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# Constrained Coverage for Mobile Sensor Networks

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***Abstract*— We consider the problem of self-deployment of a mobile sensor network. We are interested in a deployment strategy that maximizes the area coverage of the network with the constraint that each of the nodes has at least K neighbors, where K is a user-specified parameter. We propose an algorithm based on artificial potential fields which is distributed, scalable and does not require a prior map of the environment. Simulations establish that the resulting networks have the required degree with a high probability, are well connected and achieve good coverage. We present analytical results for the coverage achievable by uniform random and symmetrically tiled network configurations and use these to evaluate the performance of our algorithm.**

## I. Introduction

There has been a growing interest to study and build systems of mobile sensor networks. It is envisaged that in the near future, very large scale networks consisting of both mobile and static nodes will be deployed for applications ranging from environment monitoring to emergency search-and-rescue operations [1]. The mobile nodes in the network will enhance its capabilities - they could be used to physically collect and transport data or to recharge and repair the static nodes in the network. A key step towards realizing such networks is to develop techniques for network nodes to self-deploy and reconfigure. Further, for successful operation of the network, the deployment should result in configurations that not only provide good 'sensor coverage' but also satisfy certain local (e.g. node degree) and global (e.g. network connectivity) constraints. Informally, *Constrained Coverage* is the problem of finding a deployment configuration which maximizes the collective sensor coverage of the nodes while satisfying one or more constraints.

In this paper we consider constrained coverage for a network whose constituent nodes are all autonomous mobile robots. The constraint we consider is node degree - the number of neighbors of each node in the network. More precisely, we require each node to have a minimum degree $K$, where $K$ is parameter of the deployment algorithm. Our interest in this particular constraint is twofold:

1) In several applications of sensor networks, node degree plays an important role. For instance, several localization algorithms require a certain minimum degree for the nodes [2]. Sometimes a high degree is required for the sake of redundancy.
2) Evidence from the theory of random networks indicates that global network connectivity is strongly dependent on node degree [3], a **local constraint**. This ability to influence global network properties by manipulating purely local ones is interesting.

Pragmatically we are motivated by applications of network deployment where a global map of the environment is either unavailable or of little use because the environment is not static. We also assume that no global positioning system (GPS) is available. An example is an urban search and rescue operation where a building is on fire and first responders want to gather information from inside the building. We would like our mobile sensor network to deploy itself into the building, form a network with high sensor coverage and reliably transmit the required information to personnel outside.

Our approach to the problem of constrained coverage is based on virtual potential fields. We treat each node in the network as a virtual charged particle and define simple force laws to govern the interaction between neighboring nodes. These laws incorporate two different kinds of forces. One is a repulsive force that tries to maximize coverage while the other is an attractive force that imposes the constraint of $K$-degree.As a result of these forces, a group of nodes placed close together spreads out into a network to maximize the coverage while satisfying the constraint of $K$-degree.

## II. Related Work

In 1992, Gage introduced a taxonomy for coverage by multi-robot systems [4]. He defined three kinds of coverage: blanket coverage, barrier coverage and sweep coverage. According to this taxonomy our problem falls into the category of blanket coverage problems where the main objective is to maximize the total detection area.

The problem of dispersing a large group of mobile robots into an unknown environment has received a lot of attention. Arkin and Ali developed a behavior-based approach for dispersion of robot-teams by using a random-wandering behavior coupled with obstacle and robot avoidance behaviors [5]. More recently, Batalin and Sukhatme have addressed the problem of multi-robot area coverage [6].Their approach is based on using local dispersion of the robots to achieve good global coverage. Pearce, et al. have developed a dispersion behavior for a group of miniature robots inspired by insect colony coordination behaviors [7].connectivity is a constraint to task completion

Potential Field techniques for robot applications were first introduced by Khatib [8] and ever since have been used extensively to develop elegant solutions for path planning. Recently, they have also been applied to multi-robot domains. Reif and Wang have used the idea of 'social potentials' where the potentials for a robot are constructed with respect to the other robots [9]. They describe heuristics to design social potentials for achieving a variety of behaviors like clustering, patrolling, etc. Balch and Hybinette have also developed a method based on social potentials that allows teams of robots to autonomously arrange themselves into geometric patterns while navigating through an obstacle field [10]. These methods do not aim at maximizing the area coverage. Our algorithm is most closely related to the potential field-based deployment algorithm proposed by Howard, et al. [11] where coverage is achieved as an emergent property of the system. However, in this case there is no constraint on the deployed network.

