x^2+5, du=2xdx$

$$\int\frac{xdx}{x^2-5}=\frac{1}{2}\int\frac{du}{u}=\frac{1}{2}\ell\eta|u|+c=\frac{1}{2}\ell\eta\left|x^2-5\right|+c$$

2.61.- $\int\frac{xdx}{2x^2+3}$**,** Sea: $u=2x^2+3, du=4xdx$

$$\int\frac{xdx}{2x^2+3}=\frac{1}{4}\int\frac{du}{u}=\frac{1}{4}\ell\eta|u|+c=\frac{1}{4}\ell\eta\left|2x^2+3\right|+c$$

2.62.- $\int\frac{ax+b}{a^2x^2+b^2}dx$**,** Sea: $u=a^2x^2+b^2, du=2a^2xdx; w=ax, dw=adx$

$$\int\frac{ax+b}{a^2x^2+b^2}dx=a\int\frac{xdx}{a^2x^2+b^2}+b\int\frac{dx}{a^2x^2+b^2}=\frac{a}{2a^2}\int\frac{du}{u}+\frac{b}{a}\int\frac{dw}{w^2+b^2}$$

$$=\frac{1}{2}\ell\eta|u|+\frac{\cancel{b}}{a}\frac{1}{\cancel{b}}\operatorname{arc}\tau g\frac{w}{b}+c=\frac{1}{2}\ell\eta\left|a^2x^2+b^2\right|+\frac{1}{a}\operatorname{arc}\tau g\frac{ax}{b}+c$$

2.63.- $\int \frac{xdx}{\sqrt{a^4-x^4}}$ **,** Sea: $u=x^2, du=2xdx$

$$\int \frac{xdx}{\sqrt{a^4-x^4}}=\int \frac{xdx}{\sqrt{(\sqrt{a^2})^2-(\sqrt{x^2})^2}}=\frac{1}{2}\int \frac{du}{\sqrt{(\sqrt{a^2})^2-u^2}}=\frac{1}{2}\operatorname{arcsen}\frac{u}{a^2}+c$$

$$=\frac{1}{2}\operatorname{arcsen}\frac{x^2}{a^2}+c$$

2.64.- $\int \frac{x^2dx}{1+x^6}$ **,** Sea: $u=x^3, du=3x^2dx$

$$\int \frac{x^2dx}{1+x^6}=\int \frac{x^2dx}{1+(x^3)^2}=\frac{1}{3}\int \frac{du}{1+u^2}=\frac{1}{3}\operatorname{arc}\tau g\left|u\right|+c=\frac{1}{3}\operatorname{arc}\tau g x^3+c$$

2.65.- $\int \frac{x^2dx}{\sqrt{x^6-1}}$ **,** Sea: $u=x^3, du=3x^2dx$

$$\int \frac{x^2dx}{\sqrt{x^6-1}}=\int \frac{x^2dx}{\sqrt{(x^3)^2-1}}=\frac{1}{3}\int \frac{du}{\sqrt{u^2-1}}=\frac{1}{3}\ell\eta\left|u+\sqrt{u^2-1}\right|+c=\frac{1}{3}\ell\eta\left|x^3+\sqrt{x^6-1}\right|+c$$

2.66.- $\int \frac{x-\sqrt{\operatorname{arc}\tau g 3x}}{1+9x^2}dx$ **,** Sea: $u=1+9x^2, du=18xdx; w=\operatorname{arc}\tau g3x, dw=\frac{3dx}{1+9x^2}$

$$\int \frac{x-\sqrt{\operatorname{arc}\tau g3x}}{1+9x^2}dx=\int \frac{xdx}{1+9x^2}-\int \frac{\sqrt{\operatorname{arc}\tau g3x}}{1+9x^2}dx=\frac{1}{18}\int \frac{du}{u}-\frac{1}{3}\int w^{\frac{1}{2}}dw$$

$$=\frac{1}{18}\ell\eta\left|u\right|-\frac{1}{3}\frac{w^{\frac{3}{2}}}{\frac{3}{2}}+c=\frac{1}{18}\ell\eta\left|1+9x^2\right|-\frac{2(\operatorname{arc}\tau g3x)^{\frac{3}{2}}}{9}+c$$

2.67.- $\int \sqrt{\frac{\operatorname{arcsen} t}{4-4t^2}}dt$ **,** Sea: $u=\operatorname{arcsen} t, du=\frac{dt}{\sqrt{1-t^2}}$

$$\int \sqrt{\frac{\operatorname{arcsen} t}{4-4t^2}}dt=\frac{1}{2}\int \sqrt{\frac{\operatorname{arcsen} t}{1-t^2}}dt=\frac{1}{2}\int \frac{\sqrt{\operatorname{arcsen} t}}{\sqrt{1-t^2}}dt=\frac{1}{2}\int \sqrt{u}du=\frac{1}{\cancel{2}}\frac{u^{\frac{3}{2}}}{\frac{3}{\cancel{2}}}+c=\frac{1}{3}u^{\frac{3}{2}}+c$$

$$=\frac{1}{3}\sqrt{(\operatorname{arcsen} t)^3}+c$$

2.68.- $\int \frac{\operatorname{arc}\tau g(\frac{x}{3})}{9+x^2}dx$ **,** Sea: $u=\operatorname{arc}\tau g\,\frac{x}{3}, du=\frac{3dx}{9+x^2}$

$$\int \frac{\operatorname{arc}\tau g(\frac{x}{3})}{9+x^2}dx=\frac{1}{3}\int udu=\frac{1}{3}\frac{u^2}{2}+c=\frac{1}{6}u^2+c=\frac{\operatorname{arc}\tau g(\frac{x}{3})^2}{6}+c$$

2.69.- $\int \frac{dt}{\sqrt{(9+9t^2)\ell\eta\left|t+\sqrt{1+t^2}\right|}}$, Sea: $u=\ell\eta\left|t+\sqrt{1+t^2}\right|, du=\frac{dt}{\sqrt{1+t^2}}$

$$=\frac{1}{3}\int \frac{dt}{\sqrt{(1+t^2)}\sqrt{\ell\eta\left|t+\sqrt{1+t^2}\right|}}=\frac{1}{3}\int \frac{du}{\sqrt{u}}=\frac{1}{3}\frac{u^{\frac{1}{2}}}{\frac{1}{2}}+c=\frac{2}{3}\sqrt{u}+c=\frac{2}{3}\sqrt{\ell\eta\left|t+\sqrt{1+t^2}\right|}+c$$

2.70.- $\int ae^{-mx}dx$ **,** Sea: $u=-mx, du=-mdx$

$$\int ae^{-mx}dx = a\int e^{-mx}dx = -\frac{a}{m}\int e^u du = -\frac{a}{m}e^u + c = -\frac{a}{m}e^{-mx} + c$$

2.71.- $\int 4^{2-3x}dx$ **,** Sea: $u = 2-3x, du = -3dx; a = 4$

$$\int 4^{2-3x}dx = -\frac{1}{3}\int a^u du = -\frac{1}{3}\frac{a^u}{\ell\eta a} + c = -\frac{4^{2-3x}}{3\ell\eta 4} + c$$

2.72.- $\int (e^t - e^{-t})dt$ **,** Sea: $u = -t, du = -dt$

$$\int (e^t - e^{-t})dt = \int e^t dt - \int e^{-t}dt = \int e^t dt - \int e^u dt = e^t + e^u + c = e^t + e^{-t} + c$$

2.73.- $\int e^{-(x^2+1)}xdx$ **,** Sea: $u = -x^2 - 1, du = -2xdx$

$$\int e^{-(x^2+1)}xdx = \int e^{-x^2-1}xdx = -\frac{1}{2}\int e^u du = -\frac{1}{2}e^u + c = -\frac{1}{2}e^{-(x^2+1)} + c = -\frac{1}{2e^{x^2+1}} + c$$

2.74.- $\int (e^{x/a} - e^{-x/a})^2 dx$ **,** Sea: $u = \frac{2x}{a}, du = \frac{2dx}{a}; w = -\frac{2x}{a}, dw = -\frac{2dx}{a}$

$$\int (e^{x/a} - e^{-x/a})^2 dx = \int (e^{2x/a} + 2e^{x/a}e^{-x/a} + e^{-2x/a})dx = \int e^{2x/a}dx + 2\int dx + \int e^{-2x/a}dx$$

$$= \frac{a}{2}\int e^u du + 2\int dx - \frac{a}{2}\int e^w dw = \frac{a}{2}e^u + 2x - \frac{a}{2}e^w + c = \frac{a}{2}e^{\frac{2x}{a}} + 2x - \frac{a}{2}e^{-\frac{2x}{a}} + c$$

2.75.- $\int \frac{a^{2x}-1}{\sqrt{a^x}}dx$ **,** Sea: $u = -\frac{x}{2}, du = -\frac{dx}{2}; w = \frac{3x}{2}, dw = \frac{3dx}{2}$

$$\int \frac{a^{2x}-1}{\sqrt{a^x}}dx = \int \frac{a^{2x}dx}{\sqrt{a^x}} - \int \frac{dx}{\sqrt{a^x}} = \int a^{2x-x/2}dx - \int a^{-x/2}dx = \int a^{3x/2}dx - \int a^{-x/2}dx$$

$$= \frac{2}{3}\int a^w dw + 2\int a^u du = \frac{2}{3}\frac{a^w}{\ell\eta a} + 2\frac{a^u}{\ell\eta a} + c = \frac{2}{3}\frac{a^{3x/2}}{\ell\eta a} + 2\frac{a^{-x/2}}{\ell\eta a} + c = \frac{2}{\ell\eta a}\left(\frac{a^{3x/2}}{3} + a^{-x/2}\right) + c$$

2.76.- $\int \frac{e^{1/x}}{x^2}dx$ **,** Sea: $u = \frac{1}{x}, du = -\frac{dx}{x^2}$

$$\int \frac{e^{1/x}}{x^2}dx = -\int e^u du = -e^u + c = -e^{1/x} + c = -\sqrt[x]{e} + c$$

2.77.- $\int 5^{\sqrt{x}}\frac{dx}{\sqrt{x}}$ **,** Sea: $u = \sqrt{x}, du = \frac{dx}{2\sqrt{x}}$

$$\int 5^{\sqrt{x}}\frac{dx}{\sqrt{x}} = 2\int 5^u du = \frac{2\times 5^u}{\ell\eta 5} + c = \frac{2\times 5^{\sqrt{x}}}{\ell\eta 5} + c$$

2.78.- $\int x7^{x^2}dx$ **,** Sea: $u = x^2, du = 2xdx$

$$\int x7^{x^2}dx = \frac{1}{2}\int 7^u du = \frac{1}{2}\frac{7^u}{\ell\eta 7} + c = \frac{1}{2}\frac{7^{x^2}}{\ell\eta 7} + c$$

2.79.- $\int \frac{e^t dt}{e^t - 1}$ **,** Sea: $u = e^t - 1, du = e^t dt$

$$\int \frac{e^t dt}{e^t - 1} = \int \frac{du}{u} = \ell\eta|u| + c = \ell\eta\left|e^t - 1\right| + c$$

2.80.- $\int e^x \sqrt{a - be^x}\, dx$ **,** Sea: $u = a - be^x, du = -be^x dx$

$$\int e^x \sqrt{a - be^x}\, dx = -\frac{1}{b}\int \sqrt{u} du = -\frac{1}{b}\frac{u^{3/2}}{3/2} + c = -\frac{2}{3b}u^{3/2} + c = -\frac{2}{3b}(a - be^x)^{3/2} + c$$

2.81.- $\int (e^{x/a} + 1)^{1/3} e^{x/a} dx$ **,** Sea: $u = e^{x/a + 1}, du = \frac{e^{x/a}}{a} dx$

$$\int (e^{x/a} + 1)^{1/3} e^{x/a} dx = \int \sqrt[3]{e^{x/a} + 1} e^{x/a} dx = a\int u^{1/3} du = \frac{au^{4/3}}{4/3} + c = \frac{3a(e^{x/a} + 1)^{4/3}}{4} + c$$

2.82.- $\int \frac{dx}{2^x + 3}$ **,** Sea: $u = 2^x + 3, du = 2^x \ell\eta 2 dx$

$$\int \frac{dx}{2^x + 3} = \frac{1}{3}\int \frac{3dx}{2^x + 3} = \frac{1}{3}\int \frac{2^x + 3 - 2^x}{2^x + 3} dx = \frac{1}{3}\int \frac{2^x + 3}{2^x + 3} dx - \frac{1}{3}\int \frac{2^x}{2^x + 3} dx = \frac{1}{3}\int dx - \frac{1}{3}\int \frac{du}{u}$$

$$= \frac{1}{3}x - \frac{1}{3}\ell\eta|u| + c = \frac{1}{3}x - \frac{1}{3\ell\eta 2}\ell\eta|u| + c = \frac{1}{3}x - \frac{\ell\eta\left|2^x + 3\right|}{3\ell\eta 2} + c$$

2.83.- $\int \frac{a^x dx}{1 + a^{2x}}$ **,** Sea: $u = a^x, du = a^x \ell\eta a dx; a > 0$

$$\int \frac{a^x dx}{1 + a^{2x}} = \int \frac{a^x dx}{1 + (a^x)^2} = \frac{1}{\ell\eta a}\int \frac{du}{1 + u^2} = \frac{1}{\ell\eta a}\operatorname{arc}\tau g u + c = \frac{1}{\ell\eta a}\operatorname{arc}\tau g a^x + c$$

2.84.- $\int \frac{e^{-bx}}{1 - e^{-2bx}} dx$ **,** Sea: $u = e^{-bx}, du = -be^{-bx} dx$

$$\int \frac{e^{-bx}}{1 - e^{-2bx}} dx = \int \frac{e^{-bx}}{1 - (e^{-bx})^2} dx = -\frac{1}{b}\int \frac{du}{1 - u^2} = -\frac{1}{b}\int \frac{du}{(-1)(u^2 - 1)} = \frac{1}{2b}\ell\eta\left|\frac{u - 1}{u + 1}\right| + c$$

$$= \frac{1}{2b}\ell\eta\left|\frac{e^{-bx} - 1}{e^{-bx} + 1}\right| + c.$$

2.85.- $\int \frac{e^t dt}{\sqrt{1 - e^{2t}}}$ **,** Sea: $u = e^t, du = e^t dt$

$$\int \frac{e^t dt}{\sqrt{1 - e^{2t}}} = \int \frac{e^t dt}{\sqrt{1 - (e^t)^2}} = \int \frac{du}{\sqrt{1 - u^2}} = \operatorname{arcsen} u + c = \operatorname{arcsen} e^t + c$$

2.86.- $\int \cos\frac{x}{\sqrt{2}} dx$ **,** Sea: $u = \frac{x}{\sqrt{2}}, du = \frac{dx}{\sqrt{2}}$

$$\int \cos\frac{x}{\sqrt{2}} dx = \sqrt{2}\int \cos u du = \sqrt{2}\operatorname{sen} u + c = \sqrt{2}\operatorname{sen}\frac{x}{\sqrt{2}} + c$$

2.87.- $\int \operatorname{sen}(a + bx) dx$ **,** Sea: $u = a + bx, du = bdx$

$$\int \operatorname{sen}(a + bx) dx = \frac{1}{b}\int \operatorname{sen} u du = -\frac{1}{b}\cos u + c = -\frac{1}{b}\cos(a + bx) + c$$

2.88.- $\int \cos\sqrt{x}\,\frac{dx}{\sqrt{x}}$, Sea: $u=\sqrt{x}, du=\frac{dx}{2\sqrt{x}}$

$$\int \cos\sqrt{x}\,\frac{dx}{\sqrt{x}} = 2\int \cos u du = 2\operatorname{sen} u + c = 2\operatorname{sen}\sqrt{x}+c$$

2.89.- $\int \operatorname{sen}(\ell\eta x)\frac{dx}{x}$, Sea: $u=\ell\eta x, du=\frac{dx}{x}$

$$\int \operatorname{sen}(\ell\eta x)\frac{dx}{x} = \int \operatorname{sen} u du = -\cos u + c = -\cos \ell\eta x + c$$

2.90.- $\int (\cos ax + \operatorname{sen} ax)^2 dx$, Sea: $u=2ax, du=2adx$

$$\int (\cos ax + \operatorname{sen} ax)^2 dx = \int (\cos^2 ax + 2\cos ax \operatorname{sen} ax + \operatorname{sen}^2 ax)dx$$

$$= \int (1+2\cos ax \operatorname{sen} ax)dx = \int dx + 2\int \cos ax \operatorname{sen} axdx = \int dx + \int \operatorname{sen} 2axdx$$

$$= x - \frac{1}{2a}\cos 2ax + c$$

2.91.- $\int \operatorname{sen}^2 xdx$, Sea: $u=2x, du=2dx$

$$\int \operatorname{sen}^2 xdx = \int \frac{1-\cos 2x}{2}dx = \frac{1}{2}\int dx - \frac{1}{2}\int \cos 2xdx = \frac{1}{2}\int dx - \frac{1}{4}\int \cos udu = \frac{1}{2}x - \frac{1}{4}\operatorname{sen} u + c$$

$$= \frac{1}{2}x - \frac{1}{4}\operatorname{sen} 2x + c$$

2.92.- $\int \cos^2 xdx$, Sea: $u=2x, du=2dx$

$$\int \cos^2 xdx = \int \frac{1+\cos 2x}{2}dx = \frac{1}{2}\int dx + \frac{1}{2}\int \cos 2xdx = \frac{1}{2}\int dx + \frac{1}{4}\int \cos udu = \frac{1}{2}x + \frac{1}{4}\operatorname{sen} u + c$$

$$= \frac{1}{2}x + \frac{1}{4}\operatorname{sen} 2x + c$$

2.93.- $\int \sec^2(ax+b)dx$, Sea: $u=ax+b, du=adx$

$$\int \sec^2(ax+b)dx = \frac{1}{a}\int \sec^2 udu = \frac{1}{a}\tau gu + c = \frac{1}{a}\tau g(ax+b) = +c$$

2.94.- $\int \operatorname{co}\tau g^2 axdx$, Sea: $u=ax, du=adx$

$$\int \operatorname{co}\tau g^2 axdx = \frac{1}{a}\int \operatorname{co}\tau g^2 udu = \frac{1}{a}\int (\cos ec^2 u - 1)du = \frac{1}{a}\int \cos ec^2 udu - \frac{1}{a}\int du$$

$$= -\frac{\operatorname{co}\tau gu}{a} - \frac{u}{a} + c = -\frac{\operatorname{co}\tau gax}{a} - \frac{\cancel{a}x}{\cancel{a}} + c = -\frac{\operatorname{co}\tau gax}{a} - x + c$$

2.95.- $\int \frac{dx}{\operatorname{sen}\frac{x}{a}}$, Sea: $u = x/a, du = dx/a$

$$\int \frac{dx}{\operatorname{sen}\frac{x}{a}} = \int \cos ec\tfrac{x}{a}dx = a\int \cos ecudu = a\ell\eta\left|\cos ecu - \operatorname{co}\tau gu\right| + c$$

$$= a\ell\eta\left|\cos ec\, x/a - \operatorname{co}\tau g\, x/a\right| + c$$

2.96.- $\int \frac{dx}{3\cos(5x-\frac{\pi}{4})}$ **,** Sea: $u = 5x - \pi/4, du = 5dx$

$$\int \frac{dx}{3\cos(5x-\frac{\pi}{4})} = \frac{1}{3}\int \sec(5x-\tfrac{\pi}{4})dx = \frac{1}{15}\int \sec u du = \frac{1}{15}\ell\eta\left|\sec u + \tau g u\right| + c$$

$$= \frac{1}{15}\ell\eta\left|\sec(5x-\tfrac{\pi}{4}) + \tau g(5x-\tfrac{\pi}{4})\right| + c$$

2.97.- $\int \frac{dx}{\operatorname{sen}(ax+b)}$ **,** Sea: $u = ax+b, du = adx$

$$\int \frac{dx}{\operatorname{sen}(ax+b)} = \int \cos ec(ax+b)dx = \frac{1}{a}\int \cos ec u du = \frac{1}{a}\ell\eta\left|\cos ec u - \operatorname{co}\tau g u\right| + c$$

$$= \frac{1}{a}\ell\eta\left|\cos ec(ax+b) - \operatorname{co}\tau g(ax+b)\right| + c$$

2.98.- $\int \frac{xdx}{\cos^2 x^2}$ **,** Sea: $u = x^2, du = 2xdx$

$$\int \frac{xdx}{\cos^2 x^2} = \int x\sec^2 x^2 dx = \frac{1}{2}\int \sec^2 u du = \frac{1}{2}\tau g u + c = \frac{1}{2}\tau g x^2 + c$$

2.99.- $\int \operatorname{co}\tau g \frac{x}{a-b}dx$ **,** Sea: $u = \frac{x}{a-b}, du = \frac{dx}{a-b}$

$$\int \operatorname{co}\tau g \frac{x}{a-b}dx = (a-b)\int \operatorname{co}\tau g u du = (a-b)\ell\eta\left|\operatorname{sen} u\right| + c = (a-b)\ell\eta\left|\operatorname{sen}\frac{x}{a-b}\right| + c$$

2.100.- $\int \tau g\sqrt{x}\frac{dx}{\sqrt{x}}$ **,** Sea: $u = \sqrt{x}, du = \frac{dx}{2\sqrt{x}}$

$$\int \tau g\sqrt{x}\frac{dx}{\sqrt{x}} = 2\int \tau g u du = 2\ell\eta\left|\sec u\right| + c = 2\ell\eta\left|\sec\sqrt{x}\right| + c$$

2.101.- $\int \frac{dx}{\tau g \frac{x}{5}}$ **,** Sea: $u = x/5, du = dx/5$

$$\int \frac{dx}{\tau g \frac{x}{5}} = \int \operatorname{co}\tau g \tfrac{x}{5}dx = 5\int \operatorname{co}\tau g u du = 5\ell\eta\left|\operatorname{sen} u\right| + c = 5\ell\eta\left|\operatorname{sen} x/5\right| + c$$

2.102.- $\int \left(\frac{1}{\operatorname{sen} x\sqrt{2}} - 1\right)^2 dx$ **,** Sea: $u = x\sqrt{2}, du = \sqrt{2}dx$

$$\int \left(\frac{1}{\operatorname{sen} x\sqrt{2}} - 1\right)^2 dx = \int (\cos ecx\sqrt{2} - 1)^2 dx = \int (\cos ec^2 x\sqrt{2} - 2\cos ecx\sqrt{2} + 1)dx$$

$$= \int \cos ec^2 x\sqrt{2}dx - 2\int \cos ecx\sqrt{2}dx + \int dx = \frac{1}{\sqrt{2}}\int \cos ec^2 u du - \frac{2}{\sqrt{2}}\int \cos ec u du + \int dx$$

$$= -\frac{1}{\sqrt{2}}\operatorname{co}\tau g u - \sqrt{2}\ell\eta\left|\cos ec u - \operatorname{co}\tau g u\right| + x + c$$

$$= -\frac{1}{\sqrt{2}}\operatorname{co}\tau g x\sqrt{2} - \sqrt{2}\ell\eta\left|\cos ecx\sqrt{2} - \operatorname{co}\tau g x\sqrt{2}\right| + x + c$$

2.103.- $\int \frac{dx}{\operatorname{sen} x \cos x}$ **,** Sea: $u = 2x, du = 2dx$

$$\int \frac{dx}{\operatorname{sen} x \cos x} = \int \frac{dx}{\frac{1}{2}\operatorname{sen} 2x} = 2\int \cos ec2xdx = \int \cos ecudu = \ell\eta\left|\cos ecu - \operatorname{co}\tau gu\right| + c$$

$$= \ell\eta\left|\cos ec2x - \operatorname{co}\tau g2x\right| + c$$

2.104.- $\int \frac{\cos ax}{\operatorname{sen}^5 ax}dx$ **,** Sea: $u = \operatorname{sen} ax, du = a\cos axdx$

$$\int \frac{\cos ax}{\operatorname{sen}^5 ax}dx = \frac{1}{a}\int \frac{du}{u^5} = \frac{1}{a}\frac{u^{-4}}{-4} + c = -\frac{u^{-4}}{4a} + c = -\frac{\operatorname{sen}^{-4} ax}{4a} + c = -\frac{1}{4a\operatorname{sen}^4 ax} + c$$

2.105.- $\int t\operatorname{sen}(1-2t^2)dt$ **,** Sea: $u = 1-2t^2, du = -4tdt$

$$\int t\operatorname{sen}(1-2t^2)dt = -\frac{1}{4}\int \operatorname{sen} udu = \frac{1}{4}\cos u + c = \frac{1}{4}\cos(1-2t^2) + c$$

2.106.- $\int \frac{\operatorname{sen} 3x}{3+\cos 3x}dx$ **,** Sea: $u = 3+\cos 3x, du = -3\operatorname{sen} 3xdx$

$$\int \frac{\operatorname{sen} 3x}{3+\cos 3x}dx = -\frac{1}{3}\int \frac{du}{u} = -\frac{1}{3}\ell\eta\left|u\right| + c = -\frac{1}{3}\ell\eta\left|3+\cos 3x\right| + c$$

2.107.- $\int \tau g^3 \frac{x}{3}\sec^2 \frac{x}{3}dx$ **,** Sea: $u = \tau g(\frac{x}{3}), du = \frac{1}{3}\sec^2(\frac{x}{3})dx$

$$\int \tau g^3 \frac{x}{3}\sec^2 \frac{x}{3}dx = 3\int u^3du = \frac{3u^4}{4} + c = \frac{3\tau g^4(\frac{x}{3})}{4} + c$$

2.108.- $\int \frac{\operatorname{sen} x\cos x}{\sqrt{\cos^2 x - \operatorname{sen}^2 x}}dx$ **,** Sea: $u = \cos 2x, du = 2\operatorname{sen} 2xdx$

$$\int \frac{\operatorname{sen} x\cos x}{\sqrt{\cos^2 x - \operatorname{sen}^2 x}}dx = \int \frac{\operatorname{sen} x\cos x}{\sqrt{\cos 2x}}dx = \frac{1}{4}\int \frac{\operatorname{sen} 2x}{\sqrt{\cos 2x}} = \frac{1}{4}\int \frac{du}{\sqrt{u}} = \frac{1}{4}\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + c = \frac{u^{\frac{1}{2}}}{2} + c$$

$$= \frac{\sqrt{\cos 2x}}{2} + c$$

2.109.- $\int \frac{\sqrt{\tau gx}}{\cos^2 x}dx$ **,** Sea: $u = \tau gx, du = \sec^2 xdx$

$$\int \frac{\sqrt{\tau gx}}{\cos^2 x}dx = \int \sqrt{\tau gx}\sec^2 xdx = \int u^{\frac{1}{2}}du = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2}{3}u^{\frac{3}{2}} + c = \frac{2}{3}\tau g^{\frac{3}{2}}x + c$$

2.110.- $\int \cos\frac{x}{a}\operatorname{sen}\frac{x}{a}dx$ **,** Sea: $u = \frac{2x}{a}, du = 2dx$

$$\int \cos\frac{x}{a}\operatorname{sen}\frac{x}{a}dx = \frac{1}{2}\int \operatorname{sen}\frac{2x}{a}dx = \frac{a}{4}\int \operatorname{sen} udu = -\frac{a}{4}\cos u + c = -\frac{a}{4}\cos\frac{2x}{a} + c$$

2.111.- $\int t\operatorname{co}\tau g(2t^2-3)dt$ **,** Sea: $u = 2t^3 - 3, du = 4tdt$

$$\int t\operatorname{co}\tau g(2t^2-3)dt = \frac{1}{4}\int \operatorname{co}\tau gudu = \frac{1}{4}\ell\eta\left|\operatorname{sen} u\right| + c = \frac{1}{4}\ell\eta\left|\operatorname{sen}(2t^2-3)\right| + c$$

2.112.- $\int \frac{x^3dx}{x^8+5}$, Sea: $u = x^4, du = 4x^3dx$

$$\int \frac{x^3dx}{x^8+5} = \int \frac{x^3dx}{(x^4)^2+(\sqrt{5})^2} = \frac{1}{4}\int \frac{du}{u^2+(\sqrt{5})^2} = \frac{1}{4}\frac{1}{\sqrt{5}}\operatorname{arc}\tau g\,\frac{u}{\sqrt{5}} + c = \frac{\sqrt{5}}{20}\operatorname{arc}\tau g\,\frac{x^4}{\sqrt{5}} + c$$

2.113.- $\int \operatorname{sen}^3 6x\cos 6xdx$, Sea: $u = \operatorname{sen} 6x, du = 6\cos 6xdx$

$$\int \operatorname{sen}^3 6x\cos 6xdx = \frac{1}{6}\int u^3du = \frac{1}{6}\frac{u^4}{4} + c = \frac{u^4}{24} + c = \frac{\operatorname{sen}^4 6x}{24} + c$$

2.114.- $\int \sqrt{1+3\cos^2 x}\,\operatorname{sen} 2xdx$, Sea: $u = \frac{5+3\cos 2x}{2}, du = -3\operatorname{sen} 2xdx$

$$\int \sqrt{1+3\cos^2 x}\,\operatorname{sen} 2xdx = \int \sqrt{1+3(\frac{1+\cos 2x}{2})}\,\operatorname{sen} 2xdx = \int \sqrt{1+\frac{3+3\cos 2x}{2}}\,\operatorname{sen} 2xdx$$

$$= \int \sqrt{\frac{5+3\cos 2x}{2}}\,\operatorname{sen} 2xdx = -\frac{1}{3}\int u^{\frac{1}{2}}du = -\frac{1}{3}\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + c = -\frac{2}{9}u^{\frac{3}{2}} + c$$

$$= -\frac{2}{9}\left(\frac{5+3\cos 2x}{2}\right)^{\frac{3}{2}} + c$$

2.115.- $\int x\sqrt[5]{5-x^2}\,dx$, Sea: $u = 5-x^2, du = -2xdx$

$$\int x\sqrt[5]{5-x^2}\,dx = -\frac{1}{2}\int u^{\frac{1}{5}}du = -\frac{1}{2}\frac{u^{\frac{6}{5}}}{\frac{6}{5}} + c = -\frac{5}{12}u^{\frac{6}{5}} + c = -\frac{5(5-x^2)^{\frac{6}{5}}}{12} + c$$

2.116.- $\int \frac{1+\operatorname{sen} 3x}{\cos^2 3x}dx$, Sea: $u = \operatorname{sen} 3x, du = 3dx; w = \cos u, dw = -\operatorname{sen} udu$

$$\int \frac{1+\operatorname{sen} 3x}{\cos^2 3x}dx = \int \frac{dx}{\cos^2 3x} + \int \frac{\operatorname{sen} 3x}{\cos^2 3x}dx = \frac{1}{3}\int \sec^2 udu + \frac{1}{3}\int \frac{\operatorname{sen} u}{\cos^2 u}du$$

$$= \frac{1}{3}\int \sec^2 udu - \frac{1}{3}\int \frac{dw}{w^2} = \frac{1}{3}\tau gu + \frac{1}{3w} + c = \frac{1}{3}\tau gu + \frac{1}{3\cos u} + c = \frac{1}{3}\tau g3x + \frac{1}{3\cos 3x} + c$$

2.117.- $\int \frac{(\cos ax+\operatorname{sen} ax)^2}{\operatorname{sen} ax}dx$, Sea: $u = ax, du = adx$

$$\int \frac{(\cos ax+\operatorname{sen} ax)^2}{\operatorname{sen} ax}dx = \int \frac{\cos^2 ax + 2\cos ax\operatorname{sen} ax + \operatorname{sen}^2 ax}{\operatorname{sen} ax}dx$$

$$= \int \frac{\cos^2 ax}{\operatorname{sen} ax}dx + 2\int \frac{\cos ax\,\cancel{\operatorname{sen} ax}}{\cancel{\operatorname{sen} ax}}dx + \int \frac{\cancel{\operatorname{sen}^2 ax}}{\cancel{\operatorname{sen} ax}}dx$$

$$= \int \frac{1-\operatorname{sen}^2 ax}{\operatorname{sen} ax}dx + 2\int \cos axdx + \int \operatorname{sen} axdx$$

$$= \int \frac{dx}{\operatorname{sen} ax} + 2\int \cos axdx$$

$$= \int \operatorname{cosec} axdx + 2\int \cos axdx = \frac{1}{a}\int \operatorname{cosec} udu + \frac{2}{a}\int \cos udu$$

$$=\frac{1}{a}\ell\eta\left|\cos ecu-\operatorname{co}\tau gu\right|+\frac{2}{a}\operatorname{sen}u+c=\frac{1}{a}\ell\eta\left|\cos ecax-\operatorname{co}\tau gax\right|+\frac{2}{a}\operatorname{sen}ax+c$$

2.118.- $\int\frac{x^3-1}{x+1}dx$ **,** Sea: $u=x+1, du=dx$

$$\int\frac{x^3-1}{x+1}dx=\int(x^2-x+1-\frac{2}{x+1})dx=\int x^2dx-\int xdx+\int dx-\int\frac{2}{x+1}dx$$

$$=\int x^2dx-\int xdx+\int dx-2\int\frac{du}{u}=\frac{x^3}{3}-\frac{x^2}{2}+x-2\ell\eta\left|x+1\right|+c$$

2.119.- $\int\frac{\cos ec^2 3xdx}{b-a\operatorname{co}\tau g3x}$ **,** Sea: $u=b-a\operatorname{co}\tau g3x, du=3a\cos ec^2 3xdx$

$$\int\frac{\cos ec^2 3xdx}{b-a\operatorname{co}\tau g3x}=\frac{1}{3a}\int\frac{du}{u}=\frac{1}{3a}\ell\eta\left|u\right|+c=\frac{1}{3a}\ell\eta\left|b-a\operatorname{co}\tau g3x\right|+c$$

2.120.- $\int\frac{x^3-1}{x^4-4x+1}dx$ **,** Sea: $u=x^4-4x+1, du=(4x^3-4)dx$

$$\int\frac{x^3-1}{x^4-4x+1}dx=\frac{1}{4}\int\frac{(4x^3-4)dx}{x^4-4x+1}=\frac{1}{4}\int\frac{du}{u}=\frac{1}{4}\ell\eta\left|u\right|+c=\frac{1}{4}\ell\eta\left|x^4-4x+1\right|+c$$

2.121.- $\int xe^{-x^2}dx$ **,** Sea: $u=-x^2, du=-2xdx$

$$\int xe^{-x^2}dx=-\frac{1}{2}\int e^u du=-\frac{1}{2}e^u+c=-\frac{1}{2}e^{-x^2}+c$$

2.122.- $\int\frac{3-\sqrt{2+3x^2}}{2+3x^2}dx$ **,** Sea: $u=x\sqrt{3}, du=\sqrt{3}dx; a=\sqrt{2}$

$$\int\frac{3-\sqrt{2+3x^2}}{2+3x^2}dx=3\int\frac{dx}{(\sqrt{2})^2+(\sqrt{3}x)^2}-\int\frac{(2+3x^2)^{\frac{1}{2}}}{2+3x^2}dx$$

$$\frac{3}{\sqrt{3}}\int\frac{\sqrt{3}dx}{(\sqrt{2})^2+(\sqrt{3}x)^2}-\int\frac{\cancel{(2+3x^2)}^{\frac{1}{2}}}{\cancel{2+3x^2}}dx=\frac{3}{\sqrt{3}}\int\frac{\sqrt{3}dx}{(\sqrt{2})^2+(\sqrt{3}x)^2}-\int(2+3x^2)^{-\frac{1}{2}}dx$$

$$=\frac{3}{\sqrt{3}}\int\frac{du}{(a)^2+(u)^2}-\int(2+3x^2)^{-\frac{1}{2}}dx=\sqrt{3}\int\frac{du}{(a)^2+(u)^2}-\int\frac{dx}{\sqrt{(\sqrt{2})^2+(x\sqrt{3})^2}}$$

$$=\sqrt{3}\int\frac{du}{(a)^2+(u)^2}-\frac{1}{\sqrt{3}}\int\frac{du}{\sqrt{a^2+u^2}}=\frac{\sqrt{3}}{a}\operatorname{arc}\tau g\frac{u}{a}-\frac{1}{\sqrt{3}}\ell\eta\left|u+\sqrt{a^2+u^2}\right|+c$$

$$=\frac{\sqrt{3}}{\sqrt{2}}\operatorname{arc}\tau g\frac{x\sqrt{3}}{\sqrt{2}}-\frac{\sqrt{3}}{3}\ell\eta\left|x\sqrt{3}+\sqrt{2+3+x^2}\right|+c$$

2.123.- $\int\frac{\tau g3x-\operatorname{co}\tau g3x}{\operatorname{sen}3x}dx$ **,** Sea: $u=3x, du=3dx; w=\operatorname{sen}u, dw=\cos udu$

$$\int\frac{\tau g3x-\operatorname{co}\tau g3x}{\operatorname{sen}3x}dx=\int\frac{\frac{\operatorname{sen}3x}{\cos 3x}-\frac{\cos 3x}{\operatorname{sen}3x}}{\operatorname{sen}3x}dx=\int\frac{dx}{\cos 3x}-\int\frac{\cos 3x}{\operatorname{sen}^2 3x}dx$$

$$=\int \sec 3x dx-\int \frac{\cos 3x}{\operatorname{sen}^2 3x}dx=\frac{1}{3}\int \sec u du-\frac{1}{3}\int \frac{\cos u}{\operatorname{sen}^2 u}du=\frac{1}{3}\int \sec u du-\frac{1}{3}\int \frac{dw}{w^2}$$

$$=\frac{1}{3}\ell\eta\left|\sec u+\tau g u\right|-\frac{1}{3}\frac{w^{-1}}{-1}+c=\frac{1}{3}\ell\eta\left|\sec 3x+\tau g 3x\right|+\frac{1}{3\operatorname{sen}3x}+c$$

2.124.- $\int \frac{dx}{\sqrt{e^x}}$, Sea: $u=-\frac{x}{2}, du=-\frac{dx}{2}$

$$\int \frac{dx}{\sqrt{e^x}}=\int \frac{dx}{(e^x)^{1/2}}=\int e^{-x/2}dx=-2\int e^u du=-2e^u+c=-2e^{-x/2}+c=\frac{-2}{e^{x/2}}+c=\frac{-2}{\sqrt{e^x}}+c$$

2.125.- $\int \frac{1+\operatorname{sen}x}{x+\cos x}dx$, Sea: $u=x+\cos x, du=(1-\operatorname{sen}x)dx$

$$\int \frac{1+\operatorname{sen}x}{x+\cos x}dx=\int \frac{du}{u}=\ell\eta\left|u\right|+c=\ell\eta\left|x+\cos x\right|+c$$

2.126.- $\int \frac{\sec^2 x dx}{\sqrt{\tau g^2 x-2}}$, Sea: $u=\tau g x, du=\sec^2 x dx$

$$\int \frac{\sec^2 x dx}{\sqrt{\tau g^2 x-2}}=\int \frac{du}{\sqrt{u^2-2}}=\ell\eta\left|u+\sqrt{u^2-2}\right|+c=\ell\eta\left|\tau g x+\sqrt{\tau g x^2-2}\right|+c$$

2.127.- $\int \frac{dx}{x\ell\eta^2 x}$, Sea: $u=\ell\eta x, du=\frac{dx}{2}$

$$\int \frac{dx}{x\ell\eta^2 x}=\int \frac{dx}{x(\ell\eta x)^2}=\int \frac{du}{u^2}=\frac{u^{-1}}{-1}+c=-\frac{1}{u}+c=-\frac{1}{\ell\eta\left|x\right|}+c$$

2.128.- $\int a^{\operatorname{sen}x}\cos x dx$, Sea: $u=\operatorname{sen}x, du=\cos x dx$

$$\int a^{\operatorname{sen}x}\cos x dx=\int a^u du=\frac{a^u}{\ell\eta a}+c=\frac{a^{\operatorname{sen}x}}{\ell\eta a}+c$$

2.129.- $\int \frac{x^2}{\sqrt{x^3+1}}dx$, Sea: $u=x^3+1, du=3x^2dx$

$$\int \frac{x^2dx}{\sqrt{x^3+1}}=\int \frac{x^2dx}{(x^3+1)^{1/3}}=\frac{1}{3}\int \frac{du}{u^{1/3}}=\frac{1}{\cancel{3}}\frac{u^{2/3}}{2/\cancel{3}}+c=\frac{u^{2/3}}{2}+c=\frac{(x^2+1)^{2/3}}{2}+c=\frac{\sqrt[3]{(x^2+1)^2}}{2}+c$$

2.130.- $\int \frac{xdx}{\sqrt{1-x^4}}$, Sea: $u=x^2, du=2xdx$

$$\int \frac{xdx}{\sqrt{1-x^4}}=\int \frac{xdx}{\sqrt{1-(x^2)^2}}=\frac{1}{2}\int \frac{2xdx}{\sqrt{1-(x^2)^2}}=\frac{1}{2}\int \frac{2xdx}{\sqrt{1-(u)^2}}=\frac{1}{2}\operatorname{arcsen}u+c$$

$$=\frac{1}{2}\operatorname{arcsen}x^2+c$$

2.131.- $\int \tau g^2 ax dx$, Sea: $u=ax, du=adx$

$$\int \tau g^2 axdx = \int (\sec^2 ax - 1)dx = \int \sec^2 axdx - \int dx = \frac{1}{a}\int \sec^2 udu - \int dx = \frac{1}{a}\tau gu - x + c$$
$$= \frac{1}{a}\tau gax - x + c$$

2.132.- $\int \frac{\sec^2 xdx}{\sqrt{4-\tau g^2 x}}$**,** Sea: $u = \tau gx, du = \sec^2 xdx$

$$\int \frac{\sec^2 xdx}{\sqrt{4-\tau g^2 x}} = \int \frac{du}{\sqrt{2^2 - u^2}} = \operatorname{arcs} e\,\mathrm{n}\frac{u}{2} + c = \operatorname{arcs} e\,\mathrm{n}\frac{\tau gx}{2} + c$$

2.133.- $\int \frac{dx}{\cos x/a}$**,** Sea: $u = x/a, du = dx/a$

$$\int \frac{dx}{\cos x/a} = \int \sec x/a\, dx = a\int \sec udu = a\ell\eta\left|\sec u + \tau gu\right| + c = a\ell\eta\left|\sec x/a + \tau g\, x/a\right| + c$$

2.134.- $\int \frac{\sqrt[3]{1+\ell\eta x}}{x}dx$**,** Sea: $u = 1 + \ell\eta x, du = \frac{dx}{x}$

$$\int \frac{\sqrt[3]{1+\ell\eta x}}{x}dx = \int u^{1/3}du = \frac{u^{4/3}}{4/3} + c = \frac{3u^{4/3}}{4} + c = \frac{3(1+\ell\eta x)^{4/3}}{4} + c$$

2.135.- $\int \tau g\sqrt{x-1}\frac{dx}{\sqrt{x-1}}$**,** Sea: $u = \sqrt{x-1}, du = \frac{dx}{2\sqrt{x-1}}$

$$\int \tau g\sqrt{x-1}\frac{dx}{\sqrt{x-1}} = 2\int \tau gu\frac{du}{u} = 2\ell\eta\left|\sec\sqrt{x-1}\right| + c = -2\ell\eta\left|\cos\sqrt{x-1}\right| + c$$

2.136.- $\int \frac{xdx}{\mathrm{s}e\,\mathrm{n}\,x^2}$**,** Sea: $u = x^2, du = 2xdx$

$$\int \frac{xdx}{\mathrm{s}e\,\mathrm{n}\,x^2} = \frac{1}{2}\int \frac{du}{\mathrm{s}e\,\mathrm{n}\,u} = \frac{1}{2}\int \cos ecudu = \frac{1}{2}\ell\eta\left|\cos ecu - \mathrm{co}\,\tau gu\right| + c$$
$$= \frac{1}{2}\ell\eta\left|\cos ecx^2 - \mathrm{co}\,\tau gx^2\right| + c$$

2.137.- $\int \frac{\mathrm{s}e\,\mathrm{n}\,x - \cos x}{\mathrm{s}e\,\mathrm{n}\,x + \cos x}dx$**,** Sea: $u = \mathrm{s}e\,\mathrm{n}\,x + \cos x, du = (\cos x - \mathrm{s}e\,\mathrm{n}\,x)dx$

$$\int \frac{\mathrm{s}e\,\mathrm{n}\,x - \cos x}{\mathrm{s}e\,\mathrm{n}\,x + \cos x}dx = -\int \frac{du}{u} = -\ell\eta\left|\mathrm{s}e\,\mathrm{n}\,x + \cos x\right| + c$$

2.138.- $\int \frac{e^{\operatorname{arc}\tau gx} + x\ell\eta(1+x^2) + 1}{1+x^2}$**,** Sea: $u = \operatorname{arc}\tau gx, du = \frac{dx}{1+x^2}; w = \ell\eta(1+x^2)d, dw = \frac{2xdx}{1+x^2}$

$$\int \frac{e^{\operatorname{arc}\tau gx} + x\ell\eta(1+x^2) + 1}{1+x^2} = \int \frac{e^{\operatorname{arc}\tau gx}dx}{1+x^2} + \int \frac{x\ell\eta(1+x^2)dx}{1+x^2} + \int \frac{dx}{1+x^2}$$
$$= \int e^u du + \frac{1}{2}\int wdw + \int \frac{dx}{1+x^2} = e^u + \frac{1}{2}\frac{w^2}{2} + \operatorname{arc}\tau gx + c = e^u + \frac{\ell\eta^2(1+x^2)}{4} + \operatorname{arc}\tau gx + c$$

2.139.- $\int \frac{x^2dx}{x^2-2}$**,**

$$\int \frac{x^2dx}{x^2-2}=\int (1+\frac{2}{x^2-2})dx=\int dx+2\int \frac{dx}{x^2-2}=x+2\frac{1}{2\sqrt{2}}\ell\eta\left|\frac{x-\sqrt{2}}{x+\sqrt{2}}\right|+c$$
$$=x+\frac{\sqrt{2}}{2}\ell\eta\left|\frac{x-\sqrt{2}}{x+\sqrt{2}}\right|+c$$

2.140.- $\int e^{\operatorname{sen}^2 x}\operatorname{sen}2xdx$ **,** Sea: $u=\frac{1-\cos 2x}{2}, du=\operatorname{sen}2xdx$

$$\int e^{\operatorname{sen}^2 x}\operatorname{sen}2xdx=\int e^{\frac{1-\cos 2x}{2}}\operatorname{sen}2xdx=\int e^u du=e^u+c=e^{\operatorname{sen}^2 x}+c$$

2.141.- $\int \frac{(1-\operatorname{sen}\frac{x}{\sqrt{2}})^2}{\operatorname{sen}\frac{x}{\sqrt{2}}}dx$ **,** Sea: $u=\frac{x}{\sqrt{2}}, du=\frac{dx}{\sqrt{2}}$

$$\int \frac{(1-\operatorname{sen}\frac{x}{\sqrt{2}})^2}{\operatorname{sen}\frac{x}{\sqrt{2}}}dx=\int\left(\frac{1-2\operatorname{sen}\frac{x}{\sqrt{2}}+\operatorname{sen}^2\frac{x}{\sqrt{2}}}{\operatorname{sen}\frac{x}{\sqrt{2}}}\right)dx=\int \operatorname{cosec}\frac{x}{\sqrt{2}}dx-2\int dx+\int \operatorname{sen}\frac{x}{\sqrt{2}}dx$$
$$=\sqrt{2}\int \operatorname{cosec}udu-2\int dx+\sqrt{2}\int \operatorname{sen}udu=\sqrt{2}\ell\eta\left|\operatorname{cosec}u-\operatorname{co}\tau gu\right|-2x-\sqrt{2}\cos u+c$$
$$=\sqrt{2}\ell\eta\left|\operatorname{cosec}\tfrac{x}{\sqrt{2}}-\operatorname{co}\tau g\tfrac{x}{\sqrt{2}}\right|-2x-\sqrt{2}\cos\tfrac{x}{\sqrt{2}}+c$$

2.142.- $\int \frac{5-3x}{\sqrt{4-3x^2}}dx$ **,** Sea: $u=x\sqrt{3}, du=\sqrt{3}dx; w=4-3x^2, dw=-6xdx$

$$\int \frac{5-3x}{\sqrt{4-3x^2}}dx=5\int \frac{dx}{\sqrt{4-3x^2}}-3\int \frac{xdx}{\sqrt{4-3x^2}}=5\int \frac{dx}{\sqrt{4-(x\sqrt{3})^2}}-3\int \frac{xdx}{\sqrt{4-3x^2}}$$
$$=\frac{5}{\sqrt{3}}\int \frac{du}{\sqrt{2^2-u^2}}+\frac{3}{6}\int \frac{dw}{\sqrt{w}}=\frac{5}{\sqrt{3}}\operatorname{arcsen}\frac{u}{2}+\frac{1}{2}\frac{w^{1/2}}{1/2}+c=\frac{5\sqrt{3}}{3}\operatorname{arcsen}\frac{x\sqrt{3}}{2}+\sqrt{4-3x^2}+c$$

2.143.- $\int \frac{ds}{e^s+1}$ **,** Sea: $u=1+e^{-s}, du=-e^{-s}ds$

$$\int \frac{ds}{e^s+1}=\int \frac{e^{-s}ds}{e^{-s}+1}=-\int \frac{du}{u}=-\ell\eta\left|u\right|+c=-\ell\eta\left|e^{-s}+1\right|+c$$

2.144.- $\int \frac{d\theta}{\operatorname{sen}a\theta\cos a\theta}$ **,** Sea: $u=2a\theta, du=2ad\theta$

$$\int \frac{d\theta}{\operatorname{sen}a\theta\cos a\theta}=\int \frac{d\theta}{\frac{1}{2}\operatorname{sen}2a\theta}=2\int \operatorname{cosec}2a\theta d\theta=\frac{2}{2a}\int \operatorname{cosec}udu$$
$$=\frac{1}{a}\ell\eta\left|\operatorname{cosec}u-\operatorname{co}\tau gu\right|+c=\frac{1}{a}\ell\eta\left|\operatorname{cosec}2a\theta-\operatorname{co}\tau g2a\theta\right|+c$$

2.145.- $\int \frac{e^s}{\sqrt{e^{2s}-2}}ds$ **,** Sea: $u=e^s, du=e^sds$

$$\int \frac{e^s}{\sqrt{e^{2s}-2}}ds=\int \frac{e^s}{\sqrt{(e^s)^2-2}}ds=-\int \frac{du}{\sqrt{u^2-2}}=\ell\eta\left|u+\sqrt{u^2-2}\right|+c$$
$$=\ell\eta\left|e^s+\sqrt{(e^s)^2-2}\right|+c=\ell\eta\left|e^s+\sqrt{e^{2s}-2}\right|+c$$

2.146.- $\int \operatorname{sen}(\tfrac{2\pi t}{T}+\varphi_0)dt$ **,** Sea: $u=\frac{2\pi t}{T}+\varphi_0, du=\frac{2\pi t}{T}dt$

$$\int \operatorname{sen}(\tfrac{2\pi t}{T}+\varphi_0)dt=\frac{T}{2\pi}\int \operatorname{sen} u du=-\frac{T}{2\pi}\cos u+c=-\frac{T}{2\pi}\cos(\frac{2\pi t}{T}+\varphi_0)+c$$

2.147.- $\int \frac{\arccos x/2}{\sqrt{4-x^2}}dx$ **,** Sea: $u=\arccos\frac{x}{2}, du=-\frac{dx}{\sqrt{4-x^2}}$

$$\int \frac{\arccos x/2}{\sqrt{4-x^2}}dx=-\int udu=-\frac{u^2}{2}+c=-\frac{(\arccos x/2)^2}{2}+c$$

2.148.- $\int \frac{dx}{x(4-\ell\eta^2 x)}$ **,** Sea: $u=\ell\eta x, du=\frac{dx}{x}$

$$\int \frac{dx}{x(4-\ell\eta^2 x)}=\int \frac{dx}{x\left[2^2-(\ell\eta x)^2\right]}=\int \frac{du}{2^2-u^2}=\frac{1}{4}\ell\eta\left|\frac{2+u}{2-u}\right|+c=\frac{1}{4}\ell\eta\left|\frac{2+\ell\eta x}{2-\ell\eta x}\right|+c$$

2.149.- $\int e^{-\tau gx}\sec^2 xdx$ **,** Sea: $u=-\tau gx, du=-\sec^2 xdx$

$$\int e^{-\tau gx}\sec^2 xdx=-\int e^u du=-e^u+c=-e^{-\tau gx}+c$$

2.150.- $\int \frac{\operatorname{sen} x\cos x}{\sqrt{2-\operatorname{sen}^4 x}}dx$ **,** Sea: $u=\operatorname{sen}^2 x, du=2\operatorname{sen} x\cos xdx$

$$\int \frac{\operatorname{sen} x\cos x}{\sqrt{2-\operatorname{sen}^4 x}}dx=\int \frac{\operatorname{sen} x\cos x}{\sqrt{2-(\operatorname{sen}^2 x)^2}}dx=\frac{1}{2}\int \frac{du}{\sqrt{2-u^2}}=\frac{1}{2}\operatorname{arcsen}\frac{u}{\sqrt{2}}+c$$

$$=\frac{1}{2}\operatorname{arcsen}\frac{(\operatorname{sen}^2 x)}{\sqrt{2}}+c$$

2.151.- $\int \frac{\sec x\tau gx}{\sqrt{\sec^2 x+1}}dx$ **,** Sea: $u=\sec x, du=\sec x\tau gxdx$

$$\int \frac{\sec x\tau gx}{\sqrt{\sec^2 x+1}}dx=\int \frac{du}{\sqrt{u^2+1}}=\ell\eta\left|u+\sqrt{u^2+1}\right|+c=\ell\eta\left|\sec x+\sqrt{\sec^2 x+1}\right|+c$$

2.152.- $\int \frac{dt}{\operatorname{sen}^2 t\cos^2 t}$ **,** Sea: $u=2t, du=2dt$

$$\int \frac{dt}{\operatorname{sen}^2 t\cos^2 t}=\int \frac{dt}{(\operatorname{sen} t\cos t)^2}=\int \frac{dt}{(\frac{1}{2}\operatorname{sen} 2t)^2}=4\int \frac{dt}{\operatorname{sen}^2 2t}=4\int \operatorname{cosec}^2 2tdt$$

$$=2\int \operatorname{cosec}^2 udu=-2\operatorname{co}\tau gu+c=-2\operatorname{co}\tau g2t+c$$

2.153.- $\int \frac{\operatorname{arcsen} x+x}{\sqrt{1-x^2}}dx$ **,**

Sea: $u=\operatorname{arcsen} x, du=\frac{dx}{\sqrt{1-x^2}}; w=1-x^2, dw=-2xdx$

$$\int \frac{\operatorname{arcsen} x+x}{\sqrt{1-x^2}}dx=\int \frac{\operatorname{arcsen} x}{\sqrt{1-x^2}}dx+\int \frac{x}{\sqrt{1-x^2}}dx=\int udu-\frac{1}{2}\int \frac{dw}{\sqrt{w}}=\int udu-\frac{1}{2}\int w^{-1/2}dw$$

$$=\frac{u^2}{2}-\frac{1}{2}\frac{w^{1/2}}{1/2}+c=\frac{(\arcsin x)^2}{2}-\sqrt{1-x^2}+c$$

2.154.- $\int\frac{xdx}{\sqrt{x+1}}$, Sea: $t=\sqrt{x+1}\Rightarrow x=t^2-1; dx=2tdt$

$$\int\frac{xdx}{\sqrt{x+1}}=\int\frac{(t^2-1)2tdt}{t}=2\int(t^2-1)dt=2(\frac{t^3}{3}-t)+c=\frac{2\sqrt{(x+1)^3}}{3}-2\sqrt{x+1}+c$$

2.155.- $\int x(5x^2-3)^7dx$, Sea: $u=5x^2-3, du=10xdx$

$$\int x(5x^2-3)^7dx=\frac{1}{10}\int u^7du=\frac{1}{10}\frac{u^8}{8}+c=\frac{u^8}{80}+c=\frac{(5x^2-3)^8}{80}+c$$

2.156.- $\int\sqrt{\frac{\ell\eta(x+\sqrt{x^2+1})}{x^2+1}}dx$, Sea: $u=\ell\eta(x+\sqrt{x^2+1}), du=\frac{dx}{\sqrt{x^2+1}}$

$$\int\sqrt{\frac{\ell\eta(x+\sqrt{x^2+1})}{x^2+1}}dx=\int\frac{\sqrt{\ell\eta(x+\sqrt{x^2+1})}}{\sqrt{x^2+1}}dx=\int\sqrt{u}du=\frac{u^{3/2}}{3/2}+c$$

$$=\frac{2\sqrt{\left[\ell\eta(x+\sqrt{x^2+1})\right]^3}}{3}+c$$

2.157.- $\int\frac{\operatorname{sen}^3x}{\sqrt{\cos x}}dx$, Sea: $u=\cos x, du=-\operatorname{sen}xdx$

$$\int\frac{\operatorname{sen}^3x}{\sqrt{\cos x}}dx=\int\frac{\operatorname{sen}^2x\operatorname{sen}xdx}{\sqrt{\cos x}}=\int\frac{(1-\cos^2x)\operatorname{sen}xdx}{\sqrt{\cos x}}=\int\frac{\operatorname{sen}xdx}{\sqrt{\cos x}}-\int\frac{\cos^2x\operatorname{sen}xdx}{\sqrt{\cos x}}$$

$$=\int\cos^{-1/2}x\operatorname{sen}xdx-\int\cos^{3/2}x\operatorname{sen}xdx=-\int u^{1/2}du+\int u^{3/2}du=-\frac{u^{3/2}}{3/2}+\frac{u^{5/2}}{5/2}+c$$

$$=-\frac{2u^{3/2}}{3}+\frac{2u^{5/2}}{5}+c=-\frac{2\cos x^{3/2}}{3}+\frac{2\cos x^{5/2}}{5}+c=-\frac{2\sqrt{\cos^3x}}{3}+\frac{2\sqrt{\cos^5x}}{5}+c$$

2.158.- $\int\frac{\cos xdx}{\sqrt{1+\operatorname{sen}^2x}}$,

Sea: $t=\sqrt{1+\operatorname{sen}^2x}\Rightarrow\operatorname{sen}^2x=t^2-1; 2\operatorname{sen}x\cos xdx=2tdt$

$$\int\frac{\cos xdx}{\sqrt{1+\operatorname{sen}^2x}}=\int\frac{\frac{t}{\sqrt{t^2-1}}}{t}=\int\frac{dt}{\sqrt{t^2-1}}=\ell\eta\left|\sqrt{1+\operatorname{sen}^2x}+\operatorname{sen}x\right|+c$$

2.159.- $\int\frac{(\arcsin x)^2}{\sqrt{1-x^2}}dx$, Sea: $u=\arcsin x, du=\frac{dx}{\sqrt{1-x^2}}$

$$\int\frac{(\arcsin x)^2}{\sqrt{1-x^2}}dx=\int u^2du=\frac{u^3}{3}+c=\frac{(\arcsin x)^3}{3}+c$$

2.150.- $\int e^{x+e^x}dx$, Sea: $u=e^{e^x}, du=e^{e^x}e^xdx$

$$\int e^{x+e^x}dx=\int e^x e^{e^x}dx=\int du=u+c=e^{e^x}+c$$

2.161.- $\int t(4t+1)^7\,dt$ **,** Sea: $u=4t+1\Rightarrow t=\dfrac{u-1}{4},du=4dt$

$$\int t(4t+1)^7\,dt=\int\frac{u-1}{4}u^7\,\frac{du}{4}=\frac{1}{16}\int(u-1)u^7du=\frac{1}{16}\int(u^8-u^7)du=\frac{1}{16}\frac{u^9}{9}-\frac{1}{16}\frac{u^8}{8}+c$$
$$=\frac{(4t+1)^9}{144}-\frac{(4t+1)^8}{128}+c$$

2.162.- $\int\dfrac{2t^2-10t+12}{t^2+4}dt$ **,** Sea: $u=t^2+4,du=du=2tdt$

$$\int\frac{2t^2-10t+12}{t^2+4}dt=2\int\frac{t^2-5t+6}{t^2+4}dt=2\int\left(1+\frac{2-5t}{t^2+4}\right)dt=2\int dt+4\int\frac{dt}{t^2+4}-10\int\frac{dt}{t^2+4}$$
$$=2\int dt+4\int\frac{dt}{t^2+4}-5\int\frac{du}{u}=2t+2\operatorname{arc}\tau g\,\tfrac{t}{2}-5\ell\eta\left|u\right|+c=2t+2\operatorname{arc}\tau g\,\tfrac{t}{2}-5\ell\eta\left|t^2+4\right|+c$$

2.163.- $\int\dfrac{e^t-e^{-t}}{e^t+e^{-t}}dt$ **,**

Sea: $u=e^{2t}+1,du=2e^{2t}dt;w=1+e^{-2t},dw=-2e^{-2t}dt$

$$\int\frac{e^t-e^{-t}}{e^t+e^{-t}}dt=\int\frac{e^tdt}{e^t+e^{-t}}-\int\frac{e^{-t}dt}{e^t+e^{-t}}=\int\frac{e^{2t}dt}{e^{2t}+1}-\int\frac{e^{-2t}dt}{1+e^{-2t}}=\frac{1}{2}\int\frac{du}{u}+\frac{1}{2}\int\frac{dw}{w}$$
$$=\frac{1}{2}(\ell\eta\left|u\right|+\ell\eta\left|w\right|)+c=\frac{1}{2}\ell\eta\left|uw\right|+c=\frac{1}{2}\ell\eta(e^{2t}+1)(1+e^{-2t})+c$$

CAPITULO 3

INTEGRACION DE FUNCIONES TRIGONOMETRICAS

En esta parte, serán consideradas las integrales trigonométricas de la forma:
i) $\int \operatorname{sen}^m u \cos^n u du$
ii) $\int \tau g^m u \sec^n u du$
iii) $\int \operatorname{co}\tau g^m u \operatorname{cosec}^n u du$
O bien, formas trigonométricas reducibles a algunos de los casos ya señalados.

EJERCICIOS DESARROLLADOS

3.1.-Encontrar: $\int \cos^2 x dx$

Solución.- $\cos^2 x dx = \dfrac{1+\cos 2x}{2}$

Luego: $\int \cos^2 x dx = \int \dfrac{1+\cos 2x}{2} dx = \dfrac{1}{2}\int dx + \dfrac{1}{2}\int \cos 2x dx = \dfrac{x}{2} + \dfrac{1}{4}\operatorname{sen} 2x + c$,

Como: $\int \cosh x dx = \dfrac{1}{h}\operatorname{senh} x + c$

Respuesta: $\int \cos^2 x dx = \dfrac{1}{2}x + \dfrac{1}{4}\operatorname{sen} 2x + c$

3.2.-Encontrar: $\int \cos^4 \frac{1}{2} x dx$

Solución.- $\cos^2 \frac{1}{2} x = \dfrac{1+\cos x}{2}$

Luego: $\int \cos^4 \frac{1}{2} x dx = \int (\cos^2 \frac{1}{2} x)^2 dx = \int \left(\dfrac{1+\cos x}{2}\right)^2 dx = \dfrac{1}{4}\int (1 + 2\cos x + \cos^2 x) dx$

$= \dfrac{1}{4}\int dx + \dfrac{1}{2}\int \cos x dx + \dfrac{1}{4}\int \cos^2 x dx$, como: $\int \cos^2 x dx = \frac{1}{2} x + \frac{1}{4}\operatorname{sen} 2x + c$

$= \dfrac{1}{4}\int dx + \dfrac{1}{2}\int \cos x dx + \dfrac{1}{4}\int \cos^2 x dx = \dfrac{1}{4}x + \dfrac{1}{2}\operatorname{sen} x + \dfrac{1}{4}(\dfrac{1}{2}x + \dfrac{1}{4}\operatorname{sen} 2x) + c$

$= \dfrac{1}{4}x + \dfrac{1}{2}\operatorname{sen} x + \dfrac{1}{8}x + \dfrac{1}{16}\operatorname{sen} 2x + c = \dfrac{3}{8}x + \dfrac{1}{2}\operatorname{sen} x + \dfrac{1}{16}\operatorname{sen} 2x + c$

Respuesta: $\int \cos^4 \frac{1}{2} x dx = \dfrac{3}{8}x + \dfrac{1}{2}\operatorname{sen} x + \dfrac{1}{16}\operatorname{sen} 2x + c$

3.3.-Encontrar: $\int \cos^3 x dx$

Solución.- $\int \cos^3 x dx = \int \cos x \cos^2 x dx$, como: $\cos^2 x = 1 - \operatorname{sen}^2 x$

$$=\int \cos x\cos^2 xdx=\int \cos x(1-\operatorname{sen}^2 x)dx=\int \cos xdx-\int \cos x\operatorname{sen}^2 xdx$$

Sea: $u=\operatorname{sen} x, du=\cos xdx$

$$=\int \cos xdx-\int \cos x\operatorname{sen}^2 xdx=\int \cos xdx-\int u^2du=\operatorname{sen} x-\frac{u^3}{3}+c=\operatorname{sen} x-\frac{\operatorname{sen}^3 x}{3}+c$$

Respuesta: $\int \cos^3 xdx=\operatorname{sen} x-\frac{\operatorname{sen}^3 x}{3}+c$

3.4.-Encontrar: $\int \operatorname{sen} x^3 4xdx$

Solución.- $\int \operatorname{sen} x^3 4xdx=\int \operatorname{sen} 4x\operatorname{sen}^2 4xdx$, como: $\operatorname{sen}^2 4x=1-\cos^2 4x$

$$=\int \operatorname{sen} 4x\operatorname{sen}^2 4xdx=\int \operatorname{sen} 4x(1-\cos^2 4x)dx=\int \operatorname{sen} 4xdx-\int \operatorname{sen} 4x(\cos 4x)^2 dx$$

Sea: $u=\cos 4x, du=-4\operatorname{sen} 4xdx$

$$=\int \operatorname{sen} 4xdx+\frac{1}{4}\int u^2du=-\frac{1}{4}\cos 4x+\frac{1}{4}\frac{u^3}{3}+c=-\frac{\cos 4x}{4}+\frac{\cos^3 4x}{12}+c$$

Respuesta: $\int \operatorname{sen} x^3 4xdx=-\frac{\cos 4x}{4}+\frac{\cos^3 4x}{12}+c$

3.5.-Encontrar: $\int \operatorname{sen}^2 x\cos^3 xdx$

Solución.- $\int \operatorname{sen}^2 x\cos^3 xdx=\int \operatorname{sen}^2 x\cos^2 x\cos xdx=\int \operatorname{sen}^2 x(1-\operatorname{sen}^2 x)\cos xdx$

$=\int \operatorname{sen}^2 x\cos xdx-\int \operatorname{sen}^4 x\cos xdx$; Sea: $u=\operatorname{sen} x, du=\cos xdx$

$$=\int u^2du-\int u^4du=\frac{u^3}{3}-\frac{u^5}{5}+c=\frac{\operatorname{sen}^3 x}{3}-\frac{\operatorname{sen}^5 x}{5}+c$$

Respuesta: $\int \operatorname{sen}^2 x\cos^3 xdx=\frac{\operatorname{sen}^3 x}{3}-\frac{\operatorname{sen}^5 x}{5}+c$

3.6.-Encontrar: $\int \operatorname{sen}^3 x\cos^2 xdx$

Solución.- $\int \operatorname{sen}^3 x\cos^2 xdx=\int \operatorname{sen}^2 x\operatorname{sen} x\cos^2 xdx=\int (1-\cos^2 x)\operatorname{sen} x\cos^2 xdx$

$$=\int (1-\cos^2 x)\operatorname{sen} x\cos^2 xdx=\int \operatorname{sen} x\cos^2 xdx-\int \operatorname{sen} x\cos^4 xdx$$

Sea: $u=\cos x, du=-\operatorname{sen} xdx$

$$=\int \operatorname{sen} x\cos^2 xdx-\int \operatorname{sen} x\cos^4 xdx=-\int u^2du+\int u^4du=-\frac{u^3}{3}+\frac{u^5}{5}+c$$

$$=-\frac{\cos^3 x}{3}+\frac{\cos^5 x}{5}+c$$

Respuesta: $\int \operatorname{sen}^3 x\cos^2 xdx=-\frac{\cos^3 x}{3}+\frac{\cos^5 x}{5}+c$

3.7.-Encontrar: $\int \operatorname{sen}^2 x\cos^5 xdx$

Solución.- $\int \operatorname{sen}^2 x\cos^5 xdx=\int \operatorname{sen}^2 x(\cos^2 x)^2\cos xdx=\int \operatorname{sen}^2 x(1-\operatorname{sen}^2 x)^2\cos xdx$

$$=\int \operatorname{sen}^2 x(1-2\operatorname{sen}^2 x+\operatorname{sen}^4 x)\cos xdx$$

$=\int(\operatorname{sen} x)^2\cos xdx-2\int(\operatorname{sen} x)^4\cos xdx+\int(\operatorname{sen} x)^6\cos xdx$

Sea: $u=\operatorname{sen} x, du=\cos xdx$

$=\int u^2du-2\int u^4du+\int u^6du=\frac{u^3}{3}-2\frac{u^5}{5}+\frac{u^7}{7}+c=\frac{\operatorname{sen}^3 x}{3}-2\frac{\operatorname{sen}^5 x}{5}+\frac{\operatorname{sen}^7 x}{7}+c$

Respuesta: $\int\operatorname{sen}^2 x\cos^5 xdx=\frac{\operatorname{sen}^3 x}{3}-2\frac{\operatorname{sen}^5 x}{5}+\frac{\operatorname{sen}^7 x}{7}+c$

3.8.-Encontrar: $\int\operatorname{sen}^3 x\cos^3 xdx$

Solución.- $\int\operatorname{sen}^3 x\cos^3 xdx=\int(\operatorname{sen} x\cos x)^3dx$; como: $\operatorname{sen} 2x=2\operatorname{sen} x\cos x$,

Se tiene que: $\operatorname{sen} x\cos x=\frac{\operatorname{sen} 2x}{2}$; Luego:

$=\int(\operatorname{sen} x\cos x)^3dx=\int\left(\frac{\operatorname{sen} 2x}{2}\right)^3dx=\frac{1}{8}\int\operatorname{sen}^3 2xdx=\frac{1}{8}\int\operatorname{sen} 2x\operatorname{sen}^2 2xdx$

$=\frac{1}{8}\int\operatorname{sen} 2x(1-\cos^2 2x)dx=\frac{1}{8}\int\operatorname{sen} 2xdx-\frac{1}{8}\int\operatorname{sen} 2x(\cos 2x)^2dx$

Sea: $u=\cos 2x, du=-2\operatorname{sen} 2xdx$

$=\frac{1}{8}\int\operatorname{sen} 2xdx+\frac{1}{16}\int-2\operatorname{sen} 2x(\cos 2x)^2dx=\frac{1}{8}\int\operatorname{sen} 2xdx+\frac{1}{16}\int u^2du$

$=-\frac{1}{16}\cos 2x+\frac{1}{16}\frac{u^3}{3}+c=-\frac{1}{16}\cos 2x+\frac{\cos^3 2x}{48}+c$

Respuesta: $\int\operatorname{sen}^3 x\cos^3 xdx=-\frac{1}{16}\cos 2x+\frac{\cos^3 2x}{48}+c$

3.9.-Encontrar: $\int\operatorname{sen}^4 x\cos^4 xdx$

Solución.- $\int\operatorname{sen}^4 x\cos^4 xdx=\int(\operatorname{sen} x\cos x)^4dx=\int\left(\frac{\operatorname{sen} 2x}{2}\right)^4dx=\frac{1}{16}\int\operatorname{sen}^4 2xdx$

$=\frac{1}{16}\int(\operatorname{sen}^2 2x)^2dx=\frac{1}{16}\int\left(\frac{1-\cos 4x}{2}\right)^2dx=\frac{1}{16\times 4}\int(1-\cos 4x)^2dx$

$=\frac{1}{64}\int(1-2\cos 4x+\cos^2 4x)dx=\frac{1}{64}\int dx-\frac{1}{32}\int\cos 4xdx+\frac{1}{64}\int\cos^2 4xdx$

$=\frac{1}{64}\int dx-\frac{1}{32}\int\cos 4xdx+\frac{1}{64}\int\frac{1+\cos 8x}{2}dx$

$=\frac{1}{64}\int dx-\frac{1}{32}\int\cos 4xdx+\frac{1}{128}\int dx+\frac{1}{128}\int\cos 8xdx$

$=\frac{1}{64}x-\frac{1}{128}\operatorname{sen} 4x+\frac{1}{128}x+\frac{1}{1024}\operatorname{sen} 8x+c=\frac{3x}{128}-\frac{\operatorname{sen} 4x}{128}+\frac{\operatorname{sen} 8x}{1024}+c$

Respuesta: $\int\operatorname{sen}^4 x\cos^4 xdx=\frac{1}{128}\left(3x-\operatorname{sen} 4x+\frac{\operatorname{sen} 8x}{8}\right)+c$

3.10.-Encontrar: $\int x(\cos^3 x^2-\operatorname{sen}^3 x^2)dx$ **;** Sea: $u=x^2, du=2xdx$

$$\int x(\cos^3 x^2 - \operatorname{sen}^3 x^2)dx = \frac{1}{2}\int 2x(\cos^3 x^2 - \operatorname{sen}^3 x^2)dx = \frac{1}{2}\int (\cos^3 u - \operatorname{sen}^3 u)du$$

$$= \frac{1}{2}\int \cos^3 u du - \frac{1}{2}\int \operatorname{sen}^3 u du = \frac{1}{2}\int \cos u \cos^2 u du - \frac{1}{2}\int \operatorname{sen} u \operatorname{sen}^2 u du$$

$$= \frac{1}{2}\int \cos u(1 - \operatorname{sen}^2 u)du - \frac{1}{2}\int \operatorname{sen} u(1 - \cos^2 u)du$$

$$= \frac{1}{2}\int \cos u du - \frac{1}{2}\int \cos u \operatorname{sen}^2 u du - \frac{1}{2}\int \operatorname{sen} u du + \frac{1}{2}\int \operatorname{sen} u \cos^2 u du$$

Sea: $w = \operatorname{sen} u, dw = \cos u du; z = \cos u, dz = -\operatorname{sen} u du$

$$= \frac{1}{2}\int \cos u du - \frac{1}{2}\int w^2 dw - \frac{1}{2}\int \operatorname{sen} u du - \frac{1}{2}\int z^2 dz = \frac{1}{2}\operatorname{sen} u - \frac{1}{2}\frac{w^3}{3} + \frac{1}{2}\cos u - \frac{1}{2}\frac{z^3}{3} + c$$

$$= \frac{\operatorname{sen} u}{2} - \frac{\operatorname{sen}^3 u}{6} + \frac{\cos u}{2} - \frac{\cos^3 u}{6} + c = \frac{1}{2}(\operatorname{sen} u + \cos u) - \frac{1}{6}(\operatorname{sen}^3 u + \cos^3 u) + c$$

Dado que: $\operatorname{sen}^3 u + \cos^3 u = (\operatorname{sen} u + \cos u)(\operatorname{sen}^2 u - \operatorname{sen} u \cos u + \cos^2)$

O bien: $\operatorname{sen}^3 u + \cos^3 u = (\operatorname{sen} u + \cos u)(1 - \operatorname{sen} u \cos u)$; Lo que equivale a:

$$= \frac{1}{2}(\operatorname{sen} u + \cos u) - \frac{1}{6}(\operatorname{sen} u + \cos u)(1 - \operatorname{sen} u \cos u) + c$$

$$= \frac{1}{2}(\operatorname{sen} u + \cos u) - \frac{1}{6}(\operatorname{sen} u + \cos u)(1 - \frac{2\operatorname{sen} u \cos u}{2}) + c$$

$$= \frac{1}{2}(\operatorname{sen} u + \cos u) - \frac{1}{6}(\operatorname{sen} u + \cos u)(1 - \frac{\operatorname{sen} 2u}{2}) + c$$

$$= \frac{1}{2}(\operatorname{sen} u + \cos u) - \frac{1}{6}(\operatorname{sen} u + \cos u)\frac{1}{2}(2 - \operatorname{sen} 2u) + c$$

$$= \frac{1}{12}(\operatorname{sen} u + \cos u)(6 - (2 - \operatorname{sen} 2u)) + c = \frac{1}{12}(\operatorname{sen} u + \cos u)(4 + \operatorname{sen} 2u) + c$$

$$= \frac{1}{12}(\operatorname{sen} x^2 + \cos x^2)(4 + \operatorname{sen} 2x^2) + c$$

Respuesta: $\int x(\cos^3 x^2 - \operatorname{sen}^3 x^2)dx = \frac{1}{12}(\operatorname{sen} x^2 + \cos x^2)(4 + \operatorname{sen} 2x^2) + c$

3.11.-Encontrar: $\int \operatorname{sen} 2x \cos 4x dx$

Solución.- $\operatorname{sen}\alpha\cos\beta = \frac{1}{2}[\operatorname{sen}(\alpha - \beta) + \operatorname{sen}(\alpha + \beta)]$; Se tiene que:

$$\operatorname{sen} 2x \cos 4x = \frac{1}{2}[\operatorname{sen}(2x - 4x) + \operatorname{sen}(2x + 4x)] = \frac{1}{2}[\operatorname{sen}(-2x) + \operatorname{sen}(6x)]$$

$= \frac{1}{2}[-\operatorname{sen} 2x + \operatorname{sen} 6x]$, Luego: $\int \operatorname{sen} 2x \cos 4x dx = \int \frac{1}{2}(-\operatorname{sen} 2x + \operatorname{sen} 6x)dx$

$$= -\frac{1}{2}\int \operatorname{sen} 2x dx + \frac{1}{2}\int \operatorname{sen} 6x dx = \frac{1}{4}\cos 2x - \frac{1}{12}\cos 6x + c$$

Respuesta: $\int \operatorname{sen} 2x \cos 4x dx = \frac{1}{4}\cos 2x - \frac{1}{12}\cos 6x + c$

3.12.-Encontrar: $\int \cos 3x \cos 2x dx$

Solución.- $\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right]$; Se tiene que:

$\cos 3x \cos 2x = \frac{1}{2}\left[\cos(3x-2x)+\cos(3x+2x)\right] = \frac{1}{2}\left[\cos x + \cos 5x\right]$, Luego:

$= \int \cos 3x \cos 2x dx = \int \frac{1}{2}\left[\cos x + \cos 5x\right] dx = \frac{1}{2}\int \cos x dx + \frac{1}{2}\int \cos 5x dx$

$= \frac{1}{2}\operatorname{sen} x + \frac{1}{10}\operatorname{sen} 5x + c$

Respuesta: $\int \cos 3x \cos 2x dx = \frac{1}{2}\operatorname{sen} x + \frac{1}{10}\operatorname{sen} 5x + c$

3.13.-Encontrar: $\int \operatorname{sen} 5x \operatorname{sen} x dx$

Solución.-$\operatorname{sen}\alpha\operatorname{sen}\beta = \frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]$; Se tiene que:

$\operatorname{sen} 5x \operatorname{sen} x = \frac{1}{2}\left[\cos(5x-x)-\cos(5x+x)\right] = \frac{1}{2}\left[\cos 4x - \cos 6x\right]$; Luego:

$= \int \operatorname{sen} 5x \operatorname{sen} x dx = \int \frac{1}{2}\left[\cos 4x - \cos 6x\right] = \frac{1}{2}\int \cos 4x dx - \frac{1}{2}\int \cos 6x dx$

$= \frac{1}{8}\operatorname{sen} 4x - \frac{1}{12}\operatorname{sen} 6x + c$

Respuesta: $\int \operatorname{sen} 5x \operatorname{sen} x dx = \frac{1}{8}\operatorname{sen} 4x - \frac{1}{12}\operatorname{sen} 6x + c$

3.14.-Encontrar: $\int \tau g^4 x dx$

Solución.-$\int \tau g^4 x dx = \int \tau g^2 x \tau g^2 x dx$; como: $\tau g^2 = \sec^2 x - 1$; Luego:

$= \int \tau g^2 x \tau g^2 x dx = \int \tau g^2 x(\sec^2 x - 1) dx = \int \tau g^2 x \sec^2 x dx - \int \tau g^2 x dx$

$= \int (\tau g x)^2 \sec^2 x dx - \int \frac{\operatorname{sen}^2 x}{\cos^2 x} dx = \int (\tau g x)^2 \sec^2 x dx - \int \frac{1-\cos^2 x}{\cos^2 x} dx$

$= \int (\tau g x)^2 \sec^2 x dx - \int \sec^2 x dx + \int dx$; Sea: $w = \tau g x, dw = \sec^2 x dx$

$= \int w^2 dw - \int \sec^2 x + \int dx = \frac{w^3}{3} - \tau g x + x + c = \frac{\tau g^3}{3} - \tau g x + x + c$

Respuesta: $\int \tau g^4 x dx = \frac{\tau g^3}{3} - \tau g x + x + c$

3.15.-Encontrar: $\int \sec^6 x dx$

Solución.-$\int \sec^6 x dx = \int (\sec^2 x)^2 \sec^2 x dx$; como: $\sec^2 x dx = 1 + \tau g^2 x$

$= \int (\sec^2 x)^2 \sec^2 x dx = \int (1+\tau g^2 x)^2 \sec^2 x dx = \int (1 + 2\tau g^2 x + \tau g^4 x)\sec^2 x dx$

$= \int \sec^2 x dx + 2\int (\tau g x)^2 \sec^2 x dx + \int (\tau g x)^4 \sec^2 x dx$; Sea: $u = \tau g x, du = \sec^2 x dx$

$$=\int \sec^2 xdx+2\int u^2du+\int u^4du=\tau gx+\frac{2}{3}u^3+\frac{1}{5}u^5+c=\tau gx+\frac{2}{3}\tau g^3x+\frac{1}{5}\tau g^5x+c$$

Respuesta: $\int \sec^6 xdx=\tau gx+\frac{2}{3}\tau g^3x+\frac{1}{5}\tau g^5x+c$

3.16.-Encontrar: $\int \tau g^3 2xdx$

Solución.-

$$\int \tau g^3 2xdx=\int \tau g 2x\tau g^2 2xdx=\int \tau g 2x(\sec^2 2x-1)dx=\int \tau g 2x\sec^2 2xdx-\int \tau g 2xdx$$

Sea: $u=\tau g 2x, du=2\sec^2 2xdx$; Luego:

$$=\frac{1}{2}\int udu-\int \tau g 2xdx=\frac{1}{2}\frac{u^2}{2}-\frac{1}{2}\ell\eta\left|\sec 2x\right|+c=\frac{\tau g^2 2x}{4}-\frac{1}{2}\ell\eta\left|\frac{1}{\cos 2x}\right|+c$$

Respuesta: $\int \tau g^3 2xdx=\frac{\tau g^2 2x}{4}-\frac{1}{2}\ell\eta\left|\frac{1}{\cos 2x}\right|+c$

3.17.-Encontrar: $\int \tau g^2 5xdx$

Solución.- $\int \tau g^2 5xdx=\int (\sec^2 5x-1)dx=\int \sec^2 5xdx-\int dx=\frac{1}{5}\tau g 5x-x+c$

Respuesta: $\int \tau g^2 5xdx=\frac{1}{5}\tau g 5x-x+c$

3.18.-Encontrar: $\int \tau g^3 3x\sec 3xdx$

Solución.- $\int \tau g^3 3x\sec 3xdx=\int \tau g^2 3x\tau g 3x\sec 3xdx=\int (\sec^2 3x-1)\tau g 3x\sec 3xdx$

$=\int (\sec 3x)^2\tau g 3x\sec 3xdx-\int \tau g 3x\sec 3xdx$; Sea: $u=\sec 3x, du=3\sec 3x\tau g 3xdx$

Luego: $\frac{1}{3}\int u^2du-\frac{1}{3}\int 3\tau g 3x\sec 3xdx$; como: $d(\sec 3x)=3\tau g 3x\sec 3xdx$, se admite:

$$\frac{1}{3}\int u^2du-\frac{1}{3}\int d(\sec 3x)=\frac{1}{9}u^3-\frac{1}{3}\sec 3x+c=\frac{1}{9}\sec^3 3x-\frac{1}{3}\sec 3x+c$$

Respuesta: $\int \tau g^3 3x\sec 3xdx=\frac{1}{9}\sec^3 3x-\frac{1}{3}\sec 3x+c$

3.19.-Encontrar: $\int \tau g^{3/2}x\sec^4 xdx$

Solución.- $\int \tau g^{3/2}x\sec^4 xdx=\int \tau g^{3/2}x(\sec^2 x)\sec^2 xdx=\int \tau g^{3/2}x(1+\tau g^2x)\sec^2 xdx$

$=\int (\tau gx)^{3/2}\sec^2 xdx+\int (\tau gx)^{7/2}\sec^2 xdx$; Sea: $u=\tau gx, du=\sec^2 xdx$

Luego: $\int u^{3/2}du+\int u^{7/2}du=\frac{2}{5}u^{5/2}+\frac{2}{9}u^{9/2}+c=\frac{2}{5}\tau g^{5/2}x+\frac{2}{9}\tau g^{9/2}+c$

Respuesta: $\int \tau g^{3/2}x\sec^4 xdx=\frac{2}{5}\tau g^{5/2}x+\frac{2}{9}\tau g^{9/2}+c$

3.20.-Encontrar: $\int \tau g^4 x\sec^4 xdx$

Solución.- $\int \tau g^4 x(\sec^2 x)\sec^2 xdx=\int \tau g^4 x(1+\tau g^2 x)\sec^2 xdx$

$=\int (\tau gx)^4\sec^2 xdx+\int (\tau gx)^6\sec^2 xdx$; Sea: $u=\tau gx, du=\sec^2 xdx$

Luego: $\int u^4du+\int u^6du=\frac{u^5}{5}+\frac{u^7}{7}+c=\frac{\tau g^5x}{5}+\frac{\tau g^7x}{7}+c$

Respuesta: $\int \tau g^4x\sec^4 xdx=\frac{\tau g^5x}{5}+\frac{\tau g^7x}{7}+c$

3.21.-Encontrar: $\int \operatorname{co}\tau g^3x\operatorname{co}\sec^4 xdx$

Solución.- $\int \operatorname{co}\tau g^3x\operatorname{co}\sec^4 xdx=\int \operatorname{co}\tau g^3x(\operatorname{co}\sec^2 x)\operatorname{co}\sec^2 xdx$

Como: $\cos ec^2x=1+\operatorname{co}\tau g^2x$; Luego:

$\int \operatorname{co}\tau g^3x(1+\operatorname{co}\tau g^2x)\operatorname{co}\sec^2 xdx=\int \operatorname{co}\tau g^3x\operatorname{co}\sec^2 xdx+\int \operatorname{co}\tau g^5x\operatorname{co}\sec^2 xdx$

Sea: $u=\operatorname{co}\tau gx, du=-\cos ec^2xdx$,

Luego: $-\int u^3du-\int u^5du=-\frac{u^4}{4}-\frac{u^6}{6}+c=-\frac{\operatorname{co}\tau g^4x}{4}-\frac{\operatorname{co}\tau g^6x}{6}+c$

Respuesta: $\int \operatorname{co}\tau g^3x\operatorname{co}\sec^4 xdx=-\frac{\operatorname{co}\tau g^4x}{4}-\frac{\operatorname{co}\tau g^6x}{6}+c$

3.22.-Encontrar: $\int \operatorname{co}\tau g3x\operatorname{co}\sec^4 3xdx$

Solución.- $\int \operatorname{co}\tau g3x\operatorname{co}\sec^4 3xdx=\int \operatorname{co}\tau g3x(\operatorname{co}\sec^2 3x)\operatorname{co}\sec^2 3xdx$

$\int \operatorname{co}\tau g3x(1+\operatorname{co}\tau g^2 3x)\operatorname{co}\sec^2 3xdx=\int \operatorname{co}\tau g3x\operatorname{co}\sec^2 3xdx+\int \operatorname{co}\tau g^3 3x\operatorname{co}\sec^2 3xdx$

Sea: $u=\operatorname{co}\tau g3x, du=-3\cos ec^2 3xdx$; Luego:

$-\frac{1}{3}\int udu-\frac{1}{3}\int u^3du=-\frac{u^2}{6}-\frac{u^4}{12}+c=-\frac{\operatorname{co}\tau g^2 3x}{6}-\frac{\operatorname{co}\tau g^4 3x}{12}+c$

