P8160 - Project 3
Baysian modeling of hurrican trajectories

Group 6

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Abstract

abstract

1. Introduction

1.1. Background

1.2. Objectives

2. Methods

2.1. Data Cleaning and Exploratory Analysis

There are 0 unique hurricanes in this dataset all the occurred in the north american region between the years ∞ to $-\infty$. Data on the storm's location (longitude & latitude) and maximum wind speed were recorded every 6 hours. The number of observations we have for each storm range from ∞ to $-\infty$ with a mean value of NaN observations per hurricane. Data is also collected on the storms month, and the nature of the hurricane; (Extra Tropical (ET), Disturbance (DS), Not Rated (NR), Sub Tropical (SS), and Tropical Storm (TS)).

To conduct our analysis we require at least 7 observations for each unique hurricane. In addition, we are only concerned about observations that occurred on 6 hour intervals. The dataset includes a couple observations between the 6 hour time periods. For our analysis we are only going to include observations that are recorded on hour 0, 6, 12, and 18. In addition we will exclude all hurricane IDs that have less then 7 observations. Through this process we remove -1362 observations so we are left with 21578 observations and 681 unique hurricanes. In addition we also created variables of lag difference ($t-6$ to $t-12$) for latitude, longitude and wind speed as $\Delta_{i,1}(t), \Delta_{i,2}(t), \Delta_{i,3}(t)$ and lag of wind speed as $Y_i(t-6)$.

Table 1: Data Characteristics shown by the Nature of the Hurricane variable

Characteristic	**Overall**, N = 21,578	**DS**, N = 963	**ET**, N = 2,149	**NR**, N = 96	**SS**, N = 750	**TS**, N = 17,620
Wind.kt	45 (30, 65)	25 (20, 30)	40 (30, 50)	25 (20, 25)	40 (30, 50)	50 (35, 70)
Latitude	26 (19, 34)	24 (17, 31)	44 (38, 50)	18 (14, 22)	32 (30, 36)	25 (18, 31)
Longitude	-64 (-78, -48)	-65 (-81, -45)	-46 (-62, -28)	-64 (-69, -55)	-65 (-74, -51)	-65 (-80, -51)
Month						
January	69 (0.3%)	5 (0.5%)	0 (0%)	0 (0%)	19 (2.5%)	45 (0.3%)
February	0 (0%)	0 (0%)	0 (0%)	0 (0%)	0 (0%)	0 (0%)
March	0 (0%)	0 (0%)	0 (0%)	0 (0%)	0 (0%)	0 (0%)
April	53 (0.2%)	0 (0%)	23 (1.1%)	0 (0%)	17 (2.3%)	13 (<0.1%)
May	274 (1.3%)	38 (3.9%)	18 (0.8%)	0 (0%)	57 (7.6%)	161 (0.9%)
June	795 (3.7%)	34 (3.5%)	113 (5.3%)	0 (0%)	58 (7.7%)	590 (3.3%)
July	1,478 (6.8%)	95 (9.9%)	103 (4.8%)	19 (20%)	41 (5.5%)	1,220 (6.9%)
August	5,101 (24%)	290 (30%)	319 (15%)	14 (15%)	108 (14%)	4,370 (25%)
September	8,810 (41%)	249 (26%)	854 (40%)	39 (41%)	160 (21%)	7,508 (43%)
October	3,717 (17%)	163 (17%)	547 (25%)	11 (11%)	146 (19%)	2,850 (16%)
November	1,047 (4.9%)	58 (6.0%)	158 (7.4%)	13 (14%)	86 (11%)	732 (4.2%)
December	234 (1.1%)	31 (3.2%)	14 (0.7%)	0 (0%)	58 (7.7%)	131 (0.7%)

2.2 Bayesian Model for Hurricane Trajectories

Climate researchers are interested in modeling the hurricane trajectories to forecast the winds peed. To model the wind speed of the i^{th} hurricane at time t we will use

$$Y_i(t) = \beta_{0,i} + \beta_{1,i} Y_i(t-6) + \beta_{2,i}\Delta_{i,1}(t) + \beta_{3,i}\Delta_{i,2}(t) + \beta_{4,i}\Delta_{i,3}(t) + \epsilon_i(t)$$

Where $\Delta_{i,1}(t)$, $\Delta_{i,2}(t)$ and $\Delta_{i,3}(t)$ are changes in latitude longitude and wind speed respectively between $t-6$ and t. The $\epsilon_i(t) \sim \mathcal{N}(0, \sigma^2)$ are independent across t. Let $\beta_i = (\beta_{0,i}, \beta_{1,i}, \beta_{2,i}, \beta_{3,i}, \beta_{4,i})^T \sim \mathcal{N}(\mu, \Sigma)$

be multivariate normal distribution where $\mu \in \mathbb{R}^d$ and $\Sigma \in \mathbb{R}^{d\times d}$. For this model we will be assuming non-informative or weak prior distributions for our unknown parameters σ^2, μ and Σ .

$$\pi(\sigma^2) \propto \frac{1}{\sigma^2}, \qquad \pi(\mu) \propto 1, \qquad \pi\left(\Sigma^{-1}\right) \propto |\Sigma|^{-(d+1)} \exp\left\{-\frac{1}{2}\Sigma^{-1}\right\}$$

