You must justify your answers to receive full credit.

Some antiderivatives you may find useful:

$$\int \sec^2 x \, dx = \tan x + C, \qquad \int \csc^2 x \, dx = -\cot x + C,$$

$$\int \sec x \tan x \, dx = \sec x + C, \qquad \int \csc x \cot x \, dx = -\csc x + C,$$

$$\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + C, \qquad \int \frac{1}{1+x^2} \, dx = \tan^{-1} x + C,$$

$$\int \sin^2 x \, dx = \frac{1}{2}(x - \sin x \cos x) + C,$$

$$\int \sin^3 x \, dx = -\cos x + \frac{1}{3}\cos^3 x + C.$$

$$\int \sin^4 x \, dx = \frac{1}{8}(3x - 3\sin x \cos x - 2\sin^3 x \cos x) + C,$$

$$\int \cos^2 x \, dx = \frac{1}{2}(x + \sin x \cos x) + C,$$

$$\int \cos^3 x \, dx = \sin x - \frac{1}{3}\sin^3 x + C.$$

$$\int \cos^4 x \, dx = \frac{1}{8}(3x + 3\sin x \cos x + 2\cos^3 x \sin x) + C.$$

- 14.4: Q15, 16, 19
- 14.4: Q21, 23 Please sketch the regions also.
- 14.4: Q24, 28 (Hints: Q24 can be solved using symmetry and geometry; in Q28, use Cartesian coordinates, and geometry to find the area of the segment.)
- 14.5: Q1, 15
- 14.6: Q10, 14 Please sketch the regions also.
- 14.6: Q15, 16 Please sketch the regions also. (Hint: in Q16, first octant means $x \geq 0, y \geq 0, z \geq 0$.)

- 6.1: Q1, 8: $\displaystyle\int x\cos x\,dx$, $\displaystyle\int x^2\tan^{-1}x\,dx$
- 6.1: Q16, 17: $\displaystyle\int_0^1 \sqrt{x}\sin(\pi\sqrt{x})\,dx$, $\displaystyle\int x\sec^2 x\,dx$.

1. Let T be the tetrahedron bounded by $x = 0$, $y = 0$, $z = 0$ and $2x + 3y + 6z = 12$, with density function $\delta(x, y, z)$. Sketch T and express its mass as an iterated integral.

2. (This question is to prepare you for the following week's class, and is unrelated to the material from recent classes.) Consider the function $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \begin{cases} x^2 \sin\left(\dfrac{1}{x}\right) & \text{if } x \neq 0; \\ 0 & \text{if } x = 0. \end{cases}$$

   a) Show that f is continuous at $x = 0$.
   b) Find $f'(x)$ at $x = 0$. (Hint: you need to use the limit definition of the derivative.)
   c) Find the linearisation for f about $x = 1$.
   d) Using your answer to c), approximate $f(0.9)$.

3. **Optional problem**: This problem aims to demystify the integration by parts formula by thinking about it in terms of areas above and below the curve. The concept described here is for interest only and is non-examinable.

   a) Sketch the region bounded by the curves $y = 0$ and $y = \sin x$, and satisfying $0 \le x \le \frac{\pi}{2}$, and find its area.
   b) Sketch the region bounded by the lines $y = 0$, $y = \frac{\pi}{2}$ and the curve $y = \arcsin x$. Compare this with your sketch from part a) and find the area of the region.
   c) Sketch the region bounded by the lines $x = 0$, $x = 1$ and the curve $y = \arcsin x$. Compare this with your sketch from part b) and find the area of the region.
   d) Now compute $\displaystyle\int_0^1 \arcsin x\,dx$ by first using the substitution $u = \arcsin x$, then doing integration by parts. Observe the similarity between this and part c).

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