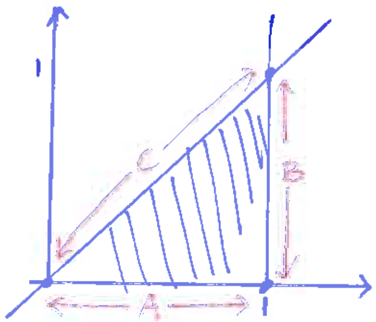
Example: Find the maximum value of $f(x, y) = x^2 + xy - 2y$ on the closed triangle with vertices $(0, 0)$, $(1, 0)$ and $(1, 1)$, and the point(s) where this maximum value is achieved.

1.



interior: $y > 0$ and $x < 1$ and $y < x$.

boundary: A: $y = 0$ $\quad 0 < x < 1$

B: $x = 1$ $\quad 0 < y < 1$

C: $y = x$ $\quad 0 < x < 1$

$(0,0)$ $\quad (1,0)$ $\quad (1,1)$

2. f is continuous and its domain is closed and bounded, so f achieves a maximum.

3. Interior critical points satisfy $\nabla f = \vec{0}$

$(2x+y)\vec{i} + (x-2)\vec{j} = \vec{0}$.

$\downarrow \qquad\qquad \downarrow$

$2x + y = 0 \qquad x = 2$ which is outside the triangle.

No singular points because f is continuous everywhere.

4. On boundary piece A: $f(x,y) = x^2 + 0 - 0 = x^2$

Candidate extrema satisfy $\frac{d}{dx}(x^2) = 0 \rightarrow x = 0$.

not in boundary piece A.

On boundary piece B: $f(x,y) = 1^2 + y - 2y = 1 - y$

Candidate extrema satisfy $\frac{d}{dy}(1-y) = 0$

$\rightarrow$ no solution.

On boundary piece C: $f(x,y) = x^2 + x^2 - 2x = 2x^2 - 2x$

Candidate extrema satisfy $\frac{d}{dx}(2x^2 - 2x) = 0$

$$4x - 2 = 0$$

$$x = \frac{1}{2}.$$

$$\left(\frac{1}{2}, \frac{1}{2}\right)$$

5. $f\left(\frac{1}{2}, \frac{1}{2}\right) = -\frac{1}{2}$

$f(0,0) = 0$

$f(1,0) = 1 \leftarrow$ maximum value of f is 1, achieved at $(1,0)$.

$f(1,1) = 0$