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# Temporal ERGMs

A simple extension

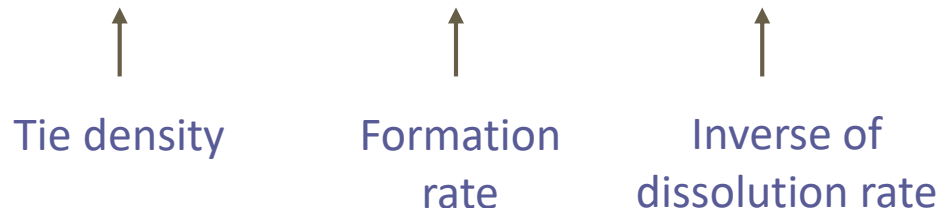
# Temporal ERGMs

- ERGMs are a general statistical framework for modeling cross-sectional network structure
  - A single ERGM predicts the *presence or absence* of a tie (and the static *prevalence* pattern of ties in the whole network)
  - Not the two dynamic processes of *tie formation* and *dissolution*
- To model network dynamics, we need temporal ERGMs (TERGMs)
- The TERGMs we are interested in can be represented by 2 ERGMs
  - One for the tie formation process
  - The other for the tie dissolution (or persistence) process

# The relationship between ERGMs & TERGMs

- Intuition: If tie density is low in the cross-section, is that because
  - Ties form infrequently? (the *tie incidence rate*)
  - Ties form frequently, but don't last long? (tie *duration*)
- Recall the classic approximation formula from epidemiology
  - The relationship between disease *prevalence*, *incidence* and *duration*
  - Applies in this context also:

$$\text{Prevalence} \approx \text{Incidence} \times \text{Duration}$$



# TERGMs: Core idea

- $Y$  (our network) is now indexed by time
- At each time step:
  - Evolution from  $Y_t$  to  $Y_{t+1}$  is a product of two processes:
    - Tie formation (on the empty dyads) and
    - Tie dissolution/persistence (on the tied dyads)
  - Each process is a draw from an ERGM
    - Two formulas: a formation formula and a dissolution/persistence formula
    - Two corresponding sets of statistics

# TERGMs

**ERGM:** Conditional log-odds of a tie *existing*

$$\text{logit}(P(Y_{ij} = 1 | \text{rest of the graph})) = \boldsymbol{\theta}' \boldsymbol{d}(\boldsymbol{g}(\boldsymbol{y}))$$

**TERGM:**

Conditional log-odds of a tie *forming*:

$$\text{logit}(P(Y_{ij,t+1} = 1 | Y_{ij,t} = 0, \text{rest of the graph})) = \boldsymbol{\theta}^{+'} \boldsymbol{d}(\boldsymbol{g}^{+}(\boldsymbol{y}))$$

Conditional log-odds of a tie *persisting*:

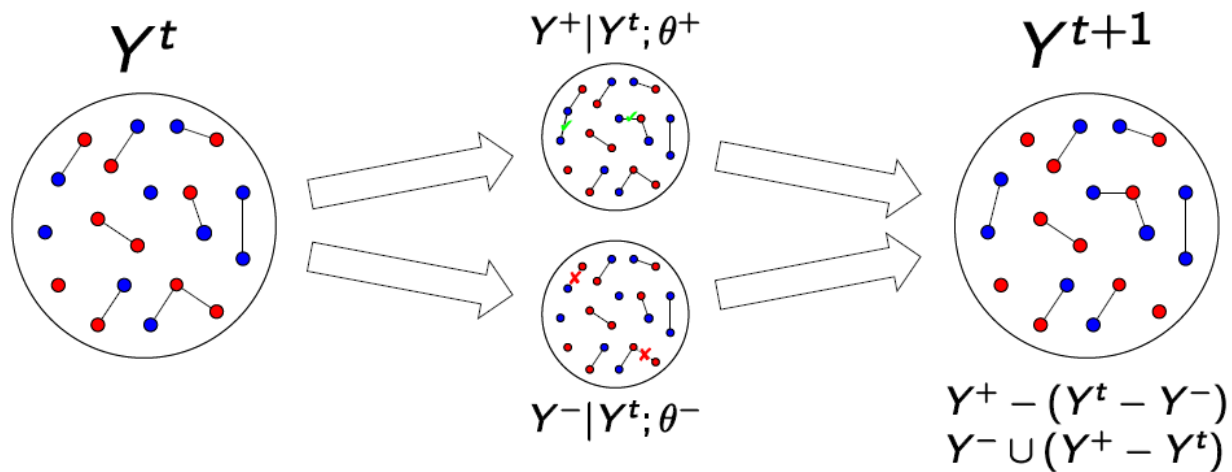
$$\text{logit}(P(Y_{ij,t+1} = 1 | Y_{ij,t} = 1, \text{rest of the graph})) = \boldsymbol{\theta}^{-'} \boldsymbol{d}(\boldsymbol{g}^{-}(\boldsymbol{y}))$$

