Step 2: find the longest eigenstrings by finding the tops $\alpha_1, \alpha_2, \dots$

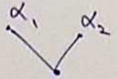
suppose $\dim \operatorname{Ker}(\sigma-\lambda_c)^m = \dim \operatorname{Ker}(\sigma-\lambda_c)^{m+1} = r$

— i.e. m is the maximal eigenstring length

(in example A, $m=2$, $r=4$)

$\therefore$ find $\alpha_i \in \operatorname{Ker}(\sigma-\lambda_c)^m \setminus \operatorname{Ker}(\sigma-\lambda_c)^{m-1}$

— this is not enough: possible to have



i.e. $(\sigma-\lambda_c)\alpha_1 = (\sigma-\lambda_c)\alpha_2$

need eigenstrings to "not intersect"

— need eigenstring bottoms to be linearly independent

$\Updownarrow$

$(\sigma-\lambda_c)^{m-1}(\operatorname{Ker}(\sigma-\lambda_c)^m)$

- Find a basis $\{\alpha_1, \dots, \alpha_r\}$ of $\operatorname{Ker}(\sigma-\lambda_c)^m$
- Consider $\{(\sigma-\lambda_c)^{m-1}(\alpha_1), \dots, (\sigma-\lambda_c)^{m-1}(\alpha_r)\}$
- Take a linearly independent subset of $\{\ \}$ (e.g. casting out algorithm) — the corresponding α_i are the tops we want.

Ex: Continue with A:

- We need basis $\{\alpha_1, \dots, \alpha_r\}$ of $\operatorname{Nul}(A-3I)^2$

  $\because (A-3I)^2 = 0$, so $\operatorname{Nul}(A-3I)^2 = \mathbb{C}^4$

  $\therefore$ we can take $\alpha_1 = e_1$, $\alpha_2 = e_2$, $\alpha_3 = e_3$, $\alpha_4 = e_4$
- Consider $\{(A-3I)^1(e_1), \dots, (A-3I)^1(e_4)\}$

  Here, $(A-3I)^1(e_i)$ are the columns of $A-3I$.

  only because $\alpha_i = e_i$
- linearly independent subset = columns with pivot i.e. 2 and 4.

$\therefore$ eigenstring tops can be e_2 and e_4.

e_2 β_2

$(A-3I)e_2$ β_1

3

$$(A-3I)e_2 = \begin{pmatrix} -2 \\ 3 \\ 3 \\ 0 \end{pmatrix}$$

e_4 β_4

$(A-3I)e_4$ β_3

3

$$(A-3I)e_4 = \begin{pmatrix} -1 \\ 2 \\ 2 \\ 0 \end{pmatrix}$$

$$J = \begin{pmatrix} 3 & 1 & \cdot & \cdot \\ \cdot & 3 & \cdot & \cdot \\ \cdot & \cdot & 3 & 1 \\ \cdot & \cdot & \cdot & 3 \end{pmatrix}$$

Jordan basis is

$$\mathcal{B} = \left\{ \begin{pmatrix} -2 \\ 3 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

i.e. $P = \begin{pmatrix} -2 & 0 & -1 & 0 \\ 3 & 1 & 2 & 0 \\ 3 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$