P8160 - Project 3
Baysian modeling of hurrican trajectories

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Motivation

Climate researchers are interested in modeling the hurricane trajectories to forecast the wind speed.

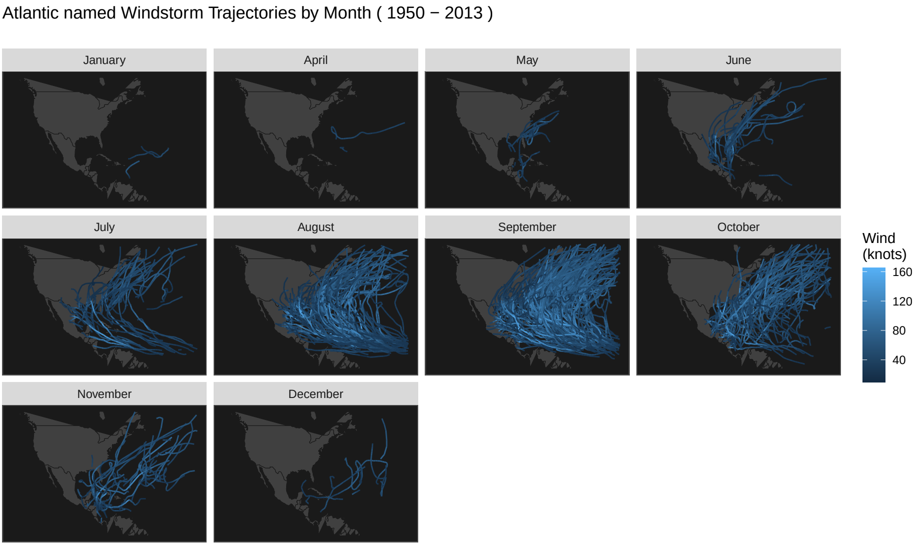
Data

- **ID**: ID of hurricanes
- **Year**: In which the hurricane occurred
- **Month**: In which the hurricane occurred
- **Nature**: Nature of the hurricane
  - ET: Extra Tropical
  - DS: Disturbance
  - NR: Not Rated
  - SS: Sub Tropical
  - TS: Tropical Storm
- **Time**: dates and time of the record
- **Latitude** and **Longitude**: The location of a hurricane check point
- **Wind.kt**: Maximum wind speed (in Knot) at each check point

Outline

1. Exploration into the Data
2. Bayesian modeling of hurricane wind speed
   - Model Equation
   - Posterior Derivation
   - MCMC Algorithm
3. How Month, Year, and the Nature of the hurricane affect wind speed
   - Explore seasonal differences
   - Explore if wind speeds is increasing over years
4. Exploring wind speeds impact on death and damages
5. As well as the characteristic of a hurricane associated with damages and deaths

Data



Atlantic named Windstorm Trajectories by Month (1950 − 2013)
January
April
May
June
July
August
September
October
November
December
Wind
(knots)
160
120
80
40

Data Cleaning

- We are only concerned about observations that occurred on 6 hour intervals. hour 0, 6, 12, and 18.
- In addition we will exclude all hurricane IDs that have less then 7 observations.
- We used the lag difference ($t - 6$ to $t - 12$) for latitude, longitude and wind speed to build $\Delta_{i,1}(t), \Delta_{i,2}(t), \Delta_{i,3}(t)$ and lag of wind speed as $Y_i(t - 6)$

Through this process we remove 460 observations so we are left with 21578 observations and 681 unique hurricanes.

