1. (3 points) Approximate the integral

$$\int_1^{10} \cos(\ln x)dx$$

by a right Riemann sum with 3 subintervals.

$\Delta x = \frac{10-1}{3} = 3$

$a = x_0 = 1$
$x_1 = 4$
$x_2 = 7$
$b = x_3 = 10$

$3\cos(\ln 4) + 3\cos(\ln 7) + 3\cos(\ln 10)$

2. (4 points) Find the derivative of the function:

$$h(x) = \int_{-x}^{x^4} \cos(t^3)dt.$$

Let $F(x)$ be an antiderivative of $\cos(t^3)$. $\cos(t^3)$ is continuous everywhere, so

so by FTC2, $h(x) = F(x^4) - F(-x)$

and by chain rule, $h'(x) = F'(x^4)\frac{d}{dx}(x^4) - F'(-x)\frac{d}{dx}(-x)$

$= \cos(x^{12})4x^3 - \cos(-x^3)(-1)$

$= \cos(x^{12})4x^3 + \cos(x^3).$

3. (5 points) The velocity of a particle at time t is given by the function

$$v(t) = -\frac{6t^3}{1+t^4}.$$

Find the total distance travelled by the particle from $t = -2$ to $t = 1$.

denominator of $v(t)$ is always positive $\therefore$ $v(t) \geq 0$ when $-6t^3 \geq 0$ i.e. $t \leq 0$
$v(t) \leq 0$ when $-6t^3 \leq 0$ i.e. $t \geq 0$

$\therefore$ distance travelled $= \int_{-2}^{1} |v(t)|\, dt$

$$= \int_{-2}^{0} v(t)\, dt + \int_{0}^{1} -v(t)\, dt$$

$$= \int_{-2}^{0} \frac{-6t^3}{1+t^4}\, dt + \int_{0}^{1} \frac{6t^3}{1+t^4}\, dt$$

$$= \left[-\frac{3}{2} \ln|1+t^4|\right]_{-2}^{0} + \left[\frac{3}{2} \ln|1+t^4|\right]_{0}^{1}$$

$$= 0 - \left(-\frac{3}{2}\ln 17\right) + \frac{3}{2}\ln 2 - 0$$

$$= \frac{3}{2}\ln 17 + \frac{3}{2}\ln 2.$$

4. (4 points) Compute the following indefinite integral:

$$\int \cos 2x + 3e^x \, dx.$$

$$= \frac{\sin 2x}{2} + 3e^x + C$$

(substitution $u = 2x$ in first term)

5. (5 points) Compute the following definite integral:

$$\int_0^4 \frac{4x^2}{\sqrt{2x+1}} dx.$$

$u = 2x+1$ $du = 2\,dx$

$u - 1 = 2x$

$(u-1)^2 = 4x^2$

when $x = 0$, $u = 1$

$x = 4$, $u = 9$

$$= \int_1^9 \frac{(u-1)^2}{\sqrt{u}} \frac{du}{2}$$

$$= \int_1^9 \frac{u^2 - 2u + 1}{2\sqrt{u}} du$$

$$= \int_1^9 \frac{1}{2}\left(u^{3/2} - 2u^{1/2} + u^{-1/2}\right) du$$

$$= \frac{1}{2}\left[\frac{u^{5/2}}{5/2} - 2\frac{u^{3/2}}{3/2} + \frac{u^{1/2}}{1/2}\right]_1^9$$

$$= \frac{9^{5/2}}{5} - \frac{2(9)^{3/2}}{3} + \frac{9^{1/2}}{1} - \left(\frac{1}{5} - 2\frac{1}{3} + \frac{1}{1}\right)$$