Final year projects with Dr. Amy Pang

This is a rough document because I'm updating whenever I think of new things. This also means the organisation of the document will keep changing. Right now this document mainly talks about presentation (written and oral), because I don't really know what advice to give about the research yet - that will be coming!

Last updates: July 7, 2017 June 27, June 24.

The FYP experience

The FYP, especially at the start, will be hard. That's because the mathematics that you're learning is hard, but in addition it will probably be the first time you learn independently, and developing the skills to do that is really hard. (Think back to the start of Mathematical Analysis class - it was hard mainly because you have never written detailed proofs before, and it requires a new type of thinking.)

Writing the FYP is another thing you're doing for the first time - a mathematical document is not like an essay for your other classes. You have to be much more precise when explaining mathematics in writing (oral presentations can be more informal).

Because it takes time to develop these skills, I strongly recommend that you start in the summer, especially for research FYPs. In my research FYPs, "starting" does not mean reading by yourself - you will have long meetings with me to get used to thinking in these new ways.

Developing these independent learning / problem solving / writing skills is an important part of the FYP. I hope that will be useful to you whatever career path you choose.

Assessment information

Your work will be assessed by your supervisor and an observer, who is a randomly chosen teacher from the department. The assessment weighting is: continuous performance 30% (by supervisor only); thesis 50% (by supervisor and observer); oral presentation 20% (by supervisor and observer).

Timeline

I prefer students to start the FYP in the summer, especially for research FYPs. We usually meet irregularly in the summer, sometimes in long group sessions. During the semester and exam period, we meet 1 hour weekly. Thesis submission is usually in late December, and oral presentations in early January, just before semester 2.

My research FYPs

In a research FYP, you will do a small part of a larger research project that I'm working on. It is "experimental pure mathematics research", which follows this process:

1. Learn the basic definitions. This will be a mixture of reading papers and talking to me, because the papers will usually require much more than you know to understand, but I will try to explain it to you without those difficult parts.

2. Do some computation, either by hand or by writing some computer code.

3. Find some "interesting" patterns in your computation, and try to prove something, or make a conjecture.

4. At the same time as 3., we start drafting the thesis. (This will help check that 1. and 2. are correct.)

5. Repeat 2,3,4 until the thesis is around 20 pages. Because it takes a lot of words to explain things well, it shouldn't take a lot of math to reach 20 pages.

Not all pieces of pure mathematics are like this - indeed, many people (students or professors) start a project aiming to prove a specific theorem, but those are more unpredictable and therefore not suitable for a 4-6 month project. VERY IMPORTANT: the idea of exploring "data" until you find a pattern should not be used for statistics or experimental science, see the cartoon `https://stats.stackexchange.com/questions/88065/` . It only works in pure mathematics because we can prove the patterns.

Your role in the projects is to come up with fresh new views and ideas that your supervisor, having been trained to think in a specific way, can never see. Any idea or thought from you is useful; nothing is too small or too stupid. Maybe many of your ideas won't be used within your project, but I believe that your ideas will help my research in two or three years. (That has happened to many of my old ideas!) In particular, if you think what I've ask you to do is unnatural, you can ask if I can "change something" - this sort of thinking has led to many good research papers!

- There is a saying that you spend two years of your PhD wondering how you would ever prove anything, and then you do it in two weeks. The reason: you spend a lot of time reading, thinking, being confused, and you feel like you're not making progress, but all the time your understanding is improving, and you're trying many ideas, and when you find an idea that works, it will be very fast. Think about how houses are built: it takes a long time to make the foundation, but once that's done the building is very fast. So it's very normal to feel confused for 2 months of your FYP, try not to worry!

- FYI if you're thinking of further study: "grown-up" pure math research is very different from an FYP. With only an undergraduate background and only 4-6 months, it is impossible to let you feel what a real research project is like (many websites say this). In an MPhil or PhD, you will learn a lot more before pursuing your project, and your project is over a longer time period, so you are likely to get a bigger result that has lots of applications, and to understand it more than your FYP.