Our goal is to estimate $\Theta = (\boldsymbol{B}, \boldsymbol{\mu}, \Sigma^{-1}, \sigma^2)$. To do this we need to establish our likelihood and prior functions. Since this is a Bayesian model we have that the likelihood will be expressed as

$$L(\boldsymbol{Y} \mid \boldsymbol{\Theta}) \propto \prod_{i=1}^{m} \left(\sigma^2\right)^{-\frac{n_i}{2}} \exp\left\{-\frac{1}{2\sigma^2}\left(\boldsymbol{Y}_i - \boldsymbol{X}_i\boldsymbol{\beta}_i\right)^T\left(\boldsymbol{Y}_i - \boldsymbol{X}_i\boldsymbol{\beta}_i\right)\right\}$$

where m is the number of hurricane and n_i is the number of observations for i^{th} hurricane. The joint prior distribution is expressed as

$$\pi\left(\boldsymbol{\Theta}\right) \propto \left(\sigma^2\right)^{-1}\left|\Sigma^{-1}\right|^{d+1} \exp\left\{-\frac{1}{2}\operatorname{tr}\left(\Sigma^{-1}\right)\right\}\prod_{i=1}^{m}\left|\Sigma^{-1}\right|^{\frac{1}{2}} \exp\left\{-\frac{1}{2}\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}\right)^T\Sigma^{-1}(\boldsymbol{\beta}_i - \boldsymbol{\mu})\right\}$$

where d is the dimension of $\boldsymbol{\mu}$. Using the likelihood and priors derived above we can calculate the posterior distribution.

$$\begin{aligned}
\pi(\boldsymbol{\Theta} \mid \boldsymbol{Y}) &\propto \left(\sigma^2\right)^{-1}\left|\Sigma^{-1}\right|^{d+1} \exp\left\{-\frac{1}{2} + \operatorname{tr}\left(\Sigma^{-1}\right)\right\} \\
&\quad\times \prod_{i=1}^{m}\left(\sigma^2\right)^{-\frac{n_i}{2}}\left|\Sigma^{-1}\right|^{\frac{1}{2}} \exp\left\{-\frac{1}{2\sigma^2}\left(\boldsymbol{Y}_i - \boldsymbol{X}_i\boldsymbol{\beta}_i\right)^T\left(\boldsymbol{Y}_i - \boldsymbol{X}_i\boldsymbol{\beta}_i\right)\right\} \exp\left\{-\frac{1}{2}\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}\right)^T\Sigma^{-1}\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}\right)\right\} \\
&= \left(\sigma^2\right)^{-\left(1+\frac{\sum_{i=1}^{m} n_i}{2}\right)}\left|\Sigma^{-1}\right|^{d+1+\frac{m}{2}} \exp\left\{-\frac{1}{2}\operatorname{tr}\left(\Sigma^{-1}\right)\right\} \exp\left\{-\frac{1}{2}\sum_{i=1}^{m}\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}\right)^T\Sigma^{-1}\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}\right)\right\} \\
&\quad\times \exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^{m}\left(\boldsymbol{Y}_i - \boldsymbol{X}_i\boldsymbol{\beta}_i\right)^T\left(\boldsymbol{Y}_i - \boldsymbol{X}_i\boldsymbol{\beta}_i\right)\right\}
\end{aligned}$$

Due to the number of unknown parameters and the complexity we will need to use method to approximate this distribution. Before describing that process we first need to calculate the conditional posterior distribution for each unknown parameter.

$$\begin{aligned}
&\boldsymbol{\beta}_i : \pi(\boldsymbol{\beta}_i \mid \boldsymbol{\Theta}_{(-\boldsymbol{\beta}_i)}\boldsymbol{Y}) \propto \exp\left\{-\frac{1}{2}\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}\right)^T\Sigma^{-1}\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}\right) - \frac{1}{2\sigma^2}\left(\boldsymbol{Y}_i - \boldsymbol{X}_i\boldsymbol{\beta}_i\right)^T\left(\boldsymbol{Y}_i - \boldsymbol{X}_i\boldsymbol{\beta}_i\right)\right\} \\
&\boldsymbol{\mu} : \pi\left(\boldsymbol{\mu} \mid \boldsymbol{\Theta}_{(-\boldsymbol{\mu})}, \boldsymbol{Y}\right) \sim N(\bar{\boldsymbol{\beta}}, \Sigma/m) \\
&\sigma^2 : \pi\left(\sigma^2 \mid \boldsymbol{\Theta}_{(-\sigma^2)}, \boldsymbol{Y}\right) \propto \left(\sigma^2\right)^{-\left(1+\frac{\sum_{i=1}^{m} n_i}{2}\right)} \times \exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^{m}\left(\boldsymbol{Y}_i - \boldsymbol{X}_i\boldsymbol{\beta}_i\right)^T\left(\boldsymbol{Y}_i - \boldsymbol{X}_i\boldsymbol{\beta}_i\right)\right\} \\
&\Sigma^{-1} : \pi\left(\Sigma^{-1} \mid \boldsymbol{\Theta}_{(-\Sigma^{-1})}, \boldsymbol{Y}\right) \sim \text{Wishart}\left(3d + 3 + m, \left(\boldsymbol{I} + \sum_{i=1}^{m}\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}\right)\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}\right)^T\right)^{-1}\right)
\end{aligned}$$

To see a more detailed description of each conditional posterior please see Appendix A.

