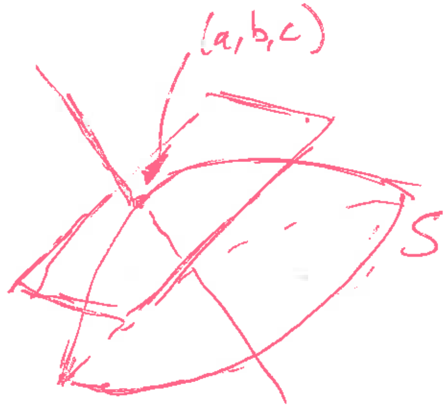
Q2b/3a Tangent plane: to a surface S at (a,b,c)

$$\nabla F(a,b,c)\cdot\left((x-a)\vec{i}+(y-b)\vec{j}+(z-c)\vec{k}\right)=0$$



Normal line: to a surface S at (a,b,c)

$$\vec{r}=a\vec{i}+b\vec{j}+c\vec{k}+t\,\nabla F(a,b,c)$$, t can take any value

and F is a function so that S is a level set of F. i.e. S is described by $F(x,y,z)=C$.

Remember not just the formulas, but what the symbols in the formula means

In this question: we are looking at the graph of $f(x,y)$.

the surface S is described by $z=f(x,y)$. i.e. $z-f(x,y)=0$

so use $F(x,y,z)=z-f(x,y)$ in the formulas above.

$F(x,y,z)$ in formula $\neq f(x,y)$ in question

function of 3 variables function of 2 variables

Q2b says tangent plane is $2x-y+3z=-13$

" $\nabla f = \cancel{2\vec{i}-\vec{j}+3\vec{k}}$ " $f(x,y)$ is a 2-variable function, so ∇f should be $?\vec{i}+?\vec{j}$, no $\vec{k}$.

Q3a " $\nabla f(1,-2) = -12\vec{i}+6\vec{j}$, so $\cancel{\vec{r} = \vec{i}-2\vec{j}+t(-12\vec{i}+6\vec{j})}$ "

this is a line in $\mathbb{R}^2$ (or horizontal line in $\mathbb{R}^3$)

but the graph of a 2-variable function is a surface in $\mathbb{R}^3$ so its normal line is in $\mathbb{R}^3$

Q3a, 3c, etc.

- E — you need to explain your work more clearly, I might take points off.

3a: explain any symbols you use that are not given in the question.
(especially important in non-routine questions — explain your calculations — I can't give you points if I don't understand your answer.)

e.g. $\nabla F(x,y,z) = (12x^3 - 6x^2 + 2x - 2y)\vec{i} + (-2x+2y)\vec{j} + \vec{k}$

where $F(x,y,z) = z - f(x,y)$.

3c: show your equation-solving in a logical order. — your solution should be readable e.g. write from top to bottom!

e.g. $f_x(x,y) = 0$ $\quad -12x^3 + 6x^2 - 2x + 2y = 0 \; (*) \Rightarrow \cancel{-12x^3 + 6x^2 - 2x + 2x = 0}$

$f_y(x,y) = 0$ $\quad 2x - 2y = 0 \Rightarrow x = y$

substitute into $(*)$: $-12x^3 + 6x^2 - 2x + 2x = 0$

See my homework solutions.

Q3d classify $(0,0)$:

second derivative test $(\cdot \cdot \cdot \cdot)$ $D_2(0,0) = \det H(0,0) = 0$

$\therefore$ inconclusive — need more information.

not automatically a saddle point

along $x=0$: $f(x,y) = -y^2 < 0 = f(0,0)$ for all $y \neq 0$.

along $y=0$: $f(x,y) = -3x^4 + 2x^3 - x^2 = x^2\underbrace{(-3x^2+2x-1)}_{\text{near } x=0, \text{ this factor is near } -1.} < 0 = f(0,0)$ for all $x \neq 0$.

Not automatically a maximum:

We found that f decreases in the directions $\vec{i}, \vec{j}, -\vec{i}, -\vec{j}$,

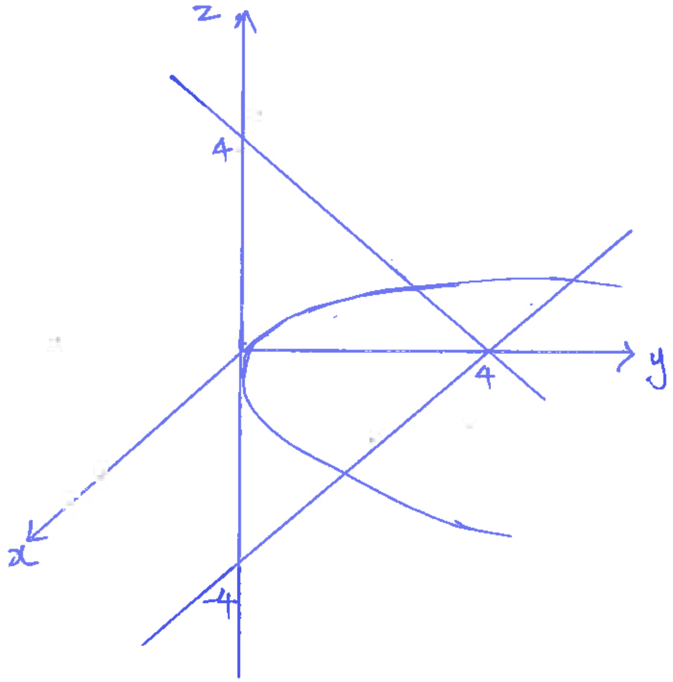
but we don't know about other directions / other paths.

To show $(0,0)$ is a maximum, we need to show $f(x,y) \leq f(0,0) = 0$ for all x, y.

A good attempt: "if I can find another path along which $f(x,y) > 0$, then $(0,0)$ is a saddle point."

midterm 1 Q6: Draw the region bounded by

$$y=x^2 \qquad y+z=4 \qquad y-z=4$$



not a 3D drawing.