

P8160 - Project 3

Bayesian modeling of hurricane trajectories

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Motivation

Climate researchers are interested in modeling the hurricane trajectories to forecast the wind speed.

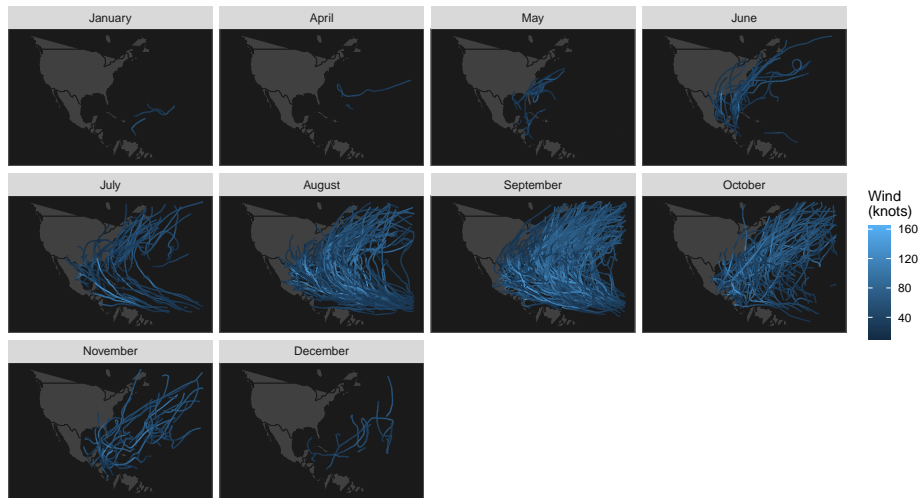
Data

- **ID:** ID of hurricanes
- **Year:** In which the hurricane occurred
- **Month:** In which the hurricane occurred
- **Nature:** Nature of the hurricane
 - ET: Extra Tropical
 - DS: Disturbance
 - NR: Not Rated
 - SS: Sub Tropical
 - TS: Tropical Storm
- **Time:** dates and time of the record
- **Latitude** and **Longitude:** The location of a hurricane check point
- **Wind.kt:** Maximum wind speed (in Knot) at each check point

- ① Exploration into the Data
- ② Bayesian modeling of hurricane wind speed
 - Model Equation
 - Posterior Derivation
 - MCMC Algorithm
- ③ How Month, Year, and the Nature of the hurricane affect wind speed
 - Explore seasonal differences
 - Explore if wind speeds is increasing over years
- ④ Exploring wind speeds impact on death and damages
- ⑤ As well as the characteristic of a hurricane associated with damages and deaths

Data

Atlantic named Windstorm Trajectories by Month (1950 – 2013)



Data Cleaning

- We are only concerned about observations that occurred on 6 hour intervals. hour 0, 6, 12, and 18.
- In addition we will exclude all hurricane IDs that have less than 7 observations.
- We used the lag difference ($t - 6$ to $t - 12$) for latitude, longitude and wind speed to build $\Delta_{i,1}(t), \Delta_{i,2}(t), \Delta_{i,3}(t)$ and lag of wind speed as $Y_i(t - 6)$

Through this process we remove 460 observations so we are left with 21578 observations and 681 unique hurricanes.

Bayesian Model for Hurricane Trajectories

To model the wind speed of the i^{th} hurricane at time t we will use

$$Y_i(t) = \beta_{0,i} + \beta_{1,i}Y_i(t-6) + \beta_{2,i}\Delta_{i,1}(t) + \beta_{3,i}\Delta_{i,2}(t) + \beta_{4,i}\Delta_{i,3}(t) + \epsilon_i(t)$$

Where

- $\Delta_{i,1}(t)$, $\Delta_{i,2}(t)$ and $\Delta_{i,3}(t)$ are changes in latitude longitude and wind speed respectively between $t-12$ and $t-6$
- $\epsilon_i(t) \sim N(0, \sigma^2)$ independent across t
- Let $\beta_i = (\beta_{0,i}, \beta_{1,i}, \beta_{2,i}, \beta_{3,i}, \beta_{4,i})^T \sim \mathcal{N}(\mu, \Sigma)$ be multivariate normal distribution where $\mu \in \mathbb{R}^d$ and $\Sigma \in \mathbb{R}^{d \times d}$.

