Prerequisites for
MATH 2205, Multivariate Calculus
Semester 1, 2017

To succeed in this class, you should easily be able to solve the following problems. This is NOT an exhaustive list: the class may also require techniques and concepts not on this list. (Tip: some of these questions may be future homework problems.)

A Single-Variable Differential Calculus

1. Find the equation of the line through $(4, 1)$ and $(-2, 3)$.
2. If $f(x) = \frac{1}{1-x}$ and $g(x) = \sqrt{x-1}$, then what is the formula for the composition $g \circ f$ and what is its domain and range?
3. Draw the graphs of the following functions:
   (a) $f(x) = 3x + 5$
   (b) $f(x) = 3x^3 + 5$
   (c) $f(x) = (x-2)(x-3)(x-5)$
   (d) $f(x) = \left|x^2 - x - 2\right|$
   (e) $f(x) = \frac{1}{x^2}$
   (f) $f(x) = \sqrt{(}x-1)$
   (g) $f(x) = 3\sin(2x) + 2$
   (h) $f(x) = e^{2x}$
   (i) $f(x) = \ln x$
4. For an arbitrary function f, how is the graph of $1 + f(-x/2)$ related to the graph of f?
5. Describe and draw the following curves:
   (a) $4x^2 + y^2 = 4$
   (b) $x^2 - y^2 = 1$
   (c) $x = y^2 - 4y + 1$
6. Evaluate $\lim\limits_{x\to-\infty} \dfrac{2x-1}{\sqrt{3x^2+x+1}}$.
7. Evaluate $\lim\limits_{x\to\infty} \dfrac{\sin x}{x}$.
8. Evaluate $\lim\limits_{h\to 0} \dfrac{3h^2 + 4h^4}{h^2 - h^3}$.
9. Evaluate $\lim\limits_{x\to 0^+} x^{1/3}e^{-x} - x^{-1/3}$.
10. What is $\lim\limits_{x\to\infty} x^{1/3}e^{-x} - x^{-1/3}$? (You don't need to prove it, just find the answer.)
11. Determine all the points where the function $f(x) = \begin{cases} \dfrac{3x+4x^2}{x^2-x^3} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ is continuous.

12. If $x\sqrt{x+y} = 8 - xy$, find dy/dx in terms of y and x.

13. Find the equation of the tangent line to $y = \ln(1 + e^x)$ when $x = 0$.

14. Find all points on the curve $y = \dfrac{1}{x^2 + x + 1}$ where the tangent is horizontal.

15. Consider the function $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0; \\ 0 & \text{if } x = 0. \end{cases}$$

    (a) Show that f is continuous at $x = 0$. (Hint: you need to use the squeeze theorem.)

    (b) Find $f'(x)$ at $x = 0$. (Hint: you need to use the limit definition of the derivative.)

    (c) Find the linearisation for f about $x = 1$.

    (d) Using your answer to c), approximate $f(0.9)$.

16. Consider the function

$$f(x) = \frac{x^2}{x^2 + 1}$$

    (a) Find the critical points of f, and determine whether each of them is a local maximum, a local minimum, or neither

    (b) Find the minimum and maximum values of f on the interval $[1, 2]$.

    (c) Find the second-order Taylor polynomial of f about the point $x = 2$.

17. Using the series expansion for e^x, find the sixth-order Taylor polynomial of xe^{-x^2} about the point $x = 0$.

B Linear Algebra

This is **only for students who are not taking Math 2207 Linear Algebra** at the same time. If you are taking Math 2207 this semester, you will see the relevant concepts in that class before we need them in Math 2205.

1. Evaluate the matrix product $\begin{pmatrix} 3 & 0 & -2 \\ 1 & 1 & 2 \\ -1 & 1 & -1 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 3 & 0 \\ 0 & -2 \end{pmatrix}$.

2. Find the determinant and inverse of the matrix $\begin{pmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 2 & 1 & 3 \end{pmatrix}$.