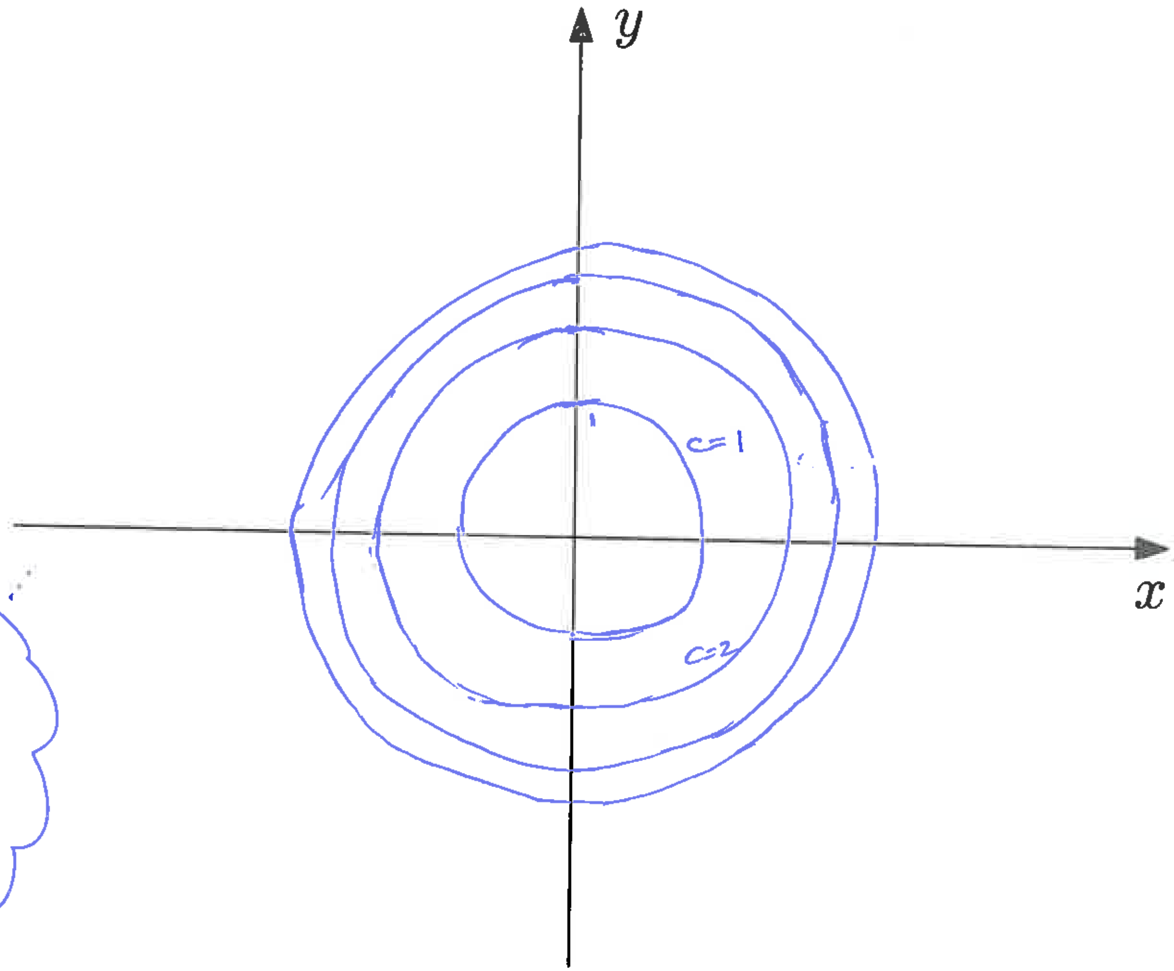
Example: Describe and sketch the level curves of $f(x,y) = x^2 + y^2$.

level curves are $c = x^2+y^2$

i.e. circles centred at the origin with radius $\sqrt{c}$.



circles are closer together as you move away from (0,0)
i.e. graph is steeper, function changes "more quickly"

