1. (3 points) Approximate the integral

$$\int_{-12}^{12} \sqrt{1+\sin x}dx$$

by a left Riemann sum with 3 subintervals.

$\Delta x = \frac{12-(-12)}{3} = 8$

$x_0 = -12$

$x_1 = -4$

$x_2 = 4$

$x_3 = 12$

$8\sqrt{1+\sin(-12)} + 8\sqrt{1+\sin(-4)} + 8\sqrt{1+\sin(4)}$

2. (4 points) Find the derivative of the function:

$$h(x) = \int_{e^x}^{e^{2x}} \arctan(\sqrt{t})dt.$$

Let $F(x)$ be an antiderivative of $\arctan\sqrt{t}$. $\arctan\sqrt{t}$ is continuous for $t>0$, so it is continuous on $[e^x, e^{2x}]$ for any x. $\therefore$ by FTC2, $h(x) = F(e^{2x}) - F(e^x)$.

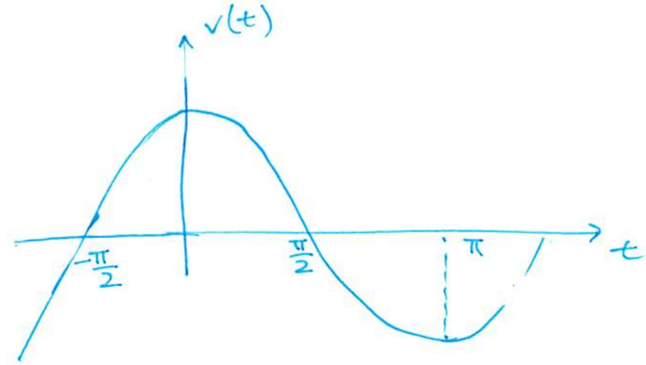
$\therefore$ by chain rule, $h'(x) = F'(e^{2x})\frac{d}{dx}(e^{2x}) - F'(e^x)\frac{d}{dx}(e^x)$

$= \arctan\sqrt{e^{2x}}\, 2e^{2x} - \arctan\sqrt{e^x}\, e^x.$

3. (5 points) The velocity of a particle at time t is given by the function

$$v(t) = \cos t.$$

Find the total distance travelled by the particle from $t = -\frac{\pi}{2}$ to $t = \pi$. **Simplify your answer as much as possible.**



From graph, $v(t) \geq 0$ on $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$v(t) \leq 0$ on $[\frac{\pi}{2}, \pi]$

so distance travelled $= \int_{-\frac{\pi}{2}}^{\pi} |v(t)|\, dt$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} v(t)\, dt + \int_{\frac{\pi}{2}}^{\pi} -v(t)\, dt$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos t\, dt + \int_{\frac{\pi}{2}}^{\pi} -\cos t\, dt$$

$$= \Big[\sin t\Big]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \Big[-\sin t\Big]_{\frac{\pi}{2}}^{\pi}$$

$$= 1 - (-1) + (-0) - (-1) = 3$$

4. (4 points) Compute the following indefinite integral:

$$\int \frac{(1+x)^2}{1+x^2}\,dx.$$

$$= \int \frac{1+2x+x^2}{1+x^2}\,dx$$

$$= \int 1 + \frac{2x}{1+x^2}\,dx$$

substitution $u = 1+x^2$ in second term.

$$= x + \ln|1+x^2| + C$$

5. (5 points) Compute the following definite integral:

$$\int_0^1 5x(x+3)^{-\frac{5}{2}}\,dx.$$

$u = x+3$ so $x = u-3$

$du = dx$

when $x=0$, $u=3$

$x=1$, $u=4$

$$= \int_3^4 5(u-3)u^{-5/2}\,du$$

$$= \int_3^4 5u^{-3/2} - 15u^{-5/2}\,du$$

$$= \left[\frac{5u^{-1/2}}{-1/2} - \frac{15u^{-3/2}}{-3/2}\right]_3^4$$

$$= \frac{-5(4)^{-1/2}}{1/2} + \frac{15(4)^{-3/2}}{3/2} - \left(-\frac{15(3)^{-1/2}}{1/2} + \frac{15(3)^{-3/2}}{3/2}\right)$$