Respuesta: $\int \operatorname{co}\tau g3x\operatorname{co}\sec^4 3xdx=-\frac{\operatorname{co}\tau g^2 3x}{6}-\frac{\operatorname{co}\tau g^4 3x}{12}+c$

3.23.-Encontrar: $\int \operatorname{co}\sec^4 2xdx$

Solución.- $\int \operatorname{co}\sec^2 2x\operatorname{co}\sec^2 2xdx=\int (1+\operatorname{co}\tau g^2 2x)\operatorname{co}\sec^2 2xdx$

$\int \operatorname{co}\sec^2 2xdx+\int \operatorname{co}\tau g^2 2x\operatorname{co}\sec^2 2xdx$; Sea: $u=\operatorname{co}\tau g2x, du=-\cos ec^2 2xdx$

Luego: $\int \operatorname{co}\sec^2 2xdx-\frac{1}{2}\int u^2du=-\frac{1}{2}\operatorname{co}\tau g2x-\frac{u^3}{3}+c=-\frac{\operatorname{co}\tau g2x}{2}-\frac{\operatorname{co}\tau g^3 2x}{6}+c$

Respuesta: $\int \operatorname{co}\sec^4 2xdx=-\frac{\operatorname{co}\tau g2x}{2}-\frac{\operatorname{co}\tau g^3 2x}{6}+c$

3.24.-Encontrar: $\int \operatorname{co}\tau g^3x\operatorname{co}\sec^3 xdx$

Solución.- $\int \operatorname{co}\tau g^3x\operatorname{co}\sec^3 xdx=\int \operatorname{co}\tau g^2x\operatorname{co}\sec^2 x\operatorname{co}\tau gx\operatorname{co}\sec xdx$

Como: $\operatorname{co}\tau g^2x=\operatorname{co}\sec^2 x-1$; Luego: $\int (\operatorname{co}\sec^2 x-1)\operatorname{co}\sec^2 x\operatorname{co}\tau gx\operatorname{co}\sec xdx$

$=\int (\operatorname{co}\sec^4 x\operatorname{co}\tau gx\operatorname{co}\sec xdx-\int \operatorname{co}\sec^2 x\operatorname{co}\tau gx\operatorname{co}\sec xdx$

Sea: $u=\cos ecx, du=-\cos ecx\operatorname{co}\tau gxdx$;

Entonces: $-\int u^4du+\int u^2du=-\frac{u^5}{5}+\frac{u^3}{3}+c=-\frac{\cos ec^5x}{5}+\frac{\cos ec^3x}{3}+c$

Respuesta: $\int \operatorname{co}\tau g^3x\operatorname{co}\sec^3 xdx=-\frac{\cos ec^5x}{5}+\frac{\cos ec^3x}{3}+c$

3.25.-Encontrar: $\int \operatorname{co}\tau g^3xdx$

Solución.- $\int \operatorname{co}\tau g^3xdx=\int \operatorname{co}\tau g^2x\operatorname{co}\tau gxdx=\int(\cos ec^2x-1)\operatorname{co}\tau gxdx$

$=\int \cos ec^2x\operatorname{co}\tau gxdx-\int \operatorname{co}\tau gxdx$; Sea: $u=\operatorname{co}\tau gx, du=-\cos ec^2xdx$

Luego: $-\int udu-\int \operatorname{co}\tau gxdx=-\frac{u^2}{2}-\ell\eta|\operatorname{sen}x|+c=-\frac{\operatorname{co}\tau g^2x}{2}-\ell\eta|\operatorname{sen}x|+c$

Respuesta: $\int \operatorname{co}\tau g^3xdx=-\frac{\operatorname{co}\tau g^2x}{2}-\ell\eta|\operatorname{sen}x|+c$

EJERCICIOS PROPUESTOS

Usando esencialmente el mecanismo tratado, encontrar las siguientes integrales:

3.26.- $\int \tau g^2 5xdx$ **3.27.-** $\int \operatorname{sen}x\cos xdx$ **3.28.-** $\int \frac{dx}{\sec 2x}$

3.29.- $\int \frac{\cos 2x}{\cos x}dx$ **3.30.-** $\int \sqrt{\cos x}\operatorname{sen}^3 xdx$ **3.31.-** $\int \tau g^2\frac{x}{3}\sec^2\frac{x}{3}dx$

3.32.- $\int \tau g^3 4x\sec 4xdx$ **3.33.-** $\int \operatorname{sen}^2\frac{x}{6}dx$ **3.34.-** $\int \frac{\operatorname{sen}2x}{\operatorname{sen}x}dx$

3.35.- $\int(\sec x+\cos ecx)^2dx$ **3.36.-** $\int \sec^3\frac{x}{4}\tau g\frac{x}{4}dx$ **3.37.-** $\int \tau g^4 2x\sec^4 2xdx$

3.38.- $\int \operatorname{sen}8x\operatorname{sen}3xdx$ **3.39.-** $\int \cos 4x\cos 5xdx$ **3.40.-** $\int \operatorname{sen}2x\cos 3xdx$

3.41.- $\int\left(\frac{\sec x}{\tau gx}\right)^4dx$ **3.42.-** $\int \frac{\cos^3 x}{\operatorname{sen}^4 x}dx$ **3.43.-** $\int \cos ec^4 3xdx$

3.44.- $\int(\tau g^3\frac{x}{3}+\tau g^4\frac{x}{3})dx$ **3.45.-** $\int \operatorname{co}\tau g^3\frac{x}{3}dx$ **3.46.-** $\int \operatorname{co}\tau g^4\frac{x}{6}dx$

3.47.- $\int \frac{dx}{\operatorname{sen}^5 x\cos x}$ **3.48.-** $\int \frac{\cos^2 x}{\operatorname{sen}^6 x}dx$ **3.49.-** $\int \frac{dx}{\operatorname{sen}^2 x\cos^4 x}$

3.50.- $\int \frac{dx}{\cos^6 4x}$ **3.51.-** $\int \frac{\cos^3 x}{1-\operatorname{sen}x}dx$ **3.52.-** $\int \cos^3\frac{x}{7}dx$

3.53.- $\int \operatorname{sen}^5\frac{x}{2}dx$ **3.54.-** $\int \sqrt{1-\cos x}dx$ **3.55.-** $\int \frac{dx}{\cos ec^4\frac{x}{3}}$

3.56.- $\int \operatorname{sen}^3\frac{x}{2}\cos^5\frac{x}{2}dx$ **3.57.-** $\int \operatorname{sen}^2x\cos^2 xdx$ **3.58.-** $\int \operatorname{sen}^4x\cos^2 xdx$

3.59.- $\int \frac{1-\cos 2x}{1+\cos 2x}dx$ **3.60.-** $\int \frac{\cos^3 x}{\sqrt{\operatorname{sen}x}}dx$ **3.61.-** $\int \operatorname{sen}^3 2xdx$

3.62.- $\int \operatorname{sen}^2 2x\cos^2 2xdx$ **3.63.-** $\int \cos^4 xdx$ **3.64.-** $\int \tau g^4x\sec^2 xdx$

3.65.- $\int \tau g^3 x \sec x dx$ **3.66.-** $\int \sec^6 a\theta d\theta$ **3.67.-** $\int \sec x dx$

3.68.- $\int co\tau g^2 2x \cos ec^2 2x dx$ **3.69.-** $\int \frac{sen^3 x}{\cos^2 x} dx$ **3.70.-** $\int \sec^4 3x \tau g 3x dx$

3.71.- $\int \sec^n x \tau g x dx; (n \neq 0)$ **3.72.-** $\int \frac{\cos^3 x}{sen^2 x} dx$ **3.73.-** $\int \frac{dx}{sen^4 x}$

3.74.- $\int \tau g^n x \sec^2 x dx; (n \neq -1)$ **3.75.-** $\int sen^6 x dx$ **3.76.-** $\int sen^4 ax dx$

3.77.- $\int sen^n x \cos x dx; (n \neq -1)$ **3.78.-** $\int co\tau g^n ax dx$ **3.79.-** $\int co\tau g^4 3x dx$

3.80.- $\int \cos x^n sen x dx; (n \neq -1)$ **3.81.-** $\int \tau g^n x dx$ **3.82.-** $\int \tau g^4 x dx$

3.83.- $\int \cos^{2n+1} x dx$

RESPUESTAS

3.26.- $\int \tau g^2 5x dx = \int (\sec^2 5x - 1) dx = \int \sec^2 5x dx + \int dx = \frac{1}{5}\tau g^5 x - x + c$

3.27.- $\int sen x \cos x dx = \frac{1}{2}\int 2 sen x \cos x dx = \frac{1}{2}\int sen 2x dx = -\frac{1}{4}\cos 2x + c$

3.28.- $\int \frac{dx}{\sec 2x} = \int \cos 2x dx = \frac{1}{2} sen 2x + c$

3.29.- $\int \frac{\cos 2x}{\cos x} dx = \int \frac{\cos^2 x - sen^2 x}{\cos x} dx = \int \frac{\cos^{\cancel{2}} x}{\cancel{\cos x}} dx - \int \frac{sen^2 x}{\cos x} dx$

$= \int \cos x dx - \int \frac{1-\cos^2 x}{\cos x} dx = \int \cos x dx - \int \frac{dx}{\cos x} + \int \cos x dx = 2\int \cos x dx - \int \sec x dx$

$= 2 sen x - \ell\eta \left|\sec x + \tau g x\right| + c$

3.30.- $\int \sqrt{\cos x}\, sen^3 x dx = \int \sqrt{\cos x}\, sen^2 x\, sen x dx = \int \sqrt{\cos x}(1-\cos^2 x) sen x dx$

$= \int \sqrt{\cos x}\, sen x dx - \int \sqrt{\cos x} \cos^2 x\, sen x dx = \int \cos^{1/2} x\, sen x dx - \int \cos^{5/2} x\, sen x dx$

Sea: $u = \cos x, du = -sen x dx$; Luego: $-\int u^{1/2} du + \int u^{5/2} du = -\frac{2}{3}u^{3/2} + \frac{2}{7}u^{7/2} + c$

$= -\frac{2}{3}\cos^{3/2} + \frac{2}{7}\cos^{7/2} + c = -\frac{2}{3}\sqrt{\cos^3 x} + \frac{2}{7}\sqrt{\cos^7 x} + c$

$= -\frac{2}{3}\cos x\sqrt{\cos x} + \frac{2}{7}\cos x^3\sqrt{\cos x} + c$

3.31.- $\int \tau g^2 \frac{x}{3} \sec^2 \frac{x}{3} dx = \int (\tau g \frac{x}{3})^2 \sec^2 \frac{x}{3} dx$; Sea: $u = \tau g \frac{x}{3}, du = \frac{1}{3}\sec^2 \frac{x}{3} dx$

$3\int (\tau g \frac{x}{3})^2 \frac{1}{3}\sec^2 \frac{x}{3} dx = 3\int u^2 du = u^3 + c = \tau g^3 \frac{x}{3} + c$

3.32.- $\int \tau g^3 4x \sec 4x dx = \int (\tau g^2 4x)\tau g 4x \sec 4x dx = \int (\sec^2 4x - 1)\tau g 4x \sec 4x dx$

$= \int \sec^2 4x \tau g 4x \sec 4x dx - \int \tau g 4x \sec 4x dx$; Sea: $u = \sec 4x, du = 4\sec 4x \tau g 4x dx$

$$=\frac{1}{4}\int u^2du-\frac{1}{4}\int du=\frac{1}{4}\frac{u^3}{3}-\frac{1}{4}u+c=\frac{\sec^3 4x}{12}-\frac{\sec 4x}{4}+c$$

3.33.- $\int \operatorname{sen}^2 \frac{x}{6}dx=\int\frac{1-\cos 2\frac{x}{6}}{2}dx=\int\frac{1-\cos\frac{x}{3}}{2}dx=\frac{1}{2}\int dx-\frac{1}{2}\int\cos\frac{x}{3}dx$

$=\frac{1}{2}x-\frac{3}{2}\operatorname{sen}\frac{x}{3}+c$

3.34.- $\int\frac{\operatorname{sen}2x}{\operatorname{sen}x}dx=\int\frac{2\cancel{\operatorname{sen}x}\cos x}{\cancel{\operatorname{sen}x}}dx=2\int\cos xdx=2\operatorname{sen}x+c$

3.35.- $\int(\sec x+\operatorname{cosec}x)^2dx=\int(\sec^2 x+2\sec x\operatorname{cosec}x+\operatorname{cosec}^2x)dx$

$=\int\sec^2 xdx+2\int\sec x\operatorname{cosec}xdx+\int\operatorname{cosec}^2xdx=\int\sec^2 xdx+2\int\frac{1}{\cos x}\frac{1}{\operatorname{sen}x}dx+\int\operatorname{cosec}^2xdx$

$=\int\sec^2 xdx+2\times 2\int\frac{dx}{2\cos x\operatorname{sen}x}+\int\operatorname{cosec}^2xdx=\int\sec^2 xdx+4\int\frac{dx}{\operatorname{sen}2x}+\int\operatorname{cosec}^2xdx$

$=\int\sec^2 xdx+4\int\operatorname{cosec}2xdx+\int\operatorname{cosec}^2xdx$

$=\tau gx+\frac{4}{2}\ell\eta\left|\operatorname{cosec}2x-\operatorname{co}\tau g2x\right|-\operatorname{co}\tau gx+c$

$=\tau gx+2\ell\eta\left|\operatorname{cosec}2x-\operatorname{co}\tau g2x\right|-\operatorname{co}\tau gx+c$

3.36.- $\int\sec^3\frac{x}{4}\tau g\frac{x}{4}dx=\int(\sec^2\frac{x}{4})\sec\frac{x}{4}\tau g\frac{x}{4}dx$

Sea: $u=\sec\frac{x}{4},du=\frac{1}{4}\sec\frac{x}{4}\tau g\frac{x}{4}dx$,

Luego: $4\int u^2du=4\frac{u^3}{3}+c=\frac{4\sec^3\frac{x}{4}}{3}+c$

3.37.- $\int\tau g^4 2x\sec^4 2xdx=\int\tau g^4 2x(\sec^2 2x)\sec^2 2xdx=\int\tau g^4 2x(1+\tau g^2 2x)\sec^2 2xdx$

$=\int(\tau g2x)^4\sec^2 2xdx+\int(\tau g2x)^6\sec^2 2xdx$

Sea: $u=\tau g2x,du=2\sec^2 2xdx$,   Luego:

$=\frac{1}{2}\int(\tau g2x)^4 2\sec^2 2xdx+\frac{1}{2}\int(\tau g2x)^6 2\sec^2 2xdx=\frac{1}{2}\int u^4du+\frac{1}{2}\int u^6du$

$=\frac{1}{2}\frac{u^5}{5}+\frac{1}{2}\frac{u^7}{7}+c=\frac{\tau g^5 2x}{10}+\frac{\tau g^7 2x}{14}+c$

3.38.- $\int\operatorname{sen}8x\operatorname{sen}3xdx$

Considerando: $\operatorname{sen}\alpha\operatorname{sen}\beta=\frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]$

Luego: $\operatorname{sen}8x\operatorname{sen}3x=\frac{1}{2}(\cos 5x-\cos 11x)$; Se tiene:

$=\frac{1}{2}\int(\cos 5x-\cos 11x)dx=\frac{1}{2}\int\cos 5xdx-\frac{1}{2}\int\cos 11xdx=\frac{\operatorname{sen}5x}{10}-\frac{\operatorname{sen}11x}{22}+c$

3.39.- $\int\cos 4x\cos 5xdx$

Considerando: $\cos\alpha\cos\beta=\frac{1}{2}\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right]$

Luego: $\cos 4x\cos 5x=\frac{1}{2}(\cos(-x)+\cos 9x)$;

Como: $\cos(-x)=\cos x \Rightarrow \frac{1}{2}(\cos x+\cos 9x)$; entonces:

$$\int \cos 4x\cos 5xdx=\frac{1}{2}\int(\cos x+\cos 9x)dx=\frac{1}{2}\int \cos xdx+\frac{1}{2}\int \cos 9xdx$$

$$=\frac{\operatorname{sen} x}{2}+\frac{\operatorname{sen} 9x}{18}+c$$

3.40.- $\int \operatorname{sen} 2x\cos 3xdx$

Considerando: $\operatorname{sen}\alpha\cos\beta=\frac{1}{2}\left[\operatorname{sen}(\alpha-\beta)+\operatorname{sen}(\alpha+\beta)\right]$

Luego: $\operatorname{sen} 2x\cos 3x=\frac{1}{2}\left[\operatorname{sen}(-x)+\operatorname{sen} 5x\right]$

Como: $\operatorname{sen}(-x)=-\operatorname{sen} x \Rightarrow \frac{1}{2}(-\operatorname{sen} x+\operatorname{sen} 5x)$; entonces:

$$\int \operatorname{sen} 2x\cos 3xdx=\frac{1}{2}\int(-\operatorname{sen} x+\operatorname{sen} 5x)dx=-\frac{1}{2}\int \operatorname{sen} xdx+\frac{1}{2}\int \operatorname{sen} 5xdx$$

$$=\frac{1}{2}\cos x-\frac{1}{10}\cos 5x+c$$

3.41.- $\int\left(\frac{\sec x}{\tau gx}\right)^4 dx=\int\left(\frac{\frac{1}{\cancel{\cos x}}}{\frac{\operatorname{sen} x}{\cancel{\cos x}}}\right)^4=\int\left(\frac{1}{\operatorname{sen} x}\right)^4 dx=\int \cos ec^4 xdx=\int \cos ec^2 x\cos ec^2 xdx$

$$=\int(1+\operatorname{co}\tau g^2 x)\cos ec^2 xdx=\int \cos ec^2 xdx+\int \operatorname{co}\tau g^2 x\cos ec^2 xdx$$

Sea: $u=\operatorname{co}\tau gx, du=-\cos ec^2 xdx$

Luego: $\int \cos ec^2 xdx-\int u^2 du=-\operatorname{co}\tau gx-\frac{u^3}{3}+c=-\operatorname{co}\tau gx-\frac{\operatorname{co}\tau g^3 x}{3}+c$

3.42.- $\int \frac{\cos^3 x}{\operatorname{sen}^4 x}dx=\int \frac{\cos^3 x}{\operatorname{sen}^3 x}\frac{1}{\operatorname{sen} x}dx=\int \operatorname{co}\tau g^3 x\cos ecxdx$

$$=\int(\operatorname{co}\tau g^2 x)\operatorname{co}\tau gx\cos ecxdx=\int(\cos ec^2 x-1)\operatorname{co}\tau gx\cos ecxdx=$$

$$=\int \cos ec^2 x\operatorname{co}\tau gx\cos ecxdx-\int \operatorname{co}\tau gx\cos ecxdx$$

Sea: $u=\cos ecx, du=-\cos ecx\operatorname{co}\tau gxdx$

Luego: $-\int u^2 du+\int du=-\frac{u^3}{3}+u+c=-\frac{\cos ec^3 x}{3}+\cos ecx+c$

3.43.- $\int \cos ec^4 3xdx=\int(\cos ec^2 3x)\cos ec^2 3xdx=\int(1+\operatorname{co}\tau g^2 3x)\cos ec^2 3x)dx$

$$=\int \cos ec^2 3xdx+\int \operatorname{co}\tau g^2 3x\cos ec^2 3xdx$$

Sea: $u=\operatorname{co}\tau g3x, du=-3\cos ec^2 3xdx$

Luego: $\int \cos ec^2 3xdx-\frac{1}{3}\int u^2 du=-\frac{1}{3}\operatorname{co}\tau g3x-\frac{1}{9}u^3+c=-\frac{\operatorname{co}\tau g3x}{3}-\frac{\operatorname{co}\tau g^3 3x}{9}+c$

3.44.- $\int(\tau g^3\tfrac{x}{3}+\tau g^4\tfrac{x}{3})dx=\int\tau g^3\tfrac{x}{3}dx+\int\tau g^4\tfrac{x}{3}dx=\int(\tau g^2\tfrac{x}{3})\tau g\tfrac{x}{3}dx+\int(\tau g^2\tfrac{x}{3})\tau g^2\tfrac{x}{3}dx$

$=\int(\sec^2\tfrac{x}{3}-1)\tau g\tfrac{x}{3}dx+\int(\sec^2\tfrac{x}{3}-1)\tau g^2\tfrac{x}{3}dx$

$=\int\sec^2\tfrac{x}{3}\tau g\tfrac{x}{3}dx-\int\tau g\tfrac{x}{3}dx+\int(\sec^2\tfrac{x}{3})\tau g^2\tfrac{x}{3}dx-\int\tau g^2\tfrac{x}{3}dx$

$=\int\sec^2\tfrac{x}{3}\tau g\tfrac{x}{3}dx-\int\tau g\tfrac{x}{3}dx+\int(\sec^2\tfrac{x}{3})\tau g^2\tfrac{x}{3}dx-\int(\sec^2\tfrac{x}{3}-1)dx$

$=\int\sec^2\tfrac{x}{3}\tau g\tfrac{x}{3}dx-\int\tau g\tfrac{x}{3}dx+\int(\sec^2\tfrac{x}{3})\tau g^2\tfrac{x}{3}dx-\int\sec^2\tfrac{x}{3}dx+\int dx$

Sea: $u=\tau g\tfrac{x}{3},du=\frac{1}{3}\sec^2\tfrac{x}{3}dx$

Luego: $3\int udu-\int\tau g\tfrac{x}{3}dx+3\int u^2du-\int\sec^2\tfrac{x}{3}dx+\int dx$

$=\frac{3}{2}u^2-3\ell\eta\left|\sec\tfrac{x}{3}\right|+u^3-3\tau g\tfrac{x}{3}+x+c=\frac{3}{2}\tau g^2\tfrac{x}{3}-3\ell\eta\left|\sec\tfrac{x}{3}\right|+\tau g^3\tfrac{x}{3}-3\tau g\tfrac{x}{3}+x+c$

3.45.- $\int\mathrm{co}\,\tau g^3\tfrac{x}{3}dx=\int(\mathrm{co}\,\tau g^2\tfrac{x}{3})\,\mathrm{co}\,\tau g\tfrac{x}{3}dx=\int(\cos ec^2\tfrac{x}{3}-1)\,\mathrm{co}\,\tau g\tfrac{x}{3}dx$

$=\int\cos ec^2\tfrac{x}{3}\,\mathrm{co}\,\tau g\tfrac{x}{3}dx-\int\mathrm{co}\,\tau g\tfrac{x}{3}dx$; Sea: $u=\cos ec\tfrac{x}{3},du=-\frac{1}{3}\cos ec\tfrac{x}{3}\,\mathrm{co}\,\tau g\tfrac{x}{3}dx$

Luego: $-3\int(\cos ec\tfrac{x}{3})(-\tfrac{1}{3}\cos ec\tfrac{x}{3}\,\mathrm{co}\,\tau g\tfrac{x}{3})dx-\int\mathrm{co}\,\tau g\tfrac{x}{3}dx=-3\int udu-\int\mathrm{co}\,\tau g\tfrac{x}{3}dx$

$=\frac{-3u^2}{2}-3\ell\eta\left|\mathrm{sen}\tfrac{x}{3}\right|+c=\frac{-3\cos ec^2\tfrac{x}{3}}{2}-3\ell\eta\left|\mathrm{sen}\tfrac{x}{3}\right|+c$

3.46.- $\int\mathrm{co}\,\tau g^4\tfrac{x}{6}dx=\int(\mathrm{co}\,\tau g^2\tfrac{x}{6})\,\mathrm{co}\,\tau g^2\tfrac{x}{6}dx=\int(\cos ec^2\tfrac{x}{6}-1)\,\mathrm{co}\,\tau g^2\tfrac{x}{6}dx$

$=\int\cos ec^2\tfrac{x}{6}\,\mathrm{co}\,\tau g^2\tfrac{x}{6}dx-\int\mathrm{co}\,\tau g^2\tfrac{x}{6}dx=\int\cos ec^2\tfrac{x}{6}\,\mathrm{co}\,\tau g^2\tfrac{x}{6}dx-\int(\cos ec^2\tfrac{x}{6}-1)dx$

$=\int\cos ec^2\tfrac{x}{6}\,\mathrm{co}\,\tau g^2\tfrac{x}{6}dx-\int\cos ec^2\tfrac{x}{6}dx+\int dx$

Sea: $u=\mathrm{co}\,\tau g\tfrac{x}{6},du=-\frac{1}{6}\cos ec^2\tfrac{x}{6}dx$

Luego: $-6\int u^2du-\int\cos ec^2\tfrac{x}{6}dx+\int dx=-2u^3+6\,\mathrm{co}\,\tau g\tfrac{x}{6}+x+c$

$=-2\,\mathrm{co}\,\tau g^3\tfrac{x}{6}+6\,\mathrm{co}\,\tau g\tfrac{x}{6}+x+c$

3.47.- $\int\frac{dx}{\mathrm{sen}^5x\cos x}$; Como: $\mathrm{sen}^2x+\cos^2x=1$,

Luego: $\int\frac{\mathrm{sen}^2x+\cos^2x}{\mathrm{sen}^5x\cos x}dx=\int\frac{dx}{\mathrm{sen}^3x\cos x}+\int\frac{\cos xdx}{\mathrm{sen}^5x}$

$=\int\frac{\mathrm{sen}^2x+\cos^2x}{\mathrm{sen}^3x\cos x}dx+\int\frac{\cos xdx}{\mathrm{sen}^5x}=\int\frac{dx}{\mathrm{sen}\,x\cos x}+\int\frac{\cos xdx}{\mathrm{sen}^3x}+\int\frac{\cos xdx}{\mathrm{sen}^5x}$

$=\int\frac{dx}{\mathrm{sen}\,x\cos x}+\int(\mathrm{sen}\,x)^{-3}\cos xdx+\int(\mathrm{sen}\,x)^{-5}\cos xdx$

$=\int\frac{dx}{\frac{\mathrm{sen}\,2x}{2}}+\int(\mathrm{sen}\,x)^{-3}\cos xdx+\int(\mathrm{sen}\,x)^{-5}\cos xdx$

$=2\int\cos ec2xdx+\int(\mathrm{sen}\,x)^{-3}\cos xdx+\int(\mathrm{sen}\,x)^{-5}\cos xdx\ (*)$

Sea: $u = \operatorname{sen} x, du = \cos x dx$, Luego:

$$(*) = 2\int \operatorname{cosec} 2x dx + \int u^{-3} du + \int u^{-5} du = \ell\eta \left|\operatorname{cosec} 2x - \operatorname{co}\tau g 2x\right| - \frac{1}{2u^2} - \frac{1}{4u^4} + c$$

$$= \ell\eta \left|\operatorname{cosec} 2x - \operatorname{co}\tau g 2x\right| - \frac{1}{2\operatorname{sen}^2 x} - \frac{1}{4\operatorname{sen}^4 x} + c$$

$$= \ell\eta \left|\operatorname{cosec} 2x - \operatorname{co}\tau g 2x\right| - \frac{\operatorname{cosec}^2 x}{2} - \frac{\operatorname{cosec}^4 x}{4} + c$$

3.48.- $\int \frac{\cos^2 x}{\operatorname{sen}^6 x} dx = \int \frac{\cos^2 x}{\operatorname{sen}^2 x} \frac{1}{\operatorname{sen}^4 x} dx = \int \operatorname{co}\tau g^2 x \operatorname{cosec}^4 x dx$

$$= \int \operatorname{co}\tau g^2 x (\operatorname{cosec}^2 x) \operatorname{cosec}^2 x dx = \int \operatorname{co}\tau g^2 x (1 + \operatorname{co}\tau g^2 x) \operatorname{cosec}^2 x dx$$

$$= \int \operatorname{co}\tau g^2 x \operatorname{cosec}^2 x dx + \int \operatorname{co}\tau g^4 x \operatorname{cosec}^2 x dx$$

Sea: $u = \operatorname{co}\tau g x, du = -\operatorname{cosec}^2 x dx$,

Luego: $-\int u^2 du - \int u^4 du = -\frac{u^3}{3} - \frac{u^5}{5} + c = -\frac{\operatorname{co}\tau g^3 x}{3} - \frac{\operatorname{co}\tau g^5 x}{5} + c$

3.49.- $\int \frac{dx}{\operatorname{sen}^2 x \cos^4 x} = \int \frac{\operatorname{sen}^2 + \cos^2 x}{\operatorname{sen}^2 x \cos^4 x} dx = \int \frac{dx}{\cos^4 x} + \int \frac{dx}{\operatorname{sen}^2 x \cos^2 x}$

$$= \int \sec^4 x dx + \int \frac{dx}{(\operatorname{sen} x \cos x)^2} = \int \sec^4 x dx + \int \frac{dx}{(\frac{\operatorname{sen} 2x}{2})^2} = \int \sec^4 x dx + 4\int \frac{dx}{\operatorname{sen}^2 2x}$$

$$= \int \sec^4 x dx + 4\int \operatorname{cosec}^2 2x dx = \int \sec^2 x \sec^2 x dx + 4\int \operatorname{cosec}^2 2x dx$$

$$= \int (1 + \tau g^2 x) \sec^2 x dx + 4\int \operatorname{cosec}^2 2x dx = \int \sec^2 x dx + \int \tau g^2 x \sec^2 x dx + 4\int \operatorname{cosec}^2 2x dx$$

Sea: $u = \tau g x, du = \sec^2 x dx$,

Luego: $\int \sec^2 x dx + \int u^2 du + 4\int \operatorname{cosec}^2 2x dx = \tau g x + \frac{u^3}{3} - 2\operatorname{co}\tau g 2x + c$

$$= \tau g x + \frac{\tau g^3 x}{3} - 2\operatorname{co}\tau g 2x + c$$

3.50.- $\int \frac{dx}{\cos^6 4x} = \int \sec^6 4x dx = \int (\sec^2 4x)^2 \sec^2 4x dx = \int (1 + \tau g^2 4x)^2 \sec^2 4x dx$

$$= \int (1 + 2\tau g^2 4x + \tau g^4 4x) \sec^2 4x dx$$

$$= \int \sec^2 4x dx + 2\int (\tau g 4x)^2 \sec^2 4x dx + \int (\tau g 4x)^4 \sec^2 4x dx$$

Sea: $u = \tau g 4x, du = 4\sec^2 4x dx$, Luego:

$$\int \sec^2 4x dx + \frac{1}{2}\int u^2 du + \frac{1}{4}\int u^4 du = \frac{\tau g 4x}{4} + \frac{1}{2}\frac{u^3}{3} + \frac{1}{4}\frac{u^5}{5} + c = \frac{\tau g 4x}{4} + \frac{\tau g^3 4x}{6} + \frac{\tau g^5 4x}{20} + c$$

3.51.- $\int \frac{\cos^3 x}{1 - \operatorname{sen} x} dx = \int \frac{\cos^3 x (1 + \operatorname{sen} x)}{1 - \operatorname{sen}^2 x} dx = \int \frac{\cos^{\cancel{3}} x (1 + \operatorname{sen} x)}{\cancel{\cos^2 x}} dx$

$$= \int \cos x (1 + \operatorname{sen} x) dx = \int \cos x dx + \int \cos x \operatorname{sen} x dx = \int \cos x dx + \frac{1}{2}\int \operatorname{sen} 2x dx$$

$= \operatorname{sen} x - \frac{1}{4}\cos 2x + c$

3.52.- $\int \cos^3 \frac{x}{7}\,dx = \int (\cos^2 \frac{x}{7})\cos\frac{x}{7}\,dx = \int (1-\operatorname{sen}^2\frac{x}{7})\cos\frac{x}{7}\,dx$

$= \int \cos\frac{x}{7}\,dx - \int \operatorname{sen}^2\frac{x}{7}\cos\frac{x}{7}\,dx$

Sea: $u = \operatorname{sen}\frac{x}{7}, du = \frac{1}{7}\cos\frac{x}{7}\,dx$

Luego: $= \int \cos\frac{x}{7}\,dx - 7\int u^2 du = 7\operatorname{sen}\frac{x}{7} - \frac{7u^3}{3} + c = 7\operatorname{sen}\frac{x}{7} - \frac{7}{3}\operatorname{sen}^3\frac{x}{7} + c$

3.53.- $\int \operatorname{sen}^5\frac{x}{2}\,dx = \int (\operatorname{sen}^2\frac{x}{2})^2\operatorname{sen}\frac{x}{2}\,dx = \int (1-\cos^2\frac{x}{2})^2\operatorname{sen}\frac{x}{2}\,dx$

$= \int (1-2\cos^2\frac{x}{2}+\cos^4\frac{x}{2})\operatorname{sen}\frac{x}{2}\,dx = \int \operatorname{sen}\frac{x}{2}\,dx - 2\int\cos^2\frac{x}{2}\operatorname{sen}\frac{x}{2}\,dx + \int\cos^4\frac{x}{2}\operatorname{sen}\frac{x}{2}\,dx$

Sea: $u = \cos\frac{x}{2}, du = -\frac{1}{2}\operatorname{sen}\frac{x}{2}\,dx$, Luego:

$= \int \operatorname{sen}\frac{x}{2}\,dx + 4\int u^2 du - 2\int u^4 du = -2\cos\frac{x}{2} + \frac{4u^3}{3} - \frac{2u^5}{5} + c$

$= -2\cos\frac{x}{2} + \frac{4\cos^3\frac{x}{2}}{3} - \frac{2\cos^5\frac{x}{2}}{5} + c$

3.54.- $\int \sqrt{1-\cos x}\,dx$

Considerando: $\operatorname{sen}^2\alpha = \frac{1-\cos 2\alpha}{2}$, y $2\alpha = x$

Se tiene: $\operatorname{sen}^2\frac{x}{2} = \frac{1-\cos 2x}{2}$; además: $1-\cos x = 2\operatorname{sen}^2\frac{x}{2}$

Luego: $\int \sqrt{2\operatorname{sen}^2\frac{x}{2}}\,dx = \sqrt{2}\int\operatorname{sen}\frac{x}{2}\,dx = -2\sqrt{2}\cos\frac{x}{2} + c$

3.55.- $\int \frac{dx}{\operatorname{cosec}^4\frac{x}{3}} = \int \operatorname{sen}^4\frac{x}{3}\,dx = \int (\operatorname{sen}^2\frac{x}{3})^2 dx = \int\left(\frac{1-\cos\frac{2x}{3}}{2}\right)^2 dx$

$= \frac{1}{4}\int(1-2\cos\frac{2x}{3}+\cos^2\frac{2x}{3})dx = \frac{1}{4}\int dx - \frac{1}{2}\int\cos\frac{2x}{3}\,dx + \frac{1}{4}\int\cos^2\frac{2x}{3}\,dx$

$= \frac{1}{4}\int dx - \frac{1}{2}\int\cos\frac{2x}{3}\,dx + \frac{1}{4}\int\frac{1+\cos\frac{4x}{3}}{2}\,dx = \frac{1}{4}\int dx - \frac{1}{2}\int\cos\frac{2x}{3}\,dx + \frac{1}{8}\int(1+\cos\frac{4x}{3})dx$

$= \frac{1}{4}\int dx - \frac{1}{2}\int\cos\frac{2x}{3}\,dx + \frac{1}{8}\int dx + \frac{1}{8}\int\cos\frac{4x}{3}\,dx = \frac{3}{8}\int dx - \frac{1}{2}\int\cos\frac{2x}{3}\,dx + \frac{1}{8}\int\cos\frac{4x}{3}\,dx$

$= \frac{3}{8}x - \frac{1}{2}\frac{3}{2}\operatorname{sen}\frac{2x}{3} + \frac{1}{8}\frac{3}{4}\operatorname{sen}\frac{4x}{3} + c = \frac{3}{8}x - \frac{3\operatorname{sen}\frac{2x}{3}}{4} + \frac{3\operatorname{sen}\frac{4x}{3}}{32} + c$

3.56.- $\int \operatorname{sen}^3\frac{x}{2}\cos^5\frac{x}{2}\,dx = \int\operatorname{sen}\frac{x}{2}\operatorname{sen}^2\frac{x}{2}\cos^5\frac{x}{2}\,dx = \int\operatorname{sen}\frac{x}{2}(1-\cos^2\frac{x}{2})\cos^5\frac{x}{2}\,dx$

$= \int\operatorname{sen}\frac{x}{2}\cos^5\frac{x}{2}\,dx - \int\cos^7\frac{x}{2}\operatorname{sen}\frac{x}{2}\,dx$

Sea: $u = \cos\frac{x}{2}, du = -\frac{1}{2}\operatorname{sen}\frac{x}{2}\,dx$

Luego: $-2\int u^5du+2\int u^7du=-\frac{2u^6}{6}+\frac{2u^8}{8}+c=-\frac{u^6}{3}+\frac{u^8}{4}+c=-\frac{\cos^6\frac{x}{2}}{3}+\frac{\cos^8\frac{x}{2}}{4}+c$

3.57.- $\int \operatorname{sen}^2x\cos^2 xdx=\int(\operatorname{sen}x\cos x)^2dx=\int\left(\frac{\operatorname{sen}2x}{2}\right)^2dx=\frac{1}{4}\int\operatorname{sen}^2 2xdx$

$=\frac{1}{4}\int\frac{1-\cos 4x}{2}dx=\frac{1}{8}\int(1-\cos 4x)dx=\frac{1}{8}\int dx-\frac{1}{8}\int\cos 4xdx=\frac{x}{8}-\frac{1}{32}\operatorname{sen}4x+c$

3.58.- $\int\operatorname{sen}^4x\cos^2 xdx=\int(\operatorname{sen}^2x\cos^2 x)\operatorname{sen}^2 xdx=\int(\operatorname{sen}x\cos x)^2\operatorname{sen}^2 xdx$

$=\int\left(\frac{\operatorname{sen}2x}{2}\right)^2\left(\frac{1-\cos 2x}{2}\right)dx=\frac{1}{4}\int\operatorname{sen}^2 2x\left(\frac{1-\cos 2x}{2}\right)dx$

$=\frac{1}{8}\int\operatorname{sen}^2 2xdx-\frac{1}{8}\int\operatorname{sen}^2 2x\cos 2xdx=\frac{1}{8}\int\frac{1-\cos 4x}{2}dx-\frac{1}{8}\int\operatorname{sen}^2 2x\cos 2xdx$

$=\frac{1}{16}\int dx-\frac{1}{16}\int\cos 4xdx-\frac{1}{8}\int\operatorname{sen}^2 2x\cos 2xdx(*)$

Sea: $u=\operatorname{sen}2x, du=2\cos 2xdx$, luego:

$(*)=\frac{1}{16}\int dx-\frac{1}{16}\int\cos 4xdx-\frac{1}{16}\int u^2du=\frac{1}{16}x-\frac{1}{64}\operatorname{sen}4x-\frac{1}{16}\frac{u^3}{3}+c$

$=\frac{1}{16}x-\frac{\operatorname{sen}4x}{64}-\frac{\operatorname{sen}^3 2x}{48}+c$

3.59.- $\int\frac{1-\cos 2x}{1+\cos 2x}dx=\int\frac{\frac{1-\cos 2x}{2}}{\frac{1+\cos 2x}{2}}dx=\int\frac{\operatorname{sen}^2 x}{\cos^2 x}dx=\int\tau g^2xdx=\int(\sec^2 x-1)dx$

$=\int\sec^2 xdx-\int dx=\tau gx-x+c$

3.60.- $\int\frac{\cos^3 x}{\sqrt{\operatorname{sen}x}}dx=\int(\operatorname{sen}x)^{-1/2}\cos^3 xdx=\int(\operatorname{sen}x)^{-1/2}\cos^2 x\cos xdx$

$=\int(\operatorname{sen}x)^{-1/2}(1-\operatorname{sen}^2 x)\cos xdx=\int(\operatorname{sen}x)^{-1/2}\cos xdx-\int\operatorname{sen}^{3/2} x\cos xdx(*)$

Sea: $u=\operatorname{sen}x, du=\cos xdx$, luego:

$(*)=\int u^{-1/2}du-\int u^{3/2}du=2u^{1/2}-\frac{2\sqrt{\operatorname{sen}^5 x}}{5}+c$

3.61.- $\int\operatorname{sen}^3 2xdx=\int\operatorname{sen}^2 2x\operatorname{sen}2xdx=\int(1-\cos^2 2x)\operatorname{sen}2xdx$

$=\int\operatorname{sen}2xdx-\int\cos^2 2x\operatorname{sen}2xdx(*)$

Sea: $u=\cos 2x, du=-2\operatorname{sen}2xdx$, luego:

$(*)=\int\operatorname{sen}2x+\frac{1}{2}\int\frac{u^2}{2}du=-\frac{1}{2}\cos 2x+\frac{1}{2}\frac{u^3}{3}+c=-\frac{1}{2}\cos 2x+\frac{u^3}{6}+c$

$=-\frac{1}{2}\cos 2x+\frac{(\cos^3 2x)}{6}+c$

3.62.- $\int sen^2 2x\cos^2 2xdx = \int\left(\frac{1-\cos 4x}{2}\right)\left(\frac{1+\cos 4x}{2}\right)dx = \frac{1}{4}\int(1-\cos^2 4x)dx$

$= \frac{1}{4}\int dx - \frac{1}{4}\int\cos^2 4xdx = \frac{1}{4}\int dx - \frac{1}{4}\int\left(\frac{1+\cos 8x}{2}\right)dx = \frac{1}{4}\int dx - \frac{1}{8}\int(1+\cos 8x)dx$

$= \frac{1}{4}\int dx - \frac{1}{8}\int dx - \frac{1}{8}\int\cos 8xdx = \frac{1}{8}\int dx - \frac{1}{8}\int\cos 8xdx = \frac{x}{8} - \frac{sen 8x}{64} + c$

3.63.- $\int\cos^4 xdx = \int(\cos^2 x)^2 dx = \int\left(\frac{1+\cos 2x}{2}\right)^2 dx = \frac{1}{4}\int(1+\cos 2x)^2 dx$

$= \frac{1}{4}\int(1+2\cos 2x+\cos^2 x)dx = \frac{1}{4}\int dx + \frac{1}{2}\int\cos 2xdx + \frac{1}{4}\int\cos^2 2xdx$

$= \frac{1}{4}\int dx + \frac{1}{2}\int\cos 2xdx + \frac{1}{4}\int\left(\frac{1+\cos 4x}{2}\right)dx = \frac{1}{4}\int dx + \frac{1}{2}\int\cos 2xdx + \frac{1}{8}\int(1+\cos 4x)dx$

$= \frac{1}{4}\int dx + \frac{1}{2}\int\cos 2xdx + \frac{1}{8}\int dx + \frac{1}{8}\int\cos 4xdx = \frac{3}{8}\int dx + \frac{1}{2}\int\cos 2xdx + \frac{1}{8}\int\cos 4xdx$

$= \frac{3}{8}x + \frac{1}{4}sen 2x + \frac{1}{32}sen 4x + c$

3.64.- $\int\tau g^4 x\sec^2 xdx$

Sea: $u = \tau gx, du = \sec^2 xdx$

Luego: $\int u^4 du = \frac{u^5}{5} + c = \frac{\tau g^5 x}{5} + c$

3.65.- $\int\tau g^3 x\sec xdx = \int\tau g^2 x\tau gx\sec xdx = \int(\sec^2 x - 1)\tau gx\sec xdx$

$= \int(\sec^2 x)\tau gx\sec xdx - \int\tau gx\sec xdx$

Sea: $u = \sec x, du = \sec x\tau gxdx$

Luego: $\int u^2 du - \int du = \frac{u^3}{3} - u + c = \frac{\sec^3 x}{3} - \sec x + c$

3.66.- $\int\sec^6 a\theta d\theta = \int\sec^4 a\theta\sec^2 a\theta d\theta = \int(\sec^2 a\theta)^2\sec^2 a\theta d\theta$

$= \int(1+\tau g^2 a\theta)^2\sec^2 a\theta d\theta = \int(1+2\tau g^2 a\theta + \tau g^4 a\theta)\sec^2 a\theta d\theta$

$= \int\sec^2 a\theta d\theta + 2\int\tau g^2 a\theta\sec^2 a\theta d\theta + \int\tau g^4 a\theta\sec^2 a\theta d\theta$

Sea: $u = \tau ga\theta, du = a\sec^2 a\theta d\theta$, Luego:

$\frac{1}{a}\int du + \frac{2}{a}\int u^2 du + \frac{1}{a}\int u^4 du = \frac{1}{a}\left[u + \frac{2u^3}{3} + \frac{u^5}{5}\right] + c = \frac{1}{a}\left[\tau ga\theta + \frac{2\tau g^3 a\theta}{3} + \frac{\tau g^5 a\theta}{5}\right] + c$

3.67.- $\int\sec xdx = \int\frac{\sec x(\tau gx+\sec x)dx}{\tau gx+\sec x} = \int\frac{\sec x\tau gx+\sec^2 x}{\tau gx+\sec x}dx$

Sea: $u = \sec x + \tau gx, du = (\sec x\tau gx + \sec^2 x)dx$

Luego: $\int\frac{du}{u} = \ell\eta|u| + c = \ell\eta|\sec x + \tau gx| + c$

3.68.- $\int \cot g^2 2x \csc^2 2x dx$

Sea: $u = \cot g 2x, du = -2\csc^2 2x dx$

Luego: $-\dfrac{1}{2}\int u^2 du = -\dfrac{u^3}{6} + c = -\dfrac{\cot g^3 2x}{6} + c$

3.69.- $\int \dfrac{\operatorname{sen}^3 x}{\cos^2 x}dx = \int \dfrac{\operatorname{sen}^2 x \operatorname{sen} x dx}{\cos^2 x} = \int \dfrac{(1-\cos^2 x)\operatorname{sen} x dx}{\cos^2 x} = \int \dfrac{\operatorname{sen} x dx}{\cos^2 x} - \int \operatorname{sen} x dx$

Sea: $u = \cos x, du = -\operatorname{sen} x dx$,

Luego: $-\int u^{-2}du - \int \operatorname{sen} x dx = \dfrac{1}{u} + \cos x + c = \dfrac{1}{\cos x} + \cos x + c = \sec x + \cos x + c$

3.70.- $\int \sec^4 3x \tau g 3x dx = \int \sec^3 3x(\sec 3x \tau g 3x)dx$

Sea: $u = \sec 3x, du = 3\sec 3x \tau g 3x dx$

Luego: $\dfrac{1}{3}\int u^3 du = \dfrac{1}{3}\dfrac{u^4}{4} + c = \dfrac{u^4}{12} + c = \dfrac{\sec^4 3x}{12} + c$

3.71.- $\int \sec^n x \tau g x dx = \int \sec^{n-1} x(\sec x \tau g x)dx$

Sea: $u = \sec x, du = \sec x \tau g x dx$,   Luego:

$\int u^{n-1}du = \dfrac{u^n}{n} + c = \dfrac{\sec^n x}{n} + c, (n \neq 0)$

3.72.- $\int \dfrac{\cos^3 x}{\operatorname{sen}^2 x}dx = \int \dfrac{\cos^2 x \cos x}{\operatorname{sen}^2 x}dx = \int \dfrac{(1-\operatorname{sen}^2 x)\cos x}{\operatorname{sen}^2 x}dx = \int \dfrac{\cos x dx}{\operatorname{sen}^2 x} - \int \cos x dx$

$-\dfrac{1}{\operatorname{sen} x} - \operatorname{sen} x + c$

3.73.- $\int \dfrac{dx}{\operatorname{sen}^4 x} = \int \dfrac{\operatorname{sen}^2 x + \cos^2 x}{\operatorname{sen}^4 x}dx = \int \dfrac{dx}{\operatorname{sen}^2 x} + \int \dfrac{\cos^2 x}{\operatorname{sen}^4 x}dx$

$= \int \csc^2 x dx + \int \dfrac{\cos^2 x}{\operatorname{sen}^2 x}\dfrac{dx}{\operatorname{sen}^2 x} = \int \csc^2 x dx + \int \cot g^2 x \csc^2 x dx$

$= -\cot g x - \dfrac{1}{3}\cot g^3 x + c$

3.74.- $\int \tau g^n x \sec^2 x dx; (n \neq -1)$

Sea: $u = \tau g x, du = \sec^2 x dx$

Luego: $\int u^n du = \dfrac{u^{n+1}}{n+1} + c = \dfrac{\tau g^{n+1} x}{n+1} + c, (n \neq -1)$

3.75.- $\int \operatorname{sen}^6 x dx = \int (\operatorname{sen}^2 x)^3 dx = \int \left(\dfrac{1-2\cos 2x}{2}\right)^3 dx$

$= \dfrac{1}{8}\int (1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x)dx$

$= \dfrac{1}{8}\left[\int dx - 3\int \cos 2x dx + 3\int \cos^2 2x dx - \int \cos^3 2x dx\right]$

$$=\frac{5x}{16}-\frac{\operatorname{sen}2x}{4}+\frac{3\operatorname{sen}4x}{64}+\frac{\operatorname{sen}^3 2x}{48}+c$$

3.76.- $\int \operatorname{sen}^4 axdx=\int(\operatorname{sen}^2 ax)^2dx=\frac{1}{4}\int(1-\cos 2ax)^2dx$

$$=\int(1-2\cos 2ax+\cos^2 2ax)dx=\frac{1}{4}\int dx-\frac{1}{2}\int\cos 2axdx+\frac{1}{4}\int\cos^2 2axdx$$

$$=\frac{1}{4}x-\frac{1}{4a}\operatorname{sen}2ax+\frac{1}{4}(\frac{1}{2}x+\frac{1}{8a}\operatorname{sen}4ax)+c=\frac{3}{8}x-\frac{1}{4a}\operatorname{sen}2ax+\frac{1}{32a}\operatorname{sen}4ax+c$$

3.77.- $\int \operatorname{sen}^n x\cos xdx=\frac{\operatorname{sen}^{n+1}x}{n+1}+c,(n\neq -1)$

3.78.- $\int \operatorname{co}\tau g^n axdx=\int \operatorname{co}\tau g^{n-2}ax\operatorname{co}\tau g^2 axdx=\int\operatorname{co}\tau g^{n-2}ax(\cos ec^2ax-1)dx$

$$=\int\operatorname{co}\tau g^{n-2}ax\cos ec^2axdx-\int\operatorname{co}\tau g^{n-2}axdx=-\frac{1}{a}\frac{\operatorname{co}\tau g^{n-1}ax}{n-1}-\int\operatorname{co}\tau g^{n-2}axdx$$

3.79.- $\int\operatorname{co}\tau g^4 3xdx$, Haciendo uso del ejercicio anterior:

$$=-\frac{\operatorname{co}\tau g^3 3x}{3\times 3}-\int\operatorname{co}\tau g^2 3xdx=-\frac{\operatorname{co}\tau g^3 3x}{9}-\int(\cos ec^2 3x-1)dx$$

$$=-\frac{\operatorname{co}\tau g^3 3x}{9}-\int\cos ec^2 3xdx+\int dx=-\frac{\operatorname{co}\tau g^3 3x}{9}-\int\cos ec^2 3xdx+\int dx$$

$$=-\frac{\operatorname{co}\tau g^3 3x}{9}+\frac{\operatorname{co}\tau g 3x}{3}+x+c$$

3.80.- $\int\cos x^n\operatorname{sen}xdx=-\frac{\cos^{n+1}x}{n+1}+c;(n\neq -1)$

3.81.- $\int\tau g^n xdx=\int\tau g^{n-2}x\tau g^2xdx=\int\tau g^{n-2}x(\sec^2 x-1)dx$

$$=\int\tau g^{n-2}x\sec^2 xdx-\int\tau g^{n-2}xdx=\frac{\tau g^{n-1}x}{n-1}-\int\tau g^{n-2}xdx$$

3.82.- $\int\tau g^4xdx=\frac{\tau g^3xdx}{3}-\int\tau g^2xdx=\frac{\tau g^3x}{3}-\int(\sec^2 x-1)dx$

$$=\frac{\tau g^3x}{3}-\int\sec^2 xdx-\int dx=\frac{\tau g^3x}{3}-\tau gx+x+c$$

3.83.- $\int\cos^{2n+1}xdx=\int\cos^{2n}x\cos xdx=\int(\cos^2 x)^n\cos xdx=\int(1-\operatorname{sen}^2 x)^n\cos xdx$

Sea: $u=\operatorname{sen}x, du=\cos xdx$ .El resultado se obtiene, evaluando $(1-u^2)^n$ por la fórmula del binomio de Newton y calculando cada sumando, cuyas integrales son del tipo: $\int u^n du$.

Las fórmulas provenientes de los ejercicios 3.78 y 3.81, se denominan **fórmulas de reducción** y su utilidad es obvia. Más adelante, en otros capítulos, usted deducirá nuevas fórmulas de reducción.

CAPITULO 4

INTEGRACION POR PARTES

Existe una variedad de integrales que se pueden desarrollar, usando la relación: $\int udv = uv - \int vdu$.

El problema es elegir u y dv, por lo cual es útil la siguiente identificación:
I: Función trigonométrica inversa.
L: Función logarítmica.
A: Función algebraica.
T: Función trigonométrica.
E: Función exponencial.
Se usa de la manera siguiente:

EJERCICIOS DESARROLLADOS

4.1.-Encontrar: $\int x\cos xdx$

Solución.- I L A T E

$$\begin{array}{cc} \downarrow & \downarrow \\ x & \cos x \end{array}$$

$$\therefore \begin{array}{ll} u = x & dv = \cos xdx \\ du = dx & v = \operatorname{sen} x \end{array}$$

$$\therefore \int x\cos xdx = x\operatorname{sen} x - \int \operatorname{sen} xdx = x\operatorname{sen} x + \cos x + c$$

Respuesta: $\int x\cos xdx = x\operatorname{sen} x + \cos x + c$

4.2.-Encontrar: $\int x\sec^2 xdx$

Solución.- I L A T E

$$\begin{array}{cc} \downarrow & \downarrow \\ x & \sec^2 3x \end{array}$$

$$\therefore \begin{array}{ll} u = x & dv = \sec^2 3xdx \\ du = dx & v = \frac{1}{3}\tau g3x \end{array}$$

$$\therefore \int x\sec^2 xdx = \frac{1}{3}x\tau g3x - \frac{1}{3}\int \tau g3xdx = \frac{x\tau g3x}{3} - \frac{1}{9}\ell\eta\left|\sec 3x\right| + c$$

Respuesta: $\int x\sec^2 xdx = \frac{x\tau g3x}{3} - \frac{1}{9}\ell\eta\left|\sec 3x\right| + c$

4.3.-Encontrar: $\int x^2 \operatorname{sen} xdx$

Solución.- I L A T E

$$\begin{array}{cc} \downarrow & \downarrow \\ x^2 & \operatorname{sen} x \end{array}$$

$$\therefore \begin{array}{ll} u = x^2 & dv = \operatorname{sen} x dx \\ du = 2x dx & v = -\cos x \end{array}$$

$\therefore \int x^2 \operatorname{sen} x dx = -x^2 \cos x + 2\int x \cos x dx$ **,** integrando por partes la segunda integral:

$\int x \cos x dx$ **;** $\begin{array}{ll} u = x & dv = \cos x dx \\ du = dx & v = \operatorname{sen} x \end{array}$

$$\therefore \int x^2 \operatorname{sen} x dx = -x^2 \cos x + 2\left[x \operatorname{sen} x - \int \operatorname{sen} x dx \right] = -x^2 \cos x + 2x \operatorname{sen} x + 2\cos x + c$$

Respuesta: $\int x^2 \operatorname{sen} x dx = -x^2 \cos x + 2x \operatorname{sen} x + 2\cos x + c$

4.4.-Encontrar: $\int (x^2 + 5x + 6)\cos 2x dx$

Solución.- I L A T E

$$\begin{array}{ll} \downarrow & \searrow \\ x^2 + 5x + 6 & \cos 2x \end{array}$$

$$\therefore \begin{array}{ll} u = x^2 + 5x + 6 & dv = \cos 2x dx \\ du = (2x + 5)dx & v = \frac{1}{2}\operatorname{sen} 2x \end{array}$$

$$\therefore \int (x^2 + 5x + 6)\cos 2x dx = \frac{(x^2 + 5x + 6)}{2}\operatorname{sen} 2x - \frac{1}{2}\int (2x + 5)\operatorname{sen} 2x dx$$

Integrando por partes la segunda integral:

I L A T E

$$\begin{array}{ll} \swarrow & \searrow \\ 2x + 5 & \operatorname{sen} 2x \end{array}$$

$$\therefore \begin{array}{ll} u = 2x + 5 & dv = \operatorname{sen} 2x dx \\ du = 2dx & v = -\frac{1}{2}\cos 2x \end{array}$$

$$\therefore \int (x^2 + 5x + 6)\cos 2x dx = \frac{1}{2}\operatorname{sen} 2x(x^2 + 5x + 6) - \frac{1}{2}\left[(2x + 5)(-\tfrac{1}{2}\cos 2x) + \int \cos 2x dx \right]$$

$$= \frac{x^2 + 5x + 6}{2}\operatorname{sen} 2x + \frac{1}{4}\cos 2x(2x + 5) - \frac{1}{2}\int \cos 2x dx$$

$$= \frac{x^2 + 5x + 6}{2}\operatorname{sen} 2x + \frac{2x + 5}{4}\cos 2x - \frac{1}{4}\operatorname{sen} 2x + c$$

Respuesta: $\int (x^2 + 5x + 6)\cos 2x dx = \frac{x^2 + 5x + 6}{2}\operatorname{sen} 2x + \frac{2x + 5}{4}\cos 2x - \frac{1}{4}\operatorname{sen} 2x + c$

Nota.-Ya se habrá dado cuenta el lector, que la elección conveniente para el u y el dv, dependerá de la ubicación de los términos funcionales en la palabra ILATE. El de la izquierda corresponde al u, y el otro será el dv.

4.5.-Encontrar: $\int \ell \eta x dx$

Solución.- I L A T E

$$\begin{array}{ll} \downarrow & \downarrow \\ \ell \eta x & 1 \end{array}$$

$$\therefore \quad \begin{array}{ll} u=\ell\eta x & dv=1dx \\ du=\dfrac{dx}{x} & v=x \end{array}$$

$$\therefore \int \ell\eta x dx = x\ell\eta x - \int dx = x\ell\eta x - x + c = x(\ell\eta x - 1) + c$$

Respuesta: $\int \ell\eta x dx = x(\ell\eta x - 1) + c$

4.6.-Encontrar: $\int \ell\eta(a^2 + x^2)dx$

Solución.- I L A T E

$$\begin{array}{ll} \downarrow & \searrow \\ \ell\eta(a^2+x^2) & 1 \end{array}$$

$$\therefore \quad \begin{array}{ll} u=\ell\eta x & dv=1dx \\ du=\dfrac{dx}{x} & v=x \end{array}$$

$$\therefore \int \ell\eta(a^2+x^2)dx = x\ell\eta(a^2+x^2) - \int \frac{2x^2dx}{a^2+x^2} = x\ell\eta(a^2+x^2) - \int (2 - \frac{2a^2}{x^2+a^2})dx$$

$$= x\ell\eta(a^2+x^2) - 2\int dx + 2a^2\int \frac{dx}{x^2+a^2} = x\ell\eta(a^2+x^2) - 2x + \frac{2\cancel{a^2}}{\cancel{a}} \operatorname{arc}\tau g \tfrac{x}{a} + c$$

$$= x\ell\eta(a^2+x^2) - 2x + 2a \operatorname{arc}\tau g \tfrac{x}{a} + c$$

Respuesta: $\int \ell\eta(a^2+x^2)dx = x\ell\eta(a^2+x^2) - 2x + 2a \operatorname{arc}\tau g \tfrac{x}{a} + c$

4.7.-Encontrar: $\int \ell\eta\left|x+\sqrt{x^2-1}\right|dx$

Solución.- I L A T E

$$\begin{array}{ll} \downarrow & \searrow \\ \ell\eta\left|x+\sqrt{x^2-1}\right| & 1 \end{array} \qquad \begin{array}{l} dv=1dx \\ v=x \end{array}$$

$$u=\ell\eta\left|x+\sqrt{x^2-1}\right|$$

$$\therefore \quad du = \frac{1+\dfrac{x}{\sqrt{x^2-1}}}{x+\sqrt{x^2-1}}d \Rightarrow du = \frac{\dfrac{\cancel{\sqrt{x^2-1}+x}}{\sqrt{x^2-1}}}{\cancel{x+\sqrt{x^2-1}}}dx \Rightarrow du = \frac{dx}{\sqrt{x^2-1}}$$

$$\therefore \int \ell\eta\left|x+\sqrt{x^2-1}\right|dx = x\ell\eta\left|x+\sqrt{x^2-1}\right| - \int \frac{xdx}{\sqrt{x^2-1}}$$

Sea : $w = x^2+1, dw = 2xdx$.

Luego: $x\ell\eta\left|x+\sqrt{x^2-1}\right| - \frac{1}{2}\int (x^2-1)^{-\frac{1}{2}}2xdx = x\ell\eta\left|x+\sqrt{x^2-1}\right| - \frac{1}{2}\int w^{-\frac{1}{2}}dw$

$$= x\ell\eta\left|x+\sqrt{x^2-1}\right| - \frac{1}{2}\frac{w^{\frac{1}{2}}}{\frac{1}{2}} + c = x\ell\eta\left|x+\sqrt{x^2-1}\right| - w^{\frac{1}{2}} + c = x\ell\eta\left|x+\sqrt{x^2-1}\right| - \sqrt{x^2-1} + c$$

Respuesta: $\int \ell\eta\left|x+\sqrt{x^2-1}\right|dx = x\ell\eta\left|x+\sqrt{x^2-1}\right| - \sqrt{x^2-1} + c$

4.8.-Encontrar: $\int \ell\eta^2 x dx$

Solución.- I L A T E

$$\begin{array}{cc} \downarrow & \downarrow \\ \ell\eta^2 x & 1 \end{array}$$

$$\therefore \begin{array}{ll} u = \ell\eta^2 x & dv = 1dx \\ du = 2\ell\eta x \dfrac{1}{x} dx & v = x \end{array}$$

$$\therefore \int \ell\eta^2 x dx = x\ell\eta^2 x - 2\int \ell\eta x \frac{1}{x} x dx = x\ell\eta^2 x - 2\int \ell\eta x dx$$

Por ejercicio 4.5, se tiene: $\int \ell\eta x dx = x(\ell\eta x - 1) + c$

Luego: $\int \ell\eta^2 x dx = x\ell\eta^2 x - 2\left[x(\ell\eta x - 1) + c\right] = x\ell\eta^2 x - 2x(\ell\eta x - 1) + c$

Respuesta: $\int \ell\eta^2 x dx = x\ell\eta^2 x - 2x(\ell\eta x - 1) + c$

4.9.-Encontrar: $\int \operatorname{arc}\tau g x dx$

Solución.- I L A T E

$$\begin{array}{cc} \downarrow & \downarrow \\ \operatorname{arc}\tau g x & 1 \end{array}$$

$$\therefore \begin{array}{ll} u = \operatorname{arc}\tau g x & dv = 1dx \\ du = \dfrac{dx}{1+x^2} & v = x \end{array}$$

$$\therefore \int \operatorname{arc}\tau g x dx = x \operatorname{arc}\tau g x - \int \frac{x dx}{1+x^2}$$

Sea: $w = 1 + x^2, dw = 2x dx$

Luego: $x \operatorname{arc}\tau g x - \dfrac{1}{2}\int \dfrac{2x dx}{1+x^2} = x \operatorname{arc}\tau g x - \dfrac{1}{2}\int \dfrac{dw}{w} = x \operatorname{arc}\tau g x - \dfrac{1}{2}\ell\eta|w| + c$

$= x \operatorname{arc}\tau g x - \dfrac{1}{2}\ell\eta\left|1+x^2\right| + c$

Respuesta: $\int \operatorname{arc}\tau g x dx = x \operatorname{arc}\tau g x - \dfrac{1}{2}\ell\eta\left|1+x^2\right| + c$

4.10.- $\int x^2 \operatorname{arc}\tau g x dx$

Solución.- I L A T E

$$\begin{array}{cc} \downarrow & \downarrow \\ \operatorname{arc}\tau g x & x^2 \end{array}$$

$$\therefore \begin{array}{ll} u = \operatorname{arc}\tau g x & dv = x^2 dx \\ du = \dfrac{dx}{1+x^2} & v = \dfrac{x^3}{3} \end{array}$$

$$\therefore \int x^2 \operatorname{arc}\tau g x dx = \frac{x^3}{3}\operatorname{arc}\tau g x - \frac{1}{3}\int \frac{x^2 dx}{1+x^2} = \frac{x^3}{3}\operatorname{arc}\tau g x - \frac{1}{3}\int (x - \frac{x}{x^2+1}) dx$$

$$=\frac{x^3}{3}\operatorname{arc}\tau gx-\frac{1}{3}\int xdx-\frac{1}{3}\int\frac{x}{x^2+1}dx$$

Por ejercicio 4.9, se tiene: $\int\frac{xdx}{x^2+1}=\frac{1}{2}\ell\eta\left|x^2+1\right|+c$

Luego: $\frac{x^3}{3}\operatorname{arc}\tau gx-\frac{1}{3}\int xdx+\frac{1}{6}\ell\eta\left|x^2+1\right|+c=\frac{x^3}{3}\operatorname{arc}\tau gx-\frac{x^2}{6}+\frac{1}{6}\ell\eta\left|x^2+1\right|+c$

Respuesta: $\int x^2\operatorname{arc}\tau gxdx=\frac{x^3}{3}\operatorname{arc}\tau gx-\frac{x^2}{6}+\frac{1}{6}\ell\eta\left|x^2+1\right|+c$

4.11.-Encontrar: $\int\operatorname{arc}\cos 2xdx$

Solución.- I L A T E

$$\begin{array}{cc}\downarrow & \downarrow\\ \operatorname{arc}\cos 2x & 1\end{array}$$

$$\therefore\quad \begin{array}{ll} u=\operatorname{arc}\cos 2x & dv=1dx\\ du=-\frac{2dx}{\sqrt{1-4x^2}} & v=x\end{array}$$

$$\therefore\int\operatorname{arc}\cos 2xdx=x\operatorname{arc}\cos 2x+2\int\frac{xdx}{\sqrt{1-4x^2}}$$

Sea: $w=1-4x^2, dw=-8xdx$

Luego: $x\operatorname{arc}\cos 2x-\frac{2}{8}\int\frac{-8xdx}{\sqrt{1-4x^2}}=x\operatorname{arc}\cos 2x-\frac{1}{4}\int w^{-1/2}dw=x\operatorname{arc}\cos 2x-\frac{1}{4}\frac{w^{1/2}}{1/2}+c$

$$=x\operatorname{arc}\cos 2x-\frac{1}{2}\sqrt{1-4x^2}+c$$

Respuesta: $\int\operatorname{arc}\cos 2xdx=x\operatorname{arc}\cos 2x-\frac{1}{2}\sqrt{1-4x^2}+c$

4.12.-Encontrar: $\int\frac{\operatorname{arcsen}\sqrt{x}}{\sqrt{x}}dx$

Solución.- I L A T E

$$\begin{array}{cc}\downarrow & \searrow\\ \operatorname{arcsen}\sqrt{x} & 1\end{array}$$

$$\therefore\quad \begin{array}{ll} u=\operatorname{arcsen}\sqrt{x} & dv=x^{-1/2}dx\\ du=\frac{1}{\sqrt{1-x}}\frac{dx}{\sqrt{x}} & v=2\sqrt{x}\end{array}$$

$$\therefore\int\operatorname{arcsen}\sqrt{x}\,x^{-1/2}dx=2\sqrt{x}\operatorname{arcsen}\sqrt{x}-\int\frac{dx}{\sqrt{1-x}}$$

Sea: $w=1-x, dw=-dx$

Luego: $2\sqrt{x}\operatorname{arcsen}\sqrt{x}+\int\frac{-dx}{\sqrt{1-x}}=2\sqrt{x}\operatorname{arcsen}\sqrt{x}+\int w^{-1/2}dw$

$$=2\sqrt{x}\operatorname{arcsen}\sqrt{x}+2w^{1/2}+c=2\sqrt{x}\operatorname{arcsen}\sqrt{x}+2\sqrt{1-x}+c$$

Respuesta: $\int \frac{\operatorname{arcsen}\sqrt{x}}{\sqrt{x}}dx = 2\sqrt{x}\operatorname{arcsen}\sqrt{x} + 2\sqrt{1-x} + c$

4.13.-Encontrar: $\int x \operatorname{arcsen} 2x^2 dx$

Solución.- I L A T E

$$\begin{array}{ll} \downarrow & \searrow \\ \operatorname{arcsen} 2x^2 & x \end{array}$$

$$\therefore \quad \begin{array}{ll} u = \operatorname{arcsen} 2x^2 & dv = xdx \\ du = \frac{4xdx}{\sqrt{1-4x^4}} & v = \frac{x^2}{2} \end{array}$$

$$\therefore \int x \operatorname{arcsen} 2x^2 dx = \frac{x^2}{2}\operatorname{arcsen} 2x^2 - 2\int \frac{x^3dx}{\sqrt{1-4x^4}}$$

Sea: $w = 1-4x^4, dw = -16x^3dx$

Luego: $\frac{x^2}{2}\operatorname{arcsen} 2x^2 + \frac{2}{16}\int \frac{(-16x^3dx)}{\sqrt{1-4x^4}} = \frac{x^2}{2}\operatorname{arcsen} 2x^2 + \frac{1}{8}\int w^{-1/2}dw$

$$= \frac{x^2}{2}\operatorname{arcsen} 2x^2 + \frac{1}{8}\frac{w^{1/2}}{1/2} + c = \frac{x^2}{2}\operatorname{arcsen} 2x^2 + \frac{1}{4}w^{1/2} + c$$

$$= \frac{x^2}{2}\operatorname{arcsen} 2x^2 + \frac{1}{4}\sqrt{1-4x^4} + c$$

Respuesta: $\int x \operatorname{arcsen} 2x^2 dx = \frac{x^2}{2}\operatorname{arcsen} 2x^2 + \frac{1}{4}\sqrt{1-4x^4} + c$

4.14.-Encontrar: $\int xe^{x/a}dx$

Sea: $w = \frac{x}{a}, dw = \frac{dx}{a}$

Luego: $\int xe^{x/a}dx = a^2\int \frac{x}{a}e^{x/a}\frac{dx}{a} = a^2\int we^w dw$, integrando por partes se tiene:

Solución.- I L A T E

$$\begin{array}{ll} \downarrow & \downarrow \\ w & e^w \end{array}$$

$$\therefore \quad \begin{array}{ll} u = w & dv = e^w dw \\ du = dw & v = e^w \end{array}$$

$$\therefore a^2\int we^w dw = a^2\left(we^w - \int e^w dw\right) = a^2\left(we^w - e^w + c\right) = a^2\left(we^w - e^w\right) + c$$

$$= a^2\left(\frac{x}{a}e^{x/a} - e^{x/a}\right) + c = a^2 e^{x/a}(\frac{x}{a} - 1) + c$$

Respuesta: $\int xe^{x/a}dx = a^2 e^{x/a}(\frac{x}{a} - 1) + c$

4.15.-Encontrar: $\int x^2 e^{-3x}dx$

Solución.- I L A T E

$$\begin{array}{cc} \downarrow & \downarrow \\ x^2 & e^{-3x} \end{array}$$

$$\therefore \quad \begin{array}{ll} u = x^2 & dv = e^{-3x}dx \\ du = 2xdx & v = -\frac{1}{3}e^{-3x} \end{array}$$

$\therefore \int x^2e^{-3x}dx = -\frac{1}{3}x^2e^{-3x} + \frac{2}{3}\int xe^{-3x}dx$, integrando por partes la segunda integral:

I L A T E

$$\begin{array}{cc} \downarrow & \downarrow \\ x & e^{-3x} \end{array}$$

$$\therefore \quad \begin{array}{ll} u = x & dv = e^{-3x}dx \\ du = dx & v = -\frac{1}{3}e^{-3x} \end{array}$$

$$\therefore \int x^2e^{-3x}dx = -\frac{1}{3}x^2e^{-3x} + \frac{2}{3}\left(-\frac{1}{3}xe^{-3x} + \frac{1}{3}\int e^{-3x}dx\right) = -\frac{x^2e^{-3x}}{3} - \frac{2}{9}xe^{-3x} + \frac{2}{9}\int e^{-3x}dx$$

$$= -\frac{x^2e^{-3x}}{3} - \frac{2}{9}xe^{-3x} - \frac{2}{27}e^{-3x} + c$$

Respuesta: $\int x^2e^{-3x}dx = \frac{-e^{-3x}}{3}\left(x^2 + \frac{2}{3}x + \frac{2}{9}\right) + c$

4.16.-Encontrar: $\int x^3e^{-x^2}dx$

Solución.- $\int x^3e^{-x^2}dx = \int x^2e^{-x^2}xdx$

Sea: $w = -x^2, dw = -2xdx$, además: $x^2 = -w$

Luego: $\int x^2e^{-x^2}xdx = -\frac{1}{2}\int x^2e^{-x^2}x(-2xdx) = -\frac{1}{2}\int -we^wdw = \frac{1}{2}\int we^wdw$, integrando por Partes se tiene:

I L A T E

$$\begin{array}{cc} \downarrow & \downarrow \\ w & e^w \end{array}$$

$$\therefore \quad \begin{array}{ll} u = w & dv = e^wdw \\ du = dw & v = e^w \end{array}$$

$$\therefore \frac{1}{2}\int we^wdw = \frac{1}{2}\left(we^w - \int e^wdw\right) = \frac{1}{2}we^w - \frac{1}{2}\int e^wdw = \frac{1}{2}we^w - \frac{1}{2}e^w + c$$

$$= -\frac{1}{2}x^2e^{-x^2} - \frac{1}{2}e^{-x^2} + c = -\frac{1}{2}e^{-x^2}(x^2+1) + c$$

Respuesta: $\int x^3e^{-x^2}dx = -\frac{1}{2}e^{-x^2}(x^2+1) + c$

4.17.-Encontrar: $\int (x^2 - 2x + 5)e^{-x}dx$

Solución.- I L A T E

$$\begin{array}{cc} \downarrow & \downarrow \end{array}$$

$x^2-2x+5 \qquad e^{-x}$

$\therefore \begin{array}{ll} u = x^2-2x+5 & dv = e^{-x}dx \\ du = (2x-2)dx & v = -e^{-x} \end{array}$

$\therefore \int (x^2-2x+5)e^{-x}dx = -e^{-x}(x^2-2x+5) + \int (2x-2)e^{-x}dx$ **,** integrando por partes la segunda integral:

I L A T E

$\downarrow \quad \downarrow$

$2x-2 \quad e^{-x}$

$\therefore \begin{array}{ll} u = 2x-2 & dv = e^{-x}dx \\ du = 2dx & v = -e^{-x} \end{array}$

$\therefore \int (x^2-2x+5)e^{-x}dx = -e^{-x}(x^2-2x+5) + \left[-e^{-x}(2x-2) + 2\int e^{-x}dx\right]$

$= -e^{-x}(x^2-2x+5) - e^{-x}(2x-2) + 2\int e^{-x}dx = -e^{-x}(x^2-2x+5) - e^{-x}(2x-2) - 2e^{-x} + c$

$= -e^{-x}(x^2 \cancel{-2x} + 5 \cancel{+2x} \cancel{-2+2}) + c = -e^{-x}(x^2+5) + c$

Respuesta: $\int (x^2-2x+5)e^{-x}dx = -e^{-x}(x^2+5) + c$

4.18.-Encontrar: $\int e^{ax}\cos bx dx$

Solución.- I L A T E

$\swarrow \quad \downarrow$

$\cos bx \quad e^{ax}$

$\therefore \begin{array}{ll} u = \cos bx & dv = e^{ax}dx \\ du = -b\,\mathrm{sen}\, bx dx & v = \dfrac{1}{a}e^{ax} \end{array}$

$\therefore \int e^{ax}\cos bx dx = \dfrac{e^{ax}\cos bx}{a} + \dfrac{b}{a}\int e^{ax}\,\mathrm{sen}\, bx dx$, Nótese que la segunda integral es semejante a la primera, salvo en la parte trigonométrica; integrando por partes la segunda integral:

I L A T E

$\swarrow \quad \downarrow$

$\mathrm{sen}\, bx \quad e^{ax}$

$\therefore \begin{array}{ll} u = \mathrm{sen}\, bx & dv = e^{ax}dx \\ du = b\cos bx dx & v = \dfrac{1}{a}e^{ax} \end{array}$

$\therefore = \dfrac{e^{ax}\cos bx}{a} + \dfrac{b}{a}\left(\dfrac{e^{ax}\,\mathrm{sen}\, bx}{a} - \dfrac{b}{a}\int e^{ax}\cos bx dx\right)$

$= \dfrac{e^{ax}\cos bx}{a} + \dfrac{be^{ax}\,\mathrm{sen}\, bx}{a^2} - \dfrac{b^2}{a^2}\int e^{ax}\cos bx dx$, Nótese que:

$\int e^{ax}\cos bx dx = \dfrac{e^{ax}\cos bx}{a} + \dfrac{be^{ax}\,\mathrm{sen}\, bx}{a^2} - \dfrac{b^2}{a^2}\int e^{ax}\cos bx dx$, la integral a encontrar aparece con coeficiente 1 en el primer miembro, y en el segundo con coeficiente:

$-\dfrac{b^2}{a^2}$. Transponiendo éste término al primer miembro y dividiendo por el nuevo coeficiente: $1+\dfrac{b^2}{a^2}=\dfrac{a^2+b^2}{a^2}$, se tiene:

$$\left(\frac{a^2+b^2}{a^2}\right)\int e^{ax}\cos bxdx=\frac{ae^{ax}\cos bx+be^{ax}\operatorname{sen} bx}{a^2}+c$$

$$\int e^{ax}\cos bxdx=\frac{\dfrac{ae^{ax}\cos bx+be^{ax}\operatorname{sen} bx}{\cancel{a^2}}}{\left(\dfrac{a^2+b^2}{\cancel{a^2}}\right)}+c=\frac{e^{ax}(a\cos bx+b\operatorname{sen} bx)}{a^2+b^2}+c$$

Respuesta: $\displaystyle\int e^{ax}\cos bxdx=\frac{e^{ax}(a\cos bx+b\operatorname{sen} bx)}{a^2+b^2}+c$

4.19.-Encontrar: $\int e^{x}\cos 2xdx$

Solución.- Este ejercicio es un caso particular del ejercicio anterior, donde: $a=1$ y $b=2$. Invitamos al lector, resolverlo por partes, aún cuando la respuesta es inmediata.