To the best of our knowledge, the problem of coverage maximization in network deployment with an explicit node degree constraint has not yet been addressed in the literature.

## III. Problem Formulation

**Problem:** *Given $N$ mobile nodes with isotropic radial sensors of range $R_s$ and isotropic radio communication of range $R_c$, how should they deploy themselves so that the resulting configuration maximizes the net sensor coverage of the network with the constraint that each node has at least $K$ neighbors?*

**Definition:** *Two nodes are considered neighbors if the Euclidean distance between them is less than or equal to the communication range $R_c$.*

We make the following assumptions:

1) The nodes are capable of omni-directional motion,
2) Each node can sense the exact relative range and bearing of its neighbors,
3) The quality of sensing (communication) is constant within $R_s(R_c)$ and is zero outside the sensing (communication) range, i.e. it follows a binary model.

We impose certain desiderata on the problem solution. The deployment algorithm should:

- be distributed and scalable
- not require a prior map or localization of nodes
- adapt to changes in the environment and the network itself

We use the following three metrics to evaluate the performance of the deployment algorithm.

1) the normalized per-node coverage. This is defined as:

$$Coverage = \frac{(\textit{Net Area Covered by the Network})}{N\pi R_s^2}$$

In the remainder of the paper, we use the term 'coverage' to mean the normalized per-node coverage as defined above.

2) the percentage of nodes in the network that have at least $K$ neighbors.
3) the average degree of the network.

## IV. The Deployment Algorithm

Potential field-based techniques have been used extensively to solve navigation problems in mobile robotics. In these methods, virtual potential fields are used to represent goals and constraints and the control law for the robot's motion is formulated such that it moves from a high potential state to a low potential state similar to the way in which a charged particle would move in an electrostatic field.

In our deployment algorithm, we construct virtual forces between nodes so that each node can attract or repel its neighbors. The forces are of two kinds. The first, $\mathbf{F_{cover}}$, causes the nodes to repel each other to increase their coverage, and the second, $\mathbf{F_{degree}}$ constrains the degree of nodes by making them attract each other when they are on the verge of being disconnected. By using a combination of these forces each node maximizes its coverage while maintaining a degree of at least $K$.

In our experiments, each node begins with more than $K$ neighbors and repels all of them using $\mathbf{F_{cover}}$ till it has only $K$ left. The resulting neighbors are called the node's *critical neighbors* and the connections between them and the node are called *critical connections*.

The node now communicates to all its neighbors that its connection with them is critical and therefore should not be broken. It then continues to repel all its neighbors using $\mathbf{F_{cover}}$ but as the distance between the node and its critical neighbor increases, $\|\mathbf{F_{cover}}\|$ decreases and $\|\mathbf{F_{degree}}\|$ increases. As a result, at some distance $\eta R_c$, where $0 < \eta < 1$, the net force $\|\mathbf{F_{cover}} + \mathbf{F_{degree}}\|$ between the node and its neighbor is zero. At this distance, the node and its neighbor are in *equilibrium* with respect to each other. We call $\frac{1}{\eta}$ the *safety factor* because the larger its value, the smaller the probability of critical neighbors losing connectivity.

The forces are constructed as inverse square law profiles - $\|\mathbf{F_{cover}}\|$ tends to infinity when the distance between nodes is zero so that collisions are avoided. Similarly, $\|\mathbf{F_{degree}}\|$ tends to infinity when the distance between the critical neighbors is $R_c$ so that loss of connectivity between them is prevented.

Figure 1 shows a node with $> K$ and exactly $K$ neighbors and Figure 2 shows the corresponding force profiles.

Mathematically, the forces can be expressed as follows. Consider a network of $n$ nodes $1, 2, 3, \ldots, n$ at positions $\mathbf{x_1}, \mathbf{x_2}, \ldots, \mathbf{x_n}$ respectively. Let $\Delta x_{ij}$ represent the Euclidean distance between nodes $i$ and $j$, i.e. $\Delta x_{ij} = \|\mathbf{x_i} - \mathbf{x_j}\|$

$\mathbf{F_{cover}}$ and $\mathbf{F_{degree}}$ are defined as follows.