2.3 Markov Chain Monte Carlo (MCMC) Algorithm

Due to the complexity of the above posterior distribution we will use a Markov chain Monte Carlo (MCMC) process. Since we could generate full conditional posterior distribution for some parameters but not all we will instead apply hybrid algorithm consisting with Metropolis-Hastings (MH) steps and Gibbs steps. We will preform a MH step for $\beta_{j,i}$, σ^2 and sample from the $\Sigma^{-1^{(t+1)}}$ distribution. We will also describe a gibbs step for μ using the $\beta_{j,i}$ gathered.

First is the MH steps for $\beta_{j,i}$. Sampling proposed $\beta'_{j,i}$, $j = 0, 1...4$ for i^{th} hurricane from proposal distribution $U\left(\beta_{j,i}^{(t)} - a_{j,i}, \beta_{j,i}^{(t)} + a_{j,i}\right)$, where $a_{j,i}$ is the search window for $\beta_{j,i}$. Since the proposals are symmetry, the accepting or rejecting the proposed $\beta'_{j,i}$ depends on the ratio of posterior distribution. Some of the parameters in $\boldsymbol{\Theta}$ could be cancelled out, so the ratio simplified to be:

$$\frac{\pi\left(\boldsymbol{\beta}'_i, \boldsymbol{\Theta}^{(t)}_{(-\boldsymbol{\beta}_i)} \mid \boldsymbol{Y}\right)}{\pi\left(\boldsymbol{\beta}^{(t)}_i, \boldsymbol{\Theta}^{(t)}_{(-\boldsymbol{\beta}_i)} \mid \boldsymbol{Y}\right)} = \frac{\exp\left\{-\frac{1}{2\sigma^{2(t)}}\left(\boldsymbol{Y}_i - \boldsymbol{X}_i\boldsymbol{\beta}'_i\right)^T\left(\boldsymbol{Y}_i - \boldsymbol{X}_i\boldsymbol{\beta}'_i\right) - \frac{1}{2}\left(\boldsymbol{\beta}'_i - \boldsymbol{\mu}^{(t)}\right)^T\Sigma^{-1^{(t)}}\left(\boldsymbol{\beta}'_i - \boldsymbol{\mu}^{(t)}\right)\right\}}{\exp\left\{-\frac{1}{2\sigma^{2(t)}}\left(\boldsymbol{Y}_i - \boldsymbol{X}_i\boldsymbol{\beta}^{(t)}_i\right)^T\left(\boldsymbol{Y}_i - \boldsymbol{X}_i\boldsymbol{\beta}^{(t)}_i\right) - \frac{1}{2}\left(\boldsymbol{\beta}^{(t)}_i - \boldsymbol{\mu}^{(t)}\right)^T\Sigma^{-1(t)}\left(\boldsymbol{\beta}^{(t)}_i - \boldsymbol{\mu}^{(t)}\right)\right\}}$$

where $\boldsymbol{\beta}'_i$ consisting with $\beta'_{j,i}$, $\beta^{(t)}_{k,i}$ for $k > j$ and $\beta^{(t+1)}_{k,i}$ for $k < j$. The log of the ratio is

$$\begin{aligned}\log\frac{\pi\left(\boldsymbol{\beta}'_i, \boldsymbol{\Theta}^{(t)}_{(-\boldsymbol{\beta}_i)} \mid \boldsymbol{Y}\right)}{\pi\left(\boldsymbol{\beta}^{(t)}_i, \boldsymbol{\Theta}^{(t)}_{(-\boldsymbol{\beta}_i)} \mid \boldsymbol{Y}\right)} = &-\frac{1}{2}\left(\frac{\left(\boldsymbol{Y}_i - \boldsymbol{X}_i\boldsymbol{\beta}'_i\right)^T\left(\boldsymbol{Y}_i - \boldsymbol{X}_i\boldsymbol{\beta}'_i\right)}{\sigma^{2(t)}} + \left(\boldsymbol{\beta}'_i - \boldsymbol{\mu}^{(t)}\right)^T\Sigma^{-1^{(t)}}\left(\boldsymbol{\beta}'_i - \boldsymbol{\mu}^{(t)}\right)\right)\\ &+\frac{1}{2}\left(\frac{\left(\boldsymbol{Y}_i - \boldsymbol{X}_i\boldsymbol{\beta}^{(t)}_i\right)^T\left(\boldsymbol{Y}_i - \boldsymbol{X}_i\boldsymbol{\beta}^{(t)}_i\right)}{\sigma^{2(t)}} + \left(\boldsymbol{\beta}^{(t)}_i - \boldsymbol{\mu}^{(t)}\right)^T\Sigma^{-1^{(t)}}\left(\boldsymbol{\beta}^{(t)}_i - \boldsymbol{\mu}^{(t)}\right)\right)\end{aligned}$$

Then we randomly sample u from $U(0,1)$ and compare $\log(u)$ with the log ratio. If the $\log(u)$ is smaller, we accept $\beta'_{j,i} = \beta^{(t+1)}_{j,i}$, otherwise we reject $\beta'_{j,i}$ and $\beta^{(t)}_{j,i} = \beta^{(t+1)}_{j,i}$.