Prior Distributions Assumptions:

- For σ^2 we assume $\pi(\sigma^2) \propto \frac{1}{\sigma^2}$
- For μ we assume $\pi(\mu) \propto 1$
- For Σ we assume $\pi(\Sigma^{-1}) \propto |\Sigma|^{-(d+1)} \exp\{-\frac{1}{2}\Sigma^{-1}\}$

Goal: Estimate $\Theta = (B, \mu, \Sigma^{-1}, \sigma^2)$

Likelihood & Prior Function

Likelihood Let $X_i = (1, Y_i(t-6), \Delta_{i,1}(t), \Delta_{i,2}(t), \Delta_{i,3}(t))$

$$L(Y | \Theta) \propto \prod_{i=1}^m (\sigma^2)^{-\frac{n_i}{2}} \exp \left\{ -\frac{1}{2\sigma^2} (Y_i - X_i \beta_i)^T (Y_i - X_i \beta_i) \right\}$$

where m is the number of hurricane and n_i is the number of observations for i^{th} hurricane

Prior Let $\Theta = (B, \mu, \Sigma^{-1}, \sigma^2)$

$$\pi(\Theta) \propto (\sigma^2)^{-1} |\Sigma^{-1}|^{d+1} \exp \left\{ -\frac{1}{2} \text{tr}(\Sigma^{-1}) \right\} \prod_{i=1}^m |\Sigma^{-1}|^{\frac{1}{2}} \exp \left\{ -\frac{1}{2} (\beta_i - \mu)^T \Sigma^{-1} (\beta_i - \mu) \right\}$$

where d is the dimension of μ

Posterior Calculation

Posterior

$$\pi(\Theta | Y) \propto (\sigma^2)^{-\left(1 + \frac{\sum_{i=1}^m n_i}{2}\right)} |\Sigma^{-1}|^{d+1+\frac{m}{2}} \exp\left\{-\frac{1}{2} \text{tr}(\Sigma^{-1})\right\} \\ \times \exp\left\{-\frac{1}{2} \sum_{i=1}^m (\beta_i - \mu)^T \Sigma^{-1} (\beta_i - \mu)\right\} \exp\left\{-\frac{1}{2\sigma^2} \sum_{i=1}^m (Y_i - X_i \beta_i)^T (Y_i - X_i \beta_i)\right\}$$

Conditional Posterior

$$\beta_i : \pi(\beta_i | \Theta_{(-\beta_i)} Y) \propto \exp\left\{-\frac{1}{2} (\beta_i - \mu)^T \Sigma^{-1} (\beta_i - \mu) - \frac{1}{2\sigma^2} (Y_i - X_i \beta_i)^T (Y_i - X_i \beta_i)\right\}$$

$$\mu : \pi(\mu | \Theta_{(-\mu)}, Y) \sim \mathcal{N}(\bar{\beta}, \Sigma/m), \bar{\beta} = (\bar{\beta}_{0,.}, \bar{\beta}_{1,.}, \bar{\beta}_{2,.}, \bar{\beta}_{3,.}, \bar{\beta}_{4,.})^T$$

$$\sigma^2 : \pi(\sigma^2 | \Theta_{(-\sigma^2)}, Y) \propto (\sigma^2)^{-\left(1 + \frac{\sum_{i=1}^m n_i}{2}\right)} \times \exp\left\{-\frac{1}{2\sigma^2} \sum_{i=1}^m (Y_i - X_i \beta_i)^T (Y_i - X_i \beta_i)\right\}$$

$$\Sigma^{-1} : \pi(\Sigma^{-1} | \Theta_{(-\Sigma^{-1})}, Y) \sim \text{Wishart}\left(3d + 3 + m, \left(I + \sum_{i=1}^m (\beta_i - \mu)(\beta_i - \mu)^T\right)^{-1}\right)$$

MCMC Algorithm

We apply hybrid algorithm consisting with Metropolis-Hastings steps and Gibbs steps.

