1. (3 points) Approximate the integral

$$\int_{-2}^{10} \arctan(x^3)dx$$

by a left Riemann sum with 4 subintervals.

$\Delta x = \frac{10-(-2)}{4} = 3$ $\therefore x_0 = -2$

$x_1 = 1$

$x_2 = 4$

$x_3 = 7$

$3\arctan((-2)^3) + 3\arctan(1^3) + 3\arctan(4^3) + 3\arctan(7^3)$

2. (4 points) Find the derivative of the function:

$$h(x) = \int_{\tan x}^{5\tan x} e^{2\sin t}dt.$$

Let $F(x)$ be an antiderivative of $e^{2\sin t}$.

$e^{2\sin t}$ is continuous everywhere, so, by FTC2,

$h(x) = F(5\tan x) - F(\tan x)$

so $h'(x) = F'(5\tan x)\frac{d}{dx}(5\tan x) - F'(\tan x)\frac{d}{dx}(\tan x)$

$= e^{2\sin(5\tan x)} 5\sec^2 x - e^{2\sin(\tan x)}\sec^2 x$

3. (5 points) The velocity of a particle at time t is given by the function

$$v(t) = \frac{4t}{1+t^2}$$

Find the total distance travelled by the particle from $t = -1$ to $t = 5$.

$v(t) \geq 0$ for $t \geq 0$, $v(t) \leq 0$ for $t \leq 0$

since denominator is always positive

$\therefore$ distance travelled $= \displaystyle\int_{-1}^{5} |v(t)|\, dt$

$$= \int_{-1}^{0} -v(t)\, dt + \int_{0}^{5} v(t)\, dt$$

$$= \int_{-1}^{0} \frac{-4t}{1+t^2}\, dt + \int_{0}^{5} \frac{4t}{1+t^2}\, dt$$

$$= \Big[-2\ln|1+t^2|\Big]_{-1}^{0} + \Big[2\ln|1+t^2|\Big]_{0}^{5}$$

substitution $u = 1+t^2$.

$$= -0 - (-2\ln 2) + 2\ln 26 - 0$$

$$= 2\ln 2 + 2\ln 26 .$$

4. (4 points) Compute the following indefinite integral:

$$\int \sec^2 x + e^{2x} dx.$$

$$= \tan x + \frac{e^{2x}}{2} + C$$

(substitution $u = 2x$ in second term)

5. (5 points) Compute the following definite integral:

$$\int_1^2 \frac{3x+2}{\sqrt{3x-2}} dx.$$

$u = 3x - 2 \Rightarrow 3x = u + 2$

$3x + 2 = u + 4$

$du = 3dx$

when $x = 1$, $u = 1$

$x = 2$, $u = 4$

$$= \int_1^4 \frac{u+4}{\sqrt{u}} \frac{du}{3}$$

$$= \frac{1}{3} \int_1^4 u^{1/2} + 4u^{-1/2} du$$

$$= \frac{1}{3} \left[\frac{u^{3/2}}{3/2} + 4 \frac{u^{1/2}}{1/2} \right]_1^4$$

$$= \frac{1}{3} \left(\frac{4^{3/2}}{3/2} + 4 \frac{4^{1/2}}{1/2} \right) - \frac{1}{3} \left(\frac{1}{3/2} + 4 \frac{1}{1/2} \right)$$