amypang.github.io/3407

black – old
blue – new

§6.1 Basic properties of vector spaces

A vector space over $\mathbb{F}$ is a set with
explain later – pretend $\mathbb{F}=\mathbb{R}$ for now
addition and scalar multiplication satisfying
10 axioms (see reference sheet)

Exs: A list of n numbers:

② $\mathbb{R}^n = \left\{ \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \middle| \; x_1, \dots, x_n \in \mathbb{R} \right\}$

usual addition and scalar multiplication

Ex 6.1-11 sequences $\{(x_1, x_2, \cdots) \mid x_i \in \mathbb{R}\}$

componentwise addition and scalar multiplication.

- $(x_1, x_2, \cdots) + (x_1', x_2', \cdots)$
  $= (x_1 + x_1', x_2 + x_2', \cdots)$
- $c(x_1, x_2, \cdots) = (cx_1, cx_2, \cdots)$

Check V4: $\vec{0} = (0, 0, \cdots)$

satisfies $(x_1, x_2, \cdots) + (0, 0, \cdots) = (x_1, x_2, \cdots)$

Ex 6.1-12 convergent sequences (i.e. $\lim_{n \to \infty} x_n$ exists)

To check V4 — need to show $(0, 0, \cdots)$ is convergent

V1 $(x_1 + x_1', x_2 + x_2', \cdots)$ is convergent

if $(x_1, x_2, \cdots), (x_1', x_2', \cdots)$ is convergent

see Functional Analysis.

③ $M_{m,n}(\mathbb{R})$, $m \times n$ matrices with entries in $\mathbb{R}$.

④ Functions from any non-empty domain S to $\mathbb{R}$.

$f+g$ defined by $(f+g)(s) = f(s) + g(s)$.

cf $\qquad (cf)(s) = c\,[f(s)]$

e.g. V4: $\vec{0}$ is the zero function: $\vec{0}(s) = 0$ for all $s \in S$.

Many interesting vector spaces come from choosing particular domains S and particular "types" of functions

(5) $S = [0,1] = \{ x \mid 0 \leq x \leq 1 \}$

$C^0([0,1]) = \{ f : [0,1] \to \mathbb{R}$ that are continuous $\}$

(7) $S = \mathbb{R}$

$\mathbb{R}[x] = \{ f : \mathbb{R} \to \mathbb{R}$ that are polynomials $\}$

the variable is x.

(8)

$P_n(\mathbb{R}) = \{ f : \mathbb{R} \to \mathbb{R}$ that are polynomials of degree $< n \}$

$= \{ a_0 + a_1 x + \dots + a_{n-1} x^{n-1} \mid a_i \in \mathbb{R} \}$

About $\mathbb{F}$: We can change the set of scalars from $\mathbb{R}$ to $\mathbb{C}$ (complex numbers)

e.g. $\mathbb{C}^n$ is a vector space over $\mathbb{C}$.

Also $M_{m,n}(\mathbb{C})$, $\mathbb{C}[x]$, $P_n(\mathbb{C})$

We write $\mathbb{F}$ to mean $\mathbb{R}$ or $\mathbb{C}$.

Non-examinable: there are other possibiliti[es] for F (a field, see §0.1) e.g. $\mathbb{F} = \{0, 1\}$ with $1+1=0$ - see algebraic coding, end of abstract algebra