Update component wise:

- Sampling proposed $\beta'_{j,i}$, $j = 0, 1 \dots 4$, for i^{th} hurricane from proposed distribution $U(\beta_{j,i}^{(t)} - a_{j,i}, \beta_{j,i}^{(t)} + a_{j,i})$, where $a_{j,i}$ is the search window for $\beta_{j,i}$. Since the proposed is symmetric, the acceptance probability is the ratio of posterior distribution.
- Then, Gibb step for μ : Sample $\mu^{(t+1)}$ from $\mathcal{N}(\bar{\beta}^{(t+1)}, \Sigma^{(t)}/m)$, where $\bar{\beta}^{(t+1)}$ is the average of $\beta_i^{(t+1)}$ over all hurricanes.
- Next, Random-walk Metropolis to generate $\sigma^{2'}$ with step size from $U(-a_{\sigma^2}, a_{\sigma^2})$ and compare the ratio of posterior distribution with $u \sim U(0, 1)$
- Finally, we sample $\Sigma^{-1(t+1)}$ from
Wishart $\left(3d + 3 + m, \left(I + \sum_{i=1}^m \left(\beta_i^{(t+1)} - \mu^{(t+1)} \right) \left(\beta_i^{(t+1)} - \mu^{(t+1)} \right)^T \right)^{-1} \right)$

Initial Starting Values

Initial Values:

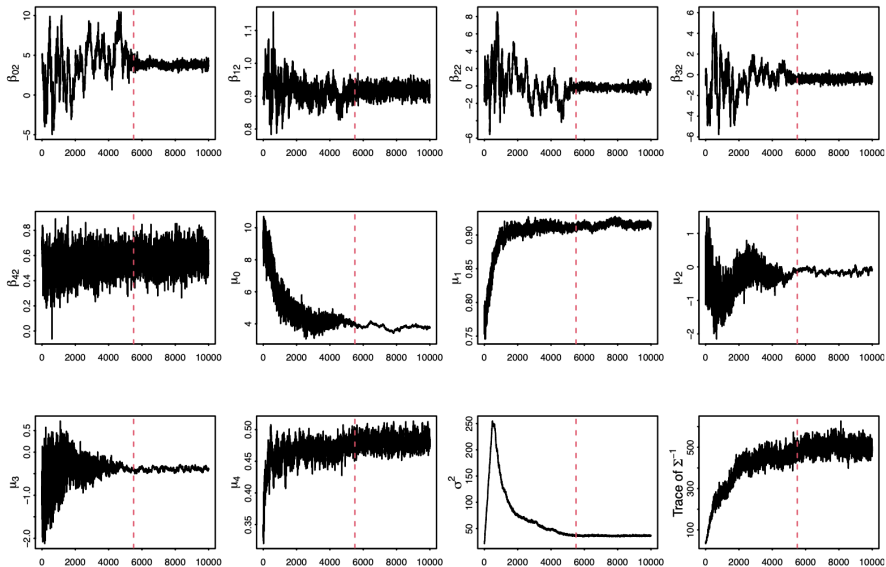
- β_i : Fit OLS multivariate linear regression (MLR) for i^{th} hurricane and use the coefficients as $\beta_i^{(0)}$
- μ : Average over all $\beta_i^{(0)}$ as $\mu^{(0)}$
- σ^2 : $\hat{\sigma}_i^2$ is the mean square residuals of the OLS model for i^{th} hurricane. Take the mean over all $\hat{\sigma}_i^2$ as $\sigma^{2(0)}$
- Σ^{-1} : Generate the covariance matrix of $\beta_i^{(0)}$ and take the inverse of the matrix as $\Sigma^{-1(0)}$

Table 1: Range of Search Window and Acceptance Rate for parameters used MH step

	Search Window	Acceptance Rate (%)
β_0	1.1	45.87 - 51.36
β_1	(0.04, 0.1)	31.67 - 63.68
β_2	(0.8, 1.0)	38.60 - 45.60
β_3	(0.5, 0.6)	33.20 - 61.32
β_4	(0.4, 0.5)	34.95 - 60.45
σ^2	2.0	44.83

