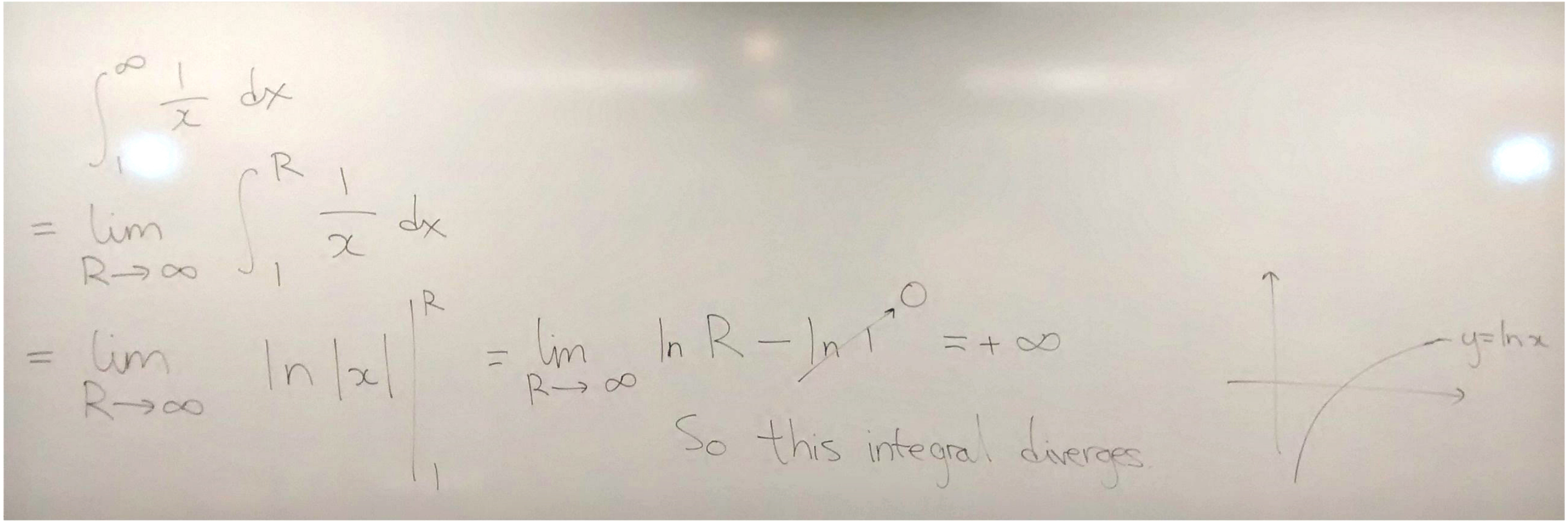
Rewriting our example on p2 in this terminology:

$$\int_1^\infty \frac{1}{x^2}\,dx = \lim_{R\to\infty}\int_1^R \frac{1}{x^2}\,dx = \lim_{R\to\infty} \left.\frac{-1}{x}\right|_1^R = \lim_{R\to\infty} 1 - \frac{1}{R} = 1.$$

Example: Evaluate $\displaystyle\int_1^\infty \frac{1}{x}\,dx$.



$$\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$$

$$= \lim_{R\to-\infty} \int_R^0 \frac{1}{1+x^2}\,dx + \lim_{R\to\infty} \int_0^R \frac{1}{1+x^2}\,dx$$

$$= \lim_{R\to-\infty} \tan^{-1}(x)\Big|_R^0 + \lim_{R\to\infty} \tan^{-1}(x)\Big|_0^R$$

$$= 0 - \left(-\frac{\pi}{2}\right) + \frac{\pi}{2} - 0 = \pi$$

$$\int_2^6 (x-3)^{-1/3}\,dx$$ integrand is not defined at $x=3$.

$$= \lim_{c\to 3^-} \int_2^c (x-3)^{-1/3}\,dx + \lim_{c\to 3^+} \int_c^6 (x-3)^{-1/3}$$

$$= \lim_{c\to 3^-} \left.\frac{(x-3)^{2/3}}{2/3}\right|_2^c + \lim_{c\to 3^+} \left.\frac{(x-3)^{2/3}}{2/3}\right|_c^6$$

substitution
$u = x-3$
$du = dx$

$$= 0 - \frac{(-1)^{2/3}}{2/3} + \frac{3^{2/3}}{2/3} - 0 = \frac{3}{2}\left(3^{2/3}-1\right)$$

Alternative: we can do substitution before writing the integral as a limit, if we use <u>one substitution</u> on the <u>whole integrand</u>

$u = x-3$
$du = dx$
$u=2 \to x=-1$
$u=6 \to x=3$

$$\int_2^6 (x-3)^{-1/3}\,dx = \int_{-1}^3 u^{-1/3}\,du = \lim_{c\to 0^-} \int_{-1}^c u^{-1/3}\,du + \lim_{c\to 0^+} \int_c^3 u^{-1/3}\,du$$

not defined at $u=0$.

$$= \lim_{c\to 0^-} \left.\frac{u^{2/3}}{2/3}\right|_{-1}^c + \lim_{c\to 0^+} \left.\frac{u^{2/3}}{2/3}\right|_c^3$$

$$= 0 - \frac{(-1)^{2/3}}{2/3} + \frac{3^{2/3}}{2/3} - 0 = \frac{3}{2}\left(3^{2/3}-1\right)$$