Example: Find the vector and scalar parametric equations for the line through $(1, 0, -1)$ parallel to $-\mathbf{i} - \mathbf{j} + 3\mathbf{k}$, and sketch this line.

vector parametric:

$$\vec{r} = (\vec{i} + 0\vec{j} - \vec{k}) + t(-\vec{i} - \vec{j} + 3\vec{k})$$

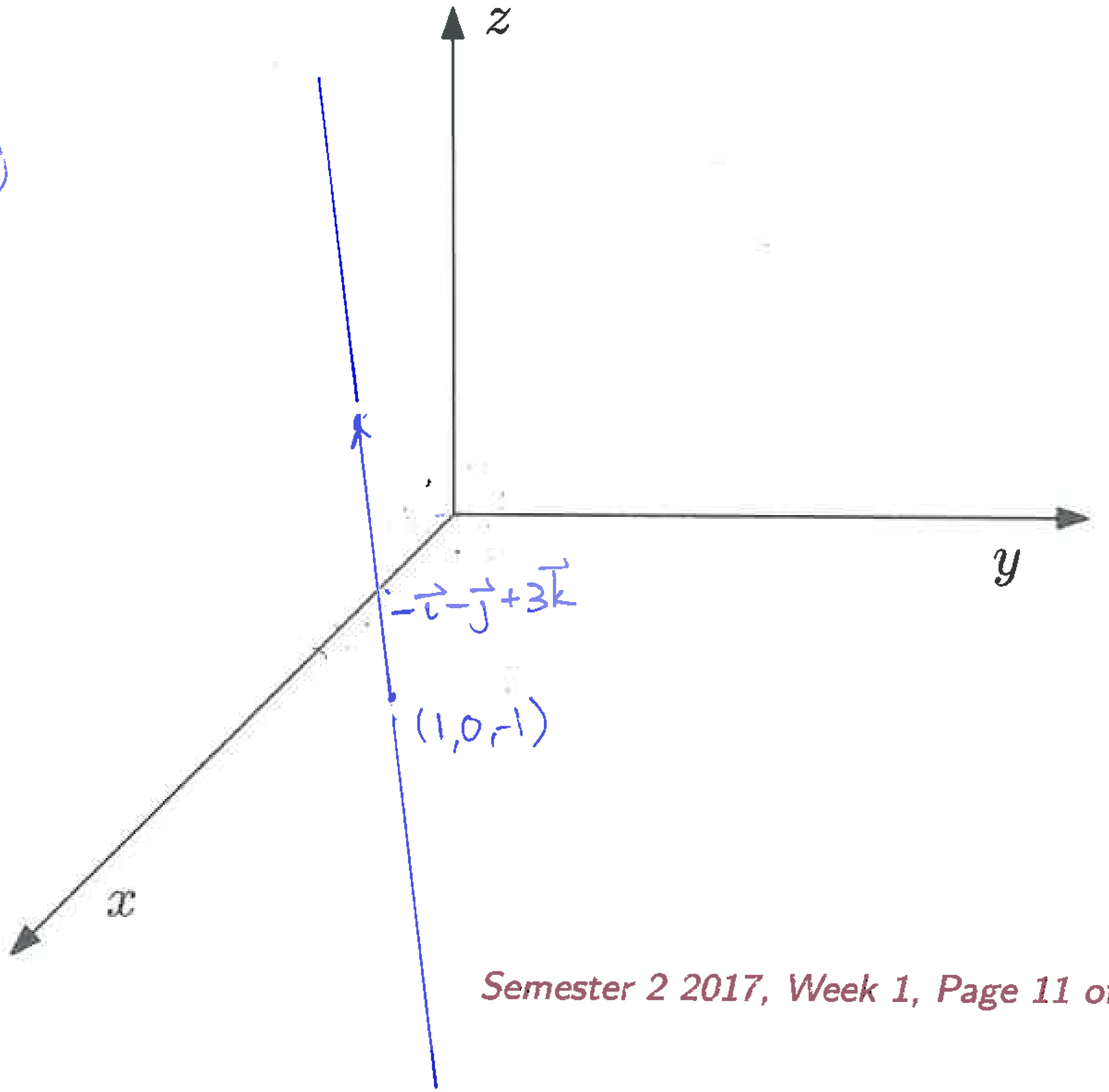
$$= (1-t)\vec{i} - t\vec{j} + (-1+3t)\vec{k}$$

