Here is an improper integral that is of both types.

Example: Evaluate $\displaystyle\int_{1/e}^{\infty} \frac{1}{x(\ln x)^2}\,dx$.

$\because$ integrand is defined when $x \neq 0$, and
$x > 0$ (so $\ln x$ is defined)
$\ln x \neq 0 \rightarrow x \neq 1$

$$= \int_{-1}^{\infty} \frac{1}{u^2}\,du$$

substitution $u = \ln x$
$du = \frac{1}{x}\,dx$
$x = 1/e \Rightarrow u = \ln\left(\frac{1}{e}\right) = -\ln e = -1$
$x \to \infty \Rightarrow u \to \infty$

$$= \lim_{c \to 0^-} \int_{-1}^{c} \frac{1}{u^2}\,du + \lim_{c \to 0^+} \int_{c}^{1} \frac{1}{u^2}\,du + \lim_{R \to \infty} \int_{1}^{R} \frac{1}{u^2}\,du$$

This converges only when all three limits converge — if we know one of them diverges we don't need to calculate the other two

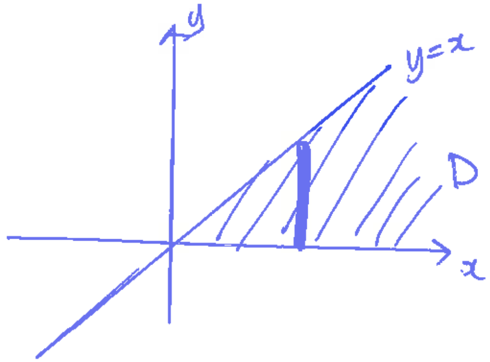
can be any number between 0 and ∞

$$\lim_{c \to 0^+} \int_{c}^{1} \frac{1}{u^2}\,du = \lim_{c \to 0^+} \left.\frac{-1}{u}\right|_{c}^{1} = \lim_{c \to 0^+} -1 + \frac{1}{c} = +\infty$$

so this integral diverges,
so the original integral diverges.

Example: (type I) Evaluate $\iint_D e^{-x-y}\,dA$, where D is the region $0 \le y \le x$.

This integral is improper because
D is unbounded.



$e^{-x-y} > 0$ for all (x,y) in D (in fact for all (x,y) in $\mathbb{R}^2$)

so we can calculate this improper integral as an
iterated integral.

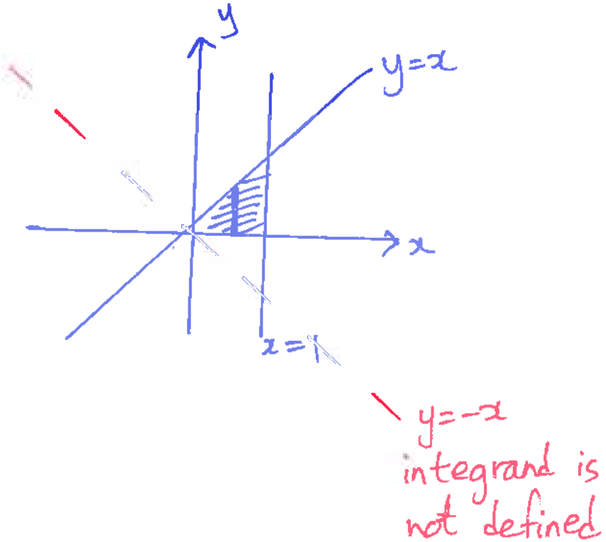
$$\iint_D e^{-x-y}\,dA = \int_0^\infty \int_0^x e^{-x-y}\,dy\,dx = \int_0^\infty -e^{-x-y}\Big|_{y=0}^{y=x} dx$$

$$= \int_0^\infty -e^{-2x} + e^{-x}\,dx$$

$$= \lim_{R\to\infty} \int_0^R -e^{-2x} + e^{-x}\,dx$$

$$= \lim_{R\to\infty} \frac{e^{-2x}}{2} - e^{-x}\Big|_0^R = 0 - \left(\frac{1}{2} - 1\right) = \frac{1}{2}$$

Example: (type II) Evaluate $\iint_D \frac{1}{(x+y)^2}\,dA$, where D is the region bounded by $y=0$, $x=1$ and $y=x$.



This integral is improper because the integrand is not defined when $x+y=0$ i.e. at $(0,0)$.

$\frac{1}{(x+y)^2} > 0$ so we can calculate this integral using iterated integrals.

$$\int_0^1 \int_0^x \frac{1}{(x+y)^2}\,dy\,dx = \int_0^1 \int_x^{2x} \frac{1}{u^2}\,du\,dx$$

substitution:
$u = x+y$
$du = dy$

$y=0 \rightarrow u=x$
$y=x \rightarrow u=2x$

$$= \int_0^1 \left. \frac{-1}{u} \right|_{u=x}^{u=2x} dx$$

$$= \int_0^1 \frac{-1}{2x} + \frac{1}{x}\,dx$$

$$= \int_0^1 \frac{1}{2x}\,dx = \lim_{c\to 0^+} \int_c^1 \frac{1}{2x}\,dx = \lim_{c\to 0^+} \left. \frac{1}{2}\ln|x| \right|_c^1 = \lim_{c\to 0^+} 0 - \frac{1}{2}\ln c$$

$$= +\infty$$

so this integral diverges.