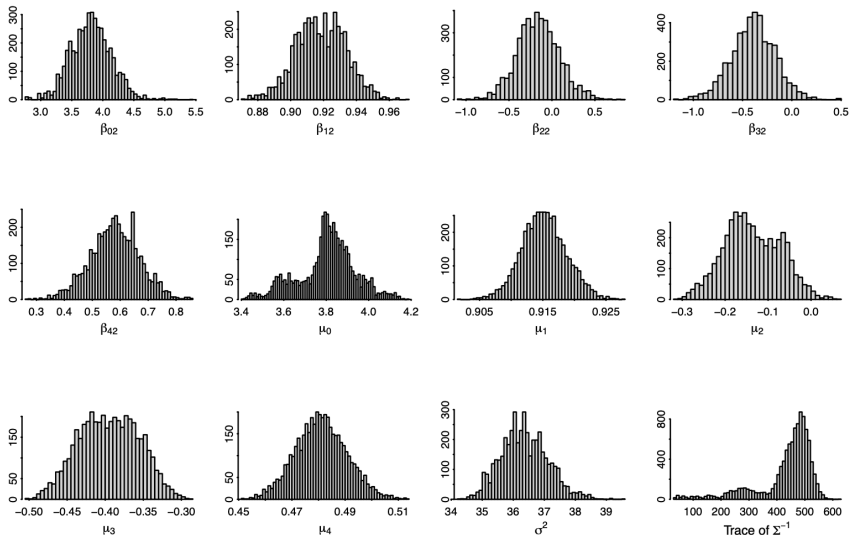
MCMC Model Convergence

Covergence Plots of Selected Parameters



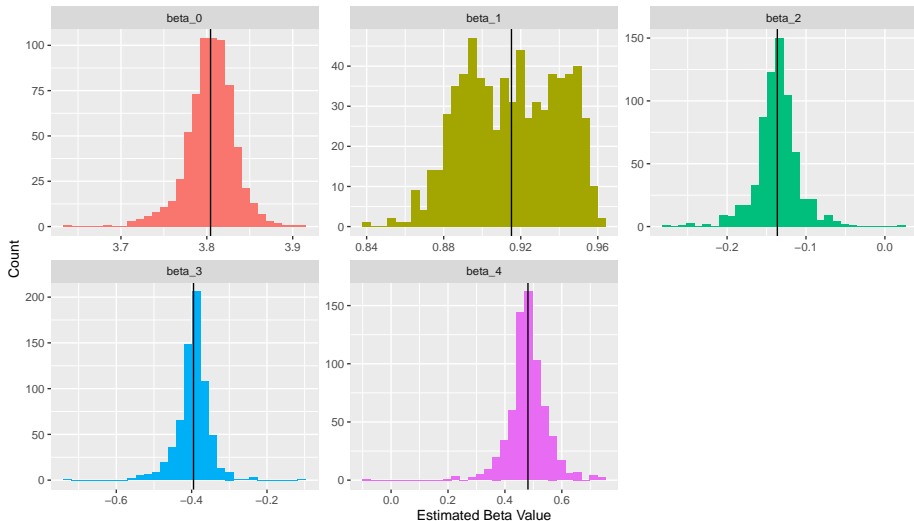
MCMC Model Convergence

Histograms of Selected Parameters



B Estimates

Histograms of Estimated Betas for all Hurricanes



The μ and σ^2 Estiamtes

Table 2: Bayesian Estiamtes for μ and σ^2

	μ_0	μ_1	μ_2	μ_3	μ_4	σ^2	σ_{00}^2	σ_{11}^2	σ_{22}^2	σ_{33}^2	σ_{44}^2
Estimates	3.8	0.92	-0.14	-0.39	0.48	36.36	0.063	0.003	0.047	0.042	0.018

$$\hat{\Sigma}^{-1} = \begin{bmatrix} 16.07 & 7.63 & 0.34 & -1.78 & 0.47 \\ 7.63 & 381.32 & 5.39 & 3.96 & -6.32 \\ 0.34 & 5.39 & 21.43 & 1.48 & 1.14 \\ -1.78 & 3.96 & 1.48 & 24.35 & -3.3 \\ 0.47 & -6.32 & 1.14 & -3.3 & 55.12 \end{bmatrix} \quad \hat{\rho} = \begin{bmatrix} 1 & -0.101 & -0.018 & 0.094 & -0.011 \\ -0.101 & 1 & -0.057 & -0.043 & 0.043 \\ -0.018 & -0.057 & 1 & -0.067 & -0.042 \\ 0.094 & -0.043 & -0.067 & 1 & 0.089 \\ -0.011 & 0.043 & -0.042 & 0.089 & 1 \end{bmatrix}$$

Model Performance

The overall adjusted R^2 of the estimated Bayesian model is 0.9524156.

