Remember that $B = PDP^{-1}$ if and only if $BP = PD$ and P is invertible. This allows us to check our answer without inverting P:

$$BP = \begin{bmatrix} -3 & 0 & 0 \\ -1 & -2 & 1 \\ -1 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -3 & -3 & 0 \\ -3 & 0 & -1 \\ 0 & -3 & -1 \end{bmatrix},$$

$$PD = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -3 & -3 & 0 \\ -3 & 0 & -1 \\ 0 & -3 & -1 \end{bmatrix} = BP, \text{ and}$$

$$\det P = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 1 \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} - 1 \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = -2 \neq 0.$$