Bayesian Model for Hurricane Trajectories

To model the wind speed of the i^{th} hurricane at time t we will use

$$Y_i(t) = \beta_{0,i} + \beta_{1,i}Y_i(t-6) + \beta_{2,i}\Delta_{i,1}(t) + \beta_{3,i}\Delta_{i,2}(t) + \beta_{4,i}\Delta_{i,3}(t) + \epsilon_i(t$$

Where

- $\Delta_{i,1}(t)$, $\Delta_{i,2}(t)$ and $\Delta_{i,3}(t)$ are changes in latitude longitude and wind speed respectively between $t-12$ and $t-6$
- $\epsilon_i(t) \sim N(0, \sigma^2)$ independent across t
- Let $\beta_i = (\beta_{0,i}, \beta_{1,i}, \beta_{2,i}, \beta_{3,i}, \beta_{4,i})^T \sim \mathcal{N}(\mu, \Sigma)$ be multivariate normal distribution where $\mu \in \mathbb{R}^d$ and $\Sigma \in \mathbb{R}^{d \times d}$.

Prior Distributions Assumptions:

- For σ^2 we assume $\pi(\sigma^2) \propto \frac{1}{\sigma^2}$
- For μ we assume $\pi(\mu) \propto 1$
- For Σ we assume $\pi\left(\Sigma^{-1}\right) \propto |\Sigma|^{-(d+1)} \exp\left\{-\frac{1}{2}\Sigma^{-1}\right\}$

Goal: Estimate $\Theta = (B, \mu, \Sigma^{-1}, \sigma^2)$

Likelihood & Prior Function

Likelihood Let $X_i = \big(\mathbf{1}, Y_i(t-6), \Delta_{i,1}(t), \Delta_{i,2}(t), \Delta_{i,3}(t)\big)$

$$L(Y \mid \Theta) \propto \prod_{i=1}^{m} \left(\sigma^2\right)^{-\frac{n_i}{2}} \exp\left\{-\frac{1}{2\sigma^2} \left(Y_i - X_i\beta_i\right)^T \left(Y_i - X_i\beta_i\right)\right\}$$

where m is the number of hurricane and n_i is the number of observations for i^{th} hurricane

Prior Let $\Theta = (B, \mu, \Sigma^{-1}, \sigma^2)$

$$\pi\left(\Theta\right) \propto \left(\sigma^2\right)^{-1} \left|\Sigma^{-1}\right|^{d+1} \exp\left\{-\frac{1}{2} \operatorname{tr}\left(\Sigma^{-1}\right)\right\} \prod_{i=1}^{m} \left|\Sigma^{-1}\right|^{\frac{1}{2}} \exp\left\{-\frac{1}{2} \left(\beta_i - \mu\right)^T \Sigma^{-1} (\beta_i - \mu)\right\}$$

where d is the dimension of μ

Posterior Calculation

Posterior

$$\pi(\Theta \mid Y) \propto \left(\sigma^2\right)^{-\left(1+\frac{\sum_{i=1}^m n_i}{2}\right)} \left|\Sigma^{-1}\right|^{d+1+\frac{m}{2}} \exp\left\{-\frac{1}{2}\operatorname{tr}\left(\Sigma^{-1}\right)\right\}$$
$$\times \exp\left\{-\frac{1}{2}\sum_{i=1}^m \left(\beta_i - \mu\right)^T \Sigma^{-1}\left(\beta_i - \mu\right)\right\} \exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^m \left(Y_i - X_i\beta_i\right)^T \left(Y_i - X_i\beta_i\right)\right\}$$

Conditional Posterior

$$\beta_i : \pi(\beta_i \mid \Theta_{(-\beta_i)} Y) \propto \exp\left\{-\frac{1}{2}\left(\beta_i - \mu\right)^T \Sigma^{-1}\left(\beta_i - \mu\right) - \frac{1}{2\sigma^2}\left(Y_i - X_i\beta_i\right)^T \left(Y_i - X_i\beta_i\right)\right\}$$

$$\mu : \pi\left(\mu \mid \Theta_{(-\mu)}, Y\right) \sim \mathcal{N}(\bar{\beta}, \Sigma/m), \bar{\beta} = \left(\bar{\beta}_{0,.}, \bar{\beta}_{1,.}, \bar{\beta}_{2,.}, \bar{\beta}_{3,.}, \bar{\beta}_{4,.}\right)^T$$

