ST 623 HW5

Amy Ly

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Problem Context

The following dataset was taken from Snedecor and Cochran (1967, p. 354) and was obtained from an experiment that wanted to determine the effects of temperature and storage time on teh loss of ascorbic acid in snap-beans. The beans were all harvested under uniform conditions at the Iowa Agricultural Station before 8 AM and then perpared and quick-frozen before noon on the same day. Three packages were assigned at ranom to each temperature and storage-time combination.

```
factors <- list(temp = c(0, 10, 20), storage = c(2, 4, 6, 8) )
combo <- cross_df(factors)
```

```
## Warning: ‘cross_df()‘ was deprecated in purrr 1.0.0.
## i Please use ‘tidyr::expand_grid()‘ instead.
## i See <https://github.com/tidyverse/purrr/issues/768>.
```

```
conc <- c(45, 45, 34, 47, 43, 28, 46, 41, 21, 46, 37, 16)
df <- cbind(combo, conc)

df$temp <- as.factor(df$temp)

df
```

```
##    temp storage conc
## 1     0       2   45
## 2    10       2   45
## 3    20       2   34
## 4     0       4   47
## 5    10       4   43
## 6    20       4   28
## 7     0       6   46
## 8    10       6   41
## 9    20       6   21
## 10    0       8   46
## 11   10       8   37
## 12   20       8   16
```

Suppose that the ascorbic acid concentration decays exponentially fast, with a decay rate that is temperature dependant. For a given storage temperature T, the expected concentration after time t (measured in weeks) is:

$\mu = E(Y) = exp(-\alpha - \beta_T t)$

Note that the initial concentration $exp(-\alpha)$ is assumed to be independent of storage condition.

Part a

Create a regression model, treating temperature as a factor and storage time as covariates. Write down the details of the model and the score equations.

Here I have a model with one factor with 3 levels and a covariate. Note that temp $= 2°F$ is the reference category.

$y_i = \alpha + \beta_0 1(temp = 10) + \beta_1 1(temp = 20) + \beta_3 x_i$

$\mu = E(Y) = e^{-\alpha - \beta_T t}$

The log likelihood would then be:

$-\frac{n}{2} log(2\pi\sigma^2) - \frac{1}{2\sigma^2} \sum_{i=1}^{n} (y_i - \mu)^2$

To get the score function, we would take the derivative of the log likelihood function.

```
mod <- glm(conc~temp:storage, data=df, family=gaussian(link="log"))

summary(mod)
```

```
##
## Call:
## glm(formula = conc ~ temp:storage, family = gaussian(link = "log"),
##     data = df)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -1.49820  -0.38546   0.08059   0.80866   0.86028
##
## Coefficients:
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)     3.8354389  0.0186464 205.694 3.49e-16 ***
## temp0:storage  -0.0007716  0.0037540  -0.206 0.842286
## temp10:storage -0.0236121  0.0040351  -5.852 0.000382 ***
## temp20:storage -0.1329785  0.0061597 -21.588 2.23e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 1.124727)
##
##     Null deviance: 1226.9167  on 11  degrees of freedom
## Residual deviance:    8.9978  on  8  degrees of freedom
## AIC: 40.599
##
## Number of Fisher Scoring iterations: 3
```

Part b

Estimate the times taken at each of the three temperature for the ascorbic acid combination to be reduced to 50% of the original value.

```
a <- mod$coef[1]
b_0 <- mod$coef[2]
```

```r
b_10 <- mod$coef[3]
b_20 <- mod$coef[4]
V <- vcov(mod)

SE_0 <- sqrt(V[2,2])
SE_10 <- sqrt(V[3,3])
SE_20 <- sqrt(V[4,4])
z <- qnorm(0.95)


t_0 <- -log(2)/b_0
t_10 <- -log(2)/b_10
t_20 <- -log(2)/b_20

CI_0 <- c(t_0-z*SE_0, t_0 +z*SE_0)
CI_10 <- c(t_10-z*SE_10, t_10 +z*SE_10)
CI_20 <- c(t_20-z*SE_20, t_20 +z*SE_20)
```

At 0 degrees Fahrenheit, it would require 898.3304904 weeks for the ascorbic acid to be reduced to 50% of its original value, with a 95% confidence interval of (898.3243156, 898.3366652) weeks.

At 10 degrees Fahrenheit, it would require 29.355637 weeks for the ascorbic acid to be reduced to 50% of its original value, with a 95% confidence interval of (29.3489999, 29.3622741) weeks.

At 20 degrees Fahrenheit, it would require 5.2124744 weeks for the ascorbic acid to be reduced to 50% of its original value, with a 95% confidence interval of (5.2023426, 5.2226063) weeks.