Is $\sum_{n=1}^{\infty} a_n$ convergent?

1. Is it a p-series? $\sum \frac{1}{n^p}$

   yes, $p \leq 1$: divergent

   yes, $p > 1$: convergent

   no : continue

2. Is it a geometric series? $\sum ar^{n-1}$ i.e. is $\underbrace{\frac{a_{n+1}}{a_n}}_{r}$ independent of n?

   yes, $r < 1$: convergent

   yes, $r \geq 1$: divergent

   no : continue

3. Can you quickly find $\lim_{n \to \infty} a_n$?

   yes, it's not zero : series is divergent by test for divergence.

   yes, it's zero : continue

   no : continue

4. What are the signs of the a_n?

   always positive : go to step 6

   alternating : go to step 5

   irregular sign changes : test for absolute convergence : apply step 6 onwards to $\sum |a_n|$

5. Alternating series test for $\sum (-1)^n b_n$:

$\lim_{n\to\infty} b_n = 0$, b_n decreasing : converges

$\lim_{n\to\infty} b_n \neq 0$: diverges, by test for divergence : $\lim_{n\to\infty} a_n$ does not exist.

b_n not decreasing : test for absolute convergence : apply step 6 onwards to $\sum |a_n|$.

6. Which best describes your positive series?

a fraction where the dominant term in the numerator and the dominant term in the denominator are both $\bigcirc^n$ or both $n^{\square}$: limit comparison test with geometric series or p-series, respectively. $b_n = \dfrac{\text{dominant term in numerator}}{\text{dominant term in denominator}}$.

if $\lim_{n\to\infty} \frac{a_n}{b_n}$ = finite positive number, then $\sum a_n$ converges if $\sum b_n$ converges,
$\sum a_n$ diverges if $\sum b_n$ diverges

a product or quotient of $\bigcirc^n$, $n^{\square}$, $n!$: ratio test

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} < 1$ — converges

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} > 1$ or is ∞ — diverges

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$ or does not exist — try another test

something easy to integrate : integral test $a_n = f(n)$ for f decreasing

$\sum a_n$ converges if $\int_1^\infty f(x)\,dx$ converges

$\sum a_n$ diverges if $\int_1^\infty f(x)\,dx$ diverges

involves a trig function : comparison test, using $-1 \leq \sin x, \cos x \leq 1$.

if $a_n \leq b_n$ and $\sum b_n$ converges, then $\sum a_n$ converges

if $a_n \geq b_n$ and $\sum b_n$ diverges, then $\sum a_n$ diverges

it might be easier to guess a b_n, see if $\sum b_n$ converges, then multiply b_n by a constant to get the inequality you want.

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something else : comparison test?