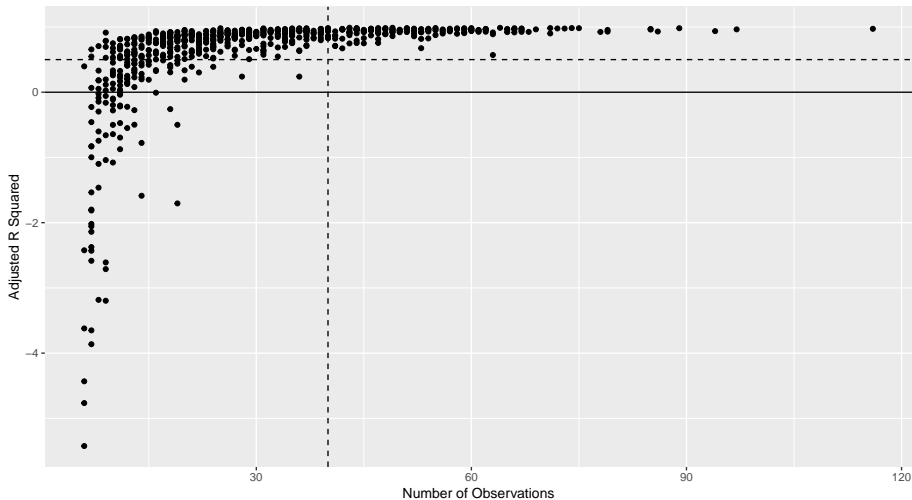
Table 3: R_{adj}^2 for each hurricane

R_{adj}^2	Count of Hurricanes	Percentage(%)
0.6-1	522	76.7
0.2-0.6	79	11.6
< 0.2	80	11.7

Model Performance

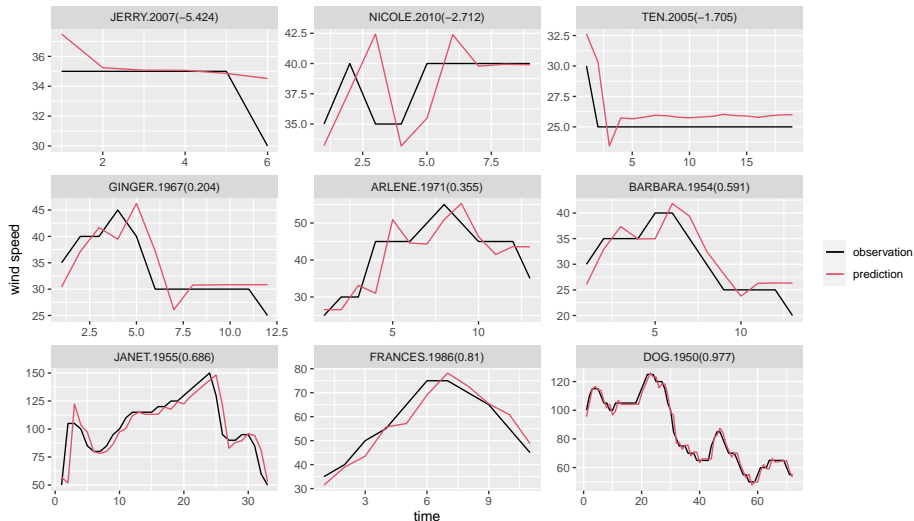
Adjusted R Squared Value for each Hurricane

