

P8160 - Project 3

Bayesian modeling of hurricane trajectories

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Motivation

Climate researchers are interested in modeling the hurricane trajectories to forecast the wind speed.

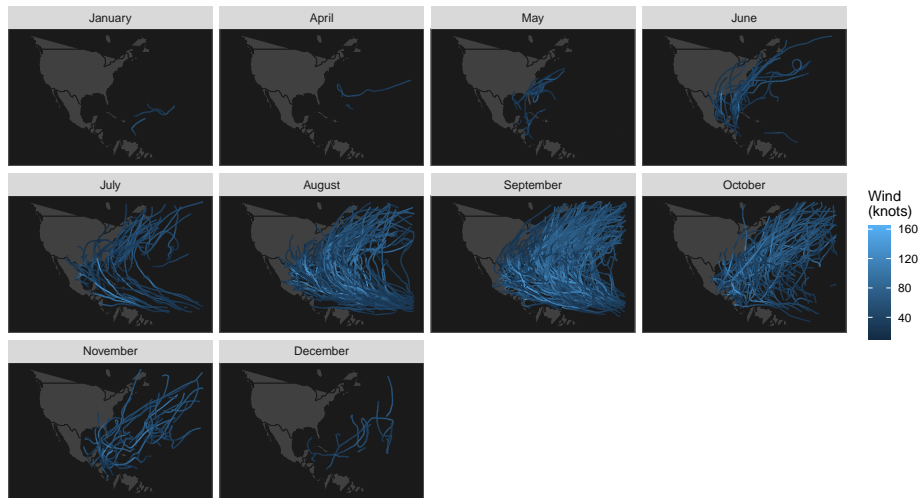
Data

- **ID:** ID of hurricanes
- **Year:** In which the hurricane occurred
- **Month:** In which the hurricane occurred
- **Nature:** Nature of the hurricane
 - ET: Extra Tropical
 - DS: Disturbance
 - NR: Not Rated
 - SS: Sub Tropical
 - TS: Tropical Storm
- **Time:** dates and time of the record
- **Latitude** and **Longitude:** The location of a hurricane check point
- **Wind.kt:** Maximum wind speed (in Knot) at each check point

- ① Exploration into the Data
- ② Bayesian modeling of hurricane wind speed
 - Model Equation
 - Posterior Derivation
 - MCMC Algorithm
- ③ How Month, Year, and the Nature of the hurricane affect wind speed
 - Explore seasonal differences
 - Explore if wind speeds is increasing over years
- ④ Exploring wind speeds impact on death and damages
- ⑤ As well as the characteristic of a hurricane associated with damages and deaths

Data

Atlantic named Windstorm Trajectories by Month (1950 – 2013)



Data Cleaning

- We are only concerned about observations that occurred on 6 hour intervals. hour 0, 6, 12, and 18.
- In addition we will exclude all hurricane IDs that have less than 7 observations.
- We used the lag difference ($t - 6$ to $t - 12$) for latitude, longitude and wind speed to build $\Delta_{i,1}(t), \Delta_{i,2}(t), \Delta_{i,3}(t)$ and lag of wind speed as $Y_i(t - 6)$

Through this process we remove 460 observations so we are left with 21578 observations and 681 unique hurricanes.

Bayesian Model for Hurricane Trajectories

To model the wind speed of the i^{th} hurricane at time t we will use

$$Y_i(t) = \beta_{0,i} + \beta_{1,i}Y_i(t-6) + \beta_{2,i}\Delta_{i,1}(t) + \beta_{3,i}\Delta_{i,2}(t) + \beta_{4,i}\Delta_{i,3}(t) + \epsilon_i(t)$$

Where

- $\Delta_{i,1}(t)$, $\Delta_{i,2}(t)$ and $\Delta_{i,3}(t)$ are changes in latitude longitude and wind speed respectively between $t-12$ and $t-6$
- $\epsilon_i(t) \sim N(0, \sigma^2)$ independent across t
- Let $\beta_i = (\beta_{0,i}, \beta_{1,i}, \beta_{2,i}, \beta_{3,i}, \beta_{4,i})^T \sim \mathcal{N}(\mu, \Sigma)$ be multivariate normal distribution where $\mu \in \mathbb{R}^d$ and $\Sigma \in \mathbb{R}^{d \times d}$.

