1. (3 points) Approximate the integral

$$\int_{-2}^{18} \frac{1}{\sqrt{1+\sin x}}dx$$

by a right Riemann sum with 4 subintervals.

$$\Delta x = \frac{18-(-2)}{4} = \frac{20}{4} = 5$$

$x_0 = -2$

$x_1 = 3$

$x_2 = 8$

$x_3 = 13$

$x_4 = 18$

$$5\frac{1}{\sqrt{1+\sin 3}} + 5\frac{1}{\sqrt{1+\sin 8}} + 5\frac{1}{\sqrt{1+\sin 13}} + 5\frac{1}{\sqrt{1+\sin 18}}$$

2. (4 points) Find the derivative of the function:

$$h(x) = \int_{\arctan x}^{2\arctan x} \cos(e^{2t} - 1)dt.$$

Let $F(x)$ be an antiderivative of $\cos(e^{2t}-1)$. $\cos(e^{2t}-1)$ is continuous everywhere,

so by FTC 2, $h(x) = F(2\arctan x) - F(\arctan x)$

$\therefore$ by chain rule, $h'(x) = F'(2\arctan x)\frac{d}{dx}(2\arctan x) - F'(\arctan x)\frac{d}{dx}\arctan x$

$$= \frac{2\cos(e^{4\arctan x}-1)}{1+x^2} - \frac{\cos(e^{2\arctan x}-1)}{1+x^2}$$

3. (5 points) The velocity of a particle at time t is given by the function

$$v(t) = \sin 2t.$$

Find the total distance travelled by the particle from $t = -\frac{\pi}{4}$ to $t = \frac{\pi}{2}$. **Simplify your answer as much as possible.**

$v(t) = 0$ when $\sin 2t = 0$

i.e. $2t = n\pi$

$t = \frac{n\pi}{2}$

$\therefore v(t) < 0$ on $-\frac{\pi}{4} < t < 0$, $v(t) > 0$ on $0 < t < \frac{\pi}{2}$.

$\therefore$ distance travelled $= \int_{-\frac{\pi}{4}}^{\frac{\pi}{2}} |v(t)|\, dt$

$$= \int_{-\frac{\pi}{4}}^{0} -v(t)\, dt + \int_{0}^{\frac{\pi}{2}} v(t)\, dt$$

$$= \int_{-\frac{\pi}{4}}^{0} -\sin 2t\, dt + \int_{0}^{\frac{\pi}{2}} \sin 2t\, dt$$

$$= \left[\frac{\cos 2t}{2}\right]_{-\frac{\pi}{4}}^{0} + \left[\frac{-\cos 2t}{2}\right]_{0}^{\frac{\pi}{2}}$$

$$= \frac{1}{2} - 0 + \left(\frac{-(-1)}{2}\right) - \left(\frac{-1}{2}\right) = \frac{3}{2}$$

4. (4 points) Compute the following indefinite integral:

$$\int \frac{1}{x}\left((\ln x)^2 + x^2\right) dx.$$

$$= \int \frac{1}{x}(\ln x)^2 + x \, dx$$

$$= \frac{(\ln x)^3}{3} + \frac{x^2}{2} + C$$

(substitution $u = \ln x$ in first term)

5. (5 points) Compute the following definite integral:

$$\int_0^1 (x+1)(x+5)^{\frac{3}{2}} dx.$$

$u = x+5$ so $x+1 = u-4$

$du = dx$

when $x = 0$, $u = 5$

when $x = 1$, $u = 6$

$$= \int_5^6 (u-4)u^{3/2} \, du$$

$$= \int_5^6 u^{5/2} - 4u^{3/2} \, du$$

$$= \left[\frac{u^{7/2}}{7/2} - \frac{4u^{5/2}}{5/2}\right]_5^6$$

$$= \left(\frac{6^{7/2}}{7/2} - \frac{4(6)^{5/2}}{5/2}\right) - \left(\frac{5^{7/2}}{7/2} - \frac{4(5)^{5/2}}{5/2}\right)$$