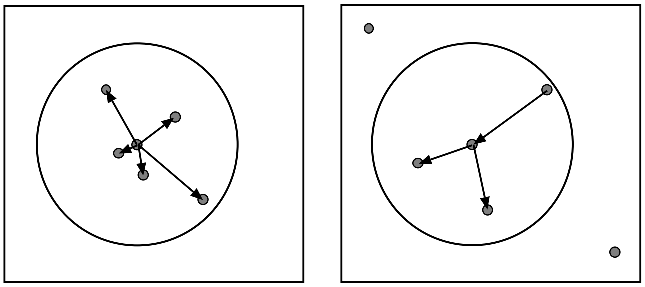
$$\mathbf{F_{cover}}(\mathbf{i}, \mathbf{j}) = \left(\frac{-\mathrm{K_{cover}}}{\Delta \mathrm{x_{ij}}^2}\right)\left(\frac{\mathbf{x_i} - \mathbf{x_j}}{\Delta \mathrm{x_{ij}}}\right)$$

$$\mathbf{F_{degree}}(\mathbf{i}, \mathbf{j}) = \begin{cases} \frac{K_{degree}}{(\Delta x_{ij} - R_c)^2}\left(\frac{\mathbf{x_i} - \mathbf{x_j}}{\Delta x_{ij}}\right) & \text{if critical connection;} \\ \mathbf{0} & \text{otherwise.} \end{cases}$$

where $K_{cover}$ and $K_{degree}$ are the force constants.

The resultant force between the nodes $i$ and $j$ is

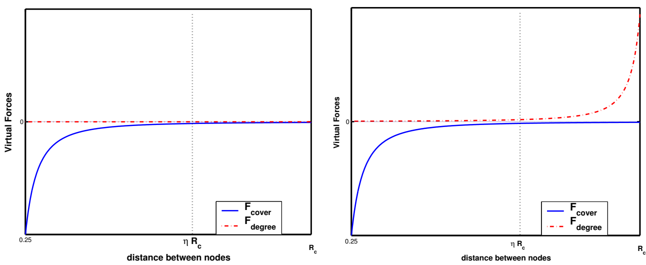
$$\mathbf{F}(\mathbf{i}, \mathbf{j}) = \mathbf{F_{cover}}(\mathbf{i}, \mathbf{j}) + \mathbf{F_{degree}}(\mathbf{i}, \mathbf{j})$$



(a) Node with $> K$ neighbors

(b) Node with exactly $K$ (critical) neighbors

Fig. 1
ILLUSTRATION OF THE ALGORITHM FOR K=3




(a) Non-Critical connection

(b) Critical connection

Fig. 2
FORCE PROFILES

and node $i$ will experience a net force of

$$\mathbf{F_i} = \sum_{\text{all neighbors j}} \mathbf{F(i,j)}$$

The equation of motion for node $i$ is formulated as:

$$\mathbf{\ddot{x}_i(t)} = \left( \frac{\mathbf{F_i} - \nu \mathbf{\dot{x}_i}}{\mathrm{m}} \right)$$

where $\nu$ is a damping factor and $m$ is the virtual mass of the node which is assumed to be 1.

*Computational Details*

Having described the equation of motion for the node, we discuss our choices of the four parameters $K_{cover}$, $K_{degree}$, $\nu$ and $\eta$.

- $K_{cover}$: Consider two nodes that are repelling each other. As the distance $d$ between them increases, the combined coverage of the nodes increases, reaches a maximum of $2\pi R_s^2$ at $d = 2R_s$ and remains constant after that. This implies that for $d > 2R_s$ repelling does not improve coverage. We therefore pick a value for $K_{cover}$ such that at
$$d = 2R_s, \|\mathbf{F_{cover}}\| \approx 0$$

