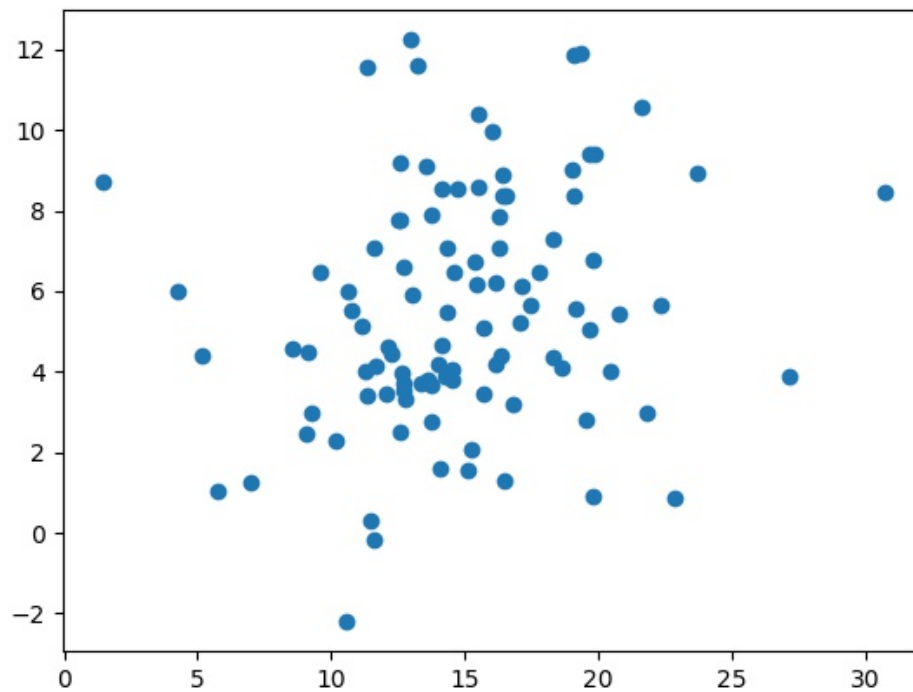
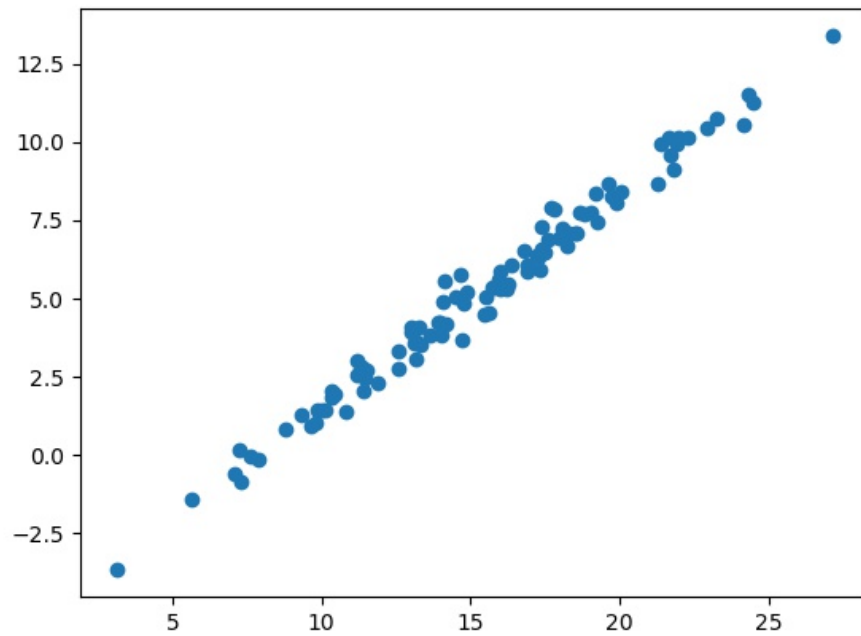


