CS280 HW3 - Multi-view Reconstruction

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1 Fundamental Matrix

To calculate the fundamental matrix, we first normalize the matching points of the two images, and perform optimization to get the optimal F, lastly we do denormalization to make F consistent with the original coordinates of the matching points. The mathematical derivation for the three steps are attached in the next pages.

The optimization objective and the residual are different, the mathematical derivation are attached in the next pages.

Here we report the fundamental matrix and residual for "house" images are:

$$F = \begin{bmatrix} -3.12 \times 10^{-8} & 7.09 \times 10^{-7} & 6.73 \times 10^{-5} \\ 3.16 \times 10^{-6} & -2.72 \times 10^{-7} & -8.12 \times 10^{-3} \\ -5.17 \times 10^{-5} & 7.46 \times 10^{-3} & -5.20 \times 10^{-3} \end{bmatrix}$$
$$residual = 2.64 \times 10^{-6}$$

The fundamental matrix and residual for "library" images are:

$$F = \begin{bmatrix} -3.00 \times 10^{-8} & 5.96 \times 10^{-7} & -8.58 \times 10^{-5} \\ -3.38 \times 10^{-6} & -3.17 \times 10^{-8} & 6.22 \times 10^{-3} \\ 8.02 \times 10^{-4} & -5.59 \times 10^{-3} & -1.51 \times 10^{-1} \end{bmatrix}$$
$$residual = 1.00 \times 10^{-5}$$

$\boxed{\text{Normalization}}$

Suppose $u = (u_1, u_2)^T$

define $T = \frac{1}{\sigma}\begin{pmatrix} 1 & & -u_1 \\ & 1 & -u_2 \\ & & \sigma \end{pmatrix}$

$$T\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \frac{1}{\sigma}\begin{pmatrix} 1 & & -u_1 \\ & 1 & -u_2 \\ & & \sigma \end{pmatrix}\begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$= \frac{1}{\sigma}\begin{pmatrix} x - u_1 \\ y - u_2 \\ \sigma \end{pmatrix} = \begin{pmatrix} \frac{1}{\sigma}(x - u_1) \\ \frac{1}{\sigma}(y - u_2) \\ 1 \end{pmatrix}$$

so we normalize (x, y) to $\left(\frac{1}{\sigma}(x-u_1), \frac{1}{\sigma}(y-u_2)\right)$

$\boxed{\text{Denormalization}}$

$$F \leftarrow T_2^t F T_1$$

Optimization

The optimization problem to be solved is:

$$\min_f \|Af\|_2 \quad s.t. \quad \|f\|_2 = 1$$

$$A = U\Sigma V^T$$

$$\|Af\|_2 = \|U\Sigma V^T f\|_2 = \|\Sigma V^T f\|_2$$

$$= \|\Sigma f'\|_2 \qquad f' = V^T f \qquad \|f'\| = 1$$

$$\geq \lambda_q$$

$$\Sigma = \mathrm{diag}(\lambda_1, \cdots, \lambda_q) \qquad \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_q.$$

when $f' = (0, \cdots, 1)^T$ we have $\|Af\|_2 = \lambda_q$.

$$f = Vf' = (V_1 \cdots V_q)\begin{pmatrix} 0 \\ \vdots \\ 1 \end{pmatrix} = V_q \qquad \text{(last column of } V\text{)}$$

$F^* = V_q$.reshape(3, 3)

And then we can find its rank 2 approximation by Eckart-Young Theorem introduced in the problem description.

$\boxed{\text{Residual}}$

$$\text{residual} = \frac{1}{2n} \sum_{i=1}^{N} \left(\frac{|x_2^T F x_1|^2}{\|F x_1\|^2} + \frac{|x_1^T F^T x_2|^2}{\|F^T x_2\|^2} \right)$$

$$\Rightarrow \underset{F}{\arg\min} \sum_{i=1}^{N} |x_2^T F x_1|^2 \left(\frac{1}{\|F x_1\|^2} + \frac{1}{\|F^T x_2\|^2} \right) \quad \text{s.t } \|F\|_F = 1 \qquad (A)$$

the optimization problem we solved with SVD earlier is :

$$\underset{f}{\arg\min} \|Af\|_2 = \underset{f}{\arg\min} \|Af\|_2^2$$

$$= \underset{f}{\arg\min} \left\| \begin{pmatrix} a_1^T \\ a_2^T \\ \vdots \\ a_N^T \end{pmatrix} f \right\|_2^2$$

$$= \underset{f}{\arg\min} \left\| \begin{matrix} a_1^T f \\ \vdots \\ a_N^T f \end{matrix} \right\|_2^2$$

$$= \underset{f}{\arg\min} \sum_{i=1}^{N} |a_i^T f|^2$$

$$= \underset{F}{\arg\min} \sum_{i=1}^{N} |x_2^T F x_1|^2 \quad \text{s.t } \|F\|_F = 1 \qquad (B)$$

