P8160 - Project 3
Baysian modeling of hurrican trajectories

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Motivation

Climate researchers are interested in modeling the hurricane trajectories to forecast the wind speed.

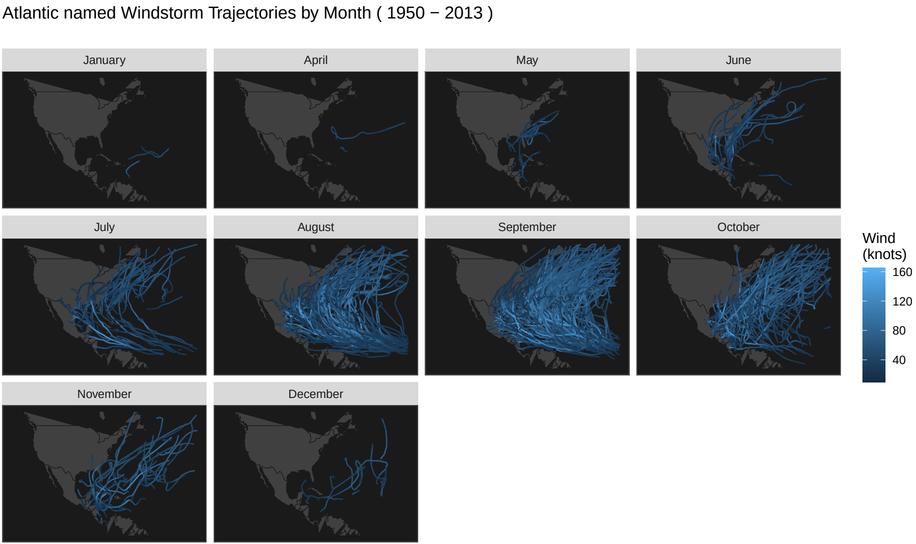
Data

- **ID**: ID of hurricanes
- **Year**: In which the hurricane occurred
- **Month**: In which the hurricane occurred
- **Nature**: Nature of the hurricane
  - ET: Extra Tropical
  - DS: Disturbance
  - NR: Not Rated
  - SS: Sub Tropical
  - TS: Tropical Storm
- **Time**: dates and time of the record
- **Latitude** and **Longitude**: The location of a hurricane check point
- **Wind.kt**: Maximum wind speed (in Knot) at each check point

Outline

1. Exploration into the Data
2. Bayesian modeling of hurricane wind speed
    - Model Equation
    - Posterior Derivation
    - MCMC Algorithm
3. How Month, Year, and the Nature of the hurricane affect wind speed
    - Explore seasonal differences
    - Explore if wind speeds is increasing over years
4. Exploring wind speeds impact on death and damages
5. As well as the characteristic of a hurricane associated with damages and deaths

Data



Atlantic named Windstorm Trajectories by Month (1950 − 2013)
January
April
May
June
July
August
September
October
November
December
Wind
(knots)
160
120
80
40

Data Cleaning

- We are only concerned about observations that occurred on 6 hour intervals. hour 0, 6, 12, and 18.
- In addition we will exclude all hurricane IDs that have less then 7 observations.
- We used a lag (t to $t-6$) for the latitude, longitude and wind speed.

Through this process we remove 460 observations so we are left with 21578 observations and 681 unique hurricanes.

Bayesian Model for Hurricane Trajectories

To model the wind speed of the i^{th} hurricane at time t we will use

$$Y_i(t+6) = \beta_{0,i} + \beta_{1,i}Y_i(t) + \beta_{2,i}\Delta_{i,1}(t) + \beta_{3,i}\Delta_{i,2}(t) + \beta_{4,i}\Delta_{i,3}(t) + \epsilon$$

Where

- $\Delta_{i,1}(t)$, $\Delta_{i,2}(t)$ and $\Delta_{i,3}(t)$ are changes in latitude longitude and wind speed respectively between $t-6$ and t
- $\epsilon_i(t) \sim N(0, \sigma^2)$ independent across t
- Let $\beta_i = (\beta_{0,i}, \beta_{1,i}, \beta_{2,i}, \beta_{3,i}, \beta_{4,i}) \sim \mathcal{N}(\mu, \Sigma)$ be multivariate normal distribution where $\mu \in \mathbb{R}^d$ and $\Sigma \in \mathbb{R}^{d\times d}$.