Respuesta: $\displaystyle\int e^{x}\cos 2xdx=\frac{e^{x}(\cos 2x+2\operatorname{sen} 2x)}{5}+c$

4.20.-Encontrar: $\int e^{ax}\operatorname{sen} bxdx$

Solución.- I L A T E

$$\swarrow \quad \downarrow$$

$$\operatorname{sen} bx \qquad e^{ax}$$

$$\therefore \quad \begin{array}{ll} u=\operatorname{sen} bx & dv=e^{ax}dx \\ du=b\cos bxdx & v=\dfrac{1}{a}e^{ax} \end{array}$$

$\therefore \displaystyle\int e^{ax}\operatorname{sen} bxdx=\frac{e^{ax}\operatorname{sen} bx}{a}-\frac{b}{a}\int e^{ax}\cos bxdx$, integrando por partes la segunda integral:

I L A T E

$$\swarrow \quad \downarrow$$

$$\cos bx \qquad e^{ax}$$

$$\therefore \quad \begin{array}{ll} u=\cos bx & dv=e^{ax}dx \\ du=-b\operatorname{sen} bxdx & v=\dfrac{1}{a}e^{ax} \end{array}$$

$$\therefore \int e^{ax}\operatorname{sen} bxdx=\frac{e^{ax}\operatorname{sen} bx}{a}-\frac{b}{a}\left(\frac{e^{ax}\cos bx}{a}+\frac{b}{a}\int e^{ax}\operatorname{sen} bxdx\right)$$

$$=\frac{e^{ax}\operatorname{sen} bx}{a}-\frac{be^{ax}\cos bx}{a^2}-\frac{b^2}{a^2}\int e^{ax}\operatorname{sen} bxdx,$$

Como habrá notado el lector, la integral a encontrar aparece con coeficiente 1 en el primer miembro, y en el segundo con coeficiente: $-\frac{b^2}{a^2}$. Transponiendo éste término al primer miembro y dividiendo por el nuevo coeficiente: $1+\frac{b^2}{a^2}=\frac{a^2+b^2}{a^2}$, se tiene:

$$\left(\frac{a^2+b^2}{a^2}\right)\int e^{ax}\operatorname{sen} bxdx=\frac{ae^{ax}\operatorname{sen} bx-be^{ax}\cos bx}{a^2}+c$$

$$\int e^{ax}\operatorname{sen} bxdx=\frac{\frac{ae^{ax}\operatorname{sen} bx-be^{ax}\cos bx}{\cancel{a^2}}}{\left(\frac{a^2+b^2}{\cancel{a^2}}\right)}+c=\int e^{ax}\operatorname{sen} bxdx=\frac{e^{ax}(a\operatorname{sen} bx-b\cos bx)}{a^2+b^2}+c$$

Respuesta: $\int e^{ax}\operatorname{sen} bxdx=\frac{e^{ax}(a\operatorname{sen} bx-b\cos bx)}{a^2+b^2}+c$

4.21.-Encontrar: $\int x\sqrt{1+x}dx$

Solución.- Cuando el integrando, está formado por el producto de funciones algebraicas, es necesario tomar como dv, la parte más fácil integrable y u como la parte más fácil derivable. Sin embargo, la opción de "más fácil" quedará a criterio del lector.

$$\therefore \quad \begin{array}{ll} u=x & dv=(1+x)^{1/2}dx \\ du=dx & v=\frac{2}{3}(1+x)^{3/2} \end{array}$$

$$\therefore \int x\sqrt{1+x}dx=\frac{2}{3}x(1+x)^{3/2}-\frac{2}{3}\int(1+x)^{3/2}dx=\frac{2}{3}x(1+x)^{3/2}-\frac{2}{3}\frac{(1+x)^{5/2}}{5/2}+c$$

$$=\frac{2}{3}x(1+x)^{3/2}-\frac{4(1+x)^{5/2}}{15}+c$$

Respuesta: $\int x\sqrt{1+x}dx=\frac{2}{3}x(1+x)^{3/2}-\frac{4(1+x)^{5/2}}{15}+c$

4.22.-Encontrar: $\int\frac{x^2dx}{\sqrt{1+x}}$

Solución.- $\int\frac{x^2dx}{\sqrt{1+x}}=\int x^2(1+x)^{-1/2}dx$

$$\therefore \quad \begin{array}{ll} u=x^2 & dv=(1+x)^{-1/2}dx \\ du=2xdx & v=2(1+x)^{1/2} \end{array}$$

$\therefore \int\frac{x^2dx}{\sqrt{1+x}}=2x^2\sqrt{1+x}-4\int x\sqrt{1+x}dx$, integrando por partes la segunda integral:

$$\therefore \quad \begin{array}{ll} u = x & dv = (1+x)^{1/2}dx \\ du = dx & v = \dfrac{2}{3}(1+x)^{3/2} \end{array}$$

$$\int \frac{x^2dx}{\sqrt{1+x}} = 2x^2\sqrt{1+x} - 4\left[\frac{2}{3}x(1+x)^{3/2} - \frac{2}{3}\int (1+x)^{3/2}\,dx\right]$$

$$= 2x^2\sqrt{1+x} - \frac{8}{3}x(1+x)^{3/2} + \frac{8}{3}\frac{(1+x)^{5/2}}{5/2} + c = 2x^2\sqrt{1+x} - \frac{8}{3}x(1+x)^{3/2} + \frac{16}{15}(1+x)^{5/2} + c$$

Respuesta: $\int \frac{x^2dx}{\sqrt{1+x}} = 2x^2\sqrt{1+x} - \frac{8}{3}x(1+x)^{3/2} + \frac{16}{15}(1+x)^{5/2} + c$

4.23.-Encontrar: $\int \frac{xdx}{e^x}$

Solución.- $\int \frac{xdx}{e^x} = \int xe^{-x}dx$

I L A T E

$$\begin{array}{cc} \downarrow & \downarrow \\ x & e^{-x} \end{array}$$

$$\therefore \quad \begin{array}{ll} u = x & dv = e^{-x}dx \\ du = dx & v = -e^{-x} \end{array}$$

$$\therefore \int xe^{-x}dx = -xe^{-x} + \int e^{-x}dx = -xe^{-x} - e^{-x} + c = e^{-x}(-x-1) + c = -e^{-x}(x+1) + c$$

Respuesta: $\int \frac{xdx}{e^x} = -e^{-x}(x+1) + c$

4.24.-Encontrar: $\int x^2 \ell\eta\left|\sqrt{1-x}\right|dx$

Solución.- $\therefore$

$$u = \ell\eta\left|\sqrt{1-x}\right| \qquad dv = x^2dx$$

$$du = \frac{1}{\left|\sqrt{1-x}\right|}\frac{1}{2}(1-x)^{-1/2}(-1)dx \Rightarrow du = \frac{-dx}{2(1-x)} \qquad v = \frac{x^3}{3}$$

$$\therefore \int x^2\ell\eta\left|\sqrt{1-x}\right|dx = \frac{x^3}{3}\ell\eta\left|\sqrt{1-x}\right| + \frac{1}{6}\int\frac{x^3}{1-x}dx = \frac{x^3}{3}\ell\eta\left|\sqrt{1-x}\right| - \frac{1}{6}\int\left(x^2 + x + 1 - \frac{1}{1-x}\right)dx$$

$$= \frac{x^3}{3}\ell\eta\left|\sqrt{1-x}\right| - \frac{1}{6}\frac{x^3}{3} - \frac{1}{6}\frac{x^2}{2} - \frac{1}{6}x - \frac{1}{6}\ell\eta|1-x| + c$$

$$= \frac{x^3}{3}\ell\eta\left|\sqrt{1-x}\right| - \frac{1}{6}\ell\eta|1-x| - \frac{x^3}{18} - \frac{x^2}{12} - \frac{x}{6} + c$$

Respuesta: $\int x^2\ell\eta\left|\sqrt{1-x}\right|dx = \frac{x^3}{3}\ell\eta\left|\sqrt{1-x}\right| - \frac{1}{6}\ell\eta|1-x| - \frac{x^3}{18} - \frac{x^2}{12} - \frac{x}{6} + c$

4.25.-Encontrar: $\int x\,\mathrm{sen}^2 x dx$

Solución.-

$$\therefore \quad \begin{array}{ll} u = x & dv = \operatorname{sen}^2 x dx \\ du = dx & v = \frac{1}{2}x - \frac{1}{4}\operatorname{sen} 2x \end{array} \qquad \left(v = \int \frac{1-\cos 2x}{2} dx \right)$$

$$\therefore \int x \operatorname{sen}^2 x dx = \frac{1}{2}x^2 - \frac{1}{4}x \operatorname{sen} 2x - \frac{1}{2}\int x dx + \frac{1}{4}\int \operatorname{sen} 2x dx$$

$$= \frac{1}{2}x^2 - \frac{1}{4}x \operatorname{sen} 2x - \frac{1}{4}x^2 - \frac{1}{8}\cos 2x + c = \frac{1}{4}x^2 - \frac{1}{4}x \operatorname{sen} 2x - \frac{1}{8}\cos 2x + c$$

Respuesta: $\int x \operatorname{sen}^2 x dx = \frac{x^2}{4} - \frac{x \operatorname{sen} 2x}{4} - \frac{\cos 2x}{8} + c$

Otra solución.-

$$\int x \operatorname{sen}^2 x dx = \int x \frac{1-\cos 2x}{2} dx = \frac{1}{2}\int x dx - \frac{1}{2}\int x \cos 2x dx = \frac{1}{2}\frac{x^2}{2} - \frac{1}{2}\int x \cos 2x dx$$

$= \frac{x^2}{4} - \frac{1}{2}\int x \cos 2x dx$ **;** integrando por partes, la segunda integral:

$$\therefore \quad \begin{array}{ll} u = x & dv = \cos 2x dx \\ du = dx & v = \frac{1}{2}\operatorname{sen} 2x \end{array}$$

$$\int x \operatorname{sen}^2 x dx = \frac{x^2}{4} - \frac{1}{2}\left(\frac{x}{2}\operatorname{sen} 2x - \frac{1}{2}\int \operatorname{sen} 2x dx\right) = \frac{x^2}{4} - \frac{x}{4}\operatorname{sen} 2x + \frac{1}{4}\int \operatorname{sen} 2x dx$$

$$= \frac{x^2}{4} - \frac{x}{4}\operatorname{sen} 2x + \frac{1}{4}(-\frac{1}{2}\cos 2x) + c = \frac{x^2}{4} - \frac{x}{4}\operatorname{sen} 2x - \frac{\cos 2x}{8} + c$$

Respuesta: $\int x \operatorname{sen}^2 x dx = \frac{x^2}{4} - \frac{x \operatorname{sen} 2x}{4} - \frac{\cos 2x}{8} + c$

4.26.-Encontrar: $\int x(3x+1)^7 dx$

Solución.-

$$\therefore \quad \begin{array}{ll} u = x & dv = (3x+1)^7 dx \\ du = dx & v = \frac{1}{24}(3x+1)^8 \end{array} \qquad \left(v = \int (3x+1)^7 dx \right)$$

$$\therefore \int x(3x+1)^7 dx = \frac{x}{24}(3x+1)^8 - \frac{1}{24}\int (3x+1)^8 dx = \frac{x}{24}(3x+1)^8 - \frac{1}{24}\frac{1}{3}\frac{(3x+1)^9}{9} + c$$

$$= \frac{x}{24}(3x+1)^8 - \frac{(3x+1)^9}{648} + c$$

Respuesta: $\int x(3x+1)^7 dx = \frac{x}{24}(3x+1)^8 - \frac{(3x+1)^9}{648} + c$

EJERCICIOS PROPUESTOS

Usando esencialmente el mecanismo presentado, encontrar las integrales siguientes:

4.27.- $\int x(2x+5)^{10}dx$ **4.28.-** $\int \arcsen xdx$ **4.29.-** $\int x\,\mathrm{sen}\,xdx$

4.30.- $\int x\cos 3xdx$ **4.31.-** $\int x2^{-x}dx$ **4.32.-** $\int x^2e^{3x}dx$

4.33.- $\int x^3e^{-x/3}dx$ **4.34.-** $\int x\,\mathrm{sen}\,x\cos xdx$ **4.35.-** $\int x^2\ell\eta xdx$

4.36.- $\int \frac{\ell\eta x}{x^3}dx$ **4.37.-** $\int \frac{\ell\eta x}{\sqrt{x}}dx$ **4.38.-** $\int x\,\mathrm{arc}\,\tau gxdx$

4.39.- $\int x\,\mathrm{arcsen}\,xdx$ **4.40.-** $\int \frac{xdx}{\mathrm{sen}^2 x}$ **4.41.-** $\int e^x\,\mathrm{sen}\,xdx$

4.42.- $\int 3^x\cos xdx$ **4.43.-** $\int \mathrm{sen}(\ell\eta x)dx$ **4.44.-** $\int (x^2-2x+3)\ell\eta xdx$

4.45.- $\int x\ell\eta\left|\frac{1-x}{1+x}\right|dx$ **4.46.-** $\int \frac{\ell\eta^2 x}{x^2}dx$ **4.47.-** $\int x^2\,\mathrm{arc}\,\tau g3xdx$

4.48.- $\int x(\mathrm{arc}\,\tau gx)^2dx$ **4.49.-** $\int (\mathrm{arcsen}\,x)^2dx$ **4.50.-** $\int \frac{\mathrm{arcsen}\,x}{x^2}dx$

4.51.- $\int \frac{\mathrm{arcsen}\sqrt{x}}{\sqrt{1-x}}dx$ **4.52.-** $\int \frac{\mathrm{sen}^2 x}{e^x}dx$ **4.53.-** $\int \tau g^2x\sec^3 xdx$

4.54.- $\int x^3\ell\eta^2xdx$ **4.55.-** $\int x\ell\eta(9+x^2)dx$ **4.56.-** $\int \mathrm{arcsen}\sqrt{x}dx$

4.57.- $\int x\,\mathrm{arc}\,\tau g(2x+3)dx$ **4.58.-** $\int e^{\sqrt{x}}dx$ **4.59.-** $\int \cos^2(\ell\eta x)dx$

4.60.- $\int \frac{\ell\eta(\ell\eta x)}{x}dx$ **4.61.-** $\int \ell\eta|x+1|dx$ **4.62.-** $\int x^2e^xdx$

4.63.- $\int \cos^n xdx$ **4.64.-** $\int \mathrm{sen}^n xdx$ **4.65.-** $\int x^m(\ell\eta x)^ndx$

4.66.- $\int x^3(\ell\eta x)^2dx$ **4.67.-** $\int x^ne^xdx$ **4.68.-** $\int x^3e^xdx$

4.69.- $\int \sec^n xdx$ **4.70.-** $\int \sec^3 xdx$ **4.71.-** $\int x\ell\eta xdx$

4.72.- $\int x^n\ell\eta|ax|dx, n\neq -1$ **4.73.-** $\int \mathrm{arcsen}\,axdx$ **4.74.-** $\int x\,\mathrm{sen}\,axdx$

4.75.- $\int x^2\cos axdx$ **4.76.-** $\int x\sec^2 axdx$ **4.77.-** $\int \cos(\ell\eta x)dx$

4.78.- $\int \ell\eta(9+x^2)dx$ **4.79.-** $\int x\cos(2x+1)dx$ **4.80.-** $\int x\,\mathrm{arc}\sec xdx$

4.81.- $\int \mathrm{arc}\sec\sqrt{x}dx$ **4.82.-** $\int \sqrt{a^2-x^2}dx$ **4.83.-** $\int \ell\eta|1-x|dx$

4.84.- $\int \ell\eta(x^2+1)dx$ **4.85.-** $\int \mathrm{arc}\,\tau g\sqrt{x}dx$ **4.86.-** $\int \frac{x\,\mathrm{arcsen}\,x}{\sqrt{1-x^2}}dx$

4.87.- $\int x\,\mathrm{arc}\,\tau g\sqrt{x^2-1}dx$ **4.88.-** $\int \frac{x\,\mathrm{arc}\,\tau gx}{(x^2+1)^2}dx$ **4.89.-** $\int \mathrm{arcsen}\,x\frac{xdx}{\sqrt{(1-x^2)^3}}$

4.90.- $\int x^2\sqrt{1-x}dx$

RESPUESTAS

4.27.- $\int x(2x+5)^{10}dx$

Solución.-

$$\therefore \quad \begin{array}{ll} u = x & dv = (2x+5)^{10}dx \\ du = dx & v = \dfrac{(2x+5)^{11}}{22} \end{array}$$

$$\int x(2x+5)^{10}dx = \frac{x}{22}(2x+5)^{11} - \frac{1}{22}\int (2x+5)^{11}dx = \frac{x}{22}(2x+5)^{11} - \frac{1}{44}(2x+5)^{12} + c$$

$$= \frac{x}{22}(2x+5)^{11} - \frac{1}{528}(2x+5)^{12} + c$$

4.28.- $\int \arcsin xdx$

Solución.-

$$\therefore \quad \begin{array}{ll} u = \arcsin x & dv = dx \\ du = \dfrac{dx}{\sqrt{1-x^2}} & v = x \end{array}$$

Además: $w = 1 - x^2, dw = -2xdx$

$$\int \arcsin xdx = x\arcsin x - \int \frac{xdx}{\sqrt{1-x^2}} = x\arcsin x + \frac{1}{2}\int \frac{dw}{w^{1/2}} = x\arcsin x + \sqrt{1-x^2} + c$$

4.29.- $\int x\operatorname{sen} xdx$

Solución.-

$$\therefore \quad \begin{array}{ll} u = x & dv = \operatorname{sen} xdx \\ du = dx & v = -\cos x \end{array}$$

$$\int x\operatorname{sen} xdx = -x\cos x + \int \cos xdx = -x\cos x + \operatorname{sen} x + c$$

4.30.- $\int x\cos 3xdx$

Solución.-

$$\therefore \quad \begin{array}{ll} u = x & dv = \cos 3xdx \\ du = dx & v = \dfrac{1}{3}\operatorname{sen} 3x \end{array}$$

$$\int x\cos 3xdx = \frac{x}{3}\operatorname{sen} 3x - \int \frac{1}{3}\operatorname{sen} 3xdx = \frac{x}{3}\operatorname{sen} 3x + \frac{\cos 3x}{9} + c$$

4.31.- $\int x2^{-x}dx$

Solución.-

$$\therefore \quad \begin{array}{ll} u = x & dv = 2^{-x}dx \\ du = dx & v = -\dfrac{2^{-x}}{\ell\eta 2} \end{array}$$

$$\int x2^{-x}dx = -\frac{x2^{-x}}{\ell\eta 2} + \frac{1}{\ell\eta 2}\int 2^{-x}dx = -\frac{x2^{-x}}{\ell\eta 2} + \frac{1}{\ell\eta 2}\left(\frac{-2^{-x}}{\ell\eta 2}\right) + c = -\frac{x}{2^x\ell\eta 2} - \frac{1}{2^{-x}\ell\eta^2 2} + c$$

4.32.- $\int x^2e^{3x}dx$

Solución.-

$\therefore \quad \begin{array}{ll} u = x^2 & dv = e^{3x}dx \\ du = 2xdx & v = \frac{1}{3}e^{3x} \end{array}$

$\int x^2 e^{3x}dx = \frac{x^2}{3}e^{3x} - \frac{2}{3}\int xe^{3x}dx$, integral la cual se desarrolla nuevamente por partes, esto es: $\therefore \quad \begin{array}{ll} u = x & dv = e^{3x}dx \\ du = dx & v = \frac{1}{3}e^{3x} \end{array}$

$$= \frac{x^2}{3}e^{3x} - \frac{2}{3}\left(\frac{x}{3}e^{3x} - \frac{1}{3}\int e^{3x}dx\right) = \frac{x^2}{3}e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{9}\int e^{3x}dx = \frac{x^2}{3}e^{3x} - \frac{2x}{9}e^{3x} + \frac{2}{27}e^{3x} + c$$

4.33.- $\int x^3 e^{-x/3}dx$

Solución.-

$\therefore \quad \begin{array}{ll} u = x^3 & dv = e^{-x/3}dx \\ du = 3x^2dx & v = -3e^{-x/3} \end{array}$

$\int x^3 e^{-x/3}dx = -3x^3 e^{-x/3} + 9\int x^2 e^{-x/3}dx$, integral la cual se desarrolla nuevamente por partes, esto es: $\therefore \quad \begin{array}{ll} u = x^2 & dv = e^{-x/3}dx \\ du = 2xdx & v = -3e^{-x/3} \end{array}$

$$= -3x^3 e^{-x/3} + 9\left(-3x^2 e^{-x/3} + 6\int xe^{-x/3}dx\right) = -3x^3 e^{-x/3} - 27x^2 e^{-x/3} + 54\int xe^{-x/3}dx$$

, la nueva integral se desarrolla por partes, esto es:

$\therefore \quad \begin{array}{ll} u = x & dv = e^{-x/3}dx \\ du = dx & v = -3e^{-x/3} \end{array}$

$$= -\frac{3x^3}{e^{x/3}} - \frac{27x^2}{e^{x/3}} + 54\left(-3xe^{-x/3} + 3\int e^{-x/3}dx\right) = -\frac{3x^3}{e^{x/3}} - \frac{27x^2}{e^{x/3}} - \frac{162x}{e^{x/3}} + 162(-3e^{-x/3}) + c$$

$$= -\frac{3x^3}{e^{x/3}} - \frac{27x^2}{e^{x/3}} - \frac{162x}{e^{x/3}} - \frac{486}{e^{x/3}} + c$$

4.34.- $\int x \operatorname{sen} x \cos x dx$

Solución.-

$\therefore \quad \begin{array}{ll} u = x & dv = \operatorname{sen} 2xdx \\ du = dx & v = -\frac{\cos 2x}{2} \end{array}$

$$\int x \operatorname{sen} x \cos x dx = \frac{1}{2}\int x \operatorname{sen} 2xdx = \frac{1}{2}\left(-\frac{x}{2}\cos 2x + \frac{1}{2}\int \cos 2xdx\right)$$

$$= -\frac{x}{4}\cos 2x + \frac{1}{4}\int \cos 2xdx = -\frac{x}{4}\cos 2x + \frac{1}{8}\operatorname{sen} 2x + c$$

4.35.- $\int x^2 \ell\eta x dx$

Solución.-

$$u = \ell\eta x \qquad dv = x^2 dx$$

$$\therefore \quad du = \frac{dx}{x} \qquad v = \frac{x^3}{3}$$

$$\int x^2 \ell\eta x dx = \frac{x^3 \ell\eta x}{3} - \frac{1}{3}\int x^2 dx = \frac{x^3 \ell\eta x}{3} - \frac{x^3}{9} + c$$

4.36.- $\int \frac{\ell\eta x}{x^3} dx$

Solución.-

$$u = \ell\eta x \qquad dv = x^{-3} dx$$

$$\therefore \quad du = \frac{dx}{x} \qquad v = -\frac{1}{2x^2}$$

$$\int \frac{\ell\eta x}{x^3} dx = \int x^{-3} \ell\eta x dx = -\frac{\ell\eta x}{2x^2} + \frac{1}{2}\int x^{-3} dx = -\frac{\ell\eta x}{2x^2} - \frac{1}{4x^2} + c$$

4.37.- $\int \frac{\ell\eta x}{\sqrt{x}} dx$

Solución.-

$$u = \ell\eta x \qquad dv = x^{-\frac{1}{2}} dx$$

$$\therefore \quad du = \frac{dx}{x} \qquad v = 2\sqrt{x}$$

$$\int \frac{\ell\eta x}{\sqrt{x}} dx = \int x^{-\frac{1}{2}} \ell\eta x dx = 2\sqrt{x}\ell\eta x - 2\int x^{-\frac{1}{2}} dx = 2\sqrt{x}\ell\eta x - 4\sqrt{x} + c$$

4.38.- $\int x \operatorname{arc}\tau g x dx$

Solución.-

$$u = \operatorname{arc}\tau g x \qquad dv = x dx$$

$$\therefore \quad du = \frac{dx}{1+x^2} \qquad v = \frac{x^2}{2}$$

$$\int x \operatorname{arc}\tau g x dx = \frac{x^2}{2}\operatorname{arc}\tau g x - \frac{1}{2}\int \frac{x^2 dx}{1+x^2} = \frac{x^2}{2}\operatorname{arc}\tau g x - \frac{1}{2}\int \left(1 - \frac{1}{1+x^2}\right) dx$$

$$= \frac{x^2}{2}\operatorname{arc}\tau g x - \frac{1}{2}\int dx + \frac{1}{2}\int \frac{dx}{1+x^2} = \frac{x^2}{2}\operatorname{arc}\tau g x - \frac{1}{2}x + \frac{\operatorname{arc}\tau g x}{2} + c$$

4.39.- $\int x \operatorname{arcs}e\operatorname{n} x dx$

Solución.-

$$u = \operatorname{arcs}e\operatorname{n} x \qquad dv = x dx$$

$$\therefore \quad du = \frac{dx}{\sqrt{1+x^2}} \qquad v = \frac{x^2}{2}$$

$\int x \operatorname{arcs}e\operatorname{n} x dx = \frac{x^2}{2}\operatorname{arcs}e\operatorname{n} x - \frac{1}{2}\int \frac{x^2 dx}{\sqrt{1+x^2}}$, integral para la cual se sugiere la sustitución siguiente: $\therefore$ $x = \operatorname{s}e\operatorname{n}\theta$, $dx = \cos\theta d\theta$

$$=\frac{x^2}{2}\operatorname{arcsen}x-\frac{1}{2}\int\frac{\operatorname{sen}^2\theta\cancel{\cos\theta}d\theta}{\cancel{\cos\theta}}$$
$$=\frac{x^2}{2}\operatorname{arcsen}x-\frac{1}{2}\int\left(\frac{1-\cos 2\theta}{2}\right)d\theta=\frac{x^2}{2}\operatorname{arcsen}x-\frac{1}{4}\int d\theta+\frac{1}{4}\int\cos 2\theta d\theta$$
$$=\frac{x^2}{2}\operatorname{arcsen}x-\frac{1}{4}\theta+\frac{1}{8}\operatorname{sen}2\theta+c=\frac{x^2}{2}\operatorname{arcsen}x-\frac{1}{4}\operatorname{arcsen}x+\frac{2\operatorname{sen}\theta\cos\theta}{8}+c$$
Como: $\operatorname{sen}\theta=x,\cos\theta=\sqrt{1-x^2}$; luego:
$$=\frac{x^2}{2}\operatorname{arcsen}x-\frac{1}{4}\operatorname{arcsen}x+\frac{1}{4}x\sqrt{1-x^2}+c$$
4.40.- $\int\frac{xdx}{\operatorname{sen}^2 x}$

Solución.-

$$\therefore\quad \begin{array}{ll} u=x & dv=\operatorname{cosec}^2 xdx \\ du=dx & v=-\operatorname{co}\tau gx \end{array}$$
$$\int\frac{xdx}{\operatorname{sen}^2 x}=\int x\operatorname{cosec}^2 xdx=-x\operatorname{co}\tau gx+\int\operatorname{co}\tau gxdx=-x\operatorname{co}\tau gx+\ell\eta\left|\operatorname{sen}x\right|+c$$
4.41.- $\int e^x\operatorname{sen}xdx$

Solución.-

$$\therefore\quad \begin{array}{ll} u=\operatorname{sen}x & dv=e^x dx \\ du=\cos xdx & v=e^x \end{array}$$
$\int e^x\operatorname{sen}xdx=e^x\operatorname{sen}x-\int e^x\cos xdx$, integral la cual se desarrolla por partes, esto es:

$$\therefore\quad \begin{array}{ll} u=\cos x & dv=e^x dx \\ du=-\operatorname{sen}xdx & v=e^x \end{array}$$
$$=e^x\operatorname{sen}x-\left(e^x\cos x+\int e^x\operatorname{sen}xdx\right)=e^x\operatorname{sen}x-e^x\cos x-\int e^x\operatorname{sen}xdx$$
Luego se tiene: $\int e^x\operatorname{sen}xdx=e^x\operatorname{sen}x-e^x\cos x-\int e^x\operatorname{sen}xdx$, de donde es inmediato:
$$2\int e^x\operatorname{sen}xdx=e^x(\operatorname{sen}x-\cos x)+c$$
$$\int e^x\operatorname{sen}xdx=\frac{e^x}{2}(\operatorname{sen}x-\cos x)+c$$
4.42.- $\int 3^x\cos xdx$

Solución.-

$$\therefore\quad \begin{array}{ll} u=\cos x & dv=3^x dx \\ du=-\operatorname{sen}xdx & v=\dfrac{3^x}{\ell\eta 3} \end{array}$$

$\int 3^x \cos xdx = \cos x \frac{3^x}{\ell\eta 3} + \frac{1}{\ell\eta 3}\int 3^x \operatorname{sen} xdx$, integral la cual se desarrolla por partes, esto es: $\therefore$

$$u = \operatorname{sen} x \qquad dv = 3^x dx$$
$$du = \cos xdx \qquad v = \frac{3^x}{\ell\eta 3}$$

$$= \cos x \frac{3^x}{\ell\eta 3} + \frac{1}{\ell\eta 3}\left(\frac{3^x}{\ell\eta 3}\operatorname{sen} x - \frac{1}{\ell\eta 3}\int 3^x \cos xdx \right)$$

$= \cos x \frac{3^x}{\ell\eta 3} + \frac{3^x \operatorname{sen} x}{\ell\eta^2 3} - \frac{1}{\ell\eta^2 3}\int 3^x \cos xdx$,luego:

$= \int 3^x \cos xdx = \frac{3^x}{\ell\eta}\left(\cos x + \frac{\operatorname{sen} x}{\ell\eta 3} \right) - \frac{1}{\ell\eta^2 3}\int 3^x \cos xdx$, de donde es inmediato:

$$= (1 + \frac{1}{\ell\eta^2 3})\int 3^x \cos xdx = \frac{3^x}{\ell\eta 3}\left(\cos x + \frac{\operatorname{sen} x}{\ell\eta 3} \right) + c$$

$$= (\frac{\ell\eta^2 3 + 1}{\ell\eta^{\cancel{2}} 3})\int 3^x \cos xdx = \frac{3^x}{\cancel{\ell\eta 3}}\left(\cos x + \frac{\operatorname{sen} x}{\ell\eta 3} \right) + c$$

$$= \int 3^x \cos xdx = \frac{3^x \ell\eta 3}{\ell\eta^2 3 + 1}\left(\cos x + \frac{\operatorname{sen} x}{\ell\eta 3} \right) + c$$

4.43.- $\int \operatorname{sen}(\ell\eta x)dx$

Solución.-

$$\therefore \quad u = \operatorname{sen}(\ell\eta x) \qquad dv = dx$$
$$du = \frac{\cos(\ell\eta x)}{x}dx \qquad v = x$$

$\int \operatorname{sen}(\ell\eta x)dx = x\operatorname{sen}(\ell\eta x) - \int \cos(\ell\eta x)dx$, integral la cual se desarrolla por partes, esto es:

$$\therefore \quad u = \cos(\ell\eta x) \qquad dv = dx$$
$$du = \frac{-\operatorname{sen}(\ell\eta x)}{x}dx \qquad v = x$$

$$= x\operatorname{sen}(\ell\eta x) - \left[x\cos(\ell\eta x) + \int \operatorname{sen}(\ell\eta x)dx \right] = x\operatorname{sen}(\ell\eta x) - x\cos(\ell\eta x) - \int \operatorname{sen}(\ell\eta x)dx$$

Se tiene por tanto:

$\int \operatorname{sen}(\ell\eta x)dx = x\left[\operatorname{sen}(\ell\eta x) - \cos(\ell\eta x)\right] - \int \operatorname{sen}(\ell\eta x)dx$, de donde es inmediato:

$$2\int \operatorname{sen}(\ell\eta x)dx = x\left[\operatorname{sen}(\ell\eta x) - \cos(\ell\eta x)\right] + c \quad \int \operatorname{sen}(\ell\eta x)dx = \frac{x}{2}\left[\operatorname{sen}(\ell\eta x) - \cos(\ell\eta x)\right] + c$$

4.44.- $\int (x^2 - 2x + 3)\ell\eta xdx$

Solución.-

$$u = \ell\eta x \qquad dv = (x^2 - 2x + 3)dx$$

$$\therefore \quad du = \frac{dx}{x} \qquad v = \frac{x^3}{3} - x^2 + 3x$$

$$\int (x^2 - 2x + 3)\ell\eta x dx = (\frac{x^3}{3} - x^2 + 3x)\ell\eta x - \int (\frac{x^2}{3} - x + 3)dx$$

$$= (\frac{x^3}{3} - x^2 + 3x)\ell\eta x - \int \frac{x^2}{3}dx - \int xdx + 3\int dx = (\frac{x^3}{3} - x^2 + 3x)\ell\eta x - \frac{x^3}{9} - \frac{x^2}{2} + 3x + c$$

4.45.- $\int x\ell\eta\left|\frac{1-x}{1+x}\right|dx$

Solución.-

$$u = \ell\eta\left|\frac{1-x}{1+x}\right| \qquad dv = xdx$$

$$\therefore \quad du = \frac{2dx}{x^2 - 1} \qquad v = \frac{x^2}{2}$$

$$\int x\ell\eta\left|\frac{1-x}{1+x}\right|dx = \frac{x^2}{2}\ell\eta\left|\frac{1-x}{1+x}\right| - \int \frac{x^2 dx}{x^2 - 1} = \frac{x^2}{2}\ell\eta\left|\frac{1-x}{1+x}\right| - \int (1 + \frac{1}{x^2 - 1})dx$$

$$= \frac{x^2}{2}\ell\eta\left|\frac{1-x}{1+x}\right| - \int dx - \int \frac{dx}{x^2 - 1} = \frac{x^2}{2}\ell\eta\left|\frac{1-x}{1+x}\right| - x - \frac{1}{2}\ell\eta\left|\frac{x-1}{x+1}\right| + c$$

4.46.- $\int \frac{\ell\eta^2 x}{x^2}dx$

Solución.-

$$u = \ell\eta^2 x \qquad dv = x^{-2}dx$$

$$\therefore \quad du = \frac{2\ell\eta x}{x}dx \qquad v = -\frac{1}{x}$$

$\int \frac{\ell\eta^2 x}{x^2}dx = -\frac{\ell\eta^2 x}{x} + 2\int \frac{\ell\eta x}{x^2}dx = -\frac{\ell\eta^2 x}{x} + 2\int x^{-2}\ell\eta x dx$, integral la cual se desarrolla por partes, esto es:

$$u = \ell\eta x \qquad dv = x^{-2}dx$$

$$\therefore \quad du = \frac{dx}{x} \qquad v = -\frac{1}{x}$$

$$= -\frac{\ell\eta^2 x}{x} + 2\left(-\frac{\ell\eta x}{x} + \int \frac{dx}{x^2}\right) = -\frac{\ell\eta^2 x}{x} - \frac{2\ell\eta x}{x} + 2\int \frac{dx}{x^2} = -\frac{\ell\eta^2 x}{x} - \frac{2\ell\eta x}{x} - \frac{2}{x} + c$$

4.47.- $\int x^2 \operatorname{arc}\tau g 3x dx$

Solución.-

$$u = \operatorname{arc}\tau g 3x \qquad dv = x^2 dx$$

$$\therefore \quad du = \frac{3dx}{1 + 9x^2} \qquad v = \frac{x^3}{3}$$

$$\int x^2 \operatorname{arc}\tau g3x dx = \frac{x^3}{3}\operatorname{arc}\tau g3x - \int \frac{x^3 dx}{1+9x^2} = \frac{x^3}{3}\operatorname{arc}\tau g3x - \frac{1}{9}\int \frac{x^3 dx}{{}^1\!/_9 + x^2}$$

$$= \frac{x^3}{3}\operatorname{arc}\tau g3x - \frac{1}{9}\left[\int\left(x - \frac{{}^1\!/_9\, x}{x^2 + {}^1\!/_9}\right)dx\right] = \frac{x^3}{3}\operatorname{arc}\tau g3x - \frac{1}{9}\frac{x^2}{2} + \frac{1}{81}\int \frac{xdx}{x^2 + {}^1\!/_9}$$

$$= \frac{x^3}{3}\operatorname{arc}\tau g3x - \frac{x^2}{18} + \frac{1}{162}\ell\eta\left|x^2 + \frac{1}{9}\right| + c$$

4.48.- $\int x(\operatorname{arc}\tau gx)^2 dx$

Solución.-

$$\therefore \quad u = (\operatorname{arc}\tau gx)^2 \qquad dv = xdx$$
$$du = \frac{2\operatorname{arc}\tau gxdx}{1+x^2} \qquad v = \frac{x^2}{2}$$

$\int x(\operatorname{arc}\tau gx)^2 dx = \frac{x^2}{2}(\operatorname{arc}\tau gx)^2 - \int (\operatorname{arc}\tau gx)\frac{x^2 dx}{1+x^2}$, integral la cual se desarrolla por partes, esto es:

$$\therefore \quad u = \operatorname{arc}\tau gx \qquad dv = \frac{x^2 dx}{1+x^2}$$
$$du = \frac{dx}{1+x^2} \qquad v = x - \operatorname{arc}\tau gx$$

$$= \frac{(x\operatorname{arc}\tau gx)^2}{2} - \left[(x - \operatorname{arc}\tau gx)\operatorname{arc}\tau gx - \int (x - \operatorname{arc}\tau gx)\frac{dx}{1+x^2}\right]$$

$$= \frac{(x\operatorname{arc}\tau gx)^2}{2} - x\operatorname{arc}\tau gx + (\operatorname{arc}\tau gx)^2 + \int \frac{xdx}{1+x^2} - \int \frac{\operatorname{arc}\tau gxdx}{1+x^2}$$

$$= \frac{(x\operatorname{arc}\tau gx)^2}{2} - x\operatorname{arc}\tau gx + (\operatorname{arc}\tau gx)^2 + \frac{1}{2}\ell\eta(1+x^2) - \frac{(\operatorname{arc}\tau gx)^2}{2} + c$$

4.49.- $\int (\operatorname{arcs}e\operatorname{n} x)^2 dx$

Solución.-

$$\therefore \quad u = (\operatorname{arcs}e\operatorname{n} x)^2 \qquad dv = dx$$
$$du = \frac{2\operatorname{arcs}e\operatorname{n} xdx}{\sqrt{1-x^2}} \qquad v = x$$

$\int (\operatorname{arcs}e\operatorname{n} x)^2 dx = x(\operatorname{arcs}e\operatorname{n} x)^2 - 2\int \operatorname{arcs}e\operatorname{n} x\frac{xdx}{\sqrt{1-x^2}}$, integral la cual se desarrolla por partes, esto es: $\therefore$

$$u = \operatorname{arcs}e\operatorname{n} x \qquad dv = \frac{xdx}{\sqrt{1-x^2}}$$
$$du = \frac{dx}{\sqrt{1-x^2}} \qquad v = -\sqrt{1-x^2}$$

$$= x(\operatorname{arcs}e\operatorname{n} x)^2 - 2\left[-\sqrt{1-x^2}\operatorname{arcs}e\operatorname{n} x + \int dx\right]$$

$$= x(\operatorname{arcs}e\operatorname{n} x)^2 + 2\sqrt{1-x^2}\operatorname{arcs}e\operatorname{n} x - 2x + c$$

4.50.- $\int \frac{\arcsen x}{x^2}dx$

Solución.-

$$\therefore \quad u = \arcsen x \qquad dv = x^{-2}dx$$
$$du = \frac{dx}{\sqrt{1-x^2}} \qquad v = -\frac{1}{x}$$

$$\int \frac{\arcsen x}{x^2}dx = \int x^{-2}\arcsen x dx = -\frac{\arcsen x}{x} + \int \frac{dx}{x\sqrt{1-x^2}}$$

$$= -\frac{\arcsen x}{x} + \ell\eta\left|\frac{x}{1+\sqrt{1-x^2}}\right| + c$$

4.51.- $\int \frac{\arcsen \sqrt{x}}{\sqrt{1-x}}dx$

Solución.-

$$\therefore \quad u = \arcsen \sqrt{x} \qquad dv = \frac{dx}{\sqrt{1-x}}$$
$$du = \frac{dx}{\sqrt{1-x}}\frac{1}{2\sqrt{x}} \qquad v = -2\sqrt{1-x}$$

$$\int \frac{\arcsen \sqrt{x}}{\sqrt{1-x}}dx = -2\sqrt{1-x}\arcsen \sqrt{x} + \int \frac{dx}{\sqrt{x}} = -2\sqrt{1-x}\arcsen \sqrt{x} + 2\sqrt{x} + c$$

4.52.- $\int \frac{\operatorname{sen}^2 x}{e^x}dx$

Solución.-

$$\therefore \quad u = \operatorname{sen}^2 x \qquad dv = e^{-x}dx$$
$$du = 2\operatorname{sen} x\cos x \qquad v = -e^{-x}$$

$$\int \frac{\operatorname{sen}^2 x}{e^x}dx = \int \operatorname{sen}^2 x e^{-x}dx = -e^{-x}\operatorname{sen}^2 x + 2\int \operatorname{sen} x\cos x e^{-x}dx$$

$= -e^{-x}\operatorname{sen}^2 x + \cancel{2}\int \frac{\operatorname{sen} 2x}{\cancel{2}}e^{-x}dx$, $*$ Integral la cual se desarrolla por partes, esto es:

$$\therefore \quad u = \operatorname{sen} 2x \qquad dv = e^{-x}dx$$
$$du = 2\cos 2x dx \qquad v = -e^{-x}$$

$= -e^{-x}\operatorname{sen}^2 x + 2\int \cos 2x e^{-x}dx$, Integral la cual se desarrolla por partes, esto es:

$$\therefore \quad u = \cos 2x \qquad dv = e^{-x}dx$$
$$du = -2\operatorname{sen} 2x dx \qquad v = -e^{-x}$$

$$\int \operatorname{sen} 2x e^{-x}dx = -e^{-x}\operatorname{sen} 2x + 2\left(-e^{-x}\cos 2x - 2\int \operatorname{sen} 2x e^{-x}dx\right)$$

$\int \operatorname{sen} 2x e^{-x}dx = -e^{-x}\operatorname{sen} 2x - 2e^{-x}\cos 2x - 4\int \operatorname{sen} 2x e^{-x}dx$, de donde:

$$5\int \operatorname{sen} 2x e^{-x}dx = -e^{-x}(\operatorname{sen} 2x + 2\cos 2x) + c$$

$\int \operatorname{sen} 2x e^{-x} dx = \frac{-e^{-x}}{5}(\operatorname{sen} 2x + 2\cos 2x) + c$, Sustituyendo en: $*$

$\int \frac{\operatorname{sen}^2 x dx}{e^x} = -e^{-x}\operatorname{sen}^2 x - \frac{2e^{-x}}{5}(\operatorname{sen} 2x + 2\cos 2x) + c$

4.53.- $\int \tau g^2 x \sec^3 x dx = \int (\sec^2 x - 1)\sec^3 x dx = \int \sec^5 x dx (*) - \int \sec^3 x dx (**)$

Solución.-

$*\int \sec^5 x dx$, Sea: $\begin{matrix} u = \sec^3 x & dv = \sec^2 x dx \\ du = 3\sec^3 x \tau g x dx & v = \tau g x \end{matrix}$

$\int \sec^5 x dx = \int \sec^3 x \sec^2 x dx = \sec^3 x \tau g x - 3\int \sec^3 x \tau g^2 x dx$

$**\int \sec^3 x dx$, Sea: $\begin{matrix} u = \sec x & dv = \sec^2 x dx \\ du = \sec x \tau g x dx & v = \tau g x \end{matrix}$

$\int \sec^3 x dx = \int \sec x \sec^2 x dx = \sec x \tau g x - \int \sec x \tau g^2 x dx = \sec x \tau g x - \int \sec x(\sec x^2 - 1)dx$

$= \sec x \tau g x - \int \sec^3 x dx + \int \sec x dx$, luego: $2\int \sec^3 x dx = \sec x \tau g x + \int \sec x dx$

Esto es: $\int \sec^3 x dx = \frac{1}{2}(\sec x \tau g x + \ell n|\sec x \tau g x|) + c$, ahora bien:

$\int \tau g^2 x \sec^3 x dx = \int \sec^5 x dx - \int \sec^3 x dx$, con ($*$ y $**$)

$\int \tau g^2 x \sec^3 x dx = \sec^3 x \tau g x - 3\int \sec^3 x \tau g^2 x dx - \frac{1}{2}(\sec x \tau g x + \ell n|\sec x \tau g x|) + c$

De lo anterior: $4\int \tau g^2 x \sec^3 x dx = \sec^3 x \tau g x - \frac{1}{2}(\sec x \tau g x + \ell n|\sec x \tau g x|) + c$

Esto es: $\int \tau g^2 x \sec^3 x dx = \frac{1}{4}\sec^3 x \tau g x - \frac{1}{8}(\sec x \tau g x + \ell n|\sec x \tau g x|) + c$

4.54.- $\int x^3 \ell \eta^2 x dx$

Solución.-

$\therefore \quad \begin{matrix} u = \ell \eta^2 x & dv = x^3 dx \\ du = \frac{2\ell \eta x}{x} dx & v = \frac{x^4}{4} \end{matrix}$

$\int x^3 \ell \eta^2 x dx = \frac{x^4}{4}\ell \eta^2 x - \frac{1}{2}\int x^3 \ell \eta x dx$, integral la cual se desarrolla por partes, esto es:

$\begin{matrix} u = \ell \eta x & dv = x^3 dx \\ du = \frac{dx}{x} & v = \frac{x^4}{4} \end{matrix}$

$= \frac{x^4}{4}\ell \eta^2 x - \frac{1}{2}\left(\frac{x^4}{4}\ell \eta x - \frac{1}{4}\int x^3 dx\right) = \frac{x^4}{4}\ell \eta^2 x - \frac{1}{8}x^4 \ell \eta x + \frac{1}{8}\frac{x^4}{4} + c$

$= \frac{x^4}{4}\ell \eta^2 x - \frac{1}{8}x^4 \ell \eta x + \frac{x^4}{32} + c$

4.55.- $\int x\ell\eta(9+x^2)dx$

Solución.-

$$u=\ell\eta(9+x^2) \qquad dv=xdx$$

$$\therefore \quad du=\frac{2xdx}{9+x^2} \qquad v=\frac{x^2}{2}$$

$$\int x\ell\eta(9+x^2)dx=\frac{x^2}{2}\ell\eta(9+x^2)-\int\frac{x^3}{9+x^2}dx=\frac{x^2}{2}\ell\eta(9+x^2)-\int\left(x-\frac{9x}{x^2+9}\right)dx$$

$$=\frac{x^2}{2}\ell\eta(9+x^2)-\int xdx+9\int\frac{xdx}{9+x^2}=\frac{x^2}{2}\ell\eta(9+x^2)-\frac{x^2}{2}+\frac{9}{2}\ell\eta(x^2+9)+c$$

$$=\frac{x^2}{2}\left[\ell\eta(9+x^2)-1\right]+\frac{9}{2}\ell\eta(x^2+9)+c$$

4.56.- $\int \operatorname{arcsen}\sqrt{x}dx$

Solución.-

$$u=\operatorname{arcsen}\sqrt{x}dx \qquad dv=dx$$

$$\therefore \quad du=\frac{dx}{\sqrt{1-x^2}}\frac{1}{2\sqrt{x}} \qquad v=x$$

$$\int\operatorname{arcsen}\sqrt{x}dx=x\operatorname{arcsen}\sqrt{x}-\int\frac{xdx}{\sqrt{1-x}}\frac{1}{2\sqrt{x}}=x\operatorname{arcsen}\sqrt{x}-\frac{1}{2}\int\frac{\sqrt{x}dx}{\sqrt{1-x}}$$

Para la integral resultante, se recomienda la siguiente sustitución: $\sqrt{1-x}=t$, de donde: $x=1-t^2$, y $dx=-2tdt$ (ver capitulo 9)

$=x\operatorname{arcsen}\sqrt{x}-\dfrac{1}{2}\dfrac{\sqrt{1-t^2}(-2\cancel{t}dt)dx}{\cancel{t}}=x\operatorname{arcsen}\sqrt{x}+\sqrt{1-t^2}dt$, Se recomienda la sustitución: $t=\operatorname{sen}\theta$, de donde: $\sqrt{1-t^2}=\cos\theta$, y $dt=\cos\theta d\theta$. Esto es:

$$=x\operatorname{arcsen}\sqrt{x}+\int\cos^2\theta d\theta=x\operatorname{arcsen}\sqrt{x}+\frac{1}{2}\int(1+\cos2\theta)d\theta$$

$$=x\operatorname{arcsen}\sqrt{x}+\frac{1}{2}\theta+\frac{1}{4}\operatorname{sen}2\theta+c=x\operatorname{arcsen}\sqrt{x}+\frac{1}{2}\theta+\frac{1}{2}\operatorname{sen}\theta\cos\theta+c$$

$$=x\operatorname{arcsen}\sqrt{x}+\frac{\operatorname{arcsen}t}{2}+\frac{t}{2}\sqrt{1-t^2}+c=x\operatorname{arcsen}\sqrt{x}+\frac{\operatorname{arcsen}\sqrt{1-x}}{2}+\frac{\sqrt{1-x}}{2}\sqrt{x}+c$$

4.57.- $\int x\operatorname{arc}\tau g(2x+3)dx$

Solución.-

$$u=\operatorname{arc}\tau g(2x+3) \qquad dv=xdx$$

$$\therefore \quad du=\frac{2dx}{1+(2x+3)^2} \qquad v=\frac{x^2}{2}$$

$$\int x\operatorname{arc}\tau g(2x+3)dx=\frac{x^2}{2}\operatorname{arc}\tau g(2x+3)-\int\frac{x^2dx}{1+4x^2+12x+9}$$

$$=\frac{x^2}{2}\operatorname{arc}\tau g(2x+3)-\int\frac{x^2dx}{4x^2+12x+10}=\frac{x^2}{2}\operatorname{arc}\tau g(2x+3)-\int\left(\frac{1}{4}-\frac{3x+5/2}{4x^2+12x+10}\right)dx$$

$$=\frac{x^2}{2}\operatorname{arc}\tau g(2x+3)-\frac{1}{4}\int dx+\int\frac{3x+5/2}{4x^2+12x+10}dx$$

$$=\frac{x^2}{2}\operatorname{arc}\tau g(2x+3)-\frac{1}{4}x+3\int\frac{x+5/6}{4x^2+12x+10}dx$$

$$=\frac{x^2}{2}\operatorname{arc}\tau g(2x+3)-\frac{1}{4}x+\frac{3}{8}\int\frac{8x+40/6}{4x^2+12x+10}dx$$

$$=\frac{x^2}{2}\operatorname{arc}\tau g(2x+3)-\frac{1}{4}x+\frac{3}{8}\int\frac{8x+12-32/6}{4x^2+12x+10}dx$$

$$=\frac{x^2}{2}\operatorname{arc}\tau g(2x+3)-\frac{1}{4}x+\frac{3}{8}\int\frac{(8x+12)dx}{4x^2+12x+10}-\frac{3}{8}\frac{32}{6}\int\frac{dx}{4x^2+12x+10}$$

$$=\frac{x^2}{2}\operatorname{arc}\tau g(2x+3)-\frac{1}{4}x+\frac{3}{8}\ell\eta\left|4x^2+12x+10\right|-2\int\frac{dx}{4x^2+12x+10}$$

$$=\frac{x^2}{2}\operatorname{arc}\tau g(2x+3)-\frac{1}{4}x+\frac{3}{8}\ell\eta\left|4x^2+12x+10\right|-2\int\frac{dx}{(2x+3)^2+1}$$

$$=\frac{x^2}{2}\operatorname{arc}\tau g(2x+3)-\frac{1}{4}x+\frac{3}{8}\ell\eta\left|4x^2+12x+10\right|-\frac{2}{2}\int\frac{2dx}{(2x+3)^2+1}$$

$$=\frac{x^2}{2}\operatorname{arc}\tau g(2x+3)-\frac{1}{4}x+\frac{3}{8}\ell\eta\left|4x^2+12x+10\right|-\operatorname{arc}\tau g(2x+3)+c$$

$$=\frac{1}{2}\left[(x^2-2)\operatorname{arc}\tau g(2x+3)-\frac{1}{2}x+\frac{3}{4}\ell\eta\left|4x^2+12x+10\right|\right]+c$$

4.58.- $\int e^{\sqrt{x}}dx$

Solución.-

$$\therefore\quad u=e^{\sqrt{x}}\qquad dv=dx$$
$$du=\frac{e^{\sqrt{x}}dx}{2\sqrt{x}}\qquad v=x$$

$\int e^{\sqrt{x}}dx=xe^{\sqrt{x}}-\frac{1}{2}\int\frac{xe^{\sqrt{x}}dx}{2\sqrt{x}}$, Se recomienda la sustitución: $z=\sqrt{x},dz=\frac{dx}{2\sqrt{x}}$

$=xe^{\sqrt{x}}-\frac{1}{2}\int z^2e^zdz$, Esta integral resultante, se desarrolla por partes:

$$\therefore\quad u=z^2\qquad dv=e^zdz$$
$$du=2zdz\qquad v=e^z$$

$=xe^{\sqrt{x}}-\frac{1}{2}\left(z^2e^z-2\int ze^zdz\right)=xe^{\sqrt{x}}-\frac{z^2e^z}{2}+\int ze^zdz$, integral que se desarrolla por partes:

$$\therefore \quad \begin{array}{ll} u = z & dv = e^z dz \\ du = dz & v = e^z \end{array}$$

$$= xe^{\sqrt{x}} - \frac{z^2e^z}{2} + ze^z - \int e^z dz = xe^{\sqrt{x}} - \frac{z^2e^z}{2} + ze^z - e^z + c = xe^{\sqrt{x}} - \frac{xe^{\sqrt{x}}}{2} + \sqrt{x}e^{\sqrt{x}} - e^{\sqrt{x}} + c$$

$$= e^{\sqrt{x}}\left(\frac{x}{2} + \sqrt{x} - 1\right) + c$$

4.59.- $\int \cos^2(\ell\eta x)dx$

Solución.-

$$\therefore \quad \begin{array}{ll} u = \cos(2\ell\eta x) & dv = dx \\ du = -\frac{[\mathrm{s}e\mathrm{n}(2\ell\eta x)]2dx}{x} & v = x \end{array}$$

$$\int \cos^2(\ell\eta x)dx = \int \frac{1+\cos(2\ell\eta x)}{2}dx = \frac{1}{2}\int dx + \frac{1}{2}\int \cos(2\ell\eta x)dx$$

$$= \frac{1}{2}x + \frac{1}{2}\left[x\cos(2\ell\eta x) + 2\int \mathrm{s}e\mathrm{n}(2\ell\eta x)dx\right] = \frac{x}{2} + \frac{x}{2}\cos(2\ell\eta x) + \int \mathrm{s}e\mathrm{n}(2\ell\eta x)dx *$$

Integral que se desarrolla por partes:

$$\therefore \quad \begin{array}{ll} u = \mathrm{s}e\mathrm{n}(2\ell\eta x) & dv = dx \\ du = -\frac{[\cos(2\ell\eta x)]2dx}{x} & v = x \end{array}$$

$$* = \frac{x}{2} + \frac{x}{2}\cos(2\ell\eta x) + x\,\mathrm{s}e\mathrm{n}(2\ell\eta x) - 2\int \cos(2\ell\eta x)dx,$$

Dado que apareció nuevamente: $\int \cos(2\ell\eta x)dx$, igualamos: $*$

$\cancel{\frac{x}{2}} + \frac{1}{2}\int \cos(2\ell\eta x)dx = \cancel{\frac{x}{2}} + \frac{x}{2}\cos(2\ell\eta x) + x\,\mathrm{s}e\mathrm{n}(2\ell\eta x) - 2\int \cos(2\ell\eta x)dx$, de donde:

$$\frac{5}{2}\int \cos(2\ell\eta x)dx = \frac{x}{2}\cos(2\ell\eta x) + x\,\mathrm{s}e\mathrm{n}(2\ell\eta x) + c$$

$\frac{1}{2}\int \cos(2\ell\eta x)dx = \frac{x}{10}\cos(2\ell\eta x) + \frac{x}{5}\mathrm{s}e\mathrm{n}(2\ell\eta x) + c$, Por tanto:

$$\int \cos^2(\ell\eta x)dx = \frac{x}{2} + \frac{x}{10}\cos(2\ell\eta x) + \frac{x}{5}\mathrm{s}e\mathrm{n}(2\ell\eta x) + c$$

4.60.- $\int \frac{\ell\eta(\ell\eta x)}{x}dx$, Sustituyendo por: $w = \ell\eta x, dw = \frac{dx}{x}$, Se tiene:

Solución.-

$\int \frac{\ell\eta(\ell\eta x)}{x}dx = \int \ell\eta w dw$, Esta integral se desarrolla por partes:

$$\therefore \quad \begin{array}{ll} u = \ell\eta w & dv = dw \\ du = \frac{dw}{w} & v = w \end{array}$$

$$= w\ell\eta w - \int dw = w\ell\eta w - w + c = w(\ell\eta w - 1) + c = \ell\eta x\left[\ell\eta(\ell\eta x) - 1\right] + c$$

4.61.- $\int \ell\eta|x+1|dx$

Solución.-

$$\therefore \quad u = \ell\eta|x+1| \qquad dv = dx$$
$$du = \frac{dx}{x+1} \qquad v = x$$

$$\int \ell\eta|x+1|dx = x\ell\eta|x+1| - \int \frac{xdx}{x+1} = x\ell\eta|x+1| - \int\left(1-\frac{1}{x+1}\right)dx$$

$$= x\ell\eta|x+1| - x + \ell\eta|x+1| + c$$

4.62.- $\int x^2e^x dx$

Solución.-

$$\therefore \quad u = x^2 \qquad dv = e^x dx$$
$$du = 2xdx \qquad v = e^x$$

$$\int x^2e^xdx = x^2e^x - 2\int xe^xdx$$

Integral que se desarrolla nuevamente por partes:

$$\therefore \quad u = x \qquad dv = e^x dx$$
$$du = dx \qquad v = e^x$$

$$= x^2e^x - 2\left[xe^x - \int e^xdx\right] = x^2e^x - 2xe^x + 2e^x + c$$

4.63.- $\int \cos^n xdx = \int \cos^{n-1} x\cos xdx$

Solución.-

$$\therefore \quad u = \cos^{n-1} x \qquad dv = \cos xdx$$
$$du = (n-1)\cos^{n-2} x(-\operatorname{sen} x)dx \qquad v = \operatorname{sen} x$$

$$= \cos^{n-1} x\operatorname{sen} x + (n-1)\int \operatorname{sen}^2 x\cos^{n-2} xdx$$

$$= \cos^{n-1} x\operatorname{sen} x + (n-1)\int (1-\cos^2 x)\cos^{n-2} xdx$$

$= \cos^{n-1} x\operatorname{sen} x + (n-1)\int \cos^{n-2} xdx - (n-1)\int \cos^n xdx$, Se tiene:

$\int \cos^n xdx = \cos^{n-1} x\operatorname{sen} x + (n-1)\int \cos^{n-2} xdx - (n-1)\int \cos^n xdx$, Esto es:

$$n\int \cos^n xdx = \cos^{n-1} x\operatorname{sen} x + (n-1)\int \cos^{n-2} xdx$$

$$\int \cos^n xdx = \frac{\cos^{n-1} x\operatorname{sen} x}{n} + \frac{(n-1)}{n}\int \cos^{n-2} xdx$$

4.64.- $\int \operatorname{sen}^n xdx = \int \operatorname{sen}^{n-1} x\operatorname{sen} xdx$

Solución.-

$$\therefore \quad u = \operatorname{sen}^{n-1} x \qquad dv = \operatorname{sen} xdx$$
$$du = (n-1)\operatorname{sen}^{n-2} x(\cos x)dx \qquad v = -\cos x$$

$$= -\operatorname{sen}^{n-1} x\cos x + (n-1)\int \cos^2 x\operatorname{sen}^{n-2} xdx$$

$$= -\operatorname{sen}^{n-1} x\cos x + (n-1)\int (1-\operatorname{sen}^2 x)\operatorname{sen}^{n-2} xdx$$

$=-\operatorname{sen}^{n-1} x\cos x+(n-1)\int \operatorname{sen}^{n-2} xdx-(n-1)\int \operatorname{sen}^{n} xdx$, Se tiene:

$\int \operatorname{sen}^{n} xdx=-\operatorname{sen}^{n-1} x\cos x+(n-1)\int \operatorname{sen}^{n-2} xdx-(n-1)\int \operatorname{sen}^{n} xdx$

$n\int \operatorname{sen}^{n} xdx=-\operatorname{sen}^{n-1} x\cos x+(n-1)\int \operatorname{sen}^{n-2} xdx$

$\int \operatorname{sen}^{n} xdx=\dfrac{-\operatorname{sen}^{n-1} x\cos x}{n}+\dfrac{(n-1)}{n}\int \operatorname{sen}^{n-2} xdx$

4.65.- $\int x^m(\ell\eta x)^n dx=x^{m+1}(\ell\eta x)^n-n\int x^m(\ell\eta x)^{n-1}dx-m\int x^m(\ell\eta x)^n dx$

Solución.-

$\therefore$ $u=x^m(\ell\eta x)^n$ $\quad dv=dx$

$du=x^m n(\ell\eta x)^{n-1}\dfrac{dx}{x}+mx^{m-1}(\ell\eta x)^n dx$ $\quad v=x$

Se tiene: $(m+1)\int x^m(\ell\eta x)^n dx=x^{m+1}(\ell\eta x)^n-n\int x^m(\ell\eta x)^{n-1}dx$

$\int x^m(\ell\eta x)^n dx=\dfrac{x^{m+1}(\ell\eta x)^n}{(m+1)}-\dfrac{n}{(m+1)}\int x^m(\ell\eta x)^{n-1}dx$

4.66.- $\int x^3(\ell\eta x)^2 dx$

Solución.-

Puede desarrollarse como caso particular del ejercicio anterior, haciendo: $m=3, n=2$

$\int x^3(\ell\eta x)^2 dx=\dfrac{x^{3+1}(\ell\eta x)^2}{3+1}-\dfrac{2}{3+1}\int x^3(\ell\eta x)^{2-1}dx=\dfrac{x^4(\ell\eta x)^2}{4}-\dfrac{1}{2}\int x^3(\ell\eta x)dx*$

Para la integral resultante: $\int x^3(\ell\eta x)dx*$

$\int x^3(\ell\eta x)dx=\dfrac{x^4(\ell\eta x)}{4}-\dfrac{1}{4}\int x^3dx=\dfrac{x^4(\ell\eta x)}{4}-\dfrac{x^4}{16}+c$, introduciendo en: $*$

$\int x^3(\ell\eta x)^2 dx=\dfrac{x^4(\ell\eta x)^2}{4}-\dfrac{x^4}{8}(\ell\eta x)+\dfrac{x^4}{32}+c$

4.67.- $\int x^n e^x dx$

Solución.-

$\therefore$ $u=x^n$ $\quad dv=e^x dx$

$du=nx^{n-1}dx$ $\quad v=e^x$

$\int x^n e^x dx=x^n e^x-n\int x^{n-1}e^x dx$

4.68.- $\int x^3 e^x dx$

Solución.-

$\therefore$ $u=x^3$ $\quad dv=e^x dx$

$du=3x^2dx$ $\quad v=e^x$

Puede desarrollarse como el ejercicio anterior, haciendo: $n=3$

$\int x^3e^x dx=x^3e^x-3\int x^2e^x dx*$, Además:

$*\int x^2e^xdx = x^2e^x - 2\int xe^xdx**$, Además: $\int xe^xdx = xe^x - \int e^xdx = xe^x - e^x + c$

Reemplazando en$**$ y luego en $*$:

$\int x^3e^xdx = x^3e^x - 3\left[x^2e^x - 2(xe^x - e^x)\right] + c$

$\int x^3e^xdx = e^x(x^3 - 3x^2 + 6x - 6) + c$

4.69.- $\int \sec^n xdx = \int \sec^{n-2} x\sec^2 xdx$

Solución.-

$$\therefore \quad \begin{array}{ll} u = \sec^{n-2} x & dv = \sec^2 xdx \\ du = (n-2)\sec^{n-3} x\sec x\tau gxdx & v = \tau gx \end{array}$$

$= \sec^{n-2} x\tau gx - (n-2)\int \tau g^2 x\sec^{n-2} xdx = \sec^{n-2} x\tau gx - (n-2)\int (\sec^2 x - 1)\sec^{n-2} xdx$

$= \sec^{n-2} x\tau gx - (n-2)\int \sec^n xdx + (n-2)\int \sec^{n-2} xdx$, Se tiene:

$\int \sec^n xdx = \sec^{n-2} x\tau gx - (n-2)\int \sec^n xdx + (n-2)\int \sec^{n-2} xdx$

$(n-1)\int \sec^n xdx = \sec^{n-2} x\tau gx + (n-2)\int \sec^{n-2} xdx$

$$\int \sec^n xdx = \frac{\sec^{n-2} x\tau gx}{(n-1)} + \frac{(n-2)}{(n-1)}\int \sec^{n-2} xdx$$

4.70.- $\int \sec^3 xdx$

Solución.-

Puede desarrollarse como caso particular del ejercicio anterior, haciendo: $n = 3$

$$\int \sec^3 xdx = \frac{\sec^{3-2} x\tau gx}{3-1} + \frac{3-2}{3-1}\int \sec^{3-2} xdx = \frac{\sec x\tau gx}{2} + \frac{1}{2}\int \sec xdx$$

$$= \frac{\sec x\tau gx}{2} + \frac{1}{2}\ell\eta\left|\sec x\tau gx\right| + c$$

4.71.- $\int x\ell\eta xdx$

Solución.-

$$\therefore \quad \begin{array}{ll} u = \ell\eta x & dv = xdx \\ du = \dfrac{dx}{x} & v = \dfrac{x^2}{2} \end{array}$$

$$\int x\ell\eta xdx = \frac{x^2}{2}\ell\eta x - \int \frac{xdx}{2} = \frac{x^2}{2}\ell\eta x - \frac{1}{4}x^2 + c$$

4.72.- $\int x^n\ell\eta\left|ax\right|dx, n \neq -1$

Solución.-

$$\therefore \quad \begin{array}{ll} u = \ell\eta\left|ax\right| & dv = xdx \\ du = \dfrac{dx}{x} & v = \dfrac{x^{n+1}}{n+1} \end{array}$$

$$\int x^n\ell\eta\left|ax\right|dx = \frac{x^{n+1}}{n+1}\ell\eta\left|ax\right| - \frac{1}{n+1}\int x^ndx = \frac{x^{n+1}}{n+1}\ell\eta\left|ax\right| - \frac{x^{n+1}}{(n+1)^2} + c$$

4.73.- $\int \arcsen axdx$

Solución.-

$$\therefore \quad u = \arcsen ax \qquad dv = dx$$
$$du = \frac{adx}{\sqrt{1-a^2x^2}} \qquad v = x$$

$$\int \arcsen axdx = x\arcsen ax - \int \frac{axdx}{\sqrt{1-a^2x^2}} = x\arcsen ax + \frac{1}{2a}\int \frac{(-2a^2x)dx}{\sqrt{1-a^2x^2}}$$

$$= x\arcsen ax + \frac{1}{2a}\frac{(1-a^2x^2)^{1/2}}{1/2} + c = x\arcsen ax + \frac{1}{a}\sqrt{1-a^2x^2} + c$$

4.74.- $\int x\,sen\,axdx$

Solución.-

$$\therefore \quad u = x \qquad dv = sen\,axdx$$
$$du = dx \qquad v = -\frac{1}{a}\cos ax$$

$$\int x\,sen\,axdx = -\frac{x}{a}\cos ax + \frac{1}{a}\int \cos axdx = -\frac{x}{a}\cos ax + \frac{1}{a^2}sen\,ax + c$$

$$= \frac{1}{a^2}sen\,ax - \frac{x}{a}\cos ax + c$$

4.75.- $\int x^2\cos axdx$

Solución.-

$$\therefore \quad u = x^2 \qquad dv = \cos axdx$$
$$du = 2xdx \qquad v = -\frac{1}{a}sen\,ax$$

$\int x^2\cos axdx = \frac{x^2}{a}sen\,ax - \frac{2}{a}\int x\,sen\,axdx$, aprovechando el ejercicio anterior:

$$= \frac{x^2}{a}sen\,ax - \frac{2}{a}\left(\frac{1}{a^2}sen\,ax - \frac{x}{a}\cos ax\right) + c = \frac{x^2}{a}sen\,ax - \frac{2}{a^3}sen\,ax - \frac{2x}{a^2}\cos ax + c$$

4.76.- $\int x\sec^2 axdx$

Solución.-

$$\therefore \quad u = x \qquad dv = \sec^2 axdx$$
$$du = dx \qquad v = \frac{1}{a}\tau gax$$

$$\int x\sec^2 axdx = \frac{x}{a}\tau gax - \frac{1}{a}\int \tau gaxdx = \frac{x}{a}\tau gax - \frac{1}{a}\frac{1}{a}\ell\eta\left|\sec ax\right| + c$$

$$= \frac{x}{a}\tau gax - \frac{1}{a^2}\ell\eta\left|\sec ax\right| + c$$

4.77.- $\int \cos(\ell\eta x)dx$

Solución.-

$$\therefore \quad \begin{array}{ll} u = \cos(\ell\eta x) & dv = dx \\ du = -\dfrac{\operatorname{sen}(\ell\eta x)}{x}dx & v = x \end{array}$$

$\int \cos(\ell\eta x)dx = x\cos(\ell\eta x) + \int \operatorname{sen}(\ell\eta x)dx$, aprovechando el ejercicio:4.43

$\int \operatorname{sen}(\ell\eta x)dx = \frac{x}{2}\left[\operatorname{sen}(\ell\eta x) - \cos(\ell\eta x)\right] + c$, Luego:

$$= x\cos(\ell\eta x) + \frac{x}{2}\left[\operatorname{sen}(\ell\eta x) - \cos(\ell\eta x)\right] + c = x\cos(\ell\eta x) + \frac{x}{2}\operatorname{sen}(\ell\eta x) - \frac{x}{2}\cos(\ell\eta x) + c$$

$$= \frac{x}{2}\left[\cos(\ell\eta x) + \operatorname{sen}(\ell\eta x)\right] + c$$

4.78.- $\int \ell\eta(9 + x^2)dx$

Solución.-

$$\therefore \quad \begin{array}{ll} u = \ell\eta(9 + x^2) & dv = dx \\ du = \dfrac{2xdx}{9 + x^2} & v = x \end{array}$$

$$\int \ell\eta(9 + x^2)dx = x\ell\eta(9 + x^2) - 2\int \frac{x^2dx}{9 + x^2} = x\ell\eta(9 + x^2) - 2\int \left(1 - \frac{9}{9 + x^2}\right)dx$$

$$= x\ell\eta(9 + x^2) - 2\int dx + 18\int \frac{dx}{9 + x^2} = x\ell\eta(9 + x^2) - 2x + 6\operatorname{arc}\tau g\, {}^{x}\!/\!{}_{3} + c$$

4.79.- $\int x\cos(2x + 1)dx$

Solución.-

$$\therefore \quad \begin{array}{ll} u = x & dv = \cos(2x + 1)dx \\ du = dx & v = \dfrac{1}{2}\operatorname{sen}(2x + 1) \end{array}$$

$$\int x\cos(2x + 1)dx = \frac{x}{2}\operatorname{sen}(2x + 1) - \frac{1}{2}\int \operatorname{sen}(2x + 1)dx$$

$$= \frac{x}{2}\operatorname{sen}(2x + 1) + \frac{1}{4}\cos(2x + 1) + c$$

4.80.- $\int x\operatorname{arcsec} xdx$

Solución.-

$$\therefore \quad \begin{array}{ll} u = \operatorname{arcsec} x & dv = xdx \\ du = \dfrac{dx}{x\sqrt{x^2 - 1}} & v = \dfrac{x^2}{2} \end{array}$$

$$\int x\operatorname{arcsec} xdx = \frac{x^2}{2}\operatorname{arcsec} x - \frac{1}{2}\int \frac{xdx}{\sqrt{x^2 - 1}} = \frac{x^2}{2}\operatorname{arcsec} x - \frac{1}{2}\sqrt{x^2 - 1} + c$$

4.81.- $\int \operatorname{arcsec}\sqrt{x}dx$

Solución.-

$$\therefore \quad \begin{array}{ll} u = \operatorname{arc}\sec x & dv = dx \\ du = \dfrac{1}{2}\dfrac{dx}{x\sqrt{x-1}} & v = x \end{array}$$

$$\int \operatorname{arc}\sec\sqrt{x}dx = x\operatorname{arc}\sec x - \frac{1}{2}\int \frac{dx}{\sqrt{x-1}} = x\operatorname{arc}\sec x - \sqrt{x-1} + c$$

4.82.- $\int \sqrt{a^2-x^2}dx = \int \frac{a^2-x^2}{\sqrt{a^2-x^2}}dx = a^2\int \frac{dx}{\sqrt{a^2-x^2}} - \int \frac{x^2dx}{\sqrt{a^2-x^2}}$

$= a^2 \operatorname{arcs}e\operatorname{n}\frac{x}{a} - \int x\frac{xdx}{\sqrt{a^2-x^2}} *$, integral que se desarrolla por partes:

Solución.-

$$\therefore \quad \begin{array}{ll} u = x & dv = \dfrac{xdx}{\sqrt{a^2-x^2}} \\ du = dx & v = -\sqrt{a^2-x^2} \end{array}$$

$* = a^2 \operatorname{arcs}e\operatorname{n}\frac{x}{a} - \left(-x\sqrt{a^2-x^2} + \int \sqrt{a^2-x^2}dx\right)$, Se tiene que:

$\int \sqrt{a^2-x^2}dx = a^2 \operatorname{arcs}e\operatorname{n}\frac{x}{a} + x\sqrt{a^2-x^2} - \int \sqrt{a^2-x^2}dx$, De donde:

$$2\int \sqrt{a^2-x^2}dx = a^2 \operatorname{arcs}e\operatorname{n}\frac{x}{a} + x\sqrt{a^2-x^2} + c$$

$$\int \sqrt{a^2-x^2}dx = \frac{a^2}{2}\operatorname{arcs}e\operatorname{n}\frac{x}{a} + \frac{x}{2}\sqrt{a^2-x^2} + c$$

4.83.- $\int \ell\eta|1-x|dx$

Solución.-

$$\therefore \quad \begin{array}{ll} u = \ell\eta|1-x| & dv = dx \\ du = -\dfrac{dx}{1-x} & v = x \end{array}$$

$$\int \ell\eta|1-x|dx = x\ell\eta|1-x| - \int \frac{xdx}{x-1} = x\ell\eta|1-x| - \int\left(1+\frac{1}{x-1}\right)dx$$

$$= x\ell\eta|1-x| - \int dx - \int \frac{dx}{x-1} = x\ell\eta|1-x| - x - \ell\eta|x-1| + c$$

4.84.- $\int \ell\eta(x^2+1)dx$

Solución.-

$$\therefore \quad \begin{array}{ll} u = \ell\eta(x^2+1) & dv = dx \\ du = \dfrac{2xdx}{x^2+1} & v = x \end{array}$$

$$\int \ell\eta(x^2+1)dx = x\ell\eta(x^2+1) - 2\int \frac{x^2dx}{x^2+1} = x\ell\eta(x^2+1) - 2\int\left(1-\frac{1}{x^2+1}\right)dx$$

$$= x\ell\eta(x^2+1) - 2x + 2\operatorname{arc}\tau gx + c$$

4.85.- $\int \operatorname{arc}\tau g\sqrt{x}dx$

Solución.-

$$\therefore \quad u = \operatorname{arc}\tau g\sqrt{x} \qquad dv = dx$$
$$du = \frac{dx}{1+x}\frac{1}{2\sqrt{x}} \qquad v = x$$

$\int \operatorname{arc}\tau g\sqrt{x}dx = x\operatorname{arc}\tau g\sqrt{x} - \frac{1}{2}\int \frac{\sqrt{x}dx}{1+x} *$ En la integral resultante, se recomienda la sustitución: $\sqrt{x} = t$, esto es $x = t^2, dx = 2tdt$

$$= x\operatorname{arc}\tau g\sqrt{x} - \frac{1}{\cancel{2}}\int \frac{t\cancel{2}tdt}{1+t^2} = x\operatorname{arc}\tau g\sqrt{x} - \int \frac{t^2dt}{1+t^2} = x\operatorname{arc}\tau g\sqrt{x} - \int\left(1-\frac{1}{1+t^2}\right)dt$$

$$= x\operatorname{arc}\tau g\sqrt{x} - \int dt + \int \frac{dt}{1+t^2} = x\operatorname{arc}\tau g\sqrt{x} - t + \operatorname{arc}\tau gt + c$$

$$= x\operatorname{arc}\tau g\sqrt{x} - \sqrt{x} + \operatorname{arc}\tau g\sqrt{x} + c$$

4.86.- $\int \frac{x\operatorname{arcs}e\operatorname{n}x}{\sqrt{1-x^2}}dx$

Solución.-

$$\therefore \quad u = \operatorname{arcs}e\operatorname{n}x \qquad dv = \frac{xdx}{\sqrt{1-x^2}}$$
$$du = \frac{dx}{\sqrt{1-x^2}} \qquad v = -\sqrt{1-x^2}$$

$$\int \frac{x\operatorname{arcs}e\operatorname{n}x}{\sqrt{1-x^2}}dx = -\sqrt{1-x^2}\operatorname{arcs}e\operatorname{n}x + \int dx = -\sqrt{1-x^2}\operatorname{arcs}e\operatorname{n}x + x + c$$

4.87.- $\int x\operatorname{arc}\tau g\sqrt{x^2-1}dx$

Solución.-

$$\therefore \quad u = \operatorname{arc}\tau g\sqrt{x^2-1} \qquad dv = xdx$$
$$du = \frac{dx}{x\sqrt{x^2-1}} \qquad v = \frac{x^2}{2}$$

$$\int x\operatorname{arc}\tau g\sqrt{x^2-1}dx = \frac{x^2}{2}\operatorname{arc}\tau g\sqrt{x^2-1} - \frac{1}{2}\int \frac{xdx}{\sqrt{x^2-1}} = \frac{x^2}{2}\operatorname{arc}\tau g\sqrt{x^2-1} - \frac{1}{2}\sqrt{x^2-1} + c$$

4.88.- $\int \frac{x\operatorname{arc}\tau gx}{(x^2+1)^2}dx$

Solución.-

$$\therefore \quad u = \operatorname{arc}\tau gx \qquad dv = \frac{xdx}{(x^2+1)^2}$$
$$du = \frac{dx}{x^2+1} \qquad v = \frac{-1}{2(x^2+1)}$$

$\int \frac{x\operatorname{arc}\tau gx}{(x^2+1)^2}dx = \frac{-\operatorname{arc}\tau gx}{2(x^2+1)} + \frac{1}{2}\int \frac{dx}{(x^2+1)^2} *$, Se recomienda la siguiente sustitución:

$x=\tau g\theta$, de donde: $dx=\sec^2\theta d\theta$; $x^2+1=\sec^2\theta$

$$*=\frac{-\operatorname{arc}\tau gx}{2(x^2+1)}+\frac{1}{2}\int\frac{\sec^2\theta d\theta}{\sec^4\theta}=-\frac{\operatorname{arc}\tau gx}{2(x^2+1)}+\frac{1}{2}\int\cos^2\theta d\theta=-\frac{\operatorname{arc}\tau gx}{2(x^2+1)}+\frac{1}{2}\int\frac{1+\cos 2\theta d\theta}{2}$$

$$=-\frac{\operatorname{arc}\tau gx}{2(x^2+1)}+\frac{1}{4}\theta+\frac{1}{8}\operatorname{sen}2\theta+c=-\frac{\operatorname{arc}\tau gx}{2(x^2+1)}+\frac{1}{4}\operatorname{arc}\tau gx+\frac{1}{4}\operatorname{sen}\theta\cos\theta+c$$

$$=-\frac{\operatorname{arc}\tau gx}{2(x^2+1)}+\frac{1}{4}\operatorname{arc}\tau gx+\frac{1}{4}\frac{x}{\sqrt{x^2+1}}\frac{1}{\sqrt{x^2+1}}+c$$

$$=-\frac{\operatorname{arc}\tau gx}{2(x^2+1)}+\frac{1}{4}\operatorname{arc}\tau gx+\frac{x}{4(x^2+1)}+c$$

4.89.- $\int \operatorname{arcsen} x\frac{xdx}{\sqrt{(1-x^2)^3}}$

Solución.-

$$\therefore\quad \begin{array}{ll} u=\operatorname{arcsen}x & dv=\dfrac{xdx}{(1-x^2)^{3/2}} \\ du=\dfrac{dx}{\sqrt{1-x^2}} & v=\dfrac{1}{\sqrt{1-x^2}} \end{array}$$

$$\int \operatorname{arcsen} x\frac{xdx}{\sqrt{(1-x^2)^3}}=\frac{\operatorname{arcsen}x}{\sqrt{1-x^2}}-\int\frac{dx}{1-x^2}=\frac{\operatorname{arcsen}x}{\sqrt{1-x^2}}+\frac{1}{2}\ell\eta\left|\frac{1-x}{1+x}\right|+c$$

4.90.- $\int x^2\sqrt{1-x}dx$

Solución.-

$$\therefore\quad \begin{array}{ll} u=\sqrt{1-x} & dv=x^2dx \\ du=-\dfrac{dx}{2\sqrt{1-x}} & v=\dfrac{x^3}{3} \end{array}$$

$\int x^2\sqrt{1-x}dx=\frac{x^3}{3}\sqrt{1-x}+\frac{1}{6}\int\frac{x^3dx}{\sqrt{1-x}}*$, Se recomienda usar la siguiente sustitución: $\sqrt{1-x}=t$, o sea: $x=1-t^2$, De donde: $dx=-2tdt$

$$=\frac{x^3}{3}\sqrt{1-x}+\frac{1}{\cancel{6}}\int\frac{(1-t^2)^3(-\cancel{2}\cancel{t}dt)}{\cancel{t}}=\frac{x^3}{3}\sqrt{1-x}-\frac{1}{3}\int(1-t^2)^3dt$$

$$=\frac{x^3}{3}\sqrt{1-x}-\frac{1}{3}\int(1-3t^2+3t^4-t^6)dt=\frac{x^3}{3}\sqrt{1-x}-\frac{1}{3}(t-t^3+\frac{3t^5}{5}-\frac{t^7}{7})+c$$

$$=\frac{x^3}{3}\sqrt{1-x}-\frac{1}{3}\left[\sqrt{1-x}-(1-x)\sqrt{1-x}+\frac{3}{5}(1-x)^2\sqrt{1-x}-\frac{3}{7}(1-x)^3\sqrt{1-x}\right]+c$$

$$=\frac{\sqrt{1-x}}{3}\left[x^3-1-(1-x)+\frac{3}{5}(1-x)^2-\frac{1}{7}(1-x)^3\right]+c$$

IMPORTANTE: En este capítulo ningún resultado, o casi ninguno, se presentaron en su forma más reducida. Esto es intencional. Una de las causas del fracaso en éstos tópicos, a veces está en el mal uso del álgebra elemental.
He aquí una oportunidad para mejorar tal eficiencia. Exprese cada resultado en su forma más reducida.

CAPITULO 5

INTEGRACION DE FUNCIONES CUADRATICAS

Una función cuadrática, es de la forma: ax^2+bx+c y si ésta aparece en el denominador, la integral que la contiene se hace fácil de encontrar, para la cual conviene diferenciar dos tipos esenciales en lo que se refiere al numerador.

EJERCICIOS DESARROLLADOS

5.1.-Encontrar: $\int \frac{dx}{x^2+2x+5}$

Solución.- Completando cuadrados, se tiene:

$x^2+2x+5=(x^2+2x+\_\_)+5-\_\_=(x^2+2x+1)+5-1=(x^2+2x+1)+4$

$x^2+2x+5=(x+1)^2+2^2$, luego se tiene:

$\int \frac{dx}{x^2+2x+5}=\int \frac{dx}{(x+1)^2+2^2}$. Sea: $w=x+1, dw=dx; a=2$

$$\int \frac{dx}{(x+1)^2+2^2}=\int \frac{dw}{w^2+2^2}=\frac{1}{2}\operatorname{arc}\tau g\frac{w}{a}+c=\frac{1}{2}\operatorname{arc}\tau g\frac{x+1}{2}+c$$

Respuesta: $\int \frac{dx}{x^2+2x+5}=\frac{1}{2}\operatorname{arc}\tau g\frac{x+1}{2}+c$

5.2.-Encontrar: $\int \frac{dx}{4x^2+4x+2}$

Solución.- $\int \frac{dx}{4x^2+4x+2}=\int \frac{dx}{4(x^2+x+\frac{1}{2})}=\frac{1}{4}\int \frac{dx}{x^2+x+\frac{1}{2}}$

Completando cuadrados:

$$x^2+x+\tfrac{1}{2}=(x^2+x+\_\_)+\frac{1}{2}-\_\_=(x^2+x+\frac{1}{4})+\frac{1}{2}-\frac{1}{4}=(x^2+x+\frac{1}{4})+\frac{1}{4}$$

$(x^2+x+\frac{1}{2})=(x+\frac{1}{2})^2+(\frac{1}{2})^2$, luego se tiene:

$\frac{1}{4}\int \frac{dx}{x^2+x+\frac{1}{2}}=\frac{1}{4}\int \frac{dx}{(x+\frac{1}{2})^2+(\frac{1}{2})^2}$, Sea: $w=x+\frac{1}{2}, dw=dx; a=\frac{1}{2}$

$$=\frac{1}{4}\int \frac{dx}{(x+\frac{1}{2})^2+(\frac{1}{2})^2}=\frac{1}{4}\int \frac{dw}{w^2+a^2}=\frac{1}{4}\frac{1}{a}\operatorname{arc}\tau g\frac{w}{a}+c=\frac{1}{4}\frac{1}{\frac{1}{2}}\operatorname{arc}\tau g\frac{x+\frac{1}{2}}{\frac{1}{2}}+c$$

$$=\frac{1}{2}\operatorname{arc}\tau g\frac{\frac{2x+1}{\cancel{2}}}{\frac{1}{\cancel{2}}}+c=\frac{1}{2}\operatorname{arc}\tau g(2x+1)+c$$

Respuesta: $\int \frac{dx}{4x^2+4x+2} = \frac{1}{2}\operatorname{arc}\tau g(2x+1)+c$

5.3.-Encontrar: $\int \frac{2xdx}{x^2-x+1}$

Solución.- $u = x^2 - x + 1, du = (2x-1)dx$

$$\int \frac{2xdx}{x^2-x+1} = \int \frac{(2x-1+1)dx}{x^2-x+1} = \int \frac{(2x-1)dx}{x^2-x+1} + \int \frac{dx}{x^2-x+1} = \int \frac{du}{u} + \int \frac{dx}{x^2-x+1}$$

Completando cuadrados:

$$x^2 - x + 1 = (x^2 - x + \_\_) + 1 \_\_ = (x^2 - x + \frac{1}{4}) + 1 - \frac{1}{4}$$

$x^2 - x + 1 = (x^2 - \frac{1}{2})^2 + \frac{3}{4}$, Luego se tiene:

$$\int \frac{du}{u} + \int \frac{dx}{x^2-x+1} = \int \frac{du}{u} + \int \frac{du}{(x-\frac{1}{2})^2 + \frac{3}{4}} = \int \frac{du}{u} + \int \frac{dx}{(x-\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2}$$

$w = x - \frac{1}{2}, dw = dx; a = \frac{\sqrt{3}}{2}$, luego:

$$\int \frac{du}{u} + \int \frac{dx}{(x-\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2} = \int \frac{du}{u} + \int \frac{dw}{w^2+a^2} = \ell\eta|u| + \frac{1}{a}\operatorname{arc}\tau g\frac{w}{a} + c$$

$$= \ell\eta\left|x^2-x+1\right| + \frac{1}{\frac{\sqrt{3}}{2}}\operatorname{arc}\tau g\frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}} + c = \ell\eta\left|x^2-x+1\right| + \frac{2\sqrt{3}}{3}\operatorname{arc}\tau g\frac{\frac{2x-1}{\cancel{2}}}{\frac{\sqrt{3}}{\cancel{2}}} + c$$

Respuesta: $\int \frac{2xdx}{x^2-x+1} = \ell\eta\left|x^2-x+1\right| + \frac{2\sqrt{3}}{3}\operatorname{arc}\tau g\frac{2x-1}{\sqrt{3}} + c$

5.4.-Encontrar: $\int \frac{x^2dx}{x^2+2x+5}$

Solución.-

$$\int \frac{x^2dx}{x^2+2x+5} = \int\left(1 - \frac{2x+5}{x^2+2x+5}\right)dx = \int dx - \int \frac{2x+5}{x^2+2x+5}dx,$$

Sea: $u = x^2 + 2x + 5, du = (2x+2)dx$

Ya se habrá dado cuenta el lector que tiene que construir en el numerador, la expresión: $(2x+2)dx$. Luego se tiene:

$$= \int dx - \int \frac{(2x+2+3)}{x^2+2x+5}dx = \int dx - \int \frac{(2x+2)dx}{x^2+2x+5} + 3\int \frac{dx}{x^2+2x+5},$$

Completando cuadrados, se tiene:

$$x^2+2x+5 = (x^2+2x+\_\_)+5-\_\_ = (x^2+2x+1)+5-1 = (x^2+2x+1)+4 = (x+1)^2+2^2$$

Luego se admite como forma equivalente a la anterior:

$\int dx - \int \frac{du}{u} - 3\int \frac{dx}{(x+1)^2+2^2}$, Sea: $w = x+1, dw = dx; a = 2$, luego:

$$= \int dx - \int \frac{du}{u} - 3\int \frac{dw}{w^2 + a^2} = x - \ell\eta|u| - 3\frac{1}{a}\operatorname{arc}\tau g\frac{w}{a} + c$$

$$= x - \ell\eta\left|x^2 + 2x + 5\right| - \frac{3}{2}\operatorname{arc}\tau g\frac{x+1}{2} + c$$

Respuesta: $\int \frac{x^2dx}{x^2 + 2x + 5} = x - \ell\eta\left|x^2 + 2x + 5\right| - \frac{3}{2}\operatorname{arc}\tau g\frac{x+1}{2} + c$

5.5.-Encontrar: $\int \frac{2x-3}{x^2 + 2x + 2}dx$

Solución.- Sea: $u = x^2 + 2x + 2, du = (2x + 2)dx$

$$\int \frac{2x-3}{x^2 + 2x + 2}dx = \int \frac{2x+2-5}{x^2 + 2x + 2}dx = \int \frac{2x+2}{x^2 + 2x + 2}dx - 5\int \frac{dx}{x^2 + 2x + 2}$$

$= \int \frac{du}{u}dx - 5\int \frac{dx}{x^2 + 2x + 2}$, Completando cuadrados:

$x^2 + 2x + 2 = (x+1)^2 + 1^2$. Luego:

$= \int \frac{du}{u}dx - 5\int \frac{dx}{(x+1)^2 + 1^2}$, Sea: $w = x + 1, du = dx; a = 1$. Entonces se tiene:

$$= \int \frac{du}{u}dx - 5\int \frac{dx}{w^2 + a^2} = \ell\eta|u| - 5\frac{1}{a}\operatorname{arc}\tau g\frac{w}{a} + c = \ell\eta\left|x^2 + 2x + 5\right| - 5\operatorname{arc}\tau g(x+1) + c$$

Respuesta: $\int \frac{2x-3}{x^2 + 2x + 2}dx = \ell\eta\left|x^2 + 2x + 5\right| - 5\operatorname{arc}\tau g(x+1) + c$

5.6.-Encontrar: $\int \frac{dx}{\sqrt{x^2 - 2x - 8}}$

Solución.- Completando cuadrados se tiene: $x^2 - 2x - 8 = (x-1)^2 - 3^2$

$\int \frac{dx}{\sqrt{x^2 - 2x - 8}} = \int \frac{dx}{\sqrt{(x-1)^2 - 3^2}}$, Sea: $w = x - 1, dw = dx; a = 3$

$$= \int \frac{dw}{\sqrt{w^2 - a^2}} = \ell\eta\left|w + \sqrt{w^2 - a^2}\right| + c = \ell\eta\left|x - 1 + \sqrt{x^2 - 2x - 8}\right| + c$$

Respuesta: $\int \frac{dx}{\sqrt{x^2 - 2x - 8}} = \ell\eta\left|x - 1 + \sqrt{x^2 - 2x - 8}\right| + c$

5.7.-Encontrar: $\int \frac{xdx}{\sqrt{x^2 - 2x + 5}}$

Solución.- Sea: $u = x^2 - 2x + 5, du = (2x - 2)dx$. Luego:

$$\int \frac{xdx}{\sqrt{x^2 - 2x + 5}} = \frac{1}{2}\int \frac{2xdx}{\sqrt{x^2 - 2x + 5}} = \frac{1}{2}\int \frac{2x-2+2}{\sqrt{x^2 - 2x + 5}}dx$$

$$= \frac{1}{2}\int \frac{(2x-2)dx}{\sqrt{x^2 - 2x + 5}} + \frac{2}{2}\int \frac{dx}{\sqrt{x^2 - 2x + 5}} = \frac{1}{2}\int \frac{du}{\sqrt{u}} + \int \frac{dx}{\sqrt{x^2 - 2x + 5}}$$

Completando cuadrados se tiene: $x^2 + 2x + 5 = (x-1)^2 + 2^2$. Por lo tanto:

$= \frac{1}{2}\int u^{-1/2}du + \int \frac{dx}{\sqrt{(x-1)^2+2^2}}$. Sea: $w = x-1, du = dx; a = 2$

$= \frac{1}{2}\int u^{-1/2}du + \int \frac{dw}{\sqrt{w^2+a^2}} = \frac{1}{\cancel{2}}\frac{u^{1/2}}{1/\cancel{2}} + \ell\eta\left|w+\sqrt{w^2+a^2}\right| + c = u^{1/2} + \ell\eta\left|w+\sqrt{w^2+a^2}\right| + c$

$= \sqrt{x^2+2x+5} + \ell\eta\left|x-1+\sqrt{x^2-2x+5}\right| + c$

Respuesta: $\int \frac{xdx}{\sqrt{x^2-2x+5}} = \sqrt{x^2-2x+5} + \ell\eta\left|x-1+\sqrt{x^2-2x+5}\right| + c$

5.8.-Encontrar: $\int \frac{(x+1)dx}{\sqrt{2x-x^2}}$

Solución.- Sea: $u = 2x - x^2, du = (2-2x)dx$ .Luego:

$\int \frac{(x+1)dx}{\sqrt{2x-x^2}} = -\frac{1}{2}\int \frac{-2(x+1)dx}{\sqrt{2x-x^2}} = -\frac{1}{2}\int \frac{(-2x-2)dx}{\sqrt{2x-x^2}} = -\frac{1}{2}\int \frac{(-2x+2-4)dx}{\sqrt{2x-x^2}}$

$= -\frac{1}{2}\int \frac{(2-2x)dx}{\sqrt{2x-x^2}} + \frac{4}{2}\int \frac{dx}{\sqrt{2x-x^2}} = -\frac{1}{2}\int \frac{du}{\sqrt{u}} + 2\int \frac{dx}{\sqrt{2x-x^2}}$

Completando cuadrados: $2x - x^2 = -(x^2-2x) = -(x^2-2x+1-1) = -(x^2-2x+1)+1$

$= -(x-1)^2 + 1 = 1-(x-1)^2$. Luego la expresión anterior es equivalente a:

$= -\frac{1}{2}\int u^{-1/2}du + 2\int \frac{dx}{\sqrt{1-(x-1)^2}}$. Sea: $w = x-1, dw = dx; a = 1$. Entonces:

$= -\frac{1}{\cancel{2}}\int \frac{u^{1/2}}{1/\cancel{2}}du + 2\int \frac{dw}{\sqrt{a^2-w^2}} = -u^{1/2} + 2\operatorname{arcsen}\frac{w}{a} + c = -\sqrt{2x-x^2} + 2\operatorname{arcsen}(x-1) + c$

Respuesta: $\int \frac{(x+1)dx}{\sqrt{2x-x^2}} = -\sqrt{2x-x^2} + 2\operatorname{arcsen}(x-1) + c$

5.9.-Encontrar: $\int \frac{xdx}{\sqrt{5x^2-2x+1}}$

Solución.- Sea: $u = 5x^2-2x+1, du = (10x-2)dx$. Luego:

$\int \frac{xdx}{\sqrt{5x^2-2x+1}} = \frac{1}{10}\int \frac{10xdx}{\sqrt{5x^2-2x+1}} = \frac{1}{10}\int \frac{(10x-2+2)dx}{\sqrt{5x^2-2x+1}}$

$= \frac{1}{10}\int \frac{(10x-2)dx}{\sqrt{5x^2-2x+1}} + \frac{2}{10}\int \frac{dx}{\sqrt{5x^2-2x+1}} = \frac{1}{10}\int \frac{du}{\sqrt{u}} + \frac{1}{5}\int \frac{dx}{\sqrt{5x^2-2x+1}}$

$= \frac{1}{10}\int \frac{du}{\sqrt{u}} + \frac{1}{5}\int \frac{dx}{\sqrt{5(x^2-\frac{2}{5}x+\frac{1}{5})}} = \frac{1}{10}\int u^{-1/2}du + \frac{1}{5\sqrt{5}}\int \frac{dx}{\sqrt{(x^2-\frac{2}{5}x+\frac{1}{5})}}$

Completando cuadrados: $x^2 - \frac{2}{5}x + \frac{1}{5} = (x^2 - \frac{2}{5}x + \_\_) + \frac{1}{5} - \_\_$

$= (x^2 - \frac{2}{5}x + \frac{1}{25}) + \frac{1}{5} - \frac{1}{25} = (x-\frac{1}{5})^2 + (\frac{2}{5})^2$, Luego es equivalente:

$$=\frac{1}{10}\int u^{-1/2}du+\frac{1}{5\sqrt{5}}\int\frac{dx}{\sqrt{(x-1/5)^2+(2/5)^2}}$$, Sea: $w=x-1/5, dw=dx; a=2/5$,

