**McGill University** | **Practice Final**
**Faculty of Science**

---

# Calculus 2

## MATH 141

December 2015

Time: 3 Hours

Examiner: Dr. C. Y. A. Pang

---

| **Student name (last, first)** | **Student number (McGill ID)** |
|---|---|
| | |

---

### INSTRUCTIONS

1. Please write all your answers in this exam booklet.
2. This is a closed book exam; any notes or private writing paper is not permitted. If you need extra room for your answers, use the back side of the question pages, and **clearly indicate that your answer continues there**. Do not detach pages from this exam.
3. You do **not** need to simplify your arithmetic unless explicitly instructed to do so. However, a final answer involving a composition of a trigonometric function and an inverse-trigonometric function (in any order) will not be accepted.
4. The use of a translation dictionary is permitted.
5. The use of electronic devices, such as cellphones and calculators, for whatever reason, is not permitted.
6. This exam booklet comprises two cover pages and 19 pages with 7 questions. The questions are divided into two parts:
   - In Part A, only your final answer will be scored; there is no partial credit. Please put **only your final answer** in the boxes provided; if there is any ambiguity, you will not receive credit.
   - In Part B, please show all of your work and justify your answers to receive full credit.

   This exam is a total of 100 points. In questions with multiple parts, each part need not be worth the same number of points.
7. On the next page are some formulae you may find useful.

**Formulae you may find useful**:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$$\sin^2(\theta) = \frac{1}{2}(1-\cos(2\theta))$$

$$\cos^2(\theta) = \frac{1}{2}(1+\cos(2\theta))$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

The following boxes are strictly for grading purposes. Please do not mark.

| Question: | A | 2 | 3 | 4 | 5 | 6 | 7 | **Total** |
|---|---|---|---|---|---|---|---|---|
| Points: | 22 | 13 | 11 | 20 | 14 | 12 | 8 | 100 |
| Score: | | | | | | | | |

PART A: ANSWER ONLY

Enter **only your final answer** in the boxes provided; if there is any ambiguity, you will not receive credit.

1. (a) (2 points) Calculate the indefinite integral

$$\int \frac{x^3 + x^5}{\sqrt{x}} dx.$$

| Answer only |
| --- |
| |

(b) (2 points) Calculate the indefinite integral

$$\int \sin(2x) + \cos(2x) dx.$$

| Answer only |
| --- |
| |

(Question 1 continues on the next page.)

(c) (2 points) Calculate the definite integral

$$\int_0^2 \frac{5x^4 - 8}{\sqrt{x^5 - 8x + 9}} dx.$$

**Simplify your answer as much as possible.** (You may wish to double-check your arithmetic, since there is no partial credit available on this question.)

| Answer only |
| --- |
| |

(Question 1 continues on the next page.)

(d) (4 points) Calculate the indefinite integral

$$\int \tan^3 x \sec x \, dx.$$

Answer only

(Question 1 continues on the next page.)

(e) (4 points) Approximate the integral

$$\int_{-1}^{3} \arctan\left(\frac{1}{x+3}\right) dx$$

by a right Riemann sum with 2 subintervals.

| Answer only |
|---|
| |

(f) (4 points) Consider the function

$$g(x) = \frac{d}{dx} \int_{3x}^{x^3} \sin(e^{2t})\, dt.$$

Give an expression for $g(x)$ without integral signs.

| Answer only |
|---|
| |

(Question 1 continues on the next page.)

(g) (4 points) The velocity of a particle at time $t$ is given by

$$v(t) = t^3 - 1.$$

Calculate the **total distance** travelled by the particle in the time interval [-1,3]. **Simplify your answer as much as possible.** (You may wish to double-check your arithmetic, since there is no partial credit available on this question.)

