# Strategies for Integration: what is $\int f(x)\,dx$?

1. Does it look like $\int \frac{g'(x)}{g(x)}\,dx$?

   yes: answer is $\ln|g(x)|$.

   no: continue

* At each "go back", remember to treat each term in your integral separately.

2. Does it look like $\int h(g(x))\,g'(x)\,dx$?

   Do you see $\sin(g(x))$, $\cos(g(x))$, $e^{g(x)}$, $\ln(g(x))$, where $g(x)$ isn't just $(\text{number})\times x$?

   yes: substitution $u = g(x)$, go back to start.

   no: continue

3. Do some algebraic manipulation, go back to start.

4. Which best describes your integrand $f$?

   a rational function: do long division if deg(numerator) $\geq$ deg(denominator), then partial fractions:

   $$\frac{A_1}{ax+b} + \frac{A_2}{(ax+b)^2} + \dots + \frac{A_r}{(ax+b)^r}$$ for each $(ax+b)^r$ in denominator

   $$\frac{A_1x+B_1}{ax^2+bx+c} + \frac{A_2x+B_2}{(ax^2+bx+c)^2} + \dots + \frac{A_rx+B_r}{(ax^2+bx+c)^r}$$ for each $(ax^2+bx+c)^r$ in denominator.

   a product of a polynomial and $\sin x$, $\cos x$, $e^x$: by parts: $u$ = polynomial, $dv = \sin x\,dx, \cos x\,dx, e^x\,dx$.

   a product of a polynomial and $\ln x$: by parts: $u = \ln x$, $dv$ = polynomial $dx$.

   $e^x \sin x$, $e^x \cos x$: by parts twice, boomerang trick product of trigonometric functions: write everything in terms of $\sin x$ and $\cos x$ and go to step 5a, or write everything in terms of $\tan x$ and $\sec x$ and go to step 5b.

contains a squareroot: go to step 6

something else: try substitutions $u=$ any function of $x$ that occurs in the integrand, go back to start.

5a. $\sin^m x \cos^n x$:

power of $\sin x$ is positive and odd: substitution $u=\cos x$

power of $\cos x$ is positive and odd: substitution $u=\sin x$

powers of $\sin x$ and $\cos x$ are both positive and even: double angle formulae, return to step 5a.

none of the above: change to $\tan x$ and $\sec x$ and go to step 5b.

5b. $\tan^m x \sec^n x$: is the power of $\sec x$ positive? If not, convert $\tan^2 x$ into $\sec^2 x - 1$ (repeat many times if necessary), or multiply top and bottom by $\sec x$.

power of $\sec x$ is positive and even: substitution $u=\tan x$

power of $\tan x$ is positive and odd: substitution $u=\sec x$

none of the above: change to $\sin x$ and $\cos x$ and go to step 5a.

6. Substitution $u=$ what's under the squareroot. If that didn't help, what's under the squareroot:

a quadratic: complete the square. Now does it look like:

$\sqrt{a^2-x^2}$: substitution $x=a\sin\theta$, go to step 5a.

$\sqrt{a^2+x^2}$: substitution $x=a\tan\theta$, go to step 5b.

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something messy: try to write it as a perfect square, then use $\sqrt{g(x)^2}=|g(x)|$