Then, Gibb step for $\boldsymbol{\mu}$: Sample $\boldsymbol{\mu}^{(t+1)}$ from $\mathcal{N}\left(\bar{\boldsymbol{\beta}}^{(t+1)}, \Sigma^{(t)}/m\right)$, where $\bar{\boldsymbol{\beta}}^{(t+1)}$ is the average $\boldsymbol{\beta}^{(t+1)}_i$ over all hurricanes.

Next, is the MH step to generate $\sigma^{2'}$ from $U\left(\sigma^{2^{(t)}} - a_{\sigma^2}, \sigma^{2^{(t)}} + a_{\sigma^2}\right)$. Firstly, check whether $\sigma^{2'}$ is positive, if not, we reject $\sigma^{2'}$. Then, we randomly sample u from $U(0,1)$ and compare $\log(u)$ with the log posterior ratio. The log posterior ratio

$$\begin{aligned}\log\frac{\pi\left(\sigma^{2'}, \boldsymbol{\Theta}^{(t)}_{(-\sigma^2)} \mid \boldsymbol{Y}\right)}{\pi\left(\sigma^{2^{(t)}}, \boldsymbol{\Theta}^{(t)}_{(-\sigma^2)} \mid \boldsymbol{Y}\right)} = &-\left(1 + \frac{M}{2}\right)\log(\sigma^{2'}) - \frac{1}{2\sigma^{2'}}\sum_{i=1}^{m}\left(\boldsymbol{Y}_i - \boldsymbol{X}_i\boldsymbol{\beta}^{(t+1)}_i\right)^T\left(\boldsymbol{Y}_i - \boldsymbol{X}_i\boldsymbol{\beta}^{(t+1)}_i\right)\\ &+\left(1 + \frac{M}{2}\right)\log(\sigma^{2^{(t)}}) + \frac{1}{2\sigma^{2^{(t)}}}\sum_{i=1}^{m}\left(\boldsymbol{Y}_i - \boldsymbol{X}_i\boldsymbol{\beta}^{(t+1)}_i\right)^T\left(\boldsymbol{Y}_i - \boldsymbol{X}_i\boldsymbol{\beta}^{(t+1)}_i\right)\end{aligned}$$

where M is total number of observation for all hurricanes. If the $\log(u)$ is smaller, we accept $\sigma^{2'} = \sigma^{2^{(t+1)}}$.

Finally, we sample $\Sigma^{-1^{(t+1)}}$ from Wishart $\left(3d + 3 + m, \left(\boldsymbol{I} + \sum_{i=1}^{m}\left(\boldsymbol{\beta}^{(t+1)}_i - \boldsymbol{\mu}^{(t+1)}\right)\left(\boldsymbol{\beta}^{(t+1)}_i - \boldsymbol{\mu}^{(t+1)}\right)^T\right)^{-1}\right)$.

For the MCMC process we will run 10,000 iterations using the code shown in Appendix B.

2.4 Initial Starting Values for MCMC

To initiate the MCMC process we need to specify the starting values. The choice of starting value is important to help with the convergence time of our algorithm. For each parameter we will chose the start values to be:

- $\boldsymbol{\beta}_i$: Fit OLS multivariate linear regression (MLR) for i^{th} hurricane and use the coefficients as $\boldsymbol{\beta}^{(0)}_i$

Table 2: Range of Search Window and Acceptance Rate for paraemters used MH step

	Search Window	Acceptance Rate (%)
$\boldsymbol{\beta}_{0,}$	1.1	45.87 - 51.36
$\boldsymbol{\beta}_{1,}$	(0.04, 0.1)	31.67 - 63.68
$\boldsymbol{\beta}_{2,}$	(0.8, 1.0)	38.60 - 45.60
$\boldsymbol{\beta}_{3,}$	(0.5, 0.6)	33.20 - 61.32
$\boldsymbol{\beta}_{4,}$	(0.4, 0.5)	34.95 - 60.45
σ^2	2.0	44.83

- $\boldsymbol{\mu}$: Average over all $\boldsymbol{\beta}_i^{(0)}$ as $\boldsymbol{\mu}^{(0)}$
- σ^2: $\hat{\sigma}_i^2$ is the mean square residuals of the OLS model for i^{th} hurricane. Take the mean over all $\hat{\sigma}_i^2$ as ${\sigma^2}^{(0)}$
- Σ^{-1}: Generate the covariance matrix of $\boldsymbol{\beta}_i^{(0)}$ and take the inverse of the matrix as ${\Sigma^{-1}}^{(0)}$

There is a MH step in the MCMC algorithm so the choice of search window is important. The acceptance rate of the MH step is around 0.317 to 0.637, so the search window of our algorithm is appropriate. We tune the search windows multiple times to achieve this result. Table 1 demonstrates the range of search windows a and associated acceptance rates for $\boldsymbol{\beta}$ and σ^2.