scalar parametric: $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

$x = 1 - t$

$y = -t$

$z = -1 + 3t$

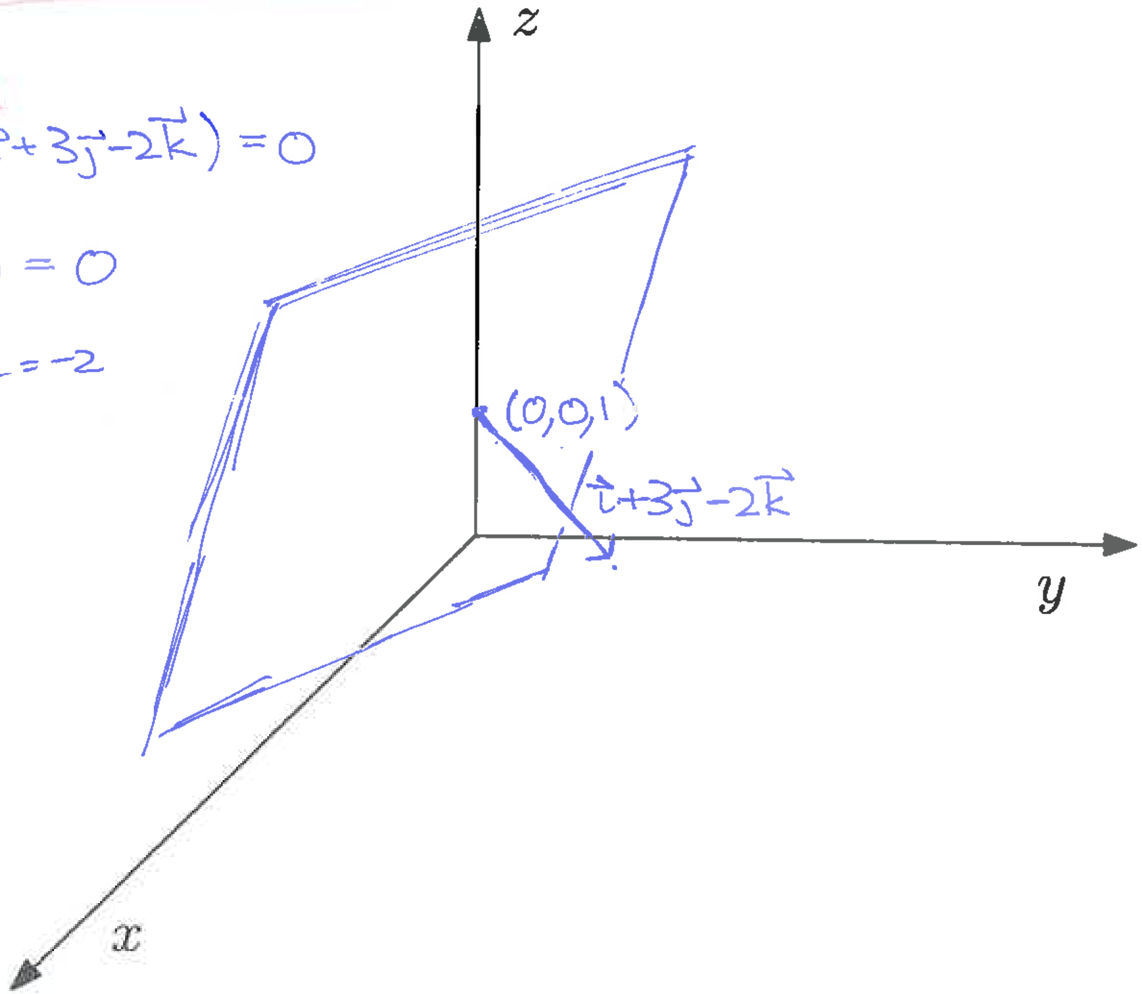


Example: Find the standard form of the plane through $\underbrace{(0,0,1)}_{}\overset{}{}$ $\left(\overset{P_0}{\parallel}\right)$ with normal vector $\underbrace{\mathbf{i}+3\mathbf{j}-2\mathbf{k}}_{\vec{n}}$, and sketch this plane.

$$\left((x\vec{i}+y\vec{j}+z\vec{k})-(0\vec{i}+0\vec{j}+\vec{k})\right)\cdot(\vec{i}+3\vec{j}-2\vec{k})=0$$