Vertical dotted line at 40, Horizontal dotted line at 0.5

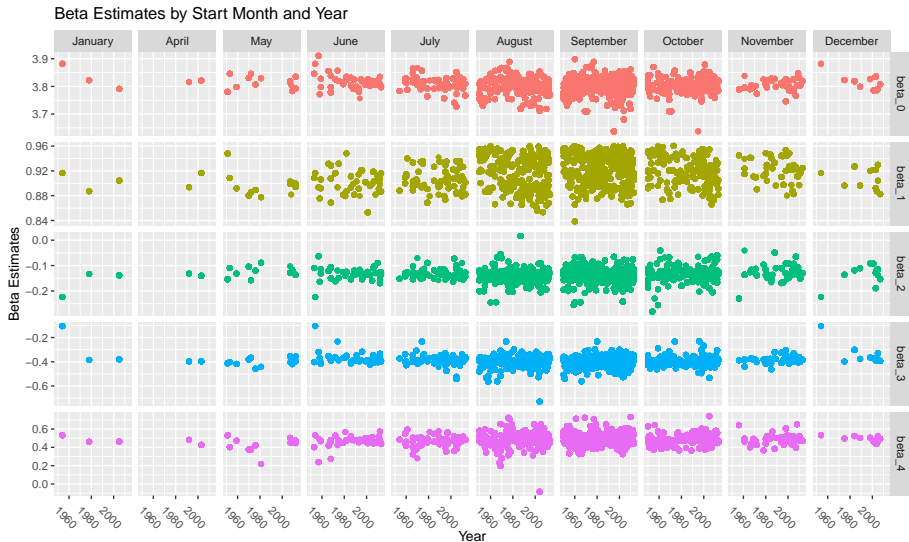


Model Performance

Observed Wind Speed vs. Bayesian Model Estimates



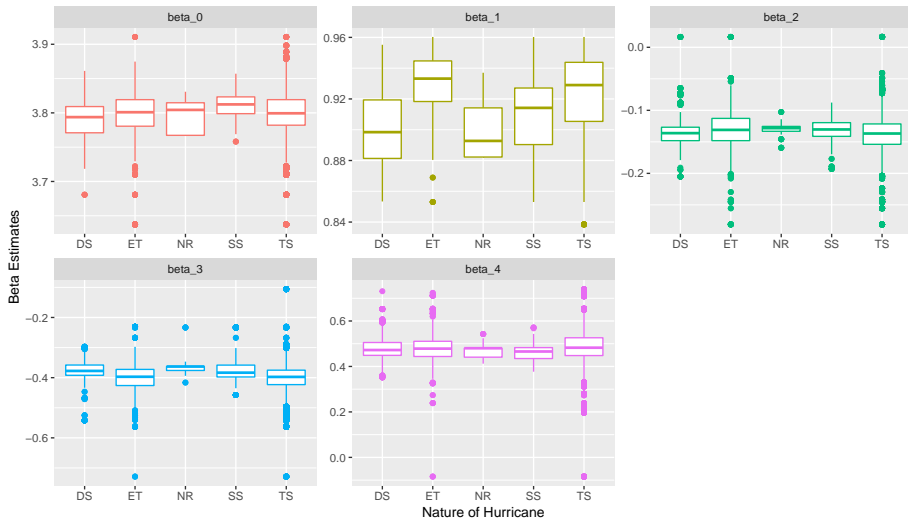
Understanding Seasonal Differences



Typical Hurricane season is June to November

Understanding Nature of Hurricane Differences

Beta Estimates by Nature of the Hurricane



Modeling Seasonal, and Nature Difference

Model:

For each β value we fit a different linear model.

$$\begin{aligned} Y_{ij} = & \alpha_{0j} + \alpha_{1j} \times \text{Decade}_i \\ & + \alpha_{(k+1)j} I(\text{Nature} = k)_i \\ & + \alpha_{(l+5)j} I(\text{Month} = l)_i + \epsilon_{ij} \end{aligned}$$

Where i is the hurricane, j is the Beta model, $k \in (ET, NR, SS, TS)$ making DS the reference group. Let $l \in (\text{April} - \text{December})$.

Results:

- Due to length each model estimate were omitted
- No Nature Indicators were significant
- No Month Indicators were significant
- For the β_1 model the decade estimated is significant -0.003 (-0.002, -0.004)

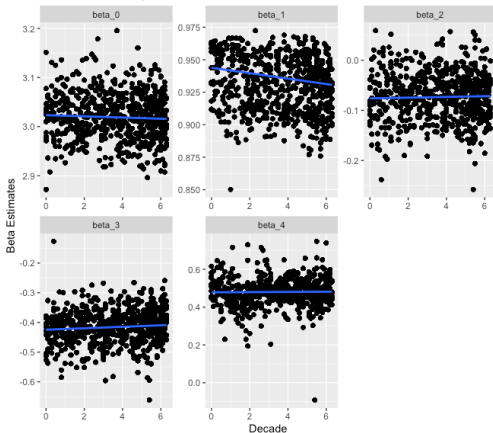
Thus Month and Nature don't have a linear association with each beta value.

