

You must justify your answers to receive full credit.

1. Find the intersection of the planes

$$x + 7y - 5z = 10$$

$$x + 4y - 2z = 7$$

$$x + 6y - 4z = 9$$

and describe this intersection geometrically.

2. Consider a linear system whose augmented matrix has the form

$$\left[ \begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & a & b \end{array} \right]$$

- a) For what values of  $a$  and  $b$  will the system have infinitely many solutions?  
b) For what values of  $a$  and  $b$  will the system be inconsistent?
3. Find a polynomial of degree 3 (a polynomial of the form  $f(t) = a + bt + ct^2 + dt^3$ ) whose graph passes through the points  $(0, 1)$ ,  $(1, 0)$ ,  $(-1, 0)$  and  $(2, -15)$ .

4. For what values of  $a$  is  $\begin{bmatrix} 8 \\ 12 \\ a \end{bmatrix}$  in the span of  $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 6 \end{bmatrix} \right\}$ ?

5. Let

$$A = \begin{bmatrix} | & | & | \\ \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \\ | & | & | \end{bmatrix} = \begin{bmatrix} 0 & 1 & -4 \\ 0 & 0 & -2 \\ 1 & 2 & 3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} -2 \\ -2 \\ 13 \end{bmatrix}$$

- a) Is  $\mathbf{b}$  in  $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ ? How many vectors are in  $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ ?  
b) Is  $\mathbf{b}$  in  $\text{Span}\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ ? How many vectors are in  $\text{Span}\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ ?  
c) Is  $\mathbf{a}_1$  in  $\text{Span}\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ ? Explain your answer.
6. State whether each of the following statements is always true or sometimes false. If it is true, give a brief justification (e.g. by referring to results from the textbook or from class); if it is false, give a counterexample with an explanation.
- a) If  $\mathbf{v}_3$  is in  $\text{Span}\{\mathbf{v}_1\}$ , then  $\mathbf{v}_3$  is in  $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$ .

- b) If  $\mathbf{v}_3$  is in  $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$ , then  $\mathbf{v}_3$  is in  $\text{Span}\{\mathbf{v}_1\}$ .
- c) If  $A\mathbf{x} = \mathbf{b}$  is an inconsistent system, then  $\text{rref}(A)$  contains a row of zeros.
- d) If  $\text{rref}(A)$  contains a row of zeros, then  $A\mathbf{x} = \mathbf{b}$  is an inconsistent system.
- e) A system of 3 equations in 4 variables cannot have a unique solution.
- f) If  $A$  is a  $4 \times 6$  matrix such that  $\text{rref}(A)$  has a pivot in each row, then every  $b$  in  $\mathbb{R}^4$  is a linear combination of the columns of  $A$ .

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