$$\sigma^2 : \pi\left(\sigma^2 \mid \Theta_{(-\sigma^2)}, Y\right) \propto \left(\sigma^2\right)^{-\left(1+\frac{\sum_{i=1}^m n_i}{2}\right)} \times \exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^m \left(Y_i - X_i\beta_i\right)^T \left(Y_i - X_i\beta_i\right)\right\}$$

$$\Sigma^{-1} : \pi\left(\Sigma^{-1} \mid \Theta_{(-\Sigma^{-1})}, Y\right) \sim \text{Wishart}\left(3d+3+m, \left(I + \sum_{i=1}^m \left(\beta_i - \mu\right)\left(\beta_i - \mu\right)^T\right)^{-1}\right)$$

MCMC Algorithm

We apply hybrid algorithm consisting with Metropolis-Hastings steps and Gibbs steps.

Update component wise:

- Sampling proposed $\beta'_{j,i}$, $j = 0, 1...4$, for i^{th} hurricane from proposed distribution $U\left(\beta_{j,i}^{(t)} - a_{j,i}, \beta_{j,i}^{(t)} + a_{j,i}\right)$, where $a_{j,i}$ is the search window for $\beta_{j,i}$. Since the proposed is symmetric, the acceptance probability is the ratio of posterior distribution.
- Then, Gibb step for μ: Sample $\mu^{(t+1)}$ from $\mathcal{N}\left(\bar{\beta}^{(t+1)}, \Sigma^{(t)}/m\right)$, where $\bar{\beta}^{(t+1)}$ is the average of $\beta_i^{(t+1)}$ over all hurricanes.
- Next, Random-walk Metropolis to generate $\sigma^{2'}$ with step size from $U\left(-a_{\sigma^2}, -a_{\sigma^2}\right)$ and compare the ratio of posterior distribution with $u \sim U(0, 1)$
- Finally, we sample $\Sigma^{-1^{(t+1)}}$ from $\text{Wishart}\left(3d + 3 + m, \left(I + \sum_{i=1}^{m}\left(\beta_i^{(t+1)} - \mu^{(t+1)}\right)\left(\beta_i^{(t+1)} - \mu^{(t+1)}\right)^T\right)^{-1}\right)$

Initial Starting Values

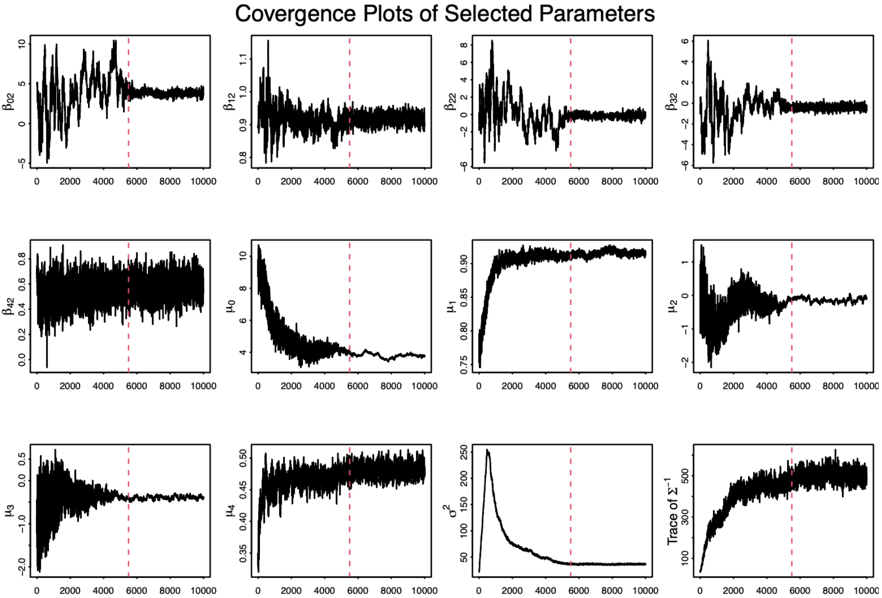
Initial Values:

