MATH 2207: Linear Algebra
Homework 1, due 13:45 Friday 3 February 2017

You must justify your answers to receive full credit.

1. Find a polynomial of degree 3 (a polynomial of the form $f(t) = a + bt + ct^2 + dt^3$) whose graph passes through the points $(0,1)$, $(1,0)$, $(-1,0)$ and $(2,-15)$.

2. Consider a linear system whose augmented matrix has the form

$$\left[\begin{array}{ccc|c} 2 & 4 & -3 & 6 \\ 0 & b & 7 & 2 \\ 0 & 0 & a & a \end{array}\right].$$

   a) For what values of a and b will the system be inconsistent? Explain your answer. (Hint: you should consider $b \neq 0$ and $b = 0$ separately.)

   b) For what values of a and b will the system have infinitely many solutions? Explain your answer.

3. Let

$$A = \begin{bmatrix} | & | & | \\ \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \\ | & | & | \end{bmatrix} = \begin{bmatrix} 1 & 3 & -4 \\ 0 & 0 & -2 \\ -2 & 6 & 3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 4 \\ 1 \\ -4 \end{bmatrix}.$$

   a) Is $\mathbf{b}$ in $\{\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}\}$? How many vectors are in $\{\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}\}$?

   b) Is $\mathbf{b}$ in Span $\{\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}\}$? How many vectors are in Span $\{\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}\}$?

   c) Is $\mathbf{a}_1$ in Span $\{\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}\}$? Explain your answer. (Hint: you do not need to do any calculation.)

4. Find the intersection of the planes

$$\begin{aligned} x + 7y - 5z &= 10 \\ x + 4y - 2z &= 7 \\ x + 6y - 4z &= 9 \end{aligned}$$

   and give your answer in parametric form. Is this intersection a point, a line or a plane?

5. Give, in parametric form, the solution to the linear system associated to the following augmented matrix:

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 1 & 6 \end{array}\right].$$

6. State whether each of the following statements is always true or sometimes false. If it is true, give a brief justification (e.g. by referring to results from the textbook or from class); if it is false, give a numerical counterexample with an explanation.

   a) If $\mathbf{v}_3$ is in Span$\{\mathbf{v_1}\}$, then $\mathbf{v}_3$ is in Span$\{\mathbf{v_1}, \mathbf{v_2}\}$.

   b) If $\mathbf{v}_3$ is in Span$\{\mathbf{v_1}, \mathbf{v_2}\}$, then $\mathbf{v}_3$ is in Span$\{\mathbf{v_1}\}$.

   c) If rref(A) contains a row of zeros, then $A\mathbf{x} = \mathbf{b}$ is an inconsistent system.

   d) If $A\mathbf{x} = \mathbf{b}$ is an inconsistent system, then rref(A) contains a row of zeros.

   e) A system of 3 equations in 5 variables cannot have a unique solution.

   f) Suppose A is an $m \times n$ matrix and there is a vector $\mathbf{b}$ in $\mathbb{R}^m$ such that $A\mathbf{x} = \mathbf{b}$ has a unique solution. Then, if $\mathbf{c}$ is any vector such that $A\mathbf{x} = \mathbf{c}$ is consistent, then $A\mathbf{x} = \mathbf{c}$ has a unique solution.

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