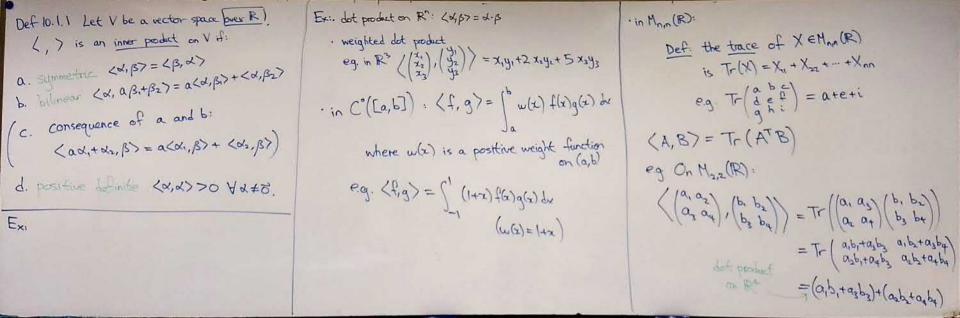
\$10.1 Inner product

Motivation: dot product on R" is useful

e.g. if [x,..., xx] is an orthogonal basis for WER

then the closest point in W to B/best approximation in W to B is Proju(B) = B.d, x, +... + B.dk dk Want to similarly approximate functions in other vector spaces, i.e. want to define d.B for d, BEV — use a symmetric bilinear form
that's positive definite.



In general  $\langle A, B \rangle = [A]_A \cdot [B]_A : f A = \{E'', E'', ..., E'''\}$ is the standard basis of Mnn(PR). Over C, we have to define inner products differently, because the condition <<, <>>>>> Vd + o does not make sense if <<, <>>>>> CIR Def 10.1.1/9.6.1 Let V be a vector space laver C. <, > is an inner product on Vif: a. Hermitian (x, B) = < B, x) where means complex conjugation (then  $\langle \alpha, \alpha \rangle = \langle \alpha, \alpha \rangle$  so  $\langle \alpha, \alpha \rangle \in \mathbb{R}$ ) b. sequelinear  $\langle \alpha, \alpha \rangle + \langle \alpha, \alpha \rangle + \langle \alpha, \beta \rangle = \alpha \langle \alpha, \beta, \gamma \rangle + \langle \alpha, \beta, \gamma \rangle$ (c. consequence of a and b: (ad, +d2, B) = a(d, B7 + (d2, B)) d. positive definite (x, x) >0 42+0.

Ex: V= f: R -> C continuous on [a, b]}