2.5 MCMC Model Performance

To assess the quality of the proposed model we will assess the overall adjusted R^2 value from the Bayesian model. In addition the model performance will be assessed for each hurricane by the adjusted R^2 value as well as a goodness of fit test. The goodness of fit test we will use residuals of Bayesian estimates, r_{ij}, of the j^{th} observation in i^{th} hurricane, to calculate the test statistics. $\chi^2_{stat} = \frac{\sum_{j=1}^{n_i} r_{ij}^2}{\sigma^2}$, where σ^2 is the estimate σ^2 from MCMC. Based on the normal assumption in intro, $\chi^2_{stat} \sim \chi^2_{n_i}$, where n_i is the number of observation for i^{th} hurricane. Visual inspection can also be used for each individual hurricane plotting their observed wind speed with the model's predicted wind speed.

2.6 Models to Explore Seasonal Differences and Yearly Trends

In the dataset information about the hurricanes start month, year and the nature of the hurricane is recorded. It is of interested to explore how these three variables affect the wind speed. Specifically we are interested in exploring the seasonal differences, and if there any evidence supporting the statement that "the hurricane wind speed has been increasing over years". To do visual inspection can first be used to help us understand how these three variables impact the beta estimates. Then we can see if there is a linear trend given the three variables using each β estimates gained from the Bayesian model described above. For each β value we fit a different linear model. We can first fit a model using the three variables (month, year, and nature) as predictors and the 5 different β values as the outcome. This will result in 5 different linear models, one for each β value.

$$Y_{ji} = \alpha_{0j} + \alpha_{1j} \times \text{Decade}_i + \alpha_{(k+1)j} I(\text{Nature} = k)_i + \alpha_{(l+5)j} I(\text{Month} = l)_i + \epsilon_{ji}$$

Where i is the hurricane, j is the Beta model (0 through 4), $k \in (ET,\ NR,\ SS,\ TS)$ making DS the reference group. Let l in (April - December) where January is the reference group. We chose to include nature and month as categorical variables and year (transformed into decade) as a continuous variable.

We are also going to consider 5 different models just using decade as a predictor. $Y_i = \alpha_{0i} + \alpha_{1i} \times \text{Decade}$ where Y_i is each β_i and $i \in (0, \ldots, 4)$.

2.7 Models to Explore Death and Damages

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3. Results

3.1 MCMC Model Convergence

Using the starting values describe above we can now examine the model performance. We are particular interested in the parameter convergence. Below we can see Figure 1 that displays each of the parameters of iterations over the 10,000 iterations. Wee see that each parameter reaches a convergence at around 5,500 iterations. Therefore, we will make 5,500 iterations our burn in period. Figure 2 displays the histogram results from the selected parameters excluding the burn in period. We see that each of the distributions is relatively normal with some skewed results for Σ^{-1} and heavy tails in μ_0.

Convergence was reach in each of the parameters of interest. Thus we can take the average estimate excluding the burn in period. When we do this we are left with the μ_i values, σ^2 values, Σ^2 matrix, and a β_i estimate for each hurricane. Figure 4 displays the μ_i values, σ^2 values, Σ^2 matrix results. We see that the μ_2, μ_3 estimate are negative while the others are positive. Figure 3 displays the β_i estimates for each hurricane in a histogram.

The average β_0 estimate for the hurricanes is 3.80 which represents the intercept of the wind speed model. The average β_1 estimate for the hurricanes is 0.915 which represents the lagged wind speed impact model. This positive estimate indicated that a larger previous wind speed will indicate a higher wind speed for the next time point. The β_2 estimate is -0.136 which represents the impact of the change in latitude. The β_3 estimate is -0.395 which represents the impact of the change in longitude. Both of the impact of change in latitude and longitude have negative beta estimated values indicating that a larger change in direction corresponds with a decrease in wind speeds. Thus hurricane speed and wind speed seem to be inversely correlated. The β_4 estimate is 0.481 which represents the impact of the change in wind speed. This positive value indicated that a larger change in wind speed corresponds with a larger wind speed value.

3.2 MCMC Model Performance

The overall adjusted R^2 of the estimated Bayesian model is 0.9524156. We can also look at the adjusted R^2 value for each individual hurricane. Table 2 displays the values grouped by the adjusted R^2 showing the number of hurricanes and the percentage. We see that most hurricanes do well with a adjusted R^2 above 0.6. Even though most of the estimated models track hurricanes well with great adjusted R^2, some estimated models track the hurricanes extremely bad. 23.3% of the estimated models do not track the hurricane well and have adjusted R^2 less than 0.6. Few models have negative adjusted R^2. We also perform the goodness-of-fit test for individual estimated model. The estimated models of 88 hurricanes have p-value less than 0.05, implying that those models do not fit well.

Figure 5 shows the adjusted R^2 for each hurricane by their number of observations. This plot shows that with more observations the better our model with do. The to the right of the vertical line shows that 40 or more observations give us an adjusted R^2 values above 0.5.

Figure 6 displays 9 randomly selected hurricanes actual wind speeds and predicted wind speeds. We see that for all of them the lines are very similar with some slight derivations away when the number of observations are small. In DOG 1950 we see very good model prediction and for this particular hurricane we have a lot of observations. However, for JERRY 2007 we do not have as many observations and see deviations at the begging and end of the storm.