Exploring if wind speeds have increased over the years

Model: $Y_i = \alpha_{0i} + \alpha_{1i} \times \text{Decade}$ where Y_i is each β_i and $i \in (0, \dots, 4)$.

Linear Model Output for Each β

Beta Estimates by Decade



Characteristic	Beta	95% CI [†]	p-value
beta0			
decade	-0.001	-0.002, 0.000	0.2
beta1			
decade	-0.003	-0.004, -0.002	<0.001
beta2			
decade	0.000	-0.001, 0.001	0.6
beta3			
decade	0.002	0.000, 0.004	0.041
beta4			
decade	0.001	-0.002, 0.004	0.5

[†] CI = Confidence Interval

• β_1 : Indicates a decrease in the change of wind speed over years

Deaths and Damages - Data Exploration

Distribution of Continuous Variables (Standardized)

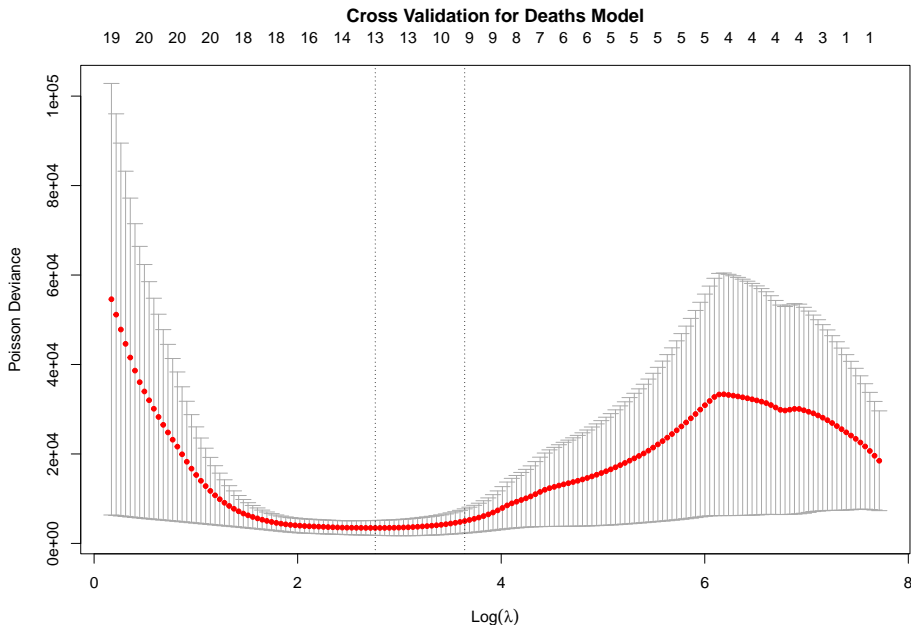
Characteristic	N = 43
damage	(-0.46, -0.43, -0.33, -0.09, 5.19)
deaths	(-0.29, -0.28, -0.27, -0.22, 5.81)
maxspeed	(-1.94, -0.81, 0.14, 0.89, 1.64)
meanspeed	(-1.67, -0.82, -0.02, 0.74, 2.18)
maxpressure	(-3.88, -0.57, 0.10, 0.32, 2.09)
meanpressure	(-3.94, 0.24, 0.31, 0.36, 0.41)
hours	(-1.87, -0.85, 0.00, 0.74, 2.50)
total_pop	(-0.98, -0.69, -0.37, 0.13, 2.74)
percent_poor	(-0.30, -0.30, -0.30, -0.29, 3.64)
percent_usa	(-1.22, -1.22, 0.14, 0.96, 1.14)

Note: For each characteristic, values denote the minimum, 25th percentile, median, 75th percentile, and maximum, respectively.

Deaths and Damages - Prediction and Inference

- Model Selection
 - Poisson regression with total population as offset
 - $y_i \sim \text{Poisson}(\mu_i)$
 - $\log(\mu_i) = \mathbf{x}_i' \gamma$
 - penalized regression via lasso
 - optimal λ selected via leave-one-out cross validation
 - feasible with small n
 - non-random
- Post-selection Inference
 - bootstrap smoothing as proposed by Efron (2014)
 - fit full model via Poisson-family GLM to obtain $\hat{\mu}$
 - for single bootstrap b , draw $\mathbf{y}_b^* \sim \text{Poisson}(\hat{\mu})$
 - execute lasso with \mathbf{y}_b^* as output to obtain distribution of $\hat{\gamma}_b^*$
 - compute empirical standard error of $\hat{\gamma}_b^*$

