Additional possible FYPs with Dr. Amy Pang, Fall 2019

Here are some other project ideas that you can do with me. Please speak to me if you are interested.

3. **Determinants of Almost-Triangular Matrices**

Let $S = \{-1, 0\}$ and consider $n \times n$ matrices of the form

$$UH(S)_n = \left\{ \begin{bmatrix} 0 & * & * & \dots & * \\ 1 & \ddots & * & \dots & * \\ 0 & \ddots & \ddots & * & \vdots \\ \vdots & \ddots & \ddots & \ddots & * \\ 0 & \dots & 0 & 1 & 0 \end{bmatrix} \middle| \; * \in S \right\}.$$

(The $*$s in different entries can be different.)

Let F_n denote the nth Fibonacci number, i.e. from the sequence $0, 1, 1, 2, 3, 5, 8, 13, \dots$. Computer calculations suggest that

a. the determinant of any matrix from $UH(S)_n$ is at most F_n;

b. the number of distinct determinants for matrices from $UH(S)_n$ is $1 + F_n$.

In this project, we will attempt to prove a. and b., by expanding the determinant along a suitable row or column and then applying induction with the recurrence $F_n = F_{n-1} + F_{n-2}$. A similar result has been proved in [2], and suitably varying the proof there should work.

[1, Conjectures 12-20] gives several other sets S for which a. above is true. So, if there is time, then we may also consider

c. What condition on the set S guarantees that the determinant on $UH(S)_n$ is at most F_n?

d. How does the number of distinct determinants for matrices from $UH(S)_n$ depend on S?

Prerequisites: Ability to write clear accurate proofs, and to work independently.

References

[1] S. E. Thornton, The Characteristic Polynomial Database, published electronically at http://bohemianmatrices.com/cpdb, January 2019.

[2] Ching, L. (1993). The maximum determinant of an $n \times n$ lower Hessenberg $(0, 1)$ matrix. Linear algebra and its applications, 183, 147-153.