- β_i: Fit OLS multivariate linear regression (MLR) for i^{th} hurricane and use the coefficients as $\beta_i^{(0)}$
- μ: Average over all $\beta_i^{(0)}$ as $\mu^{(0)}$
- σ^2: $\hat{\sigma}_i^2$ is the mean square residuals of the OLS model for i^{th} hurricane. Take the mean over all $\hat{\sigma}_i^2$ as $\sigma^{2^{(0)}}$
- Σ^{-1}: Generate the covariance matrix of $\beta_i^{(0)}$ and take the inverse of the matrix as $\Sigma^{-1^{(0)}}$

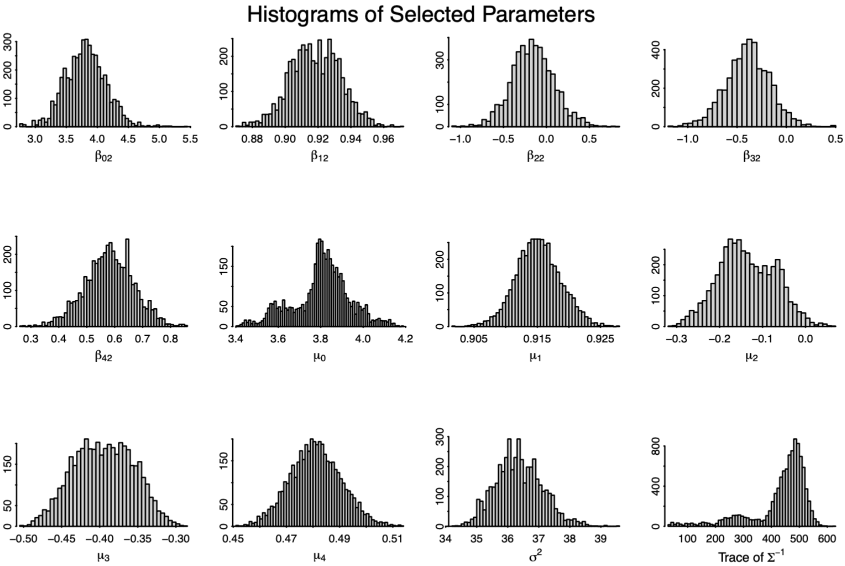
Table 1: Range of Search Window and Acceptance Rate for paraemters used MH step

	Search Window	Acceptance Rate (%)
β_0	1.1	45.87 - 51.36
β_1	(0.04, 0.1)	31.67 - 63.68
β_2	(0.8, 1.0)	38.60 - 45.60
β_3	(0.5, 0.6)	33.20 - 61.32
β_4	(0.4, 0.5)	34.95 - 60.45
σ^2	2.0	44.83