Prior Distributions Assumptions:

- For σ^2 we assume $\pi(\sigma^2) \propto \frac{1}{\sigma^2}$
- For μ we assume $\pi(\mu) \propto 1$
- For Σ we assume $\pi(\Sigma^{-1}) \propto |\Sigma|^{-(d+1)} \exp\{-\frac{1}{2}\Sigma^{-1}\}$

Goal: Estimate $\Theta = (B, \mu, \Sigma^{-1}, \sigma^2)$

Likelihood & Prior Function

Likelihood Let $X_i = (1, Y_i(t-6), \Delta_{i,1}(t), \Delta_{i,2}(t), \Delta_{i,3}(t))$

$$L(Y | \Theta) \propto \prod_{i=1}^m (\sigma^2)^{-\frac{n_i}{2}} \exp \left\{ -\frac{1}{2\sigma^2} (Y_i - X_i \beta_i)^T (Y_i - X_i \beta_i) \right\}$$

where m is the number of hurricane and n_i is the number of observations for i^{th} hurricane

Prior Let $\Theta = (B, \mu, \Sigma^{-1}, \sigma^2)$

$$\pi(\Theta) \propto (\sigma^2)^{-1} |\Sigma^{-1}|^{d+1} \exp \left\{ -\frac{1}{2} \text{tr}(\Sigma^{-1}) \right\} \prod_{i=1}^m |\Sigma^{-1}|^{\frac{1}{2}} \exp \left\{ -\frac{1}{2} (\beta_i - \mu)^T \Sigma^{-1} (\beta_i - \mu) \right\}$$

where d is the dimension of μ

Posterior Calculation

Posterior

$$\pi(\Theta | Y) \propto (\sigma^2)^{-\left(1 + \frac{\sum_{i=1}^m n_i}{2}\right)} |\Sigma^{-1}|^{d+1+\frac{m}{2}} \exp\left\{-\frac{1}{2} \text{tr}(\Sigma^{-1})\right\} \\ \times \exp\left\{-\frac{1}{2} \sum_{i=1}^m (\beta_i - \mu)^T \Sigma^{-1} (\beta_i - \mu)\right\} \exp\left\{-\frac{1}{2\sigma^2} \sum_{i=1}^m (Y_i - X_i \beta_i)^T (Y_i - X_i \beta_i)\right\}$$

Conditional Posterior

$$\beta_i : \pi(\beta_i | \Theta_{(-\beta_i)} Y) \propto \exp\left\{-\frac{1}{2} (\beta_i - \mu)^T \Sigma^{-1} (\beta_i - \mu) - \frac{1}{2\sigma^2} (Y_i - X_i \beta_i)^T (Y_i - X_i \beta_i)\right\}$$

$$\mu : \pi(\mu | \Theta_{(-\mu)}, Y) \sim \mathcal{N}(\bar{\beta}, \Sigma/m), \bar{\beta} = (\bar{\beta}_{0,.}, \bar{\beta}_{1,.}, \bar{\beta}_{2,.}, \bar{\beta}_{3,.}, \bar{\beta}_{4,.})^T$$

$$\sigma^2 : \pi(\sigma^2 | \Theta_{(-\sigma^2)}, Y) \propto (\sigma^2)^{-\left(1 + \frac{\sum_{i=1}^m n_i}{2}\right)} \times \exp\left\{-\frac{1}{2\sigma^2} \sum_{i=1}^m (Y_i - X_i \beta_i)^T (Y_i - X_i \beta_i)\right\}$$

$$\Sigma^{-1} : \pi(\Sigma^{-1} | \Theta_{(-\Sigma^{-1})}, Y) \sim \text{Wishart}\left(3d + 3 + m, \left(I + \sum_{i=1}^m (\beta_i - \mu)(\beta_i - \mu)^T\right)^{-1}\right)$$

MCMC Algorithm

We apply hybrid algorithm consisting with Metropolis-Hastings steps and Gibbs steps.