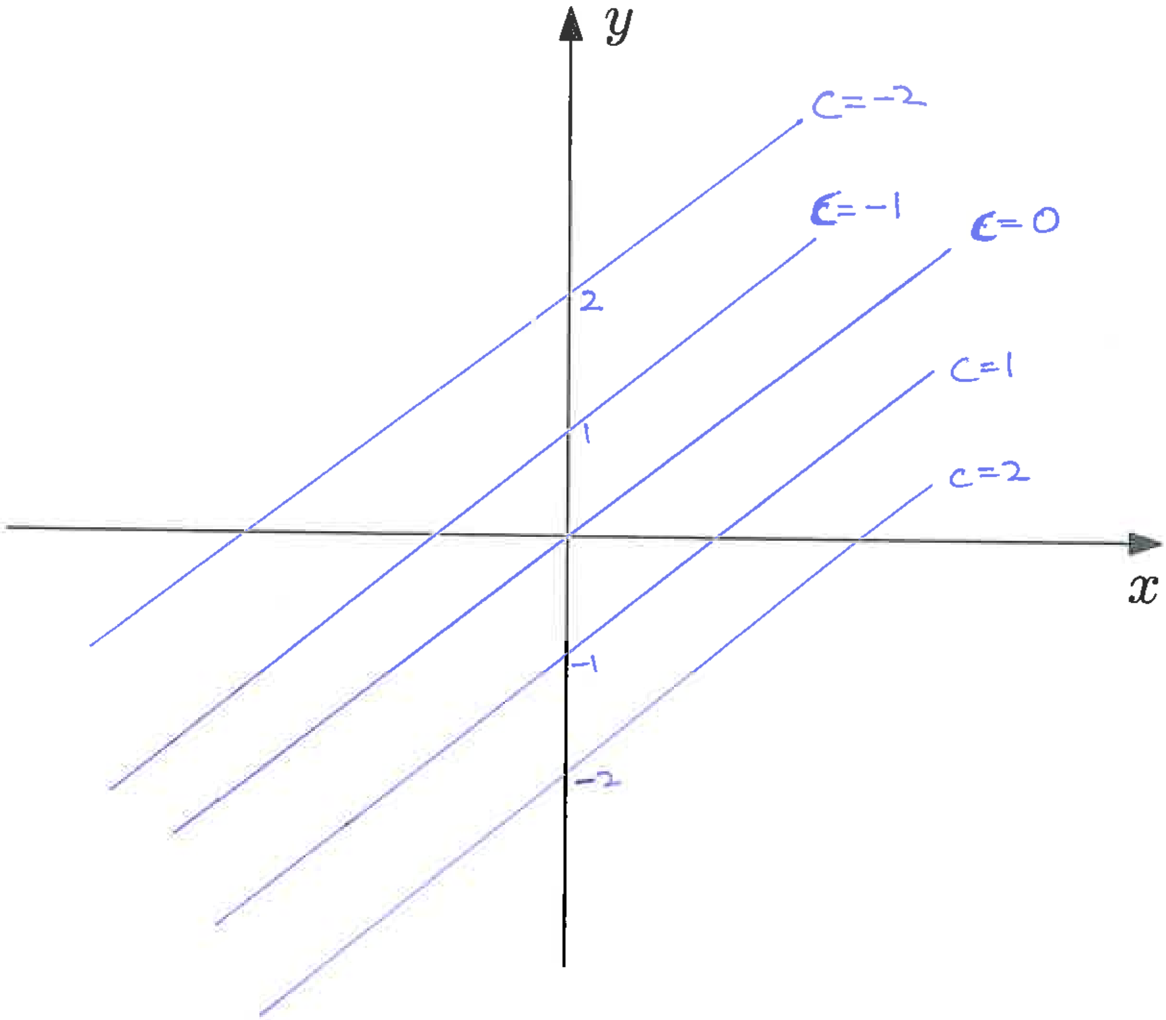
(linear function)

Example: Describe and sketch the level curves of $k(x, y) = x - y$.

level curves are $C = x - y$

$y = x - C$

parallel lines of slope 1 passing through $(0, -C)$.

