From review lecture Dec 4 2015 (Amy Pang).

Compute the following indefinite integral:

$$\int \frac{x^3 + 3x^2 + 18}{x(x^2+3)}\, dx.$$

1. top is not derivative of bottom.
2. no obvious substitution.

so run rational function algorithm:

deg(top) = deg(bottom) $\therefore$ long division:

$$\begin{array}{r} 1 \\ x^3+3x \,\overline{\big)\, x^3+3x^2+0x+18} \\ -\;\; x^3 \qquad\quad +3x \\ \hline 3x^2-3x+18 \end{array}$$

$$\text{So } \int \frac{x^3+3x^2+18}{x(x^2+3)}\, dx = \int 1 + \frac{3x^2-3x+18}{x(x^2+3)}\, dx$$

$$= x + \int \frac{3x^2-3x+18}{x(x^2+3)}\, dx$$

Partial fractions: $$\frac{3x^2-3x+18}{x(x^2+3)} = \frac{A}{x} + \frac{Bx+C}{x^2+3}$$

$$3x^2-3x+18 = A(x^2+3) + (Bx+C)x$$

$$= Ax^2 + 3A + Bx^2 + Cx$$

root of factor of denominator

$x=0$: $18 = 3A \Rightarrow A = 6$

coeff of x^2: $3 = A+B = 6+B \Rightarrow B = -3$

coeff of x: $-3 = C \Rightarrow C = -3$

can also plug in random values for x

substitution: $u = x^2+3$, $du = 2x\,dx$

$$\text{So } \int \frac{x^3+3x^2+18}{x(x^2+3)}\, dx = x + \int \frac{6}{x} - \frac{3x}{x^2+3} - \frac{3}{x^2+3}\, dx = x + 6\ln|x| - \frac{3}{2}\ln|x^2+3| - \int \frac{3}{x^2+3}\, dx$$

$$\int \frac{3}{x^2+3}\,dx \qquad x = \sqrt{3}\tan\theta \rightsquigarrow \frac{x}{\sqrt{3}} = \tan\theta \rightarrow \theta = \arctan\left(\frac{x}{\sqrt{3}}\right).$$

$$dx = \sqrt{3}\sec^2\theta\,d\theta$$

$$= \int \frac{3}{3\sec^2\theta}\sqrt{3}\sec^2\theta\,d\theta$$

$$= \int \sqrt{3}\,d\theta = \sqrt{3}\theta + C = \sqrt{3}\arctan\left(\frac{x}{\sqrt{3}}\right) + C$$

answer: $x + 6\ln|x| - \frac{3}{2}\ln|x^2+3| - \sqrt{3}\arctan\left(\frac{x}{\sqrt{3}}\right) + C.$

Compute the following indefinite integral:

$$\int 20(x\sec(x^5))^4\,dx.$$

$$= \int 20x^4 \sec^4(x^5)\,dx$$

$$= \int 4\sec^4(u)\,du$$

$$= \int 4\sec^2(u)\,\sec^2(u)\,du$$

$$= \int 4(\tan^2 u + 1)\sec^2 u\,du$$

$$= \int 4(w^2+1)\,dw$$

$$= \frac{4w^3}{3} + 4w + C$$

$$= \frac{4\tan^3 u}{3} + 4\tan u + C$$

$$= \frac{4\tan^3(x^5)}{3} + 4\tan(x^5) + C.$$

$u = x\sec(x^5)$
$du = \text{mess.}$ doesn't work.

$u = x^5$
$du = 5x^4\,dx$

$w = \tan u$
$dw = \sec^2 u\,du$

in green is the part we ran out of time for in lecture.

Let C be the parametric curve

$$x = t^3, \quad y = t^2, \quad 0 \le t \le 1.$$

Find the area of the surface obtained by rotating the curve about the x-axis.

$$\text{Surface area} = \int 2\pi y\, ds = \int_0^1 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_0^1 2\pi t^2 \sqrt{(3t^2)^2 + (2t)^2}\, dt$$

try $u = 9t^4 + 4t^2$ — doesn't help.

$$= \int_0^1 2\pi t^2 \sqrt{9t^4 + 4t^2}\, dt$$

not obvious

$$= \int_0^1 2\pi t^2 \sqrt{t^2(9t^2+4)}\, dt$$

$$= \int_0^1 2\pi t^2 \boxed{|t|} \sqrt{9t^2+4}\, dt$$

for $0 \le t \le 1$, $t > 0$

$$= \int_0^1 2\pi t^3 \sqrt{9t^2+4}\, dt$$

$\sqrt{\text{quadratic}}$: can use trig sub $3t = 2\tan\theta$

$$= \int_0^1 \frac{\pi}{9} t^2 \sqrt{9t^2+4}\, (18t\, dt)$$

easier: $u = 9t^2 + 4$.

$du = 18t\, dt$

$u - 4 = 9t^2$

$\frac{u-4}{9} = t^2$

$t = 0 \mapsto u = 4$

$t = 1 \mapsto u = 13$

$$= \int_4^{13} \frac{\pi}{9} \frac{u-4}{9} u^{1/2}\, du$$

$$= \int_4^{13} \frac{\pi}{81} \left(u^{3/2} - 4u^{1/2}\right) du$$

$$= \frac{\pi}{81} \left[\frac{u^{5/2}}{5/2} - \frac{4u^{3/2}}{3/2} \right]_4^{13}$$

$$= \frac{\pi}{81} \left(\frac{2}{5} 13^{5/2} - \frac{8}{3} 13^{3/2} - \frac{2}{5} 4^{5/2} + \frac{8}{3} 4^{3/2} \right)$$