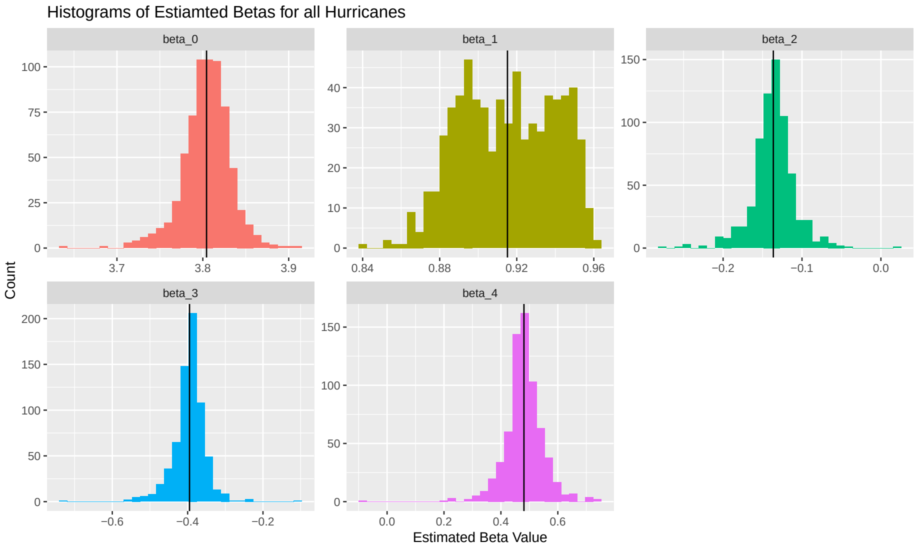
MCMC Model Convergence



MCMC Model Convergence



B Estimates



Histograms of Estiamted Betas for all Hurricanes
beta_0
beta_1
beta_2
beta_3
beta_4
Count
Estimated Beta Value
100
75
50
25
0
3.7
3.8
3.9
40
30
20
10
0
0.84
0.88
0.92
0.96
150
100
50
0
−0.2
−0.1
0.0
200
150
100
50
0
−0.6
−0.4
−0.2
150
100
50
0
0.0
0.2
0.4
0.6

The μ and σ^2 Estiamtes

Table 2: Bayesian Estiamtes for μ and σ^2

	μ_0	μ_1	μ_2	μ_3	μ_4	σ^2	σ^2_{00}	σ^2_{11}	σ^2_{22}	σ^2_{33}	σ^2_{44}
Estimates	3.8	0.92	-0.14	-0.39	0.48	36.36	0.063	0.003	0.047	0.042	0.018

$$\hat{\Sigma}^{-1} = \begin{bmatrix} 16.07 & 7.63 & 0.34 & -1.78 & 0.47 \\ 7.63 & 381.32 & 5.39 & 3.96 & -6.32 \\ 0.34 & 5.39 & 21.43 & 1.48 & 1.14 \\ -1.78 & 3.96 & 1.48 & 24.35 & -3.3 \\ 0.47 & -6.32 & 1.14 & -3.3 & 55.12 \end{bmatrix} \quad \hat{\rho} = \begin{bmatrix} 1 & -0.101 & -0.018 & 0.094 & -0.011 \\ -0.101 & 1 & -0.057 & -0.043 & 0.043 \\ -0.018 & -0.057 & 1 & -0.067 & -0.042 \\ 0.094 & -0.043 & -0.067 & 1 & 0.089 \\ -0.011 & 0.043 & -0.042 & 0.089 & 1 \end{bmatrix}$$

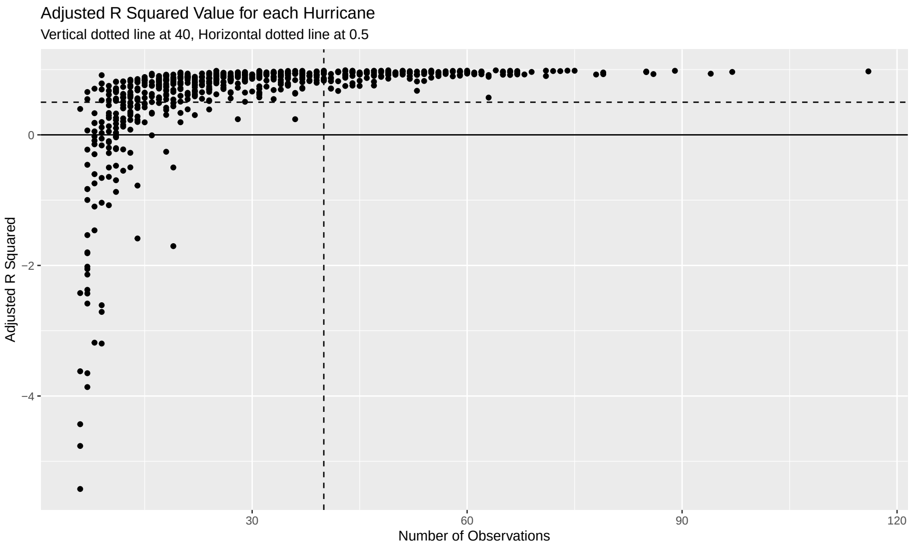
Model Performance

The overall adjusted R^2 of the estimated Bayesian model is 0.9524156.

