

i.e. $\underline{\Phi}_{V}(\sigma^{*}(A)) = \widehat{\sigma}(\underline{\Phi}_{V}(A))$ $\forall A \in V$ i.e. $\langle \sigma^*(A), - \rangle_0 = \delta \left[\langle \alpha, - \rangle_V \right]$ [definition of δ] $= \langle \alpha, \sigma(-) \rangle_V$

i.e. 1.00 = 00 = 00

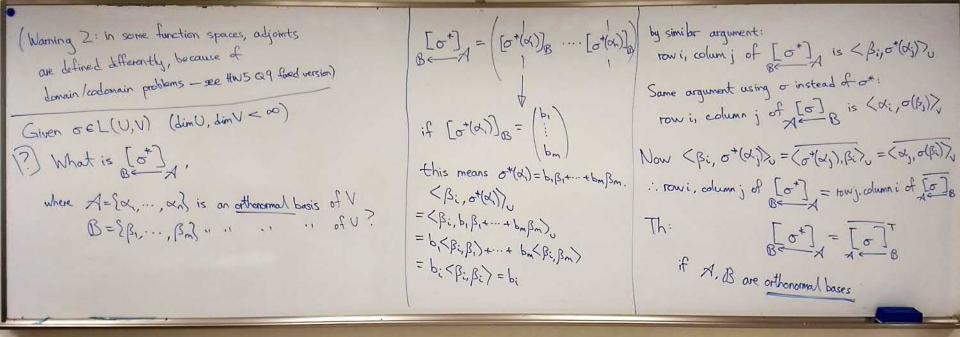
 $\langle \sigma^{\dagger}(\alpha), \beta \rangle_{\nu} = \langle \alpha, \sigma(\beta) \rangle_{\nu} \quad (*)$

Alternatively, we can define the adjoint by (+) (Th. 10.2.4)

Given $\sigma \in L(U,V)$:

(see HW5 Q9) Given $\sigma \in L(U,V)$: For each $\alpha \in V$, define $\phi^{(\alpha)}: U \rightarrow \mathbb{F}$ by $\phi^{\alpha}(\beta) = \langle \alpha, \sigma(\beta) \rangle_{V}$ i.e. $\phi'' = \phi_{x} \circ \sigma$, a composition of linear transformations : linear, i.e. \$ (x) EU. .. by Riesz representation, $\phi^{(\alpha)} = \langle f, - \rangle_U = \phi_f$ for some $f \in U$. There is such a y for each 2, so let y=0*(x). Then check or is linear. Warning 1: or depends on the choice of <, >.

(see HWS Q9)



From 2207:
$$(Row A)^{+} = Nwl A$$
 (over R)

so $(Row A^{+})^{+} = Nwl (A^{+})$

($(cl A)^{+} = Nwl (A^{+})$

Th $10.2.13:$ For $\sigma \in L(U, V)$, $(range \sigma)^{+} = ker(\sigma^{+})$

Proof: $(range \sigma)^{+} = \{\beta \mid \langle \alpha, \beta \rangle = 0 \mid \forall \alpha \in range \sigma\}$
 $= \{\beta \mid \langle \sigma(y), \beta \rangle = 0 \mid \forall y \in U\}$
 $= \{\beta \mid \langle \gamma, \sigma^{+}(\beta) \rangle = 0 \mid \forall y \in U\}$
 $= \{\beta \mid \langle \sigma^{+}(\beta) \rangle = 0 \mid \forall y \in U\}$

Def 10.2. 16 o: V -> V is self-adjoint if o=0*. Ex: Let $V = C^{\circ}([a,b])$ with $\langle f,g \rangle = \int_{a}^{b} f(x)g(x) dx$. Then $\sigma(f) = hf$ i.e. $\sigma(f)(x) = h(x)f(x)$, i.e. $\sigma = multiplication$ by h, is self-adjoint. $\langle f, \sigma(g) \rangle = \langle f, hg \rangle = \int_{\mathcal{B}} f(x)h(x)g(x) dx$ = < tp ' 3> At' 3. $\therefore \sigma^{+}(t) = fh = hf = \sigma(f).$

From theorem above: if $\sigma: V \rightarrow V$ is self-adjoint, and A is an othonormal basis of V, then [0] = [0+] = [0] : if V is over IR, then [o] is symmetric. if V is over C, then [o] is Hermitian.