where:

$\boldsymbol{g}^{+}(\boldsymbol{y})$	= vector of network statistics in the formation model
$\boldsymbol{\theta}^{+}$	= vector of parameters in the formation model
$\boldsymbol{g}^{-}(\boldsymbol{y})$	= vector of network statistics in the dissolution model
$\boldsymbol{\theta}^{-}$	= vector of parameters in the dissolution model

# These are separable TERGMs

- The two processes occur separately within a time step:



- $Y^+$  = network in the formation process after evolution
- $Y^-$  = network in the dissolution process after evolution

# Why separate formation from dissolution?

- **Intuition:** The determinants of tie formation are often different from those that influence dissolution.
- **Interpretation:** Because of this, we want model parameters that can be interpreted in terms of ties formed and ties dissolved. (We also need data that can allow us to estimate these).
- **Estimation and Simulation:** Data typically take the form of a single cross-sectional network with information on relational durations (prevalence and duration). Separability allows us to estimate incidence (formation) from prevalence and duration, and match this prevalence in simulation

# TERGMs: Example of interpretation

Term = ~edges

	$\theta$ is +	$\theta$ is –
Formation model	>50% of empty dyads have ties created during each timestep	<50% of empty dyads have ties created during each timestep
Dissolution (persistence) model	>50% of existing ties preserved (fewer dissolved); longer average duration	<50% of existing ties preserved (more dissolved); shorter average duration

Assuming time step is 1 day, what combo do you think is most common in long term partnership networks?



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# TERGMs: Example of interpretation

Term = `~concurrent` (# of nodes with degree 2+)

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Formation model	actors with exactly 1 tie are <i>more likely</i> than others to form a new tie	actors with exactly 1 tie are <i>less likely</i> than others to form a new tie
Dissolution (persistence) model	actors with 2 ties are <i>more likely</i> than others to have them persist	actors with 2 ties are <i>less likely</i> than others to have them persist

Assuming time step is 1 day, what combo do you think is most common in sexual partnership networks?

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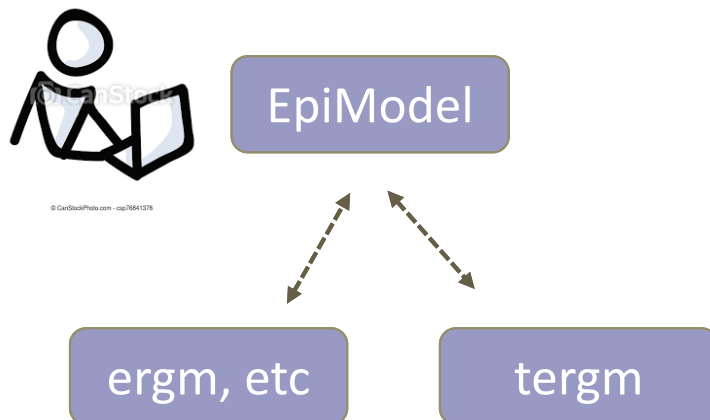
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Assuming time step is 1 day, what combo do you think is most common in sexual partnership networks?

Why 2, and not 2+ in the interpretation of dissolution ?

# No lab on tergm's now, but ...

- The Statnet package for temporal network models is called *tergm*
  - These temporal modeling tools are not implemented in `statnetWeb`
  - But TERGMs are the network model used in `EpiModel`
  - So you will learn to work with them via `EpiModel`, starting in module 4



EpiModel passes your code to the Statnet packages as needed, and gives you access to lots of additional functionality required for modeling epidemic and demographic processes

You can run these and other Statnet packages from R directly for standalone network analyses. See the online training materials at the [statnet.org](http://statnet.org) website for more information