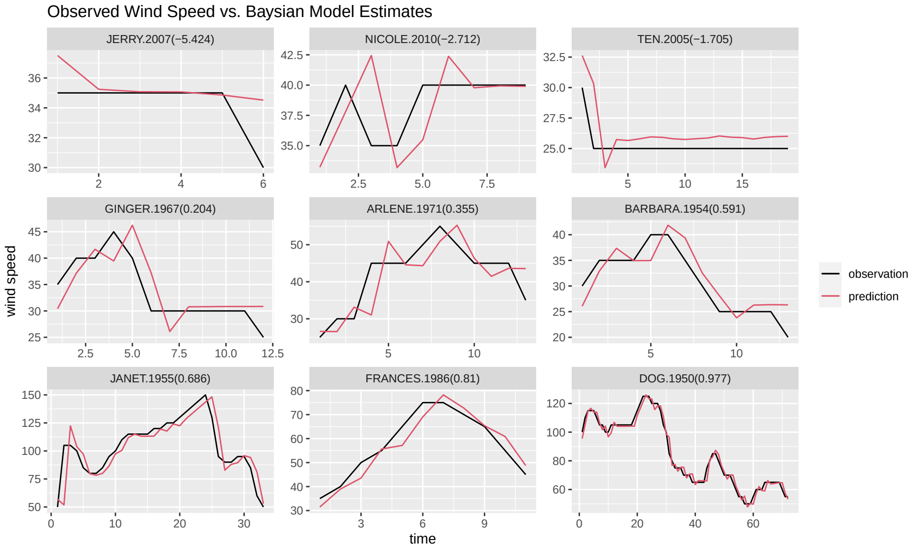
Table 3: R^2_{adj} for each hurricane

R^2_{adj}	Count of Hurricanes	Percentage($\%$)
0.6-1	522	76.7
0.2-0.6	79	11.6
< 0.2	80	11.7