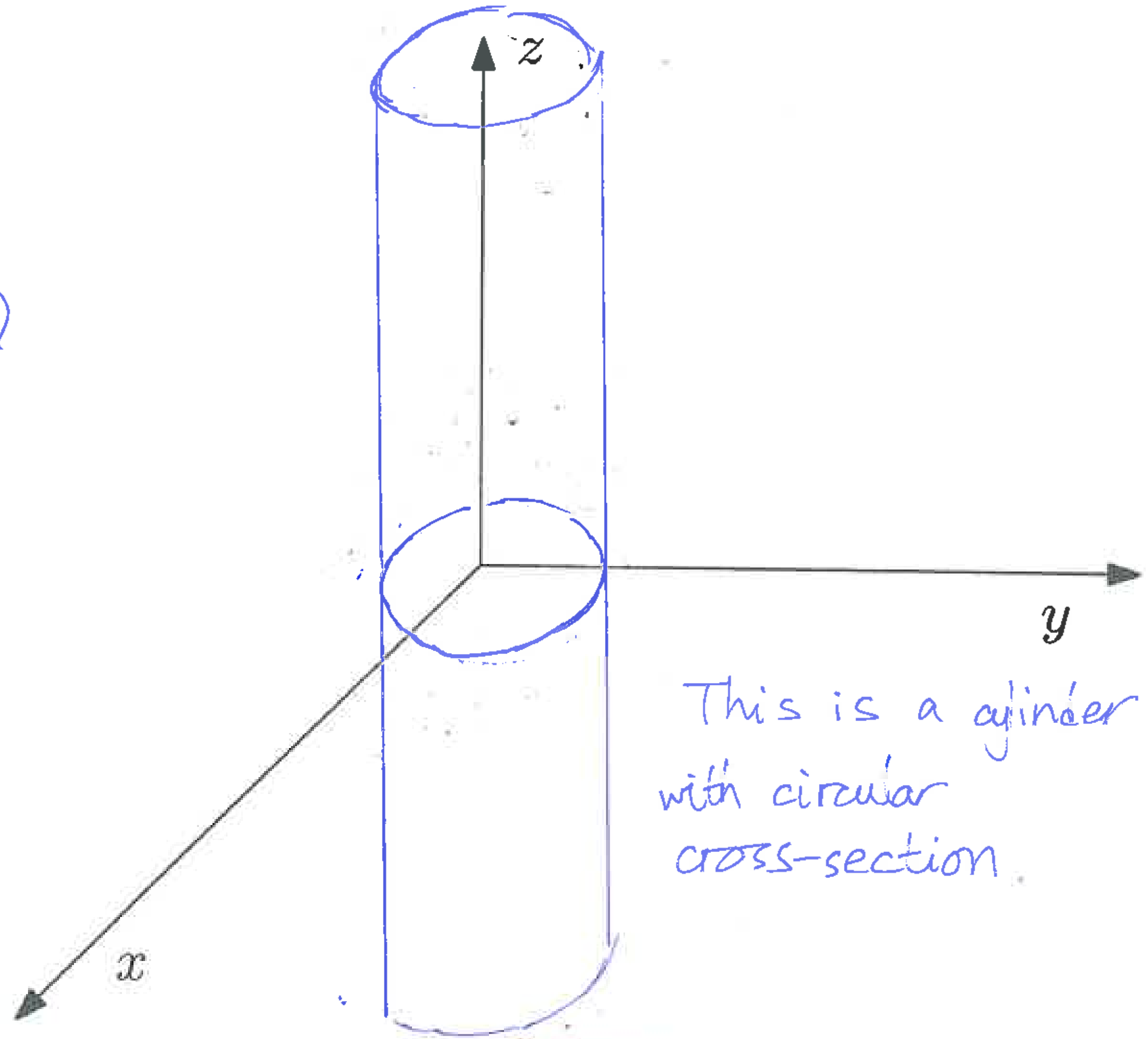
$$(x-0)1+(y-0)3+(z-1)(-2)=0$$

$$x+3y-2z=-2$$



Example: Describe and sketch the set in $\mathbb{R}^3$ satisfying $x^2 + y^2 = 1$.

z does not appear in the equation, so the set is "parallel" to the z-direction



This is a cylinder with circular cross-section.