the two optimization problems are different

2 Find rotation translation

The possible translations and rotations for "house" images are:

$$t_1 = \begin{bmatrix} -0.99 & 0.02 & -0.02 \end{bmatrix}$$
$$t_2 = \begin{bmatrix} 0.99 & -0.02 & 0.02 \end{bmatrix}$$
$$R_1 = \begin{bmatrix} 0.99 & 0.02 & -0.09 \\ 0.03 & -0.99 & 0.00 \\ -0.09 & -0.01 & -0.99 \end{bmatrix}$$
$$R_2 = \begin{bmatrix} 0.98 & 0.06 & -0.15 \\ -0.07 & 0.99 & 0.00 \\ 0.15 & 0.01 & 0.98 \end{bmatrix}$$

The possible translations and rotations for "library" images are:

$$t_1 = \begin{bmatrix} -0.99 & 0.00 & 0.05 \end{bmatrix}$$
$$t_2 = \begin{bmatrix} 0.99 & 0.00 & -0.05 \end{bmatrix}$$
$$R_1 = \begin{bmatrix} 0.91 & 0.01 & -0.39 \\ 0.01 & -0.99 & 0.00 \\ -0.39 & 0.00 & -0.91 \end{bmatrix}$$
$$R_2 = \begin{bmatrix} 0.95 & 0.02 & -0.28 \\ -0.02 & 0.99 & 0.00 \\ 0.28 & 0.00 & 0.95 \end{bmatrix}$$

3 Find 3D points

To solve the 3D point for two matching points in two images, we solve a homogeneous linear system, which is defined in the next page. We solve the linear system using SVD, also described in the next page. In total we solve N sets of linear equations.

After we have the 3D points, we re-project each point back into the two images, and then calculate the distances between the original points and the re-projected points. Take the average over the two images and over N, we have the reconstruction error.

The mathematical derivation of the reconstruction error is attached in the next page.

The reconstruction error for "house" images is:

$$Error_{reconstruction} = 0.11$$

The reconstruction error for "library" images is:

$$Error_{reconstruction} = 0.12$$

$\boxed{\text{reconstruction error}}$:

$$\text{rec-err} = \frac{1}{2n} \sum_{i=1}^{N} \left[\| x_1 - \hat{x}_1 \|^2 + \| x_2 - \hat{x}_2 \|^2 \right]$$

x_1 : 2D point in image 1

x_2 : 2D point in image 2

$\hat{x}_1$: 3D reprojection to image 1

$\hat{x}_2$: 3D reprojection to image 2

$\hat{x}_1$: $P_1 X$ in non-homogenous coordinates

$\hat{x}_2$: $P_2 X$ in non-homogenous coordinates

$\boxed{\text{Homogenous linear system}}$

$$\begin{pmatrix} P_{31}^{(1)} x^{(1)} - P_{11}^{(1)} & P_{32}^{(1)} x^{(1)} - P_{12}^{(1)} & P_{33}^{(1)} x^{(1)} - P_{13}^{(1)} & P_{34}^{(1)} x^{(1)} - P_{14}^{(1)} \\ P_{31}^{(1)} y^{(1)} - P_{21}^{(1)} & P_{32}^{(1)} y^{(1)} - P_{22}^{(1)} & P_{33}^{(1)} y^{(1)} - P_{23}^{(1)} & P_{34}^{(1)} y^{(1)} - P_{24}^{(1)} \\ P_{31}^{(2)} x^{(2)} - P_{11}^{(2)} & P_{32}^{(2)} x^{(2)} - P_{12}^{(2)} & P_{33}^{(2)} x - P_{13}^{(2)} & P_{34}^{(2)} x^{(2)} - P_{14}^{(2)} \\ P_{31}^{(2)} y^{(2)} - P_{21}^{(2)} & P_{32}^{(2)} y^{(2)} - P_{22}^{(2)} & P_{33}^{(2)} y - P_{23}^{(2)} & P_{34}^{(2)} y - P_{24}^{(2)} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ W \end{pmatrix} = 0$$

$$B \begin{pmatrix} X \\ Y \\ Z \\ W \end{pmatrix} = 0$$

$$B = U \Sigma V^T$$

$\begin{pmatrix} X \\ Y \\ Z \\ W \end{pmatrix} \rightarrow$ column of V whose singular value is zero

