§1.1: Systems of Linear Equations

Example of a linear equation: $y = 5x + 2$

Can be rearranged to give: $(-5)x + (1)y = 2$

Another example: $3(x_1 + 2x_2) + 1 = x_1 + 1$

Rearrangement: $(2)x_1 + (6)x_2 = 0$

Another example: $x_2 = \sqrt{2}(\sqrt{6} - x_1) + x_3$

Rearrangement: $\sqrt{2}x_1 + (1)x_2 + (-1)x_3 = 2\sqrt{3}$

Not linear: $x_2 = 2\sqrt{x_1}$

$xy + x = 5$

In general, a linear equation is an equation of the form

$$a_1x_1 + a_2x_2 + \ldots a_nx_n = b.$$

$x_1, x_2, \ldots x_n$ are the variables.

$a_1, a_2, \ldots a_n$ are the coefficients.

Definition: A system of linear equations (or a linear system) is a collection of linear equations involving the same set of variables.

Example:

$$\begin{array}{rcrcr} x & + & y & = & 3 \\ 3x & & +2z & = & -2 \end{array}$$

This is a system of 2 equations in 3 variables, x, y, z.

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Example:

$$\begin{array}{rcrcl} x & & +y & = & 3 \\ 3x & & +2z & = & -2 \end{array}$$

Definition: A solution of a linear system is a list $(s_1, s_2, \ldots, s_n)$ of numbers that makes each equation a true statement when the values $s_1, s_2, \ldots, s_n$ are substituted for $x_1, x_2, \ldots, x_n$ respectively.

Example: A solution to the example above is $(2, 1, -4)$. (More clearly: $x = 2, y = 1, z = -4$.)

Definition: The solution set of a linear system is the set of all possible solutions.

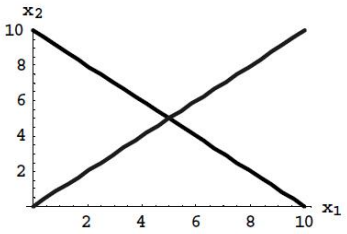
Definition: A linear system is *consistent* if it has a solution, and *inconsistent* if it does not have a solution.

Fact: A linear system has either

- exactly one solution — consistent
- infinitely many solutions — consistent
- no solutions — inconsistent

EXAMPLE Two equations in two variables:

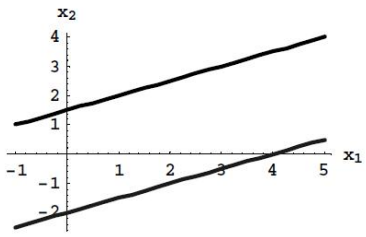
$$\begin{aligned} x_1 + x_2 &= 10 \\ -x_1 + x_2 &= 0 \end{aligned}$$



one unique solution

consistent

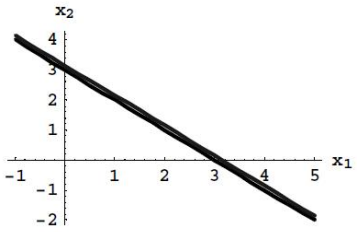
$$\begin{aligned} x_1 - 2x_2 &= -3 \\ 2x_1 - 4x_2 &= 8 \end{aligned}$$



no solution

inconsistent

$$\begin{aligned} x_1 + x_2 &= 3 \\ -2x_1 - 2x_2 &= -6 \end{aligned}$$



infinitely many solutions

consistent