Your meetings with me

- During the semester, you will meet with me roughly once a week, for at most 1 hour each time. Even if you are too busy to work on the project, you should meet me because talking generates new ideas.

- For each meeting, you should prepare a 2-minute informal presentation of what you did since our last meeting, and a list of any questions you have. This reflection will help you see the "big picture" of your project.

- I usually ask you to do many things. Don't worry: I don't expect you to finish everything before the next meeting, or even to work on all the different things. But I want you to have lots of things that you can work on because you might be stuck on one or two of them. Good ideas take time to develop.

- If you are confused about something, try to think of a precise question to ask me at the meeting. If you only say "I don't understand" then I don't know what your confusion is and it will be hard to help you. Sometimes thinking of a precise question might actually solve your confusion (it's happened to me before). Basically, try to think a bit and understand your confusion (know what you don't know) rather than give up and wait for me to explain it.

- If you cannot understand a theorem or a formula, try to look at special cases: e.g. what happens when $n = 2$? What I mean is, substitute actual numbers or objects for the variables.

- After you solve your confusion, you should write down the confusion and how you solved it, so you don't forget later and confuse yourself again!

- If a computer calculation is not working the way you want, please email me right away. There's a good chance I can fix it in 5-10 minutes, then you can work on the project again. If you wait until our next meeting, you would waste a week. Sometimes there are many small problems with one calculation, and if you wait every time you will waste many weeks.

- The above also applies to minor mathematical problems that stop you from making any progress at all (e.g. you don't know whether the formula means add X or add Y). If you think it's something that I can answer in 5 minutes, please email me (email a photo if you want to write math / draw pictures) or come to my office and ask.

- Please keep neat notes on your project. An idea you abandoned earlier might later turn out to be useful, so you need to be able to find your discarded notes and remember what you were thinking at the time. I do this by writing notes that are like "talking to myself": e.g. "I need to find an eigenvector. Formula A is not useful because the algebra is not commutative. Does Formula B work? Try $n = 2$ ... " I also think that phrasing it out as writing helps me understand better (i.e. what I said above about having specific questions to ask).

- When you start writing, you will probably want to send me drafts for comments. Please **send a .pdf** in addition to .doc or .tex (see "thesis" section below), because I'm not always on a computer that can open .doc or .tex. And please send them **at least 24 hours in advance, and highlight** the parts that you want me to read. (I think it is easy to highlight .pdf.) Otherwise I won't know which parts are different from the last draft you sent me.

Reading papers

- Google and Google Scholar (scholar.google.com) are good places to look for papers. I usually ask my FYP students to find out about applications or connections of their topics, and you can start this by simply searching your FYP topic on one of these two sites.
- If the website asks you to pay to read the paper:
  - Click “all X versions” on the Google Scholar entry, and look for a free version of the paper, e.g. arxiv.org
  - If you do your Google Scholar search on a school computer, there is sometimes a ”Check full-text @HKBULib” option.
  - Send me the link. I have access to some journals.
- Usually you are reading a paper for one of two reasons: to get a general idea of what the paper is about, so you know what problems the math community at the time was interested in; or to find a specific piece of information (e.g. a theorem or a definition). In the first case, you probably only need to read the introduction. In the second case, find out from the introduction where that information is (i.e. which section), and jump straight to that section. No one reads a whole paper from beginning to end.
- If you can’t understand something in a paper, sometimes it’s because you don’t know the definitions of some of the words. Sometimes they are defined earlier in the paper; sometimes they are “standard” definitions that most people in the field will have learnt in their PhDs. In the second case, you can try looking in Wikipedia, or google for a textbook or course notes or other writings for beginners. Masters’ theses sometimes give definitions clearly.
- If you can’t understand a theorem in a paper, you can look at papers that cite it (Google Scholar has a function for this). Sometimes a paper citing the result will summarise the result nicely.
- When looking at several papers/webpages/sources: be careful that different authors may use different notation for the same thing. Or even worse, they may use the same notation for a different thing. If you think that two papers may be using different notation, you can ask me.
- If a paper is useful, note down its author, title, journal, and other information (for your reference section), and also why you find it useful. Otherwise later you may remember that you saw this useful thing in a paper, but not remember which paper it is.