Table 3: R^2_{adj} for each hurricane

R^2_{adj}	Count of Hurricanes	Percentage(%)
0.6-1	522	76.7
0.2-0.6	79	11.6
< 0.2	80	11.7

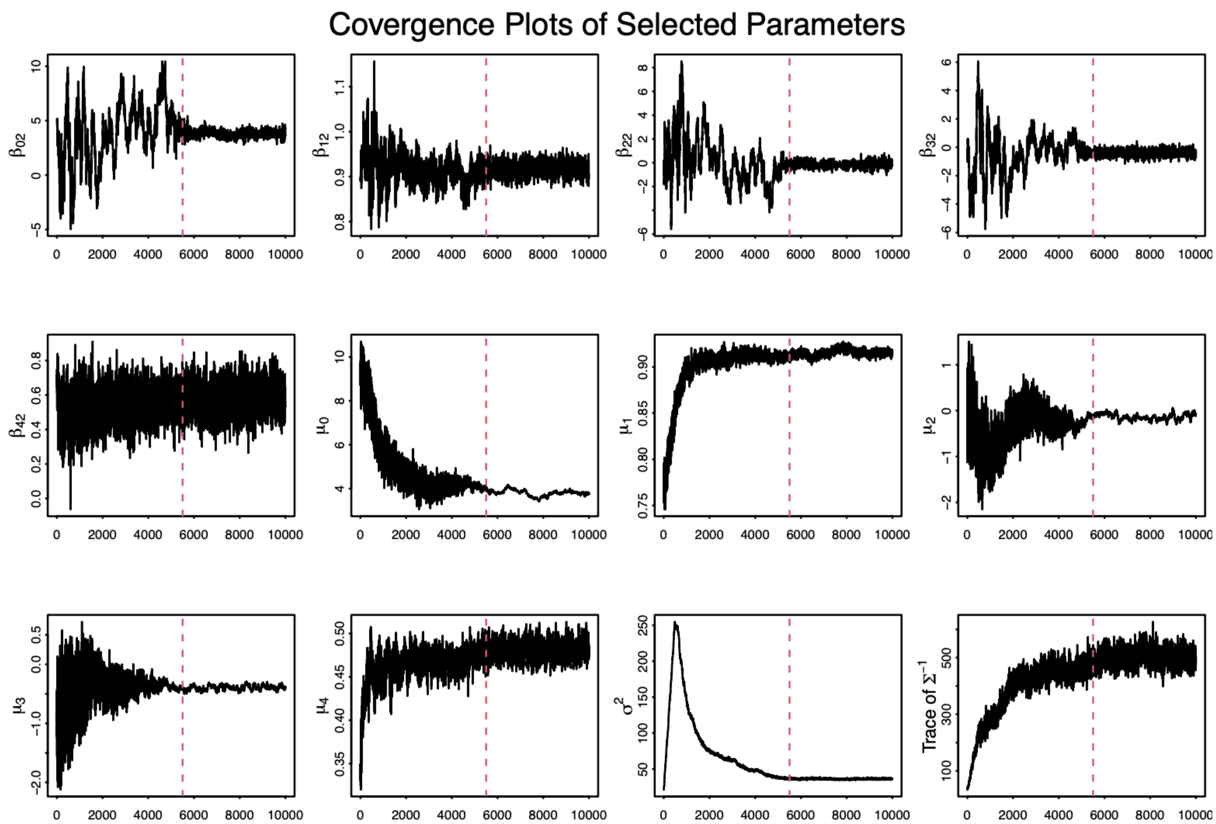

Figure 1: Convergence Plot of the Selected Parameters after 10,000 Iterations

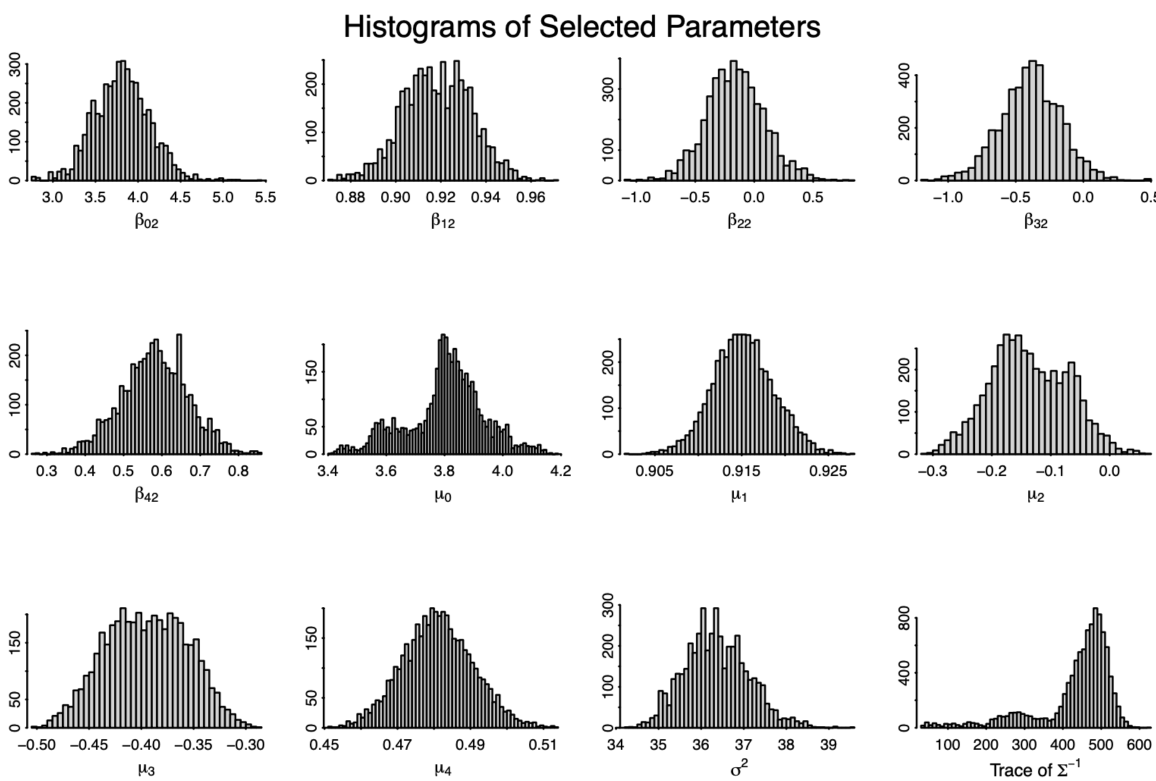