Prior Distributions Assumptions:

- For σ^2 we assume $\pi(\sigma^2) \propto \frac{1}{\sigma^2}$
- For μ we assume $\pi(\mu) \propto 1$
- For Σ we assume $\pi\left(\Sigma^{-1}\right) \propto |\Sigma|^{-(d+1)} \exp\left\{-\frac{1}{2}\Sigma^{-1}\right\}$

Goal: Estimate $\Theta = (B, \mu, \Sigma^{-1}, \sigma^2)$

Likelihood & Prior Function

Likelihood

$$L(Y \mid \theta) \propto \prod_{i=1}^{m} \left(\sigma^2\right)^{-\frac{n_i}{2}} \exp\left\{-\frac{1}{2\sigma^2}\left(Y_i - X_i\beta_i\right)^T\left(Y_i - X_i\beta_i\right)\right\}$$

where m is the number of hurricane and n_i is the number of observations for i^{th} hurricane

Prior Let $\theta = (B, \mu, \Sigma^{-1}, \sigma^2)$

$$\pi(\theta) \propto \left(\sigma^2\right)^{-1}\left|\Sigma^{-1}\right|^{d+1}\exp\left\{-\frac{1}{2}\operatorname{tr}\left(\Sigma^{-1}\right)\right\}\prod_{i=1}^{m}\left|\Sigma^{-1}\right|^{\frac{1}{2}}\exp\left\{-\frac{1}{2}\left(\beta_i-\mu\right)^T\Sigma^{-1}(\beta_i-\mu)\right\}$$

where d is the dimension of μ

Posterior Calculation

Posterior

$$\pi(\theta \mid Y) \propto \left(\sigma^2\right)^{-\left(1+\frac{\sum_{i=1}^{m} n_i}{2}\right)} \left|\Sigma^{-1}\right|^{d+1+\frac{m}{2}} \exp\left\{-\frac{1}{2}\operatorname{tr}\left(\Sigma^{-1}\right)\right\}$$
$$\times \exp\left\{-\frac{1}{2}\sum_{i=1}^{m}\left(\beta_i-\mu\right)^T \Sigma^{-1}\left(\beta_i-\mu\right)\right\} \exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^{m}\left(Y_i - X_i\beta_i\right)^T\left(Y_i - X_i\beta_i\right)\right\}$$

Conditional Posterior

$$\beta_i : \pi(\beta_i \mid \theta_{(-\beta_i)} Y) \propto \exp\left\{-\frac{1}{2}\left(\beta_i-\mu\right)^T \Sigma^{-1}\left(\beta_i-\mu\right) - \frac{1}{2\sigma^2}\left(Y_i - X_i\beta_i\right)^T\left(Y_i - X_i\beta_i\right)\right\}$$
$$\mu : \pi\left(\mu \mid \theta_{(-\mu)}, Y\right) \sim N(\bar{\beta}, \Sigma/m)$$
$$\sigma^2 : \pi\left(\sigma^2 \mid \theta_{(-\sigma^2)}, Y\right) \propto \left(\sigma^2\right)^{-\left(1+\frac{\sum_{i=1}^{m} n_i}{2}\right)} \times \exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^{m}\left(Y_i - X_i\beta_i\right)^T\left(Y_i - X_i\beta_i\right)\right\}$$
$$\Sigma^{-1} : \pi\left(\Sigma^{-1} \mid \theta_{(-\Sigma^{-1})}, Y\right) \sim \text{Wishart}\left(3d+3+m, \left(I + \sum_{i=1}^{m}\left(\beta_i-\mu\right)\left(\beta_i-\mu\right)^T\right)^{-1}\right)$$

MCMC Algorithm

We apply hybrid algorithm consisting with Metropolis-Hastings steps and Gibbs steps.

Update component wise:

- Sampling proposed β'_{ij}, $j = 0, 1...4$ for i^{th} hurricane from proposal distribution $U\left(\beta_{ij}^{(t)} - a_{ij}, \beta_{ij}^{(t)} + a_{ij}\right)$, where a_{ij} is the search window for β_{ij}. Since the proposals are symmetry, the accepting or rejecting the proposed β'_{ij} depends on the ratio of posterior distribution.
- Then, Gibb step for μ: Sample $\mu^{(t+1)}$ from $N\left(\bar{\beta}^{(t+1)}, \Sigma^{(t)}/m\right)$, where $\bar{\beta}^{(t+1)}$ is the average over $\beta_i^{(t+1)}$.
- Next, MH step to generate $\sigma^{2'}$ from $U\left(\sigma^{2^{(t)}} - a_{\sigma^2}, \sigma^{2^{(t)}} + a_{\sigma^2}\right)$.
- Finally, we sample $\Sigma^{-1^{(t+1)}}$ from $\text{Wishart}\left(3d + 3 + m, \left(I + \sum_{i=1}^{m}\left(\beta_i^{(t+1)} - \mu^{(t+1)}\right)\left(\beta_i^{(t+1)} - \mu^{(t+1)}\right)^T\right)^{-1}\right)$