Example: Describe and sketch the set satisfying $y^2 + 4z^2 = 4$.

x does not appear

$y^2 + 4z^2 = 4$

$y^2 + (2z)^2 = 2^2$

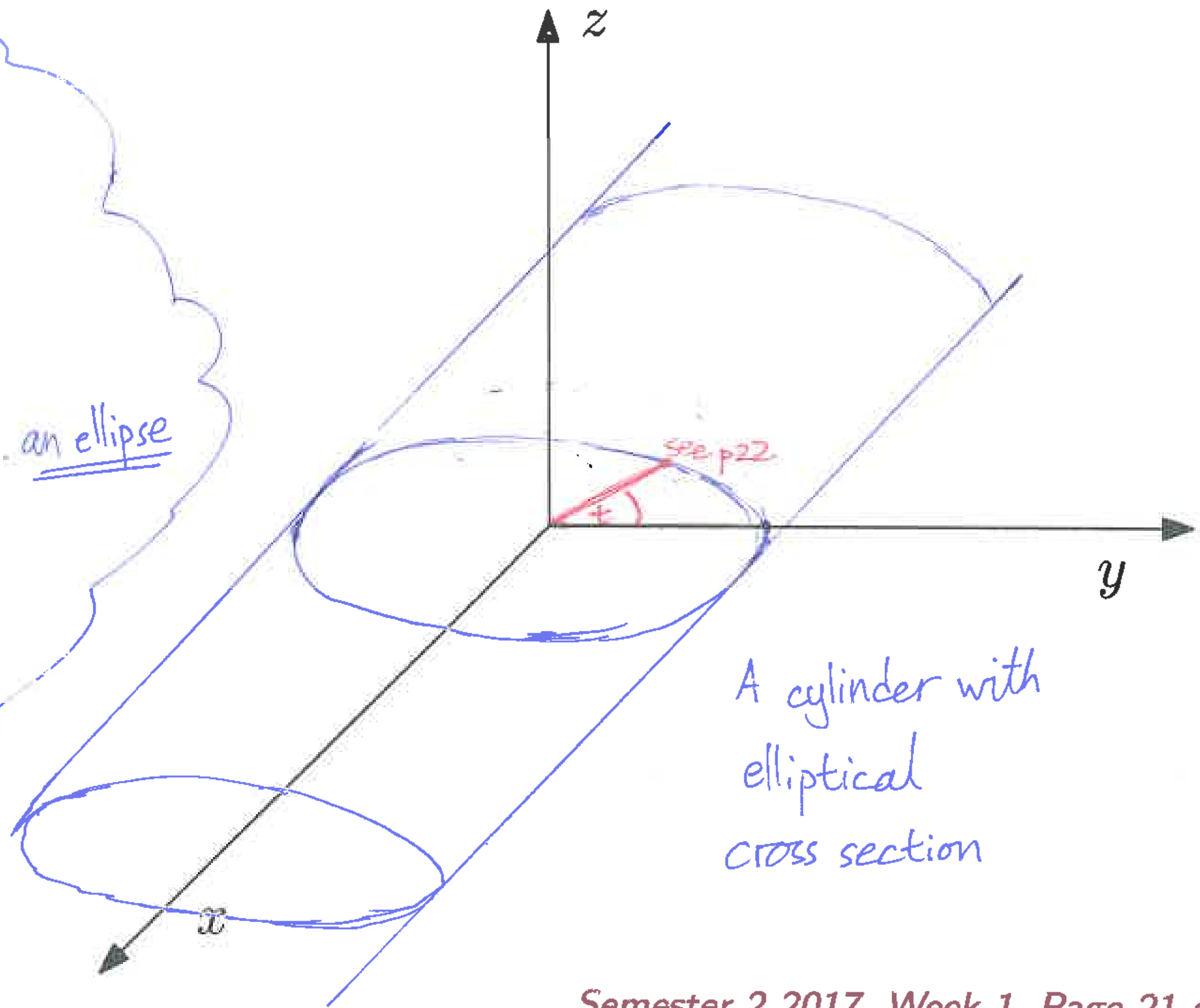
a circle of radius 2, then replace z with $2z$. i.e. an ellipse

i.e. contracts by 2 in the z-direction.

if you don't know whether it stretches or contracts:

take $y=0$: $z = \pm 1$

$z=0$: $y = \pm 2$



see p22

A cylinder with elliptical cross section

Recall that it is useful to parametrise a surface, i.e. write x, y, z explicitly as functions of a parameter.

Example: Parametrise the cylinder $y^2 + 4z^2 = 4$.

