Homework 6, **due 16:00 Saturday 9 December 2017, to Dr. Pang's mailbox on 12F FSC**

You must justify your answers to receive full credit.

You may use any (correct) method on the extremisation problems, regardless of which section the questions come from, or what techniques are specified within the question. You will always have a choice of techniques in exam questions.

- 13.2 Q1, 9
- 13.3 Q21, 22 (in Q21, you may assume that this minimum exists.)
- 12.8 Q5, 11 - in Q11, find also $\left(\dfrac{\partial^2 x}{\partial y^2}\right)_z$ when $(x, y, z, w) = \left(\frac{4}{5}, \frac{3}{5}, 0, 0\right)$.
- 12.8 Q12, 14
- 12.8 Q17, 24a - in Q17, for the computation of $\left(\dfrac{\partial y}{\partial u}\right)_v$ at $(u, v) = (1, 1)$, you may assume that $x = y = z = 1$ also.
- 14.4 Q32, 33 (Hints: in Q32, you may want to solve linear equations to solve for x and y in terms of u and v; in Q33, the first quadrant means the part with $x \geq 0$ and $y \geq 0$.)
- 14.6 Q9

1. For each of the sets below:

   i) Find its interior;
   ii) Find its boundary;
   iii) Determine whether it is closed;
   iv) Determine whether it is bounded.

   a) $\{(x, y) \in \mathbb{R}^2 \mid x + y = 1\}$;
   b) $\{(x, y) \in \mathbb{R}^2 \mid 0 < x^2 + y^2 < 1\}$;
   c) $\{(x, y, z) \in \mathbb{R}^3 \mid 0 < x^2 + y^2 < 1\}$;
   d) $\{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 < 1, y + z \geq 2\}$.

2. Consider the functions

$$f(x,y) = x^2y^2 + x^3y^2, \quad g(x,y) = x^2 - 2x + y^2.$$

   a) Show, by factoring or otherwise, that $(0,0)$ is a local minimum of f.
   b) Does f have an absolute minimum value on $\mathbb{R}^2$? Explain your answer. (Hint: can you "send f to $-\infty$"?)
   c) Does g have an absolute minimum value on $\mathbb{R}^2$? Explain your answer.

3. Find the maximum and minimum values of $f(x,y,z) = 2xy^2 + z$ over the closed region bounded by the paraboloid $z = 1 - x^2 - y^2$ and the plane $z = 0$. (Hint: the region has 3 boundary pieces, compare with the half-ball in p7 of week 12 notes. The 1D boundary piece can be parametrised by $(x,y,z) = (\cos t, \sin t, 0)$. I think there are 10 candidate extrema in total.)

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