Initial Starting Values

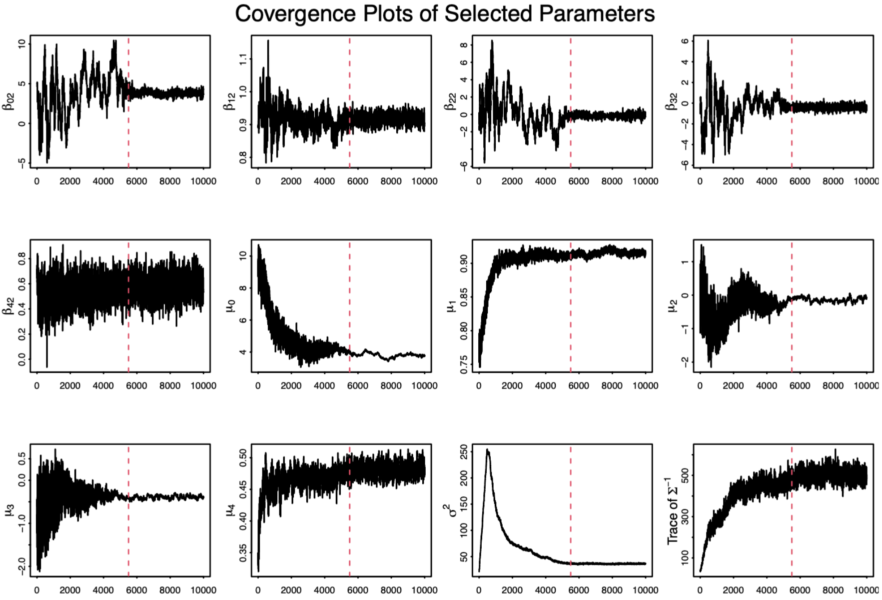
Initial Values:

- β_i: Fit multivariate linear regression (MLR) for i^{th} hurricane and use the coefficients as β_{i0}
- μ: Average over all β_{i0} as μ_0
- σ^2: $\hat{\sigma}_i^2$ is the mean square residuals of the MLR model on i^{th} hurricane. Take the mean over all $\hat{\sigma}_i^2$ as σ_0^2
- Σ^{-1}: Generate the covaraiance matrix of β_{i0} and take the inverse of the matrix as Σ_0^{-1}

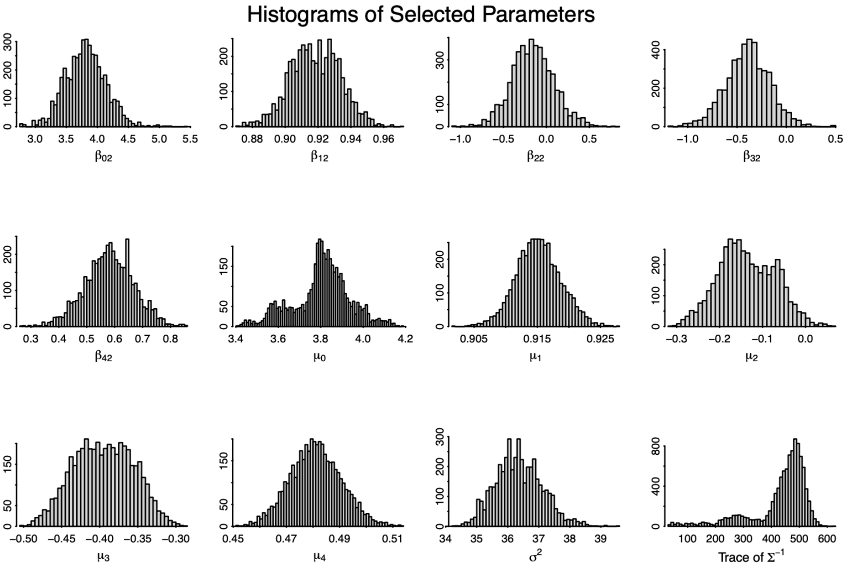
Table 1: Range of Search Window and Acceptance Rate for paraemters used MH step

