Homework 5, due 20:00 Thursday 2 May 2019 to Dr. Pang's mailbox

You must justify your answers to receive full credit.

1. Consider $\mathbb{C}^2$ with the standard dot product. Let

$$\alpha = \begin{pmatrix} 2+3i \\ -1-i \end{pmatrix}, \beta = \begin{pmatrix} -2i \\ 2 \end{pmatrix}, \gamma = \begin{pmatrix} -1+4i \\ 2-i \end{pmatrix}.$$

Calculate the following quantities:

a) $\alpha \cdot \beta$

b) $\beta \cdot \alpha$

c) the length of α

d) the distance between β and γ.

2. Prove the following "parallelogram law":

$$\|\alpha+\beta\|^2 + \|\alpha-\beta\|^2 = 2\|\alpha\|^2 + 2\|\beta\|^2.$$

3. Consider $P_{<3}(\mathbb{R})$, the vector space of polynomials over $\mathbb{R}$ of degree less than 3, with inner product

$$\langle f, g\rangle = \int_{-1}^{1} (1+3x) f(x) g(x)\, dx.$$

Construct an orthogonal basis for $P_{<3}(\mathbb{R})$. You may use the following:

$$\int_{-1}^{1} x^n\, dx = \begin{cases} 0 & \text{if } n \text{ odd} \\ \dfrac{2}{n+1} & \text{if } n \text{ even.} \end{cases}$$

(Click here for a hint)

4. Consider $M_{2,2}(\mathbb{R})$ with the inner product

$$\langle A, B\rangle = \mathrm{Tr}(A^T B).$$

Consider the subspace W with orthogonal basis $\left\{ \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix} \right\}$.

a) Calculate the orthogonal projection of $\begin{pmatrix} 4 & 3 \\ -2 & 1 \end{pmatrix}$ onto W.

b) Find the closest point in W to $\begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix}$.

5. to be released later

6. to be released later

7. to be released later

8. to be released later

Optional questions If you attempted seriously all the above questions, then your scores for the following questions may replace any lower scores for two of the above questions.

9. to be released later

10. to be released later

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