is not bounded.

is closed (has no boundary)

Example: Does $f(x, y) = 2x^3 + 6xy + 3y^2$ have a maximum on $\mathbb{R}^2$? (You found on ex. sheet #18 Q1 that $(1, -1)$ is a local minimum and $(0, 0)$ is a saddle point, and there are no other critical points.)

Answer 1: There is no local maximum, so there is no absolute maximum

You need to be careful with this argument if the domain has a boundary

Using less information: only the critical points, not the classification

Answer 2: If f has a maximum, it must be a critical point (because there are no singular points or boundary points) i.e. $(1,-1)$ or $(0,0)$.

$f(1,-1) = -1$, $f(0,0) = 0$ but $f(1,1) = 11 > f(1,-1)$, $> f(0,0)$

can be any point

so f cannot have a maximum at $(1,-1)$ or $(0,0)$, so f has no maximum.

Use even less information

$f(x,y) = 2x^3 + 6xy + 3y^2$

Answer 3: want to show that $f \to \infty$ as $x, y \to \infty$:
easier to look at just 1 direction (many choices)

along $x=0$, $f(x,y) = 0 + 0 + 3y^2 \to \infty$ as $y \to \infty$, so f has no maximum.

Be careful: a function that does not "go to infinity" might still not have a maximum or minimum.

Example: Does $f(x, y) = \dfrac{1}{1 + x^2 + y^2}$ have a maximum and a minimum on $\mathbb{R}^2$?

f does not have a minimum: $\dfrac{1}{1+x^2+y^2} > 0$

$\therefore$ as $x, y \to \infty$, $f(x,y) \to 0$, but $f(x,y)$ is never actually 0

but along $x=0$, $f(x,y) = \dfrac{1}{1+y^2} \longrightarrow 0$ as $y \to \infty$

f does have a maximum: $1 + x^2 + y^2 \geq 1$ with equality if and only if $(x,y) = (0,0)$

so $\dfrac{1}{1+x^2+y^2} \leq 1$ so f has a maximum value of 1, achieved at $(0,0)$.