Homework 3, due 14:45 Monday 23 October 2017

You must justify your answers to receive full credit.

- 14.2: Q2, 9
- 14.2: Q14, 21
- 14.2: Q16
- 14.4: Q15, 16, 19
- 14.4: Q21, 23: Please sketch the regions also.
- 14.4: Q24, 28 (Hints: Q24 can be solved using symmetry and geometry; in Q28, **use Cartesian** (x, y) **coordinates**, and geometry to find the area of the segment.)
- 14.5: Q1, 4

1. Let R be the region bounded by $x^2 + y^2 + z^2 = 4$ and $x^2 - 2x + y^2 = 0$ with density function $\delta(x, y, z)$. Describe and sketch R, and express its mass as an iterated integral. (You do **not** need to evaluate this integral.)

2. Let D be the region bounded by $z = 3 - 2x^2 - 2y^2$ and $z = 2 - x^2 - y^2$. Find the centroid of D. (Hint: symmetry may be useful.)

3. (This question is to prepare you for the following week's class, and is unrelated to the material from recent classes.) Consider the function $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0; \\ 0 & \text{if } x = 0. \end{cases}$$

   a) Show that f is continuous at $x = 0$. (Hint: you need to use the squeeze theorem.)
   b) Find $f'(x)$ at $x = 0$. (Hint: you need to use the limit definition of the derivative.)
   c) Find the linearisation for f about $x = 1$.
   d) Using your answer to c), approximate $f(0.9)$.

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