Entonces: $=\frac{1}{10}\int u^{-1/2}du+\frac{1}{5\sqrt{5}}\int\frac{dw}{\sqrt{w^2+a^2}}=\frac{1}{10}\frac{u^{1/2}}{1/2}+\frac{1}{5\sqrt{5}}\ell\eta\left|w+\sqrt{w^2+a^2}\right|+c$

$$=\frac{\sqrt{5x^2-2x+1}}{5}+\frac{1}{5\sqrt{5}}\ell\eta\left|x-\frac{1}{5}+\frac{\sqrt{5x^2-2x+1}}{\sqrt{5}}\right|+c$$

Respuesta: $\int\frac{xdx}{\sqrt{5x^2-2x+1}}=\frac{\sqrt{5x^2-2x+1}}{5}+\frac{\sqrt{5}}{25}\ell\eta\left|x-\frac{1}{5}+\frac{\sqrt{5x^2-2x+1}}{\sqrt{5}}\right|+c$

5.10.-Encontrar: $\int\frac{xdx}{\sqrt{5+4x-x^2}}$

Solución.- $u=5+4x-x^2, du=(4-2x)dx$. Luego:

$$\int\frac{xdx}{\sqrt{5+4x-x^2}}=-\frac{1}{2}\int\frac{-2xdx}{\sqrt{5+4x-x^2}}=-\frac{1}{2}\int\frac{(-2x+4-4)dx}{\sqrt{5+4x-x^2}}$$

$$=-\frac{1}{2}\int\frac{(4-2x)dx}{\sqrt{5+4x-x^2}}+\frac{4}{2}\int\frac{dx}{\sqrt{5+4x-x^2}}=-\frac{1}{2}\int\frac{du}{\sqrt{u}}+2\int\frac{dx}{\sqrt{5+4x-x^2}}$$

Completando cuadrados: $5+4x-x^2=-(x^2-4x-5)=-(x^2-4x+4-4-5)$

$=-(x^2-4x+4)+9=9-(x-2)^2=3^2-(x-2)^2$. Equivalente a:

$=-\frac{1}{2}\int u^{-1/2}du+2\int\frac{dx}{\sqrt{3^2-(x-2)^2}}$. Sea: $w=x-2, dw=dx; a=3$. Entonces:

$$=-\frac{1}{2}\int u^{-1/2}du+2\int\frac{dw}{\sqrt{a^2-w^2}}=-\frac{1}{\cancel{2}}\frac{u^{1/2}}{1/\cancel{2}}+2\operatorname{arcsen}\frac{w}{a}+c$$

$$=-\sqrt{5+4x-x^2}+2\operatorname{arcsen}\frac{x-2}{3}+c$$

Respuesta: $\int\frac{xdx}{\sqrt{5+4x-x^2}}=-\sqrt{5+4x-x^2}+2\operatorname{arcsen}\frac{x-2}{3}+c$

5.11.-Encontrar: $\int\frac{dx}{\sqrt{2+3x-2x^2}}$

Solución.- Completando cuadrados se tiene:

$$2+3x-2x^2=-(2x^2-3x-2)=-2(x^2-3/2\,x-1)=-2(x^2-\frac{3}{2}x+\frac{9}{16}-\frac{25}{16})$$

$=-2\left[(x^2-\frac{3}{2}x+\frac{9}{16})-\frac{25}{16}\right]=-2\left[(x-3/4)^2-(5/4)^2\right]=2\left[(5/4)^2-(x-3/4)^2\right]$, luego:

$$\int\frac{dx}{\sqrt{2+3x-2x^2}}=\int\frac{dx}{\sqrt{2\left[(5/4)^2-(x-3/4)^2\right]}}=\frac{1}{\sqrt{2}}\int\frac{dx}{\sqrt{(5/4)^2-(x-3/4)^2}}$$

Sea: $w=x-3/4, dw=dx, a=5/4$. Luego:

$$=\frac{1}{\sqrt{2}}\int\frac{dx}{\sqrt{(5/4)^2-(x-3/4)^2}}=\frac{1}{\sqrt{2}}\int\frac{dw}{\sqrt{a^2-w^2}}=\frac{1}{\sqrt{2}}\text{arcs}e\text{n}\frac{w}{a}+c=\frac{1}{\sqrt{2}}\text{arcs}e\text{n}\frac{x-3/4}{5/4}+c$$

$$=\frac{\sqrt{2}}{2}\text{arcs}e\text{n}\frac{4x-3}{5}+c$$

Respuesta: $\int\frac{dx}{\sqrt{2+3x-2x^2}}=\frac{\sqrt{2}}{2}\text{arcs}e\text{n}\frac{4x-3}{5}+c$

5.12.-Encontrar: $\int\frac{dx}{3x^2+12x+42}$

Solución.-

$$\int\frac{dx}{3x^2+12x+42}=\int\frac{dx}{3(x^2+4x+14)}=\frac{1}{3}\int\frac{dx}{(x^2+4x+14)}=\frac{1}{3}\int\frac{dx}{(x^2+4x+4+10)}=$$

$$=\frac{1}{3}\int\frac{dx}{(x+2)^2+10}=\frac{1}{3}\int\frac{dx}{(x+2)^2+(\sqrt{10})^2}=\frac{1}{3}\frac{1}{\sqrt{10}}\text{arc}\,\tau g\,\frac{x+2}{\sqrt{10}}+c$$

Respuesta: $\int\frac{dx}{3x^2+12x+42}=\frac{\sqrt{10}}{30}\text{arc}\,\tau g\,\frac{x+2}{\sqrt{10}}+c$

5.13.-Encontrar: $\int\frac{3x-2}{x^2-4x+5}dx$

Solución.- Sea: $u=x^2-4x+5, du=(2x-4)dx$, Luego:

$$\int\frac{3x-2}{x^2-4x+5}dx=3\int\frac{xdx}{x^2-4x+5}-2\int\frac{dx}{x^2-4x+5}=3\int\frac{(x-2)+2}{x^2-4x+5}-2\int\frac{dx}{x^2-4x+5}$$

$$=3\int\frac{(x-2)}{x^2-4x+5}+6\int\frac{dx}{x^2-4x+5}-2\int\frac{dx}{x^2-4x+5}=\frac{3}{2}\int\frac{du}{u}+4\int\frac{dx}{x^2-4x+5}$$

$$=\frac{3}{2}\int\frac{du}{u}+4\int\frac{dx}{(x^2-4x+4)+1}=\frac{3}{2}\ell\eta\left|x^2-4x+5\right|+4\int\frac{dx}{(x-2)^2+1}$$

$$=\frac{3}{2}\ell\eta\left|x^2-4x+5\right|+4\,\text{arc}\,\tau g(x-2)+c$$

Respuesta: $\int\frac{3x-2}{x^2-4x+5}dx=\frac{3}{2}\ell\eta\left|x^2-4x+5\right|+4\,\text{arc}\,\tau g(x-2)+c$

EJERCICIOS PROPUESTOS

Usando Esencialmente la técnica tratada, encontrar las integrales siguientes:

5.14.- $\int\sqrt{x^2+2x-3}dx$ **5.15.-** $\int\sqrt{12+4x-x^2}dx$ **5.16.-** $\int\sqrt{x^2+4x}dx$

5.17.- $\int\sqrt{x^2-8x}dx$ **5.18.-** $\int\sqrt{6x-x^2}dx$ **5.19.-** $\int\frac{(5-4x)dx}{\sqrt{12x-4x^2-8}}$

5.20.- $\int \frac{xdx}{\sqrt{27+6x-x^2}}$

5.21.- $\int \frac{(x-1)dx}{3x^2-4x+3}$

5.22.- $\int \frac{(2x-3)dx}{x^2+6x+15}$

5.23.- $\int \frac{dx}{4x^2+4x+10}$

5.24.- $\int \frac{(2x+2)dx}{x^2-4x+9}$

5.25.- $\int \frac{(2x+4)dx}{\sqrt{4x-x^2}}$

5.26.- $\frac{2}{3}\int \frac{(x+\frac{3}{2})dx}{9x^2-12x+8}$

5.27.- $\int \frac{(x+6)dx}{\sqrt{5-4x-x^2}}$

5.28.- $\int \frac{dx}{2x^2+20x+60}$

5.29.- $\int \frac{3dx}{\sqrt{80+32x-4x^2}}$

5.30.- $\int \frac{dx}{\sqrt{12x-4x^2-8}}$

5.31.- $\int \frac{5dx}{\sqrt{28-12x-x^2}}$

5.32.- $\int \sqrt{12-8x-4x^2}dx$

5.33.- $\sqrt{x^2-x+\frac{5}{4}}dx$

5.34.- $\int \frac{dx}{x^2-2x+5}$

5.35.- $\int \frac{(1-x)dx}{\sqrt{8+2x-x^2}}$

5.36.- $\int \frac{xdx}{x^2+4x+5}$

5.37.- $\int \frac{(2x+3)dx}{4x^2+4x+5}$

5.38.- $\int \frac{(x+2)dx}{x^2+2x+2}$

5.39.- $\int \frac{(2x+1)dx}{x^2+8x-2}$

5.40.- $\int \frac{dx}{\sqrt{-x^2-6x}}$

5.41.- $\int \frac{(x-1)dx}{x^2+2x+2}$

RESPUESTAS

5.14.- $\int \sqrt{x^2-2x-3}dx$

Solución.- Completando cuadrados se tiene:

$x^2-2x-3=(x^2-2x+1)-3-1=(x-1)^2-4=(x-1)^2-2^2$

Haciendo: $u=x-1, du=dx; a=2$, se tiene:

$$\int \sqrt{x^2-2x-3}dx=\int \sqrt{(x-1)^2-2^2}dx=\int \sqrt{u^2-a^2}du$$

$$=\frac{1}{2}u\sqrt{u^2-a^2}-\frac{1}{2}a^2\ell\eta\left|u+\sqrt{u^2-a^2}\right|+c$$

$$=\frac{1}{2}(x-1)\sqrt{(x-1)^2-2^2}-\frac{1}{2}2^2\ell\eta\left|(x-1)+\sqrt{(x-1)^2-2^2}\right|+c$$

$$=\frac{1}{2}(x-1)\sqrt{x^2-2x-3}-2\ell\eta\left|(x-1)+\sqrt{x^2-2x-3}\right|+c$$

5.15.- $\int \sqrt{12+4x-x^2}dx$

Solución.- Completando cuadrados se tiene:

$12+4x-x^2=-(x^2-4x-12)=-(x^2-4x+4-12-4)=-(x^2-4x+4)+16$

$=4^2-(x-2)^2$

Haciendo: $u=x-2, du=dx; a=4$, se tiene:

$$\int \sqrt{12+4x-x^2}dx=\int \sqrt{4^2-(x-2)^2}dx=\int \sqrt{a^2-u^2}du=\frac{1}{2}u\sqrt{a^2-u^2}+\frac{1}{2}a^2\operatorname{arcsen}\frac{u}{a}+c$$

$$=\frac{1}{2}(x-2)\sqrt{4^2-(x-2)^2}+\frac{1}{2}4^2\operatorname{arcsen}\frac{(x-2)}{4}+c$$
$$=\frac{1}{2}(x-2)\sqrt{12+4x-x^2}+8\operatorname{arcsen}\frac{(x-2)}{4}+c$$

5.16.- $\int\sqrt{x^2+4x}dx$

Solución.- Completando cuadrados se tiene:

$x^2+4x=(x^2+4x+4)-4=(x+2)^2-2^2$

Haciendo: $u=x+2, du=dx; a=2$, se tiene:

$$\int\sqrt{x^2+4x}dx=\int\sqrt{(x+2)^2-2^2}dx=\int\sqrt{u^2-a^2}du$$
$$=\frac{1}{2}u\sqrt{u^2-a^2}-\frac{1}{2}a^2\ell\eta\left|u+\sqrt{u^2-a^2}\right|+c$$
$$=\frac{1}{2}(x+2)\sqrt{(x+2)^2-2^2}-\frac{1}{2}2^2\ell\eta\left|(x+2)+\sqrt{(x+2)^2-2^2}\right|+c$$
$$=\frac{(x+2)}{2}\sqrt{x^2+4x}-2\ell\eta\left|(x+2)+\sqrt{x^2+4x}\right|+c$$

5.17.- $\int\sqrt{x^2-8x}dx$

Solución.- Completando cuadrados se tiene:

$x^2-8x=(x^2-8x+16)-16=(x-4)^2-4^2$

Haciendo: $u=x-4, du=dx; a=4$, se tiene:

$$\int\sqrt{(x-4)^2-4^2}dx=\sqrt{u^2-a^2}du=\frac{1}{2}u\sqrt{u^2-a^2}-\frac{1}{2}a^2\ell\eta\left|u+\sqrt{u^2-a^2}\right|+c$$
$$=\frac{1}{2}(x-4)\sqrt{(x-4)^2-4^2}-\frac{1}{2}4^2\ell\eta\left|(x-4)+\sqrt{(x-4)^2-4^2}\right|+c$$
$$=\frac{(x-4)}{2}\sqrt{x^2-8x}-8\ell\eta\left|(x-4)+\sqrt{x^2-8x}\right|+c$$

5.18.- $\int\sqrt{6x-x^2}dx$

Solución.- Completando cuadrados se tiene:

$6x-x^2=-(x^2-6x)=-(x^2-6x+9-9)=-(x^2-6x+9)+9=3^2-(x-3)^2$

Haciendo: $u=x-3, du=dx; a=3$, se tiene:

$$\int\sqrt{6x-x^2}dx=\sqrt{3^2-(x-3)^2}dx=\sqrt{a^2-u^2}du=\frac{1}{2}u\sqrt{a^2-u^2}+\frac{1}{2}a^2\operatorname{arcsen}\frac{u}{a}+c$$
$$=\frac{1}{2}(x-3)\sqrt{3^2-(x-3)^2}+\frac{1}{2}3^2\operatorname{arcsen}\frac{x-3}{3}+c$$
$$=\frac{(x-3)}{2}\sqrt{6x-x^2}+\frac{9}{2}\operatorname{arcsen}\frac{x-3}{3}+c$$

5.19.- $\int\frac{(5-4x)dx}{\sqrt{12x-4x^2-8}}$

Solución.- Sea: $u=12x-4x^2-8, du=(12-8x)dx$

$$\int\frac{(5-4x)dx}{\sqrt{12x-4x^2-8}}=\int\frac{(-4x+5)dx}{\sqrt{12x-4x^2-8}}=\frac{1}{2}\int\frac{2(-4x+5)dx}{\sqrt{12x-4x^2-8}}=\frac{1}{2}\int\frac{(-8x+10)dx}{\sqrt{12x-4x^2-8}}$$
$$=\frac{1}{2}\int\frac{(-8x+12-2)dx}{\sqrt{12x-4x^2-8}}=\frac{1}{2}\int\frac{(-8x+12)dx}{\sqrt{12x-4x^2-8}}-\int\frac{dx}{\sqrt{12x-4x^2-8}}$$
$$=\frac{1}{2}\int\frac{(-8x+12)dx}{\sqrt{12x-4x^2-8}}-\int\frac{dx}{\sqrt{4(3x-x^2-2)}}=\frac{1}{2}\int\frac{(-8x+12)dx}{\sqrt{12x-4x^2-8}}-\frac{1}{2}\int\frac{dx}{\sqrt{3x-x^2-2}}$$
Completando cuadrados se tiene:
$$3x-x^2-2=-(x^2-3x+2)=-(x^2-3x+\frac{9}{4}-\frac{9}{4}+2)=-(x^2-3x+\frac{9}{4})+\frac{9}{4}-2$$
$$=-(x-\tfrac{3}{2})^2+\frac{1}{4}=(\frac{1}{2})^2-(x-\frac{3}{2})^2$$
$$=\frac{1}{2}\int\frac{(-8x+12)dx}{\sqrt{12x-4x^2-8}}-\frac{1}{2}\int\frac{dx}{\sqrt{(\tfrac{1}{2})^2-(x-\tfrac{3}{2})^2}}$$
Haciendo: $u=12x-4x^2-8, du=(12-8x)dx$ y $w=x-\tfrac{3}{2}, dw=dx$, entonces:
$$=\frac{1}{2}\int\frac{du}{\sqrt{u}}-\frac{1}{2}\int\frac{dw}{\sqrt{(\tfrac{1}{2})^2-w^2}}=\frac{1}{\cancel{2}}\frac{u^{\frac{1}{2}}}{\tfrac{1}{\cancel{2}}}-\frac{1}{2}\operatorname{arcsen}\frac{w}{\tfrac{1}{2}}+c$$
$$=u^{\frac{1}{2}}-\frac{1}{2}\operatorname{arcsen}2w+c=\sqrt{12x-4x^2-8}-\frac{1}{2}\operatorname{arcsen}(2x-3)+c$$

5.20.- $\int\frac{xdx}{\sqrt{27+6x-x^2}}$

Solución.- Sea: $u=27+6x-x^2, du=(6-2x)dx$
$$\int\frac{xdx}{\sqrt{27+6x-x^2}}=-\frac{1}{2}\int\frac{-2xdx}{\sqrt{27+6x-x^2}}=-\frac{1}{2}\int\frac{(-2x+6-6)dx}{\sqrt{27+6x-x^2}}$$
$$=-\frac{1}{2}\int\frac{(-2x+6)dx}{\sqrt{27+6x-x^2}}+3\int\frac{dx}{\sqrt{27+6x-x^2}}=-\frac{1}{2}\int\frac{du}{\sqrt{u}}+3\int\frac{dx}{\sqrt{27+6x-x^2}}$$
Completando cuadrados se tiene:
$$27+6x-x^2=-(x^2-6x-27)=-(x^2-6x+9-9-27)=-(x^2-6x+9)+36$$
$=6^2-(x-3)^2$, Luego:
$$=-\frac{1}{2}\int u^{-\frac{1}{2}}du+3\int\frac{dx}{\sqrt{6^2-(x-3)^2}}=-\frac{1}{\cancel{2}}\frac{u^{\frac{1}{2}}}{\tfrac{1}{\cancel{2}}}+3\operatorname{arcsen}\frac{x-3}{6}+c$$
$$=-u^{\frac{1}{2}}+3\operatorname{arcsen}\frac{x-3}{6}+c=-\sqrt{27+6x-x^2}+3\operatorname{arcsen}\frac{x-3}{6}+c$$

5.21.- $\int\frac{(x-1)dx}{3x^2-4x+3}$

Solución.- Sea: $u=3x^2-4x+3, du=(6x-4)dx$
$$\int\frac{(x-1)dx}{3x^2-4x+3}=\frac{1}{6}\int\frac{(6x-6)dx}{3x^2-4x+3}=\frac{1}{6}\int\frac{(6x-4-2)dx}{3x^2-4x+3}=\frac{1}{6}\int\frac{(6x-4)dx}{3x^2-4x+3}-\frac{1}{3}\int\frac{dx}{3x^2-4x+3}$$

$$=\frac{1}{6}\int\frac{du}{u}-\frac{1}{3}\int\frac{dx}{3x^2-4x+3}=\frac{1}{6}\int\frac{du}{u}-\frac{1}{3}\int\frac{dx}{3(x^2-\frac{4}{3}x+1)}$$

$$=\frac{1}{6}\int\frac{du}{u}-\frac{1}{9}\int\frac{dx}{(x^2-\frac{4}{3}x+1)}$$

Completando cuadrados se tiene:

$$x^2-\frac{4}{3}x+1=(x^2-\frac{4}{3}x+\frac{4}{9})+1-\frac{4}{9}=(x^2-\frac{4}{3}x+\frac{4}{9})+\frac{5}{9}=(x-\frac{2}{3})^2+(\frac{\sqrt{5}}{3})^2$$

$$=\frac{1}{6}\int\frac{du}{u}-\frac{1}{9}\int\frac{dx}{(x-\frac{2}{3})^2+(\frac{\sqrt{5}}{3})^2}=\frac{1}{6}\ell\eta|u|-\frac{1}{9}\frac{1}{\frac{\sqrt{5}}{3}}\operatorname{arc}\tau g\frac{x-\frac{2}{3}}{\frac{\sqrt{5}}{3}}+c$$

$$=\frac{1}{6}\ell\eta\left|3x^2-4x+3\right|-\frac{\sqrt{5}}{15}\operatorname{arc}\tau g\frac{3x-2}{\sqrt{5}}+c$$

5.22.- $\int\frac{(2x-3)dx}{x^2+6x+15}$

Solución.- Sea: $u=x^2+6x+15, du=(2x+6)dx$

$$\int\frac{(2x-3)dx}{x^2+6x+15}=\int\frac{(2x+6-9)dx}{x^2+6x+15}=\int\frac{(2x+6)dx}{x^2+6x+15}-9\int\frac{dx}{x^2+6x+15}$$

$=\int\frac{du}{u}-9\int\frac{dx}{x^2+6x+15}$, Completando cuadrados se tiene:

$$x^2+6x+15=(x^2+6x+9)+15-9=(x+3)^2+6^2=(x+3)^2+(\sqrt{6})^2$$

$$=\int\frac{du}{u}-9\int\frac{dx}{(x+3)^2+(\sqrt{6})^2}=\ell\eta\left|x^2+6x+15\right|-9\frac{1}{\sqrt{6}}\operatorname{arc}\tau g\frac{x+3}{\sqrt{6}}+c$$

$$=\ell\eta\left|x^2+6x+15\right|-\frac{3\sqrt{6}}{2}\operatorname{arc}\tau g\frac{x+3}{\sqrt{6}}+c$$

5.23.- $\int\frac{dx}{4x^2+4x+10}$

Solución.-

$\int\frac{dx}{4x^2+4x+10}=\int\frac{dx}{4(x^2+x+\frac{5}{2})}=\frac{1}{4}\int\frac{dx}{(x^2+x+\frac{5}{2})}$, Completando cuadrados:

$$x^2+x+\frac{5}{2}=(x^2+x+\frac{1}{4})+\frac{5}{2}-\frac{1}{4}=(x+\frac{1}{2})^2+\frac{9}{4}=(x+\frac{1}{2})^2+(\frac{3}{2})^2$$

$$=\frac{1}{4}\int\frac{dx}{(x+\frac{1}{2})^2+(\frac{3}{2})^2}=\frac{1}{4}\frac{1}{\frac{3}{2}}\operatorname{arc}\tau g\frac{x+\frac{1}{2}}{\frac{3}{2}}+c=\frac{1}{6}\operatorname{arc}\tau g\frac{2x+1}{3}+c$$

5.24.- $\int\frac{(2x+2)dx}{x^2-4x+9}$

Solución.- Sea: $u=x^2-4x+9, du=(2x-4)dx$

$$\int \frac{(2x+2)dx}{x^2-4x+9} = \int \frac{(2x-4+6)dx}{x^2-4x+9} = \int \frac{(2x-4)dx}{x^2-4x+9} + 6\int \frac{dx}{x^2-4x+9}$$

$= \int \frac{du}{u} + 6\int \frac{dx}{x^2-4x+9}$, Completando cuadrados se tiene:

$x^2-4x+9 = (x^2-4x+4)+9-4 = (x-2)^2+5 = (x-2)^2+(\sqrt{5})^2$,

$$= \int \frac{du}{u} + 6\int \frac{dx}{(x-2)^2+(\sqrt{5})^2} = \ell\eta\left|u\right| + 6\frac{1}{\sqrt{5}}\operatorname{arc}\tau g\frac{x-2}{\sqrt{5}} + c$$

$$= \ell\eta\left|x^2-4x+9\right| + \frac{6\sqrt{5}}{5}\operatorname{arc}\tau g\frac{x-2}{\sqrt{5}} + c$$

5.25.- $\int \frac{(2x+4)dx}{\sqrt{4x-x^2}}$

Solución.- Sea: $u = 4x-x^2+9, du = (4-2x)dx$

$$\int \frac{(2x+4)dx}{\sqrt{4x-x^2}} = -\int \frac{(-2x-4)dx}{\sqrt{4x-x^2}} = -\int \frac{(-2x+4-8)dx}{\sqrt{4x-x^2}} = -\int \frac{(-2x+4)dx}{\sqrt{4x-x^2}} + 8\int \frac{dx}{\sqrt{4x-x^2}}$$

$= -\int u^{-1/2}du + 8\int \frac{dx}{\sqrt{4x-x^2}}$, Completando cuadrados se tiene:

$4x-x^2 = -(x^2-4x) = -(x^2-4x+4-4) = -(x^2-4x+4)+4 = 2^2-(x-2)^2$

$$= -\int u^{-1/2}du + 8\int \frac{dx}{\sqrt{2^2-(x-2)^2}} = -2u^{1/2} + 8\operatorname{arcs}e\mathrm{n}\frac{x-2}{2} + c$$

$$= -2\sqrt{4x-x^2} + 8\operatorname{arcs}e\mathrm{n}\frac{x-2}{2} + c$$

5.26.- $\frac{2}{3}\int \frac{(x+\frac{3}{2})dx}{9x^2-12x+8}$

Solución.- Sea: $u = 9x^2-12x+8, du = (18x-12)dx$

$$\frac{2}{3}\int \frac{(x+\frac{3}{2})dx}{9x^2-12x+8} = \frac{2}{3}\frac{1}{18}\int \frac{(18x+27)dx}{9x^2-12x+8} = \frac{1}{27}\int \frac{(18x+27)dx}{9x^2-12x+8} = \frac{1}{27}\int \frac{(18x-12+39)dx}{9x^2-12x+8}$$

$$= \frac{1}{27}\int \frac{(18x-12)dx}{9x^2-12x+8} + \frac{39}{27}\int \frac{dx}{9x^2-12x+8} = \frac{1}{27}\int \frac{du}{u} + \frac{39}{27}\int \frac{dx}{9(x^2-\frac{4}{3}x+\frac{8}{9})}$$

$$= \frac{1}{27}\int \frac{du}{u} + \frac{39}{27\times 9}\int \frac{dx}{(x^2-\frac{4}{3}x+\frac{8}{9})}$$

Completando cuadrados se tiene:

$$x^2-\frac{4}{3}+\frac{8}{9} = (x^2-\frac{4}{3}x+\frac{4}{9})+\frac{8}{9}-\frac{4}{9} = (x-\frac{2}{3})^2+\frac{4}{9} = (x-\frac{2}{3})^2+(\frac{2}{3})^2$$

$$= \frac{1}{27}\int \frac{du}{u} + \frac{39}{27\times 9}\int \frac{dx}{(x-\frac{2}{3})^2+(\frac{2}{3})^2} = \frac{1}{27}\ell\eta\left|u\right| + \frac{39}{27\times 9}\frac{1}{\frac{2}{3}}\operatorname{arc}\tau g\frac{x-\frac{2}{3}}{\frac{2}{3}} + c$$

$$=\frac{1}{27}\ell\eta\left|9x^2-12x+8\right|-\frac{13}{54}\operatorname{arc}\tau g\,\frac{3x-2}{2}+c$$

5.27.- $\int\frac{(x+6)dx}{\sqrt{5-4x-x^2}}$

Solución.- Sea: $u=5-4x-x^2, du=(-4-2x)dx$

$$\int\frac{(x+6)dx}{\sqrt{5-4x-x^2}}=-\frac{1}{2}\int\frac{(-2x-12)dx}{\sqrt{5-4x-x^2}}=-\frac{1}{2}\int\frac{(-2x-4-8)dx}{\sqrt{5-4x-x^2}}$$

$$=-\frac{1}{2}\int\frac{(-2x-4)dx}{\sqrt{5-4x-x^2}}+4\int\frac{dx}{\sqrt{5-4x-x^2}}=-\frac{1}{2}\int\frac{du}{\sqrt{u}}+4\int\frac{dx}{\sqrt{5-4x-x^2}}$$

Completando cuadrados se tiene: $5-4x-x^2=9-(x+2)^2=3^2-(x+2)^2$

$$=-\frac{1}{2}\int\frac{du}{\sqrt{u}}+4\int\frac{dx}{\sqrt{3^2-(x+2)^2}}=-\sqrt{u}+4\operatorname{arcs}e\mathrm{n}\,\frac{x+2}{3}+c$$

$$=-\sqrt{5-4x-x^2}+4\operatorname{arcs}e\mathrm{n}\,\frac{x+2}{3}+c$$

5.28.- $\int\frac{dx}{2x^2+20x+60}$

Solución.-

$\int\frac{dx}{2x^2+20x+60}=\frac{1}{2}\int\frac{dx}{x^2+10x+30}$, Completando cuadrados se tiene:

$$x^2+10x+30=(x^2+10x+25)+5=(x+5)^2+(\sqrt{5})^2$$

$$=\frac{1}{2}\int\frac{dx}{(x+5)^2+(\sqrt{5})^2}=\frac{1}{2}\frac{1}{\sqrt{5}}\operatorname{arc}\tau g\,\frac{x+5}{\sqrt{5}}+c=\frac{\sqrt{5}}{10}\operatorname{arc}\tau g\,\frac{x+5}{\sqrt{5}}+c$$

5.29.- $\int\frac{3dx}{\sqrt{80+32x-4x^2}}$

Solución.-

$$\int\frac{3dx}{\sqrt{80+32x-4x^2}}=\int\frac{3dx}{\sqrt{4(20+8x-x^2)}}=\frac{3}{2}\int\frac{dx}{\sqrt{(20+8x-x^2)}}$$

Completando cuadrados se tiene:

$$20+8x-x^2=-(x^2-8x-20)=-(x^2-8x+16-20-16)=-(x^2-8x+16)+36$$

$$=-(x-4)^2+6^2=6^2-(x-4)^2$$

$$=\frac{3}{2}\int\frac{dx}{\sqrt{6^2-(x-4)^2}}=\frac{3}{2}\operatorname{arcs}e\mathrm{n}\,\frac{x-4}{6}+c$$

5.30.- $\int\frac{dx}{\sqrt{12x-4x^2-8}}$

Solución.-

$$\int\frac{dx}{\sqrt{12x-4x^2-8}}=\int\frac{dx}{\sqrt{4(-x^2+3x-2)}}=\frac{1}{2}\int\frac{dx}{\sqrt{(-x^2+3x-2)}}$$

Completando cuadrados se tiene:

$$-x^2+3x-2=-(x^2-3x+2)=-(x^2-3x+\frac{9}{4}+2-\frac{9}{4})=-(x^2-3x+\frac{9}{4})+\frac{1}{4}$$

$$=(\tfrac{1}{2})^2-(x-\tfrac{3}{2})^2$$

$$=\frac{1}{2}\int\frac{dx}{\sqrt{(\frac{1}{2})^2-(x-\frac{3}{2})^2}}=\frac{1}{2}\operatorname{arcsen}\frac{x-\frac{3}{2}}{\frac{1}{2}}+c=\frac{1}{2}\operatorname{arcsen}(2x-3)+c$$

5.31.- $\int\frac{5dx}{\sqrt{28-12x-x^2}}$

Solución.-

$\int\frac{5dx}{\sqrt{28-12x-x^2}}=5\int\frac{dx}{\sqrt{28-12x-x^2}}$, Completando cuadrados se tiene:

$$28-12x-x^2=8^2-(x+6)^2$$

$$=5\int\frac{dx}{\sqrt{8^2-(x+6)^2}}=5\operatorname{arcsen}\frac{x+6}{8}+c$$

5.32.- $\int\sqrt{12-8x-4x^2}\,dx$

Solución.- Sea: $u=x+1, du=dx; a=2$

$$\int\sqrt{12-8x-4x^2}\,dx=\int\sqrt{4(3-2x-x^2)}\,dx=2\int\sqrt{3-2x-x^2}\,dx$$

Completando cuadrados se tiene:

$$3-2x-x^2=-(x^2+2x-3)=-(x^2+2x+1)+4=2^2-(x+1)^2$$

$$2\int\sqrt{2^2-(x+1)^2}\,dx=2\int\sqrt{a^2-u^2}\,du=2(\frac{1}{2}u\sqrt{a^2-u^2}+\frac{a^2}{2}\operatorname{arcsen}\frac{u}{a})+c$$

$$=(x+1)\sqrt{-x^2-2x+3}+4\operatorname{arcsen}\frac{x+1}{2}+c$$

5.33.- $\sqrt{x^2-x+\frac{5}{4}}\,dx$

Solución.- Sea: $u=x-\frac{1}{2}, du=dx; a=1$

Completando cuadrados se tiene:

$$x^2-x+\tfrac{5}{4}=(x-\tfrac{1}{2})^2+1$$

$$\sqrt{x^2-x+\tfrac{5}{4}}\,dx=\sqrt{(x-\tfrac{1}{2})^2+1}\,dx=\sqrt{u^2+a^2}\,du$$

$$=\frac{1}{2}u\sqrt{u^2+a^2}+\frac{1}{2}a^2\ell\eta\left|u+\sqrt{u^2+a^2}\right|+c$$

$$=\frac{1}{2}(x-\tfrac{1}{2})\sqrt{x^2-x+\tfrac{5}{4}}+\frac{1}{2}\ell\eta\left|x-\tfrac{1}{2}+\sqrt{x^2-x+\tfrac{5}{4}}\right|+c$$

$$=\frac{1}{4}(2x-1)\sqrt{x^2-x+\tfrac{5}{4}}+\frac{1}{2}\ell\eta\left|x-\tfrac{1}{2}+\sqrt{x^2-x+\tfrac{5}{4}}\right|+c$$

5.34.- $\int\frac{dx}{x^2-2x+5}$

Solución.- Completando cuadrados se tiene:

$x^2-2x+5=(x^2-2x+4)+1=(x-2)^2+1$

$$\int\frac{dx}{x^2-2x+5}=\int\frac{dx}{(x-2)^2+1}=\operatorname{arc}\tau g(x-2)+c$$

5.35.- $\int\frac{(1-x)dx}{\sqrt{8+2x-x^2}}$

Solución.- Sea: $u=8+2x-x^2, du=(2-2x)dx=2(1-x)dx$

$$\int\frac{(1-x)dx}{\sqrt{8+2x-x^2}}=\frac{1}{2}\int\frac{du}{\sqrt{u}}=\frac{1}{2}\int u^{-1/2}du=\sqrt{u}+c=\sqrt{8+2x-x^2}+c$$

5.36.- $\int\frac{xdx}{x^2+4x+5}$

Solución.- Sea: $u=x^2+4x+5, du=(2x+4)dx$

$$\int\frac{xdx}{x^2+4x+5}=\frac{1}{2}\int\frac{2xdx}{x^2+4x+5}=\frac{1}{2}\int\frac{(2x+4)-4}{x^2+4x+5}dx$$

$=\frac{1}{2}\int\frac{(2x+4)dx}{x^2+4x+5}-2\int\frac{dx}{x^2+4x+5}=\frac{1}{2}\int\frac{du}{u}-2\int\frac{dx}{x^2+4x+5}$, Completando cuadrados se tiene: $x^2+4x+5=(x^2+4x+4)+1=(x+2)^2+1$

$$=\frac{1}{2}\int\frac{du}{u}-2\int\frac{dx}{(x+2)^2+1}=\frac{1}{2}\ell\eta|u|-2\operatorname{arc}\tau g(x+2)+c$$

$$=\frac{1}{2}\ell\eta\left|x^2+4x+5\right|-2\operatorname{arc}\tau g(x+2)+c$$

5.37.- $\int\frac{(2x+3)dx}{4x^2+4x+5}$

Solución.- Sea: $u=4x^2+4x+5, du=(8x+4)dx$

$$\int\frac{(2x+3)dx}{4x^2+4x+5}=\frac{1}{4}\int\frac{(8x+12)dx}{4x^2+4x+5}=\frac{1}{4}\int\frac{(8x+4)+8}{4x^2+4x+5}dx$$

$$\frac{1}{4}\int\frac{(8x+4)dx}{4x^2+4x+5}+2\int\frac{dx}{4x^2+4x+5}=\frac{1}{4}\int\frac{du}{u}+2\int\frac{dx}{4x^2+4x+5}=\frac{1}{4}\int\frac{du}{u}+2\int\frac{dx}{4(x^2+x+5/4)}$$

$=\frac{1}{4}\int\frac{du}{u}+\frac{1}{2}\int\frac{dx}{(x^2+x+5/4)}$, Completando cuadrados se tiene:

$$x^2+x+\frac{5}{4}=(x^2+x+\frac{1}{4})+1=(x+1/2)^2+1$$

$$=\frac{1}{4}\int\frac{du}{u}+\frac{1}{2}\int\frac{dx}{(x+1/2)^2+1}=\frac{1}{4}\ell\eta|u|+\frac{1}{2}\operatorname{arc}\tau g(x+1/2)+c$$

5.38.- $\int\frac{(x+2)dx}{x^2+2x+2}$

Solución.- Sea: $u=x^2+2x+2, du=(2x+2)dx$

$$\int\frac{(x+2)dx}{x^2+2x+2}=\frac{1}{2}\int\frac{(2x+4)dx}{x^2+2x+2}=\frac{1}{2}\int\frac{(2x+2)+2}{x^2+2x+2}dx=\frac{1}{2}\int\frac{(2x+2)dx}{x^2+2x+2}+\int\frac{dx}{x^2+2x+2}$$
$$=\frac{1}{2}\int\frac{du}{u}+\int\frac{dx}{x^2+2x+2}=\frac{1}{2}\int\frac{du}{u}+\int\frac{dx}{(x+1)^2+1}$$
$$=\frac{1}{2}\ell\eta\left|u\right|+\operatorname{arc}\tau g(x+1)+c=\frac{1}{2}\ell\eta\left|x^2+2x+2\right|+\operatorname{arc}\tau g(x+1)+c$$

5.39.- $\int\frac{(2x+1)dx}{x^2+8x-2}$

Solución.- Sea: $u=x^2+8x-2,du=(2x+8)dx$
$$\int\frac{(2x+1)dx}{x^2+8x-2}=\int\frac{(2x+8)-7dx}{x^2+8x-2}=\int\frac{(2x+8)dx}{x^2+8x-2}-7\int\frac{dx}{x^2+8x-2}$$
$$=\int\frac{du}{u}-7\int\frac{dx}{(x^2+8x+16)-18}=\int\frac{du}{u}-7\int\frac{dx}{(x+4)^2-(3\sqrt{2})^2}$$
$$=\ell\eta\left|u\right|-7\frac{1}{2(3\sqrt{2})}\ell\eta\left|\frac{(x+4)-(3\sqrt{2})}{(x+4)+(3\sqrt{2})}\right|+c$$
$$=\ell\eta\left|x^2+8x-2\right|-\frac{7\sqrt{2}}{12}\ell\eta\left|\frac{(x+4)-(3\sqrt{2})}{(x+4)+(3\sqrt{2})}\right|+c$$

5.40.- $\int\frac{dx}{\sqrt{-x^2-6x}}$

Solución.- Completando cuadrados se tiene:
$$-x^2-6x=-(x^2+6x)=-(x^2+6x+9)+9=3^2-(x+3)^2$$
$$\int\frac{dx}{\sqrt{3^2-(x+3)^2}}=\operatorname{arcs}e\operatorname{n}\frac{x+3}{3}+c$$

5.41.- $\int\frac{(x-1)dx}{x^2+2x+2}$

Solución.- Sea: $u=x^2+2x+2,du=(2x+2)dx$
$$\int\frac{(x-1)dx}{x^2+2x+2}=\frac{1}{2}\int\frac{(2x+2)-4}{x^2+2x+2}dx=\frac{1}{2}\int\frac{(2x+2)dx}{x^2+2x+2}-2\int\frac{dx}{x^2+2x+2}$$
$$=\frac{1}{2}\int\frac{du}{u}-2\int\frac{dx}{x^2+2x+2}=\frac{1}{2}\int\frac{du}{u}-2\int\frac{dx}{(x+1)^2+1}=\frac{1}{2}\ell\eta\left|u\right|-2\operatorname{arc}\tau g(x+1)+c$$
$$=\frac{1}{2}\ell\eta\left|x^2+2x+2\right|-2\operatorname{arc}\tau g(x+1)+c$$

CAPITULO 6

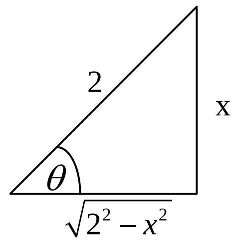
INTEGRACION POR SUSTITUCION TRIGONOMETRICA

Existen integrales que contienen expresiones de las formas: $a^2 - x^2, a^2 + x^2$
$x^2 - a^2$, las que tienen fácil solución si se hace la sustitución trigonométrica adecuada. A saber, si la expresión es: $a^2 - x^2$, la sustitución adecuada es:
$x = a\,\mathrm{sen}\,\theta$ ó $x = a\cos\theta$. Si la expresión es: $a^2 + x^2$, entonces: $x = a\sec\theta$

EJERCICIOS DESARROLLADOS

1. Encontrar: $\int \dfrac{dx}{\sqrt{(4-x^2)^3}}$

Solución.- Dada le expresión: $4 - x^2$, la forma es: $a^2 - x^2$, la sustitución adecuada es: $x = a\,\mathrm{sen}\,\theta$ o sea: $x = 2\,\mathrm{sen}\,\theta \therefore dx = 2\cos\theta d\theta$. Además: $\mathrm{sen}\,\theta = \dfrac{x}{a}$. Una figura auxiliar adecuada para ésta situación, es:



$$\int \frac{dx}{\sqrt{(4-x^2)^3}} = \int \frac{dx}{\sqrt{(2^2-x^2)^3}} = \int \frac{2\cos\theta d\theta}{\sqrt{(2^2 - 2^2\,\mathrm{sen}^2\,\theta)^3}} = \int \frac{2\cos\theta d\theta}{\sqrt{\left[(2^2(1-\mathrm{sen}^2\,\theta)\right]^3}}$$

$$= \int \frac{2\cos\theta d\theta}{\sqrt{(2^2\cos^2\theta)^3}} = \int \frac{2\cos\theta d\theta}{(2\cos\theta)^3} = \int \frac{2\cos\theta d\theta}{2^3\cos^3\theta} = \frac{1}{2^2}\int \frac{d\theta}{\cos^2\theta} = \frac{1}{4}\int \sec^2\theta d\theta$$

$= \dfrac{1}{4}\int \sec^2\theta d\theta = \dfrac{1}{4}\tau g\theta + c$. A partir de la figura triangular se tiene:

$\tau g\theta = \dfrac{x}{\sqrt{4-x^2}}$, Luego: $\dfrac{1}{4}\tau g\theta + c = \dfrac{1}{4}\dfrac{x}{\sqrt{4-x^2}} + c$

Respuesta: $\int \dfrac{dx}{\sqrt{(4-x^2)^3}} = \dfrac{1}{4}\dfrac{x}{\sqrt{4-x^2}} + c$

6.2.-Encontrar: $\int \dfrac{\sqrt{25-x^2}}{x}dx$

Solución.-

$$\int\frac{\sqrt{25-x^2}}{x}dx=\int\frac{\sqrt{5^2-x^2}}{x}dx$$, la forma es: a^2-x^2 , luego:

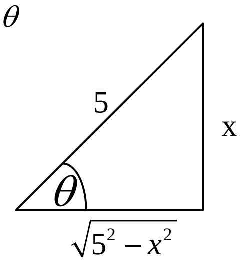
Sea: $x=5\operatorname{sen}\theta \therefore dx=5\cos\theta d\theta$, $\sqrt{5^2-x^2}=5\cos\theta$

Además: $\operatorname{sen}\theta=\dfrac{x}{5}$

$$\int\frac{\sqrt{5^2-x^2}}{x}dx=\int\frac{\cancel{5}\cos\theta 5\cos\theta d\theta}{\cancel{5}\operatorname{sen}\theta}=5\int\frac{\cos^2\theta d\theta}{\operatorname{sen}\theta}=5\int\frac{(1-\operatorname{sen}^2\theta)d\theta}{\operatorname{sen}\theta}$$

$$=5\int\frac{d\theta}{\operatorname{sen}\theta}-5\int\operatorname{sen}\theta d\theta=5\int\operatorname{cosec}\theta-5\int\operatorname{sen}\theta d\theta$$

$$=5\ell\eta\left|\operatorname{cosec}\theta-\operatorname{co}\tau g\theta\right|+5\cos\theta+c\ .$$



De la figura se tiene:

$\operatorname{cosec}\theta=\dfrac{5}{x},\operatorname{co}\tau g\theta=\dfrac{\sqrt{25-x^2}}{x}$, luego:

$$=5\ell\eta\left|\frac{5}{x}-\frac{\sqrt{25-x^2}}{x}\right|+\cancel{5}\frac{\sqrt{25-x^2}}{\cancel{5}}+c=5\ell\eta\left|\frac{5-\sqrt{25-x^2}}{x}\right|+\sqrt{25-x^2}+c$$

Respuesta: $\displaystyle\int\frac{\sqrt{25-x^2}}{x}dx=5\ell\eta\left|\frac{5-\sqrt{25-x^2}}{x}\right|+\sqrt{25-x^2}+c$

6.3.-Encontrar: $\displaystyle\int\frac{dx}{\sqrt{(4x-x^2)^3}}$

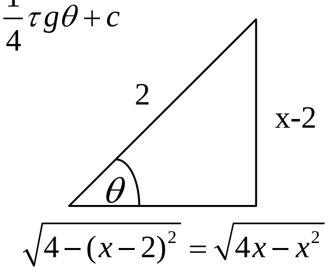
Solución.- $4x-x^2=-(x^2-4x)=-(x^2-4x+4-4)=4-(x^2-4x+4)=2^2-(x-2)^2$

$\displaystyle\int\frac{dx}{\sqrt{(4x-x^2)^3}}=\int\frac{dx}{(\sqrt{2^2-(x-2)^2})^3}$, la forma es: a^2-u^2 ,

Luego: $x-2=2\operatorname{sen}\theta \therefore dx=2\cos\theta d\theta$, $\sqrt{2^2-(x-2)^2}=2\cos\theta$

Además: $\operatorname{sen}\theta=\dfrac{x-2}{2}$

$$\int\frac{dx}{(\sqrt{2^2-(x-2)^2})^3}=\int\frac{2\cos\theta d\theta}{2^3\cos^3\theta}=\frac{1}{4}\int\frac{d\theta}{\cos^2\theta}=\frac{1}{4}\int\sec^2\theta d\theta=\frac{1}{4}\tau g\theta+c$$



De la figura se tiene:

Pero: $\tau g\theta=\dfrac{x-2}{\sqrt{4x-x^2}}$, luego: $\dfrac{1}{4}\tau g\theta+c=\dfrac{x-2}{4\sqrt{4x-x^2}}+c$

Respuesta: $\displaystyle\int\frac{dx}{\sqrt{(4x-x^2)^3}}=\frac{x-2}{4\sqrt{4x-x^2}}+c$

6.4.-Encontrar: $\int \frac{x^2 dx}{(a^2 - x^2)^{3/2}}$

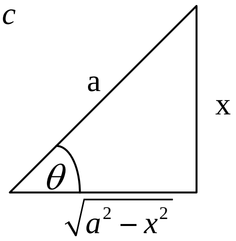
Solución.-

$\int \frac{x^2 dx}{(a^2 - x^2)^{3/2}} = \int \frac{x^2 dx}{(\sqrt{a^2 - x^2})^3}$, la forma es: $a^2 - x^2$

Luego: $x = a\,\mathrm{sen}\,\theta, dx = a\cos\theta, \sqrt{a^2 - x^2} = a\cos\theta$, además: $\mathrm{sen}\,\theta = \frac{x}{a}$

$$\int \frac{x^2 dx}{(\sqrt{a^2 - x^2})^3} = \int \frac{a^2\,\mathrm{sen}^2\,\theta a\cos\theta d\theta}{(a\cos\theta)^3} = \int \frac{\cancel{a^3}\,\mathrm{sen}^2\,\theta \cancel{\cos\theta} d\theta}{\cancel{a^3}\,\cancel{\cos\theta}\cos^2\theta} = \int \frac{\mathrm{sen}^2\,\theta d\theta}{\cos^2\theta}$$

$$= \int \frac{(1-\cos^2\theta)d\theta}{\cos^2\theta} = \int \frac{d\theta}{\cos^2\theta} - \int d\theta = \int \sec^2\theta d\theta - \int d\theta = \tau g\theta - \theta + c$$



De la figura se tiene:

Pero: $\tau g\theta = \frac{x}{\sqrt{a^2 - x^2}}$, además: $\mathrm{sen}\,\theta = \frac{x}{a}$ y $\theta = \mathrm{arcsen}\frac{x}{a}$

Luego: $\tau g\theta - \theta + c = \frac{x}{\sqrt{a^2 - x^2}} - \mathrm{arcsen}\frac{x}{a} + c$

Respuesta: $\int \frac{x^2 dx}{\sqrt{(a^2 - x^2)^3}} = \frac{x}{\sqrt{a^2 - x^2}} - \mathrm{arcsen}\frac{x}{a} + c$

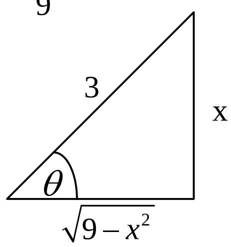
6.5.-Encontrar: $\int \frac{dx}{x^2\sqrt{9 - x^2}}$

Solución.-

$\int \frac{dx}{x^2\sqrt{9 - x^2}} = \int \frac{dx}{x^2\sqrt{3^2 - x^2}}$, la forma es: $a^2 - x^2$

Luego: $x = 3\,\mathrm{sen}\,\theta, dx = 3\cos\theta d\theta, \sqrt{3^2 - x^2} = 3\cos\theta$, además: $\mathrm{sen}\,\theta = \frac{x}{3}$

$$\int \frac{dx}{x^2\sqrt{3^2 - x^2}} = \int \frac{\cancel{3\cos\theta} d\theta}{3^2\,\mathrm{sen}^2\,\theta \cancel{3\cos\theta}} = \frac{1}{9}\int \frac{d\theta}{\mathrm{sen}^2\,\theta} = \frac{1}{9}\int \mathrm{cosec}^2\theta d\theta = -\frac{1}{9}\mathrm{co}\tau g\theta + c$$



De la figura se tiene:

Pero: $\text{co}\tau g\theta = \dfrac{\sqrt{9-x^2}}{x}$, luego: $\dfrac{1}{9}\text{co}\tau g\theta + c = -\dfrac{\sqrt{9-x^2}}{9x} + c$

Respuesta: $\displaystyle\int \frac{dx}{x^2\sqrt{9-x^2}} = -\frac{\sqrt{9-x^2}}{9x} + c$

6.6.-Encontrar: $\displaystyle\int \frac{x^2dx}{\sqrt{9-x^2}}$

Solución.-

$\displaystyle\int \frac{x^2dx}{\sqrt{9-x^2}} = \int \frac{x^2dx}{\sqrt{3^2-x^2}}$, la forma es: $a^2 - x^2$

Luego: $x = 3\,\text{sen}\,\theta, dx = 3\cos\theta d\theta, \sqrt{3^2-x^2} = 3\cos\theta$, además: $\text{sen}\,\theta = \dfrac{x}{3}$

Usaremos la misma figura anterior, luego:

$$\int \frac{x^2dx}{\sqrt{3^2-x^2}} = \int \frac{3^2\,\text{sen}^2\,\theta\,\cancel{3\cos\theta}\,d\theta}{\cancel{3\cos\theta}} = 9\int \text{sen}^2\,\theta d\theta = 9\int \frac{(1-\cos 2\theta)d\theta}{2}$$

$$\frac{9}{2}\int \theta - \frac{9}{2}\int \cos 2\theta d\theta = \frac{9}{2}\theta - \frac{9}{4}\text{sen}\,2\theta + c = \frac{9}{2}\theta - \frac{9}{4}2\,\text{sen}\,\theta\cos\theta + c$$

$= \dfrac{9}{2}\theta - \dfrac{9}{2}\text{sen}\,\theta\cos\theta + c$, de la figura se tiene que: $\text{sen}\,\theta = \dfrac{x}{3}, \cos\theta = \dfrac{\sqrt{9-x^2}}{3}$ y $\theta = \text{arcsen}\,\dfrac{x}{3}$, luego es equivalente:

$$= \frac{9}{2}\text{arcsen}\,\frac{x}{3} - \frac{9}{4}\frac{x}{3}\frac{\sqrt{9-x^2}}{3} + c = \frac{9}{2}\left(\text{arcsen}\,\frac{x}{3} - \frac{\sqrt{9-x^2}}{9}\right) + c$$

Respuesta: $\displaystyle\int \frac{x^2dx}{\sqrt{9-x^2}} = \frac{9}{2}\left(\text{arcsen}\,\frac{x}{3} - \frac{\sqrt{9-x^2}}{9}\right) + c$

6.7.-Encontrar: $\displaystyle\int \sqrt{x^2-4}dx$

Solución.-

$\displaystyle\int \sqrt{x^2-4}dx = \int \sqrt{x^2-2^2}dx$, la forma es: $x^2 - a^2$

Luego: $x = 2\sec\theta, dx = 2\sec\theta\tau g\theta d\theta, \sqrt{x^2-2^2} = 2\tau g\theta$, además: $\sec\theta = \dfrac{x}{2}$

$$\int \sqrt{x^2-2^2}dx = \int 2\tau g\theta 2\sec\theta\tau g\theta d\theta = 4\int \sec\theta\tau g^2\theta d\theta = 4\int \sec\theta(\sec^2\theta - 1)d\theta$$

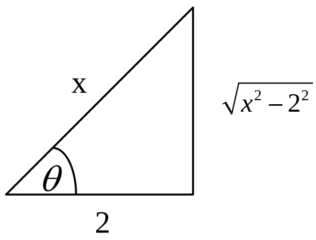
$$= 4\int \sec^3\theta d\theta - 4\int \sec\theta d\theta$$

Se sabe que: $\displaystyle\int \sec^3\theta d\theta = \frac{\sec\theta\tau g\theta}{2} + \frac{1}{2}\ell\eta\left|\sec\theta + \tau g\theta\right| + c$, luego lo anterior es equivalente a:

$$= 4\left(\frac{1}{2}\sec\theta\tau g\theta + \frac{1}{2}\ell\eta\left|\sec\theta + \tau g\theta\right|\right) - 4\ell\eta\left|\sec\theta + \tau g\theta\right| + c$$

$$= 2\sec\theta\tau g\theta + 2\ell\eta\left|\sec\theta + \tau g\theta\right| - 4\ell\eta\left|\sec\theta + \tau g\theta\right| + c$$

$$= 2\sec\theta\tau g\theta - 2\ell\eta\left|\sec\theta + \tau g\theta\right| + c$$



De la figura se tiene:

$\sec\theta = \dfrac{x}{2}, \tau g\theta = \dfrac{\sqrt{x^2-4}}{2}$, luego:

$$= \cancel{2}\,\frac{x}{\cancel{2}}\,\frac{\sqrt{x^2-4}}{2} - 2\ell\eta\left|\frac{x}{2} + \frac{\sqrt{x^2-4}}{2}\right| + c = \frac{x\sqrt{x^2-4}}{2} - 2\ell\eta\left|\frac{x+\sqrt{x^2-4}}{2}\right| + c$$

$$= \frac{x\sqrt{x^2-4}}{2} - 2\ell\eta\left|x+\sqrt{x^2-4}\right| - 2\ell\eta 2 + c$$

Respuesta: $\int\sqrt{x^2-4}\,dx = \dfrac{x\sqrt{x^2-4}}{2} - 2\ell\eta\left|x+\sqrt{x^2-4}\right| + c$

6.8.-Encontrar: $\int\dfrac{x^2dx}{\sqrt{x^2-16}}$

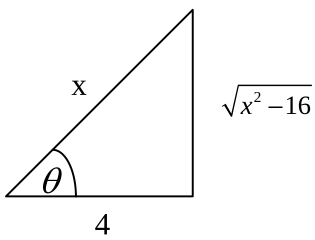
Solución.-

$\int\dfrac{x^2dx}{\sqrt{x^2-16}} = \int\dfrac{x^2dx}{\sqrt{x^2-4^2}}$, la forma es: x^2-a^2

Luego: $x = 4\sec t, dx = 4\sec t\tau gt dt, \sqrt{x^2-4^2} = 4\tau gt$, además: $\sec t = \dfrac{x}{4}$

$$\int\frac{x^2dx}{\sqrt{x^2-4^2}} = \int\frac{4^2\sec^2 t(\cancel{4}\sec t\,\cancel{\tau g t}\,dt)}{\cancel{4}\,\cancel{\tau g t}} = 16\int\sec^3 tdt$$

$$= 16\left(\frac{1}{2}\sec t\tau gt + \frac{1}{2}\ell\eta\left|\sec t + \tau gt\right| + c\right) = 8\sec t\tau gt + 8\ell\eta\left|\sec t + \tau gt\right| + c$$



De la figura se tiene:

$\sec t = \dfrac{x}{4}, \tau gt = \dfrac{\sqrt{x^2-16}}{4}$, luego equivale a:

$$= 8\frac{x}{4}\frac{\sqrt{x^2-16}}{4} + 8\ell\eta\left|\frac{x}{4} + \frac{\sqrt{x^2-16}}{4}\right| + c = \frac{x}{2}\sqrt{x^2-16} + 8\ell\eta\left|\frac{x\sqrt{x^2-16}}{4}\right| + c$$

$$= \frac{x}{2}\sqrt{x^2-16} + 8\ell\eta\left|x\sqrt{x^2-16}\right| - 8\ell\eta 4 + c = \frac{x}{2}\sqrt{x^2-16} + 8\ell\eta\left|x\sqrt{x^2-16}\right| + c$$

Respuesta: $\int \frac{x^2dx}{\sqrt{x^2-16}} = \frac{x}{2}\sqrt{x^2-16} + 8\ell\eta\left|x\sqrt{x^2-16}\right| + c$

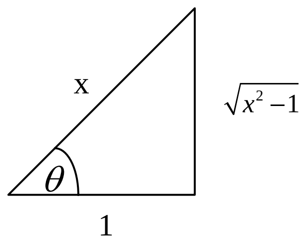
6.9.-Encontrar: $\int \frac{dx}{x\sqrt{x^2-1}}$

Solución.-

$\int \frac{dx}{x\sqrt{x^2-1}} = \int \frac{dx}{x\sqrt{x^2-1^2}}$, la forma es: x^2-a^2

Luego: $x = \sec t, dx = \sec t\tau gtdt, \sqrt{x^2-1^2} = \tau gt$, además:

$\int \frac{dx}{x\sqrt{x^2-1}} = \int \frac{\cancel{\sec t\tau gt}\,dt}{\cancel{\sec t\tau gt}} = \int dt = t + c$,



De la figura se tiene:
Dado que: $\sec t = x \Rightarrow t = \operatorname{arcsec} x$, luego:
$t + c = \operatorname{arcsec} x + c$

Respuesta: $\int \frac{dx}{x\sqrt{x^2-1}} = \operatorname{arcsec} x + c$

6.10.-Encontrar: $\int \frac{dx}{(\sqrt{4x^2-24x+27})^3}$

Solución.-

$$\int \frac{dx}{(\sqrt{4x^2-24x+27})^3} = \int \frac{dx}{\sqrt{4(x^2-6x+27/4)^3}} = \int \frac{dx}{\sqrt{4^3}\left(\sqrt{x^2-6x+27/4}\right)^3}$$

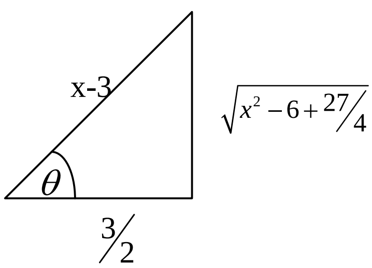
$= \frac{1}{8}\int \frac{dx}{\sqrt{(x^2-6x+27/4)^3}}$, Se tiene:

$$x^2-6x+\frac{27}{4} = (x^2-6x+\_\_) + \frac{27}{4} - \_\_ = (x^2-6x+9) + \frac{27}{4} - 9$$

$= (x^2-6x+9) - \frac{9}{4} = (x^2-6x+27/4) = (x-3)^2 - (3/2)^2$, la expresión anterior equivale a:

$\frac{1}{8}\int \frac{dx}{(\sqrt{x^2-6x+27/4})^3} = \frac{1}{8}\int \frac{dx}{\left[\sqrt{(x-3)^2-(3/2)^2}\right]^3}$, siendo la forma: u^2-a^2, luego:

$x - 3 = 3/2 \sec t, dx = 3/2 \sec t\tau gtdt$, además: $\sec t = \frac{x-3}{3/2}$



De la figura se tiene:

$\sec t = \frac{x}{4}, \tau gt = \frac{\sqrt{x^2-16}}{4}$, luego equivale a:

$$\frac{1}{8}\int \frac{dx}{\left[\sqrt{(x-3)^2-(3/2)^2}\right]^3} = \frac{1}{8}\int \frac{3/2 \sec t \tau gt dt}{(3/2)^2 \tau g^3 t} = \frac{1}{8}\frac{1}{\frac{3^2}{2^2}}\int \frac{\sec t dt}{\tau g^2 t} = \frac{1}{18}\int \frac{\frac{1}{\cos t}}{\frac{\operatorname{sen}^2 t}{\cos^2 t}}$$

$$= \frac{1}{18}\int \frac{\cos t dt}{(\operatorname{sen} t)^2} = \frac{1}{18}\int (\operatorname{sen} t)^{-2}\cos t dt = \frac{1}{18}\frac{(\operatorname{sen} t)^{-1}}{-1} + c = -\frac{1}{18}\frac{1}{(\operatorname{sen} t)} + c$$

$= -\frac{1}{18}\operatorname{cosec} t + c$, como: $\operatorname{cosec} t = \frac{x-3}{\sqrt{x^2-6x+27/4}}$, entonces:

$$= -\frac{1}{18}\frac{x-3}{\sqrt{x^2-6x+27/4}} + c = -\frac{1}{18}\frac{x-3}{\sqrt{\frac{4x^2-24x+27}{4}}} + c = -\frac{1}{18}\frac{x-3}{\frac{\sqrt{4x^2-24x+27}}{2}} + c$$

$$= -\frac{1}{9}\frac{x-3}{\sqrt{4x^2-24x+27}} + c$$

Respuesta: $\int \frac{dx}{(\sqrt{4x^2-24x+27})^3} = -\frac{1}{9}\frac{x-3}{\sqrt{4x^2-24x+27}} + c$

6.11.-Encontrar: $\int \frac{dx}{\sqrt{(16+x^2)^4}}$

Solución.-

$$\int \frac{dx}{\sqrt{(16+x^2)^4}} = \int \frac{dx}{\sqrt{(4^2+x^2)^4}}$$

Luego: $x = 4\tau gt, dx = 4\sec^2 t dt, \sqrt{4^2+x^2} = 4\sec t$, además: $\tau gt = x/4$

$$\int \frac{dx}{\sqrt{(4^2+x^2)^4}} = \int \frac{4\sec^2 t dt}{4^4 \sec^4 t} = \frac{1}{64}\int \frac{dt}{\sec^2 t} = \frac{1}{64}\int \cos^2 t dt = \frac{1}{64}\int \frac{(1+\cos 2t)}{2}dt$$

$$= \frac{1}{128}\int dt + \frac{1}{128}\int \cos 2t dt = \frac{1}{128}t + \frac{1}{256}\operatorname{sen} 2t + c$$

Como: $\tau gt = x/4 \Rightarrow t = \operatorname{arc} \tau g\, x/4$, $\operatorname{sen} 2t = 2\operatorname{sen} t \cos t$; luego:

$\frac{1}{128}t + \frac{1}{256}\operatorname{sen} 2t + c = 2\frac{x}{\sqrt{16+x^2}}\frac{4}{\sqrt{16+x^2}} = \frac{8x}{16+x^2}$, Se tiene:

$$\frac{1}{128}\operatorname{arc}\tau g\, x/4 + \frac{1}{256}\frac{8x}{16+x^2} + c = \frac{1}{128}\operatorname{arc}\tau g\, x/4 + \frac{x}{32(16+x^2)} + c$$

Respuesta: $\int \frac{dx}{\sqrt{(16+x^2)^4}} = \frac{1}{128}\operatorname{arc}\tau g\frac{x}{4} + \frac{x}{32(16+x^2)} + c$

6.12.-Encontrar: $\int \frac{x^2dx}{(x^2+100)^{3/2}}$

Solución.-

$$\int \frac{x^2dx}{(x^2+100)^{3/2}} = \int \frac{x^2dx}{(\sqrt{x^2+10^2})^3},$$

se tiene: $x = 10\tau gt, dt = 10\sec^2 tdt$, $\sqrt{x^2+10^2} = 10\sec t$;además: $\tau gt = \frac{x}{10}$, luego:

$$\int \frac{x^2dx}{(\sqrt{x^2+10^2})^3} = \int \frac{\cancel{10^2}\tau g^2t(\cancel{10}\,\cancel{\sec^2 t})dt}{(\cancel{10^3}\sec^{\cancel{3}} t)} = \int \frac{\tau g^2tdt}{\sec t} = \int \frac{\frac{\operatorname{sen}^2 t}{\cos^{\cancel{2}} t}}{\frac{1}{\cancel{\cos t}}}dt = \int \frac{\operatorname{sen}^2 t}{\cos t}dt$$

$$= \int \frac{(1-\cos^2 t)}{\cos t}dt = \int \frac{dt}{\cos t} - \int \cos tdt = \int \sec tdt - \int \cos tdt = \ell\eta\left|\sec t + \tau gt\right| - \operatorname{sen} t + c$$

Como: $\sec t = \frac{\sqrt{100+x^2}}{10}, \tau gt = \frac{x}{10}$, además: $\operatorname{sen} t = \frac{x}{\sqrt{100+x^2}}$

$$= \ell\eta\left|\frac{\sqrt{100+x^2}}{10} + \frac{x}{10}\right| - \frac{x}{\sqrt{x^2+100}} + c = \ell\eta\left|\frac{\sqrt{100+x^2}+x}{10}\right| - \frac{x}{\sqrt{x^2+100}} + c$$

$$= \ell\eta\left|\sqrt{100+x^2}+x\right| - \frac{x}{\sqrt{x^2+100}} - \ell\eta 10 + c = \ell\eta\left|\sqrt{100+x^2}+x\right| - \frac{x}{\sqrt{x^2+100}} + c$$

Respuesta: $\int \frac{x^2dx}{(x^2+100)^{3/2}} = \ell\eta\left|\sqrt{100+x^2}+x\right| - \frac{x}{\sqrt{x^2+100}} + c$

Nota: En los ejercicios 6.11 y 6.12 se ha omitido la figura (triángulo rectángulo). Conviene hacerla y ubicar los datos pertinentes. En adelante se entenderá que el estudiante agregará este complemento tan importante.

6.13.-Encontrar: $\int \frac{x^2dx}{(x^2+8^2)^{3/2}}$

Solución.-

$$\int \frac{x^2dx}{(x^2+8^2)^{3/2}} = \int \frac{x^2dx}{(\sqrt{x^2+8^2})^3},$$

se tiene: $x = 8\tau gt, dt = 8\sec^2 tdt$, $\sqrt{x^2+8^2} = 8\sec t$ además: $\tau gt = \frac{x}{8}$, luego:

$$\int \frac{x^2dx}{(\sqrt{x^2+8^2})^3} = \int \frac{\cancel{8^2}\tau g^2t(\cancel{8}\,\cancel{\sec^2 t})}{\cancel{8^3}\sec^{\cancel{3}} t}dt = \int \frac{\tau g^2t}{\sec t}dt = \int \sec tdt - \int \cos tdt$$

$= \ell\eta\left|\sec t + \tau g t\right| - \operatorname{sen} t + c$, como: $\sec t = \dfrac{\sqrt{x^2+64}}{8}, \tau g t = \dfrac{x}{8}, \operatorname{sen} t = \dfrac{x}{\sqrt{x^2+64}}$

Se tiene como expresión equivalente:

$$= \ell\eta\left|\frac{\sqrt{x^2+64}}{8} + \frac{x}{8}\right| - \frac{x}{\sqrt{x^2+64}} + c = \ell\eta\left|\frac{\sqrt{x^2+64}+x}{8}\right| - \frac{x}{\sqrt{x^2+64}} + c$$

$$= \ell\eta\left|\sqrt{x^2+64}+x\right| - \frac{x}{\sqrt{x^2+64}} + c$$

Respuesta: $\displaystyle\int \frac{x^2dx}{(x^2+8^2)^{3/2}} = \ell\eta\left|\sqrt{x^2+64}+x\right| - \frac{x}{\sqrt{x^2+64}} + c$

6.14.-Encontrar: $\displaystyle\int \frac{dx}{(\sqrt{3^2+x^2})^4}$

Solución.- se tiene: $x = 3\tau g t, dx = 3\sec^2 t dt$, $\sqrt{3^2+x^2} = 3\sec t$, además: $\tau g t = \dfrac{x}{3}$

$$\int \frac{dx}{(\sqrt{3^2+x^2})^4} = \int \frac{\cancel{3}\,\cancel{\sec^2} t\,dt}{3^{\cancel{4}} + \sec^{\cancel{4}} t} = \frac{1}{3^3}\int \frac{dt}{\sec^2 t} = \frac{1}{27}\int \cos^2 t dt = \frac{1}{54}t + \frac{1}{54}\int \cos 2t dt$$

$$= \frac{1}{54}t + \frac{1}{108}\operatorname{sen} 2t + c_1 = \frac{1}{54}t + \frac{1}{108}2\operatorname{sen} t\cos t + c = \frac{1}{54}t + \frac{1}{54}\operatorname{sen} t\cos t + c$$

Como: $\underline{\tau g t = \dfrac{x}{3} \Rightarrow t = \operatorname{arc}\tau g\dfrac{x}{3}}$, además: $\operatorname{sen} t = \dfrac{x}{\sqrt{9+x^2}}$, $\cos t = \dfrac{3}{\sqrt{9+x^2}}$

$$= \frac{1}{54}\operatorname{arc}\tau g\frac{x}{3} + \frac{1}{54}\frac{x}{\sqrt{9+x^2}}\frac{3}{\sqrt{9+x^2}} + c = \frac{1}{54}\operatorname{arc}\tau g\frac{x}{3} + \frac{x}{18(9+x^2)} + c$$

Respuesta: $\displaystyle\int \frac{dx}{(\sqrt{3^2+x^2})^4} = \frac{1}{54}\operatorname{arc}\tau g\frac{x}{3} + \frac{x}{18(9+x^2)} + c$

6.15.-Encontrar: $\displaystyle\int \frac{dx}{\sqrt{x^2-4x+13}}$

Solución.- Completando cuadrados se tiene:

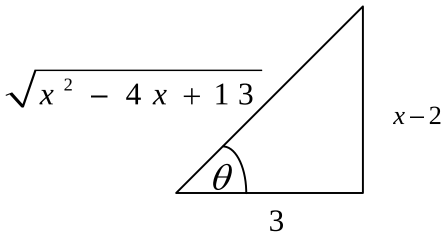
$x^2-4x+13 = (x^2-4x+\_\_)+13-\_\_ = (x^2-4x+4)+13-4 = (x-2)^2+3^2$

Se tiene: $x-2 = 3\tau g t, dx = 3\sec^2 t dt$, $\sqrt{3^2+x^2} = 3\sec t$

$\sqrt{(x-2)^2+3^2} = \sqrt{x^2-4x+13} = 3\sec t$,

Sea: $x-2 = 3\tau g t, dx = 3\sec^2 t dt$;además: $\tau g t = \dfrac{x-2}{3}$, luego:

$$\int \frac{dx}{\sqrt{(x-2)^2+3^2}} = \int \frac{\cancel{3}\sec^{\cancel{2}} t dt}{\cancel{3\sec} t} = \int \sec t dt = \ell\eta\left|\sec t + \tau g t\right| + c$$



De la figura se tiene:

$\sec t = \dfrac{\sqrt{x^2-4x+13}}{3}$**,** $\tau gt = \dfrac{x-2}{3}$, luego:

$$= \ell\eta\left|\frac{\sqrt{x^2-4x+13}}{3}+\frac{x-2}{3}\right|+c = \ell\eta\left|\frac{\sqrt{x^2-4x+13}+(x-2)}{3}\right|+c$$

$$= \ell\eta\left|\sqrt{x^2-4x+13}+(x-2)\right|+c$$

Respuesta: $\displaystyle\int \frac{dx}{\sqrt{x^2-4x+13}} = \ell\eta\left|\sqrt{x^2-4x+13}+(x-2)\right|+c$

6.16.-Encontrar: $\displaystyle\int \sqrt{1+4x^2}\,dx$

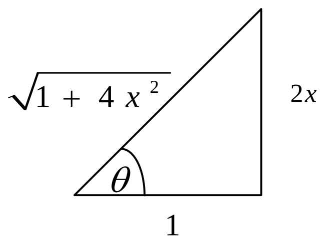
Solución.-

$$\int \sqrt{1+4x^2}\,dx = \int \sqrt{1^2+(2x)^2}\,dx$$

Se tiene: $2x = \tau gt, 2dx = \sec^2 t dt \Rightarrow dx = \dfrac{1}{2}\sec^2 t dt$, Además: $\tau gt = \dfrac{2x}{1}$

$$\int \sqrt{1^2+(2x)^2}\,dx = \int \sqrt{1^2+\tau g^2 t}\,\frac{1}{2}\sec^2 dt = \frac{1}{2}\int \sec t \sec^2 t dt = \frac{1}{2}\int \sec^3 t dt$$

$$= \frac{1}{4}\sec t \tau gt + \frac{1}{4}\ell\eta\left|\sec t \tau gt\right| + c \,,$$



De la figura se tiene:

$\sec t = \dfrac{\sqrt{1+4x^2}}{1}$, $\tau gt = 2x$

$$= \frac{1}{4}\sqrt{1+4x^2}\,2x + \frac{1}{4}\ell\eta\left|\sqrt{1+4x^2}\right| + 2x + c$$

Respuesta: $\displaystyle\int \sqrt{1+4x^2}\,dx = \frac{1}{4}\sqrt{1+4x^2}\,2x + \frac{1}{4}\ell\eta\left|\sqrt{1+4x^2}\right| + 2x + c$

EJERCICIOS PROPUESTOS:

Utilizando esencialmente la técnica de sustitución por variables trigonométricas, encontrar las integrales siguientes:

6.17.- $\displaystyle\int \sqrt{4-x^2}$ **6.18.-** $\displaystyle\int \frac{dx}{\sqrt{a^2-x^2}}$ **6.19.-** $\displaystyle\int \frac{dx}{x^2+a^2}$

6.20.- $\int \frac{dx}{x^2-a^2}$ **6.21.-** $\int \frac{dx}{\sqrt{x^2+a^2}}$ **6.22.-** $\int \frac{dx}{\sqrt{x^2-a^2}}$

6.23.- $\int \frac{dx}{x\sqrt{x^2-9}}$ **6.24.-** $\int \frac{dx}{x\sqrt{x^2-2}}$ **6.25.-** $\int \frac{dx}{x\sqrt{1+x^2}}$

6.26.- $\int \frac{x^2dx}{\sqrt{1-x^2}}$ **6.27.-** $\int \frac{x^3dx}{\sqrt{2-x^2}}$ **6.28.-** $\int \frac{\sqrt{x^2-9}}{x}dx$

6.29.- $\int \frac{dx}{x\sqrt{4x^2-16}}$ **6.30.-** $\int \frac{\sqrt{x^2+1}}{x}dx$ **6.31.-** $\int \frac{dx}{x^2\sqrt{4-x^2}}$

6.32.- $\int \sqrt{a-x^2}\,dx$ **6.33.-** $\int \sqrt{a^2-x^2}\,dx$ **6.34.-** $\int \frac{x^2dx}{\sqrt{x^2+a^2}}$

6.35.- $\int \frac{dx}{x^2\sqrt{x^2+9}}$ **6.36.-** $\int \frac{dx}{\sqrt{5-4x^2}}$ **6.37.-** $\int \frac{x^2dx}{(4-x^2)^{3/2}}$

6.38.- $\int x^2\sqrt{5-x^2}\,dx$ **6.39.-** $\int \frac{dx}{x^4\sqrt{x^2+3}}$ **6.40.-** $\int x^3\sqrt{a^2x^2+b^2}\,dx$

6.41.- $\int \frac{dx}{x^2\sqrt{x^2+a^2}}$ **6.42.-** $\int \frac{dx}{(x^2+a^2)^2}$ **6.43.-** $\int x^3\sqrt{a^2x^2-b^2}\,dx$

6.44.- $\int \frac{dx}{x^2\sqrt{a^2-x^2}}$ **6.45.-** $\int \frac{\sqrt{2x^2-5}}{x}dx$ **6.46.-** $\int \frac{x^3dx}{\sqrt{3x^2-5}}$

6.47.- $\int \frac{\sqrt{x^2-100}}{x}dx$ **6.48.-** $\int \frac{dx}{x^2\sqrt{x^2-2}}$ **6.49.-** $\int \frac{dx}{x\sqrt{9-x^2}}$

6.50.- $\int \frac{\sqrt{x^2+a^2}}{x}dx$ **6.51.-** $\int \frac{xdx}{\sqrt{a^2-x^2}}$ **6.52.-** $\int \frac{dx}{\sqrt{1-4x^2}}$

6.53.- $\int \frac{dx}{\sqrt{4+x^2}}$ **6.54.-** $\int \frac{xdx}{\sqrt{4+x^2}}$ **6.55.-** $\int \frac{dx}{x\sqrt{a^2+x^2}}$

6.56.- $\int \frac{(x+1)dx}{\sqrt{4-x^2}}$ **6.57.-** $\int \frac{dx}{\sqrt{2-5x^2}}$ **6.58.-** $\int \frac{dx}{(a^2-x^2)^{3/2}}$

6.59.- $\int \frac{dx}{\sqrt{4-(x-1)^2}}$ **6.60.-** $\int \frac{x^2dx}{\sqrt{2x-x^2}}$ **6.61.-** $\int \frac{x^2dx}{\sqrt{17-x^2}}$

6.62.- $\int \frac{x^2dx}{\sqrt{21+4x-x^2}}$ **6.63.-** $\int \frac{dx}{(x^2-2x+5)^{3/2}}$ **6.64.-** $\int \frac{(2x+1)dx}{\sqrt{(4x^2-2x+1)^3}}$

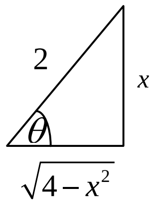
6.65.- $\int \frac{dx}{(x-1)\sqrt{x^2-3x+2}}$ **6.66.-** $\int \frac{xdx}{\sqrt{x^2-2x+5}}$ **6.67.-** $\int \frac{(x+1)dx}{\sqrt{2x-x^2}}$

6.68.- $\int \frac{(x-1)dx}{\sqrt{x^2-4x+3}}$ **6.69.-** $\int \frac{dx}{\sqrt{x^2-2x-8}}$ **6.70.-** $\int \frac{xdx}{\sqrt{x^2+4x+5}}$

RESPUESTAS

6.17.- $\int \sqrt{4-x^2}$

Solución.-



Se tiene: $x = 2\,\mathrm{s}e\mathrm{n}\,\theta, dx = 2\cos\theta d\theta$, $\sqrt{4+x^2} = 2\cos\theta$

$$\int \sqrt{4-x^2} = \int 2\cos\theta 2\cos\theta d\theta = 4\int \cos^2\theta d\theta = 2\theta + \mathrm{s}e\mathrm{n}\,2\theta + c = 2\theta + 2\,\mathrm{s}e\mathrm{n}\,\theta\cos\theta + c$$

$$= 2\,\mathrm{arcs}e\mathrm{n}\frac{x}{2} + \frac{x\sqrt{4-x^2}}{2} + c$$

6.18.- $\int \frac{dx}{\sqrt{a^2-x^2}}$

Solución.- se tiene: $x = a\,\mathrm{s}e\mathrm{n}\,\theta, dx = a\cos\theta d\theta$, $\sqrt{a^2-x^2} = a\cos\theta$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \int \frac{\cancel{a\cos\theta}\,d\theta}{\cancel{a\cos\theta}} = \int d\theta = \theta + c = \mathrm{arcs}e\mathrm{n}\frac{x}{a} + c$$

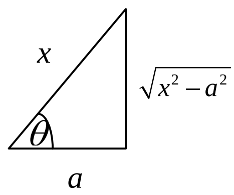
6.19.- $\int \frac{dx}{x^2+a^2}$

Solución.- se tiene: $x = a\tau g\theta, dx = a\sec^2\theta d\theta$, $\sqrt{x^2+a^2} = a\sec\theta$

$$\int \frac{dx}{x^2+a^2} = \int \frac{dx}{(\sqrt{x^2+a^2})^2} = \int \frac{\cancel{a}\,\cancel{\sec^2\theta}\,d\theta}{a^{\cancel{2}}\,\cancel{\sec^2\theta}} = \frac{1}{a}\int d\theta = \frac{1}{a}\theta + c = \frac{1}{a}\mathrm{arc}\,\tau g\,\frac{x}{a} + c$$

6.20.- $\int \frac{dx}{x^2-a^2}$

Solución.-



Se tiene: $x = a\sec\theta, dx = a\sec\theta\tau g\theta d\theta$, $\sqrt{x^2-a^2} = a\tau g\theta$

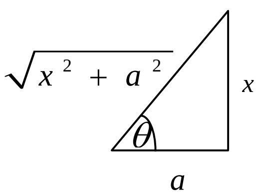
$$\int \frac{dx}{x^2-a^2} = \int \frac{dx}{(\sqrt{x^2-a^2})^2} = \int \frac{\cancel{a}\sec\theta\,\cancel{\tau g\theta}\,d\theta}{a^{\cancel{2}}\tau g^{\cancel{2}}\theta} = \frac{1}{a}\int \frac{\sec\theta d\theta}{\tau g\theta} = \frac{1}{a}\int \cos ec\theta d\theta$$

$$= \frac{1}{a}\ell\eta\left|\cos ec\theta - \mathrm{co}\,\tau g\theta\right| = \frac{1}{a}\ell\eta\left|\frac{x}{\sqrt{x^2-a^2}} - \frac{a}{\sqrt{x^2-a^2}}\right| + c$$

$$= \frac{1}{a}\ell\eta\left|\frac{x-a}{\sqrt{x^2-a^2}}\right| + c = \frac{1}{a}\ell\eta\left|\sqrt{\frac{(x-a)^2}{x^2-a^2}}\right| + c = \frac{1}{2a}\ell\eta\left|\frac{x-a}{x+a}\right| + c$$

6.21.- $\int \frac{dx}{\sqrt{x^2+a^2}}$

Solución.-



Se tiene: $x = a\tau g\theta, dx = a\sec^2\theta d\theta$, $\sqrt{x^2+a^2} = a\sec\theta$

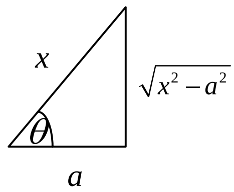
$$\int\frac{dx}{\sqrt{x^2+a^2}} = \int\frac{\cancel{a}\sec^{\cancel{2}}\theta d\theta}{\cancel{a\sec\theta}} = \int\sec\theta d\theta = \ell\eta\left|\sec\theta+\tau g\theta\right|+c$$

$$= \ell\eta\left|\frac{\sqrt{x^2+a^2}}{a}+\frac{x}{a}\right|+c = \ell\eta\left|\frac{\sqrt{x^2+a^2}+x}{a}\right|+c = \ell\eta\left|x+\sqrt{x^2+a^2}\right|-\ell\eta a+c$$

$$= \ell\eta\left|x+\sqrt{x^2+a^2}\right|+c$$

6.22.- $\int\frac{dx}{\sqrt{x^2-a^2}}$

Solución.-



Se tiene: $x = a\sec\theta, dx = a\sec\theta\tau g\theta d\theta$, $\sqrt{x^2+a^2} = a\tau g\theta$

$$\int\frac{dx}{\sqrt{x^2-a^2}} = \int\frac{\cancel{a}\sec\theta\,\cancel{\tau g\theta}d\theta}{\cancel{a\tau g\theta}} = \int\sec\theta d\theta = \ell\eta\left|\sec\theta+\tau g\theta\right|+c$$

$$= \ell\eta\left|\frac{x}{a}+\frac{\sqrt{x^2-a^2}}{a}\right|+c = \ell\eta\left|\frac{x+\sqrt{x^2-a^2}}{a}\right|+c = \ell\eta\left|x+\sqrt{x^2-a^2}\right|+c$$

6.23.- $\int\frac{dx}{x\sqrt{x^2-9}}$

Solución.-

Se tiene: $x = 3\sec\theta, dx = 3\sec\theta\tau g\theta d\theta$, $\sqrt{x^2-9} = 3\tau g\theta$

$$\int\frac{dx}{x\sqrt{x^2-9}} = \int\frac{\cancel{3\sec\theta}\,\cancel{\tau g\theta}d\theta}{\cancel{3\sec\theta}3\cancel{\tau g\theta}} = \frac{1}{3}\int d\theta = \frac{1}{3}\theta+c = \frac{\operatorname{arc}\sec x/3}{3}+c$$

6.24.- $\int\frac{dx}{x\sqrt{x^2-2}}$

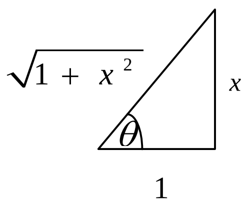
Solución.-

Se tiene: $x = \sqrt{2}\sec\theta, dx = \sqrt{2}\sec\theta\tau g\theta d\theta$, $\sqrt{x^2-2} = \sqrt{2}\tau g\theta$

$$\int\frac{dx}{x\sqrt{x^2-2}} = \int\frac{\cancel{\sqrt{2}\sec\theta}\,\cancel{\tau g\theta}d\theta}{\cancel{\sqrt{2}\sec\theta}\sqrt{2}\,\cancel{\tau g\theta}} = \frac{\sqrt{2}}{2}\int d\theta = \frac{\sqrt{2}}{2}\theta+c = \frac{\sqrt{2}}{2}\operatorname{arc}\sec\frac{\sqrt{2}}{2}x+c$$

6.25.- $\int\frac{dx}{x\sqrt{1+x^2}}$

Solución.-



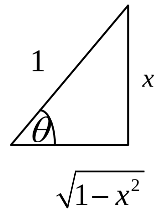
Se tiene: $x=\tau g\theta, dx=\sec^2\theta d\theta$, $\sqrt{1+x^2}=\sec\theta$

$$\int\frac{dx}{x\sqrt{1+x^2}}=\int\frac{\sec^{\cancel{2}}\theta d\theta}{\tau g\theta\cancel{\sec\theta}}=\int\frac{d\theta}{\operatorname{sen}\theta}=\int\cos ec\theta d\theta=\ell\eta\left|\cos ec\theta-\operatorname{co}\tau g\theta\right|+c$$

$$=\ell\eta\left|\frac{\sqrt{1+x^2}}{x}-\frac{1}{x}\right|+c=\ell\eta\left|\frac{\sqrt{1+x^2}-1}{x}\right|+c$$

6.26.- $\int\frac{x^2dx}{\sqrt{1-x^2}}$

Solución.-



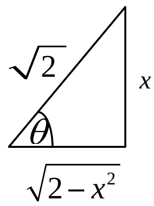
Se tiene: $x=\operatorname{sen}\theta, dx=\cos\theta d\theta$, $\sqrt{1-x^2}=\cos\theta$

$$\int\frac{x^2dx}{\sqrt{1-x^2}}=\int\frac{\operatorname{sen}^2\theta\cancel{\cos\theta}d\theta}{\cancel{\cos\theta}}=\int\operatorname{sen}^2\theta d\theta=\frac{1}{2}\theta-\frac{1}{4}\operatorname{sen}2\theta+c$$

$$=\frac{1}{2}\theta-\frac{1}{2}\operatorname{sen}\theta\cos\theta+c=\frac{\operatorname{arcsen}x}{2}-\frac{x}{2}\sqrt{1-x^2}+c$$

6.27.- $\int\frac{x^3dx}{\sqrt{2-x^2}}$

Solución.-



Se tiene: $x=\sqrt{2}\operatorname{sen}\theta, dx=\sqrt{2}\cos\theta d\theta$, $\sqrt{2-x^2}=\sqrt{2}\cos\theta$

$$\int\frac{x^3dx}{\sqrt{2-x^2}}=\int\frac{2\sqrt{2}\operatorname{sen}^3\theta\cancel{\sqrt{2}\cos\theta}d\theta}{\cancel{\sqrt{2}\cos\theta}}=2\sqrt{2}\int\operatorname{sen}^3\theta d\theta=2\sqrt{2}(-\cos\theta+\frac{\cos^3\theta}{3})+c$$

$$=2\sqrt{2}(-\frac{\sqrt{2-x^2}}{\sqrt{2}}+\frac{(\sqrt{2-x^2})^3}{3(\sqrt{2})^3})+c=-\sqrt{2(2-x^2)}+\frac{(2-x^2)\sqrt{2-x^2}}{3}+c$$