2c

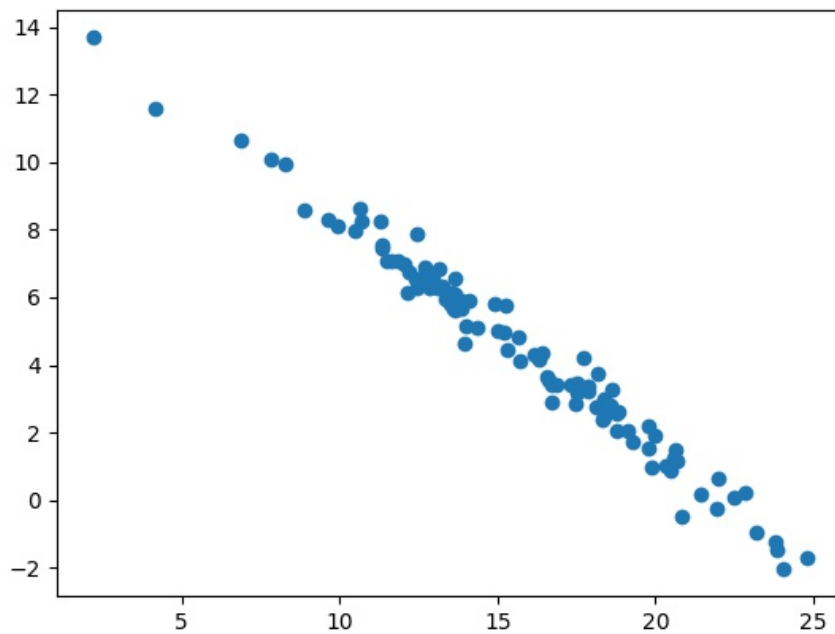
when $\text{sigma} = \begin{bmatrix} 20 & 0 \\ 0 & 10 \end{bmatrix}$, the MLE mean is $[14.62972319 \ 5.13279893]$, and
MLE covariance is $\begin{bmatrix} 13.93459582 & 2.83012681 \\ 2.83012681 & 9.93336424 \end{bmatrix}$
the singular values for sigma matrix is $[20. \ 10.]$



when $\Sigma = \begin{bmatrix} 20 & 14 \\ 14 & 10 \end{bmatrix}$, the MLE mean is $\begin{bmatrix} 14.98469043 & 5.04061468 \end{bmatrix}$,
and MLE covariance is $\begin{bmatrix} 20.46621341 & 14.35034222 \\ 14.35034222 & 10.23210116 \end{bmatrix}$
the singular values for sigma matrix is $\begin{bmatrix} 29.86606875 & 0.13393125 \end{bmatrix}$



when $\text{sigma} = \begin{bmatrix} 20 & -14 \\ -14 & 10 \end{bmatrix}$, the MLE mean is $\begin{bmatrix} 14.35926801 & 5.38991362 \end{bmatrix}$,
and MLE covariance is $\begin{bmatrix} 16.75876404 & -11.60036104 \\ -11.60036104 & 8.29960636 \end{bmatrix}$
the singular values for sigma matrix is $\begin{bmatrix} 29.86606875 & 0.13393125 \end{bmatrix}$



```

import numpy as np
import matplotlib.pyplot as plt

def mean(samples):
    return np.average(samples, axis = 0)

def covariance(samples, mu_hat):
    n = samples.shape[0]
    d = samples.shape[1]
    mu_hat = mu_hat.reshape((d, 1))
    return (samples.T-mu_hat) @ (samples.T-mu_hat).T/n

if __name__ == '__main__':
    mu = [15, 5]
    sigmas = [[[20, 0], [0, 10]], [[20, 14], [14, 10]], [[20, -14], [-14, 10]]]
    for i, sigma in enumerate(sigmas):
        samples = np.random.multivariate_normal(mu, sigma, size=100)
        mu_hat = mean(samples)
        sigma_hat = covariance(samples, mu_hat)
        print('when sigma = {}, the MLE mean is {}, and MLE covariance is
        {}'.format(sigma, mu_hat, sigma_hat))
        print('the singular values for sigma matrix is
        {}'.format(np.linalg.svd(sigma, compute_uv = False)))
        fig = plt.figure()
        plt.scatter(samples[:, 0], samples[:, 1])
        plt.savefig('sigma{}.jpg'.format(i))

```