Final year projects with Dr. Amy Pang, Fall 2017

(May 31, 2017, this is a quick draft and will be updated periodically)

These projects are designed to let you experience "experimental pure mathematics research" - the process of learning something new by yourself, computing examples to look for a pattern, generating and testing conjectures. Unlike classes, there will be a lot of "exploration" - we won't be sure of what the projects will discover until the end.

(Not all pieces of pure mathematics are like this - indeed, many people start a project aiming to prove a specific theorem, but those are more stressful and unpredictable. VERY IMPORTANT: the idea of exploring "data" until you find a pattern should not be used for statistics or experimental science, see the cartoon `https://stats.stackexchange.com/questions/88065/` . It only works in pure mathematics because we can prove the patterns.)

Each of the projects contains many many ideas that we can try. This is so that you can choose to work on something that you like, and that is giving good results. You do not have to try every idea I suggest.

I hope this experience will be useful to you whatever your career.

Your role in the projects is to come up with fresh new views and ideas that your supervisor, having been trained to think in a specific way, can never see. Any idea or thought from you is useful; nothing is too small or too stupid. Maybe many of your ideas won't be used within your project, but I believe that your ideas will help my research in two or three years. (That has happened to many of my old ideas!)

Your meetings with me

- During the semester, you will meet with me roughly once a week, for at most 1 hour each time. Even if you are too busy to work on the project, you should meet me because talking generates new ideas.

- For each meeting, you should prepare a 2-minute informal presentation of what you did since our last meeting, and a list of any questions you have. This reflection will help you see the "big picture" of your project.

- I usually ask you to do many things. Don't worry: I don't expect you to finish everything before the next meeting, or even to work on all the different things. But I want you to have lots of things that you can work on because you might be stuck on one or two of them. Good ideas take time to develop.

- If you are confused about something, try to think of a precise question to ask me at the meeting. If you only say "I don't understand" then I don't know what your confusion is and it will be hard to help you. Sometimes thinking of a precise question might actually solve your confusion (it's happened to me before). Basically, try to think a bit and understand your confusion (know what you don't know) rather than give up and wait for me to explain it.

- If a computer calculation is not working the way you want, please email me right away. There's a good chance I can fix it in 5-10 minutes, then you can work on the project again. If you wait until our next meeting, you would waste a week. Sometimes there are many small problems with one calculation, and if you wait every time you will waste many weeks.
- The above also applies to minor mathematical problems that stop you from making any progress at all (e.g. you don't know whether the formula means add X or add Y). If you think it's something that I can answer in 5 minutes, please email me (email a photo if you want to write math / draw pictures) or come to my office and ask.
- Please keep neat notes on your project. An idea you abandoned earlier might later turn out to be useful, so you need to be able to find your discarded notes and remember what you were thinking at the time.
- Because of the "exploratory" nature of these projects, we will do research and draft the thesis at the same time (instead of doing all the research first and then writing at the end). The writing will help you check that your results are correct. We will stop the research when the thesis is around 20 pages. Because it takes a lot of words to explain things well, it shouldn't take a lot of math to reach 20 pages.
- When you start writing, you will probably want to send me drafts for comments. Please send them **at least 24 hours in advance, and highlight** the parts that you want me to read. Otherwise I won't know which parts are different from the last draft you sent me.

Reading papers

- Google and Google Scholar (scholar.google.com) are good places to look for papers. I usually ask my FYP students to find out about applications or connections of their topics, and you can start this by simply searching your FYP topic on one of these two sites.
- If the website asks you to pay to read the paper:
  - Click "all X versions" on the Google Scholar entry, and look for a free version of the paper, e.g. arxiv.org
  - If you do your Google Scholar search on a school computer, there is sometimes a "Check full-text @HKBULib" option.
  - Send me the link. I have access to some journals.
- Usually you are reading a paper for one of two reasons: to get a general idea of what the paper is about, so you know what problems the math community at the time was interested in; or to find a specific piece of information. In the first case, you probably only need to read the introduction. In the second case, find out from the introduction where that information is (i.e. which section), and jump straight to that section. No one reads a whole paper from beginning to end.
- If you can't understand something in a paper, try looking at papers that cite it (Google Scholar has a function for this). Sometimes a paper citing the result will summarise the result nicely.
- If a paper is useful, note down its author, title, journal, and other information (for your reference section), and also why you find it useful. Otherwise later you may remember that you saw this useful thing in a paper, but not remember which paper it is.