6.28.- $\int\frac{\sqrt{x^2-9}}{x}dx$

Solución.-

Se tiene: $x=3\sec\theta, dx=3\sec\theta\tau g\theta d\theta$, $\sqrt{x^2-9}=3\tau g\theta$

$$\int\frac{\sqrt{x^2-9}}{x}dx=\int\frac{3\tau g\theta\cancel{3\sec\theta}\tau g\theta d\theta}{\cancel{3\sec\theta}}=3\int\tau g^2\theta d\theta=3\int(\sec^2\theta-1)d\theta$$

$$=3\int\sec^2\theta d\theta-3\int d\theta=3\tau g\theta-3\theta+c=\sqrt{x^2-9}-3\operatorname{arcsec}\frac{x}{3}+c$$

6.29.- $\int \frac{dx}{x\sqrt{4x^2-16}}$

Solución.-

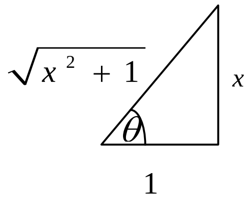
Se tiene: $\frac{x}{2}=\sec\theta, dx=2\sec\theta\tau g\theta d\theta$, $\sqrt{\frac{x^2}{4}-1}=\tau g\theta$

$$\int \frac{dx}{x\sqrt{4x^2-16}}=\frac{1}{4}\int \frac{dx}{x\sqrt{(x/2)^2-1}}=\frac{1}{4}\int \frac{\cancel{2\sec\theta\tau g\theta}d\theta}{\cancel{2\sec\theta\tau g\theta}}=\frac{1}{4}\int d\theta=\frac{1}{4}\theta+c$$

$$=\frac{1}{4}\operatorname{arcsec}\frac{x}{2}+c$$

6.30.- $\int \frac{\sqrt{x^2+1}}{x}dx$

Solución.-



Se tiene: $x=\tau g\theta, dx=\sec^2\theta d\theta$, $\sqrt{x^2+1}=\sec\theta$

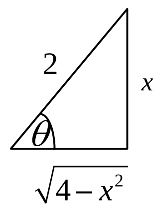
$$\int \frac{\sqrt{x^2+1}}{x}dx=\int \frac{\sec\theta\sec^2\theta d\theta}{\tau g\theta}=\int \frac{d\theta}{\cos^2\theta\,\mathrm{sen}\,\theta}=\ell\eta\left|\tau g\frac{\theta}{2}\right|+\frac{1}{\cos\theta}+c$$, o bien:

$$=\ell\eta\left|\cos ec\theta-\operatorname{co}\tau g\theta\right|+\frac{1}{\cos\theta}+c=\ell\eta\left|\frac{\sqrt{x^2+1}}{x}-\frac{1}{x}\right|+\frac{1}{1/\sqrt{x^2+1}}+c$$

$$=\ell\eta\left|\frac{\sqrt{x^2+1}-1}{x}\right|+\sqrt{x^2+1}+c$$

6.31.- $\int \frac{dx}{x^2\sqrt{4-x^2}}$

Solución.-



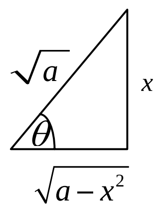
Se tiene: $x=2\,\mathrm{sen}\,\theta, dx=2\cos\theta d\theta$, $\sqrt{4-x^2}=2\cos\theta$

$$\int \frac{dx}{x^2\sqrt{4-x^2}}=\int \frac{\cancel{2\cos\theta}d\theta}{4\,\mathrm{sen}^2\theta\,\cancel{2\cos\theta}}=\frac{1}{4}\int \cos ec^2\theta d\theta=-\frac{1}{4}\operatorname{co}\tau g\theta+c$$

$$=-\frac{\sqrt{4-x^2}}{4x}+c$$

6.32.- $\int \sqrt{a-x^2}dx$

Solución.-



Se tiene: $x=\sqrt{a}\,\mathrm{sen}\,\theta, dx=\sqrt{a}\cos\theta d\theta$, $\sqrt{a-x^2}=\sqrt{a}\cos\theta$

$$\int\sqrt{a-x^2}dx=\int\sqrt{a}\cos\theta\sqrt{a}\cos\theta d\theta=a\int\cos^2\theta d\theta$$

$$\frac{a}{2}\theta+\frac{a}{2}\mathrm{sen}\,\theta\cos\theta+c=\frac{a}{2}\mathrm{arcsen}\frac{x}{\sqrt{a}}+\frac{x}{2}\sqrt{a^2-x^2}+c$$

6.33.- $\int\sqrt{a^2-x^2}dx$

Solución.-

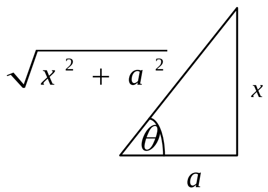
Se tiene: $x=a\,\mathrm{sen}\,\theta, dx=a\cos\theta d\theta$, $\sqrt{a^2-x^2}=a\cos\theta$

$$\int\sqrt{a^2-x^2}dx=\int a\cos\theta a\cos\theta d\theta=a^2\int\cos^2\theta d\theta$$

$$\frac{a^2}{2}\theta+\frac{a^2}{2}\mathrm{sen}\,\theta\cos\theta+c=\frac{a^2}{2}\mathrm{arcsen}\frac{x}{a}+\frac{x}{2}\sqrt{a^2-x^2}+c$$

6.34.- $\int\frac{x^2dx}{\sqrt{x^2+a^2}}$

Solución.-



Se tiene: $x=a\tau g\theta, dx=a\sec^2\theta d\theta$, $\sqrt{x^2+a^2}=a\sec\theta$

$$\int\frac{x^2dx}{\sqrt{x^2+a^2}}=\int\frac{a^2\tau g^2\theta\cancel{a}\sec^{\cancel{2}}\theta d\theta}{\cancel{a\sec\theta}}=a^2\int\tau g^2\theta\sec\theta d\theta=a^2\int\frac{\mathrm{sen}^2\theta}{\cos^3\theta}d\theta$$

$$=a^2\int\frac{(1-\cos^2\theta)}{\cos^3\theta}d\theta=a^2\int\sec^3\theta d\theta-a^2\int\sec\theta d\theta$$

$$=a^2\left(\frac{\sec\theta\tau g\theta}{2}+\frac{1}{2}\ell\eta\left|\sec\theta+\tau g\theta\right|\right)-a^2\ell\eta\left|\sec\theta+\tau g\theta\right|+c$$

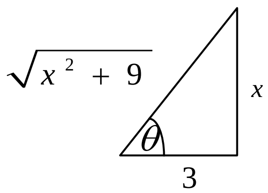
$$=\frac{a^2}{2}\sec\theta\tau g\theta+\frac{a^2}{2}\ell\eta\left|\sec\theta+\tau g\theta\right|-a^2\ell\eta\left|\sec\theta+\tau g\theta\right|+c$$

$$=\frac{a^2}{2}\sec\theta\tau g\theta-\frac{a^2}{2}\ell\eta\left|\sec\theta+\tau g\theta\right|+c$$

$$=\frac{a^{\cancel{2}}}{2}\frac{\sqrt{x^2+a^2}}{\cancel{a}}\frac{x}{\cancel{a}}-\frac{a^2}{2}\ell\eta\left|\frac{\sqrt{x^2+a^2}}{a}+\frac{x}{a}\right|+c=\frac{x\sqrt{x^2+a^2}}{2}-\frac{a^2}{2}\ell\eta\left|\sqrt{x^2+a^2}+x\right|+c$$

6.35.- $\int\frac{dx}{x^2\sqrt{x^2+9}}$

Solución.-



Se tiene: $x = 3\tau g\theta, dx = 3\sec^2\theta d\theta$, $\sqrt{x^2+9} = 3\sec\theta$

$$\int \frac{dx}{x^2\sqrt{x^2+9}} = \int \frac{\cancel{3}\sec^{\cancel{2}}\theta d\theta}{9\tau g^2\theta\cancel{3\sec\theta}} = \frac{1}{9}\int \frac{\sec\theta d\theta}{\tau g^2\theta} = \frac{1}{9}\int \frac{\cos\theta}{\operatorname{sen}^2\theta}d\theta = -\frac{1}{9\operatorname{sen}\theta}+c$$

$$= -\frac{\sqrt{x^2+9}}{9x}+c$$

6.36.- $\int \frac{dx}{\sqrt{5-4x^2}}$

Solución.-

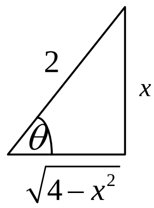
Se tiene: $x = \sqrt{5/4}\operatorname{sen}\theta, dx = \sqrt{5/4}\cos\theta d\theta$, $\sqrt{(5/4)^2 - x^2} = 5/4\cos\theta$

$$\int \frac{dx}{\sqrt{5-4x^2}} = \frac{1}{2}\int \frac{dx}{\sqrt{5/4 - x^2}} = \frac{1}{2}\int \frac{\cancel{\sqrt{5/4}\cos\theta} d\theta}{\cancel{\sqrt{5/4}\cos\theta}} = \frac{1}{2}\int d\theta = \frac{1}{2}\theta + c$$

$$= \frac{1}{2}\operatorname{arcsen}\frac{x}{\sqrt{5/4}} + c = \frac{1}{2}\operatorname{arcsen}\frac{2x}{\sqrt{5}} + c$$

6.37.- $\int \frac{x^2dx}{(4-x^2)^{3/2}}$

Solución.-



Se tiene: $x = 2\operatorname{sen}\theta, dx = 2\cos\theta d\theta$, $\sqrt{4-x^2} = 2\cos\theta$

$$\int \frac{x^2dx}{(4-x^2)^{3/2}} = \int \frac{x^2dx}{\sqrt{(4-x^2)^3}} = \int \frac{\cancel{4}\operatorname{sen}^{\cancel{2}}\theta\cancel{2}\cancel{\cos\theta}d\theta}{\cancel{8}\cos^{\cancel{3}}\theta} = \int \tau g^2\theta d\theta = \int(\sec^2\theta - 1)d\theta$$

$$= \tau g\theta - \theta + c = \frac{x}{\sqrt{4-x^2}} - \operatorname{arcsen}\frac{x}{2} + c$$

6.38.- $\int x^2\sqrt{5-x^2}dx$

Solución.-

Se tiene: $x = \sqrt{5}\operatorname{sen}\theta, dx = \sqrt{5}\cos\theta d\theta$, $\sqrt{5-x^2} = \sqrt{5}\cos\theta$

$$\int x^2\sqrt{5-x^2}dx = \int 5\operatorname{sen}^2\theta\sqrt{5}\cos\theta\sqrt{5}\cos\theta d\theta = 25\int \operatorname{sen}^2\theta\cos^2\theta d\theta = \frac{25}{4}\int \operatorname{sen}^2 2\theta d\theta$$

$$= \frac{25}{8}\int(1-\cos4\theta)d\theta = \frac{25}{8}\theta - \frac{25}{32}\operatorname{sen}4\theta + c = \frac{25}{8}\theta - \frac{25}{32}(2\operatorname{sen}2\theta\cos2\theta) + c$$

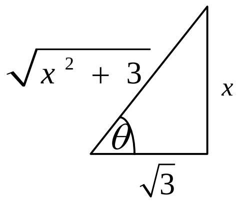
$$= \frac{25}{8}\theta - \frac{25}{32}\left[2\operatorname{sen}\theta\cos2\theta(\cos^2\theta - \operatorname{sen}^2\theta)\right] + c$$

$$=\frac{25}{8}\theta-\frac{25}{16}\left[\operatorname{sen}\theta\cos^3\theta-\operatorname{sen}^3\theta\cos\theta)\right]+c$$

$$=\frac{25}{2}\left[\operatorname{arcsen}\frac{x}{\sqrt{5}}-\frac{x(\sqrt{5-x^2})^3}{25}+\frac{x^3\sqrt{5-x^2}}{25}\right]+c$$

6.39.- $\int\frac{dx}{x^4\sqrt{x^2+3}}$

Solución.-



Se tiene: $x=\sqrt{3}\tau g\theta, dx=\sqrt{3}\sec^2\theta d\theta$, $\sqrt{x^2+3}=\sqrt{3}\sec\theta$

$$\int\frac{dx}{x^4\sqrt{x^2+3}}=\int\frac{\cancel{\sqrt{3}}\sec^{\cancel{2}}\theta d\theta}{9\tau g^4\theta\cancel{\sqrt{3}}\,\cancel{\sec\theta}}=\frac{1}{9}\int\frac{\sec\theta d\theta}{\tau g^4\theta}=\frac{1}{9}\int\frac{\cos^3\theta d\theta}{\operatorname{sen}^4\theta}=\frac{1}{9}\int\frac{(1-\operatorname{sen}^2\theta)\cos\theta d\theta}{\operatorname{sen}^4\theta}$$

$$=\frac{1}{9}\int\frac{\cos\theta d\theta}{\operatorname{sen}^4\theta}-\frac{1}{9}\int\frac{\cos\theta d\theta}{\operatorname{sen}^2\theta}=-\frac{1}{27}\cos ec^3\theta+\frac{1}{9}\cos ec\theta+c=\frac{\sqrt{x^2+3}}{9x}-\left(\frac{\sqrt{x^2+3}}{3x}\right)^3+c$$

6.40.- $\int x^3\sqrt{a^2x^2+b^2}dx$

Solución.-

Se tiene: $ax=b\tau g\theta, adx=b\sec^2\theta d\theta$, $\sqrt{a^2x^2+b^2}=b\sec\theta$

$$\int x^3\sqrt{a^2x^2+b^2}dx=\int\frac{b^3}{a^3}\tau g^3\theta b\sec\theta\frac{b}{a}\sec^2\theta d\theta=\frac{b^5}{a^4}\int\tau g^3\theta\sec^3\theta d\theta$$

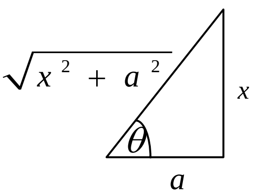
$$=\frac{b^5}{a^4}\int\tau g^2\theta\sec^2\theta\tau g\theta\sec\theta d\theta=\frac{b^5}{a^4}\int(\sec^2\theta-1)\sec^2\theta\tau g\theta\sec\theta d\theta$$

$$=\frac{b^5}{a^4}\int\sec^4\theta\tau g\theta\sec\theta d\theta-\frac{b^5}{a^4}\int\sec^2\theta\tau g\theta\sec\theta d\theta=\frac{b^5}{a^4}\frac{\sec^5\theta}{5}+\frac{b^5}{a^4}\frac{\sec^3\theta}{3}+c$$

$$=\frac{b^5}{a^4}\left[\frac{(\sqrt{a^2x^2+b^2})^5}{5b^5}+\frac{(\sqrt{a^2x^2+b^2})^3}{3b^3}\right]+c=\frac{(a^2x^2+b^2)^{5/2}}{5a^4}-\frac{(a^2x^2+b^2)^{3/2}b^2}{3a^4}+c$$

6.41.- $\int\frac{dx}{x^2\sqrt{x^2+a^2}}$

Solución.-



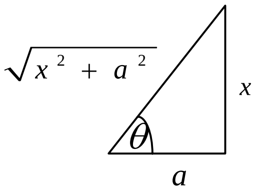
Se tiene: $x=a\tau g\theta, dx=a\sec^2\theta d\theta$, $\sqrt{x^2+a^2}=a\sec\theta$

$$\int \frac{dx}{x^2\sqrt{x^2+a^2}} = \int \frac{\cancel{a}\sec^{\cancel{2}}\theta d\theta}{a^2\tau g^2\theta\,\cancel{a\sec\theta}} = \frac{1}{a^2}\int \frac{\sec\theta d\theta}{\tau g^2\theta} = \frac{1}{a^2}\int \frac{\cos\theta d\theta}{\operatorname{sen}^2\theta}d\theta$$

$$= \frac{1}{a^2}\int \operatorname{co}\tau g\theta\operatorname{cosec}\theta d\theta = -\frac{\operatorname{cosec}\theta}{a^2} + c = -\frac{1}{a^2x}\sqrt{x^2+a^2} + c$$

6.42.- $\int \frac{dx}{(x^2+a^2)^2}$

Solución.-



Se tiene: $x = a\tau g\theta, dx = a\sec^2\theta d\theta$, $\sqrt{x^2+a^2} = a\sec\theta$

$$\int \frac{dx}{(x^2+a^2)^2} = \int \frac{dx}{(\sqrt{x^2+a^2})^4} = \int \frac{\cancel{a}\,\cancel{\sec^2}\theta d\theta}{a^{\cancel{4}}\sec^{\cancel{4}}\theta} = \frac{1}{a^3}\int \cos^2\theta d\theta = \frac{1}{2a^3}\theta + \frac{1}{2a^3}\frac{\operatorname{sen}2\theta}{2} + c$$

$$= \frac{1}{2a^3}\theta + \frac{1}{2a^3}\frac{\cancel{2}\operatorname{sen}\theta\cos\theta}{\cancel{2}} + c = \frac{1}{2a^3}\operatorname{arc}\tau g\frac{x}{a} + \frac{1}{2a^3}\left(\frac{x}{\sqrt{x^2+a^2}}\frac{a}{\sqrt{x^2+a^2}}\right) + c$$

$$= \frac{1}{2a^3}\operatorname{arc}\tau g\frac{x}{a} + \frac{1}{2a^3}\left(\frac{ax}{\sqrt{x^2+a^2}}\right) + c$$

6.43.- $\int x^3\sqrt{a^2x^2-b^2}dx$

Solución.-

Se tiene: $ax = b\sec\theta, adx = b\sec\theta\tau g\theta d\theta$, $\sqrt{a^2x^2-b^2} = b\tau g\theta$

$$\int x^3\sqrt{a^2x^2-b^2}dx = \int \frac{b^3}{a^3}\sec^3\theta b\tau g\theta\frac{b}{a}\sec\theta\tau g\theta d\theta = \frac{b^5}{a^4}\int \sec^4\theta\tau g^2\theta d\theta$$

$$= \frac{b^5}{a^4}\int \sec^4\theta(\sec^2\theta - 1)d\theta = \frac{b^5}{a^4}\int \sec^4\theta\sec^2\theta d\theta - \frac{b^5}{a^4}\int \sec^2\theta\sec^2\theta d\theta$$

$$= \frac{b^5}{a^4}\int (1+\tau g^2\theta)^2\sec^2\theta d\theta - \frac{b^5}{a^4}\int (1+\tau g^2\theta)\sec^2\theta d\theta$$

$$= \frac{b^5}{a^4}\int (1+2\tau g^2\theta+\tau g^4\theta)\sec^2\theta d\theta - \frac{b^5}{a^4}\int (1+\tau g^2\theta)\sec^2\theta d\theta$$

$$= \frac{b^5}{a^4}\left[\int \tau g^2\theta\sec^2\theta d\theta + \int \tau g^4\theta\sec^2\theta d\theta\right] = \frac{b^5}{a^4}\left[\frac{\tau g^3\theta}{3} + \frac{\tau g^5\theta}{5}\right] + c$$

$$= \frac{b^5}{a^4}\left[\frac{1}{3}\left(\frac{\sqrt{a^2x^2-b^2}}{b}\right)^3 + \frac{1}{5}\left(\frac{\sqrt{a^2x^2-b^2}}{b}\right)^5\right] + c$$

6.44.- $\int \frac{dx}{x^2\sqrt{a^2-x^2}}$

Solución.-

Se tiene: $x = a\,\mathrm{sen}\,\theta, dx = a\cos\theta d\theta$, $\sqrt{a^2 - x^2} = a\cos\theta$

$$\int \frac{dx}{x^2\sqrt{a^2-x^2}} = \int \frac{\cancel{a\cos\theta}\, d\theta}{a^2\,\mathrm{sen}^2\,\theta\,\cancel{a\cos\theta}} = \frac{1}{a^2}\int \cos ec^2\theta d\theta = -\frac{1}{a^2}\mathrm{co}\,\tau g\theta + c$$

$$= -\frac{1}{a^2}\frac{\cos\theta}{\mathrm{sen}\,\theta} + c = -\frac{1}{a^2}\left(\frac{\sqrt{a^2-x^2}}{x}\right) + c$$

6.45.- $\int \frac{\sqrt{2x^2-5}}{x}dx$

Solución.-

Se tiene: $\sqrt{2}x = \sqrt{5}\sec\theta, \sqrt{2}dx = \sqrt{5}\sec\theta\tau g\theta d\theta$, $\sqrt{2x^2-5} = \sqrt{5}\tau g\theta$

$$\int \frac{\sqrt{2x^2-5}}{x}dx = \int \frac{\sqrt{5}\tau g\theta\, \cancel{\frac{\sqrt{5}}{\sqrt{2}}\sec\theta}\,\tau g\theta d\theta}{\cancel{\frac{\sqrt{5}}{\sqrt{2}}\sec\theta}} = \sqrt{5}\int \tau g^2\theta d\theta = \sqrt{5}\int \sec^2\theta d\theta - \sqrt{5}\int d\theta$$

$$= \sqrt{5}\tau g\theta - \sqrt{5}\theta + c = \sqrt{2x^2-5} - \sqrt{5}\,\mathrm{arc}\sec\sqrt{2/3}\,x + c$$

6.46.- $\int \frac{x^3dx}{\sqrt{3x^2-5}}$

Solución.-

Se tiene: $\sqrt{3}x = \sqrt{5}\sec\theta, \sqrt{3}dx = \sqrt{5}\sec\theta\tau g\theta d\theta$, $\sqrt{3x^2-5} = \sqrt{5}\tau g\theta$

$$\int \frac{x^3dx}{\sqrt{3x^2-5}} = \int \frac{(\sqrt{5/3}\sec\theta)^3\sqrt{5/3}\sec\theta\,\cancel{\tau g\theta}\, d\theta}{\sqrt{5/3}\,\cancel{\tau g\theta}} = \frac{5\sqrt{5}}{9}\int \sec^4\theta d\theta$$

$$= \frac{5\sqrt{5}}{9}\int \sec^2\theta\sec^2\theta d\theta = \frac{5\sqrt{5}}{9}\int \sec^2\theta(1+\tau g^2\theta)d\theta$$

$$= \frac{5\sqrt{5}}{9}\left[\int \sec^2\theta d\theta + \int \sec^2\theta\tau g^2\theta d\theta\right] = \frac{5\sqrt{5}}{9}\left[\tau g\theta + \frac{\tau g^3\theta}{3}\right] + c$$

$$= \frac{5}{9}\left[\sqrt{3x^2-5} + \frac{(\sqrt{3x^2-5})^3}{15}\right] + c$$

6.47.- $\int \frac{\sqrt{x^2-100}}{x}dx$

Solución.-

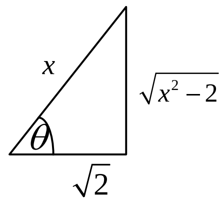
Se tiene: $x = 10\sec\theta, dx = 10\sec\theta\tau g\theta d\theta$, $\sqrt{x^2-100} = 10\tau g\theta$

$$\int \frac{\sqrt{x^2-100}}{x}dx = \int \frac{10\tau g\theta\,\cancel{10\sec\theta}\,\tau g\theta d\theta}{\cancel{10\sec\theta}} = 10\int \tau g^2\theta d\theta = 10\int \sec^2\theta - 10\int d\theta$$

$$= 10(\tau g\theta - \theta) + c = \sqrt{x^2-100} - 10\,\mathrm{arcs}\,e\,\mathrm{n}\frac{x}{10} + c$$

6.48.- $\int \frac{dx}{x^2\sqrt{x^2-2}}$

Solución.-



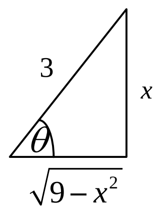
Se tiene: $x=\sqrt{2}\sec\theta, dx=\sqrt{2}\sec\theta\tau g\theta d\theta$, $\sqrt{x^2-2}=\sqrt{2}\tau g\theta$

$$\int \frac{dx}{x^2\sqrt{x^2-2}}=\int \frac{\cancel{\sqrt{2}}\,\cancel{\sec\theta}\,\cancel{\tau g\theta}\,d\theta}{2\sec^{\cancel{2}}\theta\,\cancel{\sqrt{2}\tau g\theta}}=\frac{1}{2}\int\cos\theta d\theta=\frac{1}{2}\mathrm{sen}\,\theta+c=\frac{1}{2}\frac{\sqrt{x^2-2}}{x}+c$$

$$=\frac{\sqrt{x^2-2}}{2x}+c$$

6.49.- $\int \frac{dx}{x\sqrt{9-x^2}}$

Solución.-



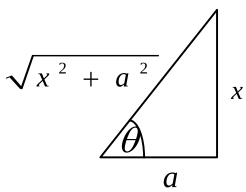
Se tiene: $x=3\mathrm{sen}\,\theta, dx=3\cos\theta d\theta$, $\sqrt{9-x^2}=3\cos\theta$

$$\int \frac{dx}{x\sqrt{9-x^2}}=\int \frac{\cancel{3\cos\theta}\,d\theta}{3\mathrm{sen}\,\theta\,\cancel{3\cos\theta}}=\frac{1}{3}\int\cos ec\theta d\theta=\frac{1}{3}\ell\eta\left|\cos ec\theta-\mathrm{co}\,\tau g\theta\right|+c$$

$$=\frac{1}{3}\ell\eta\left|\frac{3-\sqrt{9-x^2}}{x}\right|+c$$

6.50.- $\int \frac{\sqrt{x^2+a^2}}{x}dx$

Solución.-



Se tiene: $x=a\tau g\theta, dx=a\sec^2\theta d\theta$, $\sqrt{x^2+a^2}=a\sec\theta$

$$\int \frac{\sqrt{x^2+a^2}}{x}dx=\int \frac{a\sec\theta}{\cancel{a}\tau g\theta}\cancel{a}\sec^2\theta d\theta=a\int\frac{\sec^3\theta d\theta}{\tau g\theta}=a\int\frac{\sec^2\theta\sec\theta}{\tau g\theta}d\theta$$

$$=a\int\frac{(1+\tau g^2\theta)\sec\theta}{\tau g\theta}d\theta=a\int\frac{\sec\theta}{\tau g\theta}d\theta+a\int\sec\theta\tau g\theta d\theta$$

$$a\ell\eta\left|\cos ec\theta-\mathrm{co}\,\tau g\theta\right|+a\sec\theta+c=a\ell\eta\left|\frac{\sqrt{x^2+a^2}-a}{x}\right|+\sqrt{x^2+a^2}+c$$

6.51.- $\int \frac{xdx}{\sqrt{a^2-x^2}}$

Solución.-

Se tiene: $x = a\,\mathrm{sen}\,\theta, dx = a\cos\theta d\theta$, $\sqrt{a^2-x^2} = a\cos\theta$

$$\int\frac{xdx}{\sqrt{a^2-x^2}} = \int\frac{a\,\mathrm{sen}\,\theta\,\cancel{a\cos\theta}}{\cancel{a\cos\theta}}d\theta = a\int \mathrm{sen}\,\theta d\theta = -a\cos\theta + c = -\sqrt{a^2-x^2}+c$$

6.52.- $\int \frac{dx}{\sqrt{1-4x^2}}$

Solución.-

Se tiene: $2x = \mathrm{sen}\,\theta, 2dx = \cos\theta d\theta$, $\sqrt{1-4x^2} = \cos\theta$

$$\int\frac{dx}{\sqrt{1-4x^2}} = \frac{1}{2}\int\frac{\cancel{\cos\theta}}{\cancel{\cos\theta}}d\theta = \frac{1}{2}\int d\theta = \frac{1}{2}\theta + c = \frac{1}{2}\mathrm{arcsen}\,2x + c$$

6.53.- $\int \frac{dx}{\sqrt{4+x^2}}$

Solución.-

Se tiene: $x = 2\tau g\theta, dx = 2\sec^2\theta d\theta$, $\sqrt{4+x^2} = 2\sec\theta$

$$\int\frac{dx}{\sqrt{4+x^2}} = \int\frac{\cancel{2}\sec^{\cancel{2}}\theta d\theta}{\cancel{2\sec\theta}} = \int\sec\theta d\theta = \ell\eta\left|\sec\theta + \tau g\theta\right| + c = \ell\eta\left|\sqrt{4+x^2}+x\right| + c$$

6.54.- $\int \frac{xdx}{\sqrt{4+x^2}}$

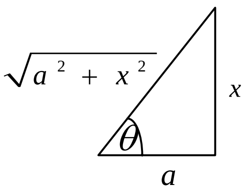
Solución.-

Se tiene: $x = 2\tau g\theta, dx = 2\sec^2\theta d\theta$, $\sqrt{4+x^2} = 2\sec\theta$

$$\int\frac{xdx}{\sqrt{4+x^2}} = \int\frac{2\tau g\theta\,\cancel{2}\sec^{\cancel{2}}\theta d\theta}{\cancel{2\sec\theta}} = 2\int\tau g\theta\sec\theta d\theta = 2\sec\theta + c = \sqrt{4+x^2}+c$$

6.55.- $\int \frac{dx}{x\sqrt{a^2+x^2}}$

Solución.-



Se tiene: $x = a\tau g\theta, dx = a\sec^2\theta d\theta$, $\sqrt{a^2+x^2} = a\sec\theta$

$$\int\frac{dx}{x\sqrt{a^2+x^2}} = \int\frac{\cancel{a}\sec^{\cancel{2}}\theta d\theta}{a\tau g\theta\,\cancel{a\sec\theta}} = \frac{1}{a}\int\frac{\sec\theta d\theta}{\tau g\theta} = \frac{1}{a}\int\mathrm{cosec}\theta d\theta$$

$$= \frac{1}{a}\ell\eta\left|\mathrm{cosec}\theta - \mathrm{co}\,\tau g\theta\right| + c = \frac{1}{a}\ell\eta\left|\frac{\sqrt{a^2+x^2}}{x} - \frac{a}{x}\right| + c = \frac{1}{a}\ell\eta\left|\frac{\sqrt{a^2+x^2}-a}{x}\right| + c$$

6.56.- $\int \frac{(x+1)dx}{\sqrt{4-x^2}}$

Solución.-

Se tiene: $x = 2\operatorname{sen}\theta, dx = 2\cos\theta d\theta, \sqrt{4-x^2} = 2\cos\theta$

$$\int \frac{(x+1)dx}{\sqrt{4-x^2}} = \int \frac{xdx}{\sqrt{4-x^2}} + \int \frac{dx}{\sqrt{4-x^2}} = \int \frac{2\operatorname{sen} \cancel{2\cos\theta} d\theta}{\cancel{2\cos\theta}} + \int \frac{\cancel{2\cos\theta} d\theta}{\cancel{2\cos\theta}}$$

$$2\int \operatorname{sen}\theta d\theta + \int d\theta = -2\cos\theta + \theta + c = -\sqrt{4-x^2} + \operatorname{arcsen}\frac{x}{2} + c$$

6.57.- $\int \frac{dx}{\sqrt{2-5x^2}}$

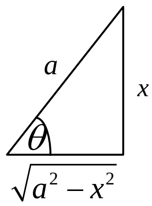
Solución.-

Se tiene: $\sqrt{5}x = \sqrt{2}\operatorname{sen}\theta, \sqrt{5}dx = \sqrt{2}\cos\theta d\theta, \sqrt{2-5x^2} = \sqrt{2}\cos\theta$

$$\int \frac{dx}{\sqrt{2-5x^2}} = \int \frac{\frac{\cancel{\sqrt{2}}}{\sqrt{5}}\cancel{\cos\theta} d\theta}{\cancel{\sqrt{2}}\cancel{\cos\theta}} = \frac{\sqrt{5}}{5}\int d\theta = \frac{\sqrt{5}}{5}\theta + c = \frac{\sqrt{5}}{5}\operatorname{arcsen}\sqrt{\frac{5}{2}}x + c$$

6.58.- $\int \frac{dx}{(a^2-x^2)^{3/2}}$

Solución.-



Se tiene: $x = a\operatorname{sen}\theta, dx = a\cos\theta d\theta, \sqrt{a^2-x^2} = a\cos\theta$

$$\int \frac{dx}{(a^2-x^2)^{3/2}} = \int \frac{dx}{(\sqrt{a^2-x^2})^3} = \int \frac{\cancel{a}\cancel{\cos\theta} d\theta}{a^{\cancel{3}}\cos^{\cancel{3}}\theta} = \frac{1}{a^2}\int \sec^2\theta d\theta = \frac{1}{a^2}\tau g\theta + c$$

$$= \frac{x}{a^2\sqrt{a^2-x^2}} + c$$

6.59.- $\int \frac{dx}{\sqrt{4-(x-1)^2}}$

Solución.-

Se tiene: $x-1 = 2\operatorname{sen}\theta, dx = 2\cos\theta d\theta, \sqrt{4-(x-1)^2} = 2\cos\theta$

$$\int \frac{dx}{\sqrt{4-(x-1)^2}} = \int \frac{\cancel{2\cos\theta} d\theta}{\cancel{2\cos\theta}} = \int d\theta = \theta + c = \operatorname{arcsen}\frac{x-1}{2} + c$$

6.60.- $\int \frac{x^2dx}{\sqrt{2x-x^2}}$

Solución.-

Se tiene: $x-1 = \operatorname{sen}\theta \Rightarrow x = \operatorname{sen}\theta + 1, dx = \cos\theta d\theta, \sqrt{1-(x-1)^2} = \cos\theta$

Completando cuadrados se tiene:

$2x - x^2 = -(x^2-2x) = -(x^2-2x+1)+1 = 1-(x-1)^2$, luego:

$$\int \frac{x^2dx}{\sqrt{2x-x^2}} = \int \frac{x^2dx}{\sqrt{1-(x-1)^2}} = \int \frac{(\operatorname{sen}\theta+1)^2\cancel{\cos\theta} d\theta}{\cancel{\cos\theta}} = \int (\operatorname{sen}\theta+1)^2 d\theta$$

$$=\int \operatorname{sen}^2\theta d\theta+2\int \operatorname{sen}\theta d\theta+\int d\theta=\frac{1}{2}\int d\theta-\frac{1}{2}\int\cos 2\theta d\theta+2\int \operatorname{sen}\theta d\theta+\int d\theta$$

$$=\frac{3}{2}\int d\theta-\frac{1}{2}\int\cos 2\theta d\theta+2\int \operatorname{sen}\theta d\theta=\frac{3}{2}\theta-\frac{1}{4}\operatorname{sen}2\theta-2\cos\theta+c$$

$$=\frac{3}{2}\theta-\frac{1}{2}\operatorname{sen}\theta\cos\theta-2\cos\theta+c=\frac{3}{2}\operatorname{arcsen}(x-1)-\frac{1}{2}(x-1)\sqrt{2x-x^2}-2\sqrt{2x-x^2}+c$$

6.61.- $\int\frac{x^2dx}{\sqrt{17-x^2}}$

Solución.-

Se tiene: $x=\sqrt{17}\operatorname{sen}\theta, dx=\sqrt{17}\cos\theta d\theta$, $\sqrt{17-x^2}=\sqrt{17}\cos\theta$

$$\int\frac{x^2dx}{\sqrt{17-x^2}}=\int\frac{17\operatorname{sen}^2\theta\cancel{\sqrt{17}\cos\theta}d\theta}{\cancel{\sqrt{17}\cos\theta}}=17\int\operatorname{sen}^2\theta d\theta=\frac{17}{2}\int d\theta-\frac{17}{2}\int\cos 2\theta d\theta$$

$$=\frac{17}{2}\theta-\frac{17}{4}\operatorname{sen}2\theta+c=\frac{17}{2}\theta-\frac{17}{2}\operatorname{sen}\theta\cos\theta+c$$

$$=\frac{17}{2}\operatorname{arcsen}\frac{x}{\sqrt{17}}-\frac{\cancel{17}}{2}\frac{x}{\cancel{\sqrt{17}}}\frac{\sqrt{17-x^2}}{\cancel{\sqrt{17}}}+c=\frac{17}{2}\operatorname{arcsen}\frac{x}{\sqrt{17}}-\frac{1}{2}x\sqrt{17-x^2}+c$$

6.62.- $\int\frac{x^2dx}{\sqrt{21+4x-x^2}}$

Solución.-

Se tiene: $x-2=5\operatorname{sen}\theta\Rightarrow x=5\operatorname{sen}\theta+2, dx=5\cos\theta d\theta$, $\sqrt{5^2-(x-2)^2}=5\cos\theta$

Completando cuadrados se tiene:

$21+4x-x^2=-(x^2-4x+4-4)+21=-(x^2-4x+4)+25=5^2-(x-2)^2$, luego:

$$\int\frac{x^2dx}{\sqrt{21+4x-x^2}}=\int\frac{x^2dx}{\sqrt{5^2-(x-2)^2}}=\int\frac{(5\operatorname{sen}\theta+2)^2\cancel{5\cos\theta}d\theta}{\cancel{5\cos\theta}}=\int(5\operatorname{sen}\theta+2)^2d\theta$$

$$=\int(25\operatorname{sen}^2\theta+20\operatorname{sen}\theta+4)d\theta=25\int\frac{1-\cos 2\theta}{2}d\theta+20\int\operatorname{sen}\theta d\theta+4\int d\theta$$

$$=\frac{25}{2}\int d\theta-\frac{25}{2}\int\cos 2\theta d\theta+20\int\operatorname{sen}\theta d\theta=\frac{25}{2}\theta-\frac{25}{4}\operatorname{sen}2\theta-20\cos\theta+4\theta+c$$

$$=\frac{33}{2}\theta-\frac{25}{2}\operatorname{sen}\theta\cos\theta-20\cos\theta+c$$

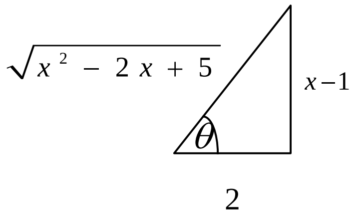
$$=\frac{33}{2}\operatorname{arcsen}\frac{x-2}{5}-\frac{25}{2}\frac{x-2}{5}\left(\frac{\sqrt{21+4x-x^2}}{5}\right)-20\left(\frac{\sqrt{21+4x-x^2}}{5}\right)+c$$

$$=\frac{33}{2}\operatorname{arcsen}\frac{x-2}{5}-\sqrt{21+4x-x^2}\left(\frac{x-2}{2}+4\right)+c$$

$$=\frac{33}{2}\operatorname{arcsen}\frac{x-2}{5}-\sqrt{21+4x-x^2}\left(\frac{x+6}{2}\right)+c$$

6.63.- $\int \frac{dx}{(x^2-2x+5)^{3/2}}$

Solución.-



Se tiene: $x-1=2\tau g\theta, dx=2\sec^2\theta d\theta$, $\sqrt{(x-1)^2+2^2}=2\sec\theta$

Completando cuadrados se tiene:

$x^2-2x+5=(x^2-2x+1)+5-1=(x^2-2x+1)+4=(x-1)^2+2^2$, luego:

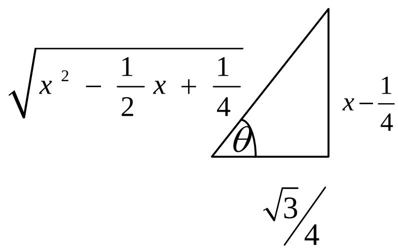
$$\int \frac{dx}{(x^2-2x+5)^{3/2}}=\int \frac{dx}{\sqrt{\left[(x-1)^2+2^2\right]^3}}=\int \frac{2\sec^2\theta d\theta}{2^3\sec^3\theta}=\frac{1}{4}\int \cos\theta d\theta=\frac{1}{4}sen\,\theta+c$$

$$=\frac{1}{4}\frac{x-1}{\sqrt{x^2-2x+5}}+c$$

6.64.- $\int \frac{(2x+1)dx}{\sqrt{(4x^2-2x+1)^3}}$

Solución.-

Sea: $u=4x^2-2x+1, du=(8x-2)dx$



Se tiene: $x-\frac{1}{4}=\frac{\sqrt{3}}{4}\tau g\theta, dx=\frac{\sqrt{3}}{4}\sec^2\theta d\theta$, $\sqrt{(x-1/4)^2+(\sqrt{3}/4)^2}=\sqrt{3}/4\,\sec\theta$

Completando cuadrados se tiene:

$x^2-\frac{1}{2}x+\frac{1}{4}=(x^2-\frac{1}{2}x+\frac{1}{16})+\frac{1}{4}-\frac{1}{16}=(x-\frac{1}{4})^2+\frac{3}{16}=(x-\frac{1}{4})^2+(\frac{\sqrt{3}}{4})^2$, luego:

$$\int \frac{(2x+1)dx}{\sqrt{(4x^2-2x+1)^3}}=\frac{1}{4}\int \frac{(8x+4)dx}{\sqrt{(4x^2-2x+1)^3}}=\frac{1}{4}\int \frac{(8x-2+6)dx}{\sqrt{(4x^2-2x+1)^3}}$$

$$=\frac{1}{4}\int \frac{(8x-2)dx}{\sqrt{(4x^2-2x+1)^3}}+\frac{3}{2}\int \frac{dx}{\sqrt{(4x^2-2x+1)^3}}$$

$$=\frac{1}{4}\int \frac{du}{(u)^{3/2}}+\frac{3}{2}\int \frac{dx}{\sqrt{4(x^2-1/2\,x+1/4)^3}}=\frac{1}{4}\int (u)^{-3/2}du+\frac{3}{2}\frac{1}{8}\int \frac{dx}{\sqrt{(x^2-1/2\,x+1/4)^3}}$$

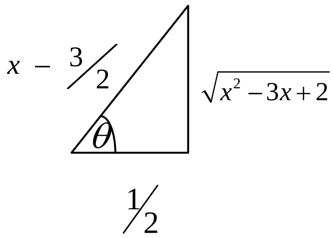
$$=\frac{1}{4}\int (u)^{-3/2}du+\frac{3}{16}\int \frac{dx}{\sqrt{\left[(x-1/4)^2+(\sqrt{3}/4)^2\right]^3}}=\frac{1}{4}\int (u)^{-3/2}du+\frac{3}{16}\int \frac{\frac{\sqrt{3}}{4}\sec^2\theta d\theta}{(\frac{\sqrt{3}}{4}\sec\theta)^3}$$

$$=\frac{1}{4}\int (u)^{-3/2}du+\int\frac{d\theta}{\sec\theta}=\frac{1}{4}\frac{u^{-1/2}}{(-1/2)}+\operatorname{sen}\theta+c=-\frac{1}{2u^{1/2}}+\operatorname{sen}\theta+c$$

$$=\frac{-1}{2\sqrt{4x^2-2x+1}}+\frac{x-1/4}{\sqrt{x^2-1/2\,x+1/4}}+c=\frac{4x-2}{4\sqrt{x^2-1/2\,x+1/4}}+c$$

6.65.- $\int\frac{dx}{(x-1)\sqrt{x^2-3x+2}}$

Solución.-



Se tiene: $x-\frac{3}{2}=\frac{1}{2}\sec\theta\Rightarrow x-1=\frac{1}{2}(\sec\theta+1),dx=\frac{1}{2}\sec\theta\tau g\theta d\theta$,

$\sqrt{(x-3/2)^2+(1/2)^2}=1/2\,\tau g\theta$

Completando cuadrados se tiene:

$x^2-3x+2=(x^2-3x+\frac{9}{4})-\frac{1}{4}=(x-\frac{3}{2})^2-(\frac{1}{2})^2$, luego:

$$\int\frac{dx}{(x-1)\sqrt{x^2-3x+2}}=\int\frac{dx}{(x-1)\sqrt{(x-\frac{3}{2})^2-(\frac{1}{2})^2}}=\int\frac{1/2\,\sec\theta\,\cancel{\tau g\theta}\,d\theta}{1/2\,(\sec\theta+1)\,\cancel{1/2\,\tau g\theta}}$$

$$=\int\frac{\sec\theta d\theta}{1/2\,(\sec\theta+1)}=2\int\frac{\sec\theta d\theta}{(\sec\theta+1)}=2\int\frac{\sec\theta(\sec\theta-1)d\theta}{\sec^2\theta-1}=2\int\frac{\sec^2\theta d\theta}{\tau g^2\theta}-2\int\frac{\sec\theta d\theta}{\tau g^2\theta}$$

$$=2\int\cos ec^2\theta d\theta-2\int\frac{\operatorname{cosec}\theta d\theta}{\operatorname{sen}^2\theta}=-2\operatorname{co}\tau g\theta+2\operatorname{cosec}\theta+c$$

$$-2\frac{1/2}{\sqrt{x^2-3x+2}}+2\frac{x-3/2}{\sqrt{x^2-3x+2}}+c=\frac{2x-4}{\sqrt{x^2-3x+2}}+c$$

6.66.- $\int\frac{xdx}{\sqrt{x^2-2x+5}}$

Solución.-

Se tiene: $x-1=2\tau g\theta,dx=2\sec^2\theta d\theta$, $\sqrt{(x-1)^2+(2)^2}=2\sec\theta$

Completando cuadrados se tiene:

$x^2-2x+5=(x^2-2x+1)+4=(x-1)^2-2^2$, luego:

$$\int \frac{xdx}{\sqrt{x^2-2x+5}} = \int \frac{xdx}{\sqrt{(x-1)^2-2^2}} = \int \frac{(2\tau g\theta+1)\cancel{2}\sec^{\cancel{2}}\theta d\theta}{\cancel{2\sec\theta}}$$

$$= 2\int \tau g\theta \sec\theta d\theta + \int \sec\theta d\theta = 2\sec\theta + \ell\eta\left|\sec\theta+\tau g\theta\right| + c$$

$$= \sqrt{x^2-2x+5} + \ell\eta\left|\frac{\sqrt{x^2-2x+5}+x-1}{2}\right| + c$$

6.67.- $\int \frac{(x+1)dx}{\sqrt{2x-x^2}}$

Solución.-

Se tiene: $x-1 = \mathrm{s}e\mathrm{n}\,\theta \Rightarrow x+1 = \mathrm{s}e\mathrm{n}\,\theta + 2, dx = \cos\theta d\theta$, $\sqrt{1-(x-1)^2} = \cos\theta$

Completando cuadrados se tiene:

$2x-x^2 = -(x^2-2x) = -(x^2-2x+1-1) = -(x^2-2x+1)+1 = 1-(x-1)^2$, luego:

$$\int \frac{(x+1)dx}{\sqrt{2x-x^2}} = \int \frac{(x+1)dx}{\sqrt{1-(x-1)^2}} = \int \frac{(\mathrm{s}e\mathrm{n}\,\theta+2)\cos\theta d\theta}{\cos\theta} = \int \mathrm{s}e\mathrm{n}\,\theta d\theta + 2\int d\theta$$

$$= -\cos\theta + 2\theta + c = -\sqrt{2x-x^2} + 2\arcsin(x-1) + c$$

6.68.- $\int \frac{(x-1)dx}{\sqrt{x^2-4x+3}}$

Solución.-

Se tiene: $x-2 = \sec\theta \Rightarrow x-1 = \sec\theta + 1, dx = \sec\theta\tau g\theta d\theta$, $\sqrt{(x-2)^2-1} = \tau g\theta$

Completando cuadrados se tiene:

$x^2-4x+3 = x^2-4x+4-1 = (x-2)^2-1$, luego:

$$\int \frac{(x-1)dx}{\sqrt{x^2-4x+3}} = \int \frac{(x-1)dx}{\sqrt{(x-2)^2-1}} = \int \frac{(\sec\theta+1)\sec\theta\,\cancel{\tau g\theta}\,d\theta}{\cancel{\tau g\theta}}$$

$$= \int \sec^2\theta d\theta + \int \sec\theta d\theta = \tau g\theta + \ell\eta\left|\sec\theta+\tau g\theta\right| + c$$

$$= \sqrt{x^2-4x+3} + \ell\eta\left|x-2+\sqrt{x^2-4x+3}\right| + c$$

6.69.- $\int \frac{dx}{\sqrt{x^2-2x-8}}$

Solución.-

Se tiene: $x-1 = 3\sec\theta, dx = 3\sec\theta\tau g\theta d\theta$, $\sqrt{(x-1)^2-3^2} = 3\tau g\theta$

Completando cuadrados se tiene:

$x^2-2x-8 = x^2-2x+1-9 = (x-1)^2-3^2$, luego:

$$\int \frac{dx}{\sqrt{x^2-2x-8}} = \int \frac{dx}{\sqrt{(x-1)^2-3^2}} = \int \frac{\cancel{3}\sec\theta\,\cancel{\tau g\theta}\,d\theta}{\cancel{3\tau g\theta}} = \int \sec\theta d\theta = \ell\eta\left|\sec\theta+\tau g\theta\right| + c$$

$$= \ell\eta\left|\frac{x-1}{3}+\frac{\sqrt{x^2-2x-8}}{3}\right| + c = \ell\eta\left|x-1+\sqrt{x^2-2x-8}\right| + c$$

6.70.- $\int \frac{xdx}{\sqrt{x^2+4x+5}}$

Solución.-

Se tiene: $x+2=\tau g\theta, dx=\sec^2\theta d\theta$, $\sqrt{(x+2)^2+1^2}=\sec\theta$

Completando cuadrados se tiene:

$x^2+4x+5=(x^2+4x+4)+1=(x+2)^2+1^2$, luego:

$$\int \frac{xdx}{\sqrt{x^2+4x+5}}=\int \frac{xdx}{\sqrt{(x+2)^2+1^2}}=\int \frac{(\tau g\theta-2)\sec^{\cancel{2}}\theta d\theta}{\cancel{\sec\theta}}=\int \tau g\theta\sec\theta d\theta-2\int \sec\theta d\theta$$

$$=\sec\theta-2\ell\eta\left|\sec\theta+\tau g\theta\right|+c=\sqrt{x^2+4x+5}-2\ell\eta\left|\sqrt{x^2+4x+5}+x+2\right|+c$$

CAPITULO 7

INTEGRACIÓN DE FUNCIONES RACIONALES

Mediante el recurso de la descomposición en fracciones simples, el proceso de integración de funciones racionales se puede simplificar notablemente.

EJERCICIOS DESARROLLADOS

7.1.-Encontrar: $\int \frac{dx}{x^2-9}$

Solución.- Descomponiendo el denominador en factores: $x^2-9=(x+3)(x-3)$,
Como los factores son ambos lineales y diferentes se tiene:

$\frac{1}{x^2-9}=\frac{A}{x+3}+\frac{B}{x-3}$, de donde:

$$\frac{1}{\cancel{x^2-9}}=\frac{A}{\cancel{x+3}}+\frac{B}{\cancel{x-3}} \Rightarrow 1=A(x-3)+B(x+3)(*) \Rightarrow 1=(A+B)x+(-3A+3B)$$

Para calcular las constantes A y B, se pueden identificar los coeficientes de igual potencia x en la última expresión, y se resuelve el sistema de ecuaciones dado; obteniendo así los valores de las constantes en referencia (método general) luego:

$$\begin{pmatrix} A+\ \ B=0 \\ -3A+3B=1 \end{pmatrix} \Rightarrow \begin{pmatrix} 3A+3B=0 \\ -3A+3B=1 \end{pmatrix} \Rightarrow 6B=1 \Rightarrow B={}^{1}\!\!/_{6}$$, además:

$$A+B=0 \Rightarrow A=-B \Longrightarrow A=-{}^{1}\!\!/_{6}$$

También es frecuente usar otro mecanismo, que consiste en la expresión $(*)$
Sustituyendo a x por los valores que anulen los denominadores de las fracciones:

$$x=3 \Rightarrow 1=6B \Rightarrow B={}^{1}\!\!/_{6}$$

$$x=-3 \Rightarrow 1=-6A \Rightarrow A=-{}^{1}\!\!/_{6}$$

Usando cualquier método de los señalados anteriormente, se establece que:

$\frac{1}{x^2-9}=\frac{-{}^{1}\!/_{6}}{x+3}+\frac{{}^{1}\!/_{6}}{x-3}$, Luego se tiene:

$$\int \frac{dx}{x^2-9}=-\frac{1}{6}\int\frac{dx}{x+3}+\frac{1}{6}\int\frac{dx}{x-3}=-\frac{1}{6}\ell\eta|x+3|+\frac{1}{6}\ell\eta|x-3|+c$$

$$=\frac{1}{6}\left(\ell\eta|x-3|-\ell\eta|x+3|\right)+c$$

Respuesta: $\displaystyle\int \frac{dx}{x^2-9} = \frac{1}{6}\ell\eta\left|\frac{x-3}{x+3}\right| + c$

7.2.-Encontrar: $\displaystyle\int \frac{dx}{x^2+7x-6}$

Solución.- Sea: $x^2+7x+6=(x+6)(x+1)$, factores lineales y diferentes; luego:

$\dfrac{1}{x^2+7x+6} = \dfrac{A}{x+6} + \dfrac{B}{x+1}$,

De donde:

$1 = A(x+1)+B(x+6)(*) \Rightarrow 1 = (A+B)x+(A+6B)$, calculando las constantes A y B por el método general, se tiene: $1=(A+B)x+(A+6B)$

$\begin{pmatrix} A+\ \ B=0 \\ A+6B=1 \end{pmatrix} \Rightarrow - \begin{pmatrix} -A-\ \ B=0 \\ A+6B=1 \end{pmatrix} \Rightarrow 5B=1 \Rightarrow B=\frac{1}{5}$, además:

$A+B=0 \Rightarrow A=-B \Longrightarrow A=-\frac{1}{5}$

Ahora utilizando el método abreviado se tiene:

$x=-1 \Rightarrow 1=5B \Rightarrow B=\frac{1}{5}$

$x=-6 \Rightarrow 1=-5A \Rightarrow A=-\frac{1}{5}$

Usando cualquier método se puede establecer:

$\dfrac{1}{x^2+7x+6} = \dfrac{-\frac{1}{5}}{x+6} + \dfrac{\frac{1}{5}}{x+1}$, Luego se tiene:

$$\int \frac{dx}{x^2+7x+6} = -\frac{1}{5}\int\frac{dx}{x+6} + \frac{1}{5}\int\frac{dx}{x+1} = -\frac{1}{5}\ell\eta|x+6| + \frac{1}{5}\ell\eta|x+1| + c$$

$$= \frac{1}{5}\left(\ell\eta|x+1| - \ell\eta|x+6|\right) + c$$

Respuesta: $\displaystyle\int \frac{dx}{x^2+7x+6} = \frac{1}{5}\ell\eta\left|\frac{x+1}{x+6}\right| + c$

7.3.-Encontrar: $\displaystyle\int \frac{xdx}{x^2-4x+4}$

Solución.- Sea: $x^2-4x+4=(x-2)^2$, factores lineales con repetición; luego:

$\dfrac{x}{x^2-x+4} = \dfrac{A}{x-2} + \dfrac{B}{(x-2)^2} \Rightarrow \dfrac{x}{\cancel{x^2-x+4}} = \dfrac{A(x-2)+B}{\cancel{(x-2)^2}}$,

De donde:

$x = A(x-2)+B(*)$, calculando las constantes A y B por el método general, se tiene: $x = Ax+(-2A+B)$, luego:

$\begin{pmatrix} A \qquad =1 \\ -2A+B=0 \end{pmatrix} \Rightarrow B=2A \Rightarrow B=2(1) \Rightarrow B=2$

Usando el método abreviado, se sustituye en x, el valor que anula el denominador(o los denominadores), y si este no es suficiente se usan para sustituir cualquier valor conveniente de x, esto es: $x=0, x=-1$; luego en $(*)$

$x=2 \Rightarrow 2=B \Rightarrow B=2$

$x=0 \Rightarrow 0=-2A+B \Rightarrow 2A+B \Rightarrow A=B/2 \Rightarrow A=1$

Usando cualquier método se establece:

$$\int \frac{xdx}{x^2-4x+4}=\int \frac{dx}{x-2}+2\int \frac{dx}{(x-2)^2}=\ell\eta\left|x-2\right|-\frac{2}{x-2}+c$$

Respuesta: $\int \frac{xdx}{x^2-4x+4}=\ell\eta\left|x-2\right|-\frac{2}{x-2}+c$

7.4.-Encontrar: $\int \frac{(2x^2+3)dx}{x^3-2x^2+x}$

Solución.- Sea: $x^3-2x^2+x=x(x^2-2x+1)=x(x-1)^2$, factores lineales: $x, x-1$; donde este último es con repetición; luego:

$$\frac{2x^2+3}{x^3-2x^2+x}=\frac{A}{x}+\frac{B}{(x-1)}+\frac{C}{(x-1)^2} \Rightarrow \frac{2x^2+3}{\cancel{x^3-2x^2+x}}=\frac{A(x-1)^2+Bx(x-1)+Cx}{\cancel{x(x-1)^2}}$$

De donde:

$2x^2+3=A(x-1)^2+Bx(x-1)+Cx(*)$, calculando las constantes A y B por el método general, se tiene: $2x^2+3=(A+B)x^2+(-2A-B+C)x+A$, de donde identificando los coeficientes de igual potencia de x se puede obtener el siguiente sistema de ecuaciones:

$$\begin{pmatrix} A+B & =2 \\ -2A-B+C & =0 \\ A & =3 \end{pmatrix} \Rightarrow B=2-A \Rightarrow B=2-3 \Rightarrow B=-1$$, tomando la segunda ecuación del sistema: $C=2A+B \Rightarrow C=2(3)-1 \Rightarrow C=5$,también es posible usar el método abreviado, utilizando para ello la expresión $(*)$ en la cual:

$x=1 \Rightarrow 2(1)+3=C \Rightarrow C=5$

$x=0 \Rightarrow 3=A \Rightarrow A=3$

Usando un valor arbitrario para x, sea este $x=-1$:

$x=-1 \Rightarrow 2(-1)^2+3=A(-2)^2+B(-1)(-2)+C(-1) \Rightarrow 5=4A+2B-C$, luego:

$2B=5-4A+C \Rightarrow 2B=5-4(3)+5 \Rightarrow 2B=-2 \Rightarrow B=-1$, S, e establece que:

$$\frac{2x^2+3}{x^3-2x^2+x}=\frac{3}{x}-\frac{1}{x-1}+\frac{5}{(x-1)^2}$$, entonces:

$$\frac{2x^2+3}{x^3-2x^2+x}=3\int \frac{dx}{x}-\int \frac{dx}{x-1}+5\int \frac{dx}{(x-1)^2}=3\ell\eta\left|x\right|-\ell\eta\left|x-1\right|-\frac{5}{x-1}+c$$

Respuesta: $\int \frac{(2x^2+3)dx}{x^3-2x^2+x}=\ell\eta\left|\frac{x^3}{x-1}\right|-\frac{5}{x-1}+c$

7.5.-Encontrar: $\int \frac{dx}{x^3-2x^2+x}$

Solución.- $x^3-2x^2+x=x(x-1)^2$,factores lineales:
$x, x-1$; donde este último es con repetición; luego:

$$\frac{1}{x^3-2x^2+x}=\frac{A}{x}+\frac{B}{(x-1)}+\frac{C}{(x-1)^2}\Rightarrow \frac{1}{\cancel{x^3-2x^2+x}}=\frac{A(x-1)^2+Bx(x-1)+Cx}{\cancel{x(x-1)^2}}$$

De donde:
$1=A(x-1)^2+Bx(x-1)+Cx(*)$, calculando las constantes A y B por el método general, se tiene: $1=(A+B)x^2+(-2A-B+C)x+A$, de donde identificando los coeficientes de igual potencia de x se puede obtener el siguiente sistema de ecuaciones:

$$\begin{pmatrix} A+B & =0 \\ -2A-B+C & =0 \\ A & =1 \end{pmatrix}\Rightarrow B=-A\Rightarrow B=-1$$, tomando la segunda ecuación del sistema: $C=2A+B\Rightarrow C=2(1)-1\Rightarrow C=1$, a partir de lo cual se tiene:

$$\frac{1}{x^3-2x^2+x}=\frac{1}{x}-\frac{1}{x-1}+\frac{1}{(x-1)^2}$$

$$\int\frac{dx}{x^3-2x^2+x}=\int\frac{dx}{x}-\int\frac{dx}{x-1}+\int\frac{dx}{(x-1)^2}=\ell\eta|x|-\ell\eta|x-1|-\frac{1}{x-1}+c$$

Respuesta: $\int\frac{dx}{x^3-2x^2+x}=\ell\eta\left|\frac{x}{x-1}\right|-\frac{1}{x-1}+c$

7.6.-Encontrar: $\int\frac{x^4-6x^3+12x^2+6}{x^3-6x^2+12x-8}dx$

Solución.- Se sabe que si el grado del polinomio dividendo, es igual o superior al grado del polinomio divisor, previamente conviene efectuar la división de tales polinomios.

$$\begin{array}{l|l} x^4-6x^3+12x^2+0x+6 & x^3-6x^2+12x-8 \\ \cline{2-2} -x^4+6x^3-12x^2+8x & \quad x \\ \hline \qquad\qquad\quad \boxed{8x+6} & \end{array}$$

Luego se tiene: $\int\frac{x^4-6x^3+12x^2+6}{x^3-6x^2+12x-8}dx=\int xdx+\int\frac{(8x+6)dx}{x^3-6x^2+12x-8}$

La descomposición de: $x^3-6x^2+12x-8$:

$$\begin{array}{r|rrrr} & 1 & -6 & 12 & -8 \\ 2 & & 2 & -8 & 8 \\ \hline & 1 & -4 & 4 & \boxed{0} \end{array}\qquad x=2\Rightarrow(x-2)$$

$$x^2-4x+4=(x-2)^2$$

$$x^3-6x^2+12x-8=(x-2)^3$$

Esto es factores lineales: $[(x-2)]$ con repetición por tanto:

$$\frac{8x+6}{x^3-6x^2+12x-8}=\frac{A}{x-2}+\frac{B}{(x-2)^2}+\frac{C}{(x-2)^3}$$

$$\frac{8x+6}{\cancel{x^3-6x^2+12x-8}}=\frac{A(x-2)^2+B((x-2)+C}{\cancel{(x-2)^3}}$$

Luego:

$$8x+6=A(x-2)^2+B(x-2)+C\Rightarrow 8x+6=A(x^2-4x+4)+B(x-2)+C$$

$$8x+6=Ax^2+(-4A+B)x+(4A-2B+C)$$

Calculando las constantes A y B por el método general, se tiene:

$$\begin{pmatrix} A & & =0 \\ -4A+ & B & =8 \\ +4A-2B+C=6 & & \end{pmatrix}\Rightarrow B=8+4A\Rightarrow B=8+4(0)\Rightarrow B=8,$$

Resolviendo el sistema: $C=6-4A+2B\Rightarrow C=6-4(0)+2(8)\Rightarrow C=22$, luego:

$$\frac{8x+6}{x^3-6x^2+12x-8}=\cancelto{0}{\frac{0}{x-2}}+\frac{8}{(x-1)^2}+\frac{22}{(x-1)^3}$$, de donde:

$$\int\frac{(8x+6)dx}{x^3-6x^2+12x-8}=8\int\frac{dx}{(x-2)^2}+22\int\frac{dx}{(x-2)^3}$$, o sea:

$$=\int xdx+8\int\frac{dx}{(x-2)^2}+22\int\frac{dx}{(x-2)^3}=\int xdx+8\int(x-2)^{-2}dx+22\int(x-2)^{-3}dx$$

$$\frac{x^2}{2}-\frac{8}{x-2}-\frac{11}{(x-2)^2}+c$$

Respuesta: $\int\frac{x^4-6x^3+12x^2+6}{x^3-6x^2+12x-8}dx=\frac{x^2}{2}-\frac{8}{x-2}-\frac{11}{(x-2)^2}+c$

7.7.-Encontrar: $\int\frac{x^3+x^2+x+3}{x^4+4x^2+3}dx$

Solución.- $x^4+4x^2+3=(x^2+3)(x^2+1)$, la descomposición es en factores cuadráticos sin repetición, por lo tanto:

$$\frac{x^3+x^2+x+3}{x^4+4x^2+3}=\frac{Ax+B}{x^2+3}+\frac{Cx+D}{x^2+1}$$

$$\frac{x^3+x^2+x+3}{\cancel{x^4+4x^2+3}}=\frac{(Ax+B)(x^2+1)+(Cx+D)(x^2+3)}{\cancel{(x^2+3)(x^2+1)}}$$

$$x^3+x^2+x+3=A(x^3+x)+B(x^2+1)+C(x^3+3x)+D(x^2+3)$$

$$x^3+x^2+x+3=(A+C)x^3+(B+D)x^2+(A+3C)x+(B+3D)$$, luego:

$$\begin{array}{l} (1) \\ (2) \\ (3) \\ (4) \end{array}\begin{pmatrix} A & & +C & & =1 \\ & B & & +D & =1 \\ A & & +3C & & =1 \\ & B & & +3D & =3 \end{pmatrix}$$

Con (1) y (3), se tiene: $\begin{pmatrix} A+ & C=1 \\ A+3C & =1 \end{pmatrix} \Rightarrow A=1, C=0$

Con (2) y (4), se tiene: $\begin{pmatrix} B+ & D=1 \\ B+3D & =3 \end{pmatrix} \Rightarrow B=0, D=1$

Por lo tanto: $\dfrac{x^3+x^2+x+3}{x^4+4x^2+3} = \dfrac{x}{x+3} + \dfrac{1}{x^2+1}$, o sea:

$\int \dfrac{x^3+x^2+x+3}{x^4+4x^2+3} dx = \int \dfrac{xdx}{x+3} + \int \dfrac{dx}{x^2+1}$, sea: $u = x^2+3, du = 2xdx$, luego:

$$\int \frac{x^3+x^2+x+3}{x^4+4x^2+3} dx = \frac{1}{2}\int \frac{2xdx}{x+3} + \int \frac{dx}{x^2+1^2} = \frac{1}{2}\int \frac{du}{u} + \int \frac{dx}{x^2+1^2}$$

$$= \frac{1}{2}\ell\eta|u| + \operatorname{arc}\tau gx + c = \frac{1}{2}\ell\eta\left|x^2+3\right| + \operatorname{arc}\tau gx + c$$

Respuesta: $\int \dfrac{x^3+x^2+x+3}{x^4+4x^2+3} dx = \dfrac{1}{2}\ell\eta\left|x^2+3\right| + \operatorname{arc}\tau gx + c$

7.8.-Encontrar: $\int \dfrac{x^4dx}{x^4+2x^2+1}$

Solución.-

$$\begin{array}{l|l} x^4 & x^4+2x^2+1 \\ \cline{2-2} -x^4-2x^2-1 & 1 \\ \hline \boxed{-2x^2-1} & \end{array}$$

Luego $\int \dfrac{x^4dx}{x^4+2x^2+1} = \int \left(1 - \dfrac{2x^2+1}{x^4+2x^2+1}\right)dx = \int dx - \int \dfrac{2x^2+1}{x^4+2x^2+1}dx$

La descomposición del denominador es: $x^4+2x^2+1 = (x^2+1)^2$, entonces:

$$\frac{2x^2+1}{x^4+2x^2+1} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+1)^2} \Rightarrow \frac{2x^2+1}{\cancel{x^4+2x^2+1}} = \frac{(Ax+B)(x^2+1)(Cx+D)}{\cancel{(x^2+1)^2}}$$

$$2x^2+1 = (Ax+B)(x^2+1) + (Cx+D) \Rightarrow 2x^2+1 = A(x^3+x) + B(x^2+1) + Cx + D$$

$$2x^2+1 = Ax^3 + Bx^2 + (A+C)x + (B+D)$$

Calculando las constantes por el método general, se tiene:

$$\begin{pmatrix} A & & & & =0 \\ & B & & & =2 \\ A & & +C & & =0 \\ & B & & +D & =1 \end{pmatrix}$$

Resolviendo el sistema: $C=-A \Rightarrow A=0 \therefore C=0$, $B+D=1 \Rightarrow D=1-B \Rightarrow D=-1$
luego:

$$\frac{2x^2+1}{x^4+2x^2+1}=\frac{2}{x^2+1}-\frac{1}{(x^2+1)^2}$$, o sea:

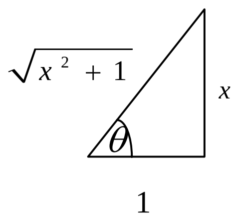
$$\int\frac{2x^2+1}{x^4+2x^2+1}=2\int\frac{dx}{x^2+1^2}-\int\frac{dx}{(x^2+1)^2}=2\int\frac{dx}{x^2+1^2}-\int\frac{dx}{(\sqrt{x^2+1})^4}$$

Sea: $x=\tau g\theta, dx=\sec^2\theta d\theta; \sqrt{x^2+1}=\sec\theta$, luego:

$$=2\operatorname{arc}\tau gx-\int\frac{\sec^2\theta}{\sec^4\theta}d\theta=2\operatorname{arc}\tau gx-\int\frac{d\theta}{\sec^2\theta}=2\operatorname{arc}\tau gx-\int\cos^2\theta$$

$$=2\operatorname{arc}\tau gx-\int\frac{1+\cos 2\theta}{2}d\theta=2\operatorname{arc}\tau gx-\frac{1}{2}\int d\theta-\frac{1}{2}\int\cos 2\theta d\theta$$

$$\operatorname{arc}\tau gx-\frac{1}{2}\theta-\frac{1}{2}\operatorname{sen}2\theta+c=2\operatorname{arc}\tau gx-\frac{1}{2}\theta-\frac{1}{2}\operatorname{sen}\theta\cos\theta+c$$



De la figura se tiene que:

$$\tau g\theta=x, \theta\operatorname{arc}\tau g\theta, \operatorname{sen}\theta=\frac{x}{\sqrt{x^2+1}}, \cos\theta=\frac{1}{\sqrt{x^2+1}}$$

Luego: $=2\operatorname{arc}\tau gx-\frac{1}{2}\operatorname{arc}\tau gx-\frac{1}{2}\frac{x}{\sqrt{x^2+1}}\frac{1}{\sqrt{x^2+1}}+c=2\operatorname{arc}\tau gx-\frac{1}{2}\operatorname{arc}\tau gx-\frac{x}{2(x^2+1)}+c$

Recordando que:

$$\frac{x^4dx}{x^4+2x^2+1}=\int dx-\int\frac{(2x^2+1)dx}{x^4+2x^2+1}=x-2\operatorname{arc}\tau gx+\frac{1}{2}\operatorname{arc}\tau gx+\frac{1}{2}\frac{x}{(x^2+1)}+c$$

Respuesta: $\int\frac{x^4dx}{x^4+2x^2+1}=x-\frac{3}{2}\operatorname{arc}\tau gx+\frac{x}{2(x^2+1)}+c$

7.9.-Encontrar: $\int\frac{x^4dx}{x^4-1}$

Solución.-

$$\begin{array}{l|l} x^4 & x^4-1 \\ \hline -x^4+1 & 1 \\ \hline \quad\boxed{1} & \end{array}$$

Luego:

$$\int\frac{x^4dx}{x^4-1}=\int\left(1+\frac{1}{x^4-1}\right)dx=\int dx+\int\frac{dx}{x^4-1}$$

Descomponiendo en factores el denominador:

$x^4-1=(x^2-1)(x^2+1)=(x^2+1)(x+1)(x-1)$, es decir factores lineales y cuadráticos sin repetición por tanto:

$$\frac{1}{x^4-1}=\frac{Ax+B}{x^2+1}+\frac{C}{x+1}+\frac{D}{x-1}$$

$$\frac{1}{\cancel{x^4-1}}=\frac{(Ax+B)(x^2-1)+C(x^2+1)(x-1)+D(x+1)(x^2+1)}{\cancel{(x^2+1)(x+1)(x+1)}}$$

$$1=A(x^3-x)+B(x^2+1)+C(x^3-x^2+x-1)+D(x^3+x^2+x+1)$$

$$1=(A+C+D)x^3+(B-C+D)x^2+(-A+C+D)x+(-B-C+D)$$

Luego:

$$\begin{array}{l} (1) \\ (2) \\ (3) \\ (4) \end{array}\begin{pmatrix} A \quad +C+D=0 \\ B-C+D=0 \\ -A \quad +C+D=0 \\ -B-C+D=1 \end{pmatrix}$$

Con (1) y (3), se tiene: $\begin{pmatrix} A+C+D=0 \\ -A+C+D=0 \end{pmatrix} \Rightarrow 2C+2D=0$ **(5)**

Con (2) y (4), se tiene: $\begin{pmatrix} B-C+D=0 \\ -B-C+D=1 \end{pmatrix} \Rightarrow -2C+2D=1$ **(6)**

Con (5) y (6), se tiene: $\begin{pmatrix} 2C+2D=0 \\ -2C+2D=1 \end{pmatrix} \Rightarrow C=-\frac{1}{4}, D=\frac{1}{4}$

Además: $A=0, B=-\frac{1}{2}$, luego:

$\frac{1}{x^4-1}=-\frac{1}{2(x^2+1)}-\frac{1}{4(x+1)}+\frac{1}{4(x-1)}$, con lo cual:

$$\int\frac{dx}{x^4-1}=-\frac{1}{2}\int\frac{dx}{(x^2+1)}-\frac{1}{4}\int\frac{dx}{(x+1)}+\frac{1}{4}\int\frac{dx}{(x-1)}$$

$$=-\frac{1}{2}\operatorname{arc}\tau g x-\frac{1}{4}\ell\eta|x+1|+\frac{1}{4}\ell\eta|x-1|+c$$

Dado que: $\int\frac{x^4dx}{x^4-1}=\int dx+\int\frac{dx}{x^4-1}=x-\frac{1}{2}\operatorname{arc}\tau g x+\frac{1}{4}\ell\eta\left|\frac{x-1}{x+1}\right|+c$, entonces:

Respuesta: $\int\frac{1}{x^4-1}=x-\frac{1}{2}\operatorname{arc}\tau g x+\frac{1}{4}\ell\eta\left|\frac{x-1}{x+1}\right|+c$

7.10.-Encontrar: $\int\frac{x^4-2x^3+3x^2-x+3}{x^3-2x^2+3x}dx$

Solución.-

$$\begin{array}{l|l} x^4-2x^3+3x^2-x+3 & x^3-2x^2+3x \\ \cline{2-2} \underline{-x^4+2x^3-3x^2\qquad\quad} & x \\ \qquad\qquad\boxed{-x+3} & \end{array}$$

Luego:

$$\int \frac{x^4-2x^3+3x^2-x+3}{x^3-2x^2+3x}dx = \int\left(x-\frac{x-3}{x^3-2x^2+3x}\right)dx = \int xdx - \int\frac{x-3}{x^3-2x^2+3x}dx$$

Descomponiendo en factores el denominador:

$x^3-2x^2+3x = x(x^2-2x+3)$, es decir un factor lineal y uno cuadrático; por lo cual:

$$\frac{x-3}{x^3-2x^2+3x} = \frac{A}{x}+\frac{Bx+C}{x^2-2x+3} \Rightarrow \frac{x-3}{\cancel{x^3-2x^2+3x}} = \frac{A(x^2-2x+3)+(Bx+C)x}{\cancel{x(x^2-2x+3)}}$$

$$x-3 = A(x^2-2x+3)+(Bx+C)x \Rightarrow x-3 = (A+B)x^2+(-2A+C)x+3A$$

De donde:

$$\begin{pmatrix} A+B & & = & 0 \\ -2A & +C & = & 1 \\ 3A & & = & -3 \end{pmatrix} \Rightarrow \begin{cases} A=-1 \\ B=-A \Rightarrow B=1 \\ C=1+2A \Rightarrow C=-1 \end{cases}$$

Luego:

$\dfrac{x-3}{x^3-2x^2+3x} = -\dfrac{1}{x}+\dfrac{x-1}{x^2-2x+3}$, de donde:

$$\int\frac{x-3}{x^3-2x^2+3x}dx = -\int\frac{dx}{x}+\int\frac{x-1}{x^2-2x+3}dx = -\ell\eta|x|+\int\frac{x-1}{x^2-2x+3}dx$$

$$\int\frac{x^4-2x^3+3x^2-x+3}{x^3-2x^2+3x}dx = \int xdx+\ell\eta|x|-\int\frac{x-1}{x^2-2x+3}dx$$

$$=\frac{x^2}{2}+\ell\eta|x|-\int\frac{x-1}{x^2-2x+3}dx = \frac{x^2}{2}+\ell\eta|x|-\frac{1}{2}\int\frac{2(x-1)dx}{x^2-2x+3}$$

Sea: $u = x^2-2x+3, du = (2x-2)dx \Rightarrow du = 2(x-1)dx$

$$=\frac{x^2}{2}+\ell\eta|x|-\frac{1}{2}\int\frac{du}{u} = \frac{x^2}{2}+\ell\eta|x|-\frac{1}{2}\ell\eta\left|x^2-2x+3\right|+c$$

Respuesta: $\displaystyle\int\frac{x^4-2x^3+3x^2-x+3}{x^3-2x^2+3x}dx = \frac{x^2}{2}+\ell\eta\left|\frac{x}{\sqrt{x^2-2x+3}}\right|+c$

EJERCICICOS PROPUESTOS

Usando La técnica de descomposición en fracciones simples parciales, calcular las siguientes integrales:

7.11.- $\displaystyle\int\frac{(x^5+2)dx}{x^2-1}$ **7.12.-** $\displaystyle\int\frac{xdx}{(x+1)^2}$ **7.13.-** $\displaystyle\int\frac{x^3dx}{x^2-2x-3}$

7.14.- $\displaystyle\int\frac{(3x+7)dx}{(x-1)(x-2)(x-3)}$ **7.15.-** $\displaystyle\int\frac{dx}{x^3+1}dx$ **7.16.-** $\displaystyle\int\frac{(x+5)dx}{x^2-x+6}$

7.17.- $\displaystyle\int\frac{(x^2+1)dx}{x^3+1}$ **7.18.-** $\displaystyle\int\frac{(x^2+6)dx}{(x-1)^2(x-2)}$ **7.19.-** $\displaystyle\int\frac{(x^2-1)dx}{(x^2+1)(x-2)}$

7.20.- $\int \frac{xdx}{x^2-4x-5}$ **7.21.-** $\int \frac{xdx}{x^2-2x-3}$ **7.22.-** $\int \frac{(x+1)dx}{x^2+4x-5}$

7.23.- $\int \frac{x^2dx}{x^2+2x+1}$ **7.24.-** $\int \frac{dx}{x(x+1)^2}$ **7.25.-** $\int \frac{dx}{(x+1)(x^2+1)}$

7.26.- $\int \frac{dx}{x(x^2+x+1)}$ **7.27.-** $\int \frac{2x^2+5x-1}{x^3+x^2-2x}dx$ **7.28.-** $\int \frac{(x^2+2x+3)dx}{(x-1)(x+1)^2}$

7.29.- $\int \frac{3x^2+2x-2}{x^3-1}dx$ **7.30.-** $\int \frac{x^4-x^3+2x^2-x+2}{(x-1)(x^2+2)^2}dx$ **7.31.-** $\int \frac{(2x^2-7x-1)dx}{x^3+x^2-x-1}$

7.32.- $\int \frac{3x^2+3x+1}{x^3+2x^2+2x+1}dx$ **7.33.-** $\int \frac{x^3+7x^2-5x+5}{(x-1)^2(x+1)^2}dx$ **7.34.-** $\int \frac{2xdx}{(x^2+x+1)^2}$

7.35.- $\int \frac{x^2+2x+3}{x^3-x}dx$ **7.36.-** $\int \frac{(2x^2-3x+5)dx}{(x+2)(x-1)(x-3)}$ **7.37.-** $\int \frac{(3x^2+x-2)dx}{(x-1)(x^2+1)}$

7.38.- $\int \frac{(x+5)dx}{x^3-3x+2}$ **7.39.-** $\int \frac{2x^3+3x^2+x-1}{(x+1)(x^2+2x+2)^2}dx$ **7.40.-** $\int \frac{(2x+1)dx}{3x^3+2x-1}$

7.41.- $\int \frac{(2x^2+3x-1)dx}{x^3+2x^2+4x+2}$ **7.42.-** $\int \frac{x^4-2x^2+3x+4}{(x-1)^3(x^2+2x+2)}dx$ **7.43.-** $\int \frac{e^t dt}{e^{2t}+3e^t+2}$

7.44.- $\int \frac{\operatorname{sen}\theta d\theta}{\cos^2\theta+\cos\theta-2}$ **7.45.-** $\int \frac{4x^4-2x^3-x^2+3x+1}{(x^3+x^2-x-1)}dx$ **7.46.-** $\int \frac{3x^4dx}{(x^2+1)^2}$

7.47.- $\int \frac{(2x^2+41x-91)dx}{x^3-2x^2-11x+12}$ **7.48.-** $\int \frac{(2x^4+3x^3-x-1)dx}{(x-1)(x^2+2x+2)^2}$ **7.49.-** $\int \frac{dx}{e^{2x}+e^x-2}$

7.50.- $\int \frac{\operatorname{sen} xdx}{\cos x(1+\cos^2 x)}$ **7.51.-** $\int \frac{(2+\tau g^2\theta)\sec^2\theta d\theta}{1+\tau g^3\theta}$ **7.52.-** $\int \frac{(5x^3+2)dx}{x^3-5x^2+4x}$

7.53.- $\int \frac{x^5dx}{(x^3+1)(x^3+8)}$

RESPUESTAS

7.11.- $\int \frac{(x^5+2)dx}{x^2-1}$

Solución.-

$$\int \frac{(x^5+2)dx}{x^2-1} = \int\left(x^3+x+\frac{x+2}{x^2-1}\right)dx = \int x^3dx+\int xdx+\int\frac{x+2}{x^2-1}dx$$

$= \frac{x^4}{4}+\frac{x^2}{2}+\int\frac{(x+2)dx}{(x+1)(x-1)}$ $(*)$, luego:

$$\frac{x+2}{\cancel{x^2-1}} = \frac{A}{\cancel{x+1}}+\frac{B}{\cancel{x-1}} \Rightarrow x+2 = A(x-1)+B(x+1)$$

$$\therefore \begin{cases} x=1 \Rightarrow 3=2B \Rightarrow B={}^{3}\!/_{2} \\ x=-1 \Rightarrow 1=-2A \Rightarrow A=-{}^{1}\!/_{2} \end{cases}$$

$$(*) = \frac{x^4}{4}+\frac{x^2}{2}-\frac{1}{2}\int\frac{dx}{x+1}+\frac{3}{2}\int\frac{dx}{x-1} = \frac{x^4}{4}+\frac{x^2}{2}-\frac{1}{2}\ell\eta\left|x+1\right|+\frac{3}{2}\ell\eta\left|x-1\right|+c$$

$$= \frac{x^4}{4}+\frac{x^2}{2}+\eta\left|\frac{(x-1)^{3/2}}{\sqrt{x+1}}\right|+c$$

7.12.- $\int\frac{xdx}{(x+1)^2}$

Solución.-

$\int\frac{xdx}{(x+1)^2}=\int\frac{Adx}{x+1}+\int\frac{Bdx}{(x+1)^2}$ $(*)$, luego:

$$\frac{x}{(x+1)^2}=\frac{A}{x+1}+\frac{B}{(x+1)^2} \Rightarrow x=A(x+1)+B$$

$$\therefore \begin{cases} x=-1 \Rightarrow -1=B \\ x=0 \Rightarrow 0=A+B \Rightarrow A=-B \Rightarrow A=-1 \end{cases}$$

$$(*) \int\frac{dx}{x+1}-\int\frac{dx}{(x+1)^2}=\ell\eta\left|x+1\right|+(x+1)^{-1}+c=\ell\eta\left|x+1\right|+\frac{1}{x+1}+c$$

7.13.- $\int\frac{x^3dx}{x^2-2x-3}$

Solución.-

$$\int\frac{x^3dx}{x^2-2x-3}=\int\left(x+2+\frac{7x+6}{x^2-2x-3}\right)dx=\int xdx+2\int dx+\int\frac{(7x+6)dx}{x^2-2x-3}$$

$= \frac{x^2}{2}+2x+\int\frac{(7x+6)dx}{(x-3)(x+1)}$ $(*)$, luego:

$$\frac{(7x+6)}{(x-3)(x+1)}=\frac{A}{x-3}+\frac{B}{x+1} \Rightarrow 7x+6=A(x+1)+B(x-3)$$

$$\therefore \begin{cases} x=3 \Rightarrow 27=4A \Rightarrow A={}^{27}\!/_{4} \\ x=-1 \Rightarrow -1=-4B \Rightarrow B={}^{1}\!/_{4} \end{cases}$$

$$(*) = \frac{x^2}{2}+2x+\frac{27}{4}\int\frac{dx}{x-3}+\frac{1}{4}\int\frac{dx}{x+1}=\frac{x^2}{2}+2x+\frac{27}{4}\ell\eta\left|x-3\right|+\frac{1}{4}\ell\eta\left|x+1\right|+c$$

$$= \frac{x^2}{2}+2x+\frac{1}{4}\ell\eta\left|(x-3)^{27}(x+1)\right|+c$$

7.14.- $\int\frac{(3x+7)dx}{(x-1)(x-2)(x-3)}$

Solución.-

$$\int\frac{(3x+7)dx}{(x-1)(x-2)(x-3)}=\int\frac{Adx}{x-1}+\int\frac{Bdx}{x-2}+\int\frac{Cdx}{x-3} \ (*)$$

$$\frac{(3x+7)}{(x-1)(x-2)(x-3)}=\frac{A}{x-1}+\frac{B}{x-2}+\frac{C}{x-3}$$

$3x-7=A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2)$, luego:

$$\therefore\begin{cases}x=1\Rightarrow -4=2A\Rightarrow A=-2\\ x=2\Rightarrow -1=-B\Rightarrow B=1\\ x=3\Rightarrow 2=2C\Rightarrow C=1\end{cases}$$

$$(*)=-2\int\frac{dx}{x-1}+\int\frac{dx}{x-2}+\int\frac{dx}{x-3}=-2\ell\eta\left|x-1\right|+\ell\eta\left|x-2\right|+\ell\eta\left|x-3\right|+c$$

$$=\ell\eta\left|\frac{(x-2)(x-3)}{(x-1)^2}\right|+c$$

7.15.- $\int\frac{dx}{x^3+1}dx$

Solución.-

$\int\frac{dx}{x^3+1}dx=\int\frac{dx}{(x+1)(x^2-x+1)}=\int\frac{Adx}{x+1}+\int\frac{(Bx+C)dx}{(x^2-x+1)}$ $(*)$, luego:

$$\frac{1}{(x+1)(x^2-x+1)}=\frac{A}{x+1}+\frac{(Bx+C)}{(x^2-x+1)}\Rightarrow 1=A(x^2-x+1)+(Bx+C)(x+1)$$

$$\therefore\begin{cases}x=-1\Rightarrow 1=3A\Rightarrow A=1/3\\ x=0\Rightarrow 1=A+C\Rightarrow C=1-A\Rightarrow C=2/3\\ x=1\Rightarrow 1=A+(B+C)2\Rightarrow 1=1/3+2B+2C\Rightarrow 1/3=B+C\Rightarrow B=1/3-C\end{cases}$$

$$\Rightarrow B=-1/3$$

$$(*)=\frac{1}{3}\int\frac{dx}{x+1}+\int\frac{(-1/3\,x+2/3)dx}{(x^2-x+1)}=\frac{1}{3}\ell\eta\left|x+1\right|-\frac{1}{3}\int\frac{(x-2)dx}{x^2-x+1}$$

$$=\frac{1}{3}\ell\eta\left|x+1\right|-\frac{1}{6}\int\frac{(2x-4)dx}{x^2-x+1}=\frac{1}{3}\ell\eta\left|x+1\right|-\frac{1}{6}\int\frac{(2x-1-3)dx}{x^2-x+1}$$

$$=\frac{1}{3}\ell\eta\left|x+1\right|-\frac{1}{6}\int\frac{(2x-1)dx}{x^2-x+1}+\frac{1}{2}\int\frac{dx}{x^2-x+1}$$

$$=\frac{1}{3}\ell\eta\left|x+1\right|-\frac{1}{6}\ell\eta\left|x^2-x+1\right|+\frac{1}{2}\int\frac{dx}{(x^2-x+1/4)+3/4}$$

$$=\frac{1}{3}\ell\eta\left|x+1\right|-\frac{1}{6}\ell\eta\left|x^2-x+1\right|+\frac{1}{2}\int\frac{dx}{(x-1/2)^2+(\sqrt{3}/2)^2}$$

$$=\frac{1}{3}\ell\eta\left|x+1\right|-\frac{1}{6}\ell\eta\left|x^2-x+1\right|+\frac{1}{2}\frac{1}{\sqrt{3}/2}\operatorname{arc}\tau g\,\frac{x-1/2}{\sqrt{3}/2}+c$$

$$=\frac{1}{3}\ell\eta\left|x+1\right|-\frac{1}{6}\ell\eta\left|x^2-x+1\right|+\frac{\sqrt{3}}{3}\operatorname{arc}\tau g\,\frac{2x-1}{\sqrt{3}}+c$$

$$=\ell\eta\left|\frac{\sqrt[3]{x+1}}{\sqrt[6]{x^2-x+1}}\right|+\frac{\sqrt{3}}{3}\operatorname{arc}\tau g\,\frac{2x-1}{\sqrt{3}}+c$$

