3. (1) **homography**

$$\begin{pmatrix} v_x \\ v_y \\ 1 \end{pmatrix} \sim \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{pmatrix} \begin{pmatrix} u_x \\ u_y \\ 1 \end{pmatrix}$$

$$v_x = \frac{h_{11}u_x + h_{12}u_y + h_{13}}{h_{31}u_x + h_{32}u_y + 1}$$

$$v_y = \frac{h_{21}u_x + h_{22}u_y + h_{23}}{h_{31}u_x + h_{32}u_y + 1}$$

$$\Rightarrow \begin{aligned} &h_{11}u_x + h_{12}u_y + h_{13} - h_{31}u_x v_x - h_{32}u_y v_x = v_x \\ &h_{21}u_x + h_{22}u_y + h_{23} - h_{31}u_x v_y - h_{32}u_y v_y = v_y \end{aligned}$$

$$\begin{pmatrix} u_x & u_y & 1 & 0 & 0 & 0 & -u_x v_x & -u_y v_x \\ 0 & 0 & 0 & u_x & u_y & 1 & -u_x v_y & -u_y v_y \end{pmatrix} \begin{pmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \end{pmatrix} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

four points:

$$\begin{pmatrix} u_{1x} & u_{1y} & 1 & 0 & 0 & 0 & -u_{1x}v_{1x} & -u_{1y}v_{1x} \\ 0 & 0 & 0 & u_{1x} & u_{1y} & 1 & -u_{1x}v_{1y} & -u_{1y}v_{1y} \\ u_{2x} & u_{2y} & 1 & 0 & 0 & 0 & -u_{2x}v_{2x} & -u_{2y}v_{2x} \\ 0 & 0 & 0 & u_{1x} & u_{1y} & 1 & -u_{2x}v_{2y} & -u_{2y}v_{2y} \\ u_{3x} & u_{3y} & 1 & 0 & 0 & 0 & -u_{3x}v_{3x} & -u_{3y}v_{3x} \\ 0 & 0 & 0 & u_{1x} & u_{1y} & 1 & -u_{3x}v_{3y} & -u_{3y}v_{3y} \\ u_{4x} & u_{4y} & 1 & 0 & 0 & 0 & -u_{4x}v_{4x} & -u_{4y}v_{4x} \\ 0 & 0 & 0 & u_{1x} & u_{1y} & 1 & -u_{4x}v_{4y} & -u_{4y}v_{2y} \end{pmatrix} \begin{pmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \end{pmatrix} = \begin{pmatrix} u_{1x} \\ v_{1y} \\ v_{2x} \\ v_{2y} \\ v_{3x} \\ v_{3y} \\ v_{4x} \\ v_{4y} \end{pmatrix}$$

A h V

$$Ah = V$$