The thesis

First, some advice that applies to any piece of writing:

- Go to the student area of the math department website and read a few previous FYPs of the same "type" as yours (some common FYP "types": statistical analysis of some data; numerically solve a problem and comment on the speed and accuracy of the method; about a theorem in pure mathematics; summarise some known results about a topic, ...). You probably find some of them easier to understand than others. Think about why those are easy to understand - that would be what you are trying to do in your thesis.
- Look at how previous theses are organised - what are the chapters, and how do they break each chapter into sections?
- Once you've drafted a small part, ask a friend to read it and give you comments - what do they find hard to understand, what do they think can be improved. To check that you have explained things clearly, you can even ask your friend to explain the main ideas of your FYP back to you! Your friend should have taken the prerequisite classes of the FYP, but not know anything specific to the FYP (because your observer won't know anything specific to the FYP).

Now, some advice specific to FYPs:

- **Put away all books, papers and other references when you write your FYP** (including notes that I wrote in group meetings). This avoids accidental plagiarism.
- Your level of explanation should be between a textbook and a paper. Read some textbooks and notice how they use a whole paragraph to explain one simple thing. Read a paper and notice how they use one sentence to explain a complicated thing, and how that's very hard to understand. You want to write something in the middle.
- In general, students don't give enough explanation. If you don't know whether you've explained enough, say more. Remember that it's easy for you to understand your writing because you have been thinking about the mathematical ideas for a long time, but your reader has not.
- One way to "give more explanation" is to give an example after every definition. If you can use the same one or two examples through the entire thesis, that is nice (this isn't always possible, it depends on the mathematics).
- A piece of writing in pure mathematics usually goes like this (but it doesn't have to, if you have good reasons to structure it differently):

  1. Introduction - what is this paper about, and why it is important (e.g. important applications or connections to other topics). If your paper has one main theorem that is simple to understand, you may want to put it here in full detail. The last paragraph of the introduction usually lists what is in each section.
  2. Preliminaries - definitions and lemmas that you will need to understand the rest of the paper. (If you only need very few definitions, you may want to put them in the Introduction and remove this section.) In combinatorics, we usually give some examples after each definition. It is sometimes nice to explain multiple definitions and theorems with the same example, so you want to choose your examples carefully.

  3., 4., ... The main part of the paper - how this is structured will depend on the paper.

  $n-1$. Future questions - this is optional. Use this if there are questions you would like to investigate. Usually in pure mathematics we don't have a conclusion section.

n. References.

- I usually start writing in Section 3 or 4. As I write, I make a list of what definitions I will need. Then I go back to write the definitions in Section 2. The Introduction is the last thing I write.
- If you need something from a much earlier section, the reader may have forgotten, so you should reference the earlier section, or remind the reader quickly in one sentence.
- There are rules about how to format your references in Section n and how to cite those references in the main text. I won't write the details here, have a look at previous FYPs.
- Once you have written 1-2 pages, you should show me, so I can tell you about any major problems with your writing before you write more.

The presentation

- The presentation is **not** a class or a lecture - your audience does not have to completely understand every detail of your project. If you simply read parts of your thesis, it will be both hard to understand and boring.
- To make an interesting and understandable presentation, first imagine that you have 5 minutes to tell your classmate about your FYP. What would you say? This should help you find out what are the important parts of your FYP.
- In 5 minutes, you can probably only say the basic ideas informally, without any precise definitions. Now think about how you can explain it more precisely if you have 20 minutes, what small clarifications you can add. (Or maybe: actually tell someone about your FYP for 5 minutes, and see what questions they have, what they don't understand.)
- Think about the difference between a book and a movie based on the book. That's the difference between your thesis and your presentation.