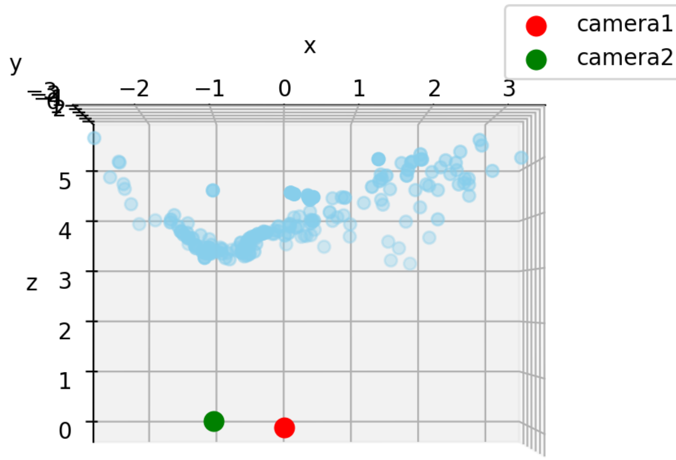

Figure 1: House 3D reconstruction

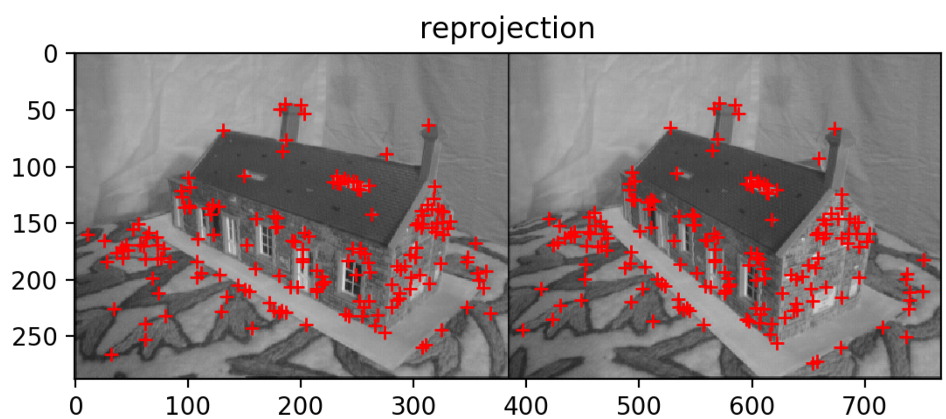

Figure 2: House re-projection

4 Plot 3D

Fig.1 is the 3D reconstruction for "house". Fig.2 shows the re-projection of 3D points into the two images for "house". Fig.3 is the 3D reconstruction for "library". Fig.4 shows the re-projection of 3D points into the two images for "library".

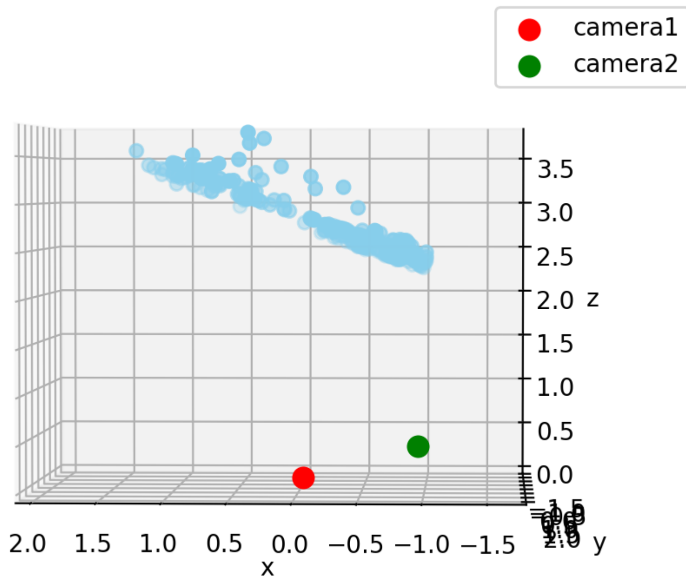

Figure 3: Library 3D reconstruction

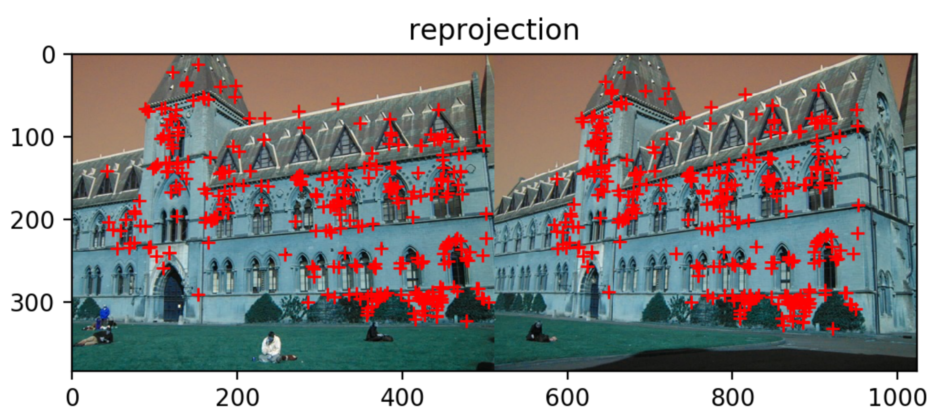

Figure 4: Library re-projection