The thesis

First, some advice that applies to any piece of writing:

- Go to the student area of the math department website and read a few previous FYPs of the same "type" as yours (some common FYP "types": statistical analysis of some data; numerically solve a problem and comment on the speed and accuracy of the method; about a theorem in pure mathematics; summarise some known results about a topic, ...). You probably find some of them easier to understand than others. Think about why those are easy to understand - that would be what you are trying to do in your thesis.

- Look at how previous theses are organised - what are the chapters, and how do they break each chapter into sections?

- Once you've drafted a small part, ask a friend to read it and give you comments - what do they find hard to understand, what do they think can be improved. To check that you have explained things clearly, you can even ask your friend to explain the main ideas of your FYP back to you! Your friend should have taken the prerequisite classes of the FYP, but not know anything specific to the FYP (because your observer won't know anything specific to the FYP).

Now, some advice specific to FYPs:

- **Put away all books, papers and other references when you write your FYP** (including notes that I wrote in group meetings). This avoids accidental plagiarism. Also, you should understand the project well enough to write it out in your own words.

- The audience of your thesis is **not** your supervisor, or any experts in this field. Remember, your observer will likely know nothing about the field. You should explain clearly everything that you learned while doing the project - impress your observer with how much you learned!

- Your level of explanation should be between a textbook and a paper. Read some textbooks and notice how they use a whole paragraph to explain one simple thing. Read a paper and notice how they use one sentence to explain a complicated thing, and how that's very hard to understand. You want to write something in the middle.

- Notice that, even in something very concise like a paper, there are a lot more words than formulae.

- In general, students don't give enough explanation. If you don't know whether you've explained enough, say more. Remember that it's easy for you to understand your writing because you have been thinking about the mathematical ideas for a long time, but your reader has not. Imagine you are teaching this to someone - what information to do have to give them so that they can reproduce your calculations and proofs?

- One way to "give more explanation" is to give an example after every definition. If you can use the same one or two examples through the entire thesis, that is nice (this isn't always possible, it depends on the mathematics).

- It's confusing to have two separate mathematical symbols separated only by a punctuation mark. You can avoid this by describing the second symbol before the symbol, e.g. "For u, v in W, their sum $u + v$ is in W." There are many more examples in my class notes.

- If you need something from a much earlier section, the reader may have forgotten, so you should reference the earlier section, or remind the reader quickly in one sentence.

- A piece of writing in pure mathematics usually goes like this (but it doesn't have to, if you have good reasons to structure it differently):

  1. Introduction - what is this paper about, and why it is important (e.g. important applications or connections to other topics). If your paper has one main theorem that is simple to understand, you may want to put it here in full detail. The last paragraph of the introduction usually lists what is in each section.
  2. Preliminaries - definitions and lemmas that you will need to understand the rest of the paper. (If you only need very few definitions, you may want to put them in the Introduction and remove this section.) In combinatorics, we usually give some examples after each definition. It is sometimes nice to explain multiple definitions and theorems with the same example, so you want to choose your examples carefully.

  3., 4., ... The main part of the paper - how this is structured will depend on the paper.

  $n-1$. Future questions - this is optional. Use this if there are questions you would like to investigate. Usually in pure mathematics we don't have a conclusion section.

  n. References.

- I usually start writing by deciding my notation. Think about what are the major mathematical objects in your thesis, and what letters you want to use for them. Make sure you don't use the same letter for two different objects!

- I usually start writing in Section 3 or 4. As I write, I make a list of what definitions I will need. Then I go back to write the definitions in Section 2. The Introduction is the last thing I write.