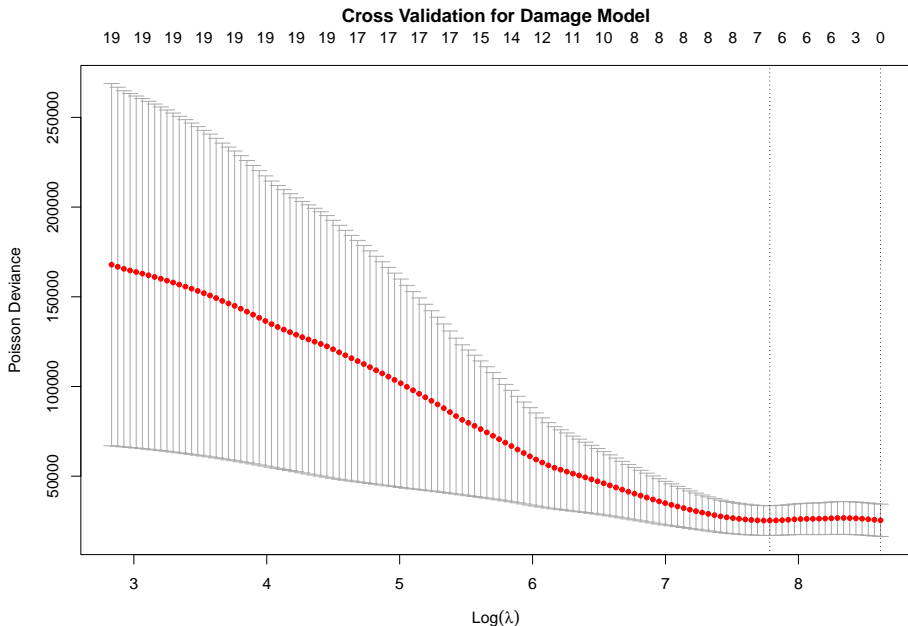
Hurricane Deaths - Model Selection



Hurricane Deaths - Inference

Covariate	Estimate	SE	p-value	Left CI	Right CI
season	0.0385	0.0013	0.0000	0.0359	0.0410
monthJuly	-3.7255	0.1095	0.0000	-3.9401	-3.5108
monthOctober	0.8073	0.0484	0.0000	0.7124	0.9022
natureNR	3.6402	0.1214	0.0000	3.4022	3.8782
natureTS	3.1078	0.0774	0.0000	2.9560	3.2596
maxpressure	-0.0379	0.0049	0.0000	-0.0474	-0.0283
hours	0.0054	0.0002	0.0000	0.0050	0.0057
percent_poor	0.0568	0.0005	0.0000	0.0558	0.0578
percent_usa	-0.0007	0.0005	0.1436	-0.0016	0.0002
beta_0	23.0045	0.5643	0.0000	21.8985	24.1104
beta_1	-10.9904	2.0393	0.0000	-14.9873	-6.9935
beta_2	1.1669	0.7284	0.1092	-0.2608	2.5945
beta_3	17.3277	0.3498	0.0000	16.6421	18.0132

Hurricane Damages - Model Selection



Hurricane Damages - Inference

Covariate	Estimate	SE	p-value	Left CI	Right CI
season	0.0399	0.0004	0.0000	0.0392	0.0406
monthJuly	-0.5762	0.0192	0.0000	-0.6137	-0.5386
monthOctober	0.4680	0.0074	0.0000	0.4536	0.4824
percent_usa	0.0001	0.0001	0.1052	0.0000	0.0003
beta_1	10.5907	0.2533	0.0000	10.0943	11.0871
beta_2	-3.3668	0.1255	0.0000	-3.6128	-3.1208
beta_3	2.4291	0.0669	0.0000	2.2979	2.5603

Conclusions

- Hurricanes with more observations enjoy better MCMC estimates
- With such a high-dimensional parameter space:
 - burn-in takes longer than expected
 - very sensitive to starting values
- Estimated β coefficients generally prove useful in predicting damage and deaths
 - particularly, $(\beta_1, \beta_2, \beta_3)$ are always selected
 - however, β_2 is only inferentially significant in the damage model