	Search Window	Acceptance Rate (%)
β_0	1.1	45.87 - 51.36
β_1	(0.04, 0.1)	31.67 - 63.68
β_2	(0.8, 1.0)	38.60 - 45.60
β_3	(0.5, 0.6)	33.20 - 61.32
β_4	(0.4, 0.5)	34.95 - 60.45
σ^2	2.0	44.83

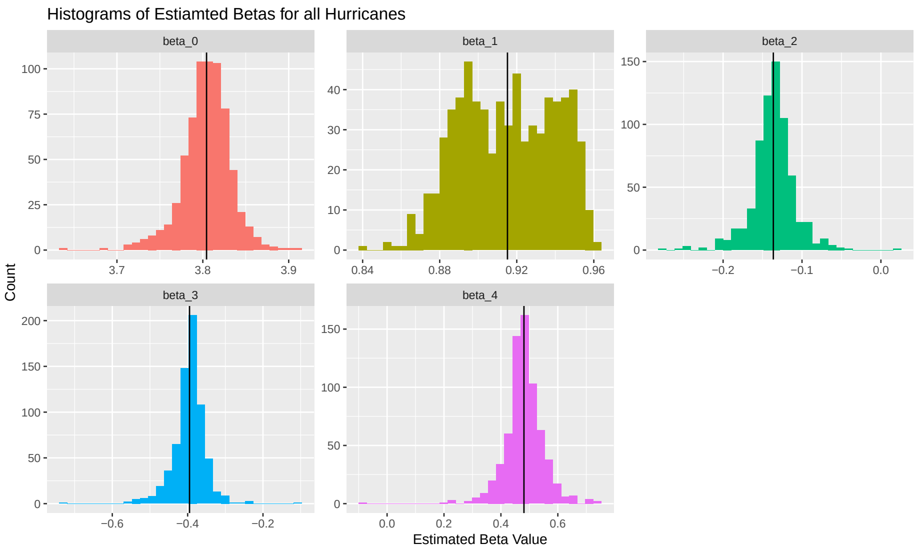
MCMC Model Convergence



MCMC Model Convergence



B Estimates


Histograms of Estiamted Betas for all Hurricanes
beta_0
beta_1
beta_2
beta_3
beta_4
Count
Estimated Beta Value

The μ and σ^2 Estiamtes

Table 2: Bayesian Estiamtes for μ and σ^2

	μ_0	μ_1	μ_2	μ_3	μ_4	σ^2	σ^2_{00}	σ^2_{11}	σ^2_{22}	σ^2_{33}	σ^2_{44}
Estimates	3.8	0.92	-0.14	-0.39	0.48	36.36	0.063	0.003	0.047	0.042	0.018

$$\hat{\Sigma}^{-1} = \begin{bmatrix} 16.07 & 7.63 & 0.34 & -1.78 & 0.47 \\ 7.63 & 381.32 & 5.39 & 3.96 & -6.32 \\ 0.34 & 5.39 & 21.43 & 1.48 & 1.14 \\ -1.78 & 3.96 & 1.48 & 24.35 & -3.3 \\ 0.47 & -6.32 & 1.14 & -3.3 & 55.12 \end{bmatrix} \quad \hat{\rho} = \begin{bmatrix} 1 & -0.101 & -0.018 & 0.094 & -0.011 \\ -0.101 & 1 & -0.057 & -0.043 & 0.043 \\ -0.018 & -0.057 & 1 & -0.067 & -0.042 \\ 0.094 & -0.043 & -0.067 & 1 & 0.089 \\ -0.011 & 0.043 & -0.042 & 0.089 & 1 \end{bmatrix}$$