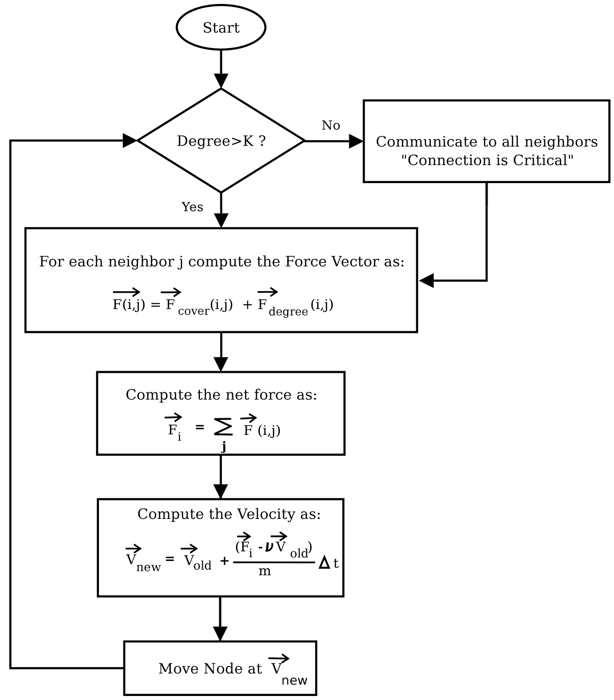



Fig. 3
THE DEPLOYMENT ALGORITHM

- $K_{degree}$: At $d = \eta R_c$ we want $\|\mathbf{F_{cover}} + \mathbf{F_{degree}}\| = 0$, i.e.
$$\frac{-K_{cover}}{\eta^2 {R_c}^2} + \frac{K_{degree}}{(\eta - 1)^2 {R_c}^2} = 0$$
$$K_{degree} = \frac{(1-\eta)^2}{\eta^2} K_{cover}$$
- $\eta$: A large $\eta$ increases the probability of critical neighbors getting disconnected while a small $\eta$ results in lesser coverage. We used a value of 0.8 for $\eta$. This is a heuristic choice based on experimental experience.
- $\nu$: We conducted a simple 'two-body' variant of our scenario by implementing our algorithm on two nodes to study the variation in their interaction for different values of $\nu$. For these experiments we fixed the values of $K_{cover}$, $K_{degree}$ and $\eta$ as explained above. We found that for small values of $\nu$ the system oscillates. We picked the smallest value of $\nu$ that does not lead to oscillations. This value corresponds roughly to the *critically damped*case for our system. In our experiments, we used $\nu = 0.25$.

In the following two sections, we analyze the coverage achievable by uniform random and symmetrically tiled network configurations that satisfy the constraint of $K$-degree. These will serve as reference points to evaluate the performance of our algorithm.

## V. COVERAGE OF UNIFORM RANDOM NETWORKS

A uniform random network is one in which the nodes are distributed randomly and with a uniform density. For such networks, the probability of finding $i$ nodes in a specified domain depends only on the area of the domain and not on its shape or location. Given any area $S$, the probability that it will contain exactly $i$ nodes is given by

$$P(i) = \frac{(\rho S)^i}{i!} e^{-\rho S}$$

where $\rho$ is the density of node deployment [12].

The probability that a randomly chosen node will have a degree of at least $K$ is the probability that there will be at least $K$ nodes in the area $\pi R_c^2$ around it.

$$P(i \geq K) = 1 - \sum_{i=0}^{i=(K-1)} \frac{(\rho \pi R_c^2)^i}{i!} e^{-\rho \pi R_c^2}$$

As the density of deployment increases, the probability that each node will have a degree of at least $K$, increases but the per-node coverage of the network decreases. Therefore the best coverage achieved by a random network that satisfies the constraint of $K$-degree with a high probability, say 0.95, will correspond to the smallest density for which $P(i \geq K) \geq 0.95$. Let this density be $\rho'$.

The coverage of a uniform random network is a function of its density and $R_s$. For a network with density $\rho'$ the normalized per node coverage is:

$$Coverage = \frac{1 - e^{-\rho' \pi R_s^2}}{\rho' \pi R_s^2}$$

Note that this expression is independent of the number of nodes in the network.

