c.

$$x_1\vec{v}_1 + \cdots + x_p\vec{v}_p = \vec{0}$$

only solution is

$x_1 = x_2 = \cdots = x_p = 0$

$\boxed{\text{Linear independence}}$

there is a solution with some $x_i \neq 0$.

(infinitely many solutions)

$\boxed{\text{linear dependence}}$

"totally different directions"

"no relationship"

"similar directions"

easy example: $\{\vec{v}, c\vec{v}\}$

$$(c)\vec{v} + (-1)(c\vec{v}) = \vec{0}$$

$(c,-1)$ is a non-trivial solution

other examples: $\left\{ \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix} \right\}$

$$1\begin{bmatrix}1\\0\\0\end{bmatrix} + 1\begin{bmatrix}0\\1\\0\end{bmatrix} + (-1)\begin{bmatrix}1\\1\\0\end{bmatrix} = \vec{0}$$

- adding any vectors to a linearly dependent set still makes a linearly dependent set.

e.g. if $\{\vec{v}_1, \vec{v}_2\}$ is linearly dependent, then $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly dependent

$$c_1\vec{v}_1 + c_2\vec{v}_2 = \vec{0} \longrightarrow c_1\vec{v}_1 + c_2\vec{v}_2 + \underset{\overset{\|}{0}}{c_3}\vec{v}_3 = \vec{0}$$

for some c_1, c_2 not both zero.

In this question: $7\begin{bmatrix}1\\1\\2\end{bmatrix} + (-1)\begin{bmatrix}7\\7\\14\end{bmatrix} = \vec{0}$

so: $7\begin{bmatrix}1\\1\\2\end{bmatrix} + (-1)\begin{bmatrix}7\\7\\14\end{bmatrix} + 0\begin{bmatrix}-1\\d\\d^2\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

$(7,-1,0)$ is a non-trivial solution, so linearly dependent.

Or:

$$\left[\begin{array}{ccc|c} 1 & 7 & -1 & 0 \\ 1 & 7 & d & 0 \\ 2 & 14 & d^2 & 0 \end{array}\right]$$

independent = unique solution
= no free variables = pivot in every column

dependent = infinitely many solutions
= has free variables.

$$\begin{array}{cc} \left[\begin{array}{ccc|c} 1 & 7 & 1 & 0 \\ 0 & 0 & d+1 & 0 \\ 0 & 0 & d^2+2 & 0 \end{array}\right] & \begin{array}{l} \\ R_2 - R_1 \\ R_3 - 2R_1 \end{array} \end{array}$$

1 is a pivot

no pivot

maybe a pivot

x_2 is always a free variable / column 2 never has a pivot.

So the columns are always linearly dependent.