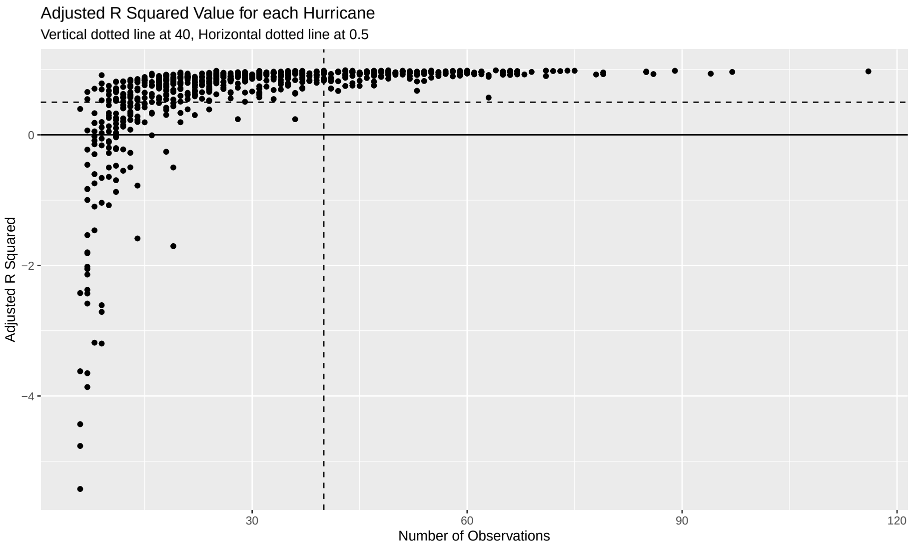
Model Performance

The overall adjusted R^2 of the estimated Bayesian model is 0.9524156.

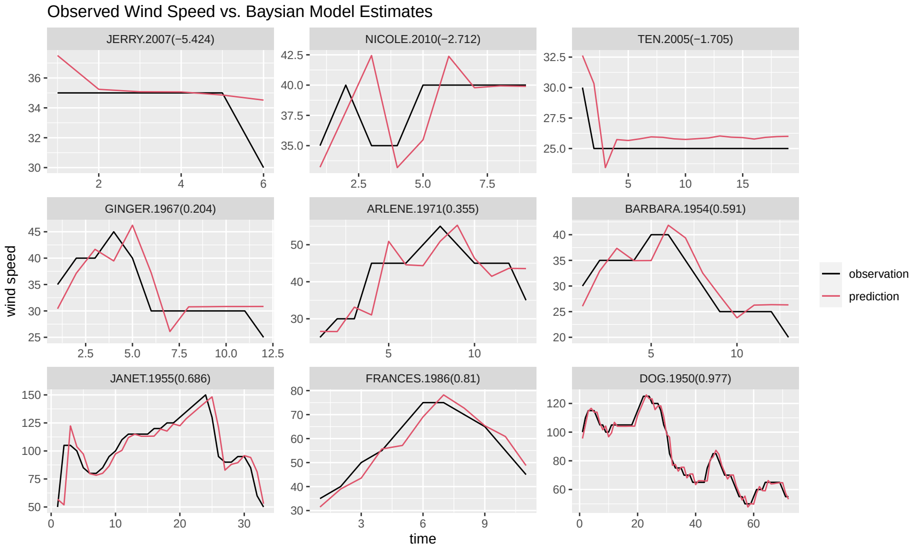
Table 3: ${R^2}_{adj}$ for each hurricane

R^2_{adj}	Count of Hurricanes	Percentage(%)
0.6-1	522	76.7
0.2-0.6	79	11.6
< 0.2	80	11.7