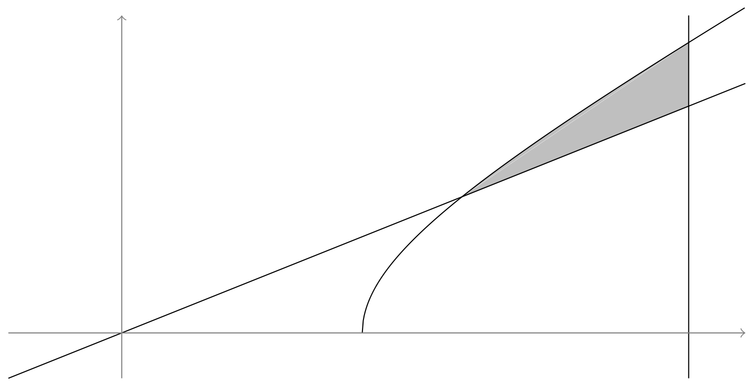
Answer only

PART B: FULL SOLUTIONS REQUIRED

In order to receive full credit, please show all of your work and justify your answers.

2. (13 points) The shaded region in the diagram below is bounded by

$$y = \sqrt{2x^2 - 9}, \quad y = x \quad \text{and } x = 5.$$

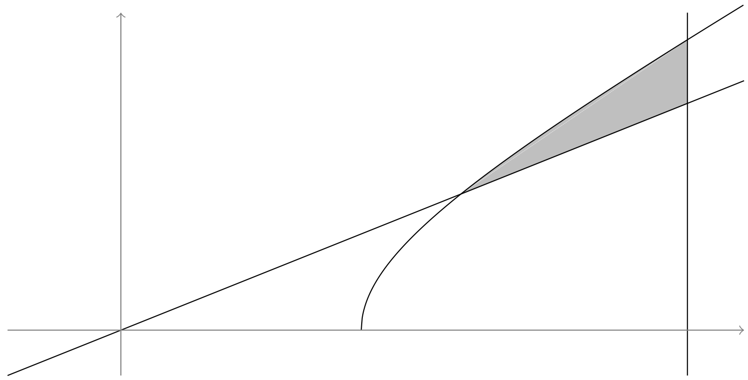


(a) Calculate the volume of the solid formed by rotating this region **about the $x$-axis**.

(Question 2 continues on the next page.)

(b) For your convenience, here again is the diagram of the region bounded by

$$y = \sqrt{2x^2 - 9}, \quad y = x \quad \text{and } x = 5.$$

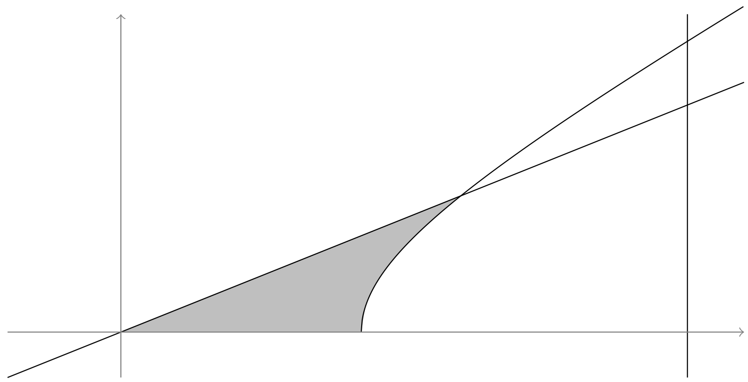


Calculate the volume of the solid formed by rotating this region **about the $y$-axis**.

(Question 2 continues on the next page.)

(c) Here are the same curves and a **different** region, bounded by

$$y = \sqrt{2x^2 - 9}, \quad y = x \quad \text{and } y = 0.$$



Find an expression, in terms of integrals, for the **area** of the shaded region. You do **not** need to evaluate your expression.

3. (11 points) (a) Compute the following indefinite integral:

$$\int \frac{x^2}{(x+1)(x^2+1)} dx.$$

(Question 3 continues on the next page.)

(b) Using part a or otherwise, compute the following improper integral, or explain why it diverges:

$$\int_0^\infty \frac{x^2}{(x+1)(x^2+1)}dx.$$

4. (20 points) (a) Compute the following improper integral, or explain why it diverges:

$$\int_{-4}^{0} \frac{1}{\sqrt{4+x}}dx.$$

(Question 4 continues on the next page.)

(b) Compute the following indefinite integral:

$$\int \frac{1}{x^2\sqrt{4+x^2}}dx.$$

(Hint: You may find it easier to calculate this **without** using part a.)

(Question 4 continues on the next page.)

