2.(1) if $\hat{s} = \begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix}$ then $S = \begin{pmatrix} 0 & -s_3 & s_2 \\ s_3 & 0 & -s_1 \\ -s_2 & s_1 & 0 \end{pmatrix}$

Proof: $\vec{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

$$\begin{array}{cccccc} s_1 & s_2 & s_3 & s_1 & s_2 & s_3 \\ b_1 & b_2 & b_3 & b_1 & b_2 & b_3 \end{array}$$

$$\hat{s} \times \vec{b} = \begin{pmatrix} s_2 b_3 - b_2 s_3 \\ s_3 b_1 - b_3 s_1 \\ s_1 b_2 - b_1 s_2 \end{pmatrix} \qquad S\vec{b} = \begin{pmatrix} 0 & -s_3 & s_2 \\ s_3 & 0 & -s_1 \\ -s_2 & s_1 & 0 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} -b_2 s_3 + s_2 b_3 \\ b_1 s_3 - b_3 s_1 \\ -b_1 s_2 + b_2 s_1 \end{pmatrix}$$

(2) $R = \exp(\phi S)$

$$= I + \phi S + \frac{1}{2!}(\phi S)^2 + \frac{1}{3!}(\phi S)^3 + \cdots$$

$$= I + \phi S + \frac{1}{2!}\phi^2 S^2 - \frac{1}{3!}\phi^3 S - \frac{1}{4!}\phi^4 S^2 + \frac{1}{5!}\phi^5 S +$$

$$= I + \left(\phi S - \frac{1}{3!}\phi^3 S + \frac{1}{5!}\phi^5 S\right) + \left(\frac{1}{2!}\phi^2 S^2 - \frac{1}{4}\phi^4 S^2 + \cdots\right)$$

$$= I + \sin\phi S + (1 - \cos\phi) S^2$$

$$S^2 = \begin{pmatrix} 0 & -s_3 & s_2 \\ s_3 & 0 & -s_1 \\ -s_2 & s_1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -s_3 & s_2 \\ s_3 & 0 & -s_1 \\ -s_2 & s_1 & 0 \end{pmatrix} = \begin{pmatrix} -s_2^2 - s_3^2 & s_1 s_2 & s_1 s_3 \\ s_1 s_2 & -s_3^2 - s_1^2 & s_2 s_3 \\ s_1 s_3 & s_2 s_3 & -s_1^2 - s_2^2 \end{pmatrix}$$

$$= \begin{pmatrix} s_1^2 - 1 & s_1 s_2 & s_1 s_3 \\ s_1 s_2 & s_2^2 - 1 & s_2 s_3 \\ s_1 s_3 & s_2 s_3 & s_3^2 - 1 \end{pmatrix}$$

$$S^3 = \begin{pmatrix} 0 & -s_3 & s_2 \\ s_3 & 0 & -s_1 \\ -s_2 & s_1 & 0 \end{pmatrix} \begin{pmatrix} s_1^2 - 1 & s_1 s_2 & s_1 s_3 \\ s_1 s_2 & s_2^2 - 1 & s_2 s_3 \\ s_1 s_3 & s_2 s_3 & s_3^2 - 1 \end{pmatrix} = \begin{pmatrix} 0 & s_3 & -s_2 \\ -s_3 & 0 & s_1 \\ s_2 & -s_1 & 0 \end{pmatrix} = -S$$

$$S^4 = S^3 \cdot S = -S \cdot S = -S^2 \qquad S^5 = S^3 \cdot S^2 = -S \cdot S^2 = -S^3 = S$$