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In general, a linear equation is an equation of the form

$$a_1x_1 + a_2x_2 + \dots a_nx_n = b.$$

$x_1, x_2, \dots x_n$ are the variables.

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Definition: A system of linear equations (or a linear system) is a collection of linear equations involving the same set of variables.

Example:

$$\begin{array}{rrrcr} x & +y & & = & 3 \\ 3x & & +2z & = & -2 \end{array}$$

This is a system of 2 equations in 3 variables, x, y, z.

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Definition: A solution of a linear system is a list $(s_1, s_2, \ldots, s_n)$ of numbers that makes each equation a true statement when the values $s_1, s_2, \ldots, s_n$ are substituted for $x_1, x_2, \ldots, x_n$ respectively.

Example: A solution to the example above is $(2, 1, -4)$.
(More clearly: $x = 2, y = 1, z = -4$.)

Definition: The solution set of a linear system is the set of all possible solutions.

Definition: A linear system is *consistent* if it has a solution, and *inconsistent* if it does not have a solution.

Fact: A linear system has either

• exactly one solution	consistent
• infinitely many solutions	consistent
• no solutions	inconsistent

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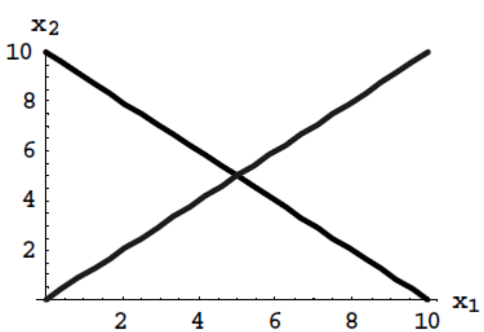
EXAMPLE Two equations in two variables:

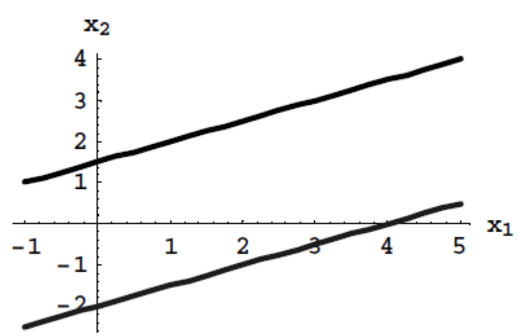
$$\begin{aligned} x_1 + x_2 &= 10 \\ -x_1 + x_2 &= 0 \end{aligned}$$

$$\begin{aligned} x_1 - 2x_2 &= -3 \\ 2x_1 - 4x_2 &= 8 \end{aligned}$$

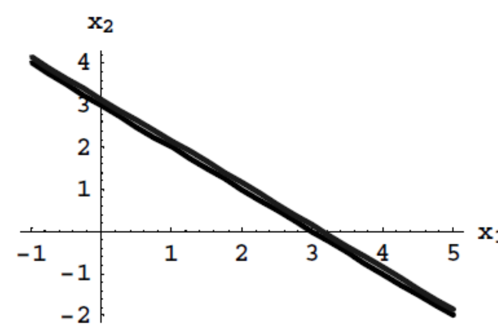
$$\begin{aligned} x_1 + x_2 &= 3 \\ -2x_1 - 2x_2 &= -6 \end{aligned}$$



one unique solution
consistent



no solution
inconsistent



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