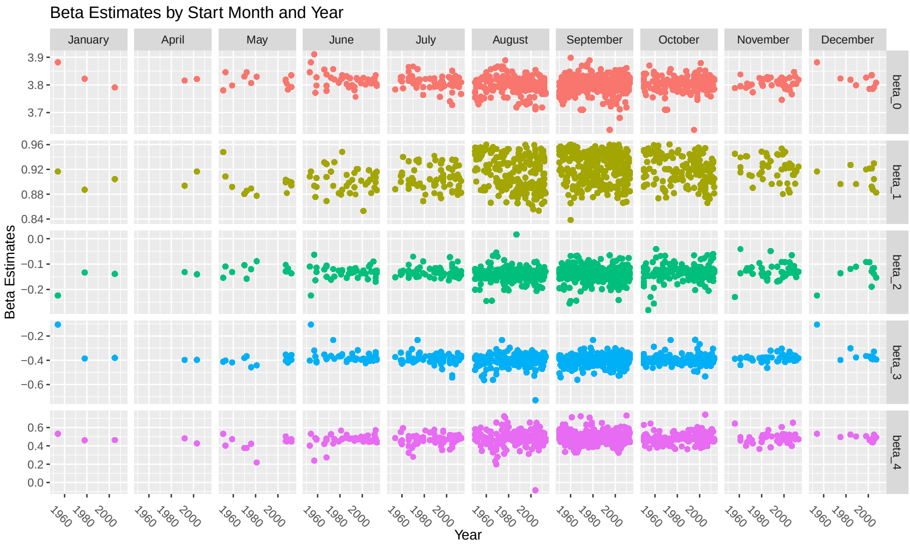
Model Performance



Model Performance

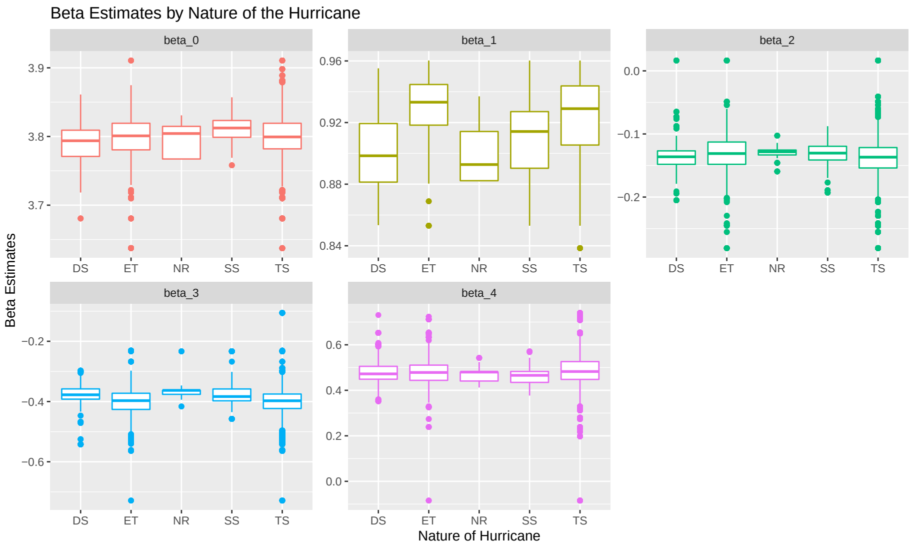


Understanding Seasonal Differences



Typical Hurricane season is June to November

Understanding Nature of Hurricane Differences



Modeling Seasonal, and Nature Difference

Model:

For each β value we fit a different linear model.

$$
\begin{aligned}
Y_{ij} = \alpha_{0j} + \alpha_{1j} \times \text{Decade}_i \\
+ \alpha_{(k+1)j} I(\text{Nature} = k)_i \\
+ \alpha_{(l+5)j} I(\text{Month} = l)_i + \epsilon_{ij}
\end{aligned}
$$

Where i is the hurricane, j is the Beta model, $k \in (ET,\ NR,\ SS,\ TS)$ making DS the reference group. Let $l \in$(April - December).

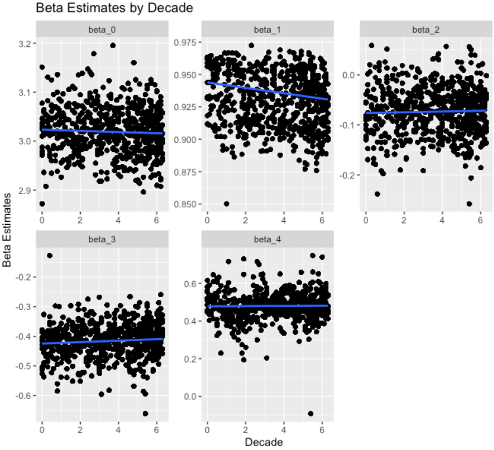
Results:

- Due to length each model estimate were omitted
- No Nature Indicators were significant
- No Month Indicators were significant
- For the β_1 model the decade estimated is significant -0.003 (-0.002, -0.004)

Thus Month and Nature don't have a linear association with each beta value.

Exploring if wind speeds have increased over the years

Model: $Y_i = \alpha_{0i} + \alpha_{1i} \times \text{Decade}$ where Y_i is each β_i and $i \in (0, \ldots, 4)$.



Linear Model Output for Each β

Characteristic	Beta	95% CI[1]	p-value
beta0			
decade	-0.001	-0.002, 0.000	0.2
beta1			
decade	-0.003	-0.004, -0.002	**<0.001**
beta2			
decade	0.000	-0.001, 0.001	0.6
beta3			
decade	0.002	0.000, 0.004	**0.041**
beta4			
decade	0.001	-0.002, 0.004	0.5

[1] CI = Confidence Interval

- β_1: Indicates a decrease in the change of wind speed over years

Deaths and Damages Data Exploration

Hurricane Deaths

Hurricane Damages

Conclusions

- Largers Samples give better estimates
- High dim sample space, burn in takes longer than expected
  - very sensitive to starting values