- Everyone writes differently, and my personal writing process (once I've planned the structure and notation) is in three parts:

  1. I work out what I want to say: in what order do I explain my results, what are the steps of my proof? I write this down as a list of rough bullet points. (You might want to use Chinese at this stage.)
  2. I write it on paper - I take time to find the right words to say each bullet point.
  3. I type it up. I like to write and type separately because typing equations requires extra thinking and disrupts my writing. Also, I can improve my writing slightly when I type it.

- Formatting: the thesis must be typed. If you are considering further study in mathematics, you should use TeX. TeX is a typing system and there are many programs that support it (for beginners, I recommend LyX https://www.lyx.org), as well as many online editors that will save you the trouble of installing it. To see how it's different from Word, you can look at this https://en.wikipedia.org/wiki/TeX#Mathematical_example . If you will not study more mathematics then Microsoft Word or similar is fine.

- Theorem formatting in Word etc.: theorems, definitions, examples, proofs etc. need to have a heading in **bold** (see next point). Theorem statements are usually in *italics*. Definitions and examples and proofs can be in regular font. The word that is being defined should look different from the rest of the sentence in some way - you can use *italics*, or <u>underline</u>. This also applies to new technical words defined outside of Definitions. Proofs should end with a □ symbol. Have a look at any of my papers (which are typed in TeX, but you should try to copy the formatting in Word).

- If you write down a theorem that is not your own, you should cite it e.g "**Theorem 1.2** [XY16 Th. 4.15] *If V is a vector space and ...*"
If the theorem is not yet in a paper, you should at least say who proved it (probably this person will be me).

- Reference formatting in Word etc.: your references in Section n should be listed in alphabetical order of author last name. To make sure you list the complete information (journal, publication date, etc.), you can go to http://www.ams.org/mathscinet/ (on campus, otherwise it will ask you for login), search for the paper, then copy the info. Some new papers won't be in that database (also if you want to cite online lecture notes or other things that aren't papers), so you'll have to type the reference yourself. To cite that reference in your main text, there are two conventions: either the references are numbered and you write "[1] proves that ...", or you write the initials of the authors and the year of publication e.g. [XY16]. I recommend the second because it means you don't have to retype your citations when you add a reference in the middle of the list. (Another advantage outside of FYPs: people in the field will recognise some widely cited references, e.g. everyone in combinatorics knows that [Sta99] is the famous textbook "Enumerative Combinatorics" by Stanley.)

- Reference formatting in TeX: you don't have to type reference information manually, I'll put more info about this later.

- Once you have written 1-2 pages, you should show me, so I can tell you about any major problems with your writing before you write more.

- Please proofread your thesis carefully from beginning to end before submitting it. Look for spelling and grammar errors, and also mathematical errors. Sometimes there are mathematical errors that you did not see while working on each section separately.

- To submit your thesis: hand me a hard copy, preferably in person, otherwise put it in my mailbox on 12/F and email me to let me know that it's there (because I don't always remember to check my mailbox). If it's a research project, also email me the .doc or .tex, so that, if I decide later to use your work in a joint paper, I can copy-and-paste it. You should contact your observer to arrange how to hand it in to him/her.

The presentation

- The presentation is **not** a class or a lecture - your audience does not have to completely understand every detail of your project. If you simply read parts of your thesis, it will be both hard to understand and boring.

- To make an interesting and understandable presentation, first imagine that you have 5 minutes to tell your classmate about your FYP. What would you say? This should help you find out what are the important parts of your FYP.

- In 5 minutes, you can probably only say the basic ideas informally, without any precise definitions. Now think about how you can explain it more precisely if you have 20 minutes, what small clarifications you can add. (Or maybe: actually tell someone about your FYP for 5 minutes, and see what questions they have, what they don't understand.)

- Think about the difference between a book and a movie based on the book. That's the difference between your thesis and your presentation.

- Now plan a short version of your presentation, based on the 5-minute imaginary explanation to your friend. Plan something that you think will take 12-15 minutes.