## VI. Coverage of Symmetrically Tiled Networks

We define a *Symmetrically Tiled Network* as one in which

- each node has exactly $K$ neighbors
- the distance between any two neighboring nodes is exactly $R_c$
- a node's neighbors are placed symmetrically around it

Figure 4 shows some network configurations that satisfy the above properties. A striking feature of these configurations is that the lines connecting neighboring nodes form regular polygons that tile the plane. For instance, in a network with $K = 3$ the angle between two neighbors of a node is $\frac{360^\circ}{K} = 120^\circ$ and therefore configuration represents a hexagonal tiling. However, such configurations are only possible for $K = 0, 1, 2, 3, 4$ and 6. For instance, if $K = 5$, then to form a symmetrically tiled configuration, the corresponding regular polygon should have an interior angle of $\frac{360^\circ}{K} = 72^\circ$ which is not possible.

Given a symmetrically tiled configuration, we can compute the per node coverage as a function of $R_s$ and $R_c$. If $R_c \geq 2R_s$, $Coverage = 1$ because there will be no overlap between the nodes. For the case when $R_c < 2R_s$ and $K = 1$, $Coverage = \frac{\left(\pi - \theta/2 + \sin(\theta/2)\right)}{\pi}$ where $\theta = \cos^{-1}(\frac{R_r}{2R_s})$. We can derive similar expressions for the other values of $K$.

It is our conjecture that the coverage of these Symmetrically Tiled networks is an upper bound on the coverage achievable by networks given the constraint of $K$-degree.

## VII. Experiments and Results

This section presents a set of experiments designed to study the performance of the proposed system for different values of the input parameters. The simulations were conducted using the Player/Stage[1] software platform which simulates the behavior of real sensors and actuators with a high degree of fidelity [13]. Each of the nodes in our simulations is capable of omni-directional motion and sensing (using a laser range finder). Further, each node has a retro-reflective beacon so that it can be distinguished from the obstacles in the environment. In most of our simulations we used a 2-d obstacle-less environment.

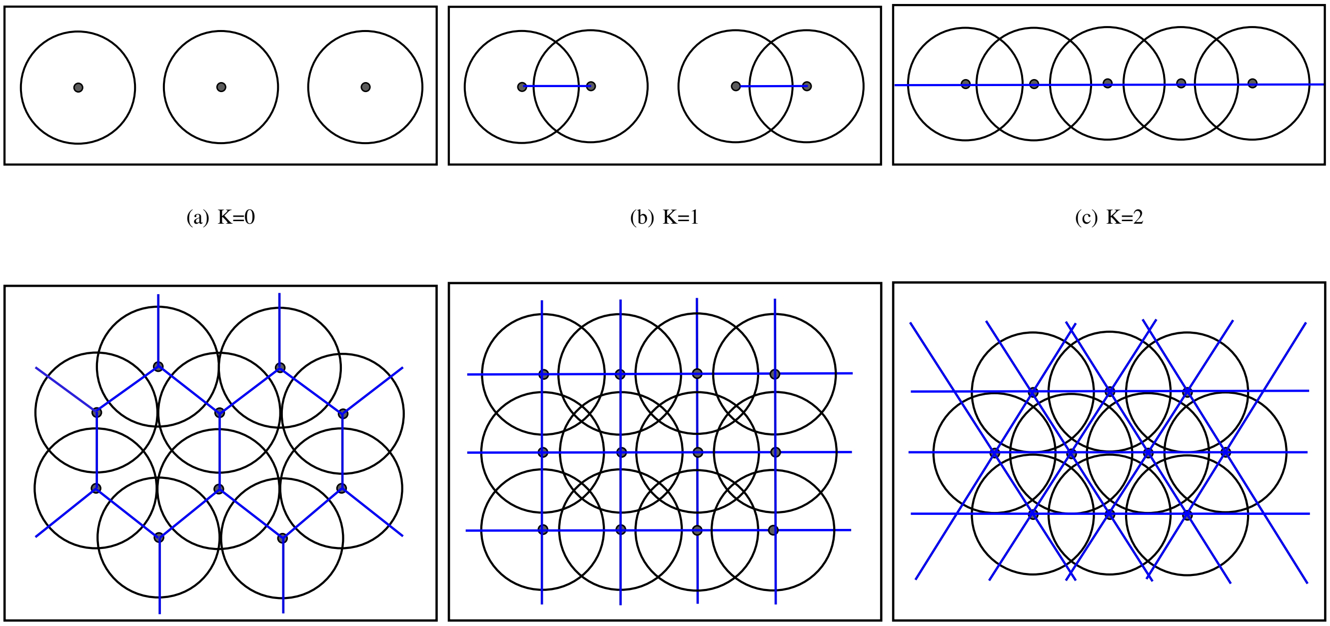
Figure 5 shows the initial and final network configuration for a typical deployment. The circles in the figure represent the coverage areas of individual nodes. The nodes start in a compact grid like configuration at the center of the environment and spread out to cover a large portion of the environment. In the particular instance shown, the sensing range of the nodes is equal to their communication range.