$y^2 + (2z)^2 = 2^2$

We cannot solve for y or z in terms of the other variable (taking square-root only gives positive values)

because: $\cos^2 t + \sin^2 t = 1$

$(2\cos t)^2 + (2\sin t)^2 = 2^2$

so $y = 2\cos t$

$2z = 2\sin t \longrightarrow z = \sin t$ } works

$$\boxed{\begin{aligned} x &= x \\ y &= 2\cos t \\ z &= \sin t \end{aligned}}$$

The next simplest quadric surface is when one of the variables only has degree 1.

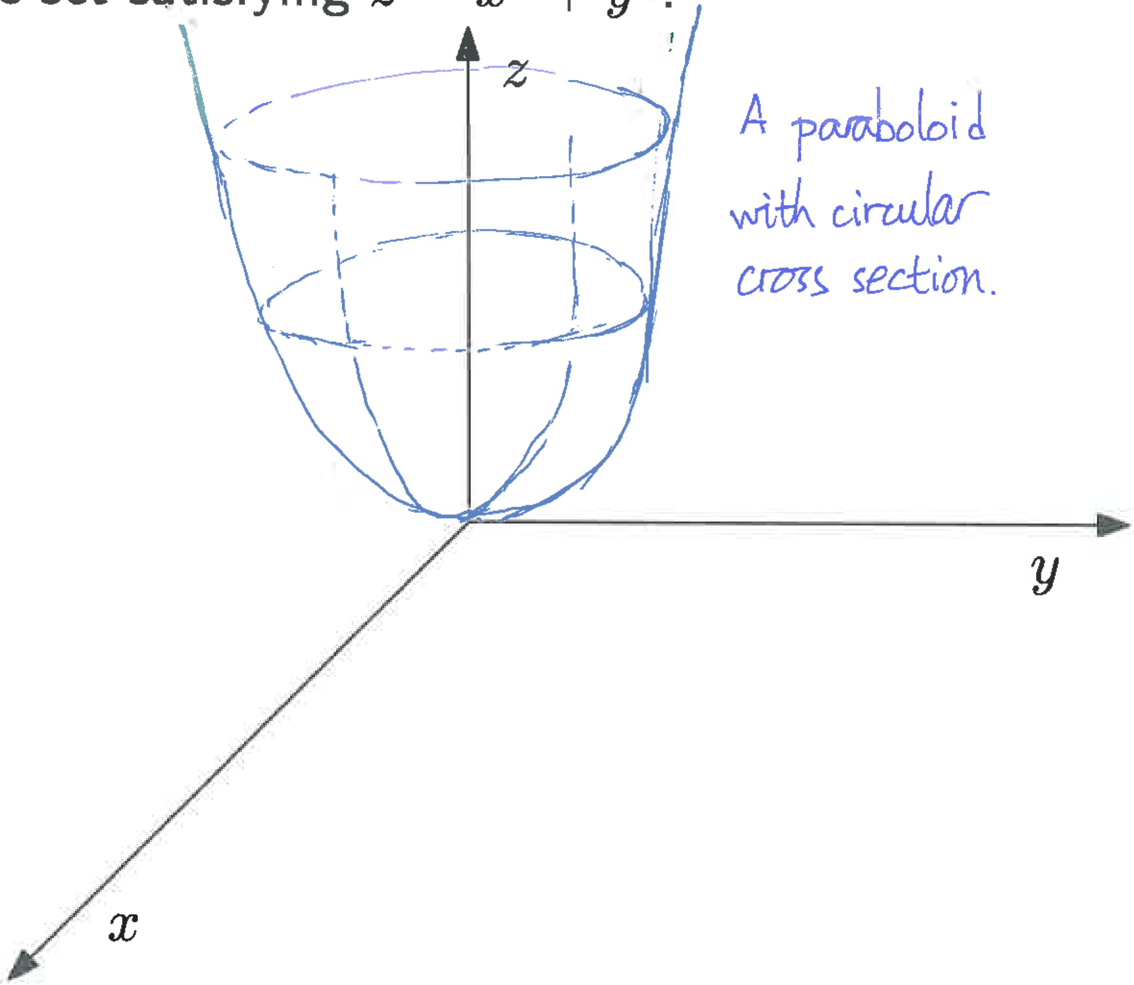
Example: Describe and sketch the set satisfying $z = x^2 + y^2$.

