Homework 4, due 14:45 Monday, 6 November 2017

You must justify your answers to receive full credit.

- 14.6: Q10, 14 Please describe and sketch the regions also.
- 14.6: Q15, 16 Please describe and sketch the regions also. (Hint: in Q16, first octant means $x \geq 0, y \geq 0, z \geq 0$.)
- 6.1: Q1, 8, 16, 17
- 6.5: Q9, 12, 15, 22
- 14.3 Q4, 12
- 12.2 Q3, 5, 11, 12
- 12.3 Q11, 36.
- 12.6 Q2, 5

The following two questions are to prepare you for the following week's class, and is unrelated to the material from recent classes.

1. Let $\mathbf{v}$ be the vector $4\mathbf{i} - \mathbf{j}$.
   a) Calculate the dot product $\mathbf{v} \bullet (3\mathbf{i} + 2\mathbf{j})$.
   b) Find a vector that is perpendicular to $\mathbf{v}$.
   c) Find a unit vector in the same direction as $\mathbf{v}$.

2. Consider the function

$$f(x) = \frac{x^2}{x^2 + 1}$$

   a) Find the critical points of f, and determine whether each of them is a local maximum, a local minimum, or neither.
   b) Find the minimum and maximum values of f on the interval $[1, 2]$.
   c) Find the second order Taylor polynomial of f about the point $x = 2$.
   d) (Unrelated to $f(x)$ above.) Using the series expansion for e^x, find the sixth-order Taylor polynomial of xe^{-x^2} about the point $x = 0$.

3. **Optional problem**: This problem aims to demystify the integration by parts formula by thinking about it in terms of areas above and below the curve. The concept described here is for interest only and is non-examinable.

   a) Sketch the region bounded by the curves $y = 0$ and $y = \sin x$, and satisfying $0 \leq x \leq \frac{\pi}{2}$, and find its area.

   b) Sketch the region bounded by the lines $y = 0$, $y = \frac{\pi}{2}$ and the curve $y = \arcsin x$. Compare this with your sketch from part a) and find the area of the region.

   c) Sketch the region bounded by the lines $x = 0$, $x = 1$ and the curve $y = \arcsin x$. Compare this with your sketch from part b) and find the area of the region.

   d) Now compute $\displaystyle\int_0^1 \arcsin x \, dx$ by first using the substitution $u = \arcsin x$, then doing integration by parts. Observe the similarity between this and part c).

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