Figure 6 shows the variation of the coverage and average degree of the network with time for different values of $K$. The coverage (average degree) increases (decreases) rapidly in the first 1-2 minutes and then saturates to a stable value within 4-5 minutes. This is because, initially all the nodes have more than $K$ neighbors and so they spread out uninhibitedly to improve the coverage until the degree constraints activate and restrict their motion. Further, since these constraints activate at different stages for different values of $K$, the coverage (average degree) graphs for the different values of $K$, start off identically but branch off at different points and settle at different values of final coverage (degree).

Figures 7 and 8 compare the performance of our algorithm in terms of the coverage and average node degree with the uniform random and symmetrically tiled network configurations for three different regimes - $R_c > 2R_s$, $R_c = 2R_s$ and $R_c < 2R_s$. Clearly, the configurations we obtain outperform the random network. Note that while computing the coverage of the symmetrically tiled configurations we assume that the size of the network is infinite. The values thus obtained are in reality an upper bound on the coverage that can be achieved with finite networks because in the latter case, we will have to take into account the edge effects.

Our third performance metric (as discussed in section III) is the percentage of the nodes in the network that have a minimum degree of $K$. This we found was at least $95\%$ in all the network configurations resulting form our deployment algorithm. This is also the case with the random networks. Recall that while finding the density of deployment for the random network we explicitly imposed the constraint that each node should have at least $K$ neighbors with a probability of at least 0.95. In the symmetrically tiled configurations however this probability is 1 since all the nodes have exactly

[1] Player/Stage was developed jointly at the USC Robotics Research Labs and HRL Labs and is freely available under the GNU General Public License from http://playerstage.sourceforge.net



(a) K=0 (b) K=1 (c) K=2

(d) K=3 (e) K=4 (f) K=6

Fig. 4

THE SYMMETRICALLY TILED NETWORK CONFIGURATIONS FOR $R_s < R_r < 2R_s$

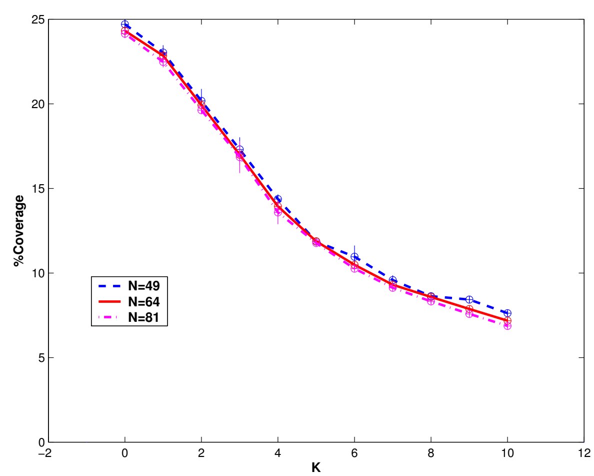



Fig. 9

VARIATION OF COVERAGE WITH NETWORK SIZE (N) FOR $R_s = 4$ AND $R_c = 8$ (AVERAGED OVER 10 TRIALS)

$K$ neighbors.