Take cross-section:

when $y=0$: $z = x^2$
when $x=0$: $z = y^2$
} parabolas

($z=0$: not useful $- x=y=0$)

when $z=C$: $C = x^2+y^2$
circle



A paraboloid with circular cross section.

Example: Describe and sketch the set satisfying $y = x^2 - 2x + z^2$.

When there is a linear and a quadratic term in the same variable: first complete the square.

$$y = \underline{(x-1)}^2 + z^2 \underline{-1}$$

$\uparrow$ move by 1 in x-direction

$\leftarrow$ move by 1 in y-direction

To find out which direction (+ or −), make both squares into 0:

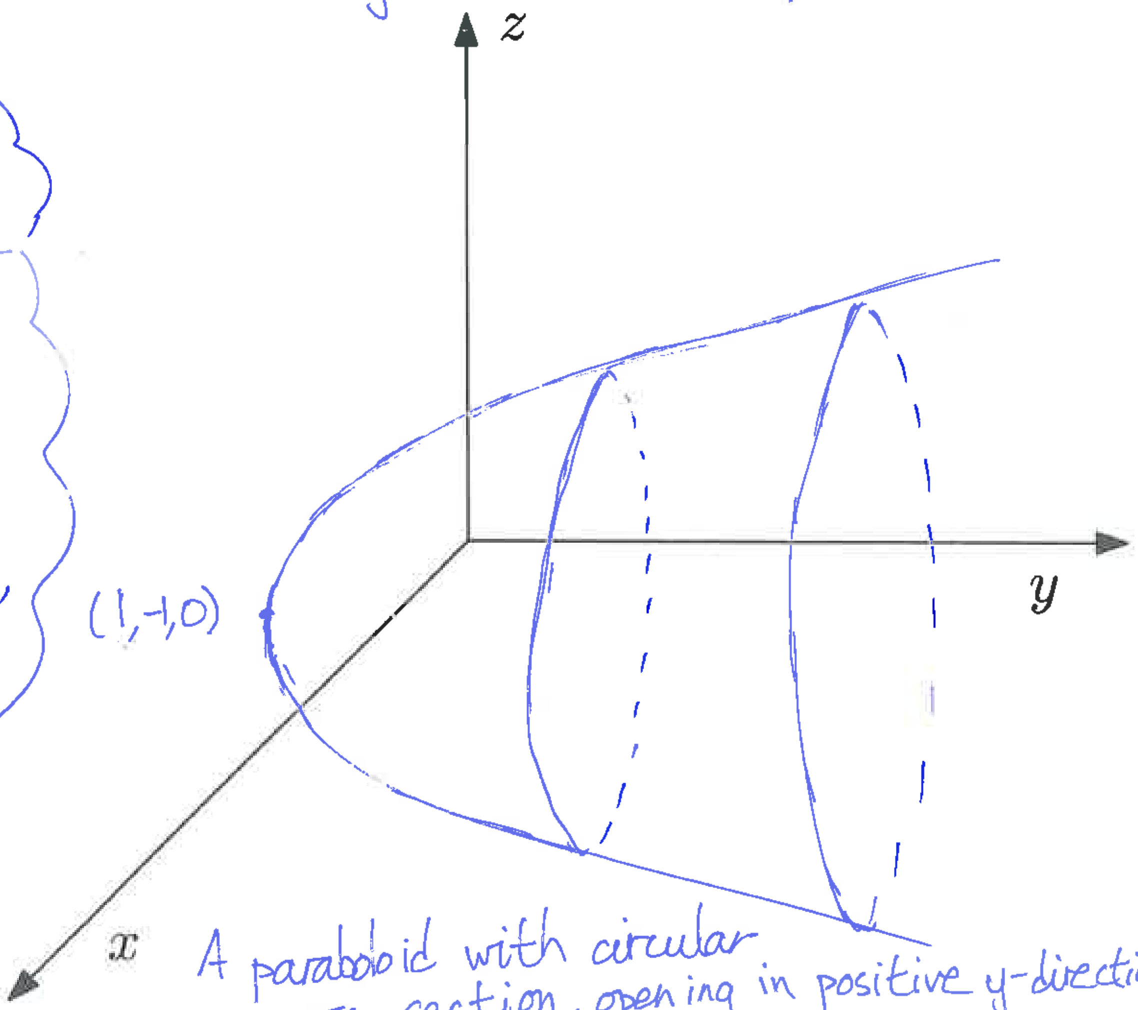
$x - 1 = 0 \rightarrow x = 1$

$z = 0$

$y = 0^2 + 0^2 - 1 \rightarrow y = -1$

$(1, -1, 0)$

$y = (x-1)^2 + z^2 - 1$



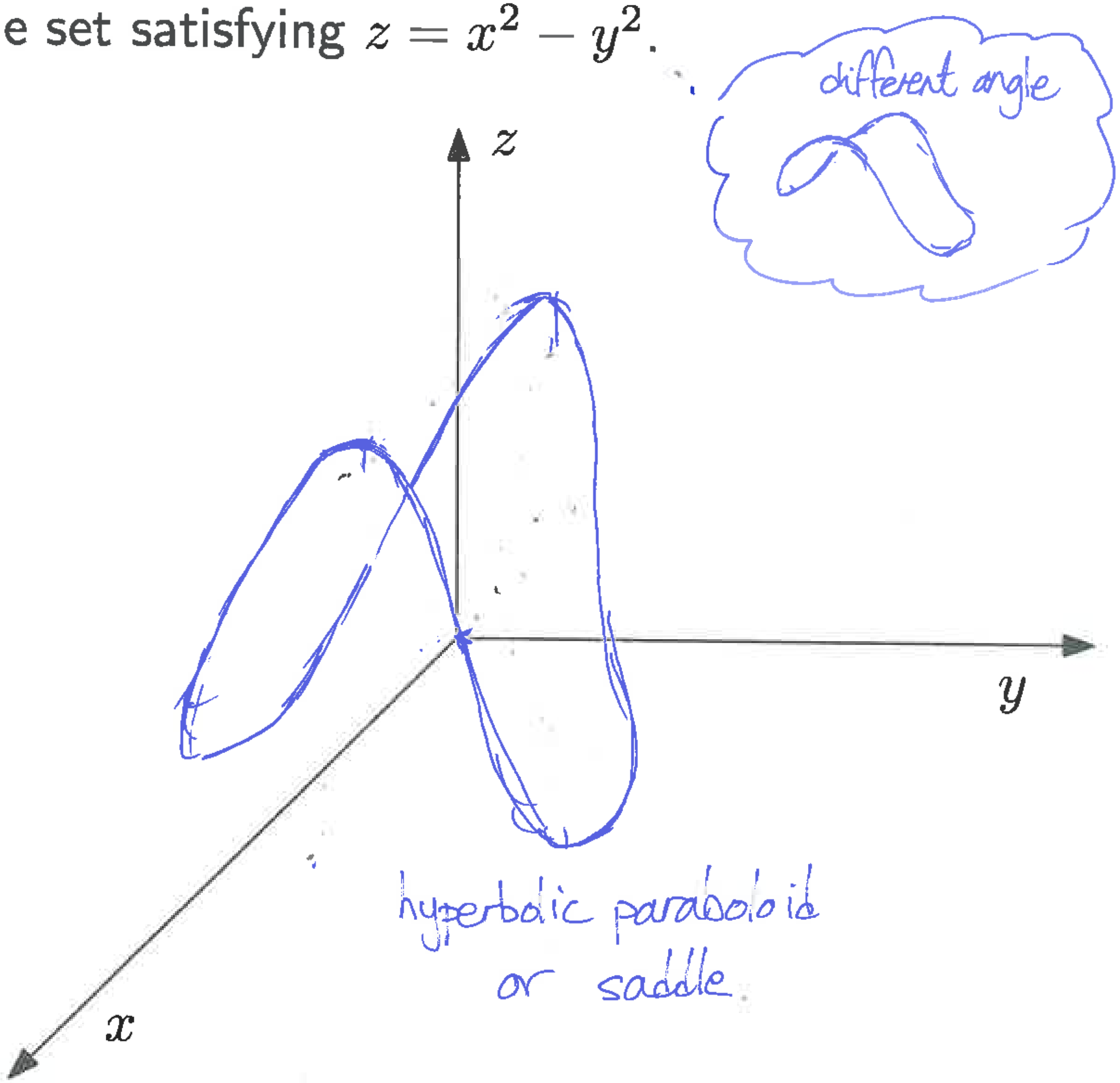
A paraboloid with circular cross section, opening in positive y-direction.

Example: Describe and sketch the set satisfying $z = x^2 - y^2$.

Cross sections:

when $y = 0$: $z = x^2$

when $x = 0$: $z = -y^2$



hyperbolic paraboloid or saddle.