7.16.- $\int\frac{(x+5)dx}{x^2-x+6}$

Solución.-

$\int\frac{(x+5)dx}{x^2-x+6}=\int\frac{(x+5)dx}{(x+3)(x-2)}=\int\frac{Adx}{(x+3)}+\int\frac{Bdx}{(x-2)}$ $(*)$, luego:

$$\frac{(x+5)}{(x^2+x-6)}=\frac{A}{(x+3)}+\frac{B}{(x-2)}\Rightarrow x+5=A(x-2)+B(x+3)$$

$$\therefore\begin{cases}x=2\Rightarrow 7=5B\Rightarrow B=7/5\\ x=-3\Rightarrow 2=-5A\Rightarrow A=-2/5\end{cases}$$

$$(*)=-\frac{2}{5}\int\frac{dx}{x+3}+\frac{7}{5}\int\frac{dx}{x-2}=-\frac{2}{5}\ell\eta|x+3|+\frac{2}{5}\ell\eta|x-2|+c=\frac{1}{5}\ell\eta\left|\frac{(x-2)^7}{(x+3)^2}\right|+c$$

7.17.- $\int\frac{(x^2+1)dx}{x^3+1}$

Solución.-

$\int\frac{(x^2+1)dx}{x^3+1}=\int\frac{(x^2+1)dx}{(x+1)(x^2-x+1)}=\int\frac{Adx}{(x+1)}+\int\frac{(Bx+C)dx}{(x^2-x+1)}$ $(*)$, luego:

$$\frac{(x^2+1)}{x^3+1}=\frac{A}{(x+1)}+\frac{Bx+C}{(x^2-x+1)}\Rightarrow x^2+1=A(x^2-x+1)+(Bx+C)(x+1)$$

$$\therefore\begin{cases}x=-1\Rightarrow 2=3A\Rightarrow A=2/3\\ x=0\Rightarrow 1=A+C\Rightarrow C=1/3\\ x=1\Rightarrow 2=A+(B+C)2\Rightarrow B=1/3\end{cases}$$

$$(*)\int\frac{(x^2+1)dx}{x^3+1}=\int\frac{(x^2+1)dx}{(x+1)(x^2-x+1)}=\frac{2}{3}\int\frac{dx}{(x+1)}+\frac{1}{3}\int\frac{(x+1)dx}{(x^2-x+1)}$$

$$=\frac{2}{3}\ell\eta|x+1|+\frac{1}{3}\int\frac{\left[1/2\,(2x-1)+2/3\right]dx}{(x^2-x+1)}=\frac{2}{3}\ell\eta|x+1|+\frac{1}{6}\int\frac{(2x-1)dx}{(x^2-x+1)}+\frac{1}{2}\int\frac{dx}{(x^2-x+1)}$$

$$=\frac{2}{3}\ell\eta|x+1|+\frac{1}{6}\ell\eta|x^2-x+1|+\frac{1}{2}\int\frac{dx}{(x^2-x+1)}$$

$$=\frac{2}{3}\ell\eta|x+1|+\frac{1}{6}\ell\eta|x^2-x+1|+\frac{1}{2}\int\frac{dx}{(x^2-x+1/4)+3/4}$$

$$=\frac{4}{6}\ell\eta|x+1|+\frac{1}{6}\ell\eta|x^2-x+1|+\frac{1}{2}\int\frac{dx}{(x-1/2)^2+(\sqrt{3}/2)^2}$$

$$=\frac{1}{6}\ell\eta\left|(x+1)^4(x^2-x+1)\right|+\frac{1}{2}\frac{1}{\frac{\sqrt{3}}{2}}\operatorname{arc}\tau g\,\frac{x-1/2}{\frac{\sqrt{3}}{2}}+c$$

$$=\frac{1}{6}\ell\eta\left|(x+1)^4(x^2-x+1)\right|+\frac{\sqrt{3}}{3}\operatorname{arc}\tau g\,\frac{2x-1}{\sqrt{3}}+c$$

7.18.- $\int\frac{(x^2+6)dx}{(x-1)^2(x-2)}$

Solución.-

$\int\frac{(x^2+6)dx}{(x-1)^2(x-2)}=\int\frac{Adx}{(x+1)}+\int\frac{Bdx}{(x-1)^2}+\int\frac{Cdx}{(x+2)}$ $(*)$, luego:

$$\frac{(x^2+6)}{(x-1)^2(x-2)}=\frac{A}{(x+1)}+\frac{B}{(x-1)^2}+\frac{C}{(x+2)}$$

$$x^2+6=A(x+1)+(x+2)+B(x+2)+C(x-1)^2$$

$$\therefore\begin{cases}x=1\Rightarrow 7=3B\Rightarrow B=7/3\\ x=-2\Rightarrow 10=9C\Rightarrow C=10/9\\ x=0\Rightarrow 6=-2A+B+C\Rightarrow A=-1/9\end{cases}$$

$$(*)=-\frac{1}{9}\int\frac{dx}{(x+1)}+\frac{7}{3}\int\frac{dx}{(x-1)^2}+\frac{10}{9}\int\frac{dx}{(x+2)}=-\frac{1}{9}\ell\eta|x-1|-\frac{7}{3}\frac{1}{x-1}+\frac{10}{9}\ell\eta|x+2|+c$$

$$=\frac{1}{9}\ell\eta\left|\frac{(x+2)^{10}}{x-1}\right|-\frac{7}{3(x-1)}+c$$

7.19.- $\int\frac{(x^2-1)dx}{(x^2+1)(x-2)}$

Solución.-

$\int\frac{(x^2-1)dx}{(x^2+1)(x-2)}=\int\frac{Ax+B}{(x^2+1)}dx+\int\frac{Cdx}{(x-2)}$ $(*)$, luego:

$$\frac{(x^2-1)}{(x^2+1)(x-2)}=\frac{Ax+B}{(x^2+1)}+\frac{C}{(x-2)}\Rightarrow x^2-1=(Ax+B)(x-2)+C(x^2+1)$$

$$\therefore\begin{cases}x=2\Rightarrow 3=5C\Rightarrow C=3/5\\ x=0\Rightarrow -1=-2B+C\Rightarrow B=4/5\\ x=1\Rightarrow 0=-(A+B)+2C\Rightarrow A=2/5\end{cases}$$

$$(*)=\int\frac{(2/5\,x+4/5)dx}{(x^2+1)}+\int\frac{3/5\,dx}{(x-2)}=\frac{1}{5}\int\frac{2xdx}{(x^2+1)}+\frac{4}{5}\int\frac{dx}{(x^2+1)}+\frac{3}{5}\int\frac{dx}{x-2}$$

$$=\frac{1}{5}\ell\eta\left|x^2+1\right|+\frac{4}{5}\operatorname{arc}x+\frac{3}{5}\ell\eta|x-2|+c=\frac{1}{5}\ell\eta\left|(x^2+1)(x-2)^3\right|+\frac{4}{5}\operatorname{arc}x+c$$

7.20.- $\int \frac{xdx}{x^2-4x-5}$

Solución.-

$$\int \frac{xdx}{x^2-4x-5} = \int \frac{xdx}{(x+5)(x-1)} = \int \frac{Adx}{(x+5)} + \int \frac{Bdx}{(x-1)}$$ $(*)$, luego:

$$\frac{x}{(x+5)(x-1)} = \frac{A}{(x+5)} + \frac{B}{(x-1)} \Rightarrow x = A(x-1) + B(x+5)$$

$$\therefore \begin{cases} x = 1 \Rightarrow 1 = 6B \Rightarrow B = 1/6 \\ x = -5 \Rightarrow -5 = -6A \Rightarrow A = 5/6 \end{cases}$$

$$(*) = \frac{5}{6}\int \frac{dx}{(x+5)} + \frac{1}{6}\int \frac{dx}{(x-1)} = \frac{5}{6}\ell\eta\left|x+5\right| + \frac{1}{6}\ell\eta\left|x-1\right| + c = \frac{5}{6}\ell\eta\left|(x+5)^5(x-1)\right| + c$$

7.21.- $\int \frac{xdx}{x^2-2x-3}$

Solución.-

$$\int \frac{xdx}{x^2-2x-3} = \int \frac{xdx}{(x-3)(x+1)} = \int \frac{Adx}{(x-3)} + \int \frac{Bdx}{(x+1)}$$ $(*)$, luego:

$$\frac{x}{(x-3)(x+1)} = \frac{A}{(x-3)} + \frac{B}{(x+1)} \Rightarrow x = A(x+1) + B(x-3)$$

$$\therefore \begin{cases} x = -1 \Rightarrow -1 = -4B \Rightarrow B = 1/4 \\ x = 3 \Rightarrow 3 = 4A \Rightarrow A = 3/4 \end{cases}$$

$$(*) = \frac{3}{4}\int \frac{dx}{(x-3)} + \frac{1}{4}\int \frac{B}{(x+1)} = \frac{3}{4}\ell\eta\left|x-3\right| + \frac{1}{4}\ell\eta\left|x+1\right| + c = \frac{1}{4}\ell\eta\left|(x-3)^3(x+1)\right| + c$$

7.22.- $\int \frac{(x+1)dx}{x^2+4x-5}$

Solución.-

$$\int \frac{(x+1)dx}{x^2+4x-5} = \int \frac{(x+1)dx}{(x+5)(x-1)} = \int \frac{Adx}{(x+5)} + \int \frac{Bdx}{(x-1)}$$ $(*)$, luego:

$$\frac{x+1}{(x^2+4x-5)} = \frac{A}{(x+5)} + \frac{B}{(x-1)} \Rightarrow x+1 = A(x-1) + B(x+5)$$

$$\therefore \begin{cases} x = 1 \Rightarrow 2 = 6B \Rightarrow B = 1/3 \\ x = -5 \Rightarrow 3 = -4A \Rightarrow -6A = 2/3 \end{cases}$$

$$(*) = \frac{2}{3}\int \frac{dx}{(x+5)} + \frac{1}{3}\int \frac{B}{(x-1)} = \frac{2}{3}\ell\eta\left|x+5\right| + \frac{1}{3}\ell\eta\left|x-1\right| + c = \frac{1}{3}\ell\eta\left|(x+5)^2(x-1)\right| + c$$

7.23.- $\int \frac{x^2dx}{x^2+2x+1}$

Solución.-

$$\int \frac{x^2dx}{x^2+2x+1} = \int \left(1 - \frac{2x+1}{x^2+2x+1}\right)dx = \int dx - \int \frac{(2x+1)dx}{x^2+2x+1} = \int dx - \int \frac{(2x+1)dx}{(x+1)^2}$$

$= x - \left[\int \frac{Adx}{(x+1)} + \int \frac{Bdx}{(x+1)^2}\right]$ $(*)$, luego:

$$\frac{2x+1}{(x+1)^2} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} \Rightarrow 2x+1 = A(x+1) + B$$

$$\therefore \begin{cases} x = -1 \Rightarrow -1 = B \Rightarrow B = -1 \\ x = 0 \Rightarrow 1 = A + B \Rightarrow A = 2 \end{cases}$$

$$(*) = x - \left[2\int \frac{dx}{(x+1)} - \int \frac{dx}{(x+1)^2}\right] = x - \left[2\ell\eta|x+1| + \frac{1}{x+5}\right] + c = x - 2\ell\eta|x+1| - \frac{1}{x+5} + c$$

7.24.- $\int \frac{dx}{x(x+1)^2}$

Solución.-

$\int \frac{dx}{x(x+1)^2} = \int \frac{Adx}{x} + \int \frac{Bdx}{(x+1)} + \int \frac{Cdx}{(x+1)^2}$ $(*)$, luego:

$$\frac{1}{x(x+1)^2} = \frac{A}{x} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2} \Rightarrow 1 = A(x+1)^2 + Bx(x+1) + Cx$$

$$\therefore \begin{cases} x = -1 \Rightarrow 1 = -C \Rightarrow C = -1 \\ x = 0 \Rightarrow 1 = A \Rightarrow A = 1 \\ x = 1 \Rightarrow 1 = 4A + 2B + C \Rightarrow B = -1 \end{cases}$$

$$(*) = \int \frac{dx}{x} - \int \frac{dx}{(x+1)} - \int \frac{dx}{(x+1)^2} = \ell\eta|x| - \ell\eta|x+1| + \frac{1}{x+1} + c = \ell\eta\left|\frac{x}{x+1}\right| + \frac{1}{x+1} + c$$

7.25.- $\int \frac{dx}{(x+1)(x^2+1)}$

Solución.-

$\int \frac{dx}{(x+1)(x+1)^2} = \int \frac{Adx}{x+1} + \int \frac{Bx+C}{(x^2+1)}dx$ $(*)$, luego:

$$\frac{1}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{(x^2+1)} \Rightarrow 1 = A(x^2+1) + (Bx+C)(x+1)$$

$$\therefore \begin{cases} x = -1 \Rightarrow 1 = 2A \Rightarrow A = \frac{1}{2} \\ x = 0 \Rightarrow 1 = A + C \Rightarrow C = \frac{1}{2} \\ x = 1 \Rightarrow 1 = 2A + (B+C)2 \Rightarrow B = \frac{-1}{2} \end{cases}$$

$$(*) \quad = \frac{1}{2}\int \frac{dx}{(x+1)} + \int \frac{(-\frac{1}{2}x + \frac{1}{2})dx}{(x^2+1)} = \frac{1}{2}\ell\eta|x+1| - \frac{1}{2}\int \frac{x-1}{(x^2+1)}dx$$

$$= \frac{1}{2}\ell\eta|x+1| - \frac{1}{4}\int \frac{2xdx}{(x^2+1)} + \frac{1}{2}\int \frac{dx}{(x^2+1)} = \frac{1}{2}\ell\eta|x+1| - \frac{1}{4}\ell\eta|x^2+1| + \frac{1}{2}\operatorname{arc}\tau gx + c$$

$$= \frac{1}{4}\ell\eta\left|\frac{(x+1)^2}{x^2+1}\right| + \frac{1}{2}\operatorname{arc}\tau g x + c$$

7.26.- $\int \frac{dx}{x(x^2+x+1)}$

Solución.-

$\int \frac{dx}{x(x^2+x+1)} = \int \frac{Adx}{x} + \int \frac{Bx+C}{(x^2+x+1)}dx$ $(*)$, luego:

$$\frac{1}{x(x^2+x+1)} = \frac{A}{x} + \frac{Bx+C}{(x^2+x+1)} \Rightarrow 1 = A(x^2+x+1) + (Bx+C)x$$

$$\therefore \begin{cases} x=0 \Rightarrow 1 = A \Rightarrow A = 1 \\ x=1 \Rightarrow 1 = 3A+B+C \Rightarrow B+C = -2 \\ x=-1 \Rightarrow 1 = A+B-C \Rightarrow B-C = 0 \end{cases}$$

$$(*) \quad = \int \frac{dx}{x} - \int \frac{(x+1)dx}{(x^2+x+1)} = \ell\eta|x+1| - \frac{1}{2}\int \frac{(2x+2)dx}{(x^2+x+1)}$$

$$= \ell\eta|x| - \frac{1}{2}\int \frac{(2x+1)+1}{(x^2+x+1)}dx = \ell\eta|x| - \frac{1}{2}\int \frac{(2x+1)dx}{(x^2+x+1)} - \frac{1}{2}\int \frac{dx}{(x^2+x+1)}$$

$$= \ell\eta|x| - \frac{1}{2}\ell\eta|x^2+x+1| - \frac{1}{2}\int \frac{dx}{(x^2+x+1/4)+3/4}$$

$$= \ell\eta|x| - \frac{1}{2}\ell\eta|x^2+x+1| - \frac{1}{2}\int \frac{dx}{(x+1/2)^2+(\sqrt{3}/2)^2}$$

$$= \ell\eta|x| - \frac{1}{2}\ell\eta|x^2+x+1| - \frac{1}{2}\frac{1}{\sqrt{3}/2}\operatorname{arc}\tau g\,\frac{x+1/2}{\sqrt{3}/2} + c$$

$$= \ell\eta|x| - \frac{1}{2}\ell\eta|x^2+x+1| - \frac{\sqrt{3}}{3}\operatorname{arc}\tau g\,\frac{2x+1}{\sqrt{3}} + c$$

7.27.- $\int \frac{2x^2+5x-1}{x^3+x^2-2x}dx$

Solución.-

$\int \frac{(2x^2+5x-1)dx}{(x^3+x^2-2x)} = \int \frac{Adx}{x} + \int \frac{Bdx}{(x-1)} + \int \frac{Cdx}{(x+2)}$ $(*)$, luego:

$$\frac{2x^2+5x-1}{(x^3+x^2-2x)} = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x+2)}$$

$$2x^2+5x-1 = A(x-1)(x+2) + Bx(x+2) + Cx(x-1)$$

$$\therefore \begin{cases} x=0 \Rightarrow -1 = -2A \Rightarrow A = 1/2 \\ x=1 \Rightarrow 6 = 3B \Rightarrow B = 2 \\ x=-2 \Rightarrow -3 = 6C \Rightarrow C = -1/2 \end{cases}$$

$$(*)=\frac{1}{2}\int\frac{dx}{x}+2\int\frac{dx}{(x-1)}-\frac{1}{2}\int\frac{dx}{(x+2)}=\frac{1}{2}\ell\eta\left|x\right|+2\ell\eta\left|x-1\right|-\frac{1}{2}\ell\eta\left|x+2\right|+c$$

7.28.- $\int\frac{x^2+2x+3}{(x-1)(x+1)^2}dx$

Solución.-

$\int\frac{x^2+2x+3}{(x-1)(x+1)^2}dx=\int\frac{Adx}{(x-1)}+\int\frac{Bdx}{(x+1)}+\int\frac{Cdx}{(x+1)^2}$ $(*)$, luego:

$$\frac{x^2+2x+3}{(x-1)(x+1)^2}=\frac{A}{(x-1)}+\frac{B}{(x-1)}+\frac{C}{(x+1)^2}$$

$$x^2+2x+3=A(x+1)^2+B(x-1)(x+1)+C(x-1)$$

$$\therefore\begin{cases}x=1\Rightarrow 6=4A\Rightarrow A=\frac{3}{2}\\ x=-1\Rightarrow 2=-2C\Rightarrow C=-1\\ x=0\Rightarrow 3=A-B-C\Rightarrow B=-\frac{1}{2}\end{cases}$$

$$(*)=\frac{3}{2}\int\frac{dx}{x-1}-\frac{1}{2}\int\frac{dx}{x+1}-\int\frac{dx}{(x+1)^2}=\frac{3}{2}\ell\eta\left|x-1\right|-\frac{1}{2}\ell\eta\left|x+1\right|+\frac{1}{x+1}+c$$

$$=\frac{1}{2}\ell\eta\left|\frac{(x-1)^3}{x+1}\right|+\frac{1}{x+1}+c$$

7.29.- $\int\frac{3x^2+2x-2}{x^3-1}dx$

Solución.-

$\int\frac{3x^2+2x-2}{x^3-1}dx=\int\frac{3x^2+2x-2}{(x-1)(x^2+x+1)}dx=\int\frac{Adx}{x-1}+\int\frac{(Bx+C)dx}{(x^2+x+1)}$ $(*)$, luego:

$$\frac{3x^2+2x-2}{(x-1)(x^2+x+1)}=\frac{A}{x-1}+\frac{Bx+C}{(x^2+x+1)}$$

$$3x^2+2x-2=A(x^2+x+1)+(Bx+C)(x-1)$$

$$\therefore\begin{cases}x=1\Rightarrow 3=3A\Rightarrow A=1\\ x=0\Rightarrow -2=A-C\Rightarrow C=3\\ x=-1\Rightarrow -1=A+(-B+C)(-2)\Rightarrow B=2\end{cases}$$

$$(*)=\int\frac{dx}{x-1}+\int\frac{(2x+3)dx}{(x^2+x+1)}=\ell\eta\left|x-1\right|+\int\frac{(2x+1)+2}{(x^2+x+1)}dx$$

$$=\ell\eta\left|x-1\right|+\int\frac{(2x+1)dx}{(x^2+x+1)}+2\int\frac{dx}{(x^2+x+1)}$$

$$=\ell\eta\left|x-1\right|+\ell\eta\left|x^2+x+1\right|+2\int\frac{dx}{(x+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}$$

$$= \ell\eta\left|(x-1)(x^2+x+1)\right| + 2\frac{1}{\sqrt{3}/2}\operatorname{arc}\tau g\,\frac{x+1/2}{\sqrt{3}/2}+c$$

$$= \ell\eta\left|(x-1)(x^2+x+1)\right| + \frac{4\sqrt{3}}{3}\operatorname{arc}\tau g\,\frac{2x+1}{\sqrt{3}}+c$$

7.30.- $\int \frac{x^4-x^3+2x^2-x+2}{(x-1)(x^2+2)^2}dx$

Solución.-

$\int \frac{x^4-x^3+2x^2-x+2}{(x-1)(x^2+2)^2}dx = \int \frac{Adx}{x-1}+\int\frac{(Bx+C)dx}{(x^2+2)}+\int\frac{(Dx+E)dx}{(x^2+2)^2}$ $(*)$, luego:

$$\frac{x^4-x^3+2x^2-x+2}{(x-1)(x^2+2)^2} = \frac{A}{x-1}+\frac{Bx+C}{(x^2+2)}+\frac{Dx+E}{(x^2+2)^2}$$

$$x^4-x^3+2x^2-x+2 = A(x^2+2)^2+(Bx+C)(x-1)(x^2+2)+(Dx+E)(x-1)$$

$$= A(x^4+4x^2+4)+(Bx+C)(x^3+2x-x^2-2)+Dx^2-Dx+Ex-E$$

$$= Ax^4+4Ax^2+4A+Bx^4+2Bx^2-Bx^3-2Bx+Cx^3+2Cx-Cx^2-2C$$

$$\Rightarrow +Dx^2-Dx+Ex-E$$

$$=(A+B)x^4+(C-B)x^3+(4A-C+2B+D)x^2+(-2B+2C-D+E)x+(4A-2C-E)$$

Igualando coeficientes, se tiene:

$$\begin{pmatrix} A+B & =1 \\ -B+C & =-1 \\ 4A+2B-C+D & =2 \\ -2B+2C-D+E & =-1 \\ 4A-2C-E & =2 \end{pmatrix} \therefore A=1/3, B=2/3, C=-1/3, D=-1, E=0$$

$$(*) = \frac{1}{3}\int\frac{dx}{x-1}+\int\frac{(\frac{2}{3}x-\frac{1}{3})dx}{(x^2+2)}-\int\frac{xdx}{(x^2+2)^2}$$

$$=\frac{1}{3}\int\frac{dx}{x-1}+\frac{1}{3}\int\frac{2xdx}{(x^2+2)}-\frac{1}{3}\int\frac{dx}{(x^2+2)}-\frac{1}{2}\int\frac{2xdx}{(x^2+2)^2}$$

$$=\frac{1}{3}\ell\eta\left|x-1\right|+\frac{1}{3}\ell\eta\left|x^2+2\right|-\frac{\sqrt{2}}{6}\operatorname{arc}\tau g\,\frac{x}{\sqrt{2}}+\frac{1}{2}\frac{1}{x^2+2}+c$$

$$=\frac{1}{3}\ell\eta\left|(x-1)(x^2+2)\right|-\frac{\sqrt{2}}{6}\operatorname{arc}\tau g\,\frac{x}{\sqrt{2}}+\frac{1}{2(x^2+2)}+c$$

7.31.- $\int\frac{2x^2-7x-1}{x^3+x^2-x-1}dx$

Solución.-

$\int\frac{2x^2-7x-1}{x^3+x^2-x-1}dx = \int\frac{2x^2-7x-1}{(x-1)(x+1)^2}dx = \int\frac{Adx}{x-1}+\int\frac{Bdx}{(x+1)}+\int\frac{Cdx}{(x+1)^2}$ $(*)$, luego:

$$\frac{2x^2-7x-1}{(x^3+x^2-x-1)}=\frac{A}{x-1}+\frac{B}{(x+1)}+\frac{C}{(x+1)^2}$$

$$2x^2-7x-1=A(x+1)^2+B(x-1)(x+1)+C(x-1)$$

$$\therefore\begin{cases} x=-1\Rightarrow 8=-2C\Rightarrow C=-4\\ x=1\Rightarrow -6=4A\Rightarrow A=-3/2\\ x=0\Rightarrow -1=A-B-C\Rightarrow B=7/2\end{cases}$$

$$(*)=-\frac{3}{2}\int\frac{dx}{x-1}+\frac{7}{2}\int\frac{dx}{x+1}-4\int\frac{dx}{(x+1)^2}=-\frac{3}{2}\ell\eta\left|x-1\right|+\frac{7}{2}\ell\eta\left|x+1\right|+\frac{4}{x+1}+c$$

$$=-\frac{1}{2}\ell\eta\left|\frac{(x+1)^7}{(x-1)^3}\right|+\frac{4}{x+1}+c$$

7.32.- $\int\frac{3x^2+3x+1}{x^3+2x^2+2x+1}dx$

Solución.-

$$\int\frac{3x^2+3x+1}{x^3+2x^2+2x+1}dx=\int\frac{(3x^2+3x+1)dx}{(x+1)(x^2+x+1)}=\int\frac{Adx}{x+1}+\int\frac{(Bx+C)dx}{(x^2+x+1)}$$ $(*)$, luego:

$$\frac{3x^2+3x+1}{(x+1)(x^2+x+1)}=\frac{A}{x+1}+\frac{Bx+C}{(x^2+x+1)}$$

$$3x^2+3x+1=A(x^2+x+1)+(Bx+C)(x+1)$$

$$\therefore\begin{cases} x=-1\Rightarrow A=1\\ x=0\Rightarrow 1=A+C\Rightarrow C=0\\ x=1\Rightarrow 7=3A+(B+C)(2)\Rightarrow B=2\end{cases}$$

$$(*)=\int\frac{dx}{x+1}+\int\frac{2xdx}{(x^2+x+1)}=\ell\eta\left|x+1\right|+\int\frac{(2x+1)-1}{(x^2+x+1)}dx$$

$$=\ell\eta\left|x+1\right|+\int\frac{(2x+1)dx}{(x^2+x+1)}-\int\frac{dx}{(x^2+x+1)}$$

$$=\ell\eta\left|x+1\right|+\ell\eta\left|x^2+x+1\right|-\int\frac{dx}{(x^2+x+1/4)+(\sqrt{3}/2)^2}$$

$$=\ell\eta\left|x+1\right|+\ell\eta\left|x^2+x+1\right|-\frac{1}{\sqrt{3}/2}\operatorname{arc}\tau g\,\frac{x+1/2}{\sqrt{3}/2}+c$$

$$=\ell\eta\left|(x+1)(x^2+x+1)\right|-\frac{2\sqrt{3}}{3}\operatorname{arc}\tau g\,\frac{2x+1}{\sqrt{3}}+c$$

7.33.- $\int\frac{x^3+7x^2-5x+5}{(x-1)^2(x+1)^2}dx$

Solución.-

$$\int \frac{x^3+7x^2-5x+5}{(x-1)^2(x+1)^3}dx = \int \frac{Adx}{x-1} + \int \frac{Bdx}{(x-1)^2} + \int \frac{Cdx}{(x+1)} + \int \frac{Ddx}{(x+1)^2} + \int \frac{Edx}{(x+1)^3}$$ $(*)$, luego:

$$\frac{x^3+7x^2-5x+5}{(x-1)^2(x+1)^3} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1} + \frac{D}{(x+1)^2} + \frac{E}{(x+1)^3}$$

$x^3+7x^2-5x+5 = A(x-1)(x+1)^3 + B(x+1)^3 + C(x-1)^2(x+1)^2$

$\Rightarrow +D(x-1)^2(x+1) + E(x-1)^2$

$= Ax^4 + 2Ax^3 - 2Ax - A + Bx^3 + 3Bx^2 + 3Bx + B + Cx^4 - 2Cx^2 + C$

$\Rightarrow +Dx^3 - Dx^2 - Dx + D + Ex^2 - 2Ex + E$

$= (A+C)x^4 + (2A+B+D)x^3 + (3B-2C-D+E)x^2$

$\Rightarrow +(-2A+3B-D-2E)x + (-A+B+C+D+E)$

Igualando coeficientes, se tiene:

$$\begin{pmatrix} A & & +C & & & = & 0 \\ 2A & +B & & +D & & = & 1 \\ & +3B & -2C & -D & +E & = & 7 \\ -2A & +3B & & -D & -2E & = & -5 \\ -A & +B & +C & +D & +E & = & 2 \end{pmatrix} \therefore A=0, B=1, C=0, D=0, E=4$$

$$(*) = \int \frac{dx}{(x-1)^2} + 4\int \frac{dx}{(x+1)^3} = -\frac{1}{x-1} - \frac{2}{(x+1)^2} + c = -\frac{x^2-4x-1}{(x-1)(x+1)^2} + c$$

7.34.- $\int \frac{2xdx}{(x^2+x+1)^2}$

Solución.-

$$\int \frac{2xdx}{(x^2+x+1)^2} = \int \frac{(Ax+B)dx}{x^2+x+1} + \int \frac{(Cx+D)dx}{(x^2+x+1)^2}$$ $(*)$, luego:

$$\frac{2x}{(x^2+x+1)^2} = \frac{Ax+B}{x^2+x+1} + \frac{Cx+D}{(x^2+x+1)^2}$$

$2x = (Ax+B)(x^2+x+1) + Cx + D \Rightarrow 2x = Ax^3 + Ax^2 + Ax + Bx^2 + Bx + B + Cx + D$

$= Ax^3 + (A+B)x^2 + (A+B+C)x + B + D$, igualando coeficientes se tiene:

$$\begin{pmatrix} A & & & & = 0 \\ A+B & & & & = 0 \\ A+B & +C & & & = 2 \\ & & & +D & = 0 \end{pmatrix}$$

$\therefore A=0, B=0, C=2, D=0$

$(*) = \int \frac{2xdx}{(x^2+x+1)}$, de donde el método sugerido pierde aplicabilidad; tal como se había planteado la técnica trabajada debe ser sustituida por otra:

$$\int \frac{2xdx}{(x^2+x+1)} = \int \frac{(2x+1)dx}{(x^2+x+1)} - \int \frac{dx}{(x^2+x+1)^2}$$

$$=\int\frac{(2x+1)dx}{(x^2+x+1)}-\frac{16}{9}\int\frac{dx}{\left\{\left[\frac{2}{\sqrt{3}}(x+\frac{1}{2})\right]^2+1\right\}}(**)$$

sea: $u=\frac{2}{\sqrt{3}}(x+\frac{1}{2}), dx=\frac{\sqrt{3}}{2}du$, entonces:

$(**)\ -\frac{1}{x^2+x+1}-\frac{16}{9}\frac{\sqrt{3}}{2}\int\frac{du}{(u^2+1)^2}$, trabajando la integral sustituyendo trigonométricamente:

$=-\frac{1}{x^2+x+1}-\frac{8\sqrt{3}}{9}\int\frac{\sec^2\theta d\theta}{\sec^4\theta}$, ya que: $u=\tau g\theta, du=\sec^2\theta d\theta$

$$=-\frac{1}{x^2+x+1}-\frac{8\sqrt{3}}{9}\left[\frac{1}{2}\operatorname{arc}\tau g u+\frac{1}{2}\frac{u}{(u^2+1)}\right]$$

$$=-\frac{1}{x^2+x+1}-\frac{8\sqrt{3}}{9}\left\{\frac{1}{2}\operatorname{arc}\tau g\frac{2}{\sqrt{3}}(x+\frac{1}{2})+\frac{\frac{2}{\sqrt{3}}(x+\frac{1}{2})}{2\left[\frac{4}{3}(x+\frac{1}{2})^2+1\right]}\right\}+c$$

$$=-\frac{1}{x^2+x+1}-\frac{8\sqrt{3}}{9}\left\{\frac{1}{2}\operatorname{arc}\tau g\frac{2}{\sqrt{3}}(x+\frac{1}{2})+\frac{x+\frac{1}{2}}{\sqrt{3}\left[\frac{4}{3}(x+\frac{1}{2})^2+1\right]}\right\}+c$$

$$=-\frac{1}{x^2+x+1}-\frac{4\sqrt{3}}{9}\operatorname{arc}\tau g\frac{2}{\sqrt{3}}(x+\frac{1}{2})-\frac{8}{9}\frac{(x+\frac{1}{2})}{\left[\frac{4}{3}(x+\frac{1}{2})^2+1\right]}+c$$

7.35.- $\int\frac{x^2+2x+3}{x^3-x}dx$

Solución.-

$\int\frac{x^2+2x+3}{x^3-x}dx=\int\frac{x^2+2x+3}{x(x-1)(x+1)}dx=\int\frac{Adx}{x}+\int\frac{Bdx}{(x-1)}+\int\frac{Cdx}{(x+1)}\ (*)$, luego:

$$\frac{x^2+2x+3}{x(x-1)(x+1)}=\frac{A}{x}+\frac{B}{(x-1)}+\frac{C}{(x+1)}$$

$$x^2+2x+3=A(x-1)(x+1)+Bx(x+1)+Cx(x-1)$$

$$\therefore\begin{cases}x=0\Rightarrow 3=-A\Rightarrow A=-3\\x=-1\Rightarrow 2=2C\Rightarrow C=1\\x=1\Rightarrow 6=2B\Rightarrow B=3\end{cases}$$

$$(*)=-3\int\frac{dx}{x}+3\int\frac{dx}{(x-1)}+\int\frac{dx}{(x+1)}=-3\ell\eta|x|+3\ell\eta|x-1|+\ell\eta|x+1|+c$$

$$=\ell\eta\left|\frac{(x-1)^3(x+1)}{x^3}\right|+c$$

7.36.- $\int \frac{(2x^2-3x+5)dx}{(x+2)(x-1)(x-3)}$

Solución.-

$$\int \frac{2x^2-3x+5}{(x+2)(x-1)(x-3)}dx = \int \frac{Adx}{(x+2)} + \int \frac{Bdx}{(x-1)} + \int \frac{Cdx}{(x-3)}$$ $(*)$, luego:

$$\frac{2x^2-3x+5}{(x+2)(x-1)(x-3)} = \frac{A}{x+2} + \frac{B}{x-1} + \frac{C}{x-3}$$

$$2x^2-3x+5 = A(x-1)(x-3) + B(x+2)(x-3) + C(x+2)(x-1)$$

$$\therefore \begin{cases} x=1 \Rightarrow 4=-6B \Rightarrow B=-2/3 \\ x=3 \Rightarrow 14=10C \Rightarrow C=7/5 \\ x=-2 \Rightarrow 19=15A \Rightarrow A=19/15 \end{cases}$$

$$(*) = \frac{19}{15}\int \frac{dx}{x+2} - \frac{2}{3}\int \frac{dx}{x-1} + \frac{7}{5}\int \frac{dx}{x-3} = \frac{19}{15}\ell\eta\left|x+2\right| - \frac{2}{3}\ell\eta\left|x-1\right| + \frac{7}{5}\ell\eta\left|x-3\right| + c$$

7.37.- $\int \frac{3x^2+x-2}{(x-1)(x^2+1)}dx$

Solución.-

$$\int \frac{3x^2+x-2}{(x-1)(x^2+1)}dx = \int \frac{Adx}{(x-1)} + \int \frac{(Bx+C)dx}{(x^2+1)}$$ $(*)$, luego:

$$\frac{3x^2+x-2}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

$$3x^2+x-2 = A(x^2+1) + (Bx+C)(x-1)$$

$$\therefore \begin{cases} x=1 \Rightarrow 2=2A \Rightarrow A=1 \\ x=0 \Rightarrow -2=A-C \Rightarrow C=3 \\ x=2 \Rightarrow 12=5A+2B+C \Rightarrow B=2 \end{cases}$$

$$(*) = \int \frac{dx}{x-1} + \int \frac{(2x+3)dx}{x^2+1} = \int \frac{dx}{x-1} + \int \frac{2xdx}{x^2+1} + 3\int \frac{dx}{x^2+1}$$

$$= \ell\eta\left|x-1\right| + \ell\eta\left|x^2+1\right| + 3\operatorname{arc}\tau gx + c = \ell\eta\left|(x-1)(x^2+1)\right| + 3\operatorname{arc}\tau gx + c$$

7.38.- $\int \frac{(x+5)dx}{x^3-3x+2}$

Solución.-

$$\int \frac{(x+5)dx}{x^3-3x+2} = \int \frac{(x+5)dx}{(x-1)^2(x+2)} = \int \frac{Adx}{(x-1)} + \int \frac{Bdx}{(x-1)^2} + \int \frac{Cdx}{(x+2)}$$ $(*)$, luego:

$$\frac{x+5}{x^3-3x+2} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)}$$

$$x+5 = A(x-1)(x+2) + B(x+2) + C(x-1)^2$$

$$\therefore \begin{cases} x=1 \Rightarrow 6=3B \Rightarrow B=2 \\ x=-2 \Rightarrow 3=9C \Rightarrow C=\frac{1}{3} \\ x=0 \Rightarrow 5=-2A+B+C \Rightarrow A=-\frac{1}{3} \end{cases}$$

$$(*) = -\frac{1}{3}\int\frac{dx}{(x-1)}+2\int\frac{dx}{(x-1)^2}+\frac{1}{3}\int\frac{dx}{(x+2)} = -\frac{1}{3}\ell\eta|x-1|-\frac{2}{x-1}+\frac{1}{3}\ell\eta|x+2|+c$$

$$=\frac{1}{3}\ell\eta\left|\frac{x+2}{x-1}\right|-\frac{2}{x-1}+c$$

7.39.- $\int\frac{2x^3+3x^2+x-1}{(x+1)(x^2+2x+2)^2}dx$

Solución.-

$\int\frac{(2x^3+3x^2+x-1)dx}{(x+1)(x^2+2x+2)^2}=\int\frac{Adx}{x+1}+\int\frac{(Bx+C)dx}{(x^2+2x+2)}+\int\frac{(Dx+E)dx}{(x^2+2x+2)^2}$ $(*)$, luego:

$$\frac{2x^3+3x^2+x-1}{(x+1)(x^2+2x+2)^2}=\frac{A}{x+1}+\frac{Bx+C}{(x^2+2x+2)}+\frac{Dx+E}{(x^2+2x+2)^2}$$

$$2x^3+3x^2+x-1=A(x^2+2x+2)^2+(Bx+C)(x^2+2x+2)(x+1)+(Dx+E)(x+1)$$

$$=Ax^4+4Ax^3+8Ax^2+8Ax+4A+Bx^4+3Bx^3+4Bx^2+2Bx+Cx^3+3Cx^2+4Cx$$

$$\Rightarrow +2C+Dx^2+Dx+Ex+E$$

$$=(A+B)x^4+(4A+3B+C)x^3+(+8A+4B+3C+D)x^2$$

$$\Rightarrow +(8A+2B+4C+D+E)x+(4A+2C+E)$$

Igualando coeficientes, se tiene:

$$\begin{pmatrix} A & +B & & & & = & 0 \\ 4A & +3B & +C & & & = & 2 \\ 8A & +4B & +3C & +D & & = & 3 \\ 8A & +2B & +4C & +D & +E & = & 1 \\ 4A & & +2C & & +E & = & -1 \end{pmatrix} \therefore A=-1, B=1, C=3, D=-2, E=-3$$

$$(*) = -\int\frac{dx}{x+1}+\int\frac{(x+3)dx}{(x^2+2x+2)}-\int\frac{(2x+3)dx}{(x^2+2x+2)^2}$$

$$=-\ell\eta|x-1|+\frac{1}{2}\int\frac{(2x+6)dx}{(x^2+2x+2)}-\int\frac{(2x+2)+1dx}{(x^2+2x+2)^2}$$

$$=-\ell\eta|x-1|+\frac{1}{2}\int\frac{(2x+2)+4}{(x^2+2x+2)}dx-\int\frac{(2x+2)dx}{(x^2+2x+2)^2}-\int\frac{dx}{(x^2+2x+2)^2}$$

$$=-\ell\eta|x-1|+\frac{1}{2}\int\frac{(2x+2)dx}{(x^2+2x+2)}+2\int\frac{dx}{(x^2+2x+2)}-\int\frac{(2x+2)dx}{(x^2+2x+2)^2}-\int\frac{dx}{(x^2+2x+2)^2}$$

$$=-\ell\eta|x-1|+\frac{1}{2}\ell\eta|x^2+2x+2|+2\int\frac{dx}{(x+1)^2+1}+\frac{1}{2}\frac{1}{x^2+2x+2}-\int\frac{dx}{\left[(x+1)^2+1\right]^2}$$

$$=-\ell\eta\left|x-1\right|+\frac{1}{2}\ell\eta\left|x^2+2x+2\right|+2\operatorname{arc}\tau g(x+1)$$

$$\Rightarrow +\frac{1}{2}\frac{1}{x^2+2x+2}-\frac{1}{2}\frac{x+1}{x^2+2x+2}-\frac{1}{2}\operatorname{arc}\tau g(x+1)+c$$

$$=\ell\eta\left|\frac{\sqrt{x^2+2x+2}}{x+1}\right|+\frac{3}{2}\operatorname{arc}\tau g(x+1)-\frac{1}{2}\frac{x}{x^2+2x+2}+c$$

7.40.- $\int\frac{(2x^2+3x-1)dx}{x^3+2x^2+4x+2}$

Solución.-

$\int\frac{(2x^2+3x-1)dx}{x^3+2x^2+4x+2}=\int\frac{(2x^2+3x-1)dx}{(x+1)(x^2+2x+2)}=\int\frac{Adx}{(x+1)}+\int\frac{(Bx+C)dx}{(x^2+2x+2)}\ (*)$, luego:

$$\frac{(2x^2+3x-1)}{(x+1)(x^2+2x+2)}=\frac{A}{(x+1)}+\frac{(Bx+C)}{(x^2+2x+2)}$$

$$2x^2+3x-1=A(x^2+2x+2)+(Bx+C)(x+1)$$

$$\therefore\begin{cases}x=-1\Rightarrow -2=A\Rightarrow A=-2\\ x=0\Rightarrow -1=2A+C\Rightarrow C=3\\ x=1\Rightarrow 4=5A+(B+C)(2)\Rightarrow B=4\end{cases}$$

$$(*)=-2\int\frac{dx}{(x+1)}+\int\frac{(4x+3)dx}{x^2+2x+2}=-2\ell\eta\left|x+1\right|+2\int\frac{(2x+2)-1}{x^2+2x+2}dx$$

$$=-2\ell\eta\left|x+1\right|+2\int\frac{(2x+2)dx}{x^2+2x+2}-2\int\frac{dx}{x^2+2x+2}$$

$$=-2\ell\eta\left|x+1\right|+2\ell\eta\left|x^2+2x+2\right|-2\operatorname{arc}\tau g(x+1)+c$$

7.41.- $\int\frac{(2x+1)dx}{3x^3+2x-1}$

Solución.-

$\int\frac{(2x+1)dx}{3x^3-2x-1}=\int\frac{(2x+1)dx}{(x-1)(3x^2+3x+1)}=\int\frac{Adx}{(x-1)}+\int\frac{(Bx+C)dx}{(3x^2+3x+1)}\ (*)$, luego:

$$\frac{(2x+1)}{(3x^3-2x-1)}=\frac{A}{(x-1)}+\frac{(Bx+C)}{(3x^2+3x+1)}$$

$$2x+1=A(3x^2+3x+1)+(Bx+C)(x-1)$$

$$\therefore\begin{cases}x=1\Rightarrow 3=7A\Rightarrow A=3/7\\ x=0\Rightarrow 1=A-C\Rightarrow C=-4/7\\ x=-1\Rightarrow -1=A+(-B+C)(-2)\Rightarrow B=-9/7\end{cases}$$

$$(*)=\frac{3}{7}\int\frac{dx}{(x-1)}-\frac{1}{7}\int\frac{(9x+4)dx}{3x^2+3x+1}=\frac{3}{7}\ell\eta\left|x-1\right|-\frac{1}{7}\frac{9}{6}\int\frac{(6x+3-1/3)dx}{3x^2+3x+1}$$

$$=\frac{3}{7}\ell\eta|x-1|-\frac{3}{14}\int\frac{(6x+3)dx}{3x^2+3x+1}+\frac{1}{14}\int\frac{dx}{3x^2+3x+1}$$
$$=\frac{3}{7}\ell\eta|x-1|-\frac{3}{14}\ell\eta\left|3x^2+3x+1\right|+\frac{1}{14}\int\frac{dx}{3(x+\frac{1}{2})^2+\frac{1}{4}}$$
$$=\frac{3}{7}\ell\eta|x-1|-\frac{3}{14}\ell\eta\left|3x^2+3x+1\right|+\frac{2}{7}\int\frac{dx}{12(x+\frac{1}{2})^2+1}$$
$$=\frac{3}{7}\ell\eta|x-1|-\frac{3}{14}\ell\eta\left|3x^2+3x+1\right|+\frac{\sqrt{3}}{21}\operatorname{arc}\tau g\,2\sqrt{3}(x+\frac{1}{2})+c$$

7.42.- $\int\frac{x^4-2x^2+3x+4}{(x-1)^3(x^2+2x+2)}dx$

Solución.-

$\int\frac{x^4-2x^2+3x+4}{(x-1)^3(x^2+2x+2)}dx=\int\frac{Adx}{(x-1)}+\int\frac{Bdx}{(x-1)^2}+\int\frac{Cdx}{(x-1)^3}+\int\frac{(Dx+E)dx}{(x^2+2x+2)}$ $(*)$, luego:

$$\frac{x^4-2x^2+3x+4}{(x-1)^3(x^2+2x+2)}=\frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{C}{(x-1)^3}+\frac{Dx+E}{(x^2+2x+2)}$$
$$x^4-2x^2+3x+4=A(x-1)^2(x^2+2x+2)+B(x-1)(x^2+2x+2)$$
$$\Rightarrow +C(x^2+2x+2)+(Dx+E)(x-1)^3$$
$$x^4-2x^2+3x+4=A(x^2-2x+1)(x^2+2x+2)+B(x^3+2x^2+2x-x^2-2x-2)$$
$$\Rightarrow +C(x^2+2x+2)+(Dx+E)(x^3-3x^2+3x-1)$$
$$x^4-2x^2+3x+4=Ax^4-Ax^2-2Ax+2A+Bx^3+Bx^2-2B+Cx^2+2Cx+2C$$
$$\Rightarrow +Dx^4-3Dx^3+3Dx^2-Dx+Ex^3-3Ex^2+3Ex-E$$
$$x^4-2x^2+3x+4=(A+D)x^4+(B-3D+E)x^3+(-A+B+C+3D-3E)x^2$$
$$\Rightarrow +(-2A+2C-D+3E)x+(-2A-2B+2C-E)$$

Igualando coeficientes se tiene:

$$\begin{pmatrix} A & & & +D & & = & 1\\ & B & & -3D & +E & = & 0\\ -A & +B & +C & +3D & -3E & = & -2\\ -2A & & +2C & -D & +3E & = & 3\\ 2A & -2B & +2C & & -E & = & 4 \end{pmatrix}$$

$$\therefore A=\frac{106}{125},B=\frac{9}{25},C=\frac{6}{5},D=\frac{19}{125},E=\frac{102}{125}$$

$$(*)=\frac{106}{125}\int\frac{dx}{x+1}-\frac{9}{25}\int\frac{dx}{(x-1)^2}+\frac{6}{5}\int\frac{dx}{(x-1)^3}+\frac{1}{125}\int\frac{(19x+102)dx}{(x^2+2x+2)}$$
$$=\frac{106}{125}\ell\eta|x-1|+\frac{9}{25}\frac{1}{x-1}+\frac{6}{5}\frac{1}{(-2)(x-1)^2}+\frac{19}{125}\int\frac{(x+\frac{102}{19})dx}{(x^2+2x+2)}$$

$$=\frac{106}{125}\ell\eta\left|x-1\right|+\frac{9}{25(x-1)}-\frac{3}{5(x-1)^2}+\frac{19}{250}\int\frac{(2x+2)+8\frac{14}{19}}{(x^2+2x+2)}dx$$

$$=\frac{106}{125}\ell\eta\left|x-1\right|+\frac{9}{25(x-1)}-\frac{3}{5(x-1)^2}+\frac{19}{250}\ell\eta\left|x^2+2x+2\right|+\frac{\cancel{19}}{250}\frac{166}{\cancel{19}}\int\frac{dx}{(x^2+2x+1)+1}$$

$$=\frac{106}{125}\ell\eta\left|x-1\right|+\frac{9}{25(x-1)}-\frac{3}{5(x-1)^2}+\frac{19}{250}\ell\eta\left|x^2+2x+2\right|+\frac{166}{250}\int\frac{dx}{(x+1)^2+1}$$

$$=\frac{106}{125}\ell\eta\left|x-1\right|+\frac{9}{25(x-1)}-\frac{3}{5(x-1)^2}+\frac{19}{250}\ell\eta\left|x^2+2x+2\right|+\frac{166}{250}\operatorname{arc}\tau g(x+1)+c$$

7.43.- $\int\frac{e^t dt}{e^{2t}+3e^t+2}$

Solución.-

$\int\frac{e^t dt}{e^{2t}+3e^t+2}=\int\frac{e^t dt}{(e^t+2)(e^t+2)}\ (*)$, Sea: $u=e^t+1, du=e^t dt; e^t+2=u+1$

Luego:

$$(*)\int\frac{du}{(u+1)u}=\int\frac{Adu}{(u+1)}+\int\frac{Bdu}{u}\ (**)$$

$$\frac{1}{(u+1)u}=\frac{A}{(u+1)}+\frac{B}{u}\Rightarrow 1=Au+B(u+1)$$

$$\therefore\begin{cases}u=0\Rightarrow 1=B\Rightarrow B=1\\ u=-1\Rightarrow 1=-A\Rightarrow A=-1\end{cases}$$

$$(**)=-\int\frac{du}{(u+1)}+\int\frac{du}{u}=-\ell\eta\left|u+1\right|+\ell\eta\left|u\right|+c=-\ell\eta\left|e^t+2\right|+\ell\eta\left|e^t+1\right|+c$$

$$=\ell\eta\left|\frac{e^t+1}{e^t+2}\right|+c$$

7.44.- $\int\frac{\operatorname{sen}\theta d\theta}{\cos^2\theta+\cos\theta-2}$

Solución.-

$\int\frac{\operatorname{sen}\theta d\theta}{\cos^2\theta+\cos\theta-2}=\int\frac{\operatorname{sen}\theta d\theta}{(\cos\theta+2)(\cos\theta-1)}\ (*)$,

Sea: $u=\cos\theta-1, du=-\operatorname{sen}\theta d\theta, \cos\theta+2=u+3$

Luego:

$$(*)\int\frac{-du}{(u+3)u}=-\int\frac{du}{u(u+3)}=-\int\frac{Adu}{u}-\int\frac{Bdu}{u+3}\ (**)$$

$$\frac{1}{u(u+3)}=\frac{A}{u}+\frac{B}{u+3}\Rightarrow 1=A(u+3)+Bu$$

$$\therefore\begin{cases}u=0\Rightarrow 1=3A\Rightarrow A=\frac{1}{3}\\ u=-3\Rightarrow 1=-3B\Rightarrow B=-\frac{1}{3}\end{cases}$$

$$(**) = -\frac{1}{3}\int\frac{du}{u}+\frac{1}{3}\int\frac{du}{(u+3)} = -\frac{1}{3}\ell\eta|u|+\frac{1}{3}\ell\eta|u+3|+c$$

$= -\frac{1}{3}\ell\eta|\cos\theta-1|+\frac{1}{3}\ell\eta|\cos\theta+2|+c$, Como: $|\cos\theta|<1$, se tiene:

$$= -\frac{1}{3}\ell\eta|1-\cos\theta|+\frac{1}{3}\ell\eta|2+\cos\theta|+c = \frac{1}{3}\ell\eta\left|\frac{2+\cos\theta}{1-\cos\theta}\right|+c$$

7.45.- $\int\frac{4x^4-2x^3-x^2+3x+1}{(x^3+x^2-x-1)}dx$

Solución.-

$$\int\frac{4x^4-2x^3-x^2+3x+1}{(x^3+x^2-x-1)}dx = \int\left(4x-6+\frac{9x^2+x-5}{x^3+x^2-x-1}\right)dx$$

$$= \int 4dx - \int 6dx + \int\frac{(9x^2+x-5)dx}{x^3+x^2-x-1} = 2x^2-6x+\int\frac{(9x^2+x-5)dx}{x^3+x^2-x-1}\ (*)$$

Trabajando sólo la integral resultante:

$\int\frac{(9x^2+x-5)dx}{x^3+x^2-x-1} = \int\frac{(9x^2+x-5)dx}{(x+1)^2(x-1)} = \int\frac{Adx}{(x+1)}+\int\frac{Bdx}{(x+1)^2}+\int\frac{Cdx}{(x-1)}\ (**)$, luego:

$$\frac{(9x^2+x-5)}{(x^3+x^2-x-1)} = \frac{A}{(x+1)}+\frac{B}{(x+1)^2}+\frac{C}{x-1}$$

$$= 9x^2+x-5 = A(x+1)(x-1)+B(x-1)+C(x+1)^2$$

$$\therefore\begin{cases} x=1 \Rightarrow 5=4C \Rightarrow C = 5/4 \\ x=-1 \Rightarrow 3=-2B \Rightarrow B=-3/2 \\ x=0 \Rightarrow -5=-A-B+C \Rightarrow A = 31/4 \end{cases}$$

$$(**) = \frac{31}{4}\int\frac{dx}{(x+1)}-\frac{3}{2}\int\frac{dx}{(x+1)^2}+\frac{5}{4}\int\frac{dx}{(x-1)} = \frac{31}{4}\ell\eta|x+1|+\frac{3}{2(x+1)}+\frac{5}{4}\ell\eta|x-1|+c$$

$$(*) = 2x^2-6x+\frac{31}{4}\ell\eta|x+1|+\frac{3}{2(x+1)}+\frac{5}{4}\ell\eta|x-1|+c$$

7.46.- $\int\frac{3x^4dx}{(x^2+1)^2}$

Solución.-

$$\int\frac{3x^4dx}{(x^2+1)^2} = \int\frac{3x^4dx}{(x^4+2x^2+1)} = 3\int\left[1-\frac{2x^2+1}{(x^2+1)^2}\right]dx = 3\int dx - 3\int\frac{2x^2+1}{(x^2+1)^2}dx$$

$$= 3x - 3\int\frac{2x^2+1}{(x^2+1)^2}dx\ (*)$$

Trabajando sólo la integral resultante:

$\int\frac{(2x^2+1)dx}{(x^2+1)^2} = \int\frac{(Ax+B)dx}{(x^2+1)}+\int\frac{(Cx+D)dx}{(x^2+1)^2}\ (**)$, luego:

$$\frac{(2x^2+1)}{(x^2+1)^2}=\frac{Ax+B}{(x^2+1)}+\frac{Cx+D}{(x^2+1)^2}\Rightarrow 2x^2+1=(Ax+B)(x^2+1)+Cx+D$$

$$\Rightarrow 2x^2+1=Ax^3+Ax+Bx^2+B+Cx+D\Rightarrow 2x^2+1=Ax^3+Bx^2+(A+C)x+(B+D)$$

Igualando coeficientes: $A=0, B=2, A+C=0\Rightarrow C=0, B+D=1\Rightarrow D=-1$

$$(**)=2\int\frac{dx}{(x^2+1)}-\int\frac{dx}{(x^2+1)^2}=2\operatorname{arc}\tau gx-\frac{1}{2}\left(\operatorname{arc}\tau gx+\frac{x}{1+x^2}\right)+c$$

$$=\frac{3}{2}\operatorname{arc}\tau gx-\frac{x}{2(1+x^2)}+c$$

$$(*)=3x-\frac{9}{2}\operatorname{arc}\tau gx-\frac{x}{2(1+x^2)}+c$$

7.47.- $\int\frac{(2x^2+41x-91)dx}{x^3-2x^2-11x+12}$

Solución.-

$$\int\frac{(2x^2+41x-91)dx}{x^3-2x^2-11x+12}=\int\frac{(2x^2+41x-91)dx}{(x-1)(x+3)(x-4)}$$

$$=\int\frac{(2x^2+41x-91)dx}{(x-1)(x+3)(x-4)}=\int\frac{Adx}{x-1}+\int\frac{Bdx}{x+3}+\int\frac{Cdx}{x-4}\ (*)$$

$$\frac{(2x^2+41x-91)}{(x-1)(x+3)(x-4)}=\frac{A}{x-1}+\frac{B}{x+3}+\frac{C}{x-4}$$

$$(2x^2+41x-91)=A(x+3)(x-4)+B(x-1)(x-4)+C(x-1)(x+3)$$

$$\therefore\begin{cases}x=-3\Rightarrow 18-123-91=B(-4)(-7)\Rightarrow B=-7\\ x=4\Rightarrow 32+164-91=C(3)(7)\Rightarrow C=5\\ x=1\Rightarrow 2+41-91=A(4)(-3)\Rightarrow A=4\end{cases}$$

$$(*)=4\int\frac{dx}{(x-1)}-7\int\frac{dx}{(x+3)}+5\int\frac{dx}{(x-4)}=4\ell\eta\left|x-1\right|-7\ell\eta\left|x+3\right|+5\ell\eta\left|x-4\right|+c$$

$$=\ell\eta\left|\frac{(x-1)^4(x-4)^5}{(x+3)^7}\right|+c$$

7.48.- $\int\frac{(2x^4+3x^3-x-1)dx}{(x-1)(x^2+2x+2)^2}$

Solución.-

$$\int\frac{2x^4+3x^3-x-1}{(x-1)(x^2+2x+2)^2}dx=\int\frac{Adx}{(x-1)}+\int\frac{(Bx+C)dx}{(x^2+2x+2)}+\int\frac{(Dx+E)dx}{(x^2+2x+2)^2}\ (*)$$, luego:

$$\frac{2x^4+3x^2-x-1}{(x-1)(x^2+2x+2)^2}=\frac{A}{(x-1)}+\frac{Bx+C}{(x^2+2x+2)}+\frac{Dx+E}{(x^2+2x+2)^2}$$

$$2x^4+3x^3-x-1=A(x^2+2x+2)^2+(Bx+C)(x-1)(x^2+2x+2)+(Dx+E)(x-1)$$

$$2x^4+3x^3-x-1=A(x^4+4x^2+4+4x^3+4x^2+8x)+B(x^4+2x^3+2x^2-x^3-2x^2-2x)$$

$$\Rightarrow +C(x^3+2x^2+2x-x^2-2x-2)+D(x^2-x)+E(x-1)$$

$$2x^4+3x^3-x-1=(A+B)x^4+(4A+B+C)x^3+(8A+C+D)x^2$$
$$\Rightarrow +(8A-2B-D+E)x+(4A-2C-E)$$
Igualando coeficientes se tiene:
$$\begin{pmatrix} A & + & B & & & & & = & 2 \\ 4A & + & B & + & C & & & = & 3 \\ 8A & & & + & C & +D & & = & 0 \\ 8A & & -2B & & & -D & +E & = & -1 \\ 4A & & & & -2C & & -E & = & -1 \end{pmatrix}$$

$$\therefore A=3/25, B=47/25, C=16/25, D=-8/5, E=1/5$$
$$(*)=\frac{3}{25}\int\frac{dx}{x-1}+\frac{1}{25}\int\frac{(47x+16)dx}{(x^2+2x+2)}-\frac{1}{5}\int\frac{(8x-1)dx}{(x^2+2x+2)^2}$$
$$=\frac{3}{25}\ell\eta|x-1|+\frac{47}{25}\int\frac{(x+16/47)dx}{(x^2+2x+2)}-\frac{8}{5}\int\frac{(x-1/8)dx}{(x^2+2x+2)^2}$$
$$=\frac{3}{25}\ell\eta|x-1|+\frac{47}{50}\int\frac{(2x+2)-62/47}{(x^2+2x+2)}dx-\frac{4}{5}\int\frac{(2x+2)-9/4}{(x^2+2x+2)^2}dx$$
$$=\frac{3}{25}\ell\eta|x-1|+\frac{47}{50}\int\frac{(2x+2)dx}{(x^2+2x+2)}-\frac{62}{50}\int\frac{dx}{(x^2+2x+2)}-\frac{4}{5}\int\frac{(2x+2)dx}{(x^2+2x+2)^2}$$
$$\Rightarrow +\frac{9}{5}\int\frac{dx}{(x^2+2x+2)^2}$$
$$=\frac{3}{25}\ell\eta|x-1|+\frac{47}{50}\ell\eta|x^2+2x+2|-\frac{62}{50}\int\frac{dx}{(x+1)^2+1}+\frac{4}{5}\int\frac{1}{(x^2+2x+2)}$$
$$\Rightarrow +\frac{9}{5}\int\frac{dx}{\left[(x+1)^2+1\right]^2}$$
$$=\frac{3}{25}\ell\eta|x-1|+\frac{47}{50}\ell\eta|x^2+2x+2|-\frac{62}{50}\operatorname{arc}\tau g(x+1)+\frac{4}{5(x^2+2x+2)}$$
$$\Rightarrow +\frac{9}{5}\left[\frac{1}{2}\operatorname{arc}\tau g(x+1)+\frac{1}{2}\frac{x+1}{x^2+2x+2}\right]+c$$
$$=\frac{3}{25}\ell\eta|x-1|+\frac{47}{50}\ell\eta|x^2+2x+2|-\frac{17}{50}\operatorname{arc}\tau g(x+1)+\frac{9x+17}{10(x^2+2x+2)}+c$$

7.49.- $\int\frac{dx}{e^{2x}+e^x-2}$

Solución.-
$$\int\frac{dx}{e^{2x}+e^x-2}=\int\frac{dx}{(e^x)^2+e^x-2}=\int\frac{dx}{\left[(e^x)^2+e^x+1/4\right]-2-1/4}$$

$$=\int\frac{dx}{\left[e^x+\frac{1}{2}\right]^2-(\frac{3}{2})^2}\ (*)$$, Sea: $u=e^x+\frac{1}{2}, du=e^x dx\Rightarrow dx=\frac{du}{u-\frac{1}{2}}$

Luego:

$$(*)\int\frac{\frac{du}{u-\frac{1}{2}}}{u^2-(\frac{3}{2})^2}=\int\frac{du}{(u-\frac{1}{2})(u+\frac{3}{2})(u-\frac{3}{2})}=\int\frac{Adu}{u-\frac{1}{2}}-\int\frac{Bdu}{(u+\frac{3}{2})}+\int\frac{Cdu}{(u-\frac{3}{2})}\ (**)$$

$$\frac{1}{(u-\frac{1}{2})(u+\frac{3}{2})(u-\frac{3}{2})}=\frac{A}{(u-\frac{1}{2})}-\frac{B}{(u+\frac{3}{2})}+\frac{C}{(u-\frac{3}{2})}$$

$$1=A(u+\tfrac{3}{2})(u-\tfrac{3}{2})-B(u-\tfrac{1}{2})(u-\tfrac{3}{2})+C(u-\tfrac{1}{2})(u+\tfrac{3}{2})$$

$$\therefore\begin{cases}u=\frac{1}{2}\Rightarrow 1=A(2)(-1)\Rightarrow A=-\frac{1}{2}\\ u=-\frac{3}{2}\Rightarrow 1=B(-2)(-3)\Rightarrow B=\frac{1}{6}\\ u=\frac{3}{2}\Rightarrow 1=C(1)(3)\Rightarrow C=\frac{1}{3}\end{cases}$$

$$(**)=-\frac{1}{2}\int\frac{du}{(u-\frac{1}{2})}+\frac{1}{6}\int\frac{du}{(u+\frac{3}{2})}+\frac{1}{3}\int\frac{du}{(u-\frac{3}{2})}$$

$$=-\frac{1}{2}\ell\eta\left|(u-\tfrac{1}{2})\right|+\frac{1}{6}\ell\eta\left|(u+\tfrac{3}{2})\right|+\frac{1}{3}\ell\eta\left|(u-\tfrac{3}{2})\right|+c$$

$$=\frac{1}{6}\ell\eta\left|\frac{(u+\frac{3}{2})(u-\frac{3}{2})^2}{(u-\frac{1}{2})^3}\right|+c=\frac{1}{6}\ell\eta\left|\frac{(e^x+2)(e^x-1)^2}{(e^x)^3}\right|+c=\frac{1}{6}\ell\eta\left|\frac{(e^x+2)(e^x-1)^2}{e^{3x}}\right|+c$$

7.50.- $\int\frac{\operatorname{sen}xdx}{\cos x(1+\cos^2x)}$

Solución.-

$$\int\frac{\operatorname{sen}xdx}{\cos x(1+\cos^2x)}=\int\frac{-\operatorname{sen}xdx}{\cos x(1+\cos^2x)}=-\int\frac{du}{u(1+u^2)}=-\int\frac{Adu}{u}-\int\frac{(Bu+C)du}{(1+u^2)}\ (*)$$

Sea: $u=\cos x, du=-\operatorname{sen}xdx$

$$\frac{1}{u(1+u^2)}=\frac{A}{u}+\frac{(Bu+C)}{(1+u^2)}\Rightarrow 1=A(1+u^2)+(Bu+C)u$$

$$1=A+Au^2+Bu^2+Cu\Rightarrow 1=(A+B)u^2+Cu+A$$

Igualando Coeficientes se tiene:

$$\therefore\begin{cases}A+B=0\Rightarrow B=-A\Rightarrow B=-(1)\Rightarrow B=-1\\ C=0,\\ A=1\end{cases}$$

$$(*)=-\int\frac{du}{u}+\int\frac{udu}{1+u^2}=-\ell\eta|u|+\ell\eta\left|\sqrt{1+u^2}\right|+c=-\ell\eta|\cos x|+\ell\eta\left|\sqrt{1+(\cos x)^2}\right|+c$$

$$= \ell\eta\left|\frac{\sqrt{1+(\cos x)^2}}{\cos x}\right| + c$$

7.51.- $\int \frac{(2+\tau g^2\theta)\sec^2\theta d\theta}{1+\tau g^3\theta}$

Solución.-

$$\int \frac{(2+\tau g^2\theta)\sec^2\theta d\theta}{1+\tau g^3\theta} = \int \frac{(2+u^2)du}{(1+u^3)} = \int \frac{(2+u^2)du}{(1+u)(u^2-u+1)} \quad (*)$$

Sea: $u = \tau g\theta, du = -\sec^2\theta d\theta$

$$\int \frac{(2+u^2)du}{(1+u^3)} = \int \frac{Adu}{(1+u)} + \int \frac{Bu+C}{(u^2-u+1)}$$, luego:

$$\frac{(2+u^2)}{(1+u^3)} = \frac{A}{(1+u)} + \frac{Bu+C}{(u^2-u+1)} \Rightarrow (2+u^2) = A(u^2-u+1) + (Bu+C)(1+u)$$

$$(2+u^2) = Au^2 - Au + A + Bu^2 + Bu + C + Cu$$

$$(2+u^2) = (A+B)u^2 + (-A+B+C)u + A + C$$

Igualando Coeficientes se tiene:

$$\begin{pmatrix} A+B & & =1 \\ -A+B & +C & =0 \\ A & +C & =2 \end{pmatrix} \therefore A=1, B=0, C=1$$

$$(*) = \int \frac{du}{1+u} + \int \frac{du}{u^2-u+1} = \int \frac{du}{1+u} + \int \frac{du}{(u-\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2}$$

$$= \ell\eta|1+u| + \frac{1}{\frac{\sqrt{3}}{2}} \operatorname{arc}\tau g \frac{u-\frac{1}{2}}{\frac{\sqrt{3}}{2}} + c = \ell\eta|1+u| + \frac{2}{\sqrt{3}} \operatorname{arc}\tau g \frac{2u-1}{\sqrt{3}} + c$$

$$= \ell\eta|1+\tau g\theta| + \frac{2}{\sqrt{3}} \operatorname{arc}\tau g \frac{(2\tau g\theta-1)}{\sqrt{3}} + c$$

7.52.- $\int \frac{(5x^3+2)dx}{x^3-5x^2+4x}$

Solución.-

$$\int \frac{(5x^3+2)dx}{x^3-5x^2+4x} = \int \frac{(5x^3+2)dx}{x(x-1)(x-4)} = \int \frac{Adx}{x} + \int \frac{Bdx}{(x-1)} + \int \frac{Cdx}{(x-4)} \quad (*)$$

$$\frac{(5x^3+2)}{x(x-1)(x-4)} = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x-4)}$$, Luego:

$$(5x^3+2) = A(x-1)(x-4) + Bx(x-4) + Cx(x-1)$$

Igualando Coeficientes se tiene:

$$\therefore \begin{cases} x=0 \Rightarrow 2=4A \Rightarrow A = \frac{1}{2} \\ x=1 \Rightarrow 7=-3B \Rightarrow B=-\frac{7}{3} \\ x=4 \Rightarrow 322=12C \Rightarrow C=\frac{161}{6} \end{cases}$$

$$(*) = \frac{1}{2}\int\frac{dx}{x} - \frac{7}{3}\int\frac{dx}{x-1} + \frac{161}{6}\int\frac{dx}{x-4} = \frac{1}{2}\ell\eta|x| - \frac{7}{3}\ell\eta|x-1| + \frac{161}{6}\ell\eta|x-4| + c$$

$$= \frac{3}{6}\ell\eta|x| - \frac{14}{3}\ell\eta|x-1| + \frac{161}{6}\ell\eta|x-4| + c = \frac{1}{6}\ell\eta\left|\frac{x^3(x-4)^{161}}{(x-1)^{14}}\right| + c$$

7.53.- $\int\frac{x^5dx}{(x^3+1)(x^3+8)}$

Solución.-

$$\int\frac{x^5dx}{(x^3+1)(x^3+8)} = \int\frac{x^5dx}{(x+1)(x^2-x+1)(x+2)(x^2-2x+4)}$$

$$= \int\frac{Adx}{(x+1)} + \int\frac{Bdx}{(x+2)} + \int\frac{(Cx+D)dx}{(x^2-x+1)} + \int\frac{(Ex+F)dx}{(x^2-2x+4)}\ (*)$$, luego:

$$\frac{x^5}{(x^3+1)(x^3+8)} = \frac{A}{(x+1)} + \frac{B}{(x+2)} + \frac{Cx+D}{(x^2-x+1)} + \frac{Ex+F}{(x^2-2x+4)}$$, luego:

$$x^5 = A(x+2)(x^2-x+1)(x^2-2x+4) + B(x+1)(x^2-x+1)(x^2-2x+4)$$

$$\Rightarrow +(Cx+D)(x+1)(x+2)(x^2-2x+4) + (Ex+F)(x+1)(x+1)(x^2-x+1)$$

$$x^5 = A(x^5+8x^2-x^4-8x+x^3+8) + B(x^5-2x^4+4x^3+x^2-2x+4)$$

$$\Rightarrow +(Cx+D)(x^4+8x+x^3+8) + (Ex+F)(x^4+2x^3+x+2)$$

$$x^5 = (A+B+C+E)x^5 + (-A-2B+C+D+2E+F)x^4 + (A+4B+D+2F)x^3$$

$$\Rightarrow +(8A+B+8C+E)x^2 + (-8A-2B+8C+8D+2E+F)x + (8A+4B+8D+2F)$$

Igualando coeficientes se tiene:

$$\begin{pmatrix} A & +B & +C & & +E & & =1 \\ -A & -2B & +C & +D & +2E & +F & =0 \\ A & +4B & & +D & & +2F & =0 \\ 8A & +B & +8C & & +E & & =0 \\ 8A & -2B & +8C & +8D & +2E & +F & =0 \\ 8A & +4B & & +8D & & +2F & =0 \end{pmatrix}$$

$$\therefore A = -\frac{1}{21}, B = \frac{8}{21}, C = -\frac{2}{21}, D = \frac{1}{21}, E = \frac{16}{21}, F = -\frac{16}{21}$$

$$(*) = -\frac{1}{21}\int\frac{dx}{x+1} + \frac{8}{21}\int\frac{dx}{(x+2)} - \frac{1}{21}\int\frac{(2x-1)dx}{(x^2-x+1)} + \frac{16}{21}\int\frac{(x-1)dx}{(x^2-2x+4)}$$

$$= -\frac{1}{21}\ell\eta|x+1| + \frac{8}{21}\ell\eta|x+2| - \frac{1}{21}\ell\eta|x^2-x+1| + \frac{8}{21}\int\frac{(2x-2)dx}{x^2-2x+4}$$

$$= -\frac{1}{21}\ell\eta\left|x+1\right| + \frac{8}{21}\ell\eta\left|x+2\right| - \frac{1}{21}\ell\eta\left|x^2-x+1\right| - \frac{8}{21}\ell\eta\left|x^2-2x+4\right| + c$$

$$= \frac{1}{21}\ell\eta\left|\frac{\left[(x+2)(x^2-2x+4)\right]^8}{(x+1)(x^2-x+1)}\right| + c$$

CAPITULO 8

INTEGRACION DE FUNCIONES RACIONALES D SENO Y COSENO

Existen funciones racionales que conllevan formas trigonométricas, reducibles por si a: seno y coseno. Lo conveniente en tales casos es usar las siguientes sustituciones: $z=\tau g\frac{x}{2}$, de donde: $x=2\operatorname{arc}\tau gz$ y $dx=\frac{2dz}{1+z^2}$. Es fácil llegar a verificar que de lo anterior se consigue: $\operatorname{sen}x=\frac{2z}{1+z^2}$ y $\cos x=\frac{1-z^2}{1+z^2}$

EJERCICIOS DESARROLLADOS

8.1.-Encontrar: $\int\frac{dx}{2-\cos x}$

Solución.- La función racional con expresión trigonométrica es: $\frac{1}{2-\cos x}$, y su solución se hace sencilla, usando sustituciones recomendadas, este es:

$z=\tau g\frac{x}{2}$, $x=2\operatorname{arc}\tau gz$, $dx=\frac{2dz}{1+z^2}$, $\cos x=\frac{1-z^2}{1+z^2}$ $\therefore$

$$\int\frac{dx}{2-\cos x}=\int\frac{\frac{2dz}{1+z^2}}{2-\frac{1-z^2}{1+z^2}}=\int\frac{\frac{2dz}{\cancel{1+z^2}}}{\frac{2+2z-1+z^2}{\cancel{1+z^2}}}=\int\frac{2dz}{3z^2+1}=\int\frac{2dz}{3(z^2+\frac{1}{3})}$$

$$=\frac{2}{3}\int\frac{dz}{z^2+(\sqrt{\frac{1}{3}})^2}=\frac{2}{3}\sqrt{3}\operatorname{arc}\tau g\sqrt{3}z+c$$, recordando que: $z=\tau g\frac{x}{2}$, se tiene:

$$=\frac{2}{3}\sqrt{3}\operatorname{arc}\tau g\sqrt{3}\tau g\frac{x}{2}+c$$

Respuesta: $\int\frac{dx}{2-\cos x}=\frac{2}{3}\operatorname{arc}\tau g\sqrt{3}\tau g\frac{x}{2}+c$

8.2.-Encontrar: $\int\frac{dx}{2-\operatorname{sen}x}$

Solución.- Forma racional: $\frac{1}{2-\operatorname{sen}x}$,

sustituciones: $z=\tau g\frac{x}{2}$, $x=2\operatorname{arc}\tau gz$, $dx=\frac{2dz}{1+z^2}$, $\operatorname{sen}x=\frac{2z}{1+z^2}$ $\therefore$

$$\int\frac{dx}{2-\operatorname{sen}x}=\int\frac{\frac{2dz}{1+z^2}}{2-\frac{2z}{1+z^2}}=\int\frac{\frac{2dz}{\cancel{1+z^2}}}{\frac{2+2z^2-2z}{\cancel{1+z^2}}}=\int\frac{\cancel{2}dz}{\cancel{2}(1+z^2-z)}=\int\frac{dz}{(z^2-z+1)}$$

Ahora bien: $z^2 - z + 1 = (z^2 - z + \frac{1}{4}) + 1 - \frac{1}{4} = (z - \frac{1}{2})^2 + \frac{3}{4} = (z - \frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2$

$$\therefore \int \frac{dx}{(z-\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2} = \frac{1}{\frac{\sqrt{3}}{2}} \operatorname{arc}\tau g \frac{z-\frac{1}{2}}{\frac{\sqrt{3}}{2}} + c = \frac{2}{\sqrt{3}} \operatorname{arc}\tau g \frac{\frac{2z-1}{\cancel{2}}}{\frac{\sqrt{3}}{\cancel{2}}} + c$$

$= \frac{2}{\sqrt{3}} \operatorname{arc}\tau g \frac{2z-1}{\sqrt{3}} + c$,recordando que: $z = \tau g \frac{x}{2}$, se tiene:

$$= \frac{2\sqrt{3}}{3} \operatorname{arc}\tau g \frac{2\tau g \frac{x}{2} - 1}{\sqrt{3}} + c$$

Respuesta: $\int \frac{dx}{2 - \operatorname{sen} x} = \frac{2\sqrt{3}}{3} \operatorname{arc}\tau g \frac{2\tau g \frac{x}{2} - 1}{\sqrt{3}} + c$

8.3.-Encontrar: $\int \frac{d\theta}{4 - 5\cos\theta}$

Solución.- Forma racional: $\frac{1}{4 - 5\cos\theta}$,

sustituciones: $z = \tau g \frac{\theta}{2}$, $x = 2\operatorname{arc}\tau g z$, $dx = \frac{2dz}{1+z^2}$, $\cos x = \frac{1-z^2}{1+z^2}$

$$\therefore \int \frac{dx}{4 - 5\cos\theta} = \int \frac{\frac{2dz}{1+z^2}}{4 - 5\left(\frac{1-z^2}{1+z^2}\right)} = \int \frac{\frac{2dz}{\cancel{1+z^2}}}{\frac{4+4z^2-5+5z^2}{\cancel{1+z^2}}} = \int \frac{2dz}{9z^2 - 1} = \int \frac{2dz}{9(z^2 - \frac{1}{9})}$$

$$= \frac{2}{9} \int \frac{dz}{z^2 - (\frac{1}{3})^2} = \frac{\cancel{2}}{9} \frac{1}{\cancel{2}(\frac{1}{3})} \ell\eta \left| \frac{z - \frac{1}{3}}{z + \frac{1}{3}} \right| + c = \frac{1}{3} \ell\eta \left| \frac{3z-1}{3z+1} \right| + c$$

Recordando que: $z = \tau g \frac{\theta}{2}$, se tiene: $= \frac{1}{3} \ell\eta \left| \frac{3\tau g \frac{\theta}{2} - 1}{3\tau g \frac{\theta}{2} + 1} \right| + c$

Respuesta: $\int \frac{d\theta}{4 - 5\cos\theta} = \frac{1}{3} \ell\eta \left| \frac{3\tau g \frac{\theta}{2} - 1}{3\tau g \frac{\theta}{2} + 1} \right| + c$

8.4.-Encontrar: $\int \frac{d\theta}{3\cos\theta + 4\operatorname{sen}\theta}$

Solución.- usando las sustituciones recomendadas:

$$\int \frac{d\theta}{3\cos\theta + 4\operatorname{sen}\theta} = \int \frac{\frac{2dz}{1+z^2}}{3\left(\frac{1-z^2}{1+z^2}\right) + 4\left(\frac{2z}{1+z^2}\right)} = \int \frac{\frac{2dz}{\cancel{1+z^2}}}{\frac{3-3z^2+8z}{\cancel{1+z^2}}}$$

$$=\int\frac{2dz}{-3(z^2-\frac{8}{3}z-1)}=-\frac{2}{3}\int\frac{dz}{z^2-\frac{8}{3}z-1}$$, pero:

$z^2-\frac{8}{3}z-1=(z^2-\frac{8}{3}z+\frac{16}{9})-1-\frac{16}{9}=(z-\frac{4}{3})^2-(\frac{5}{3})^2$, luego:

$$=-\frac{2}{3}\int\frac{dz}{(z-\frac{4}{3})^2-(\frac{5}{3})^2}$$, sea: $w=z-\frac{4}{3}, dw=dz$; de donde:

$$=-\frac{2}{3}\frac{1}{2(\frac{5}{3})}\ell\eta\left|\frac{z-\frac{4}{3}-\frac{5}{3}}{z-\frac{4}{3}+\frac{5}{3}}\right|+c=-\frac{1}{5}\ell\eta\left|\frac{3z-9}{3z+1}\right|+c$$, como: $z=\tau g\frac{\theta}{2}$, se tiene:

$$=-\frac{1}{5}\ell\eta\left|\frac{3\tau g\frac{\theta}{2}-9}{3\tau g\frac{\theta}{2}+1}\right|+c$$

Respuesta: $\int\frac{d\theta}{3\cos\theta+4\operatorname{sen}\theta}=-\frac{1}{5}\ell\eta\left|\frac{3\tau g\frac{\theta}{2}-9}{3\tau g\frac{\theta}{2}+1}\right|+c$

8.5.-Encontrar: $\int\frac{d\theta}{3+2\cos\theta+2\operatorname{sen}\theta}$

Solución.- usando las sustituciones recomendadas:

$$\int\frac{d\theta}{3+2\cos\theta+2\operatorname{sen}\theta}=\int\frac{\frac{2dz}{1+z^2}}{3+2\left(\frac{1-z^2}{1+z^2}\right)+2\left(\frac{2z}{1+z^2}\right)}=\int\frac{\frac{2dz}{1+z^2}}{3+\frac{2-2z^2}{1+z^2}+\frac{4z}{1+z^2}}$$

$$=\int\frac{\frac{2dz}{\cancel{1+z^2}}}{\frac{3+3z^2+2-2z^2+4z}{\cancel{1+z^2}}}=\int\frac{2dz}{z^2+4z+5}=\int\frac{2dz}{(z+2)^2+1}=2\operatorname{arc}\tau g(z+2)+c$$

Como: $z=\tau g\frac{\theta}{2}$, se tiene: $=2\operatorname{arc}\tau g(\tau g\frac{\theta}{2}+2)+c$

Respuesta: $\int\frac{d\theta}{3+2\cos\theta+2\operatorname{sen}\theta}=2\operatorname{arc}\tau g(\tau g\frac{\theta}{2}+2)+c$

8.6.-Encontrar: $\int\frac{dx}{\tau g\theta-\operatorname{sen}\theta}$

Solución.- Antes de hacer las sustituciones recomendadas, se buscará la equivalencia correspondiente a $\tau g\theta$

$$\tau g\theta=\frac{\operatorname{sen}\theta}{\cos\theta}=\frac{\frac{2z}{\cancel{1+z^2}}}{\frac{1-z^2}{\cancel{1+z^2}}}=\frac{2z}{1-z^2}$$, procédase ahora como antes:

$$\int \frac{dx}{\tau g\theta - \operatorname{sen}\theta} = \int \frac{\frac{2dz}{1+z^2}}{\frac{2z}{1-z^2} + \frac{2z}{1+z^2}} = \int \frac{\frac{2dz}{\cancel{1+z^2}}}{\frac{2z(1+z^2)-2z(1-z^2)}{(1-z^2)\cancel{(1+z^2)}}} = \int \frac{2(1-z^2)dz}{\cancel{2z}+2z^3 \cancel{-2z}+2z^3}$$

$$= \int \frac{(2-2z^2)dz}{4z^3} = \frac{1}{2}\int z^{-3}dz - \frac{1}{2}\int \frac{dz}{z} = -\frac{1}{4z^2} - \frac{1}{2}\ell\eta|z| + c$$

Como: $z = \tau g\, \theta/2$, se tiene: $= -\frac{1}{4}(\operatorname{co}\tau g^2\, \theta/2) - \frac{1}{2}\ell\eta\left|\tau g\, \theta/2\right| + c$

Respuesta: $\int \frac{dx}{\tau g\theta - \operatorname{sen}\theta} = -\frac{1}{4}(\operatorname{co}\tau g^2\, \theta/2) - \frac{1}{2}\ell\eta\left|\tau g\, \theta/2\right| + c$

8.7.-Encontrar: $\int \frac{dx}{2+\operatorname{sen} x}$

Solución.- usando las sustituciones recomendadas:

$$\int \frac{dx}{2+\operatorname{sen} x} = \int \frac{\frac{2dz}{1+z^2}}{2+\frac{2z}{1+z^2}} = \int \frac{\frac{2dz}{\cancel{1+z^2}}}{\frac{2+2z^2+2z}{\cancel{1+z^2}}} = \int \frac{dz}{z^2+z+1} = \int \frac{dz}{(z^2+z+1/4)+3/4}$$

$$= \int \frac{2dz}{(z+1/2)^2+(\sqrt{3}/2)^2} = \frac{1}{\sqrt{3}/2}\operatorname{arc}\tau g\frac{(z+1/2)}{\sqrt{3}/2} + c = \frac{2}{\sqrt{3}}\operatorname{arc}\tau g\frac{2z+1}{\sqrt{3}} + c$$

Como: $z = \tau g\, x/2$, se tiene: $= \frac{2}{\sqrt{3}}\operatorname{arc}\tau g\frac{2\tau g\, x/2 + 1}{\sqrt{3}} + c$

Respuesta: $\int \frac{dx}{2+\operatorname{sen} x} = \frac{2}{\sqrt{3}}\operatorname{arc}\tau g\frac{2\tau g\, x/2 + 1}{\sqrt{3}} + c$

8.8.-Encontrar: $\int \frac{\cos x dx}{1+\cos x}$

Solución.-usando las sustituciones recomendadas:

$$\int \frac{\cos x dx}{1+\cos x} = \int \frac{\left(\frac{1-z^2}{1+z^2}\right)\left(\frac{2dz}{1+z^2}\right)}{1+\frac{1-z^2}{1+z^2}} = \int \frac{\left(\frac{1-z^2}{1+z^2}\right)\left(\frac{2dz}{\cancel{1+z^2}}\right)}{\frac{1\cancel{+z^2}+1\cancel{-z^2}}{\cancel{1+z^2}}} = \int \frac{\cancel{2}(1-z^2)dz}{(1+z^2)\cancel{2}} = \int \frac{(1-z^2)dz}{(1+z^2)}$$

$$= \int \frac{(-z^2+1)dz}{(z^2+1)} = \int\left(-1+\frac{2}{z^2+1}\right)dz = \int dz + 2\int \frac{dz}{z^2+1} = -z + 2\operatorname{arc}\tau g z + c$$

Como: $z = \tau g\, x/2$, se tiene: $= -\tau g\frac{x}{2} + 2\operatorname{arc}\tau g(\tau g\frac{x}{2}) + c$

Respuesta: $\int \frac{\cos x dx}{1+\cos x} = -\tau g\frac{x}{2} + x + c$

8.9.-Encontrar: $\int \frac{dx}{1+\operatorname{sen} x+\cos x}$

Solución.- usando las sustituciones recomendadas:

$$\int \frac{dx}{1+\operatorname{sen} x+\cos x}=\int \frac{\frac{2dz}{1+z^2}}{1+\left(\frac{2z}{1+z^2}\right)+\left(\frac{1-z^2}{1+z^2}\right)}=\int \frac{2dz}{1\cancel{+z^2}+2z+1\cancel{-z^2}}$$

$=\int \frac{2dz}{2z+2}=\int \frac{dz}{z+1}=\ell\eta\left|z+1\right|+c$, como: $z=\tau g\, x/2$, se tiene: $=\ell\eta\left|\tau g\, x/2+1\right|+c$

Respuesta: $\int \frac{dx}{1+\operatorname{sen} x+\cos x}=\ell\eta\left|\tau g\, x/2+1\right|+c$

