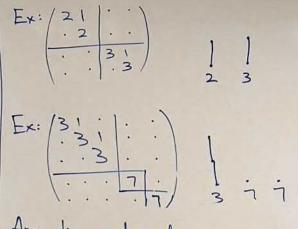
E_{X} : $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix}$, $m_A = ?$ (A-I) (A-2I) = (00;)(-1-1; X, (x) = (1-x) (1-x)(2-x) so, by above, my can be = (00) - (x-1)(x-5) or (x-1)2(x-5) : ma(x) = (x-1)(x-2) this has lover degree: In general Th. 5.3.10 if A satisfies (x-D(x-2), then OEL(V,V) is diagonalisable $\Leftrightarrow m_{\sigma}(x) = (x-\lambda_1)\cdots(x-\lambda_k),$ all 7: distinct.

Proof (very sketch):

=> similar to example above. € : either consequence of Jordan form $\ker((\sigma-\lambda_i)...(\sigma-\lambda_j c)) = \ker(\sigma-\lambda_i) \oplus ... \oplus \ker(\sigma-\lambda_i c)$ by induction on j. :. if $(\sigma - \lambda, \epsilon) \cdots (\sigma - \lambda \kappa \epsilon) = zero function,$ then V=ker(1) = ker(0-7,100... @ker(0-2,0) chaose a basis for each Ez; — their union is a basis for V consisting of eigenvectors

§ 8.2 Jordan form. Recall: not all matrices are diagonalisable all matrices over C are triangularisable i.e. P'AP is triangular BUT: an arbitrary triangular matrix is complicated. Can we make P'AP into a particularly simple triangular matrix? Def: A 2- Jordan block of sizem (ZEF, METN) is the mxm matrix with λ on diagonal immediately above the diagonal O elsewhere

e.g. size 3: (2 1 ·) size 4: $\begin{pmatrix} \lambda & 1 & 1 \\ \vdots & \lambda & 1 & 1 \\ \vdots & \ddots & \lambda & 1 \end{pmatrix}$ size 1: (x) A matrix is in Jordan form if it is where each Ji is a Jordan block. (different Ji may have same eigenvalues,



Any diagonal matrix is in Tordan form, where all blocks have size I, i.e. i, i.e. in

Th. Let V be a finite-dimensional vector space over C, o EL(V,V) Then Ibasis B of V such that LoJB is in Jordan form, and the Jordan form is unique up to reordering of blocks (but many choices of B) — i.e. the number of blocks of each size and each eigenvalue is unique.

(Norks for any field where all polynomials have solutions.)

Th. in terms of matrices: for each $A \in M_{n,n}(C)$, \exists invertible P, and $\exists J$ in J ordan form, such that A = PJP'. $P = \begin{bmatrix} C \\ B \end{bmatrix} = \begin{pmatrix} B \\ B \end{pmatrix}$ Where B= {B, B2,...,Bn} 1? I how to find P (i.e. find B)