Homework 3, due 12:00 Wednesday, 20 March 2019 to Dr. Pang's mailbox

You must justify your answers to receive full credit.

1. Let V be a vector space over $\mathbb{F}$ and $\sigma \in L(V, V)$. Let $\alpha \in V$ be an eigenvector of σ with eigenvalue λ.

   a) Show that α is an eigenvector of σ^n with eigenvalue λ^n (for $n = 1, 2, \dots$).

   Let $f(x) = a_0 + a_1 x + \cdots + a_n x^n$, for some $a_i \in \mathbb{F}$.

   b) Show that α is an eigenvector of $f(\sigma)$ with eigenvalue $f(\lambda)$.

   c) Show that, if $f(\sigma) = 0$ (zero function), then $f(\lambda) = 0$ (zero number).

2. Let $C^0(\mathbb{R})$ denote the vector space of continuous functions on $\mathbb{R}$. Consider $\sigma : C^0(\mathbb{R}) \to C^0(\mathbb{R})$ given by

$$\sigma(f)(x) = f''(x) + f(x) + f(0).$$

   a) Show that, for each n, the function $f(x) = \sin(nx)$ is an eigenvector of σ, and find its corresponding eigenvalue.

   b) Let $W = P_{<3}(\mathbb{R})$ be the subspace of polynomials with degree less than 3. Find three linearly independent eigenvectors of the restriction $\sigma|_W$. (Click here for a hint)

3. to be released later

4. to be released later

5. to be released later

6. Find, with explanation, all the possible Jordan forms of the following matrices:

   a) The characteristic polynomial of A is $-(x-2)^3(x+7)^2$, and the minimal polynomial of A has degree 3.

   b) The characteristic polynomial of B is $-(x-3)^5$, and $\ker(B-3I)$ is 3-dimensional.

   c) The characteristic polynomial of C is $(x+2)^4(x-1)^2$, and $\ker(C+2I)^2$ is 3-dimensional.

7. to be released later

8. to be released later

Optional question If you attempted seriously all the above questions, then your scores for the following question may replace any lower scores for one of the above questions.

9. to be released later

10. to be released later

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