- Now plan a short version of your presentation, based on the 5-minute imaginary explanation to your friend. Plan something that you think will take 12-15 minutes.
- In general, a pure mathematics talk has the same outline as papers (i.e. what I wrote above). One main difference: some definitions may be in the main part (Sections 3 to $n-2$) instead of Section 2, if they aren't used very often. When I read a paper I can always look back at Section 2, but if I'm listening to a talk I can't rewind, so sometimes it's good to define things just before they are used.
- Remember that your observer will probably not know much about the topic of your FYP. (This is especially true for my research FYPs, because no one in the department does any similar research.) So go through the basic background and important definitions slowly, as if we have never seen these things before - it will be impossible to follow the rest of the presentation if we don't understand the basics.
- If you have a technical theorem or definition, try to give an informal view either before or after (e.g. "linear independence is a way of saying that a set of vectors are "very different", and here is the formal definition ... "). If the formal definition is very long and technical, an example will probably be better.
- Do not think about proofs at this point, unless they are the main part of your FYP. Concentrate on the theorems, applications and examples. If you have time later, you can show the main ideas of the proofs via an example, but we rarely need to see the technical details in the presentation (we can read it in the thesis).
- Once you have written this short version, rehearse it. Do **not** wait until you have written the whole thing. When you say it out loud, you will realise some parts sound bad and need to be rewritten. Also, it probably takes a lot longer than you thought - people usually underestimate how long it takes to explain things. If you took over 22 minutes, think about what you can remove. Do **not** try to simply say it faster, then no one will understand. Even if you have several main ideas in your FYP, you don't need to say them all in your presentation - just explain in detail a simple one or an important one, and mention the others informally. It is better for the listener to understand one theorem / topic well than understand none of them at all. If you have time left over, you can add some little comments, but do not add a whole new section.

- As with the thesis, ask a friend to watch your presentation and give you comments. Your friend should make notes during your presentation of any questions, comments or improvement ideas; it's hard to remember it all at the end.

- Your slides should not be full of text. Your audience is listening to you as well as looking at your slides, so, for some easy things, you can just say them and you don't need to write them on the slide. Your slides should not look like my class slides - they have a lot of text because they also serve as notes, for you to read after class when you can't listen to me.
- Do not read out your formulae. That's hard to understand because your listener might not remember what all the symbols represent. Instead, explain the symbols in the formulae. I do this in class: when I write $\nabla f = \lambda \nabla g$, I don't say "nabla f equals lambda nabla g", I say "the gradient of the objective function f is a multiple of the gradient of g".
- If your slide has a lot of data or a lot of mathematical symbols, it will be difficult to understand. Highlight the parts that are important, the parts that you want the audience to look at. (e.g. if you have a big table of data and you want to show that one numerical method is faster than another, highlight the two times.) Maybe also point to the important parts as you explain them - point using your finger, not a laser pointer or a mouse, which are hard to see.
- I think in most of the FYP presentation rooms, you can use the whiteboard and slides at the same time. Whiteboards might be useful because you can draw something as you explain it. (You can also do animations on slides, but that takes a lot of time!)
- Don't worry too much about the appearance of your slides. If you need to draw a picture but you don't know how to draw it with the computer, it's fine to draw neatly by hand and scan it in. In fact, some of the best pure mathematics talks I've heard had slides that were entirely handwritten and scanned.

Everyone gets nervous about presentations. One thing that's helped me is to try to forget that I'm being evaluated, and concentrate on sharing the exciting mathematics in my project. Think of the mathematics as the main character of your presentation, rather than yourself.

You can invite your friends to your presentation! I always ask lots of friends to come to my presentations, because it makes me less nervous to see supportive, friendly faces in the audience.