8.10.-Encontrar: $\int \frac{dx}{\cos x+2\operatorname{sen} x+3}$

Solución.- usando las sustituciones recomendadas:

$$\int \frac{dx}{\cos x+2\operatorname{sen} x+3}=\int \frac{\frac{2dz}{1+z^2}}{\left(\frac{1-z^2}{1+z^2}\right)+\left(\frac{4z}{1+z^2}\right)+3}=\int \frac{2dz}{1-z^2+4z+3+3z^2}=\int \frac{2dz}{2z^2+2z+2}$$

$=\int \frac{dz}{z^2+2z+2}=\int \frac{dz}{(z+1)^2+1}=\operatorname{arc}\tau g(z+1)+c$, como: $z=\tau g\, \theta/2$,

Se tiene: $=\operatorname{arc}\tau g(\tau g\, x/2+1)+c$

Respuesta: $\int \frac{dx}{\cos x+2\operatorname{sen} x+3}=\operatorname{arc}\tau g(\tau g\, x/2+1)+c$

8.11.-Encontrar: $\int \frac{\operatorname{sen} x dx}{1+\operatorname{sen}^2 x}$

Solución.- usando las sustituciones recomendadas:

$$\int \frac{\operatorname{sen} x dx}{1+\operatorname{sen}^2 x}=\int \frac{\left(\frac{2z}{1+z^2}\right)\left(\frac{2dz}{1+z^2}\right)}{1+\left(\frac{2z}{1+z^2}\right)^2}\int \frac{\frac{4zdz}{(1+z^2)^2}}{1+\frac{4z^2}{(1+z^2)^2}}=\int \frac{4zdz}{(1+z^2)^2+4z^2}=\int \frac{4zdz}{1+2z^2+z^4+4z^2}$$

$$=\int \frac{4zdz}{z^4+6z^2+1}=\int \frac{4zdz}{(z^4+6z^2+9)-8}=\int \frac{4zdz}{(z^2+3)^2-(\sqrt{8})^2}$$

Sea: $w=z^2+3, dw=2zdz$

$$=2\int \frac{dw}{w^2-(\sqrt{8})^2}=\frac{\cancel{2}}{\cancel{2}\sqrt{8}}\ell\eta\left|\frac{w-\sqrt{8}}{w+\sqrt{8}}\right|+c=\frac{\sqrt{8}}{8}\ell\eta\left|\frac{w-\sqrt{8}}{w+\sqrt{8}}\right|+c=\frac{\sqrt{8}}{8}\ell\eta\left|\frac{z^2+3-\sqrt{8}}{z^2+3+\sqrt{8}}\right|+c$$

Como: $z=\tau g\, \theta/2$, se tiene: $=\frac{\sqrt{2}}{4}\ell\eta\left|\frac{z^2+3-\sqrt{8}}{z^2+3+\sqrt{8}}\right|+c=\frac{\sqrt{2}}{4}\ell\eta\left|\frac{\tau g^2\, x/2+3-2\sqrt{2}}{\tau g^2\, x/2+3+2\sqrt{2}}\right|+c$

Respuesta: $\int \frac{\operatorname{sen} x dx}{1+\operatorname{sen}^2 x}=\frac{\sqrt{2}}{4}\ell\eta\left|\frac{\tau g^2 \, {}^{x}\!/_{2}+3-2\sqrt{2}}{\tau g^2 \, {}^{x}\!/_{2}+3+2\sqrt{2}}\right|+c$

8.12.-Encontrar: $\int \frac{d\theta}{5+4\cos\theta}$

Solución.-usando las sustituciones recomendadas:

$$\int \frac{dx}{5+4\cos\theta}=\int \frac{\frac{2dz}{1+z^2}}{5+4\left(\frac{1-z^2}{1+z^2}\right)}=\int \frac{2dz}{5+5z^2+4-4z^2}=\int \frac{2dz}{z^2+9}=2\int \frac{dz}{z^2+3^2}$$

$=\frac{2}{3}\operatorname{arc}\tau g\frac{z}{3}+c$, como: $z=\tau g\frac{\theta}{2}$, se tiene: $=\frac{2}{3}\operatorname{arc}\tau g\frac{\tau g \, {}^{\theta}\!/_{2}}{3}+c$

Respuesta: $\int \frac{d\theta}{5+4\cos\theta}=\frac{2}{3}\operatorname{arc}\tau g\frac{\tau g \, {}^{\theta}\!/_{2}}{3}+c$

8.14.-Encontrar: $\int \frac{dx}{\operatorname{sen} x+\cos x}$

Solución.- usando las sustituciones recomendadas:

$$\int \frac{dx}{\operatorname{sen} x+\cos x}=\int \frac{\frac{2dz}{1+z^2}}{\left(\frac{2z}{1+z^2}\right)+\left(\frac{1-z^2}{1+z^2}\right)}=\int \frac{2dz}{2z+1-z^2}=2\int \frac{dz}{(-z^2+2z+1)}$$

$$=-2\int \frac{dz}{(z^2-2z+1)-2}=-2\int \frac{dz}{(z-1)^2-(\sqrt{2})^2}=-\cancel{2}\frac{1}{\cancel{2}\sqrt{2}}\ell\eta\left|\frac{z-1-\sqrt{2}}{z-1+\sqrt{2}}\right|+c$$

$=-\frac{\sqrt{2}}{2}\ell\eta\left|\frac{z-1-\sqrt{2}}{z-1+\sqrt{2}}\right|+c$, como: $z=\tau g \, {}^{x}\!/_{2}$, se tiene: $=-\frac{\sqrt{2}}{2}\ell\eta\left|\frac{\tau g \, {}^{x}\!/_{2}-1-\sqrt{2}}{\tau g \, {}^{x}\!/_{2}-1+\sqrt{2}}\right|+c$

Respuesta: $\int \frac{dx}{\operatorname{sen} x+\cos x}=-\frac{\sqrt{2}}{2}\ell\eta\left|\frac{\tau g \, {}^{x}\!/_{2}-1-\sqrt{2}}{\tau g \, {}^{x}\!/_{2}-1+\sqrt{2}}\right|+c$

8.14.-Encontrar: $\int \frac{\sec x dx}{\sec x+2\tau g x-1}$

Solución.- usando las sustituciones recomendadas:

$$\int \frac{\sec x dx}{\sec x+2\tau g x-1}=\int \frac{\frac{1}{\cos x}dx}{\frac{1}{\cos x}+\frac{2\operatorname{sen} x}{\cos x}-1}=\int \frac{dx}{1+2\operatorname{sen} x-\cos x}=\int \frac{\frac{2dz}{1+z^2}}{1+\left(\frac{4z}{1+z^2}\right)-\left(\frac{1-z^2}{1+z^2}\right)}$$

$$=\int\frac{\frac{2dz}{\cancel{1+z^2}}}{\frac{\cancel{1}+z^2+4z\cancel{-1}+z^2}{\cancel{1+z^2}}}=\int\frac{2dz}{2z^2+4z}=\int\frac{\cancel{2}dz}{\cancel{2}(z^2+2z)}=\int\frac{dz}{z(z+2)}\ (*)$$

Ahora bien: $\frac{1}{z(z+2)}=\frac{A}{z}+\frac{B}{z+2}$, de donde:

$\frac{1}{\cancel{z(z+2)}}=\frac{A(z+2)+B(z)}{\cancel{z(z+2)}}\Rightarrow 1=A(z+2)+B(z)$, de donde: $A=\frac{1}{2}, B=-\frac{1}{2}$

$$(*)\ \int\frac{dz}{z(z+2)}=\int\frac{\frac{1}{2}dz}{z}-\int\frac{\frac{1}{2}dz}{z+2}=\frac{1}{2}\int\frac{dz}{z}-\frac{1}{2}\int\frac{dz}{z+2}=\frac{1}{2}\ell\eta|z|-\frac{1}{2}\ell\eta|z+2|+c$$

$=\frac{1}{2}\ell\eta\left|\frac{z}{z+2}\right|+c$, como: $z=\tau g\frac{x}{2}$, se tiene: $=\frac{1}{2}\ell\eta\left|\frac{\tau g\frac{x}{2}}{\tau g\frac{x}{2}+2}\right|+c$

Respuesta: $\int\frac{\sec xdx}{\sec x+2\tau gx-1}=\frac{1}{2}\ell\eta\left|\frac{\tau g\frac{x}{2}}{\tau g\frac{x}{2}+2}\right|+c$

8.15.-Encontrar: $\int\frac{dx}{1-\cos x+\operatorname{sen} x}$

Solución.- usando las sustituciones recomendadas:

$$\int\frac{dx}{1-\cos x+\operatorname{sen} x}=\int\frac{\frac{2dz}{1+z^2}}{1-\left(\frac{1-z^2}{1+z^2}\right)+\left(\frac{2z}{1+z^2}\right)}=\int\frac{\frac{2dz}{\cancel{1+z^2}}}{\frac{\cancel{1}+z^2\cancel{-1}+z^2+2z}{\cancel{1+z^2}}}=\int\frac{2dz}{2z^2+2z}$$

$$=\int\frac{\cancel{2}dz}{\cancel{2}(z^2+z)}=\int\frac{dz}{z(z+1)}\ (*)$$

Ahora bien: $\frac{1}{z(z+1)}=\frac{A}{z}+\frac{B}{z+1}$, de donde se tiene:

$\frac{1}{\cancel{z(z+1)}}=\frac{A(z+1)+B(z)}{\cancel{z(z+1)}}\Rightarrow 1=A(z+1)+B(z)$, de donde: $A=1, B=-1$, luego:

$\int\frac{dz}{z(z+1)}=\int\frac{dz}{z}-\int\frac{dz}{z+1}=\ell\eta|z|-\ell\eta|z+1|+c=\ell\eta\left|\frac{z}{z+1}\right|+c$, como: $z=\tau g\frac{x}{2}$,

Se tiene: $=\ell\eta\left|\frac{\tau g\frac{x}{2}}{\tau g\frac{x}{2}+1}\right|+c$

Respuesta: $\int\frac{dx}{1-\cos x+\operatorname{sen} x}=\ell\eta\left|\frac{\tau g\frac{x}{2}}{\tau g\frac{x}{2}+1}\right|+c$

8.16.-Encontrar: $\int\frac{dx}{8-4\operatorname{sen} x+7\cos x}$

Solución.- usando las sustituciones recomendadas:

$$\int \frac{dx}{8-4\operatorname{sen} x+7\cos x}=\int \frac{\frac{2dz}{1+z^2}}{8-\left(\frac{8z}{1+z^2}\right)+7\left(\frac{1-z^2}{1+z^2}\right)}=\int \frac{\frac{2dz}{\cancel{1+z^2}}}{\frac{8+8z^2-8z+7-7z^2}{\cancel{1+z^2}}}$$

$$=\int \frac{2dz}{z^2-8z+15}=\int \frac{2dz}{(z-3)(z-5)}\ (*)$$

Ahora bien: $\frac{2}{(z-3)(z-5)}=\frac{A}{(z-3)}+\frac{B}{(z-5)}$, de donde se tiene:

$\Rightarrow 2=A(z-5)+B(z-3)$, de donde: $A=-1, B=1$, luego:

$$\int \frac{2dz}{(z-3)(z-5)}=-\int \frac{dz}{z-3}+\int \frac{dz}{z-5}=-\ell\eta\left|z-3\right|+\ell\eta\left|z-5\right|+c=\ell\eta\left|\frac{z-5}{z-3}\right|+c,$$

como: $z=\tau g\, {}^{x}\!/_{2}$, se tiene: $=\ell\eta\left|\frac{\tau g\, {}^{x}\!/_{2}-5}{\tau g\, {}^{x}\!/_{2}-3}\right|+c$

Respuesta: $\int \frac{dx}{8-4\operatorname{sen} x+7\cos x}=\ell\eta\left|\frac{\tau g\, {}^{x}\!/_{2}-5}{\tau g\, {}^{x}\!/_{2}-3}\right|+c$

EJERCICIOS PROPUESTOS

8.17.- $\int \frac{dx}{1+\cos x}$ **8.18.-** $\int \frac{dx}{1-\cos x}$ **8.19.-** $\int \frac{\operatorname{sen} xdx}{1+\cos x}$

8.20.- $\int \frac{\cos xdx}{2-\cos x}$ **8.21.-** $\int \frac{d\theta}{5-4\cos\theta}$ **8.22.-** $\int \frac{\operatorname{sen}\theta d\theta}{\cos^2\theta-\cos\theta-2}$

8.23.- $\int \sec xdx$ **8.24.-** $\int \frac{\cos\theta d\theta}{5+4\cos\theta}$ **8.25.-** $\int \frac{d\theta}{\cos\theta+\operatorname{co}\tau g\theta}$

RESPUESTAS

8.17.- $\int \frac{dx}{1+\cos x}$

Solución.-

$$\int \frac{dx}{1+\cos x}=\int \frac{\frac{2dz}{1+z^2}}{1+\left(\frac{1-z^2}{1+z^2}\right)}=\int \frac{\frac{2dz}{\cancel{1+z^2}}}{\frac{1+z^2+1-z^2}{\cancel{1+z^2}}}=\int dz=z+c=\tau g\, {}^{x}\!/_{2}+c$$

8.18.- $\int \frac{dx}{1-\cos x}$

Solución.-

$$\int\frac{dx}{1-\cos x}=\int\frac{\frac{2dz}{1+z^2}}{1-\left(\frac{1-z^2}{1+z^2}\right)}=\int\frac{\frac{2dz}{\cancel{1+z^2}}}{\frac{\cancel{1}+z^2\cancel{-1}-z^2}{\cancel{1+z^2}}}=\int\frac{\cancel{2}dz}{\cancel{2}z^2}=-\frac{1}{z}+c=-\operatorname{co}\tau g\,{}^{x}\!/_{2}+c$$

8.19.- $\int\frac{\operatorname{sen}xdx}{1+\cos x}$

Solución.-

$$\int\frac{\operatorname{sen}xdx}{1+\cos x}=\int\frac{\left(\frac{2z}{1+z^2}\right)\left(\frac{2dz}{1+z^2}\right)}{1+\left(\frac{1-z^2}{1+z^2}\right)}=\int\frac{\frac{4zdz}{\cancel{(1+z^2)^2}}}{\frac{1+\cancel{z^2}+1\cancel{-z^2}}{\cancel{1+z^2}}}=\int\frac{4zdz}{2(1+z^2)}=\int\frac{2zdz}{(1+z^2)}$$

$$=\ell\eta\left|1+z^2\right|+c=\ell\eta\left|1+\tau g^2\,{}^{x}\!/_{2}\right|+c$$

8.20.- $\int\frac{\cos xdx}{2-\cos x}$

Solución.-

$$\int\frac{\cos xdx}{2-\cos x}=\int\left(-1+\frac{2}{2-\cos x}\right)dx=-\int dx+2\int\frac{dx}{2-\cos x}=-\int dx+2\int\frac{\left(\frac{2dz}{1+z^2}\right)}{2-\left(\frac{1-z^2}{1+z^2}\right)}$$

$$=-\int dx+2\int\frac{\frac{2dz}{\cancel{(1+z^2)}}}{\frac{2+2z^2-1+z^2}{\cancel{1+z^2}}}=-\int dx+2\int\frac{2dz}{3z^2+1}=-\int dx+\frac{4}{3}\int\frac{dz}{(z^2+{}^{1}\!/_{3})}$$

$$=-\int dx+\frac{4}{3}\int\frac{dz}{z^2+({}^{1}\!/_{\sqrt{3}})^2}=-x+{}^{4}\!/_{3}\frac{1}{{}^{1}\!/_{\sqrt{3}}}\operatorname{arc}\tau g\frac{z}{{}^{1}\!/_{\sqrt{3}}}+c=-x+\frac{4\sqrt{3}}{3}\operatorname{arc}\tau g\sqrt{3}z+c$$

$$=-x+\frac{4\sqrt{3}}{3}\operatorname{arc}\tau g(\sqrt{3}\tau g\,{}^{x}\!/_{2})+c$$

8.21.- $\int\frac{d\theta}{5-4\cos\theta}$

Solución.-

$$\int\frac{d\theta}{5-4\cos\theta}=\int\frac{\left(\frac{2dz}{1+z^2}\right)}{5-4\left(\frac{1-z^2}{1+z^2}\right)}=\int\frac{\frac{2dz}{\cancel{(1+z^2)}}}{\frac{5+5z^2-4+4z^2}{\cancel{1+z^2}}}=\int\frac{2dz}{9z^2+1}=\frac{2}{9}\int\frac{dz}{(z^2+1)}$$

$$=\frac{2}{9}\int\frac{dz}{z^2+({}^{1}\!/_{3})^2}=\frac{2}{9}\frac{1}{{}^{1}\!/_{3}}\operatorname{arc}\tau g\frac{z}{{}^{1}\!/_{3}}+c=\frac{2}{3}\operatorname{arc}\tau g3z+c=\frac{2}{3}\operatorname{arc}\tau g(3\tau g\,{}^{x}\!/_{2})+c$$

8.22.- $\int \frac{\operatorname{sen}\theta d\theta}{\cos^2\theta-\cos\theta-2}$

Solución.-

$$\int \frac{\operatorname{sen}\theta d\theta}{\cos^2\theta-\cos\theta-2}=\int \frac{\left(\frac{2z}{1+z^2}\right)\left(\frac{2dz}{1+z^2}\right)}{\left(\frac{1-z^2}{1+z^2}\right)^2-\left(\frac{1-z^2}{1+z^2}\right)-2}=\int \frac{\frac{4zdz}{\cancel{(1+z^2)^2}}}{\frac{(1-z^2)^2-(1-z^2)(1+z^2)-2(1+z^2)^2}{\cancel{(1+z^2)^2}}}$$

$$=\int \frac{4zdz}{-6z^2-2}=-\frac{1}{3}\int \frac{2zdz}{(z^2-\frac{1}{3})}=-\frac{1}{3}\ell\eta\left|z^2-\frac{1}{3}\right|+c=-\frac{1}{3}\ell\eta\left|\tau g^2\frac{x}{2}-\frac{1}{3}\right|+c$$

8.23.- $\int \sec xdx$

Solución.-

$$\int \sec xdx=\int \frac{dx}{\cos x}=\int \frac{\frac{2dz}{\cancel{1+z^2}}}{\frac{1-z^2}{\cancel{1+z^2}}}=\int \frac{2dz}{(1-z^2)}=\int \frac{2dz}{(1+z)(1-z)}\ (*)$$

Ahora bien: $\frac{2}{(1+z)(1-z)}=\frac{A}{1+z}+\frac{B}{1-z}$, de donde: $A=1, B=1$, luego:

$$(*)\ \int \frac{2dz}{(1+z)(1-z)}=\int \frac{dz}{1+z}-\int \frac{dz}{1-z}=\ell\eta|1+z|-\ell\eta|1-z|+c=\ell\eta\left|\frac{1+z}{1-z}\right|+c$$

Como: $z=\tau g\frac{x}{2}$, Se tiene: $=\ell\eta\left|\frac{1+\tau g\frac{x}{2}}{1-\tau g\frac{x}{2}}\right|+c$

8.24.- $\int \frac{\cos\theta d\theta}{5+4\cos\theta}$

Solución.-

$$\int \frac{d\theta}{5+4\cos\theta}=\int \frac{\left(\frac{1-z^2}{1+z^2}\right)\left(\frac{2dz}{1+z^2}\right)}{5+4\left(\frac{1-z^2}{1+z^2}\right)}=\int \frac{\frac{2(1-z^2)dz}{(1+z^2)^{\cancel{2}}}}{\frac{(5+5z^2+4-4z^2)}{\cancel{(1+z^2)}}}=\int \frac{(2-2z^2)dz}{(1+z^2)(9+z^2)}$$

Ahora bien: $\frac{2-2z^2}{(z^2+1)(z^2+9)}=\frac{Az+B}{z^2+1}+\frac{Cz+D}{z^2+9}$, de donde: $A=0, B=\frac{1}{2}, C=0, D=-\frac{5}{2}$, luego:

$$\int \frac{(2-2z^2)}{(z^2+1)(z^2+9)}=\frac{1}{2}\int \frac{dz}{z^2+1}-\frac{5}{2}\int \frac{dz}{z^2+9}=\frac{1}{2}\operatorname{arc}\tau gz+\frac{5}{2}\operatorname{arc}\tau g\frac{z}{3}+c$$

$$=\frac{1}{2}\operatorname{arc}\tau g\frac{\theta}{2}-\frac{5}{6}\operatorname{arc}\tau g(\frac{\tau g\frac{\theta}{2}}{3})+c=\frac{\theta}{4}-\frac{5}{6}\operatorname{arc}\tau g(\frac{\tau g\frac{\theta}{2}}{3})+c$$

8.25.- $\int \frac{d\theta}{\cos\theta+\operatorname{co}\tau g\theta}$

Solución.-

$$\int\frac{d\theta}{\cos\theta+\mathrm{co}\,\tau g\theta}=\int\frac{\left(\dfrac{2dz}{1+z^2}\right)}{\left(\dfrac{1-z^2}{1+z^2}\right)+\left(\dfrac{1-z^2}{2z}\right)}=\int\frac{\dfrac{2dz}{\cancel{(1+z^2)}}}{\dfrac{2z(1-z^2)+(1-z^2)(1+z^2)}{\cancel{(1+z^2)}2z}}$$

$$=\int\frac{4zdz}{2z(1-z^2)+(1-z^2)(1+z^2)}=\int\frac{4zdz}{(1-z^2)(z^2+2z+1)}=\int\frac{4zdz}{(1+z^3)(1-z)}(*)$$

Ahora bien: $\dfrac{4z}{(1+z^3)(1-z)}=\dfrac{A}{1+z}+\dfrac{B}{(1+z)^2}+\dfrac{C}{(1+z)^3}+\dfrac{D}{(1-z)}$

De donde: $A=\frac{1}{2}, B=1, C=-2, D=\frac{1}{2}$, luego:

$$(*)\int\frac{4z}{(1+z^3)(1-z)}=\frac{1}{2}\int\frac{dz}{1+z}+\int\frac{dz}{(1+z)^2}-2\int\frac{dz}{(1+z)^3}+\frac{1}{2}\int\frac{dz}{1-z}$$

$$=\frac{1}{2}\ell\eta\left|1+z\right|-\frac{1}{1+z}+\frac{1}{(1+z)^2}-\frac{1}{2}\ell\eta\left|1-z\right|+c=\frac{1}{2}\ell\eta\left|\frac{1+z}{1-z}\right|-\frac{1}{1+z}+\frac{1}{(1+z)^2}+c$$

$$=\frac{1}{2}\ell\eta\left|\frac{1+z}{1-z}\right|+\frac{-(1+z)+1}{(1+z)^2}+c=\frac{1}{2}\ell\eta\left|\frac{1+z}{1-z}\right|-\frac{z}{(1+z)^2}+c=\frac{1}{2}\ell\eta\left|\frac{1+\tau g\,\theta/2}{1-\tau g\,\theta/2}\right|-\frac{\tau g\,\theta/2}{(1+\tau g\,\theta/2)^2}+c$$

CAPITULO 9

INTEGRACION DE FUNCONES IRRACIONALES

En el caso de que el integrando contiene potencias faccionarias de la variable de integración, estas se simplifican usando una sustitución del tipo:
$x = t^n, \sqrt[n]{x} = t$ **,** siendo "n "el m.c.m de los denominadores de los exponentes.

EJERCICIOS DESARROLLADOS

9.1.-Encontrar: $\int \frac{\sqrt{x}dx}{1+x}$

Solución.- La única expresión "irracional" es $\sqrt{x}$, por lo tanto:
$\sqrt{x} = t \Rightarrow x = t^2, dx = 2tdt$, luego:

$$\int \frac{\sqrt{x}dx}{1+x} = \int \frac{t(2tdt)}{1+t^2} = 2\int \frac{t^2dt}{1+t^2} = 2\int \left(1 - \frac{1}{1+t^2}\right)dt = 2\int dt - 2\int \frac{dt}{t^2+1} = 2t - 2\operatorname{arc}\tau g t + c$$

Dado que: $t = \sqrt{x}$, se tiene: $= 2\sqrt{x} - 2\operatorname{arc}\tau g\sqrt{x} + c$

Respuesta: $\int \frac{\sqrt{x}dx}{1+x} = 2\sqrt{x} - 2\operatorname{arc}\tau g\sqrt{x} + c$

9.2.-Encontrar: $\int \frac{dx}{\sqrt{x}(1+\sqrt{x})}$

Solución.- Análogamente al caso anterior: $\sqrt{x} = t \Rightarrow x = t^2, dx = 2tdt$, luego:

$$\int \frac{dx}{\sqrt{x}(1+\sqrt{x})} = \int \frac{2\cancel{t}dt}{\cancel{t}(1+t)} = \int \frac{2dt}{1+t} = 2\ell\eta\left|t+1\right| + c$$

Dado que: $t = \sqrt{x}$, se tiene: $= 2\ell\eta\left|\sqrt{x}+1\right| + c$

Respuesta: $\int \frac{dx}{\sqrt{x}(1+\sqrt{x})} = 2\ell\eta\left|\sqrt{x}+1\right| + c$

9.3.-Encontrar: $\int \frac{dx}{3+\sqrt{x+2}}$

Solución.- La expresión "irracional" es ahora $\sqrt{x+2}$, por lo tanto:
$\sqrt{x+2} = t \Rightarrow x = t^2 - 2, dx = 2tdt$, luego:

$$\int \frac{dx}{3+\sqrt{x+2}} = \int \frac{2tdt}{3+t} = 2\int \left(1 - \frac{3}{t+3}\right)dt = 2\int dt - 6\int \frac{dt}{t+3} = 2t - 6\ell\eta\left|t+3\right| + c$$

Dado que: $t = \sqrt{x+2}$, se tiene: $= 2\sqrt{x+2} - 6\ell\eta\left|\sqrt{x+2}+3\right| + c$

Respuesta: $\int \frac{dx}{3+\sqrt{x+2}} = 2\sqrt{x+2} - 6\ell\eta\left|\sqrt{x+2}+3\right| + c$

9.4.-Encontrar: $\int \frac{1-\sqrt{3x+2}}{1+\sqrt{3x+2}}dx$

Solución.- La expresión "irracional" es ahora $\sqrt{3x+2}$, por lo tanto:

$\sqrt{3x+2}=t \Rightarrow 3x=t^2-2, dx=\frac{2}{3}tdt$, luego:

$$\int \frac{1-\sqrt{3x+2}}{1+\sqrt{3x+2}}dx=\int \frac{1-t}{1+t}\frac{2}{3}tdt=\frac{2}{3}\int \frac{t-t^2}{1+t}dt=\frac{2}{3}\int\left(-t+2-\frac{2}{t+1}\right)dt$$

$$=-\frac{2}{3}\int tdt+\frac{4}{3}\int dt-\frac{4}{3}\int \frac{dt}{t+1}=-\frac{1}{3}t^2+\frac{4}{3}t-\frac{4}{3}\ell\eta\left|t+1\right|+c$$

Dado que: $t=\sqrt{3x+2}$, se tiene:

$$=-\frac{1}{3}(3x+2)+\frac{4}{3}\sqrt{3x+2}-\frac{4}{3}\ell\eta\left|\sqrt{3x+2}+1\right|+c$$

$$=-x-\frac{2}{3}+\frac{4}{3}\sqrt{3x+2}-\frac{4}{3}\ell\eta\left|\sqrt{3x+2}+1\right|+c=-x-\frac{2}{3}+\frac{4}{3}\left(\sqrt{3x+2}-\ell\eta\left|\sqrt{3x+2}+1\right|\right)+c$$

Respuesta: $\int \frac{1-\sqrt{3x+2}}{1+\sqrt{3x+2}}dx=-x-\frac{2}{3}+\frac{4}{3}\left(\sqrt{3x+2}-\ell\eta\left|\sqrt{3x+2}+1\right|\right)+c$

9.5.- Encontrar: $\int \sqrt{1+\sqrt{x}}\,dx$

Solución.- La expresión "irracional" es ahora $\sqrt{x}$, por lo tanto:

$\sqrt{x}=t \Rightarrow x=t^2, dx=2tdt$, luego: $\int (\sqrt{1+\sqrt{x}})dx=\int \sqrt{1+t}\,2tdt$, como apareció la expresión: $\sqrt{1+t}$; se procede análogamente: $w=\sqrt{1+t} \Rightarrow t=w^2-1, dt=2wdw$, esto es: $\sqrt{1+t}\,2tdt=\int w2(w^2-1)2wdw=4\int (w^4-w^2)dw=\frac{4w^5}{5}-\frac{4w^3}{3}+c$

Dado que: $w=\sqrt{1+t}$, se tiene: $=\frac{4(1+t)^{5/2}}{5}-\frac{4(1+t)^{3/2}}{3}+c$

Respuesta: $\int \sqrt{1+\sqrt{x}}\,dx=\frac{4(1+\sqrt{x})^{5/2}}{5}-\frac{4(1+\sqrt{x})^{3/2}}{3}+c$

9.6.-Encontrar: $\int \frac{dx}{\sqrt{x+1}+\sqrt[4]{x+1}}$

Solución.- Previamente se tiene que el m.c.m. de los índices de Las raíces es: 4, por lo cual: $x+1=t^4, dx=4t^3dt$, de donde:

$$\int \frac{dx}{\sqrt{x+1}+\sqrt[4]{x+1}}=\int \frac{4t^3dt}{t^2+t}=4\int\left(t-1+\frac{t}{t^2+t}\right)dt=4\int tdt-4\int dt+4\int \frac{dt}{t+1}$$

$=2t^2-4t+4\ell\eta\left|t+1\right|+c$, dado que: $t=\sqrt[4]{x+1}$

Se tiene: $=2(x+1)^{1/2}-4(x+1)^{1/2}+4\ell\eta\left|(x+1)^{1/2}+1\right|+c$

Respuesta: $\int \frac{dx}{\sqrt{x+1}+\sqrt[4]{x+1}}=2(x+1)^{1/2}-4(x+1)^{1/2}+4\ell\eta\left|(x+1)^{1/2}+1\right|+c$

9.7.-Encontrar: $\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}$

Solución.- Previamente se tiene que el m.c.m. de los índices de Las raíces es: 6, por lo cual: $x=t^6 \Rightarrow t=\sqrt[6]{x}, dx=6t^5dt$, de donde:

$$\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}=\int \frac{6t^5dt}{t^3+t^2}=6\int \frac{t^3dt}{t+1}=6\int\left(t^2-t+1-\frac{1}{t+1}\right)dt=6\int t^2dt-6\int tdt+6\int dt-6\int \frac{dt}{t+1}$$

$$=2t^3-3t^2+6t-6\ell\eta\left|t+1\right|+c$$

Dado que: $t=\sqrt[6]{x}$

Se tiene: $=2(\sqrt[6]{x})^3-3(\sqrt[6]{x})^2+6\sqrt[6]{x}-6\ell\eta\left|\sqrt[6]{x}+1\right|+c$

Respuesta: $\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}=2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\ell\eta\left|\sqrt[6]{x}+1\right|+c$

9.8.-Encontrar: $\int \frac{dx}{\sqrt{x+1}+\sqrt{(x+1)^3}}$

Solución.- Previamente se tiene igual índice por lo cual: $\sqrt{x+1}=t \Rightarrow x=t^2-1, dx=2tdt$, de donde:

$$\int \frac{dx}{\sqrt{x+1}+\sqrt{(x+1)^3}}=\int \frac{2tdt}{t+t^3}=2\int \frac{dt}{1+t^2}=2\operatorname{arc}\tau gt+c$$

Dado que: $t=\sqrt{x+1}$, Se tiene: $=2\operatorname{arc}\tau g\sqrt{x+1}+c$

Respuesta: $\int \frac{dx}{\sqrt{x+1}+\sqrt{(x+1)^3}}=2\operatorname{arc}\tau g\sqrt{x+1}+c$

9.9.-Encontrar: $\int \frac{\sqrt{x}-1}{\sqrt[3]{x}+1}dx$

Solución.- Previamente se tiene que el m.c.m. de los índices de Las raíces es: 6, por lo cual: $x=t^6 \Rightarrow t=\sqrt[6]{x}, dx=6t^5dt$, de donde:

$$\int \frac{\sqrt{x}-1}{\sqrt[3]{x}+1}dx=\int \frac{t^3-1}{t^2+1}6t^5dt=6\int \frac{t^8-t^5}{t^2+1}dt=6\int\left(t^6-t^4-t^3+t^2+t-1-\frac{t-1}{t^2+1}\right)dt$$

$$=\frac{6}{7}t^7-\frac{6}{5}t^5-\frac{3}{2}t^4+2t^3+3t^2-6t+c_1-3\int \frac{2t-2}{t^2+1}dt$$

$$=\frac{6}{7}t^7-\frac{6}{5}t^5-\frac{3}{2}t^4+2t^3+3t^2-6t+c_1-3\int \frac{2t-2}{t^2+1}dt+6\int \frac{dt}{t^2+1}$$

$$=\frac{6}{7}t^7-\frac{6}{5}t^5-\frac{3}{2}t^4+2t^3+3t^2-6t-3\ell\eta\left|t^2+1\right|+6\operatorname{arc}\tau gt+c$$

Dado que: $t=\sqrt[6]{x}$, se tiene:

$$=\frac{6}{7}x\sqrt[6]{x}-\frac{6}{5}\sqrt[6]{x^5}-\frac{3}{2}\sqrt[3]{x^2}+2\sqrt{x}+3\sqrt[3]{x}-6\sqrt[6]{x}-3\ell\eta\left|1+\sqrt[3]{x}\right|+6\operatorname{arc}\tau g\sqrt[6]{x}+c$$

Respuesta:
$\int \frac{\sqrt{x}-1}{\sqrt[3]{x}+1}dx = \frac{6}{7}x\sqrt[6]{x} - \frac{6}{5}\sqrt[6]{x^5} - \frac{3}{2}\sqrt[3]{x^2} + 2\sqrt{x} + 3\sqrt[3]{x} - 6\sqrt[6]{x} - 3\ell\eta\left|1+\sqrt[3]{x}\right| + 6\operatorname{arc}\tau g\sqrt[6]{x} + c$

9.10.-Encontrar: $\int \frac{\sqrt{x}dx}{x+2}$

Solución.- La expresión "irracional" es $\sqrt{x}$, por lo tanto:
$\sqrt{x} = t \Rightarrow x = t^2, dx = 2tdt$,

luego: $\int \frac{\sqrt{x}dx}{x+2} = \int \frac{t(2tdt)}{t^2+2} = 2\int \frac{t^2dt}{t^2+2} = 2\int\left(1-\frac{2}{t^2+2}\right)dt = 2\int dt - 4\int \frac{dt}{t^2+2}$

$= 2t - \frac{4}{\sqrt{2}}\operatorname{arc}\tau g\frac{t}{\sqrt{2}} + c$, dado que: $t = \sqrt{x}$, se tiene: $= 2\sqrt{x} - 2\sqrt{2}\operatorname{arc}\tau g\sqrt{x/2} + c$

Respuesta: $\int \frac{\sqrt{x}dx}{x+2} = 2\sqrt{x} - 2\sqrt{2}\operatorname{arc}\tau g\sqrt{x/2} + c$

9.11.-Encontrar: $\int \frac{(\sqrt{x+1}+2)dx}{(x+1)^2-\sqrt{x+1}}$

Solución.- Previamente se tiene igual índice por lo cual: $\sqrt{x+1} = t \Rightarrow x = t^2-1, dx = 2tdt$, de donde:

$$\int \frac{(\sqrt{x+1}+2)dx}{(x+1)^2-\sqrt{x+1}} = \int \frac{\left[(x+1)^{1/2}+2\right]dx}{(x+1)^2-(x+1)^{1/2}} = \int \frac{t+2}{t^4-t}2tdt = 2\int \frac{(t+2)\cancel{t}dt}{\cancel{t}(t^3-1)}$$

$= 2\int \frac{(t+2)dt}{(t-1)(t^2+t+1)}$ $(*)$, considerando que:

$$\frac{t+2}{(t-1)(t^2+t+1)} = \frac{A}{(t-1)} + \frac{Bt+C}{(t^2+t+1)} \Rightarrow A=1, B=-1, C=-1$$

Dado que: $t = \sqrt{x+1}$ Se tiene: $= 2\operatorname{arc}\tau g\sqrt{x+1} + c$

$$(*)\ 2\int \frac{(t+2)dt}{(t-1)(t^2+t+1)} = 2\int \frac{dt}{(t-1)} + 2\int \frac{-t-1}{(t^2+t+1)}dt = 2\int \frac{dt}{(t-1)} - 2\int \frac{t+1}{(t^2+t+1)}dt$$

$$= 2\int \frac{dt}{(t-1)} - 2\int \frac{\frac{1}{2}(2t+1)+\frac{1}{2}}{(t^2+t+1)}dt = 2\int \frac{dt}{(t-1)} - \int \frac{(2t+1)dt}{(t^2+t+1)} - \int \frac{dt}{(t^2+t+1)}$$

$$= 2\int \frac{dt}{(t-1)} - \int \frac{(2t+1)dt}{(t^2+t+1)} - \int \frac{dt}{(t^2+t+\frac{1}{4})+\frac{3}{4}}$$

$$= 2\ell\eta\left|t-1\right| - \ell\eta\left|t^2+t+1\right| - \frac{2}{\sqrt{3}}\operatorname{arc}\tau g\frac{2t+1}{\sqrt{3}} + c$$

$$= \ell\eta\left|\frac{(t-1)^2}{(t^2+t+1)}\right| - \frac{2}{\sqrt{3}}\operatorname{arc}\tau g\frac{2t+1}{\sqrt{3}} + c$$

Dado que: $t = \sqrt{x+1}$, se tiene

Respuesta: $\int \frac{(\sqrt{x+1}+2)dx}{(x+1)^2-\sqrt{x+1}} = \ell\eta\left|\frac{(\sqrt{x+1}-1)^2}{(\sqrt{x+1}+x+2)}\right| - \frac{2}{\sqrt{3}}\operatorname{arc}\tau g\,\frac{2\sqrt{x+1}+1}{\sqrt{3}}+c$

EJERCICIOS PROPUESTOS

9.12.- $\int \frac{1+x}{1+\sqrt{x}}dx$ **9.13.-** $\int \frac{1-x}{1+\sqrt{x}}dx$ **9.14.-** $\int \frac{dx}{a+b\sqrt{x}}$

9.15.- $\int \frac{\sqrt{x+a}}{x+a}dx$ **9.16.-** $\int \frac{\sqrt{x}dx}{1+\sqrt[4]{x}}$ **9.17.-** $\int \frac{\sqrt{x}-\sqrt[6]{x}}{\sqrt[3]{x}+1}dx$

9.18.- $\int \frac{dx}{x-2-\sqrt{x}}dx$ **9.19.-** $\int \sqrt{\frac{1+x}{1-x}}dx$ **9.20.-** $\int \frac{\sqrt{x+a}}{x+b}dx$

9.21.- $\int \frac{\sqrt[3]{x+1}}{x}dx$ **9.22.-** $\int \frac{\sqrt{a^2-x^2}}{x^3}dx$ **9.23.-** $\int x^2\sqrt{x+a}\,dx$

9.24.- $\int \frac{dx}{\sqrt{x}+\sqrt[4]{x}+2\sqrt[8]{x}}$ **9.25.-** $\int x^3\sqrt{x^2+a^2}\,dx$

RESPUESTAS

9.12.- $\int \frac{1+x}{1+\sqrt{x}}dx$

Solución.- Sea: $\sqrt{x}=t \Rightarrow x=t^2, dx=2tdt$

$$\int \frac{1+x}{1+\sqrt{x}}dx = \int \frac{1+t^2}{1+t}2tdt = 2\int \frac{t+t^3}{1+t}dt = 2\int\left(t^2-t+2-\frac{2}{t+1}\right)dt$$

$$=2\int t^2dt-2\int tdt+4\int dt-4\int\frac{dt}{t+1}=\frac{2t^3}{3}-\frac{\cancel{2}t^2}{\cancel{2}}+4t-4\ell\eta\left|t+1\right|+c$$

$$=\frac{2\sqrt{x^3}}{3}-x+4\sqrt{x}-4\ell\eta\left|\sqrt{x}+1\right|+c$$

9.13.- $\int \frac{1-x}{1+\sqrt{x}}dx$

Solución.- Sea: $\sqrt{x}=t \Rightarrow x=t^2, dx=2tdt$

$$\int \frac{1-x}{1+\sqrt{x}}dx = \int \frac{1-t}{1+t}2tdt = 2\int \frac{t-t^2}{1+t}dt = -2\int tdt+4\int dt-4\int\frac{dt}{t+1} = -t^2+4t-4\ell\eta\left|t+1\right|+c$$

$$=-x+4\sqrt{x}-4\ell\eta\left|\sqrt{x}+1\right|+c$$

9.14.- $\int \frac{dx}{a+b\sqrt{x}}$

Solución.- Sea: $\sqrt{x}=t \Rightarrow x=t^2, dx=2tdt$

$$\int \frac{dx}{a+b\sqrt{x}} = \int \frac{2tdt}{a+bt} = 2\int \frac{tdt}{a+bt} = 2\int \left(\frac{1}{b} - \frac{a}{b}\frac{1}{a+bt} \right) dt = \frac{2}{b}\int dt - \frac{2a}{b^2}\int \frac{bdt}{a+bt}$$

$$= \frac{2}{b}t - \frac{2a}{b^2}\ell\eta|a+bt| + c = \frac{2}{b}\sqrt{x} - \frac{2a}{b^2}\ell\eta\left|a+b\sqrt{x}\right| + c$$

9.15.- $\int \frac{\sqrt{x+a}}{x+a}dx$

Solución.- Sea: $\sqrt{x+a} = t \Rightarrow x = t^2 - a, dx = 2tdt$

$$\int \frac{\sqrt{x+a}}{x+a}dx = \int \frac{\cancel{t}\,2\cancel{t}\,dt}{\cancel{t^2}} = 2\int dt = 2t + c = 2\sqrt{x+a} + c$$

9.16.- $\int \frac{\sqrt{x}dx}{1+\sqrt[4]{x}}$

Solución.- m.c.m: 4 ; Sea: $\sqrt[4]{x} = t \Rightarrow x = t^4, dx = 4t^3dt$

$$\int \frac{\sqrt{x}dx}{1+\sqrt[4]{x}} = \int \frac{t^2 4t^3dt}{1+t} = 4\int \frac{t^5dt}{1+t} = 4\int \left(t^4 - t^3 + t^2 - t + 1 - \frac{1}{t+1} \right) dt$$

$$= 4\left(\frac{t^5}{5} - \frac{t^4}{4} + \frac{t^3}{3} - \frac{t^2}{2} + t - \ell\eta|t+1| \right) + c = \frac{4t^5}{5} - t^4 + \frac{4t^3}{3} - 2t^2 + 4t - 4\ell\eta|t+1|$$

$$= \frac{4x^{5/4}}{5} - x + \frac{4x^{3/4}}{3} - 2x^{1/2} + 4x^{1/4} - 4\ell\eta\left|x^{1/4}+1\right|$$

9.17.- $\int \frac{\sqrt{x}-\sqrt[6]{x}}{\sqrt[3]{x}+1}dx$

Solución.- m.c.m: 6 ; Sea: $\sqrt[6]{x} = t \Rightarrow x = t^6, dx = 6t^5dt$

$$\int \frac{\sqrt{x}-\sqrt[6]{x}}{\sqrt[3]{x}+1}dx = \int \frac{t^3-t}{t^2+1}6t^5dt = 6\int \frac{(t^8-t^6)dt}{t^2+1} = 6\int t^6dt - 2\int t^4dt + 2\int t^2dt - 2\int dt + 2\int \frac{dt}{1+t^2}$$

$$= 6\left(\frac{t^7}{7} - \frac{2t^5}{5} + \frac{2t^3}{3} - 2t + 2\operatorname{arc}\tau gt \right) + c = \frac{6t^7}{7} - \frac{12t^5}{5} + 4t^3 - 12t + 12\operatorname{arc}\tau gt + c$$

$$= \frac{6x^{7/6}}{7} - \frac{12x^{5/2}}{5} + 4x^{1/2} - 12x^{1/6} + 12\operatorname{arc}\tau gx^{1/6} + c$$

9.18.- $\int \frac{dx}{x-2-\sqrt{x}}dx$

Solución.- Sea: $\sqrt{x} = t \Rightarrow x = t^2, dx = 2tdt$

$$\int \frac{dx}{x-2-\sqrt{x}}dx = \int \frac{2tdt}{t^2-2-t} = \int \frac{(2t-1)+1}{t^2-t-2}dt = \int \frac{2t-1}{t^2-t-2}dt + \int \frac{dt}{t^2-t-2}$$

$$= \int \frac{2t-1}{t^2-t-2}dt + \int \frac{dt}{(t-1/2)^2 - 9/4} = \ell\eta\left|t^2-t-2\right| + \frac{1}{\cancel{2}\,\frac{3}{\cancel{2}}}\ell\eta\left|\frac{t-3/2}{t+3/2}\right| + c$$

$$= \ell\eta\left|t^2 - t - 2\right| + \frac{1}{3}\ell\eta\left|\frac{2t-3}{2t+3}\right| + c = \ell\eta\left|x - \sqrt{x} - 2\right| + \frac{1}{3}\ell\eta\left|\frac{2\sqrt{x}-3}{2\sqrt{x}+3}\right| + c$$

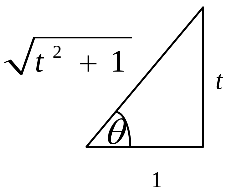
9.19.- $\int\sqrt{\frac{1+x}{1-x}}dx$

Solución.- Notará el lector, que este caso se diferencia de los anteriores, sin embargo la técnica que se seguirá, tiene la misma fundamentación y la información que se consiga es valiosa. $(*)$

Sea: $\sqrt{\frac{1+x}{1-x}} = t \Rightarrow \frac{1+x}{1-x} = t^2 \Rightarrow 1 + x = t^2 - t^2x \Rightarrow x(1+t^2) = t^2 - 1$

$x = \frac{t^2-1}{t^2+1} \Rightarrow dx = \frac{4tdt}{(t^2+1)^2}$, luego:

$(*)\ \int\sqrt{\frac{1+x}{1-x}}dx = \int\frac{t4tdt}{(t^2+1)^2} = \int\frac{4t^2dt}{(t^2+1)^2} = 4\int\frac{t^2dt}{(\sqrt{t^2+1})^4}(**)$, haciendo uso de sustituciones trigonométricas convenientes en $(**)$, y de la figura se tiene:



Se tiene: $t = \tau g\theta, dt = \sec^2\theta d\theta; \sqrt{t^2+1} = \sec\theta$

$$(**)\ 4\int\frac{t^2dt}{(\sqrt{t^2+1})^4} = \int\frac{4\tau g^2\theta\cancel{\sec^2}\theta d\theta}{\sec^{\cancel{4}}\theta} = 4\int\frac{\tau g^2\theta}{\sec^2\theta}d\theta$$

$$= 4\int \mathrm{sen}^2\theta d\theta = 2\int d\theta - 2\int\cos 2\theta d\theta = 2\theta - \mathrm{sen}\,2\theta + c = 2\theta - 2\,\mathrm{sen}\,\theta\cos\theta + c$$

$$= 2\operatorname{arc}\tau g t - 2\frac{t}{\sqrt{t^2+1}}\frac{1}{\sqrt{t^2+1}} + c = 2\operatorname{arc}\tau g t - \frac{2t}{t^2+1} + c = 2\operatorname{arc}\tau g\sqrt{\frac{1+x}{1-x}} - \frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{1+x}{1-x}+1} + c$$

$$= 2\operatorname{arc}\tau g\sqrt{\frac{1+x}{1-x}} - (1-x)\sqrt{\frac{1+x}{1-x}} + c$$

9.20.- $\int\frac{\sqrt{x+a}}{x+b}dx$

Solución.- Sea: $\sqrt{x+a} = t \Rightarrow x = t^2 - a, dx = 2tdt$

$$\int\frac{\sqrt{x+a}}{x+b}dx = \int\frac{t2tdt}{t^2-a+b} = 2\int\frac{t^2dt}{t^2+(b-a)} = 2\int\left(1 - \frac{b-a}{t^2+(b-a)}\right)dt$$

$$= 2\int dt - 2(b-a)\int\frac{dt}{t^2+(b-a)} = 2t - 2(b-a)\frac{1}{\sqrt{b-a}}\operatorname{arc}\tau g\frac{t}{\sqrt{b-a}} + c$$

$$= 2\sqrt{x+a} - 2\sqrt{b-a}\,\operatorname{arc}\tau g\sqrt{\frac{x+a}{b-a}} + c$$

9.21.- $\int \frac{\sqrt[3]{x+1}}{x}dx$

Solución.- Sea: $\sqrt[3]{x+1} = t \Rightarrow x = t^3 - 1, dx = 3t^2dt$

$$\int \frac{\sqrt[3]{x+1}}{x}dx = \int \frac{t3t^2dt}{t^3-1} = 3\int \frac{t^3dt}{t^3-1} = 3\int\left(1+\frac{1}{t^3-1}\right)dt = 3\int dt + 3\int \frac{dt}{t^3-1}$$

$= 3\int dt + 3\int \frac{dt}{(t-1)(t^2+t+1)}(*)$, por fracciones parciales:

$\frac{3}{(t-1)(t^2+t+1)} = \frac{A}{(t-1)} + \frac{Bt+C}{(t^2+t+1)} \Rightarrow 3 = A(t^2+t+1) + (Bt+C)(t-1)$, de donde:

$A = 1, B = -1, C = -2$, luego:

$$(*) = 3\int dt + \int \frac{dt}{t-1} - \int \frac{t+2}{t^2+t+1}dt = 3t + \ell\eta|t-1| - \tfrac{1}{2}\ell\eta|t^2+t+1| - \sqrt{3}\,\operatorname{arc}\tau g\left(\frac{2t+1}{\sqrt{3}}\right) + c$$

9.22.- $\int \frac{\sqrt{a^2-x^2}}{x^3}dx$

Solución.- Sea: $\sqrt{a^2-x^2} = t \Rightarrow x^2 = a^2 - t^2, xdx = -tdt$

$$\int \frac{\sqrt{a^2-x^2}}{x^3}dx = \int \frac{\sqrt{a^2-x^2}\,xdx}{x^4} = -\int \frac{ttdt}{(a^2-t^2)^2} = \int \frac{-t^2dt}{(a^2-t^2)^2} = \int \frac{-t^2dt}{(a+t)^2(a-t)^2}(*)$$

Por fracciones parciales:

$\frac{-t^2}{(t+a)^2(t-a)^2} = \frac{A}{(t+a)} + \frac{B}{(t+a)^2} + \frac{C}{(t-a)} + \frac{D}{(t-a)^2}$, de donde:

$A = \tfrac{1}{4}a, B = -\tfrac{1}{4}, C = -\tfrac{1}{4}a, D = -\tfrac{1}{4}$, luego:

$$(*)\int \frac{-t^2dt}{(a+t)^2(a-t)^2} = \frac{1}{4a}\int \frac{dt}{(t+a)} - \frac{1}{4a}\int \frac{dt}{(t+a)^2} - \frac{1}{4a}\int \frac{dt}{(t-a)} - \frac{1}{4a}\int \frac{dt}{(t-a)^2}$$

$$= \frac{1}{4a}\ell\eta|(t+a)| + \frac{1}{4(t+a)} - \frac{1}{4a}\ell\eta|(t-a)| + \frac{1}{4(t-a)} + c$$

$$= \frac{1}{4a}\ell\eta\left|\frac{(t+a)}{(t-a)}\right| + \frac{1}{4(t+a)} + \frac{1}{4(t-a)} + c$$

$$= \frac{1}{4a}\ell\eta\left|\frac{\sqrt{a^2-x^2}+a}{\sqrt{a^2-x^2}-a}\right| + \frac{\sqrt{a^2-x^2}}{2(\cancel{a^2}-x^2-\cancel{a^2})} + c = \frac{1}{4a}\ell\eta\left|\frac{\sqrt{a^2-x^2}+a}{\sqrt{a^2-x^2}-a}\right| - \frac{\sqrt{a^2-x^2}}{2x^2} + c$$

$$= \frac{1}{4a}\ell\eta\left|\frac{(\sqrt{a^2-x^2}+a)^2}{\cancel{a^2}-x^2-\cancel{a^2}}\right| - \frac{\sqrt{a^2-x^2}}{2x^2} + c = \frac{1}{2a}\ell\eta\left|\sqrt{a^2-x^2}+a\right| - \frac{1}{2a}\ell\eta|x| - \frac{\sqrt{a^2-x^2}}{2x^2} + c$$

9.23.- $\int x^2\sqrt{x+a}\,dx$

Solución.- Sea: $\sqrt{x+a} = t \Rightarrow x = t^2 - a, dx = 2tdt$

$$\int x^2\sqrt{x+a}\,dx=\int (t^2-a)^2 t2tdt=2\int t^2(t^2-a)^2dt=2\int (t^6-2at^4+a^2t^2)dt$$

$$=2\int t^6dt-4a\int t^4dt+2a^2\int t^2dt=\frac{2t^7}{7}-\frac{4at^5}{5}+\frac{2a^2t^3}{3}+c$$

$$=\frac{2(x+a)^{7/2}}{7}-\frac{4a(x+a)^{5/2}}{5}+\frac{2a^2(x+a)^{3/2}}{3}+c$$

9.24.- $\int \frac{dx}{\sqrt{x}+\sqrt[4]{x}+2\sqrt[8]{x}}$

Solución.- Sea: $\sqrt[8]{x}=t\Rightarrow x=t^8, dx=8t^7dt$

$$\int \frac{dx}{\sqrt{x}+\sqrt[4]{x}+2\sqrt[8]{x}}=\int \frac{8t^7dt}{t^4+t^2+2t}=8\int \frac{t^6dt}{t^3+t+2}=8\int\left(t^3-t-2+\frac{t^2+4t+4}{t^3+t+2}\right)dt$$

$$=8\int t^3-8\int tdt-16\int dt+8\int \frac{t^2+4t+4}{t^3+t+2}dt=8\frac{t^4}{4}-\frac{8t^2}{2}-16t+8\int \frac{t^2+4t+4}{t^3+t+2}dt$$

$=2t^4-4t^2-16t+8\int \frac{t^2+4t+4}{t^3+t+2}dt(*)$, por fracciones parciales:

$\frac{t^2+4t+4}{(t^3+t+2)}=\frac{t^2+4t+4}{(t+1)(t^2-t+2)}=\frac{A}{(t+1)}+\frac{Bt+C}{(t^2-t+2)}\Rightarrow A=1/4, B=3/4, C=14/4$, luego:

$$(*)=2t^4-4t^2-16t+8\left(\int \frac{1/4\,dt}{t+1}+\int \frac{3/4\,t+14/4}{t^2-t+2}dt\right)$$

$$=2t^4-4t^2-16t+8\left(\frac{1}{4}\int \frac{dt}{t+1}+\frac{1}{4}\int \frac{3t+14}{t^2-t+2}dt\right)=2t^4-4t^2-16t+2\int \frac{dt}{t+1}+2\int \frac{3t+14}{t^2-t+2}dt$$

$$=2t^4-4t^2-16t+2\ell\eta|t+1|+\cancel{2}\,\frac{3}{\cancel{2}}\int \frac{2t+28/3-31/3+31/3}{t^2-t+2}dt$$

$$=2t^4-4t^2-16t+2\ell\eta|t+1|+3\int \frac{(2t-1)}{t^2-t+2}dt+31\int \frac{dt}{t^2-t+2}$$

$$=2t^4-4t^2-16t+2\ell\eta|t+1|+3\ell\eta\left|t^2-t+2\right|+31\int \frac{dt}{(t-1/2)^2+7/4}$$

$$=2t^4-4t^2-16t+2\ell\eta|t+1|+3\ell\eta\left|t^2-t+2\right|+31\frac{2}{\sqrt{7}}\operatorname{arc}\tau g\frac{t-1/2}{\sqrt{7}/2}+c$$

$$=2t^4-4t^2-16t+2\ell\eta|t+1|+3\ell\eta\left|t^2-t+2\right|+\frac{62}{\sqrt{7}}\operatorname{arc}\tau g\frac{2t-1}{\sqrt{7}}+c$$

$$=2x^{1/2}-4x^{1/4}-16x^{1/8}+2\ell\eta\left|x^{1/8}+1\right|+3\ell\eta\left|x^{1/4}-x^{1/8}+2\right|+\frac{62}{\sqrt{7}}\operatorname{arc}\tau g\frac{2x^{1/8}-1}{\sqrt{7}}+c$$

9.25.- $\int x^3\sqrt{x^2+a^2}\,dx$

Solución.- Sea: $\sqrt{x^2+a^2}=t\Rightarrow x^2=t^2-a^2, xdx=tdt$

$$\int x^3\sqrt{x^2+a^2}\,dx=\int x^2\sqrt{x^2+a^2}\,xdx=\int (t^2-a^2)ttdt=\int (t^2-a^2)t^2dt=\int (t^4-a^2t^2)dt$$
$$=\frac{t^5}{5}-\frac{a^2t^3}{3}+c=\frac{(x^2+a^2)^{5/2}}{5}-\frac{a^2(x^2+a^2)^{3/2}}{3}+c=(x^2+a^2)^{3/2}\left(\frac{x^2+a^2}{5}-\frac{a^2}{3}\right)+c$$
$$=(x^2+a^2)^{3/2}\left(\frac{3x^2-2a^2}{15}\right)+c$$

EJERCICIOS COMPLEMENTARIOS

A continuación, se adjunta un listado de ejercicios que se proponen al lector. Observará que no se indica técnica alguna solicitada para el desarrollo de los mismos, y que además no se han respetado normas relativas a niveles de dificultad, ni a las técnicas mismas. Como siempre, se adjuntaran las soluciones cuyos desarrollos pueden diferir de los aquí presentados. No importa, eso es posible; además una consulta con su profesor aclarará cualquier discrepancia.

Encontrar:

1.- $\int t^3 e^{\operatorname{sen} t^4}\cos t^4 dt$ **2.-** $\int \frac{\theta d\theta}{(1+\theta)^2}$ **3.-** $\int \frac{\theta e^{\theta} d\theta}{(1+\theta)^2}$

4.- $\int e^{\tau g 3\theta}\sec^2 3\theta d\theta$ **5.-** $\int \frac{xdx}{\sqrt[3]{ax+b}}$ **6.-** $\int \sqrt{\frac{x^2-1}{x+1}}$

7.- $\int \frac{dx}{(2-x)\sqrt{1-x}}$ **8.-** $\int e^{2-x}dx$ **9.-** $\int \frac{e^x dx}{ae^x-b}$

10.- $\int \frac{(t+1)dt}{t^2+2t-5}$ **11.-** $\int \sec\frac{\varphi}{2}d\varphi$ **12.-** $\int \tau g\theta d\theta$

13.- $\int \frac{\eta^2}{a}\operatorname{sen}\frac{\eta}{b}d\eta$ **14.-** $\int \varphi\sec^2\varphi d\varphi$ **15.-** $\int \frac{dx}{5^x}$

16.- $\int \sec^2(1-x)dx$ **17.-** $\int \frac{xdx}{\sqrt{16-x^4}}$ **18.-** $\int \frac{dy}{\sqrt{1+\sqrt{1+y}}}$

19.- $\int \frac{dx}{\sqrt{x+4}-\sqrt{x+3}}$ **20.-** $\int \operatorname{cosec}\theta d\theta$ **21.-** $\int t(1-t^2)^{1/2}dt$

22.- $\int t(1-t^2)^{1/2}\operatorname{arcsen} t\,dt$ **23.-** $\int \frac{1+\cos 2x}{\operatorname{sen}^2 2x}dx$ **24.-** $\int \frac{x^2+1}{x^3-x}dx$

25.- $\int \frac{e^x dx}{\sqrt{9-e^{2x}}}$ **26.-** $\int \frac{dx}{(x-1)^3}$ **27.-** $\int \frac{(3x+4)dx}{\sqrt{2x+x^2}}$

28.- $\int \frac{ds}{\sqrt{4-s^2}}$ **29.-** $\int \frac{dx}{x^2\sqrt{x^2+e}}$ **30.-** $\int \frac{xdx}{\sqrt{1+x}}$

31.- $\int \frac{y^2 dy}{\sqrt{y+1}}$ **32.-** $\int \frac{y^3 dy}{\sqrt{y^2-1}}$ **33.-** $\int \frac{d\theta}{1+2\cos\theta}$

34.- $\int \frac{t^4 - t^3 + 4t^2 - 2t + 1}{t^3+1}dt$ **35.-** $\int \frac{d\varphi}{\ell\eta e}$ **36.-** $\int x(10+8x^2)^9 dx$

37.- $\int \frac{dx}{\sqrt{(16+x^2)^3}}$ **38.-** $\int \frac{x^3 dx}{\sqrt{x^2+4}}$ **39.-** $\int \frac{x^3 dx}{\sqrt{16-x^2}}$

40.- $\int a(x^2+1)^{1/2} dy$ **41.-** $\int \frac{dx}{(\sqrt{6-x^2})^3}$ **42.-** $\int \frac{dx}{x(3+\ell\eta x)}$

43.- $\int \frac{e^x}{16+e^{2x}}dx$ **44.-** $\int \cos\sqrt{1-x}\,dx$ **45.-** $\int \frac{x^3 dx}{\sqrt{x-1}}$

46.- $\int \frac{2y^5 - 7y^4 + 7y^3 - 19y^2 + 7y - 6}{(y-1)^2(y^2+1)^2}dy$ **47.-** $\int \mathrm{sen}\sqrt{x+1}\,dx$ **48.-** $\int \frac{9x^2+7x-6}{x^3-x}dx$

49.- $\int \frac{5w^3 - 5w^2 + 2w - 1}{w^4+w^2}dw$ **50.-** $\int \frac{3dx}{1+2x}$ **51.-** $\int \frac{(1-x)^2 dx}{x}$

52.- $\int \frac{xe^{-2x^2}}{2}dx$ **53.-** $\int e^{2t}\cos(e^t)dt$ **54.-** $\int \sqrt{x}(x^{3/2}-4)^3 dx$

55.- $\int \frac{\mathrm{sen}\, x e^{\sec x}}{\cos^2 x}dx$ **56.-** $\int \frac{ds}{s^{1/3}(1+s^{2/3})}$ **57.-** $\int \frac{1}{z^3}\left(\frac{1-z^2}{z^2}\right)^{10} dz$

58.- $\int \frac{x\ell\eta(1+x^2)}{1+x^2}dx$ **59.-** $\int \frac{\mathrm{co}\,\tau g x dx}{\ell\eta|\mathrm{sen}\, x|}$ **60.-** $\int \frac{ax^2-bx+c}{ax^2+bx-c}dx$

61.- $\int \frac{dx}{\cos^2 5x}$ **62.-** $\int \frac{dx}{12-7x}$ **63.-** $\int \tau g 16x dx$

64.- $\int \tau g 4\theta \sec^2 4\theta d\theta$ **65.-** $\int \frac{xdx}{\sqrt{x-5}}$ **66.-** $\int \frac{7t-2}{\sqrt{7-2t^2}}dt$

67.- $\int (1+x)\cos\sqrt{x}\,dx$ **68.-** $\int \frac{dx}{x(\sqrt{1+x}-1)}$ **69.-** $\int \frac{dx}{\mathrm{co}\,\tau g 6x}$

70.- $\int \mathrm{co}\,\tau g(2x-4)dx$ **71.-** $\int (e^t - e^{-2t})^2 dt$ **72.-** $\int \frac{(x+1)dx}{(x+2)^2(x+3)}$

73.- $\int (\mathrm{co}\,\tau g e^x)e^x dx$ **74.-** $\int \frac{\mathrm{sen}\,\theta+\theta}{\cos\theta+1}d\theta$ **75.-** $\int \frac{\mathrm{arc}\,\tau g x dx}{(1+x^2)^{3/2}}$

76.- $\int x\,\mathrm{co}\,\tau g(x^2/5)dx$ **77.-** $\int x\sqrt{4x^2-2}\,dx$ **78.-** $\int \frac{(x^2+9)^{1/2}dx}{x^4}$

79.- $\int x^2\,\mathrm{sen}^5 x^3 \cos x^3 dx$ **80.-** $\int \frac{xdx}{\sqrt{5x^2+7}}$ **81.-** $\int \frac{x^3 dx}{x^2-x-6}$

82.- $\int \mathrm{sen}\, 2\theta e^{\mathrm{sen}^2\theta} d\theta$ **83.-** $\int \frac{dx}{e^x - 9e^{-x}}$ **84.-** $\int \frac{dw}{1+\cos w}$

85.- $\int e^{\left(\frac{1-\operatorname{sen}^2 x/2}{3}\right)^2}(\cos^3 x/2\,\operatorname{sen} x/2)dx$

86.- $\int \frac{x^3dx}{\sqrt{19-x^2}}$

87.- $\int \frac{\operatorname{sen}\varphi d\varphi}{\cos^{1/2}\varphi}$

88.- $\int (\sec\varphi+\tau g\varphi)^2 d\varphi$

89.- $\int \frac{dt}{t(4+\ell\eta^2 t)^{1/2}}$

90.- $\int a^{\theta}b^{2\theta}c^{3\theta}d\theta$

91.- $\int \operatorname{sen}^{1/2}\varphi\cos^3\varphi d\varphi$

92.- $\int \frac{\sec^2\theta d\theta}{9+\tau g^2\theta}$

93.- $\int \frac{dx}{\sqrt{e^{2x}-16}}$

94.- $\int (e^{2s}-1)(e^{2s}+1)ds$

95.- $\int \frac{dx}{5x^2+8x+5}$

96.- $\int \frac{x^3+1}{x^3-x}dx$

97.- $\int (\operatorname{arcsen}\sqrt{1-x^2})^0 dx$

98.- $\int \frac{3dy}{1+\sqrt{y}}$

99.- $\int x(1+x)^{1/5}dx$

100.- $\int \frac{d\varphi}{a^2\operatorname{sen}^2\varphi+b^2\cos^2\varphi}$

101.- $\int \frac{tdt}{(2t+1)^{1/2}}$

102.- $\int \frac{s\ell\eta|s|ds}{(1-s^2)^{1/2}}$

103.- $\int (2\cos\alpha\operatorname{sen}\alpha-\operatorname{sen}2\alpha)d\alpha$

104.- $\int t^4\ell\eta^2 tdt$

105.- $\int u^2(1+v)^{11}dx$

106.- $\int \frac{(\varphi+\operatorname{sen}3\varphi)d\varphi}{3\varphi^2-2\cos3\varphi}$

107.- $\int \frac{(y^{1/2}+1)dy}{y^{1/2}(y+1)}$

108.- $\int \frac{ds}{s^3(s^2-4)^{1/2}}$

109.- $\int \sqrt{u}(1+u^2)^2du$

110.- $\int \frac{(x^3+x^2)dx}{x^2+x-2}$

111- $\int adb$

112.- $\int \frac{dx}{\sqrt{x^2-2x-8}}$

113.- $\int \frac{(x+1)dx}{\sqrt{2x-x^2}}$

114.- $\int f(x)f'(x)dx$

115.- $\int \frac{x^3+7x^2-5x+5}{x^2+2x-3}dx$

116.- $\int e^{\ell\eta|1+x+x^2|}dx$

117.- $\int \frac{(x-1)dx}{\sqrt{x^2-4x+3}}$

118.- $\int \frac{xdx}{\sqrt{x^2+4x+5}}$

119.- $\int \frac{4dx}{x^3+4x}$

120.- $\int \frac{\operatorname{co}\tau gxdx}{\ell\eta|\operatorname{sen}x|}$

121.- $\int \ell\eta\exp\sqrt{x-1}dx$

122.- $\int \frac{\sqrt{1+x^3}}{x}dx$

123.- $\int \sqrt{\frac{x-1}{x+1}}\frac{1}{x}dx$

124.- $\int \frac{\operatorname{sen}xdx}{1+\operatorname{sen}x+\cos x}$

125.- $\int \frac{dx}{3+2\cos x}$

126.- $\int \frac{xdx}{\sqrt{x^2-2x+5}}$

127.- $\int \frac{(1+\operatorname{sen}x)dx}{\operatorname{sen}x(2+\cos x)}$

128.- $\int \frac{dx}{x^4+4}$

RESPUESTAS

1.- $\int t^3e^{\operatorname{sen}t^4}\cos t^4dt$

Solución.- Sea: $u=\operatorname{sen}t^4, du=(\cos t^4)4t^3dt$; luego:

$$\int t^3e^{\operatorname{sen}t^4}\cos t^4dt=\frac{1}{4}\int 4t^3e^{\operatorname{sen}t^4}\cos t^4dt=\frac{1}{4}\int e^udu=\frac{1}{4}e^u+c=\frac{1}{4}e^{\operatorname{sen}t^4}+c$$

2.- $\int \frac{\theta d\theta}{(1+\theta)^2}$

Solución.-

$$\int \frac{\theta d\theta}{(1+\theta)^2} = \int \frac{Ad\theta}{1+\theta} + \int \frac{Bd\theta}{(1+\theta)^2} (*)$$

$\frac{\theta}{(1+\theta)^2} = \frac{A}{1+\theta} + \frac{B}{(1+\theta)^2} \Rightarrow \theta = A(1+\theta) + B \Rightarrow \theta = A\theta + (A+B)$, de donde:

$A=1, B=-1$, entonces: $(*) \int \frac{\theta d\theta}{(1+\theta)^2} = \int \frac{d\theta}{1+\theta} - \int \frac{d\theta}{(1+\theta)^2} = \ell\eta\left|1+\theta\right| + \frac{1}{1+\theta} + c$

3.- $\int \frac{\theta e^{\theta} d\theta}{(1+\theta)^2}$

Solución.-

Sea: $u = e^{\theta}$ $\quad dv = \frac{\theta d\theta}{(1+\theta)^2}$

$du = e^{\theta} d\theta$ $\quad v = \ell\eta\left|1+\theta\right| + \frac{1}{1+\theta}$

$$\int \frac{\theta e^{\theta} d\theta}{(1+\theta)^2} = e^{\theta}\ell\eta\left|1+\theta\right| + \frac{e^{\theta}}{1+\theta} - \int (\ell\eta\left|1+\theta\right| + \frac{1}{1+\theta}) e^{\theta} d\theta$$

$= e^{\theta}\ell\eta\left|1+\theta\right| + \frac{e^{\theta}}{1+\theta} - \int e^{\theta}\ell\eta\left|1+\theta\right| d\theta - \int \frac{e^{\theta} d\theta}{1+\theta} (*)$, resolviendo por partes la segunda integral se tiene: $u = e^{\theta}$ $\quad dv = \frac{\theta d\theta}{1+\theta}$

$du = e^{\theta} d\theta$ $\quad v = \ell\eta\left|1+\theta\right|$

Luego: $\int \frac{e^{\theta} d\theta}{1+\theta} = e^{\theta}\ell\eta\left|1+\theta\right| - \int e^{\theta}\ell\eta\left|1+\theta\right| d\theta$, esto es:

$$(*) = \cancel{e^{\theta}\ell\eta\left|1+\theta\right|} + \frac{e^{\theta}}{1+\theta} \cancel{- \int e^{\theta}\ell\eta\left|1+\theta\right| d\theta} \cancel{- e^{\theta}\ell\eta\left|1+\theta\right|} \cancel{+ \int e^{\theta}\ell\eta\left|1+\theta\right| d\theta}$$

$$= \frac{e^{\theta}}{1+\theta}$$

4.- $\int e^{\tau g 3\theta} \sec^2 3\theta d\theta$

Solución.- Sea: $u = \tau g 3\theta, du = 3\sec^2 3\theta d\theta$

$$\int e^{\tau g 3\theta} \sec^2 3\theta d\theta = \frac{1}{3}\int e^u du = \frac{1}{3} e^u + c = \frac{e^{\tau g 3\theta}}{3} + c$$

5.- $\int \frac{xdx}{\sqrt[3]{ax+b}}$

Solución.- Sea: $ax + b = t^3 \Rightarrow x = \frac{t^3 - b}{a}, dx = \frac{3t^2}{a} dt$

$$\int \frac{xdx}{\sqrt[3]{ax+b}} = \int \frac{\left(\frac{t^3-b}{a}\right)\frac{3t^2}{a}dt}{t} = \int \frac{3t(t^3-b)}{a^2}dt = \frac{3}{a^2}\int (t^4-bt)dt = \frac{3}{a^2}\left(\frac{t^5}{5}-\frac{bt^2}{2}\right)+c$$

$$= \frac{3t^5}{5a^2}-\frac{3bt^2}{2a^2}+c = \frac{3(ax+b)^{5/3}}{5a^2}-\frac{3b(ax+b)^{2/3}}{2a^2}+c$$

$$= \frac{3(ax+b)\sqrt[3]{(ax+b)^2}}{5a^2}-\frac{3b\sqrt[3]{(ax+b)^2}}{2a^2}+c$$

6.- $\int \sqrt{\frac{x^2-1}{x+1}}dx$

Solución.-

$$\int \sqrt{\frac{x^2-1}{x+1}}dx = \int \sqrt{\frac{\cancel{(x+1)}(x-1)}{\cancel{x+1}}} = \int (x-1)^{1/2}dx = \frac{(x-1)^{3/2}}{3/2}+c = \frac{2(x-1)^{3/2}}{3}+c$$

$$= \frac{2(x-1)\sqrt{x-1}}{3}+c$$

7.- $\int \frac{dx}{(2-x)\sqrt{1-x}}$

Solución.- Sea: $1-x=t^2 \Rightarrow x=1-t^2, dx=-2tdt$

$$\int \frac{dx}{(2-x)\sqrt{1-x}} = \int \frac{-2tdt}{\left[2-(1-t^2)\right]t} = -2\int \frac{dt}{1+t^2} = -2\operatorname{arc}\tau gt+c = -2\operatorname{arc}\tau g\sqrt{1-x}+c$$

8.- $\int e^{2-x}dx$

Solución.- Sea: $u=2-x, du=-dx$

$$\int e^{2-x}dx = -\int e^u du = -e^u+c = -e^{2-x}+c$$

9.- $\int \frac{e^x dx}{ae^x-b}$

Solución.- Sea: $u=ae^x-b, du=ae^x dx$

$$\int \frac{e^x dx}{ae^x-b} = \frac{1}{a}\int \frac{du}{u} = \frac{1}{a}\ell\eta|u|+c = \frac{1}{a}\ell\eta\left|ae^x-b\right|+c$$

10.- $\int \frac{(t+1)dt}{t^2+2t-5}$

Solución.- Sea: $u=t^2+2t-5, du=2(t+1)dt$

$$\int \frac{(t+1)dt}{t^2+2t-5} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ell\eta|u|+c = \frac{1}{2}\ell\eta\left|t^2+2t-5\right|+c$$

11.- $\int \sec\frac{\varphi}{2}d\varphi$

Solución.- Sea: $u=\sec\frac{\varphi}{2}+\tau g\frac{\varphi}{2}, du=\frac{1}{2}(\sec\frac{\varphi}{2}\tau g\frac{\varphi}{2}+\sec^2\frac{\varphi}{2})d\varphi$

$$\int \sec\frac{\varphi}{2}d\varphi=\int\frac{\sec\varphi/2\,(\sec\varphi/2+\tau g\,\varphi/2)}{\sec\varphi/2+\tau g\,\varphi/2}d\varphi=\int\frac{\sec^2\varphi/2+\sec\varphi/2\,\tau g\,\varphi/2}{\sec\varphi/2+\tau g\,\varphi/2}d\varphi$$

$$=2\int\frac{du}{u}=2\ell\eta|u|+c=2\ell\eta\left|\sec\varphi/2+\tau g\,\varphi/2\right|+c$$

12.- $\int \tau g\theta d\theta$

Solución.- Sea: $u=\cos\theta, du=-\mathrm{sen}\,\theta d\theta$

$$\int \tau g\theta d\theta=\int\frac{\mathrm{sen}\,\theta}{\cos\theta}d\theta=-\int\frac{du}{u}=-\ell\eta|u|+c=-\ell\eta|\cos\theta|+c=-\ell\eta\left|\frac{1}{\mathrm{s}ec\theta}\right|+c$$

$$=-\cancel{\ell\eta 1}^{0}+\ell\eta|\mathrm{s}ec\theta|+c=\ell\eta|\mathrm{s}ec\theta|+c$$

13.- $\int\frac{\eta^2}{a}\mathrm{sen}\frac{\eta}{b}d\eta$

Solución.-

Sea: $u=\frac{\eta^2}{a}$ $\quad dv=\mathrm{sen}\frac{\eta}{b}d\eta$

$du=\frac{2\eta d\eta}{a}$ $\quad v=-b\cos\frac{\eta}{b}$

$\int\frac{\eta^2}{a}\mathrm{sen}\frac{\eta}{b}d\eta=-\frac{a}{b}\eta^2\cos\frac{\eta}{b}+\frac{2b}{a}\int\eta\cos\frac{\eta}{b}d\eta(*)$, resolviendo por partes la segunda integral se tiene: $u=\eta$ $\quad dv=\cos\frac{\eta}{b}d\eta$

$du=d\eta$ $\quad v=b\,\mathrm{sen}\frac{\eta}{b}$

$$(*)=-\frac{a}{b}\eta^2\cos\frac{\eta}{b}+\frac{2b}{a}\left(b\eta\,\mathrm{sen}\frac{\eta}{b}-b\int\mathrm{sen}\frac{\eta}{b}d\eta\right)$$

$$=-\frac{a}{b}\eta^2\cos\frac{\eta}{b}+\frac{2b^2}{a}\eta\,\mathrm{sen}\frac{\eta}{b}+\frac{2b^3}{a}\cos\frac{\eta}{b}+c$$

14.- $\int\varphi\sec^2\varphi d\varphi$

Solución.-

Sea: $u=\varphi$ $\quad dv=\sec^2\varphi d\varphi$

$du=d\varphi$ $\quad v=\tau g\varphi$

$$\int\varphi\sec^2\varphi d\varphi=\varphi\tau g\varphi-\int\tau g\varphi d\varphi=\varphi\tau g\varphi-\ell\eta|\sec\varphi|+c$$

15.- $\int\frac{dx}{5^x}$

Solución.- Sea: $u=-x, du=-dx$

$$\int\frac{dx}{5^x}=\int 5^{-x}dx=-\int 5^u du=-\frac{5^u}{\ell\eta 5}+c=-\frac{5^{-x}}{\ell\eta 5}+c=-\frac{1}{5^x\ell\eta 5}+c$$

16.- $\int \sec^2(1-x)dx$

Solución.- Sea: $u = 1-x, du = -dx$

$\int \sec^2(1-x)dx = -\int \sec^2 u du = -\tau g u + c = -\tau g(1-x) + c$

17.- $\int \frac{xdx}{\sqrt{16-x^4}}$

Solución.- Sea: $u = x^2, du = 2xdx$

$\int \frac{xdx}{\sqrt{16-x^4}} = \int \frac{xdx}{\sqrt{4^2-(x^2)^2}} = \frac{1}{2}\int \frac{2xdx}{\sqrt{4^2-(x^2)^2}} = \frac{1}{2}\int \frac{du}{\sqrt{4^2-u^2}} = \frac{1}{2}\text{arcs}e\text{n}\frac{u}{4} + c$

$= \frac{1}{2}\text{arcs}e\text{n}\frac{x^2}{4} + c$

18.- $\int \frac{dy}{\sqrt{1+\sqrt{1+y}}}$

Solución.- Sea: $t = \left[1+(1+y)^{1/2}\right]^{1/2} \Rightarrow t^2 = 1+(1+y)^{1/2} \Rightarrow t^2 - 1 = (1+y)^{1/2}$

$\Rightarrow (t^2-1)^2 = 1+y \Rightarrow y = (t^2-1)^2 - 1, dy = 4t(t^2-1)dt$

$\int \frac{dy}{\sqrt{1+\sqrt{1+y}}} = \int \frac{4\cancel{t}(t^2-1)dt}{\cancel{t}} = 4\int (t^2-1)dt = 4(\frac{t^3}{3} - t) + c = 4t(\frac{t^2}{3} - 1) + c$

$= 4\sqrt{1\sqrt{1+y}}(\frac{1+\sqrt{1+y}}{3} - 1) + c = \frac{4}{3}\sqrt{1\sqrt{1+y}}(\sqrt{1+y} - 2) + c$

19.- $\int \frac{dx}{\sqrt{x+4} - \sqrt{x+3}}$

Solución.-

$\int \frac{dx}{\sqrt{x+4}-\sqrt{x+3}} = \int \frac{(x+4)^{1/2} + (x+3)^{1/2}}{(x+4)-(x+3)}dx = \int \left[(x+4)^{1/2} + (x+3)^{1/2}\right]dx$

$\int (x+4)^{1/2} + \int (x+3)^{1/2} = \frac{(x+4)^{3/2}}{3/2} + \frac{(x+3)^{3/2}}{3/2} + c = \frac{2\sqrt{(x+4)^3}}{3} + \frac{2\sqrt{(x+3)^3}}{3} + c$

$= \frac{2}{3}\left(\sqrt{(x+4)^3} + \sqrt{(x+3)^3}\right) + c$

20.- $\int \cos ec\theta d\theta$

Solución.- Sea: $u = \cos ec\theta + \text{co}\,\tau g\theta, du = -(\cos ec\theta\,\text{co}\,\tau g\theta + \cos ec^2\theta)d\theta$

$\int \cos ec\theta d\theta = \int \frac{\cos ec\theta(\cos ec\theta + \text{co}\,\tau g\theta)d\theta}{\cos ec\theta + \text{co}\,\tau g\theta} = \int \frac{\cos ec^2\theta + \cos ec\theta\,\text{co}\,\tau g\theta d\theta}{\cos ec\theta + \text{co}\,\tau g\theta}$

$= -\int \frac{du}{u} = -\ell\eta|u| + c = -\ell\eta\left|(\cos ec\theta + \text{co}\,\tau g\theta)\right| + c$

21.- $\int t(1-t^2)^{1/2}dt$

Solución.- Sea: $u = 1-t^2, du = -2tdt$

$$\int t(1-t^2)^{1/2}dt = -\frac{1}{2}\int u^{1/2}du = -\frac{1}{\cancel{2}}\frac{u^{3/2}}{3/\cancel{2}} + c = -\frac{1}{3}u^{3/2} + c = -\frac{1}{3}(1-t^2)^{3/2} + c$$

22.- $\int t(1-t^2)^{1/2} \operatorname{arcsen} t\,dt$

Solución.-

Sea: $u = \operatorname{arcsen} t \qquad dv = t(1-t^2)^{1/2}dt$

$du = \dfrac{dt}{\sqrt{1-t^2}} \qquad v = -\dfrac{1}{3}(1-t^2)^{3/2}$

$$\int t(1-t^2)^{1/2} \operatorname{arcsen} t\,dt = -\frac{1}{3}(1-t^2)^{3/2} \operatorname{arcsen} t + \frac{1}{3}\int (1-t^2)\cancel{\sqrt{1-t^2}}\,\frac{dt}{\cancel{\sqrt{1-t^2}}}$$

$$= -\frac{(1-t^2)^{3/2}}{3}\operatorname{arcsen} t + \frac{1}{3}\int (1-t^2)dt = -\frac{(1-t^2)^{3/2}}{3}\operatorname{arcsen} t + \frac{1}{3}(t - \frac{t^3}{3}) + c$$

$$= -\frac{1}{3}\left[(1-t^2)^{3/2}\operatorname{arcsen} t - t + \frac{t^3}{3}\right] + c$$

23.- $\int \dfrac{1+\cos 2x}{\operatorname{sen}^2 2x}dx$

Solución.-

$$\int \frac{1+\cos 2x}{\operatorname{sen}^2 2x}dx = \int \frac{1+\cos 2x}{1-\cos^2 x}dx = \int \frac{dx}{1-\cos 2x} = \int \frac{dx}{2\left(\frac{1-\cos 2x}{2}\right)} = \frac{1}{2}\int \frac{dx}{\operatorname{sen}^2 x}$$

$$= \frac{1}{2}\int \operatorname{cosec}^2 x\,dx = -\frac{1}{2}\operatorname{co\tau g} x + c$$

24.- $\int \dfrac{x^2+1}{x^3-x}dx$

Solución.-

$$\int \frac{x^2+1}{x^3-x}dx = \int \frac{(x^2+1)dx}{x(x^2-1)} = \int \frac{(x^2+1)dx}{x(x+1)(x-1)} = \int \frac{Adx}{x} + \int \frac{Bdx}{(x+1)} + \int \frac{Cdx}{(x-1)} (*)$$

$$\frac{(x^2+1)}{x(x+1)(x-1)} = \frac{A}{x} + \frac{B}{(x+1)} + \frac{C}{(x-1)} \Rightarrow (x^2+1) = A(x^2-1) + Bx(x-1) + Cx(x+1)$$

$x = 0 \Rightarrow 1 = -A \Rightarrow A = -1$

De donde: $x = -1 \Rightarrow 2 = B(-1)(-2) \Rightarrow B = 1$

$x = 1 \Rightarrow 2 = C(1)(2) \Rightarrow C = 1$

Entonces:

$$(*)\int \frac{(x^2+1)dx}{x(x+1)(x-1)} = -\int \frac{dx}{x} + \int \frac{dx}{(x+1)} + \int \frac{dx}{(x-1)} = -\ell\eta|x| + \ell\eta|x+1| + \ell\eta|x-1| + c$$

$$= \ell\eta\left|\frac{x^2-1}{x}\right| + c$$

25.- $\int \frac{e^x dx}{\sqrt{9-e^{2x}}}$

Solución.- Sea: $u = e^x, du = e^x dx$

$$\int \frac{e^x dx}{\sqrt{9-e^{2x}}} = \int \frac{e^x dx}{\sqrt{3^2-(e^x)^2}} = \int \frac{du}{\sqrt{3^2-u^2}} = \arcsin\frac{u}{3} + c = \arcsin\frac{e^x}{3} + c$$

26.- $\int \frac{dx}{(x-1)^3}$

Solución.-

$$\int \frac{dx}{(x-1)^3} = \int (x-1)^{-3} dx = -\frac{(x-1)^{-2}}{2} + c = -\frac{1}{(x-1)^2} + c$$

27.- $\int \frac{(3x+4)dx}{\sqrt{2x+x^2}}$

Solución.- Sea: $u = 2x+x^2, du = 2(1+x)dx$

$$\int \frac{(3x+4)dx}{\sqrt{2x+x^2}} = \int \frac{(3x+3)+1}{\sqrt{2x+x^2}}dx = 3\int \frac{(x+1)dx}{\sqrt{2x+x^2}} + \int \frac{dx}{\sqrt{2x+x^2}} = \frac{3}{2}\int \frac{du}{u^{1/2}} + \int \frac{dx}{\sqrt{2x+x^2}}$$

$$= \frac{3}{2}\int \frac{du}{u^{1/2}} + \int \frac{dx}{\sqrt{(x^2+2x+1)-1}} = \frac{3}{\cancel{2}}\frac{u^{1/2}}{1/\cancel{2}} + \int \frac{dx}{\sqrt{(x+1)^2-1}} = 3\sqrt{2x+x^2} + \int \frac{dx}{\sqrt{(x+1)^2-1}}$$

Sustituyendo por: $x+1 = \sec\theta, dx = \sec\theta\tau g\theta d\theta, \sqrt{(x+1)^2-1} = \tau g\theta$

$$= 3\sqrt{2x+x^2} + \int \frac{\sec\theta \cancel{\tau g\theta}}{\cancel{\tau g\theta}}d\theta = 3\sqrt{2x+x^2} + \int \sec\theta d\theta = 3\sqrt{2x+x^2} + \ell\eta\left|\sec\theta + \tau g\theta\right| + c$$

$$= 3\sqrt{2x+x^2} + \ell\eta\left|x+1+\sqrt{2x+x^2}\right| + c$$

28.- $\int \frac{ds}{\sqrt{4-s^2}}$

Solución.- Sea: $s = 2\operatorname{sen}\theta, ds = 2\cos\theta d\theta, \sqrt{4-s^2} = 2\cos\theta$

$$\int \frac{ds}{\sqrt{4-s^2}} = \int \frac{\cancel{2\cos\theta} d\theta}{\cancel{2\cos\theta}} = \int d\theta = \theta = \arcsin s/2 + c$$