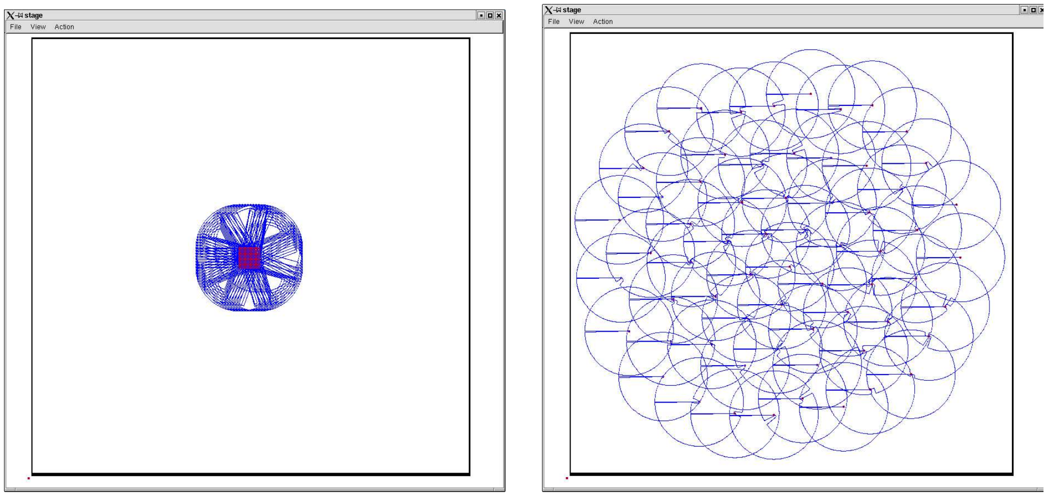
An appealing and unexpected result is that there is no significant change in the per-node coverage obtained when the size of the network $N$ was varied from 49 to 81 (Figure 9). One would expect that for smaller values of $N$ the edge effects will be more significant and therefore as $N$ increases the per-node coverage will increase. We speculate that either the edge effects do not vary significantly with the network size or a 49 node network is large enough to make edge effects negligible. In future, work we plan to fully characterize this relationship.

## VIII. CONCLUSIONS AND FUTURE WORK

We have presented a deployment algorithm for mobile sensor networks that is designed to maximize the collective sensor coverage while simultaneously constraining the degree of the network nodes. The pair-wise interaction between nodes is governed by two kinds of virtual forces - one causes the nodes to repel each other to improve their coverage and the other is an attractive force that prevents the nodes from losing connectivity. By using a combination of these two forces, every node tries to maximize its coverage while maintaining the required number of neighbors.

We have tested the algorithm extensively in simulation. Starting with configurations in which each node has a degree greater than the required degree $K$, the algorithm results in networks in which more than 95% of the nodes have a degree of at least $K$. Our analysis of uniform random and symmetrically tiled networks proves that the algorithm results in reasonably good coverage. We are working towards validating these results through experiments on real robots.

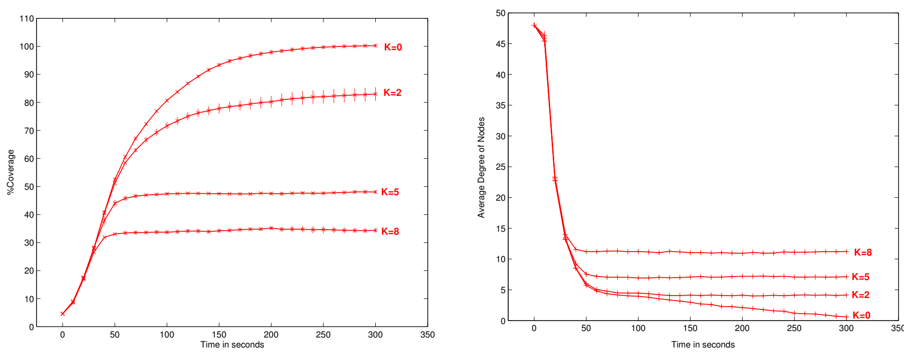
Possible criticisms of the algorithm are the strong assumptions it makes on the capabilities of the nodes - in particular the ability of each node to measure the exact range and bearing




(a) Initial Configuration (b) Final Configuration

Fig. 5

TYPICAL NETWORK CONFIGURATIONS FOR A 64 NODE DEPLOYMENT WITH K=2




(a) Coverage vs. Time (b) Average Node Degree vs. Time

Fig. 6

THE TEMPORAL PERFORMANCE FOR $N = 64$, $R_s = 4$ AND $R_c = 8$ (AVERAGED OVER 10 TRIALS)

of the neighboring nodes and obstacles. In future, we plan to extend the algorithm to work with approximate estimates of range and bearing readings.