Figure 2: Histogram Plot of the Selected Parameters not including the 5,500 burn in period results

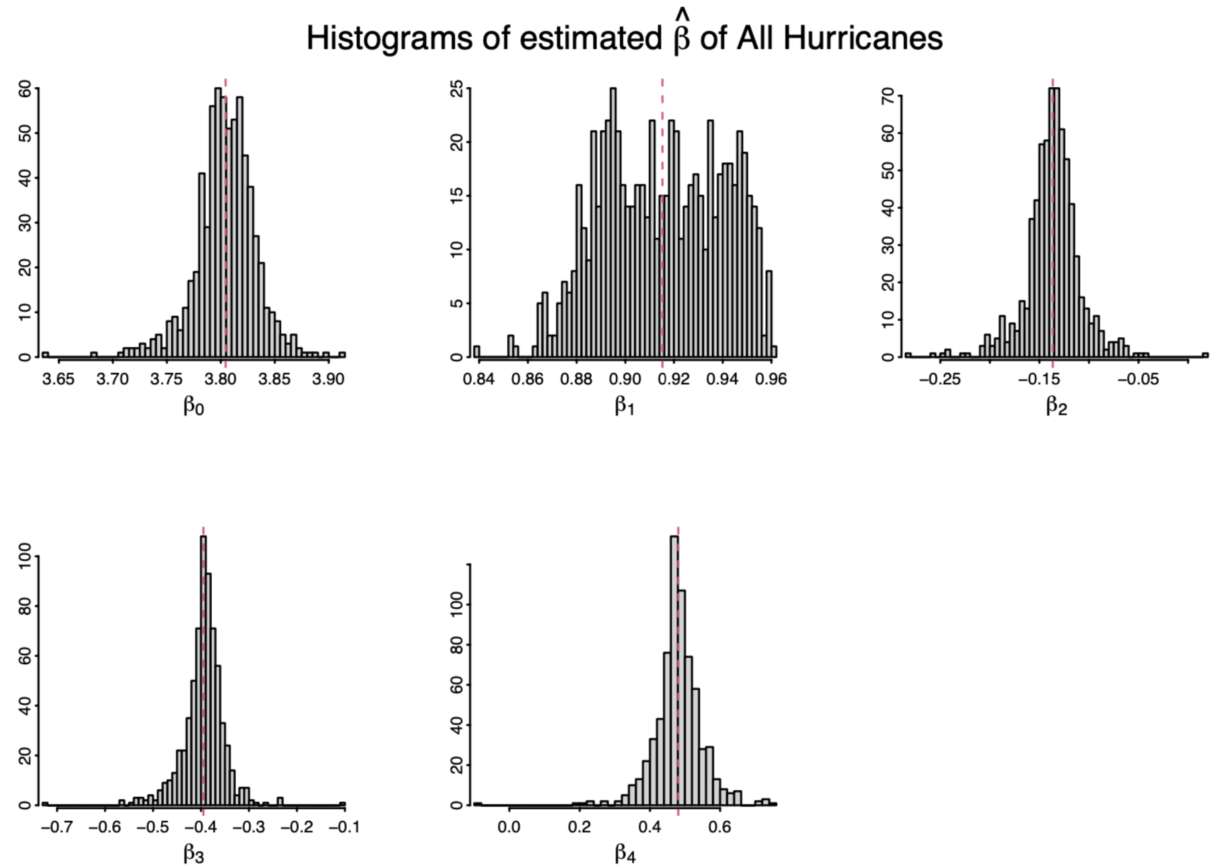

Figure 3: Bayesian Estiamtes for the β values for all Hurricane

	μ_0	μ_1	μ_2	μ_3	μ_4	σ^2	σ^2_{00}	σ^2_{11}	σ^2_{22}	σ^2_{33}	σ^2_{44}
Estimates	3.8	0.92	-0.14	-0.39	0.48	36.36	0.063	0.003	0.047	0.042	0.018