- In general, a pure mathematics talk has the same outline as papers (i.e. what I wrote above). One main difference: some definitions may be in the main part (Sections 3 to $n-2$) instead of Section 2, if they aren't used very often. When I read a paper I can always look back at Section 2, but if I'm listening to a talk I can't rewind, so sometimes it's good to define things just before they are used.

- Remember that your observer will probably not know much about the topic of your FYP. (This is especially true for my research FYPs, because no one in the department does any similar research.) So go through the basic background and important definitions slowly, as if we have never seen these things before - it will be impossible to follow the rest of the presentation if we don't understand the basics.

- If you have a technical theorem or definition, try to give an informal view either before or after (e.g. "linear independence is a way of saying that a set of vectors are "very different", and here is the formal definition ... "). If the formal definition is very long and technical, an example will probably be better.

- Do not think about proofs at this point, unless they are the main part of your FYP. Concentrate on the theorems, applications and examples. If you have time later, you can show the main ideas of the proofs via an example, but we rarely need to see the technical details in the presentation (we can read it in the thesis).

- Everyone prepares differently for presentations, but one thing that has worked for me is to write out a full script of what I'm going to say, but then I don't read from it. Reading from a script sounds boring. But it's hard to think of the right words during the presentation if I had not written something beforehand. So writing the script makes me think about how to phrase what I need to say. Some people like to have their important points on notecards during their presentation - this is ok, as long as you remember to look up from your notes and look at the audience. I personally read my script the night before and then give my presentation with no notes - but don't do this if it makes you nervous!

- Once you have written this short version, rehearse it. Do **not** wait until you have written the whole thing. When you say it out loud, you will realise some parts sound bad and need to be rewritten. Also, it probably takes a lot longer than you thought - people usually underestimate how long it takes to explain things. If you took over 22 minutes, think about what you can remove. Do **not** try to simply say it faster, then no one will understand. Even if you have several main ideas in your FYP, you don't need to say them all in your presentation - just explain in detail a simple one or an important one, and mention the others informally. It is better for the listener to understand one theorem / topic well than understand none of them at all. If you have time left over, you can add some little comments, but do not add a whole new section.

- As with the thesis, ask a friend to watch your presentation and give you comments. Your friend should make notes during your presentation of any questions, comments or improvement ideas; it's hard to remember it all at the end.

- Your slides should not be full of text. Your audience is listening to you as well as looking at your slides, so, for some easy things, you can just say them and you don't need to write them on the slide. Your slides should not look like my class slides - they have a lot of text because they also serve as notes, for you to read after class when you can't listen to me.

- Do not read out your formulae. That's hard to understand because your listener might not remember what all the symbols represent. Instead, explain the symbols in the formulae. I do this in class: when I write $\nabla f = \lambda \nabla g$, I don't say "nabla f equals lambda nabla g", I say "the gradient of the objective function f is a multiple of the gradient of g".

- If your slide has a lot of data or a lot of mathematical symbols, it will be difficult to understand. Highlight the parts that are important, the parts that you want the audience to look at. (E.g. if you have a big table of data and you want to show that one numerical method is faster than another, highlight the two times.) Maybe also point to the important parts as you explain them - point using your finger, not a laser pointer or a mouse, which are hard to see.

- I think in most of the FYP presentation rooms, you can use the whiteboard and slides at the same time. Whiteboards might be useful because you can draw something as you explain it. (You can also do animations on slides, but that takes a lot of time!)

- If you are considering further study in mathematics, you should make your slides using some form of TeX - I recommend IPE ipe.otfried.org. If you will not study more mathematics then Powerpoint or similar is fine. Pictures drawn by hand and scanned are fine.

Everyone gets nervous about presentations. One thing that's helped me is to try to forget that I'm being evaluated, and concentrate on sharing the exciting mathematics in my project. Think of the mathematics as the main character of your presentation, rather than yourself.

You can invite your friends to your presentation! I always ask lots of friends to come to my presentations, because it makes me less nervous to see supportive, friendly faces in the audience.