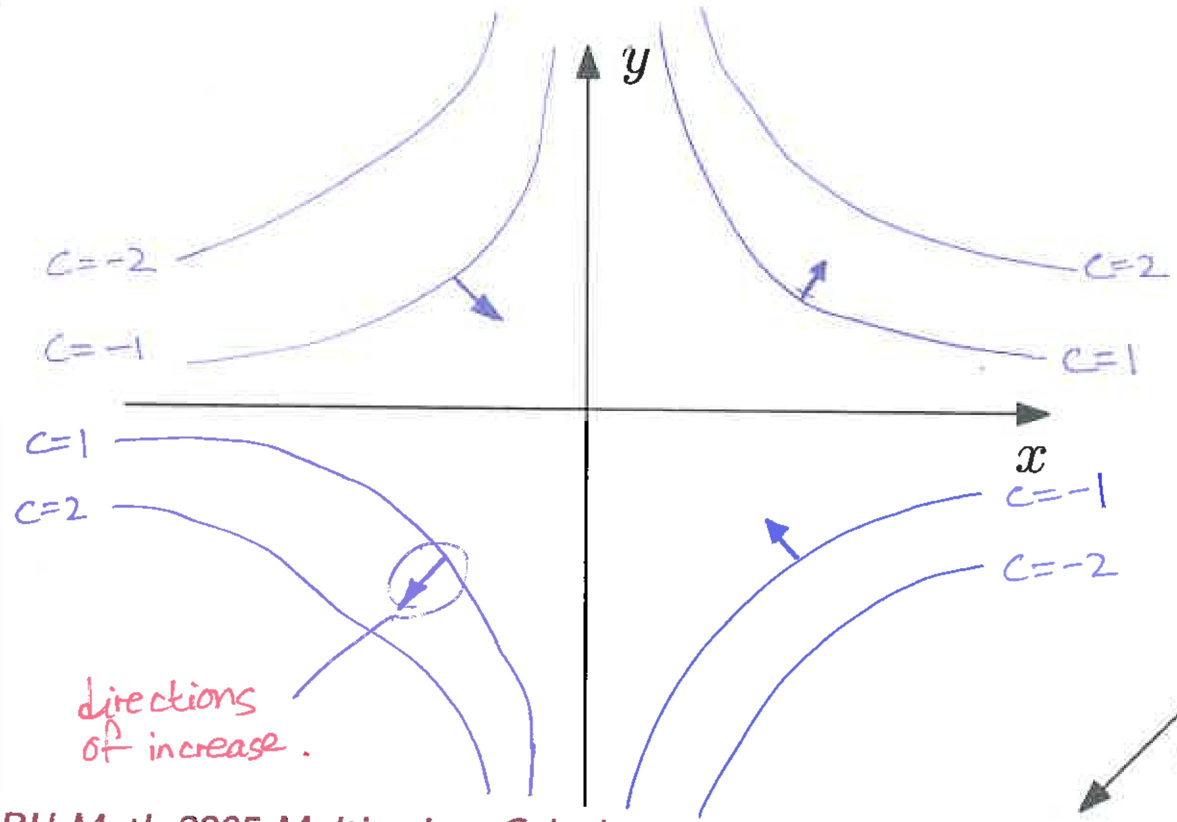


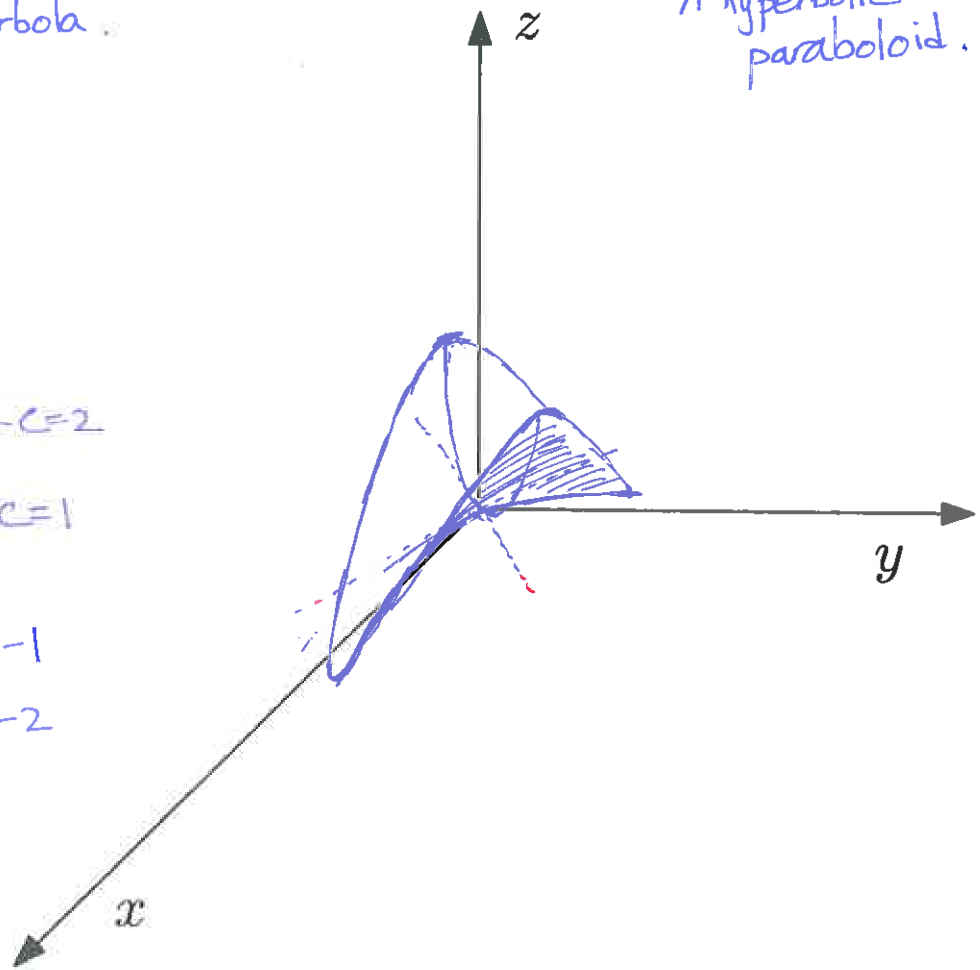
The level curves of a linear function are evenly spaced parallel lines

