1. (3 points) Approximate the integral

$$\int_1^7 \frac{\sin x}{x} dx$$

by a right Riemann sum with 3 subintervals.

$$\Delta x = \frac{7-1}{3} = \frac{6}{3} = 2$$

$x_0 = 1$
$x_1 = 3$
$x_2 = 5$
$x_3 = 7$

$$2\,\frac{\sin 3}{3} + 2\,\frac{\sin 5}{5} + 2\,\frac{\sin 7}{7}$$

2. (4 points) Find the derivative of the function:

$$h(x) = \int_{-x}^{3x^2+2} \frac{\sin t}{1+t^2} dt.$$

Let $F(x)$ be an antiderivative of $\frac{\sin t}{1+t^2}$. $\frac{\sin t}{1+t^2}$ is continuous everywhere

$\therefore$ by FTC2,

$h(x) = F(3x^2+2) - F(-x)$

so $h'(x) = F'(3x^2+2)\frac{d}{dx}(3x^2+2) - F'(-x)\frac{d}{dx}(-x)$ by chain rule

$= \frac{\sin(3x^2+2)}{1+(3x^2+2)^2}\, 6x - \frac{\sin(-x)}{1+(-x)^2}(-1)$

3. (5 points) The velocity of a particle at time t is given by the function

$$v(t) = (t+1)(t-1). = t^2 - 1$$

Find the total distance travelled by the particle from $t = -1$ to $t = 4$.

For $-1 < t < 4$, we have $t+1 \geq 0$ always, so the sign of $v(t)$ is the sign of $t-1$

i.e. $v(t) \geq 0$ on $[1,4]$

$v(t) \leq 0$ on $[-1,1]$

so distance travelled $= \int_{-1}^{4} |v(t)|\, dt$

$$= \int_{-1}^{1} -v(t)\, dt + \int_{1}^{4} v(t)\, dt$$

$$= \int_{-1}^{1} -t^2+1\, dt + \int_{1}^{4} t^2-1\, dt$$

$$= \left[\frac{-t^3}{3} + t\right]_{-1}^{1} + \left[\frac{t^3}{3} - t\right]_{1}^{4}$$

$$= \left(\frac{-1}{3}+1\right) - \left(\frac{1}{3}-1\right) + \left(\frac{4^3}{3}-4\right) - \left(\frac{1}{3}-1\right)$$

$$= \frac{58}{3}$$

4. (4 points) Compute the following indefinite integral:

$$\int 5x\cos(x^2-5)dx.$$

$u = x^2 - 5$

$\frac{du}{dx} = 2x$

$= \frac{5}{2}\int \cos u \, du$

$= \frac{5}{2}\sin u + C$

$= \frac{5}{2}\sin(x^2-5) + C.$

5. (5 points) Compute the following definite integral:

$$\int_0^1 e^{2x}\sqrt{e^x+1}dx.$$

$u = e^x + 1$

$\frac{du}{dx} = e^x$

$= \int_2^{1+e} (u-1)\sqrt{u}\, du$

when $x = 0$, $u = e^0 + 1 = 2$

$x = 1$, $u = e^1 + 1$

$= \int_2^{1+e} u^{3/2} - u^{1/2} \, du$

$= \left[\frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2}\right]_2^{1+e}$

$= \frac{(1+e)^{5/2}}{5/2} - \frac{(1+e)^{3/2}}{3/2} - \left(\frac{2^{5/2}}{5/2} - \frac{2^{3/2}}{3/2}\right)$