(c) Compute the following indefinite integral:

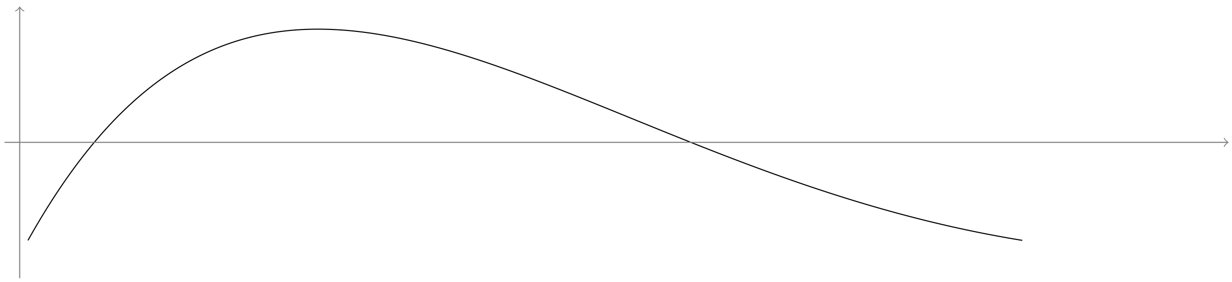
$$\int \frac{1}{x\sqrt{4+x}}dx.$$

(Hint: You may wish to start with the substitution $u = \sqrt{4+x}$. You may find it easier to calculate this **without** using parts a and b.)

5. (14 points) Let $C$ be the parametrised curve with equation

$$x = t^2, \quad y = -\cos t, \quad \frac{\pi}{6} \leq t \leq \frac{11\pi}{6},$$

as shown in the diagram below.

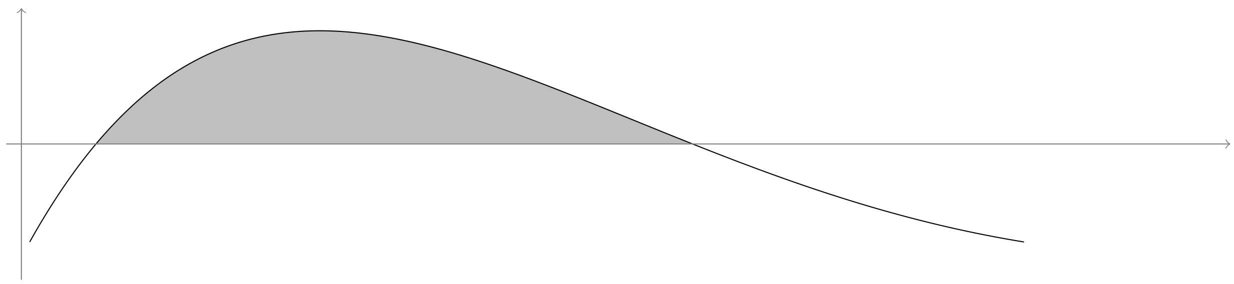


(a) Find the point(s) where $C$ has a horizontal tangent.

(Question 5 continues on the next page.)

(b) For your convenience, here again is the information about the parametrised curve $C$:

$$x = t^2, \quad y = -\cos t, \quad \frac{\pi}{6} \le t \le \frac{11\pi}{6}.$$



Find the area of the shaded region. (Hint: if you cannot determine the limits of the associated integral, you can earn partial credit by computing the indefinite integral.)

6. (12 points)

   (a) Determine whether the following series is convergent or divergent. Clearly state which tests you are using.

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3}.$$

(Question 6 continues on the next page.)

(b) Determine whether the following series is convergent or divergent. Clearly state which tests you are using.

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3 + 1}.$$

(Question 6 continues on the next page.)

(c) Determine whether the following series is convergent or divergent. Clearly state which tests you are using.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3+n}.$$

7. (8 points) Let $Q$ be the polar curve with equation

$$r = \theta^2.$$

Compute the arc length of the part of $Q$ between $(0, 0)$ and $(4, 2)$. **Simplify your answer as much as possible.**

(Hint: you may wish to use the identity $\sqrt{x^2 + x^k} = |x|\sqrt{1 + x^{k-2}}$, for $k \geq 2$.)