Update component wise:

- Sampling proposed $\beta'_{j,i}$, $j = 0, 1 \dots 4$, for i^{th} hurricane from proposal distribution $U(\beta_{j,i}^{(t)} - a_{j,i}, \beta_{j,i}^{(t)} + a_{j,i})$, where $a_{j,i}$ is the search window for $\beta_{j,i}$. Since the proposals are symmetry, the accepting or rejecting the proposed $\beta'_{j,i}$ depends on the ratio of posterior distribution.
- Then, Gibb step for μ : Sample $\mu^{(t+1)}$ from $\mathcal{N}(\bar{\beta}^{(t+1)}, \Sigma^{(t)}/m)$, where $\bar{\beta}^{(t+1)}$ is the average of $\beta_i^{(t+1)}$ over all hurricanes.
- Next, MH step to generate $\sigma^{2'}$ from $U(\sigma^{2(t)} - a_{\sigma^2}, \sigma^{2(t)} + a_{\sigma^2})$.
- Finally, we sample $\Sigma^{-1(t+1)}$ from
$$\text{Wishart}\left(3d + 3 + m, \left(I + \sum_{i=1}^m (\beta_i^{(t+1)} - \mu^{(t+1)}) (\beta_i^{(t+1)} - \mu^{(t+1)})^T\right)^{-1}\right)$$

Initial Starting Values

Initial Values:

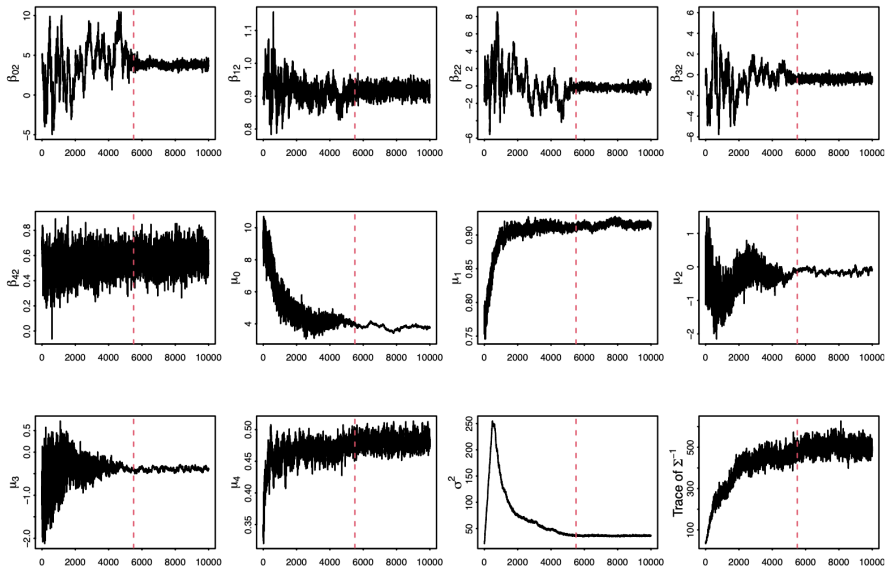
- β_i : Fit OLS multivariate linear regression (MLR) for i^{th} hurricane and use the coefficients as $\beta_i^{(0)}$
- μ : Average over all $\beta_i^{(0)}$ as $\mu^{(0)}$
- σ^2 : $\hat{\sigma}_i^2$ is the mean square residuals of the OLS model for i^{th} hurricane. Take the mean over all $\hat{\sigma}_i^2$ as $\sigma^{2(0)}$
- Σ^{-1} : Generate the covariance matrix of $\beta_i^{(0)}$ and take the inverse of the matrix as $\Sigma^{-1(0)}$

Table 1: Range of Search Window and Acceptance Rate for parameters used MH step

	Search Window	Acceptance Rate (%)
β_0	1.1	45.87 - 51.36
β_1	(0.04, 0.1)	31.67 - 63.68
β_2	(0.8, 1.0)	38.60 - 45.60
β_3	(0.5, 0.6)	33.20 - 61.32
β_4	(0.4, 0.5)	34.95 - 60.45
σ^2	2.0	44.83