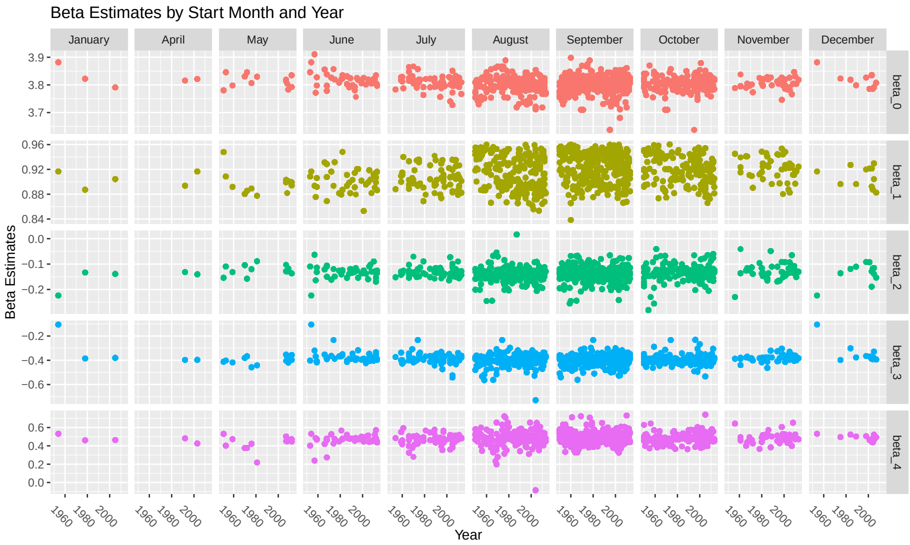
Moodel Performance



Model Performance

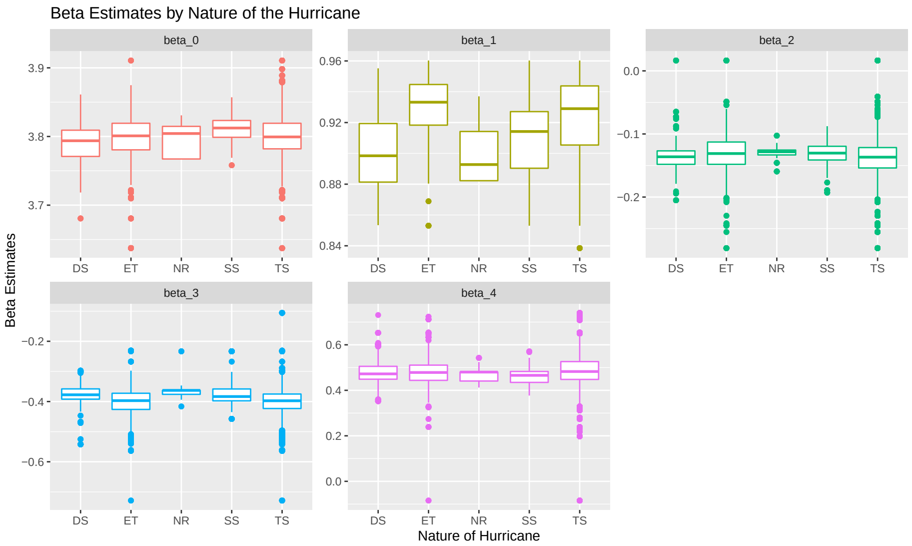


Understanding Seasonal Differences



Typical Hurricane season is June to November

Understanding Nature of Hurricane Differences



Modeling Seasonal, and Nature Difference

Model:
For each β value we fit a different linear model.

$$
\begin{aligned}
Y_{ij} = \alpha_{0j} + \alpha_{1j} \times \text{Decade}_i \\
+ \alpha_{(k+1)j} I(\text{Nature} = k)_i \\
+ \alpha_{(l+5)j} I(\text{Month} = l)_i + \epsilon_{ij}
\end{aligned}
$$

Where i is the hurricane, j is the Beta model, $k \in (ET,\ NR,\ SS,\ TS)$ making DS the reference group. Let $l \in$(April - December).

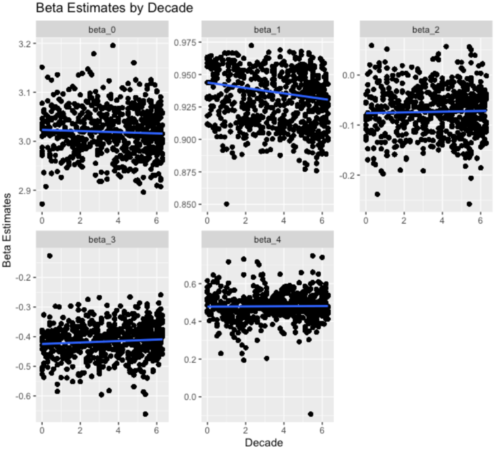
Results:

- Due to length each model estimate were omitted
- No Nature Indicators were significant
- No Month Indicators were significant
- For the β_1 model the decade estimated is significant -0.003 (-0.002, -0.004)

Thus Month and Nature don't have a linear association with each beta value.

Exploring if wind speeds have increased over the years

Model: $Y_i = \alpha_{0i} + \alpha_{1i} \times \text{Decade}$ where Y_i is each β_i and $i \in (0, \ldots, 4)$.



Linear Model Output for Each β

Characteristic	Beta	95% CI[1]	p-value
beta0			
decade	-0.001	-0.002, 0.000	0.2
beta1			
decade	-0.003	-0.004, -0.002	**<0.001**
beta2			
decade	0.000	-0.001, 0.001	0.6
beta3			
decade	0.002	0.000, 0.004	**0.041**
beta4			
decade	0.001	-0.002, 0.004	0.5

[1] CI = Confidence Interval

- β_1: Indicates a decrease in the change of wind speed over years

Deaths and Damages Data Exploration

Hurricane Deaths

Hurricane Damages

Conclusions

- Largers Samples give better estimates
- High dim sample space, burn in takes longer than expected
  - very sensitive to starting values