Example: Describe and sketch the level curves of $h(x, y) = xy$, and use this to sketch the graph of h.

level curves : $C = xy$ i.e. $y = \frac{C}{x}$ These are hyperbola.

A hyperbolic paraboloid.





(redrawn)

Example: Describe and sketch the level curves of $g(x,y) = \dfrac{\sqrt{x}}{1+y}$, and use this to sketch the graph of g.

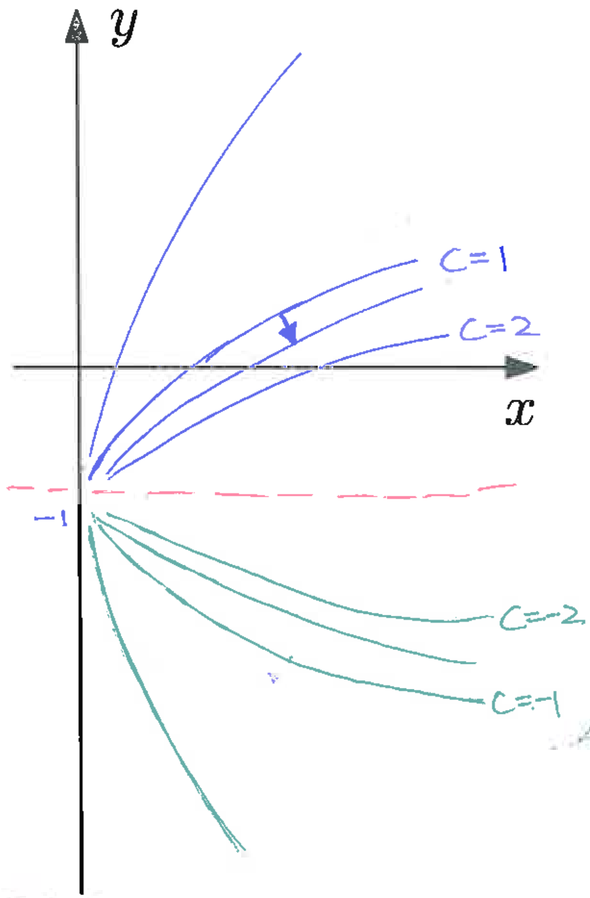
level curves $C = \frac{\sqrt{x}}{1+y}$

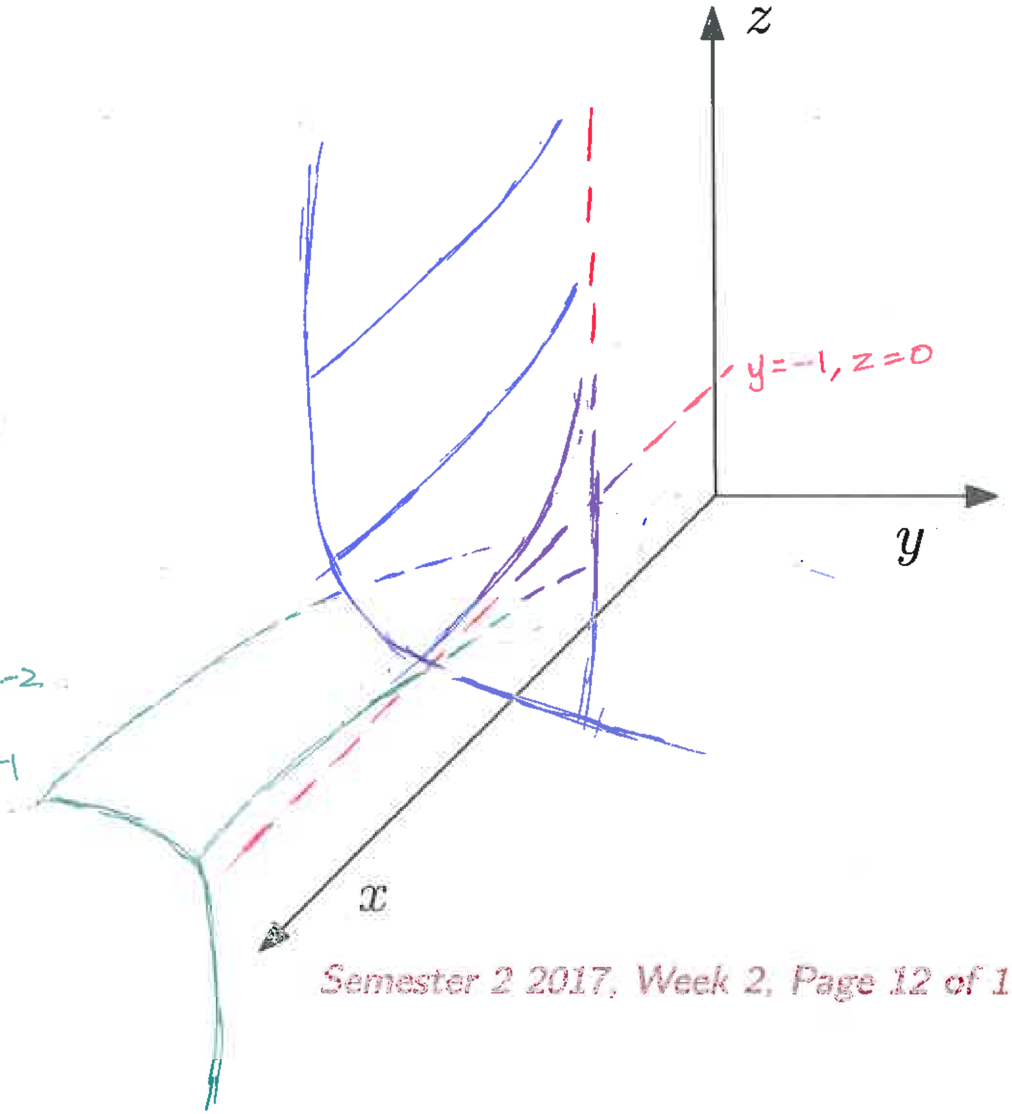
$$(1+y)C = \sqrt{x}$$

$$(1+y)^2 C^2 = x$$

$\therefore$ parabola — for each value of $\underline{C^2}$.

C has the same sign as $1+y$.





For 3-variable functions, the level sets are generally level surfaces.

Example: Describe the level surfaces of

$$G(x, y, z) = e^{-x^2-y^2-z^2}.$$

level surfaces are $C = e^{-x^2-y^2-z^2}$

$$\ln C = -x^2-y^2-z^2$$

$$-\ln C = x^2+y^2+z^2$$

$\therefore$ level surfaces are spheres centred at the origin, with radius $\sqrt{-\ln C}$.

$\because$ $-\ln C$ is ≥ 0 when $C \in (0,1]$

Not every surface in $\mathbb{R}^3$ is the graph of a (2-variable) function, but most surfaces in $\mathbb{R}^3$ can be expressed as a level set of a (3-variable) function, and this is often useful.

Example: Express the surface $2x + 2\ln y = 9 - z^2$ as the level set of a suitable function.

Take everything to one side.

$2x + 2\ln y - 9 + z^2 = 0$

This is a level set of

$F(x,y,z) = 2x + 2\ln y - 9 + z^2$ (at $C=0$).

Alternative answer:

$2x + 2\ln y + z^2 = 9$

This is a level set of $f(x,y,z) = 2x + 2\ln y + z^2$, with $C=9$.