Taylor polynomials about (a,b) are polynomials in $(x-a)$ and $(y-b)$.

When using known Taylor polynomials: the "variable" must "contain" $(x-a)$ and $(y-b)$.

Ex: (ex sheet #17 Q2) $e^x\sqrt{y}$ about $(0,1)$:

$$e^x\sqrt{y} = e^x(1+(y-1))^{1/2}$$

$$= \underbrace{\left(1+x+\frac{x^2}{2!}+\cdots\right)}_{\text{powers of } x-0}\underbrace{\left(1+\frac{1}{2}(y-1)+\frac{1}{2!}\frac{1}{2}\left(\frac{-1}{2}\right)(y-1)^2+\cdots\right)}_{\text{powers of } y-1} = \cdots$$

Ex: $e^x\sqrt{y}$ about $(2,1)$:

$$e^x\sqrt{y} = e^2e^{x-2}(1+(y-1))^{1/2} = e^2\left(1+(x-2)+\frac{(x-2)^2}{2!}+\cdots\right)\left(1+\frac{1}{2}(y-1)+\frac{1}{2!}\frac{1}{2}\left(\frac{-1}{2}\right)(y-1)^2+\cdots\right)$$

$$= \cdots$$

need powers of $x-2$.
i.e. variable needs to be $x-2$

Equivalent method:

let $u = x-2$ $\quad v = y-1$

$u+2 = x$ $\quad v+1 = y$

aim for a polynomial in u and v

$$e^{u+2}\sqrt{1+v} = e^2e^u\sqrt{1+v}$$

$$= e^2\left(1+u+\frac{u^2}{2!}+\cdots\right)\left(1+\frac{1}{2}v+\frac{1}{2!}\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)v^2+\cdots\right)$$

$$= e^2+e^2u+\frac{1}{2}e^2v+\cdots = e^2+e^2(x-2)+\frac{1}{2}e^2(y-1)+\cdots$$