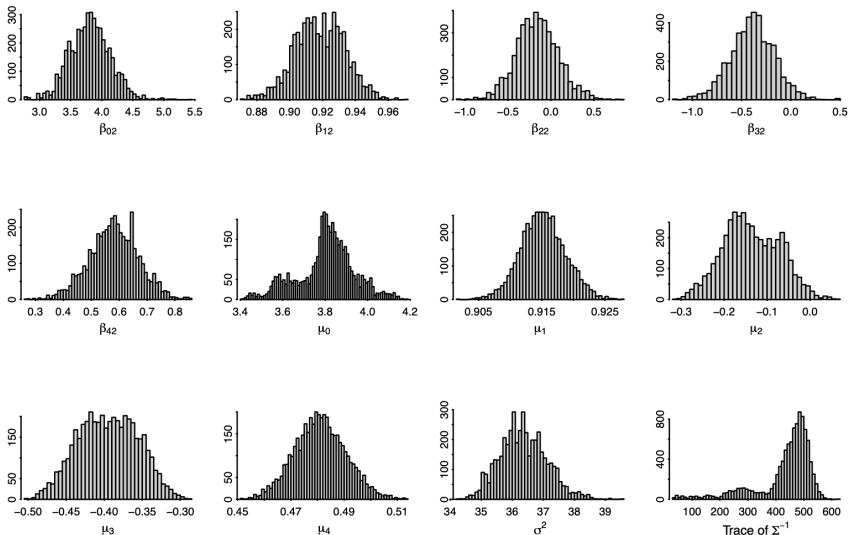
MCMC Model Convergence

Covergence Plots of Selected Parameters



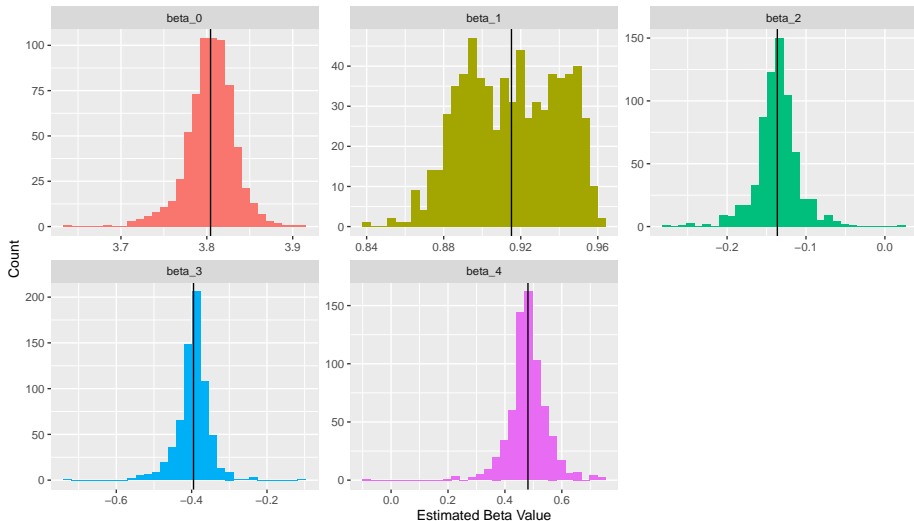
MCMC Model Convergence

Histograms of Selected Parameters



B Estimates

Histograms of Estimated Betas for all Hurricanes



The μ and σ^2 Estiamtes

Table 2: Bayesian Estiamtes for μ and σ^2

	μ_0	μ_1	μ_2	μ_3	μ_4	σ^2	σ_{00}^2	σ_{11}^2	σ_{22}^2	σ_{33}^2	σ_{44}^2
Estimates	3.8	0.92	-0.14	-0.39	0.48	36.36	0.063	0.003	0.047	0.042	0.018

$$\hat{\Sigma}^{-1} = \begin{bmatrix} 16.07 & 7.63 & 0.34 & -1.78 & 0.47 \\ 7.63 & 381.32 & 5.39 & 3.96 & -6.32 \\ 0.34 & 5.39 & 21.43 & 1.48 & 1.14 \\ -1.78 & 3.96 & 1.48 & 24.35 & -3.3 \\ 0.47 & -6.32 & 1.14 & -3.3 & 55.12 \end{bmatrix} \quad \hat{\rho} = \begin{bmatrix} 1 & -0.101 & -0.018 & 0.094 & -0.011 \\ -0.101 & 1 & -0.057 & -0.043 & 0.043 \\ -0.018 & -0.057 & 1 & -0.067 & -0.042 \\ 0.094 & -0.043 & -0.067 & 1 & 0.089 \\ -0.011 & 0.043 & -0.042 & 0.089 & 1 \end{bmatrix}$$

Model Performance

The overall adjusted R^2 of the estimated Bayesian model is 0.9524156.