The resulting network will reconfigure on addition of nodes. We would like it to be able to reconfigure even when some of the nodes cease to function (e.g. due to energy depletion) or are removed (e.g. due to malicious intervention). A simple solution could be that when a node has less than $K$ neighbors, it moves towards its closest neighbor till it gets connected to some of the neighbor's neighbors. This might result in it having more than $K$ neighbors - at which point the repulsive forces will cause the nodes to spread out again. We plan to explore this approach further.

## ACKNOWLEDGMENTS

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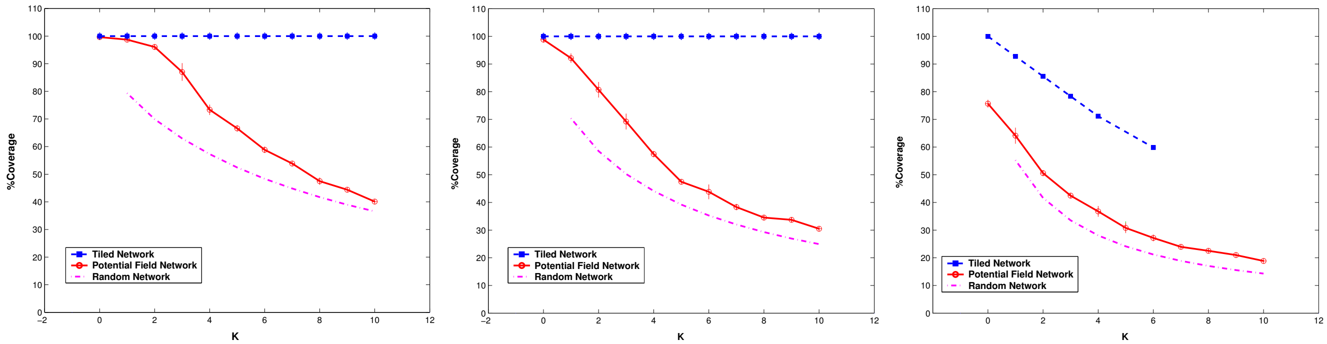
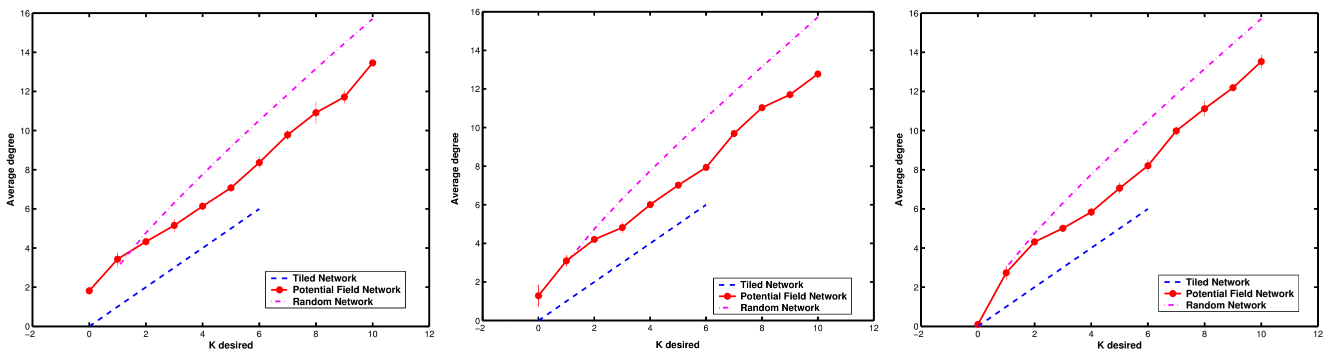



(a) $R_c = 10$ ($R_c > 2R_s$) (b) $R_c = 8$ ($R_c = 2R_s$) (c) $R_c = 6$ ($R_c < 2R_s$)

Fig. 7

COVERAGE FOR $N = 49$ AND $R_s = 4$ FOR DIFFERENT VALUES OF $R_c$ (AVERAGED OVER 10 TRIALS)




(a) $R_c = 10$ ($R_c > 2R_s$) (b) $R_c = 8$ ($R_c = 2R_s$) (c) $R_c = 6$ ($R_c < 2R_s$)

Fig. 8

AVERAGE DEGREE FOR $N = 49$ AND $R_s = 4$ FOR DIFFERENT VALUES OF $R_c$ (AVERAGED OVER 10 TRIALS)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