29.- $\int \frac{dx}{x^2\sqrt{x^2+e}}$

Solución.- Sea: $x = \sqrt{e}\tau g\theta, dx = \sqrt{e}\sec^2\theta d\theta, \sqrt{x^2+e} = \sqrt{e}\sec\theta$

$$\int \frac{dx}{x^2\sqrt{x^2+e}} = \int \frac{\cancel{\sqrt{e}}\sec^{\cancel{2}}\theta d\theta}{e\tau g^2 \cancel{\sqrt{e}\sec\theta}} = \frac{1}{e}\int \frac{\sec\theta d\theta}{\tau g^2} = \frac{1}{e}\int \frac{\frac{1}{\cancel{\cos\theta}}d\theta}{\frac{\operatorname{sen}^2\theta}{\cos^{\cancel{2}}\theta}} = \frac{1}{e}\int \frac{\cos\theta}{\operatorname{sen}^2\theta}(*)$$

Sea: $u = \operatorname{sen}\theta, du = \cos\theta d\theta$, luego:

$$(*)=\frac{1}{e}\int\frac{du}{u^2}=\frac{1}{e}\int u^{-2}du=\frac{1}{e}\frac{u^{-1}}{-1}+c=-\frac{1}{eu}+c=-\frac{1}{e\,\mathrm{sen}\,\theta}+c=-\frac{1}{e\frac{x}{\sqrt{x^2+e}}}+c$$

$$=-\frac{\sqrt{x^2+e}}{ex}+c$$

30.- $\int\frac{xdx}{\sqrt{1+x}}$

Solución.- Sea: $x+1=t^2\Rightarrow x=t^2-1, dx=2tdt$

$$\int\frac{xdx}{\sqrt{1+x}}=\int\frac{(t^2-1)2\cancel{t}dt}{\cancel{t}}=2\int(t^2-1)dt=2(\frac{t^3}{3}-t)+c=2t(\frac{t^2}{3}-1)+c$$

$$=2\sqrt{x+1}(\frac{x+1}{3}-1)+c=2\sqrt{x+1}(\frac{x-2}{3})+c$$

31.- $\int\frac{y^2dy}{\sqrt{y+1}}$

Solución.- Sea: $y+1=t^2\Rightarrow y=t^2-1, dy=2tdt$

$$\int\frac{y^2dy}{\sqrt{y+1}}=\int\frac{(t^2-1)^2 2\cancel{t}dt}{\cancel{t}}=2\int(t^2-1)^2dt=2\int(t^4-2t^2+1)dt=2\left(\frac{t^5}{5}-\frac{2t^3}{3}+t\right)+c$$

$$=2t\left(\frac{t^4}{5}-\frac{2t^2}{3}+1\right)+c=2\sqrt{y+1}\left(\frac{(\sqrt{y+1})^4}{5}-\frac{2(\sqrt{y+1})^2}{3}+1\right)+c$$

$$=2\sqrt{y+1}\left(\frac{(y+1)^2}{5}-\frac{2y+2}{3}+1\right)+c=2\sqrt{y+1}\left(\frac{y^2+2y+1}{5}-\frac{2y+2}{3}+1\right)+c$$

$$=2\sqrt{y+1}\left(\frac{3y^2-4y+8}{15}\right)+c$$

32.- $\int\frac{y^3dy}{\sqrt{y^2-1}}$

Solución.- Sea: $u=y^2-1\Rightarrow y^2=u+1, dy=2ydy$

$$\int\frac{y^3dy}{\sqrt{y^2-1}}=\int\frac{y^2ydy}{\sqrt{y^2-1}}=\frac{1}{2}\int\frac{(u+1)du}{u^{1/2}}=\frac{1}{2}\int(u^{1/2}+u^{-1/2})du=\frac{1}{\cancel{2}}\left(\frac{u^{3/2}}{3/\cancel{2}}+\frac{u^{1/2}}{1/\cancel{2}}\right)+c$$

$$=\frac{u^{3/2}}{3}+u^{1/2}+c=u^{1/2}(1/3\,u+1)+c=\sqrt{y^2-1}\left(\frac{y^2-1}{3}+1\right)+c=\sqrt{y^2-1}\left(\frac{y^2+2}{3}\right)+c$$

33.- $\int\frac{d\theta}{1+2\cos\theta}$

Solución.- Sea: $d\theta=\frac{2dz}{1+z^2},\cos\theta=\frac{1-z^2}{1+z^2},\theta=2\,\mathrm{arc}\,\tau gz$

$$\int \frac{d\theta}{1+2\cos\theta} = \int \frac{\frac{2dz}{1+z^2}}{1+\frac{2(1-z^2)}{1+z^2}} = \int \frac{2dz}{1+z^2+2(1-z^2)} = \int \frac{2dz}{1+z^2+2-2z^2} = \int \frac{2dz}{3-z^2}$$

$$= \int \frac{2dz}{3-z^2} = -2\int \frac{dz}{z^2-3} = -2\int \frac{dz}{z^2-(\sqrt{3})^2} = -\cancel{2}\,\frac{1}{\cancel{2}\sqrt{3}}\ell\eta\left|\frac{z-\sqrt{3}}{z+\sqrt{3}}\right| + c$$

$$= -\frac{1}{\sqrt{3}}\ell\eta\left|\frac{\tau g\,\theta/2-\sqrt{3}}{\tau g\,\theta/2+\sqrt{3}}\right| + c$$

34.- $\int \frac{t^4-t^3+4t^2-2t+1}{t^3+1}dt$

Solución.-

$$\int \frac{t^4-t^3+4t^2-2t+1}{t^3+1}dt = \int\left(t-1+\frac{3t^2-t+1}{t^3+t}\right)dt = \int tdt - \int dt + \int \frac{3t^2-t+1}{t^3+t}dt$$

$$= \frac{t^2}{2} - t + \int \frac{3t^2-t+1}{t^3+t}dt (*)$$

$$\frac{3t^2-t+1}{t(t^2+1)} = \frac{A}{t} + \frac{Bt+C}{(t^2+1)} \Rightarrow 3t^2-t+1 = A(t^2+1)+(Bt+C)t$$

De donde: $\left.\begin{array}{l} t=0 \Rightarrow 1=A \Rightarrow A=1 \\ t=1 \Rightarrow 3=2A+B+C \Rightarrow B+C=1 \\ t=-1 \Rightarrow 5=2A-(C-B) \Rightarrow B-C=3 \end{array}\right\} B=2, C=-1$

$$(*) = \frac{t^2}{2} - t + \int \frac{Adt}{t} + \int \frac{Bt+C}{t^2+1}dt = \frac{t^2}{2} - t + \int \frac{dt}{t} + \int \frac{2t-1}{t^2+1}dt$$

$$= \frac{t^2}{2} - t + \ell\eta|t| + \int \frac{2tdt}{t^2+1} - \int \frac{dt}{t^2+1} = \frac{t^2}{2} - t + \ell\eta|t| + \ell\eta|t^2+1| - \operatorname{arc}\tau gt + c$$

$$= \frac{t^2}{2} - t + \ell\eta|t(t^2+1)| - \operatorname{arc}\tau gt + c$$

35.- $\int \frac{d\varphi}{\ell\eta e}$

Solución.-

$$\int \frac{d\varphi}{\ell\eta e} = \int d\varphi = \varphi + c$$

36.- $\int x(10+8x^2)^9dx$

Solución.- Sea: $u = 10+8x^2, du = 16xdx$

$$\int x(10+8x^2)^9dx = \frac{1}{16}\int 16x(10+8x^2)^9dx = \frac{1}{16}\int u^9 ddu = \frac{1}{16}\frac{u^{10}}{10} + c = \frac{u^{10}}{160} + c$$

$$= \frac{(10+8x^2)^{10}}{160} + c$$

37.- $\int \frac{dx}{\sqrt{(16+x^2)^3}}$

Solución.- Sea: $x = 4\tau g\theta, dx = 4\sec^2\theta d\theta$

$$\int \frac{dx}{\sqrt{(16+x^2)^3}} = \int \frac{\cancel{4\sec^2}\theta d\theta}{4^{\cancel{3}}\sec^{\cancel{3}}\theta} = \frac{1}{16}\int \frac{d\theta}{\sec\theta} = \frac{1}{16}\int \cos\theta d\theta = \frac{1}{16}\mathrm{sen}\,\theta + c = \frac{x}{16\sqrt{16+x^2}} + c$$

38.- $\int \frac{x^3 dx}{\sqrt{x^2+4}}$

Solución.- Sea: $u = x^2 + 4 \Rightarrow x^2 = u - 4, du = 2xdx$

$$\int \frac{x^3 dx}{\sqrt{x^2+4}} = \int \frac{x^2 x dx}{\sqrt{x^2+4}} = \frac{1}{2}\int \frac{(u-4)du}{u^{1/2}} = \frac{1}{2}\int (u^{1/2} - 4u^{-1/2})du = \frac{1}{2}\int u^{1/2}du - 2\int u^{-1/2}du$$

$$= \frac{1}{\cancel{2}}\frac{u^{3/2}}{3/\cancel{2}} - \frac{2u^{1/2}}{1/2} + c = \frac{u^{3/2}}{3} - 4u^{1/2} + c = u^{1/2}(\frac{u}{3} - 4) + c = \sqrt{x^2+4}(\frac{x^2+4}{3} - 4) + c$$

$$= \sqrt{x^2+4}(\frac{x^2-8}{3}) + c$$

39.- $\int \frac{x^3 dx}{\sqrt{16-x^2}}$

Solución.- Sea: $u = 16 - x^2 \Rightarrow x^2 = 16 - u, du = -2xdx$

$$\int \frac{x^3 dx}{\sqrt{16-x^2}} = \int \frac{x^2 x dx}{\sqrt{16-x^2}} = -\frac{1}{2}\int \frac{(16-u)du}{u^{1/2}} = -\frac{1}{2}\int (16u^{-1/2} - u^{1/2})du$$

$$= -\frac{1}{\cancel{2}}\frac{16u^{1/2}}{1/\cancel{2}} + \frac{1}{\cancel{2}}\frac{u^{3/2}}{3/\cancel{2}} = -16u^{1/2} + \frac{u^{3/2}}{3} + c = -16u^{1/2} + \frac{\sqrt{u}u}{3} + c = \sqrt{u}(-16 + \frac{u}{3}) + c$$

$$= \sqrt{16-x^2}(-16 + \frac{16-x^2}{3}) + c = -\sqrt{16-x^2}(\frac{32+x^2}{3}) + c$$

40.- $\int a(x^2+1)^{1/2}dy$

Solución.-

$$\int a(x^2+1)^{1/2}dy = a(x^2+1)^{1/2}\int dy = a(x^2+1)^{1/2}y + c$$

41.- $\int \frac{dx}{(\sqrt{6-x^2})^3}$

Solución.- Sea: $x = \sqrt{6}\,\mathrm{sen}\,\theta, dx = \sqrt{6}\cos\theta d\theta, \sqrt{6-x^2} = \sqrt{6}\cos\theta$

$$\int \frac{dx}{(\sqrt{6-x^2})^3} = \int \frac{\cancel{\sqrt{6}}\ \cancel{\cos\theta} d\theta}{(\sqrt{6})^{\cancel{3}}\cos^{\cancel{3}}\theta} = \frac{1}{6}\int \frac{d\theta}{\cos^2\theta} = \frac{1}{6}\sec^2\theta d\theta = \frac{1}{6}\tau g\theta + c = \frac{1}{6}\frac{x}{\sqrt{6-x^2}} + c$$

42.- $\int \frac{dx}{x(3+\ell\eta x)}$

Solución.- Sea: $u=3+\ell\eta x, du=\frac{dx}{x}$

$$\int\frac{dx}{x(3+\ell\eta x)}=\int\frac{du}{u}=\ell\eta|u|+c=\ell\eta|3+\ell\eta x|+c$$

43.- $\int\frac{e^x}{16+e^{2x}}dx$

Solución.- Sea: $u=e^x, du=e^x dx$

$$\int\frac{e^x}{16+e^{2x}}dx=\int\frac{du}{4^2+u^2}=\frac{1}{4}\operatorname{arc}\tau g\frac{u}{4}+c=\frac{1}{4}\operatorname{arc}\tau g\frac{e^x}{4}+c$$

44.- $\int\cos\sqrt{1-x}dx$

Solución.- Sea: $1-x=t^2\Rightarrow x=1-t^2, dx=-2tdt$

$\int\cos\sqrt{1-x}dx=-2\int\cos tdt(*)$, integrando por partes se tiene:

Sea: $\begin{array}{ll} u=t & dv=\cos tdt \\ du=dt & v=\operatorname{sen} t \end{array}$

$$(*)=-2\left(t\operatorname{sen}t-\int\operatorname{sen}tdt\right)=-2t\operatorname{sen}t+2\int\operatorname{sen}tdt=-2t\operatorname{sen}t-2\cos t+c$$

$$=-2\sqrt{1-x}\operatorname{sen}\sqrt{1-x}-2\cos\sqrt{1-x}+c$$

45.- $\int\frac{x^3dx}{\sqrt{x-1}}$

Solución.- Sea: $x-1=t^2\Rightarrow x=t^2+1, dx=2tdt$

$$\int\frac{x^3dx}{\sqrt{x-1}}=\int\frac{(t^2+1)^3 2\cancel{t}dt}{\cancel{t}}=2\int(t^6+3t^4+3t^2+1)dt=\frac{2t^7}{7}+\frac{6t^5}{5}+2t^3+2t+c$$

$$=t(\frac{2t^6}{7}+\frac{6t^4}{5}+2t^2+2)+c=\sqrt{x-1}\left[\frac{2(x-1)^3}{7}+\frac{6(x-1)^2}{5}+2(x-1)+2\right]+c$$

$$=2\sqrt{x-1}\left[\frac{(x-1)^3}{7}+\frac{3(x-1)^2}{5}+x\right]+c$$

46.- $\int\frac{2y^5-7y^4+7y^3-19y^2+7y-6}{(y-1)^2(y^2+1)^2}dy$

Solución.-

$$\int\frac{2y^5-7y^4+7y^3-19y^2+7y-6}{(y-1)^2(y^2+1)^2}dy\ (*)$$

$$\frac{2y^5-7y^4+7y^3-19y^2+7y-6}{(y-1)^2(y^2+1)^2}=\frac{A}{y-1}+\frac{B}{(y-1)^2}+\frac{Cy+D}{(y^2+1)}+\frac{Ey+F}{(y^2+1)^2}$$

$$2y^5-7y^4+7y^3-19y^2+7y-6=A(y-1)(y^2+1)^2+B(y^2+1)^2$$

$\Rightarrow+(Cy+D)(y-1)^2(y^2+1)+(Ey+F)(y-1)^2$, luego:

$$2y^5-7y^4+7y^3-19y^2+7y-6=(A+C)y^5+(-A+B-2C+D)y^4$$

$$\Rightarrow+(2A+2C-2D+E)y^3+(-2A+2B-2C+2D-2E+F)y^2$$

$\Rightarrow +(A+C-2D+E-2F)y+(-A+B+D+F)$, Igualando coeficientes se tiene:

$$\left(\begin{array}{lllllll} A & & +C & & & & = 2 \\ -A & +B & -2C & +D & & & =-7 \\ 2A & & +2C & -2D & +E & & = 7 \\ -2A & +2B & -2C & +2D & -2E & +F & =-19 \\ A & & +C & -2D & +E & -2F & = 7 \\ -A & +B & & +D & & +F & =-6 \end{array}\right) \Rightarrow \begin{array}{l} A=1, B=-4, C=1 \\ D=0, E=3, F=-1 \end{array}$$

$$(*)\int \frac{2y^5-7y^4+7y^3-19y^2+7y-6}{(y-1)^2(y^2+1)^2}dy=\int\frac{dy}{y-1}-4\int\frac{dy}{(y-1)^2}+\int\frac{ydy}{(y^2+1)}+\int\frac{(3y-1)dy}{(y^2+1)^2}$$

$$=\ell\eta|y-1|+\frac{4}{y-1}+\frac{1}{2}\ell\eta|y^2+1|+3\int\frac{ydy}{(y^2+1)}-\int\frac{dy}{(y^2+1)^2}$$

$$=\ell\eta|y-1|+\frac{4}{y-1}+\ell\eta\left|\sqrt{y^2+1}\right|-\frac{3}{2}\ell\eta|y^2+1|-\left[\frac{1}{2}\frac{y}{y^2+1}+\frac{1}{2}\operatorname{arc}\tau gy\right]+c$$

$$=\ell\eta\left|(y-1)\sqrt{y^2+1}\right|+\frac{4}{y-1}-\frac{3}{2}\ell\eta|y^2+1|-\frac{y}{2(y^2+1)}-\frac{1}{2}\operatorname{arc}\tau gy+c$$

$$=\ell\eta\left|\frac{(y-1)}{\sqrt{y^2+1}}\right|+\frac{4}{y-1}-\frac{y}{2(y^2+1)}-\frac{1}{2}\operatorname{arc}\tau gy+c$$

47.- $\int \operatorname{sen}\sqrt{x+1}dx$

Solución.- Sea: $x+1=t^2 \Rightarrow x=t^2-1, dx=2tdt$

$\int \operatorname{sen}\sqrt{x+1}dx=2\int(\operatorname{sen}t)tdt(*)$, trabajando por partes

Sea: $\begin{array}{ll} u=t & dv=\operatorname{sen}tdt \\ du=dt & v=-\cos t \end{array}$

$$(*)2\int(\operatorname{sen}t)tdt=2\left(-t\cos t+\int\cos tdt\right)=-2t\cos t+2\operatorname{sen}t+c$$

$$=-2\sqrt{x+1}\cos\sqrt{x+1}+2\operatorname{sen}\sqrt{x+1}+c$$

48.- $\int\frac{9x^2+7x-6}{x^3-x}dx$

Solución.-

$$\int\frac{9x^2+7x-6}{x^3-x}dx=\int\frac{9x^2+7x-6}{x(x+1)(x-1)}dx=\int\frac{Adx}{x}+\int\frac{Bdx}{x+1}+\int\frac{Cdx}{x-1}(*)$$

$$\frac{9x^2+7x-6}{x^3-x}=\frac{A}{x}+\frac{B}{x+1}+\frac{C}{x-1}\Rightarrow 9x^2+7x-6=A(x+1)(x-1)+Bx(x-1)+Cx(x+1)$$

De donde: $\begin{cases} x=0\Rightarrow -6=-A\Rightarrow A=6 \\ x=1\Rightarrow 10=2C\Rightarrow C=5 \\ x=-1\Rightarrow -4=2B\Rightarrow B=-2 \end{cases}$

$$(*)=6\int\frac{dx}{x}-2\int\frac{dx}{x+1}+5\int\frac{dx}{x-1}=6\ell\eta|x|-2\ell\eta|x+1|+5\ell\eta|x-1|+c$$

$$= \ell\eta\left|x^6\right| - \ell\eta\left|(x+1)^2\right| + \ell\eta\left|(x-1)^5\right| + c = \ell\eta\left|\frac{x^6(x-1)^5}{(x+1)^2}\right| + c$$

49.- $\int \frac{5w^3-5w^2+2w-1}{w^4+w^2}dw$

Solución.-

$$\int \frac{5w^3-5w^2+2w-1}{w^4+w^2}dw = \int \frac{5w^3-5w^2+2w-1}{w^2(w^2+1)}dw(*)$$

$$\frac{5w^3-5w^2+2w-1}{w^2(w^2+1)} = \frac{Aw+B}{w^2} + \frac{Cw+D}{w^2+1}$$

$$5w^3-5w^2+2w-1 = (Aw+B)(w^2+1) + (Cw+D)w^2$$

$$\Rightarrow Aw^3 + Aw + Bw^2 + B + Cw^3 + Dw^2 \Rightarrow (A+C)w^3 + (B+D)w^2 + Aw + B$$

Igualando coeficientes se tiene:

$$\begin{pmatrix} A & & +C & & = & 5 \\ & B & & +D & = & -5 \\ A & & & & = & 2 \\ & B & & & = & -1 \end{pmatrix} \Rightarrow A=2, B=-1, C=3, D=-4$$

$$(*) \int \frac{Aw+B}{w^2}dw + \int \frac{Cw+D}{w^2+1}dw = \int \frac{2w-1}{w^2}dw + \int \frac{3w-4}{w^2+1}dw$$

$$= \int \frac{2wdw}{w^2} - \int w^{-2}dw + \frac{3}{2}\int \frac{2wdw}{w^2+1} - 4\int \frac{dw}{w^2+1}$$

$$= \ell\eta\left|w^2\right| + \frac{1}{w} + \ell\eta\left|\sqrt{(w^2+1)^3}\right| - 4\,\text{arc}\,\tau gw + c = \ell\eta\left|w^2\sqrt{(w^2+1)^3}\right| + \frac{1}{w} - 4\,\text{arc}\,\tau gw + c$$

50.- $\int \frac{3dx}{1+2x}$

Solución.- Sea: $u = 1+2x, du = 2dx$

$$\int \frac{3dx}{1+2x} = 3\int \frac{dx}{1+2x} = \frac{3}{2}\int \frac{du}{u} = \frac{3}{2}\ell\eta|u| + c = \frac{3}{2}\ell\eta|1+2x| + c = \ell\eta\left|\sqrt{(1+2x)^3}\right| + c$$

51.- $\int \frac{(1-x)^2dx}{x}$

Solución.-

$$\int \frac{(1-x)^2dx}{x} = \int \frac{1-2x+x^2dx}{x} = \int \frac{dx}{x} - 2\int dx + \int xdx = \ell\eta|x| - 2x + \frac{x^2}{2} + c$$

52.- $\int \frac{xe^{-2x^2}}{2}dx$

Solución.- Sea: $u = -2x^2, du = -4xdx$

$$\int \frac{xe^{-2x^2}}{2}dx = \frac{1}{2}\int xe^{-2x^2}dx = -\frac{1}{8}\int e^u du = -\frac{1}{8}e^u + c = -\frac{1}{8}e^{-2x^2} + c$$

53.- $\int e^{2t}\cos(e^t)dt$

Solución.- Sea: $w = e^t, dw = e^t dt$

$\int e^t \cos(e^t) e^t dt = \int w \cos w dw (*)$, trabajando por partes

Sea: $u = w$ $\quad dv = \cos w dw$

$du = dw$ $\quad v = \operatorname{sen} w$

$(*) \int w \cos w dw = w \operatorname{sen} w - \int \operatorname{sen} w dw = w \operatorname{sen} w + \cos w + c = e^t \operatorname{sen}(e^t) + \cos(e^t) + c$

54.- $\int \sqrt{x}(x^{3/2} - 4)^3 dx$

Solución.- Sea: $u = x^{3/2} - 4, du = \frac{3}{2}\sqrt{x} dx$

$$\int \sqrt{x}(x^{3/2} - 4)^3 dx = \frac{2}{3}\int u^3 du = \frac{2}{3}\frac{u^4}{4} + c = \frac{1}{6}u^4 + c = \frac{(x^{3/2} - 4)^4}{6} + c$$

55.- $\int \frac{\operatorname{sen} x e^{\sec x}}{\cos^2 x} dx = \int \frac{\operatorname{sen} x}{\cos x}\frac{1}{\cos} e^{\sec x} dx = \int \tau g x \sec x e^{\sec x} dx (*)$

Solución.- Sea: $u = \sec x, du = \sec x \tau g x dx$

$(*) = \int e^u du = e^u + c = e^{\sec x} + c$

56.- $\int \frac{ds}{s^{1/3}(1 + s^{2/3})}$

Solución.- Sea: $t = s^{1/3} \Rightarrow s = t^3, ds = 3t^2 dt$

$$\int \frac{ds}{s^{1/3}(1 + s^{2/3})} = \int \frac{3t^{\cancel{2}} dt}{\cancel{t}(1 + t^2)} = \int \frac{3t dt}{(1 + t^2)} = 3\int \frac{t dt}{(1 + t^2)} = \frac{3}{2}\ell\eta\left|1 + t^2\right| + c$$

57.- $\int \frac{1}{z^3}\left(\frac{1 - z^2}{z^2}\right)^{10} dz$

Solución.- Sea: $u = \frac{1 - z^2}{z^2}, du = \frac{-2dz}{z^3}$

$$\int \frac{1}{z^3}\left(\frac{1 - z^2}{z^2}\right)^{10} dz = -\frac{1}{2}\int u^{10} du = -\frac{1}{2}\frac{u^{11}}{11} + c = -\frac{u^{11}}{22} + c = -\frac{1}{22}\left(\frac{1 - z^2}{z^2}\right)^{11} + c$$

58.- $\int \frac{x\ell\eta(1 + x^2)}{1 + x^2} dx$

Solución.- Sea: $u = \ell\eta(1 + x^2), du = \frac{2x dx}{1 + x^2}$

$$\int \frac{x\ell\eta(1 + x^2)}{1 + x^2} dx = \frac{1}{2}\int u du = \frac{1}{2}\frac{u^2}{2} + c = \frac{u^2}{4} + c = \frac{\left[\ell\eta(1 + x^2)\right]^2}{4} + c$$

59.- $\int \frac{\operatorname{co}\tau g x dx}{\ell\eta\left|\operatorname{sen} x\right|}$

Solución.- Sea: $u = \ell\eta\left|\operatorname{sen} x\right|, du = \operatorname{co}\tau g x dx$

$$\int \frac{\operatorname{co}\tau g x dx}{\ell\eta\left|\operatorname{sen} x\right|} = \int \frac{du}{u} = \ell\eta\left|u\right| + c = \ell\eta\left|\ell\eta\left|\operatorname{sen} x\right|\right| + c$$

60.- $\int \frac{ax^2-bx+c}{ax^2+bx-c}dx$

Solución.-

$$\int \frac{ax^2-bx+c}{ax^2+bx-c}dx = \frac{ax^2-bx+c}{ax^2+bx-c}\int dt = \frac{ax^2-bx+c}{ax^2+bx-c}t + c$$

61.- $\int \frac{dx}{\cos^2 5x}$

Solución.- Sea: $u = 5x, du = 5dx$

$$\int \frac{dx}{\cos^2 5x} = \int \sec^2 5xdx = \frac{1}{5}\int \sec^2 udu = \frac{1}{5}\tau gu + c = \frac{1}{5}\tau g5x + c$$

62.- $\int \frac{dx}{12-7x}$

Solución.- Sea: $u = 12-7x, du = -7dx$

$$\int \frac{dx}{12-7x} = -\frac{1}{7}\int \frac{du}{u} = -\frac{1}{7}\ell\eta|u| + c = -\frac{1}{7}\ell\eta|12-7x| + c$$

63.- $\int \tau g16xdx$

Solución.- Sea: $u = \cos(16x), du = -16\operatorname{sen}(16x)dx$

$$\int \tau g16xdx = \int \frac{\operatorname{sen}(16x)}{\cos(16x)}dx = -\frac{1}{16}\int \frac{du}{u} = -\frac{1}{16}\ell\eta|u| + c = -\frac{1}{16}\ell\eta|\cos(16x)| + c$$

64.- $\int \tau g4\theta \sec^2 4\theta d\theta$

Solución.- Sea: $u = \tau g4\theta, du = 4\sec^2 4\theta d\theta$

$$\int \tau g4\theta \sec^2 4\theta d\theta = \frac{1}{4}\int udu = \frac{1}{4}\frac{u^2}{2} + c = \frac{u^2}{8} + c = \frac{\tau g^2 4\theta}{8} + c$$

65.- $\int \frac{xdx}{\sqrt{x-5}}$

Solución.- Sea: $u = x-5 \Rightarrow x = u+5, du = dx$

$$\int \frac{xdx}{\sqrt{x-5}} = \int \frac{u+5}{u^{1/2}}du = \int u^{1/2}du + 5\int u^{-1/2}du = \frac{u^{3/2}}{3/2} + 5\frac{u^{1/2}}{1/2} + c = \frac{2u^{3/2}}{3} + 10u^{1/2} + c$$

$$= \frac{2}{3}u\sqrt{u} + 10\sqrt{u} + c = \frac{2}{3}(x-5)\sqrt{x-5} + 10\sqrt{x-5} + c = 2\sqrt{x-5}\left(\frac{x+10}{3}\right) + c$$

66.- $\int \frac{7t-2}{\sqrt{7-2t^2}}dt$

Solución.-

$$\int \frac{7t-2}{\sqrt{7-2t^2}}dt = \int \frac{7tdt}{\sqrt{7-2t^2}} - \int \frac{2dt}{\sqrt{7-2t^2}} = -\frac{7}{4}\int \frac{-4tdt}{\sqrt{7-2t^2}} - \sqrt{2}\int \frac{dt}{\sqrt{7/2 - t^2}}$$

$$= -\frac{7}{2}\sqrt{7-2t^2} - \sqrt{2}\operatorname{arcsen}\sqrt{2/7}\,t + c$$

67.- $\int (1+x)\cos\sqrt{x}dx$

Solución.- Sea: $\sqrt{x}=t\Rightarrow x=t^2, dx=2tdt$

$$\int(1+x)\cos\sqrt{x}dx=\int(1+t^2)(\cos t)2tdt=2\int(t+t^3)(\cos t)dt=2\int t\cos tdt+2\int t^3\cos tdt\ (*)$$

Trabajando por partes: $\int t^3\cos tdt$

Sea: $u=t^3$ $\quad dv=\cos tdt$
$du=3t^2dt$ $\quad v=\operatorname{sen} t$

$$\int t^3\cos tdt=t^3\operatorname{sen} t-3\int t^2\operatorname{sen} tdt$$

Trabajando por partes: $\int t^2\operatorname{sen} tdt$

Sea: $u=t^2$ $\quad dv=\operatorname{sen} tdt$
$du=2tdt$ $\quad v=-\cos t$

$$\int t^2\operatorname{sen} tdt=-t^2\cos t+2\int t\cos tdt$$

Trabajando por partes: $\int t\cos tdt$

Sea: $u=t$ $\quad dv=\cos tdt$
$du=dt$ $\quad v=\operatorname{sen} t$

$$\int t\cos tdt=t\operatorname{sen} t-\int\operatorname{sen} tdt=t\operatorname{sen} t+\cos t+c_1$$

$$(*)\ 2\int t\cos tdt+2\int t^3\cos tdt=2\int t\cos tdt+2\left(t^3\operatorname{sen} t-3\int t^2\operatorname{sen} tdt\right)$$

$$=2\int t\cos tdt+2t^3\operatorname{sen} t-6\int t^2\operatorname{sen} tdt=2\int t\cos tdt+2t^3\operatorname{sen} t-6\left(-t^2\cos t+2\int t\cos tdt\right)$$

$$=2\int t\cos tdt+2t^3\operatorname{sen} t+6t^2\cos t-12\int t\cos tdt=2t^3\operatorname{sen} t+6t^2\cos t-10\int t\cos tdt$$

$$=2t^3\operatorname{sen} t+6t^2\cos t-10(t\operatorname{sen} t+\cos t)+c$$

$$=2t^3\operatorname{sen} t+6t^2\cos t-10t\operatorname{sen} t-10\cos t+c$$

$$=2\sqrt{x^3}\operatorname{sen}\sqrt{x}+6x\cos\sqrt{x}-10\sqrt{x}\operatorname{sen}\sqrt{x}-10\cos\sqrt{x}+c$$

68.- $\displaystyle\int\frac{dx}{x(\sqrt{1+x}-1)}$

Solución.- Sea: $(1+x)^{\frac{1}{2}}=t\Rightarrow 1+x=t^2\Rightarrow x=t^2-1, dx=2tdt$

$$\int\frac{dx}{x(\sqrt{1+x}-1)}=\int\frac{2tdt}{(t^2-1)(t-1)}(*)$$

$$\frac{t}{(t+1)(t^2-1)}=\frac{A}{t+1}+\frac{B}{t-1}+\frac{C}{(t-1)^2}\Rightarrow t=A(t-1)^2+B(t^2-1)+C(t+1)$$

De donde: $\begin{cases} t=1\Rightarrow 1=2C\Rightarrow C=\frac{1}{2} \\ t=-1\Rightarrow -1=4A\Rightarrow A=-\frac{1}{4} \\ t=0\Rightarrow 0=A-B+C\Rightarrow B=\frac{1}{4}\end{cases}$

$$(*)=2\left[\int\frac{Adt}{t+1}+\int\frac{Bdt}{t-1}+\int\frac{Cdt}{(t-1)^2}\right]=2\left[-\frac{1}{4}\int\frac{dt}{t+1}+\frac{1}{4}\int\frac{dt}{t-1}+\frac{1}{2}\int\frac{dt}{(t-1)^2}\right]$$

$$=-\frac{1}{2}\int\frac{dt}{t+1}+\frac{1}{2}\int\frac{dt}{t-1}+\int\frac{dt}{(t-1)^2}=-\frac{1}{2}\ell\eta|t+1|+\frac{1}{2}\ell\eta|t-1|-\frac{1}{t-1}+c$$

$$=\frac{1}{2}\ell\eta\left|\frac{t-1}{t+1}\right|-\frac{1}{t-1}+c=\frac{1}{2}\ell\eta\left|\frac{\sqrt{1+x}-1}{\sqrt{1+x}+1}\right|-\frac{1}{\sqrt{1+x}-1}+c$$

69.- $\int\frac{dx}{\mathrm{co}\,\tau g6x}$

Solución.- Sea: $u=\cos 6x, du=-6\,\mathrm{s}e\mathrm{n}\,6xdx$

$$\int\frac{dx}{\mathrm{co}\,\tau g6x}=\int\tau g6xdx=\int\frac{\mathrm{s}e\mathrm{n}\,6x}{\cos 6x}dx=-\frac{1}{6}\int\frac{du}{u}=-\frac{1}{6}\ell\eta|u|+c=-\frac{1}{6}\ell\eta|\cos 6x|+c$$

70.- $\int\mathrm{co}\,\tau g(2x-4)dx$

Solución.- Sea: $u=\mathrm{s}e\mathrm{n}(2x-4), du=2\cos(2x-4)dx$

$$\int\mathrm{co}\,\tau g(2x-4)dx=\int\frac{\cos(2x-4)}{\mathrm{s}e\mathrm{n}(2x-4)}dx=\frac{1}{2}\int\frac{du}{u}=\frac{1}{2}\ell\eta|u|+c=\frac{1}{2}\ell\eta|(2x-4)|+c$$

71.- $\int(e^t-e^{-2t})^2dt$

Solución.-

$$\int(e^t-e^{-2t})^2dt=\int(e^{2t}-2e^{t-2t}+e^{-4t})dt=\int e^{2t}dt-2\int e^{-t}dt+\int e^{-4t}dt$$

$$=\frac{1}{2}e^{2t}+2e^{-t}-\frac{1}{2}e^{-4t}+c$$

72.- $\int\frac{(x+1)dx}{(x+2)^2(x+3)}$

Solución.-

$$\int\frac{(x+1)dx}{(x+2)^2(x+3)}\Rightarrow\frac{(x+1)}{(x+2)^2(x+3)}=\frac{A}{x+2}+\frac{B}{(x+2)^2}+\frac{C}{x+3}\ (*)$$

$$\Rightarrow x+1=A(x+2)(x+3)+B(x+3)+C(x+2)^2$$

De donde: $\begin{cases}x=-2\Rightarrow -1=B\Rightarrow B=-1\\ x=-3\Rightarrow -2=C\Rightarrow C=-2\\ x=0\Rightarrow 1=6A+3B+4C\Rightarrow A=2\end{cases}$

$$(*)\ \int\frac{Adx}{x+2}+\int\frac{Bdx}{(x+2)^2}+\int\frac{Cdx}{x+3}=2\int\frac{dx}{x+2}-\int\frac{dx}{(x+2)^2}-2\int\frac{dx}{x+3}$$

$$=2\ell\eta|x+2|+\frac{1}{x+2}-2\ell\eta|x+3|+c=\ell\eta\left|\frac{x+2}{x+3}\right|^3+\frac{1}{x+2}+c$$

73.- $\int(\mathrm{co}\,\tau ge^x)e^xdx$

Solución.- Sea: $u=\mathrm{s}e\mathrm{n}\,e^x, du=(\cos e^x)e^xdx$

$$\int(\mathrm{co}\,\tau ge^x)e^xdx=\int\frac{(\cos e^x)e^xdx}{\mathrm{s}e\mathrm{n}\,e^x}=\int\frac{du}{u}=\ell\eta|u|+c=\ell\eta|\mathrm{s}e\mathrm{n}\,e^x|+c$$

74.- $\int\frac{\mathrm{s}e\mathrm{n}\,\theta+\theta}{\cos\theta+1}d\theta$

Solución.-

$$\int \frac{\operatorname{sen}\theta+\theta}{\cos\theta+1}d\theta=\int\frac{\operatorname{sen}\theta d\theta}{\cos\theta+1}+\int\frac{\theta d\theta}{\cos\theta+1}=-\int\frac{-\operatorname{sen}\theta d\theta}{\cos\theta+1}+\int\frac{\theta(\cos\theta-1)d\theta}{\cos^2\theta+1}$$

$$=-\ell\eta\left|\cos\theta+1\right|-\int\frac{\theta\cos\theta d\theta}{\operatorname{sen}^2\theta}+\int\frac{\theta d\theta}{\operatorname{sen}^2\theta}$$

$$=-\ell\eta\left|\cos\theta+1\right|-\int\theta\operatorname{co}\tau g\theta\cos ec\theta d\theta+\int\theta\cos ec^2\theta d\theta \ (*)$$

Trabajando por partes: $\int\theta\operatorname{co}\tau g\theta\cos ec\theta d\theta$

Sea: $u=\theta \qquad dv=\operatorname{co}\tau g\theta\cos ec\theta d\theta$

$du=d\theta \qquad v=-\cos ec\theta$

$$\int\theta\operatorname{co}\tau g\theta\cos ec\theta d\theta=-\theta\cos ec\theta+\int\cos ec\theta d\theta=-\theta\cos ec\theta-\ell\eta\left|\cos ec\theta-\operatorname{co}\tau g\theta\right|+c_1$$

Trabajando por partes: $\int\theta\cos ec^2\theta d\theta$

Sea: $u=\theta \qquad dv=\cos ec^2\theta d\theta$

$du=d\theta \qquad v=-t\operatorname{co}\tau g\theta$

$$\int\theta\cos ec^2\theta d\theta=-\theta\operatorname{co}\tau g\theta+\int\operatorname{co}\tau g\theta d\theta=-\theta\operatorname{co}\tau g\theta+\ell\eta\left|\operatorname{sen}\theta\right|+c_2$$

$$(*)=-\ell\eta\left|\cos\theta+1\right|+\theta\cos ec\theta+\ell\eta\left|\cos ec\theta-\operatorname{co}\tau g\theta\right|-\theta\operatorname{co}\tau g\theta+\ell\eta\left|\operatorname{sen}\theta\right|+c$$

$$=\ell\eta\left|\frac{(\cos ec\theta-\operatorname{co}\tau g\theta)\operatorname{sen}\theta}{\cos\theta+1}\right|+\theta(\cos ec\theta-\operatorname{co}\tau g\theta)+c$$

$$=\ell\eta\left|\frac{1-\cos\theta}{1+\cos\theta}\right|+\theta\left(\frac{1-\cos\theta}{\operatorname{sen}\theta}\right)+c$$

75.- $\int\frac{\operatorname{arc}\tau gxdx}{(1+x^2)^{3/2}}$

Solución.- Sea: $x=\tau g\theta\Rightarrow\theta=\operatorname{arc}\tau gx, dx=\sec^2\theta d\theta, \sqrt{1+x^2}=\sec\theta$

$\int\frac{\operatorname{arc}\tau gxdx}{(1+x^2)^{3/2}}=\int\frac{\theta\cancel{\sec^2}\theta d\theta}{\sec^{\cancel{3}}\theta}=\int\frac{\theta d\theta}{\sec\theta}=\int\theta\cos\theta d\theta(*)$, trabajando por partes

Sea: $u=\theta \qquad dv=\cos\theta d\theta$

$du=d\theta \qquad v=\operatorname{sen}\theta$

$$=\theta\operatorname{sen}\theta-\int\operatorname{sen}\theta d\theta=\theta\operatorname{sen}\theta+\cos\theta+c=(\operatorname{arc}\tau gx)\frac{x}{\sqrt{1+x^2}}+\frac{1}{\sqrt{1+x^2}}+c$$

$$=\frac{1}{\sqrt{1+x^2}}\left(x\operatorname{arc}\tau gx+1\right)+c$$

76.- $\int x\operatorname{co}\tau g(x^2/5)dx$

Solución.- Sea: $u=\operatorname{sen}\frac{x^2}{5}, du=\frac{2}{5}x\cos\frac{x^2}{5}dx$

$$\int x\cos\tau g(x^2/5)dx=\int \frac{x\cos\frac{x^2}{5}}{\operatorname{sen}\frac{x^2}{5}}dx=\frac{5}{2}\int\frac{du}{u}=\frac{5}{2}\ell\eta|u|+c=\frac{5}{2}\ell\eta\left|\operatorname{sen}\frac{x^2}{5}\right|+c$$

77.- $\int x\sqrt{4x^2-2}dx$

Solución.- Sea: $u=4x^2-2, dx=8xdx$

$$\int x\sqrt{4x^2-2}dx=\frac{1}{8}\int u^{1/2}du=\frac{1}{8}\frac{u^{3/2}}{3/2}+c=\frac{u^{3/2}}{12}+c=\frac{\sqrt{(4x^2-2)^3}}{12}+c$$

78.- $\int\frac{(x^2+9)^{1/2}dx}{x^4}$

Solución.- Sea: $x=3\tau g\theta, dx=3\sec^2\theta, \sqrt{x^2+9}=3\sec\theta$

$$\int\frac{(x^2+9)^{1/2}dx}{x^4}=\int\frac{3\sec\theta 3\sec^2\theta d\theta}{3^4\tau g^4\theta}=\frac{1}{9}\int\frac{\sec^3\theta d\theta}{\tau g^4\theta}=\frac{1}{9}\int\frac{\frac{1}{\cos^3\theta}d\theta}{\frac{\operatorname{sen}^4\theta}{\cos^4\theta}}=\frac{1}{9}\int\frac{\cos\theta d\theta}{\operatorname{sen}^4\theta}$$

$$=\frac{1}{9}\left(-\frac{1}{3}\frac{1}{\operatorname{sen}^3\theta}\right)+c=-\frac{1}{27\operatorname{sen}^3\theta}+c=-\frac{\cos ec^3\theta}{27}+c$$

$$=-\frac{1}{27}\left(\frac{\sqrt{x^2+9}}{x}\right)^3+c=-\frac{x^2+9}{27x^3}\sqrt{x^2+9}+c$$

79.- $\int x^2\operatorname{sen}^5x^3\cos x^3dx$

Solución.- Sea: $u=\operatorname{sen}x^3, du=3x^2\cos x^3dx$

$$\int x^2\operatorname{sen}^5x^3\cos x^3dx=\frac{1}{3}\int u^5du=\frac{1}{3}\frac{u^6}{6}+c=\frac{u^6}{18}+c=\frac{\operatorname{sen}^6x^3}{18}+c$$

80.- $\int\frac{xdx}{\sqrt{5x^2+7}}$

Solución.- Sea: $u=5x^2+7, du=10xdx$

$$\int\frac{xdx}{\sqrt{5x^2+7}}=\frac{1}{10}\int\frac{du}{u^{1/2}}=\frac{1}{10}\frac{u^{1/2}}{1/2}+c=\frac{u^{1/2}}{5}+c=\frac{(5x^2+7)^{1/2}}{5}+c=\frac{\sqrt{5x^2+7}}{5}+c$$

81.- $\int\frac{x^3dx}{x^2-x-6}$

Solución.-

$$\int\frac{x^3dx}{x^2-x-6}=\int\left(x+1+\frac{7x+6}{x^2-x-6}\right)dx=\int xdx+\int dx+\int\frac{(7x+6)dx}{(x-3)(x+2)}$$

$$=\frac{x^2}{2}+x+\int\frac{(7x+6)dx}{(x-3)(x+2)}(*)$$

$$\frac{(7x+6)}{(x-3)(x+2)}=\frac{A}{x-3}+\frac{B}{x+2}\Rightarrow 7x+6=A(x+2)+B(x-3)$$

De donde: $\begin{cases} x=-2\Rightarrow -8=-5B\Rightarrow B={}^{8}\!/\!_{5} \\ x=3\Rightarrow 27=5A\Rightarrow A={}^{27}\!/\!_{5} \end{cases}$

$$(*)=\frac{x^2}{2}+x+\int\frac{Adx}{x-3}+\int\frac{Bdx}{x+2}=\frac{x^2}{2}+x+\frac{27}{5}\int\frac{dx}{x-3}+\frac{8}{5}\int\frac{dx}{x+2}$$

$$=\frac{x^2}{2}+x+\frac{27}{5}\ell\eta\left|x-3\right|+\frac{8}{5}\ell\eta\left|x+2\right|+c$$

82.- $\int \operatorname{sen} 2\theta e^{\operatorname{sen}^2\theta}d\theta$

Solución.- Sea: $u=\operatorname{sen}^2\theta, du=2\operatorname{sen}\theta\cos\theta d\theta$

$$\int \operatorname{sen} 2\theta e^{\operatorname{sen}^2\theta}d\theta=\int 2\operatorname{sen}\theta\cos\theta e^{\operatorname{sen}^2\theta}d\theta=\int e^u du=e^u+c=e^{\operatorname{sen}^2\theta}+c$$

83.- $\int\frac{dx}{e^x-9e^{-x}}$

Solución.- Sea: $u=e^x, du=e^x dx$

$$\int\frac{dx}{e^x-9e^{-x}}=\int\frac{e^x dx}{e^{2x}-9}=\int\frac{e^x dx}{(e^x)^2-9}=\int\frac{du}{u^2-9}=\frac{1}{6}\ell\eta\left|\frac{u-3}{u+3}\right|+c=\frac{1}{6}\ell\eta\left|\frac{e^x-3}{e^x+3}\right|+c$$

84.- $\int\frac{dw}{1+\cos w}$

Solución.-

$$\int\frac{dw}{1+\cos w}=\int\frac{(1-\cos w)dw}{1-\cos^2 w}=\int\frac{(1-\cos w)dw}{\operatorname{sen}^2 w}=\int\cos ec^2 wdw-\int\frac{\cos wdw}{\operatorname{sen}^2 w}$$

$$=-\operatorname{co}\tau gw-\frac{(\operatorname{sen} w)^{-1}}{-1}+c=-\operatorname{co}\tau gw+\frac{1}{\operatorname{sen} w}+c=-\operatorname{co}\tau gw+\cos ecw+c$$

Nota: Este ejercicio esta desarrollado diferente en el capitulo 8.

85.- $\int e^{\left(\frac{1-\operatorname{sen}^2 x/2}{3}\right)^2}(\cos^3 {}^{x}\!/\!_{2}\operatorname{sen} {}^{x}\!/\!_{2})dx$

Solución.- Sea: $u=\left(\frac{1-\operatorname{sen}^2 x/2}{3}\right)^2, du=-\frac{2}{9}\cos^3\frac{x}{2}\operatorname{sen}\frac{x}{2}dx$

$$\int e^{\left(\frac{1-\operatorname{sen}^2 x/2}{3}\right)^2}(\cos^3 {}^{x}\!/\!_{2}\operatorname{sen} {}^{x}\!/\!_{2})dx=-\frac{9}{2}\int e^u du=-\frac{2}{9}e^u+c=-\frac{2}{9}e^{\left(\frac{1-\operatorname{sen}^2 x/2}{3}\right)^2}+c$$

86.- $\int\frac{x^3dx}{\sqrt{19-x^2}}$

Solución.- Sea: $x=\sqrt{19}\operatorname{sen}\theta, dx=\sqrt{19}\cos\theta d\theta, \sqrt{19-x^2}=\sqrt{19}\cos\theta$

$$\int\frac{x^3dx}{\sqrt{19-x^2}}=\int\frac{(\sqrt{19})^3\operatorname{sen}^3\theta\,\cancel{\sqrt{19}\cos\theta}\,d\theta}{\cancel{\sqrt{19}\cos\theta}}=19\sqrt{19}\int\operatorname{sen}\theta(1-\cos^2\theta)d\theta$$

$$=19\sqrt{19}\int \operatorname{sen}\theta d\theta-19\sqrt{19}\int \operatorname{sen}\theta\cos^2\theta d\theta=-19\sqrt{19}\cos\theta+\frac{19\sqrt{19}}{3}\cos^3\theta+c$$

$$=-19\cancel{\sqrt{19}}\,\frac{\sqrt{19-x^2}}{\cancel{\sqrt{19}}}+\frac{\cancel{19\sqrt{19}}}{3}\,\frac{\sqrt{(19-x^2)^3}}{\cancel{(\sqrt{19})^3}}+c=-19\sqrt{19-x^2}+\sqrt{(19-x^2)^3}+c$$

87.- $\int \frac{\operatorname{sen}\varphi d\varphi}{\cos^{1/2}\varphi}$

Solución.- Sea: $u=\cos\varphi, du=-\operatorname{sen}\varphi d\varphi$

$$\int \frac{\operatorname{sen}\varphi d\varphi}{\cos^{1/2}\varphi}=-\int\frac{du}{u^{1/2}}=-\int u^{-1/2}du=-\frac{u^{1/2}}{1/2}+c=-2u^{1/2}+c=-2\sqrt{\cos\varphi}+c$$

88.- $\int(\sec\varphi+\tau g\varphi)^2 d\varphi$

Solución.-

$$\int(\sec\varphi+\tau g\varphi)^2 d\varphi=\int(\sec^2\varphi+2\sec\varphi\tau g\varphi+\tau g^2\varphi)d\varphi$$

$$=\int(\sec^2\varphi+2\sec\varphi\tau g\varphi+\sec^2\varphi-1)d\varphi=\int(2\sec^2\varphi+2\sec\varphi\tau g\varphi-1)d\varphi$$

$$=2\int\sec^2\varphi d\varphi+2\int\sec\varphi\tau g\varphi d\varphi-\int d\varphi=2\tau g\varphi+2\sec\varphi-\varphi+c$$

89.- $\int\frac{dt}{t(4+\ell\eta^2 t)^{1/2}}$

Solución.- Sea: $u=\ell\eta t, du=\frac{dt}{t}$, además: $u=2\tau g\theta, du=2\sec^2\theta d\theta, \sqrt{4+u^2}=2\sec\theta$

$$\int\frac{dt}{t(4+\ell\eta^2 t)^{1/2}}=\int\frac{du}{\sqrt{4+u^2}}=\int\frac{\cancel{2}\sec^{\cancel{2}}\theta d\theta}{\cancel{2\sec\theta}}=\int\sec\theta d\theta=\ell\eta\left|\sec\theta+\tau g\theta\right|+c$$

$$=\ell\eta\left|\frac{\sqrt{4+u^2}}{2}+\frac{u}{2}\right|+c=\ell\eta\left|\frac{\sqrt{4+u^2}+u}{2}\right|+c=\ell\eta\left|\frac{\sqrt{4+\ell\eta^2 t}+\ell\eta t}{2}\right|+c$$

90.- $\int a^\theta b^{2\theta}c^{3\theta}d\theta$

Solución.- Sea: $ab^2c^3=k$,

$$\int a^\theta b^{2\theta}c^{3\theta}d\theta=\int a^\theta(b^2)^\theta(c^3)^\theta d\theta=\int(ab^2c^3)^\theta d\theta=\int k^\theta d\theta=\frac{k^\theta}{\ell\eta|k|}+c=\frac{(ab^2c^3)^\theta}{\ell\eta\left|(ab^2c^3)\right|}+c$$

91.- $\int\operatorname{sen}^{1/2}\varphi\cos^3\varphi d\varphi$

Solución.-

$$\int\operatorname{sen}^{1/2}\varphi\cos^3\varphi d\varphi=\int\operatorname{sen}^{1/2}\varphi\cos^2\varphi\cos\varphi d\varphi=\int\operatorname{sen}^{1/2}\varphi(1-\operatorname{sen}^2\varphi)\cos\varphi d\varphi$$

$$=\int\operatorname{sen}^{1/2}\varphi\cos\varphi d\varphi-\int\operatorname{sen}^{5/2}\varphi\cos\varphi d\varphi=\frac{\operatorname{sen}^{3/2}\varphi}{3/2}-\frac{\operatorname{sen}^{7/2}\varphi}{7/2}+c$$

$$=\frac{2\operatorname{sen}^{3/2}\varphi}{3}-\frac{2\operatorname{sen}^{7/2}\varphi}{7}+c$$

92.- $\int \frac{\sec^2\theta d\theta}{9+\tau g^2\theta}$

Solución.- Sea: $u=\tau g\theta, du=\sec^2\theta d\theta$

$$\int \frac{\sec^2\theta d\theta}{9+\tau g^2\theta}=\int \frac{du}{9+u^2}=\frac{1}{3}\operatorname{arc}\tau g\frac{u}{3}+c=\frac{1}{3}\operatorname{arc}\tau g\frac{(\tau g\theta)}{3}+c$$

93.- $\int \frac{dx}{\sqrt{e^{2x}-16}}$

Solución.-Sea: $u=e^x, du=e^x dx \Rightarrow dx=\frac{du}{u}$

Además: $u=4\sec\theta, du=4\sec\theta\tau g\theta d\theta, \sqrt{u^2-16}=4\tau g\theta$

$$\int \frac{dx}{\sqrt{e^{2x}-16}}=\int \frac{du/u}{\sqrt{u^2-16}}=\int \frac{du}{u\sqrt{u^2-16}}=\int \frac{\cancel{4\sec\theta}\ \cancel{\tau g\theta}d\theta}{\cancel{4\sec\theta}\,4\,\cancel{\tau g\theta}}=\frac{1}{4}\int d\theta=\frac{1}{4}\theta+c$$

$$=\frac{1}{4}\operatorname{arc}\sec\frac{u}{4}+c=\frac{1}{4}\operatorname{arc}\sec\frac{e^x}{4}+c$$

94.- $\int (e^{2s}-1)(e^{2s}+1)ds$

Solución.-

$$\int (e^{2s}-1)(e^{2s}+1)ds=\int\left[(e^{2s})^2-1\right]ds=\int e^{4s}ds-\int ds=\frac{1}{4}e^{4s}+s+c$$

95.- $\int \frac{dx}{5x^2+8x+5}$

Solución.-

$\int \frac{dx}{5x^2+8x+5}=\int \frac{dx}{5(x^2+8/5\,x+1)}=\frac{1}{5}\int \frac{dx}{x^2+8/5\,x+1}(*)$, completando cuadrados:

$$x^2+8/5\,x+1=(x^2+\frac{8}{5}x+\frac{16}{25})+1-\frac{16}{25}=(x+4/5)^2+9/25=(x+4/5)^2+(3/5)^2$$

$$(*)=\frac{1}{5}\int \frac{dx}{(x+4/5)^2+(3/5)^2}=\frac{1}{\cancel{5}}\frac{1}{3/\cancel{5}}\operatorname{arc}\tau g\frac{x+4/5}{3/5}+c=\frac{1}{3}\operatorname{arc}\tau g\frac{5x+4}{3}+c$$

96.- $\int \frac{x^3+1}{x^3-x}dx$

Solución.-

$$\int \frac{x^3+1}{x^3-x}dx=\int\left(1+\frac{x+1}{x^3-x}\right)dx=\int dx+\int \frac{x+1}{x^3-x}dx=x+\int \frac{(x+1)dx}{x(x^2-1)}$$

$$=x+\int \frac{\cancel{(x+1)}dx}{x\cancel{(x+1)}(x-1)}=x+\int \frac{dx}{x(x-1)}=x+\int \frac{Adx}{x}+\int \frac{Bdx}{x-1}(*)$$

$$\frac{1}{x(x-1)}=\frac{A}{x}+\frac{B}{x-1}\Rightarrow 1=A(x-1)+Bx$$

De donde: $\begin{cases} x=0 \Rightarrow 1=-A \Rightarrow A=-1 \\ x=1 \Rightarrow 1=B \Rightarrow B=1 \end{cases}$

$$(*)=x-\int\frac{dx}{x}+\int\frac{dx}{x-1}=x-\ell\eta|x|+\ell\eta|x-1|+c=x+\ell\eta\left|\frac{x-1}{x}\right|+c$$

97.- $\int(\text{arcsen}\sqrt{1-x^2})^0dx$

Solución.-

$$\int(\text{arcsen}\sqrt{1-x^2})^0dx=\int dx=x+c$$

98.- $\int\frac{3dy}{1+\sqrt{y}}$

Solución.-Sea: $y^{1/2}=t\Rightarrow y=t^2, dy=2tdt$

$$\int\frac{3dy}{1+\sqrt{y}}=3\int\frac{dy}{1+\sqrt{y}}=3\int\frac{2tdt}{1+t}=6\int\frac{tdt}{1+t}=6\int\left(1-\frac{1}{1+t}\right)dt=6\int dt-6\int\frac{dt}{1+t}$$

$$=6t-6\ell\eta|1+t|+c=6\sqrt{y}-6\ell\eta\left|1+\sqrt{y}\right|+c=6\left(\sqrt{y}-\ell\eta\left|1+\sqrt{y}\right|\right)+c$$

99.- $\int x(1+x)^{1/5}dx$

Solución.-Sea: $u=1+x\Rightarrow x=u-1, du=dx$

$$\int x(1+x)^{1/5}dx=\int(u-1)u^{1/5}du=\int(u^{6/5}-u^{1/5})du=\int u^{6/5}du-\int u^{1/5}du=\frac{u^{11/5}}{11/5}-\frac{u^{6/5}}{6/5}+c$$

$$=\left(\frac{5u^2}{11}-\frac{5u}{6}\right)u^{1/5}+c=\left(\frac{5(1+x)^2}{11}-\frac{5(1+x)}{6}\right)(1+x)^{1/5}+c$$

100.- $\int\frac{d\varphi}{a^2\,\text{sen}^2\varphi+b^2\cos^2\varphi}$

Solución.-Sea: $u=\tau g\varphi, du=\sec^2\varphi d\varphi$

$$\int\frac{d\varphi}{a^2\,\text{sen}^2\varphi+b^2\cos^2\varphi}=\int\frac{\text{sen}^4\varphi d\varphi}{\frac{1}{\cos^2\varphi}(a^2\tau g^2\varphi+b^2)}=\int\frac{\text{sen}^2\varphi d\varphi}{(a^2\tau g^2\varphi+b^2)}=\int\frac{du}{(a^2u^2+b^2)}$$

$$=\frac{1}{a^2}\int\frac{du}{u^2+(b/a)^2}=\frac{1}{a^2}\frac{1}{b/a}\text{arc}\tau g\frac{u}{b/a}+c=\frac{1}{ab}\text{arc}\tau g\frac{au}{b}+c=\frac{1}{ab}\text{arc}\tau g\left(\frac{a\tau g\varphi}{b}\right)+c$$

101.- $\int\frac{tdt}{(2t+1)^{1/2}}$

Solución.-

Sea: $u=t$, $du=dt$; $dv=\frac{dt}{\sqrt{2t+1}}$, $v=\sqrt{2t+1}$

$$\int \frac{tdt}{(2t+1)^{1/2}} = t\sqrt{2t+1} - \int \sqrt{2t+1}dt = t\sqrt{2t+1} - \frac{1}{\cancel{2}} \frac{(2t+1)^{3/2}}{3/\cancel{2}} + c = t\sqrt{2t+1} - \frac{(2t+1)^{3/2}}{3} + c$$

$$= \sqrt{2t+1}\left(t - \frac{2t+1}{3} \right) + c = \frac{\sqrt{2t+1}}{3}(t-1) + c$$

102.- $\int \frac{s\ell\eta|s|ds}{(1-s^2)^{1/2}}$

Solución.-

Sea: $u = \ell\eta|s|$ $\quad dv = \frac{sds}{(1-s^2)^{1/2}}$, además: $s = \operatorname{sen}\theta, ds = \cos\theta, \sqrt{1-s^2} = \cos\theta$

$du = \frac{ds}{s}$ $\quad v = -(1-s^2)^{1/2}$

$$\int \frac{s\ell\eta|s|ds}{(1-s^2)^{1/2}} = -\sqrt{1-s^2}\,\ell\eta|s| + \int \frac{\sqrt{1-s^2}}{s}ds = -\sqrt{1-s^2}\,\ell\eta|s| + \int \frac{\cos\theta\cos\theta d\theta}{\operatorname{sen}\theta}$$

$$= -\sqrt{1-s^2}\,\ell\eta|s| + \int \frac{(1-\operatorname{sen}^2\theta)d\theta}{\operatorname{sen}\theta} = -\sqrt{1-s^2}\,\ell\eta|s| + \int \operatorname{cosec}\theta d\theta - \int \operatorname{sen}\theta d\theta$$

$$= -\sqrt{1-s^2}\,\ell\eta|s| + \ell\eta|\operatorname{cosec}\theta - \operatorname{co}\tau g\theta| + \cos\theta + c$$

$$= -\sqrt{1-s^2}\,\ell\eta|s| + \ell\eta\left|\frac{1-\sqrt{1-s^2}}{s}\right| + \sqrt{1-s^2} + c$$

103.- $\int (2\cos\alpha\operatorname{sen}\alpha - \operatorname{sen}\alpha)d\alpha$

Solución.-

$$\int (2\cos\alpha\operatorname{sen}\alpha - \operatorname{sen}2\alpha)d\alpha = \int \overbrace{\cancel{(\operatorname{sen}2\alpha - \operatorname{sen}2\alpha)}}^{0} d\alpha = \int 0 d\alpha = c$$

104.- $\int t^4\ell\eta^2 tdt$

Sea: $u = \ell\eta^2 t$ $\quad dv = t^4dt$

$du = 2\ell\eta t\frac{dt}{t}$ $\quad v = \frac{t^5}{5}$

$\int t^4\ell\eta^2 tdt = \frac{t^5}{5}\ell\eta^2 t - \frac{2}{5}\int t^4\ell\eta tdt(*)$, trabajando por partes nuevamente:

Sea: $u = \ell\eta t$ $\quad dv = t^4dt$

$du = \frac{dt}{t}$ $\quad v = \frac{t^5}{5}$

$$(*) = \frac{t^5}{5}\ell\eta^2 t - \frac{2}{5}\left(\frac{t^5}{5}\ell\eta t - \frac{1}{5}\int t^4dt \right) = \frac{t^5}{5}\ell\eta^2 t - \frac{2t^5}{25}\ell\eta t + \frac{2}{25}\frac{t^5}{5} + c$$

$$= \frac{t^5}{5}\ell\eta^2 t - \frac{2t^5}{25}\ell\eta t + \frac{2t^5}{125} + c$$

105.- $\int u^2(1+v)^{11}dx$

Solución.-
$$\int u^2(1+v)^{11}dx = u^2(1+v)^{11}\int dx = u^2(1+v)^{11}x + c$$

106.- $\displaystyle\int \frac{(\varphi + \operatorname{sen} 3\varphi)d\varphi}{3\varphi^2 - 2\cos 3\varphi}$

Solución.-Sea: $u = 3\varphi^2 - 2\cos 3\varphi, du = 6(\varphi + \operatorname{sen} 3\varphi)d\varphi$
$$\int \frac{(\varphi + \operatorname{sen} 3\varphi)d\varphi}{3\varphi^2 - 2\cos 3\varphi} = \frac{1}{6}\int \frac{du}{u} = \frac{1}{6}\ell\eta\left|u\right| + c = \frac{1}{6}\ell\eta\left|3\varphi^2 - 2\cos 3\varphi\right| + c$$

107.- $\displaystyle\int \frac{(y^{1/2}+1)dy}{y^{1/2}(y+1)}$

Solución.-Sea: $y^{1/2} = t \Rightarrow y = t^2, dy = 2tdt$
$$\int \frac{(y^{1/2}+1)dy}{y^{1/2}(y+1)} = \int \frac{(t+1)2\cancel{t}dt}{\cancel{t}(t^2+1)} = 2\int \frac{(t+1)dt}{(t^2+1)} = \int \frac{2tdt}{(t^2+1)} + \int \frac{dt}{(t^2+1)} = \ell\eta\left|t^2+1\right| + 2\operatorname{arc}\tau g t + c$$
$$= \ell\eta\left|y+1\right| + 2\operatorname{arc}\tau g\sqrt{y} + c$$

108.- $\displaystyle\int \frac{ds}{s^3(s^2-4)^{1/2}}$

Solución.-Sea: $s = 2\sec\theta, ds = 2\sec\theta\tau g\theta d\theta$
$$\int \frac{ds}{s^3(s^2-4)^{1/2}} = \int \frac{\cancel{2}\cancel{\sec\theta}\,\cancel{\tau g\theta}d\theta}{8\sec^{\cancel{3}}\theta\,\cancel{2}\,\cancel{\tau g\theta}} = \frac{1}{8}\int \frac{d\theta}{\sec^2\theta} = \frac{1}{8}\int \cos^2\theta d\theta = \frac{1}{16}\int(1+\cos 2\theta)d\theta$$
$$= \frac{1}{16}\theta + \frac{1}{32}\operatorname{sen} 2\theta + c = \frac{1}{16}\left(\theta + \frac{\operatorname{sen} 2\theta}{2}\right) + c = \frac{1}{16}\left(\theta + \operatorname{sen}\theta\cos\theta\right) + c$$
$$= \frac{1}{16}\left(\operatorname{arcsec}\frac{s}{2} + \frac{2\sqrt{s^2-4}}{s^2}\right) + c$$

109.- $\displaystyle\int \sqrt{u}(1+u^2)^2du$

Solución.-
$$\int \sqrt{u}(1+u^2)^2du = \int \sqrt{u}(1+2u^2+u^4)du = \int u^{1/2}du + 2\int u^{5/2}du + \int u^{9/2}du$$
$$= \frac{u^{3/2}}{3/2} + 2\frac{u^{7/2}}{7/2} + \frac{u^{11/2}}{11/2} + c = \frac{2u^{3/2}}{3} + \frac{4u^{7/2}}{7} + \frac{2u^{11/2}}{11} + c = \frac{2u\sqrt{u}}{3} + \frac{4u^3\sqrt{u}}{7} + \frac{2u^5\sqrt{u}}{11} + c$$
$$= \sqrt{u}\left(\frac{2u}{3} + \frac{4u^3}{7} + \frac{2u^5}{11}\right) + c$$

110.- $\displaystyle\int \frac{(x^3+x^2)dx}{x^2+x-2}$

Solución.-
$$\int \frac{(x^3+x^2)dx}{x^2+x-2} = \int\left(x + \frac{2x}{x^2+x-2}\right)dx = \int xdx + \int \frac{2xdx}{(x+2)(x-1)} = \frac{x^2}{2} + \int \frac{2xdx}{(x+2)(x-1)}$$

$$=\frac{x^2}{2}+\int\frac{2xdx}{(x+2)(x-1)}=\frac{x^2}{2}+\int\frac{Adx}{x+2}+\int\frac{Bdx}{x-1}(*)$$

$$\frac{2x}{(x+2)(x-1)}=\frac{A}{x+2}+\frac{B}{x-1}\Rightarrow 2x=A(x-1)+B(x+2)$$

De donde: $\begin{cases} x=1\Rightarrow 2=3B\Rightarrow B=2/3 \\ x=-2\Rightarrow -4=-3A\Rightarrow A=4/3 \end{cases}$

$$(*)=\frac{x^2}{2}+\frac{4}{3}\int\frac{dx}{x+2}+\frac{2}{3}\int\frac{dx}{x-1}=\frac{x^2}{2}+\frac{4}{3}\ell\eta|x+2|+\frac{2}{3}\ell\eta|x-1|+c$$

$$=\frac{x^2}{2}+\frac{2}{3}\ell\eta\left|(x+2)^2(x-1)\right|+c$$

111- $\int adb$

Solución.-

$$\int adb=a\int db=ab+c$$

112.- $\int\frac{dx}{\sqrt{x^2-2x-8}}$

Solución.-

Completando cuadrados se tiene: $x^2-2x-8=(x^2-2x+1)-9=(x-1)^2-3^2$

Sea: $x-1=3\sec\theta, dx=3\sec\theta\tau g\theta d\theta, \sqrt{(x-1)^2-3^2}=3\tau g\theta$, luego:

$$\int\frac{dx}{\sqrt{x^2-2x-8}}=\int\frac{dx}{\sqrt{(x-1)^2-3^2}}=\int\frac{\cancel{3}\sec\theta\,\cancel{\tau g\theta}\,d\theta}{\cancel{3}\,\cancel{\tau g\theta}}=\int\sec\theta d\theta=\ell\eta|\sec\theta+\tau g\theta|+c$$

$$=\ell\eta\left|\frac{x-1}{3}+\frac{\sqrt{x^2-2x-8}}{3}\right|+c$$

113.- $\int\frac{(x+1)dx}{\sqrt{2x-x^2}}$

Solución.-

Completando cuadrados se tiene:

$$2x-x^2=-(x^2-2x)=-(x^2-2x+1-1)=-(x^2-2x+1)+1=1-(x^2-1)$$

Sea: $x-1=\operatorname{sen}\theta, dx=\cos\theta d\theta, \sqrt{1-(x-1)^2}=\cos\theta$, luego:

$$\int\frac{(x+1)dx}{\sqrt{2x-x^2}}=-\frac{1}{2}\int\frac{(2-2x)-4}{\sqrt{2x-x^2}}dx=-\frac{1}{2}\int\frac{(2-2x)dx}{\sqrt{2x-x^2}}+2\int\frac{dx}{\sqrt{2x-x^2}}$$

$$=-\sqrt{2x-x^2}+2\int\frac{dx}{\sqrt{2x-x^2}}=-\sqrt{2x-x^2}+2\int\frac{dx}{\sqrt{1-(x-1)^2}}$$

$$=-\sqrt{2x-x^2}+2\int\frac{\cancel{\cos\theta}\,d\theta}{\cancel{\cos\theta}}=-\sqrt{2x-x^2}+2\theta+c=-\sqrt{2x-x^2}+2\operatorname{arcsen}(x-1)+c$$

114.- $\int f(x)f'(x)dx$

Solución.- Sea: $u = f(x), du = f´(x)dx$

$$\int f(x) f´(x)dx = \int udu = \frac{u^2}{2} + c = \frac{[f(x)]^2}{2} + c$$

115.- $\int \frac{x^3 + 7x^2 - 5x + 5}{x^2 + 2x - 3} dx$

Solución.-

$$\int \frac{x^3 + 7x^2 - 5x + 5}{x^2 + 2x - 3} dx = \int \left(x + 5 + \frac{20 - 12x}{x^2 + 2x - 3} \right) dx = \int xdx + 5\int dx + \int \frac{(20 - 12x)dx}{x^2 + 2x - 3}$$

$$\int xdx + 5\int dx + \int \frac{(20 - 12x)dx}{(x+3)(x-1)} = \frac{x^2}{2} + 5x + \int \frac{Adx}{x+3} + \int \frac{B}{x-1} (*)$$

$$20 - 12x = A(x-1) + B(x+3)$$

De donde: $\begin{cases} x = 1 \Rightarrow 8 = 4B \Rightarrow B = 2 \\ x = -3 \Rightarrow 56 = -4A \Rightarrow A = -14 \end{cases}$

$$(*) = \frac{x^2}{2} + 5x - 14\int \frac{dx}{x+3} + 2\int \frac{dx}{x-1} = \frac{x^2}{2} + 5x + 14\ell\eta|x+3| + 2\ell\eta|x-1| + c$$

116.- $\int e^{\ell\eta|1+x+x^2|} dx$

Solución.-

$$\int e^{\ell\eta|1+x+x^2|} dx = \int (1 + x + x^2)dx = x + \frac{x^2}{2} + \frac{x^3}{3} + c$$

117.- $\int \frac{(x-1)dx}{\sqrt{x^2 - 4x + 3}}$

Solución.-

Completando cuadrados se tiene: $x^2 - 4x + 3 = x^2 - 4x + 4 - 1 = (x-2)^2 - 1$

Sea: $x - 2 = \sec\theta, dx = \sec\theta\tau g\theta d\theta, \sqrt{(x-2)^2 - 1} = \tau g\theta$, luego:

$$\int \frac{(x-1)dx}{\sqrt{x^2 - 4x + 3}} = \frac{1}{2}\int \frac{(2x-4) + 2}{\sqrt{x^2 - 4x + 3}} dx = \frac{1}{2}\int \frac{(2x-4)dx}{\sqrt{x^2 - 4x + 3}} + \int \frac{dx}{\sqrt{x^2 - 4x + 3}}$$

$$= \sqrt{x^2 - 4x + 3} + \int \frac{dx}{\sqrt{x^2 - 4x + 3}} = \sqrt{x^2 - 4x + 3} + \int \frac{dx}{\sqrt{(x-2)^2 - 1}}$$

$$= \sqrt{x^2 - 4x + 3} + \int \frac{\sec\theta \cancel{\tau g\theta} d\theta}{\cancel{\tau g\theta}} = \sqrt{x^2 - 4x + 3} + \int \sec\theta d\theta$$

$$= \sqrt{x^2 - 4x + 3} + \ell\eta|\sec\theta + \tau g\theta| + c$$

$$= \sqrt{x^2 - 4x + 3} + \ell\eta\left|x - 2 + \sqrt{x^2 - 4x + 3}\right| + c$$

118.- $\int \frac{xdx}{\sqrt{x^2 + 4x + 5}}$

Solución.-

Completando cuadrados se tiene: $x^2+4x+5=x^2+4x+4+1=(x+2)^2+1$

Sea: $x+2=\tau g\theta, dx=\sec^2\theta d\theta, \sqrt{(x+2)^2+1}=\sec\theta$, luego:

$$\int\frac{xdx}{\sqrt{x^2+4x+5}}=\int\frac{xdx}{\sqrt{(x+2)^2+1}}=\int\frac{(\tau g\theta-2)\sec^{\cancel{2}}\theta d\theta}{\cancel{\sec\theta}}=\int\tau g\theta\sec\theta d\theta-2\int\sec\theta d\theta$$

$$=\sec\theta-2\ell\eta\left|\sec\theta+\tau g\theta\right|+c=\sqrt{x^2+4x+5}-2\ell\eta\left|\sqrt{x^2+4x+5}+x+2\right|+c$$

119.- $\int\frac{4dx}{x^3+4x}$

Solución.-

$$\int\frac{4dx}{x^3+4x}=\int\frac{(3x^2+4)-3x^2}{x^3+4x}dx=\int\frac{(3x^2+4)dx}{x^3+4x}-3\int\frac{x^2dx}{x^3+4x}$$

$$=\ell\eta\left|x^3+4x\right|-\frac{3}{2}\int\frac{2xdx}{x^2+4}=\ell\eta\left|x^3+4x\right|-\frac{3}{2}\ell\eta\left|x^2+4\right|+c$$

$$=\ell\eta\left|\frac{x(x^2+4)}{(x^2+4)^{3/2}}\right|+c=\ell\eta\left|\frac{x}{\sqrt{x^2+4}}\right|+c$$

120.- $\int\frac{\operatorname{co}\tau gxdx}{\ell\eta\left|\operatorname{sen}x\right|}$

Solución.- Sea: $u=\ell\eta\left|\operatorname{sen}x\right|, du=\operatorname{co}\tau gxdx$

$$\int\frac{\operatorname{co}\tau gxdx}{\ell\eta\left|\operatorname{sen}x\right|}=\int\frac{du}{u}=\ell\eta\left|u\right|+c=\ell\eta\left|\ell\eta\left|\operatorname{sen}x\right|\right|+c$$

121.- $\int\ell\eta\exp\sqrt{x-1}dx$

Solución.-

$$\int\ell\eta\exp\sqrt{x-1}dx=\int\sqrt{x-1}dx=\frac{(x-1)^{3/2}}{3/2}+c=\frac{2(x-1)\sqrt{(x-1)}}{3}+c$$

122.- $\int\frac{\sqrt{1+x^3}}{x}dx$

Solución.- Sea: $\sqrt{1+x^3}=t\Rightarrow t^2=1+x^3\Rightarrow x=\sqrt[3]{t^2-1}, dx=\frac{2tdt}{3(t^2-1)^{2/3}}$

$$\int\frac{\sqrt{1+x^3}}{x}dx=\int\frac{t\frac{2tdt}{3(t^2-1)^{2/3}}}{(t^2-1)^{1/3}}=\frac{2}{3}\int\frac{t^2dt}{t^2-1}=\frac{2}{3}\int\left(1+\frac{1}{t^2-1}\right)dt=\frac{2}{3}\int dt+\frac{2}{3}\int\frac{dt}{t^2-1}$$

$$=\frac{2}{3}t+\frac{1}{3}\ell\eta\left|\frac{t-1}{t+1}\right|+c=\frac{2}{3}\sqrt{1+x^3}+\frac{1}{3}\ell\eta\left|\frac{\sqrt{1+x^3}-1}{\sqrt{1+x^3}+1}\right|+c$$

123.- $\int\sqrt{\frac{x-1}{x+1}}\frac{1}{x}dx$

Solución.- Sea: $\sqrt{\dfrac{x-1}{x+1}}=t\Rightarrow t^2=\dfrac{x-1}{x+1}\Rightarrow x(1-t^2)=t^2\Rightarrow x=\dfrac{1+t^2}{1-t^2},dx=\dfrac{4tdt}{(1-t^2)^2}$

$$\int\sqrt{\frac{x-1}{x+1}}\frac{1}{x}dx=\int t\frac{(1-t^2)}{(1+t^2)}\frac{4tdt}{(1-t^2)^2}=4\int\frac{t^2\cancel{(1-t^2)}dt}{(1+t^2)(1-t^2)^{\cancel{2}}}=4\int\frac{t^2dt}{(1+t^2)(1-t^2)}$$

$$=4\int\frac{t^2dt}{(1+t)(1-t)(1+t^2)}=4\left[\int\frac{Adt}{1+t}+\int\frac{Bdt}{1-t}+\int\frac{Ct+D}{1+t^2}dt\right](*)$$

$$\frac{t^2}{(1+t)(1-t)(1+t^2)}=\frac{A}{1+t}+\frac{B}{1-t}+\frac{Ct+D}{1+t^2}$$

$$\Rightarrow t^2=A(1-t)(1+t^2)+B(1+t)(1+t^2)+(Ct+D)(1-t^2)$$

De donde: $\begin{cases}t=1\Rightarrow 1=4B\Rightarrow B=1/4\\ t=-1\Rightarrow 1=4A\Rightarrow A=1/4\\ t=0\Rightarrow 0=A+B+D\Rightarrow D=-1/2\end{cases}$

$$t=2\Rightarrow 4=-5A+15B+(2C+D)(-3)\Rightarrow C=0$$

$$(*)=4\left(\frac{1}{4}\int\frac{dt}{1+t}+\frac{1}{4}\int\frac{dt}{1-t}-\frac{1}{2}\int\frac{dt}{1+t^2}\right)=\int\frac{dt}{1+t}-\int\frac{dt}{t-1}-2\int\frac{dt}{1+t^2}$$

$$=\ell\eta\left|t+1\right|-\ell\eta\left|t-1\right|-2\operatorname{arc}\tau gt+c=\ell\eta\left|\frac{t+1}{t-1}\right|-2\operatorname{arc}\tau gt+c$$

$$=\ell\eta\left|\frac{\sqrt{\frac{x+1}{x-1}}+1}{\sqrt{\frac{x+1}{x-1}}-1}\right|-2\operatorname{arc}\tau g\sqrt{\frac{x+1}{x-1}}+c=\ell\eta\left|\frac{\sqrt{x-1}+\sqrt{x+1}}{\sqrt{x-1}-\sqrt{x+1}}\right|-2\operatorname{arc}\tau g\sqrt{\frac{x+1}{x-1}}+c$$

124.- $\displaystyle\int\frac{\operatorname{sen}xdx}{1+\operatorname{sen}x+\cos x}$

Solución.- Sea: $\operatorname{sen}x=\dfrac{2z}{1+z^2},\cos x=\dfrac{1-z^2}{1+z^2},z=\tau g\dfrac{x}{2},dx=\dfrac{2dz}{1+z^2}$

$$\int\frac{\operatorname{sen}xdx}{1+\operatorname{sen}x+\cos x}=\int\frac{\left(\frac{2z}{1+z^2}\right)\left(\frac{2}{1+z^2}\right)}{1+\left(\frac{2z}{1+z^2}\right)\left(\frac{1-z^2}{1+z^2}\right)}dz=\int\frac{\frac{4z}{1+z^2}dz}{1+z^2+2z+1-z^2}$$

$$\int\frac{4zdz}{(1+z^2)(2+2z)}=\int\frac{2zdz}{(1+z)(1+z^2)}=\int\frac{Adz}{1+z}+\int\frac{Bz+C}{1+z^2}dz(*)$$

$$\frac{2z}{(1+z)(1+z^2)}=\frac{A}{1+z}+\frac{Bz+C}{1+z^2}$$

De donde: $\begin{cases}z=-1\Rightarrow -2=2A\Rightarrow A=-1\\ z=0\Rightarrow 0=A+C\Rightarrow C=1\\ z=1\Rightarrow 2=2A+2B+2C\Rightarrow B=1\end{cases}$

$$(*) = -\int \frac{dz}{1+z} + \int \frac{z+1}{1+z^2}dz = -\ell\eta|1+z| + \frac{1}{2}\int \frac{2zdz}{z^2+1} + \int \frac{dz}{z^2+1}$$

$$= -\ell\eta|1+z| + \frac{1}{2}\ell\eta|z^2+1| + \operatorname{arc}\tau gz + c = \ell\eta\left|\frac{\sqrt{z^2+1}}{z+1}\right| + \operatorname{arc}\tau gz + c$$

$$= \ell\eta\left|\frac{\sqrt{\tau g^2 x/2 + 1}}{\tau g\, x/2 + 1}\right| + \operatorname{arc}\tau gz + c$$

125.- $\int \frac{dx}{3+2\cos x}$

Solución.- Sea: $\operatorname{s}e\operatorname{n} x = \frac{2z}{1+z^2}, \cos x = \frac{1-z^2}{1+z^2}, z = \tau g\frac{x}{2}, dx = \frac{2dz}{1+z^2}$

$$\int \frac{dx}{3+2\cos x} = \int \frac{\frac{2z}{1+z^2}}{3+2\left(\frac{1-z^2}{1+z^2}\right)}dz = \int \frac{2dz}{3+3z^2+2-2z^2} = 2\int \frac{dz}{5+z^2} = \frac{2}{\sqrt{5}}\operatorname{arc}\tau g\frac{z}{\sqrt{5}} + c$$

$$= \frac{2\sqrt{5}}{5}\operatorname{arc}\tau g\left(\frac{\sqrt{5}}{5}\tau g\frac{x}{2}\right) + c$$

126.- $\int \frac{xdx}{\sqrt{x^2-2x+5}}$

Solución.-

Completando cuadrados se tiene: $x^2-2x+5 = x^2-2x+1+4 = (x-1)^2+2^2$,

Sea: $x-1 = 2\tau g\theta, dx = 2\sec^2\theta d\theta, \sqrt{(x-1)^2+2^2} = 2\sec\theta$,luego:

$$\int \frac{xdx}{\sqrt{x^2-2x+5}} = \frac{1}{2}\int \frac{(2x-2+2)dx}{\sqrt{x^2-2x+5}} = \frac{1}{2}\int \frac{(2x-2)dx}{\sqrt{x^2-2x+5}} + \int \frac{dx}{\sqrt{x^2-2x+5}}$$

$$= \sqrt{x^2-2x+5} + \int \frac{dx}{\sqrt{x^2-2x+5}} = \sqrt{x^2-2x+5} + \int \frac{dx}{\sqrt{(x-1)^2+2^2}}$$

$$= \sqrt{x^2-2x+5} + \int \frac{\cancel{2}\sec^{\cancel{2}}\theta d\theta}{\cancel{2\sec\theta}} = \sqrt{x^2-2x+5} + \int \sec\theta d\theta$$

$$= \sqrt{x^2-2x+5} + \ell\eta|\sec\theta + \tau g\theta| + c$$

127.- $\int \frac{(1+\operatorname{s}e\operatorname{n} x)dx}{\operatorname{s}e\operatorname{n} x(2+\cos x)}$

Solución.- Sea: $\operatorname{s}e\operatorname{n} x = \frac{2z}{1+z^2}, \cos x = \frac{1-z^2}{1+z^2}, z = \tau g\frac{x}{2}, dx = \frac{2dz}{1+z^2}$

$$\int\frac{(1+\operatorname{sen}x)dx}{\operatorname{sen}x(2+\cos x)}=\int\frac{\left(1+\frac{2z}{1+z^2}\right)\frac{\cancel{2}}{\cancel{1+z^2}}}{\frac{\cancel{2}z}{\cancel{1+z^2}}\left(2+\frac{1-z^2}{1+z^2}\right)}dz=\int\frac{(1+z^2+2z)dz}{2z(1+z^2)+z(1-z^2)}$$

$$=\int\frac{(z^2+2z+1)dz}{z^3+3z}=\int\frac{(z^2+2z+1)dz}{z(z^2+3)}=\int\frac{Adz}{z}+\int\frac{Bz+C}{(z^2+3)}dz(*)\}$$

$$\frac{(z^2+2z+1)}{z(z^2+3)}=\frac{A}{z}+\frac{Bz+C}{(z^2+3)}\Rightarrow z^2+2z+1=A(z^2+3)+(Bz+C)z$$

$\Rightarrow Az^2+3A+Bz^2+Cz\Rightarrow(A+B)z^2+Cz+3A$, igualando coeficientes se tiene:

$$\begin{pmatrix} A & +B & & =1 \\ & & C & =2 \\ 3A & & & =1 \end{pmatrix}\Rightarrow A=\frac{1}{3},B=\frac{2}{3},C=2$$

$$(*)=\frac{1}{3}\int\frac{dz}{z}+\int\frac{\frac{2}{3}z+2}{(z^2+3)}dz=\frac{1}{3}\int\frac{dz}{z}+\frac{1}{3}\int\frac{2zdz}{(z^2+3)}+2\int\frac{dz}{(z^2+3)}$$

$$=\frac{1}{3}\ell\eta\left|\tau g\,\frac{x}{2}\right|+\frac{1}{3}\ell\eta\left|\tau g^2\,\frac{x}{2}+3\right|+\frac{2}{\sqrt{3}}\operatorname{arc}\tau g\left(\frac{\tau g^2\,\frac{x}{2}}{\sqrt{3}}\right)+c$$

128.- $\int\frac{dx}{x^4+4}$

Solución.- Sea: $x^4+4=x^4+4x^2+4-4x^2=(x^2+2)^2-(2x)^2=(x^2+2x+2)(x^2-2x+2)$

$$\int\frac{dx}{x^4+4}=\int\frac{dx}{(x^2+2x+2)(x^2-2x+2)}=\int\frac{(Ax+B)dx}{(x^2+2x+2)}+\int\frac{(Cx+D)dx}{(x^2-2x+2)}(*)$$

$$\frac{1}{(x^4+4)}=\frac{(Ax+B)}{(x^2+2x+2)}+\frac{(Cx+D)}{(x^2-2x+2)}$$

$$1=(Ax+B)(x^2-2x+2)+(Cx+D)(x^2+2x+2)$$

$$1=(A+C)x^3+(-2A+B+2C+D)x^2+(2A-2B+2C+2D)x+(2B+2D)$$

Igualando coeficientes se tiene:

$$\begin{pmatrix} A & & +C & & =0 \\ -2A & +B & +2C & +D & =0 \\ 2A & -2B & +2C & +2D & =0 \\ & 2B & & +2D & =1 \end{pmatrix}\Rightarrow A=\frac{1}{8},B=\frac{1}{4},C=-\frac{1}{8},D=\frac{1}{4}$$

$$(*)=\frac{1}{8}\int\frac{(x+2)dx}{(x^2+2x+2)}-\frac{1}{8}\int\frac{(x-2)dx}{(x^2-2x+2)}$$

$$=\frac{1}{8}\int\frac{(x+1)dx}{(x+1)^2+1}+\frac{1}{8}\int\frac{dx}{(x+1)^2+1}-\frac{1}{8}\int\frac{(x-1)dx}{(x-1)^2+1}+\frac{1}{8}\int\frac{dx}{(x-1)^2+1}$$

$$=\frac{1}{16}\ell\eta\left|x^2+2x+2\right|+\frac{1}{8}\operatorname{arc}\tau g(x+1)-\frac{1}{16}\ell\eta\left|x^2-2x+2\right|+\frac{1}{8}\operatorname{arc}\tau g(x-1)+c$$

$$=\frac{1}{16}\ell\eta\left|\frac{x^2+2x+2}{x^2-2x+2}\right|+\frac{1}{8}\left[\operatorname{arc}\tau g(x+1)+\operatorname{arc}\tau g(x-1)\right]+c$$

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