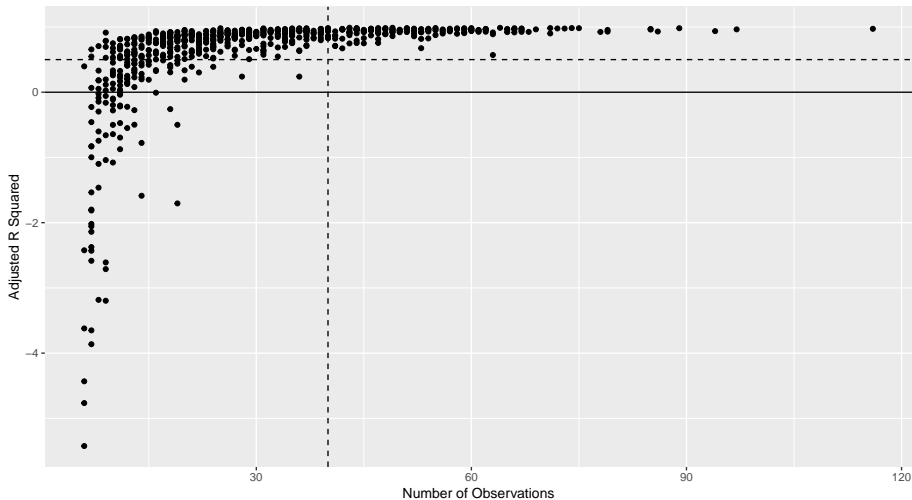
Table 3: R^2_{adj} for each hurricane

R^2_{adj}	Count of Hurricanes	Percentage(%)
0.6-1	522	76.7
0.2-0.6	79	11.6
< 0.2	80	11.7

Model Performance

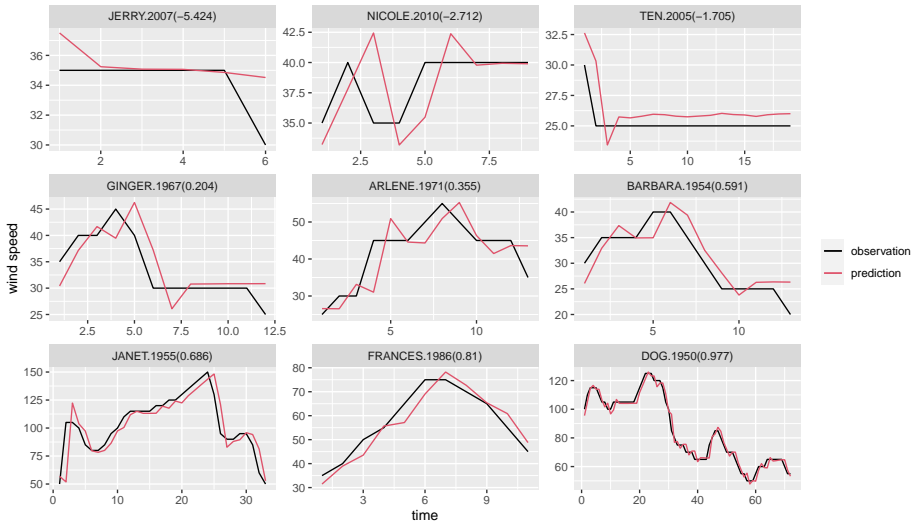
Adjusted R Squared Value for each Hurricane

