Deformations of Comonoids-in-Species, Coalgebras, Hopf Algebras and their Quasisymmetric Invariants

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Deformations of Quasisymmetric Invariants

Wanted: people who work with quasisymmetric invariants

e.g. chromatic symmetric functions for graphs,
Billera-Jia-Reiner invariant for matroids,
Ehrenboug rank-generating function for posets, ...

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Idea:
- Quasisymmetric invariants are the image of the Aguiar-Bergeron-Sottile morphism: coalgebra $\to QSym$.
- There is a general framework to q-deform the coalgebra $\to$ q-deformation of quasisymmetric invariants.
- I would like applications for the deformed invariants.

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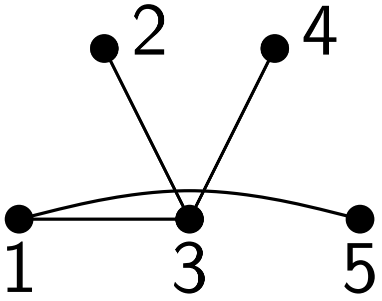
Currently, each coalgebra deformation is calculated case-by-case (if the coalgebra is noncocommutative).

Dream: Find a uniform deformation formula.

Example: q-chromatic quasisymmetric function (qCQSF)

how it arises from a coalgebra of graphs

Consider graphs on vertices labelled $\{1, 2, \ldots, n\}$,

e.g. $x =$ 

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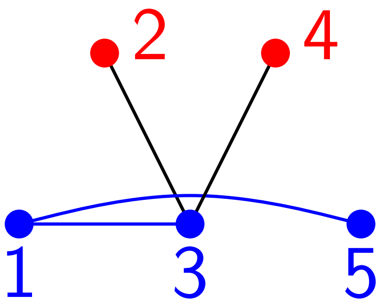
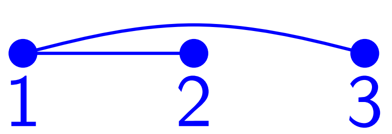
e.g. $x =$ 

Coalgebra structure:

$\Delta_{\{2,4\},\{1,3,5\}}(x) =$ 

$\otimes$ 

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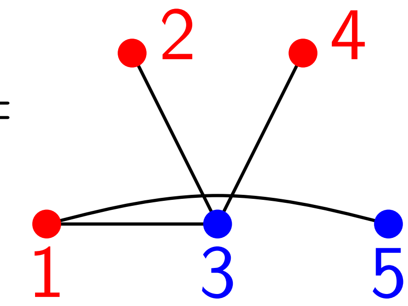
e.g. $x =$ 

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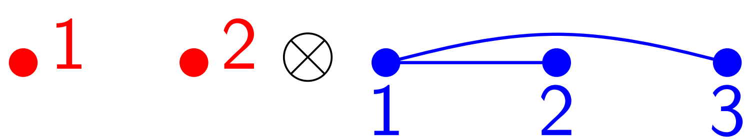
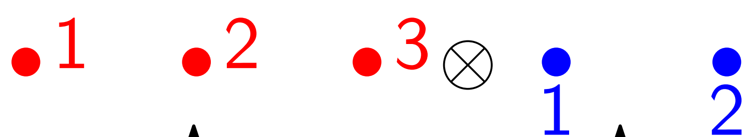
$\Delta_{\{2,4\},\{1,3,5\}}(x) =$ 

$\Delta_{\{1,2,4\},\{3,5\}}(x) =$ 

both are disconnected graphs

$\therefore$ contributes $x_1^3 x_2^2$ to CQSF

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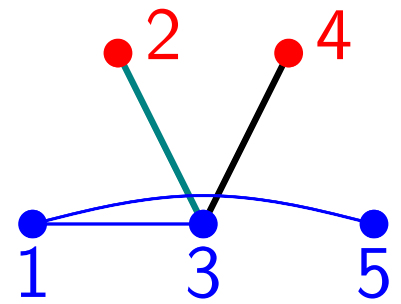
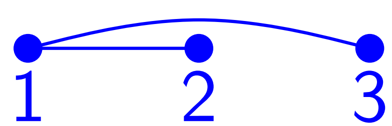
e.g. $x =$ 

Coalgebra structure:

Define $\gamma_{S,T}(x) =$ number of edges $s - t$ in x with $s < t$.
Add $q^{\gamma_{S,T}}(x)$ as a coefficient in $\Delta_{S,T}(x)$.

$\Delta_{\{2,4\},\{1,3,5\}}(x) = q$  $\otimes$ 

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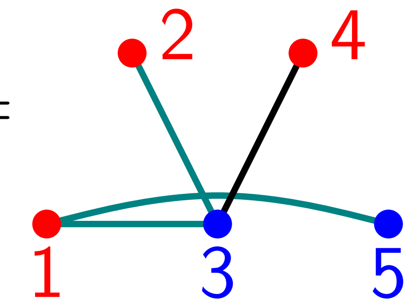
e.g. $x =$ 

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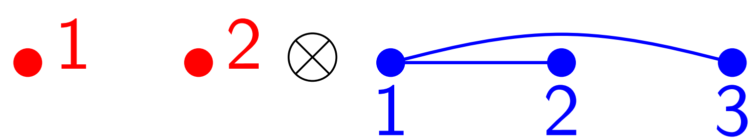
$\Delta_{\{2,4\},\{1,3,5\}}(x) = q$ 

$\Delta_{\{1,2,4\},\{3,5\}}(x) = q^3$ 

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