Vertical dotted line at 40, Horizontal dotted line at 0.5

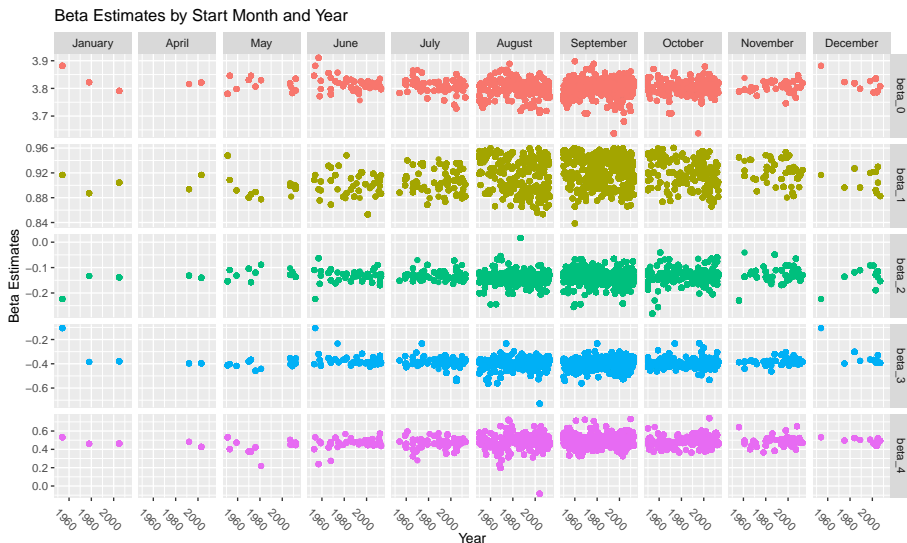


Model Performance

Observed Wind Speed vs. Bayesian Model Estimates



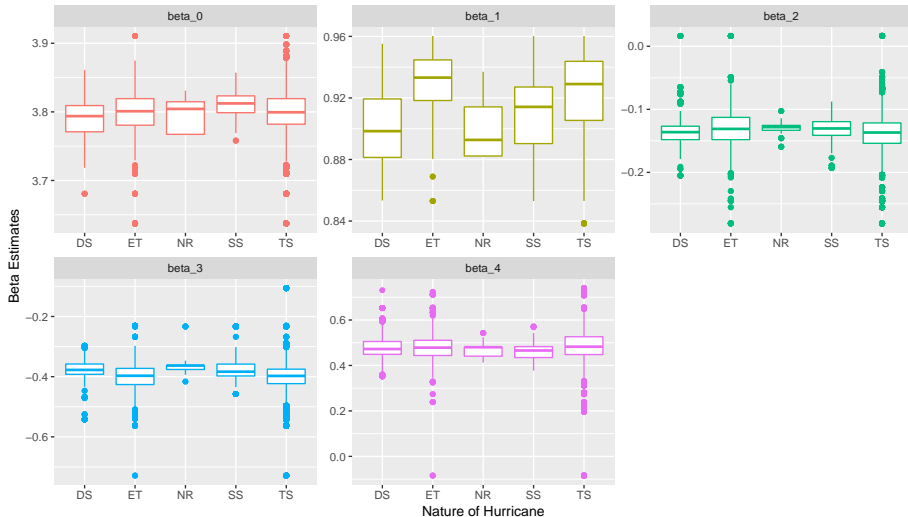
Understanding Seasonal Differences



Typical Hurricane season is June to November

Understanding Nature of Hurricane Differences

Beta Estimates by Nature of the Hurricane



Modeling Seasonal, and Nature Difference

Model:

For each β value we fit a different linear model.

$$\begin{aligned} Y_{ij} = & \alpha_{0j} + \alpha_{1j} \times \text{Decade}_i \\ & + \alpha_{(k+1)j} I(\text{Nature} = k)_i \\ & + \alpha_{(l+5)j} I(\text{Month} = l)_i + \epsilon_{ij} \end{aligned}$$

Where i is the hurricane, j is the Beta model, $k \in (ET, NR, SS, TS)$ making DS the reference group. Let $l \in (\text{April} - \text{December})$.

Results:

- Due to length each model estimate were omitted
- No Nature Indicators were significant
- No Month Indicators were significant
- For the β_1 model the decade estimated is significant -0.003 (-0.002, -0.004)

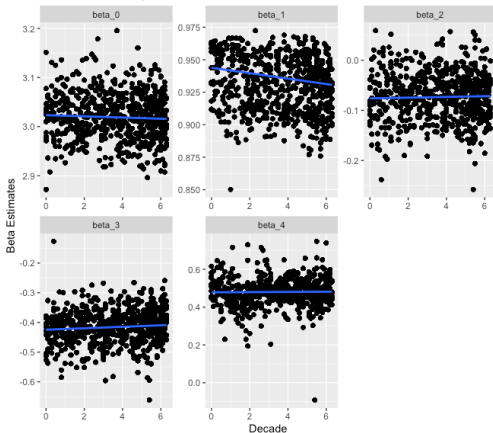
Thus Month and Nature don't have a linear association with each beta value.

Exploring if wind speeds have increased over the years

Model: $Y_i = \alpha_{0i} + \alpha_{1i} \times \text{Decade}$ where Y_i is each β_i and $i \in (0, \dots, 4)$.

Linear Model Output for Each β

Beta Estimates by Decade



Characteristic	Beta	95% CI [†]	p-value
beta0			
decade	-0.001	-0.002, 0.000	0.2
beta1			
decade	-0.003	-0.004, -0.002	<0.001
beta2			
decade	0.000	-0.001, 0.001	0.6
beta3			
decade	0.002	0.000, 0.004	0.041
beta4			
decade	0.001	-0.002, 0.004	0.5

[†] CI = Confidence Interval

• β_1 : Indicates a decrease in the change of wind speed over years

Deaths and Damages Data Exploration

Hurricane Deaths

Hurricane Damages

Conclusions

- Larger Samples give better estimates
- High dim sample space, burn in takes longer than expected
 - very sensitive to starting values