$$\hat{\Sigma}^{-1} = \begin{bmatrix} 16.07 & 7.63 & 0.34 & -1.78 & 0.47 \\ 7.63 & 381.32 & 5.39 & 3.96 & -6.32 \\ 0.34 & 5.39 & 21.43 & 1.48 & 1.14 \\ -1.78 & 3.96 & 1.48 & 24.35 & -3.3 \\ 0.47 & -6.32 & 1.14 & -3.3 & 55.12 \end{bmatrix} \quad \hat{\rho} = \begin{bmatrix} 1 & -0.101 & -0.018 & 0.094 & -0.011 \\ -0.101 & 1 & -0.057 & -0.043 & 0.043 \\ -0.018 & -0.057 & 1 & -0.067 & -0.042 \\ 0.094 & -0.043 & -0.067 & 1 & 0.089 \\ -0.011 & 0.043 & -0.042 & 0.089 & 1 \end{bmatrix}$$

Figure 4: Bayesian Estiamtes for μ and σ^2

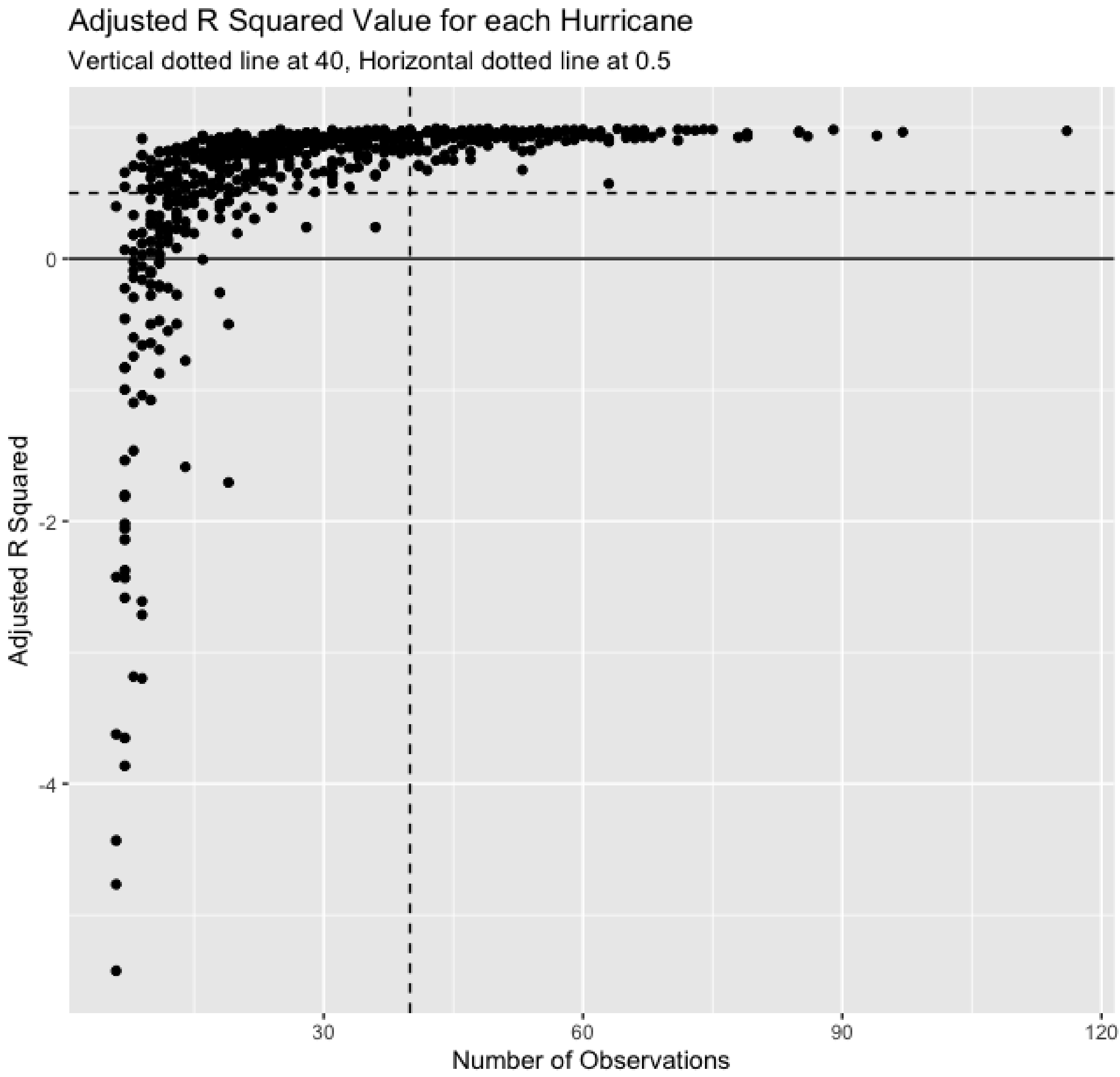

Figure 5: Adjusted R squared for each Hurricane plotted by the number of observations.

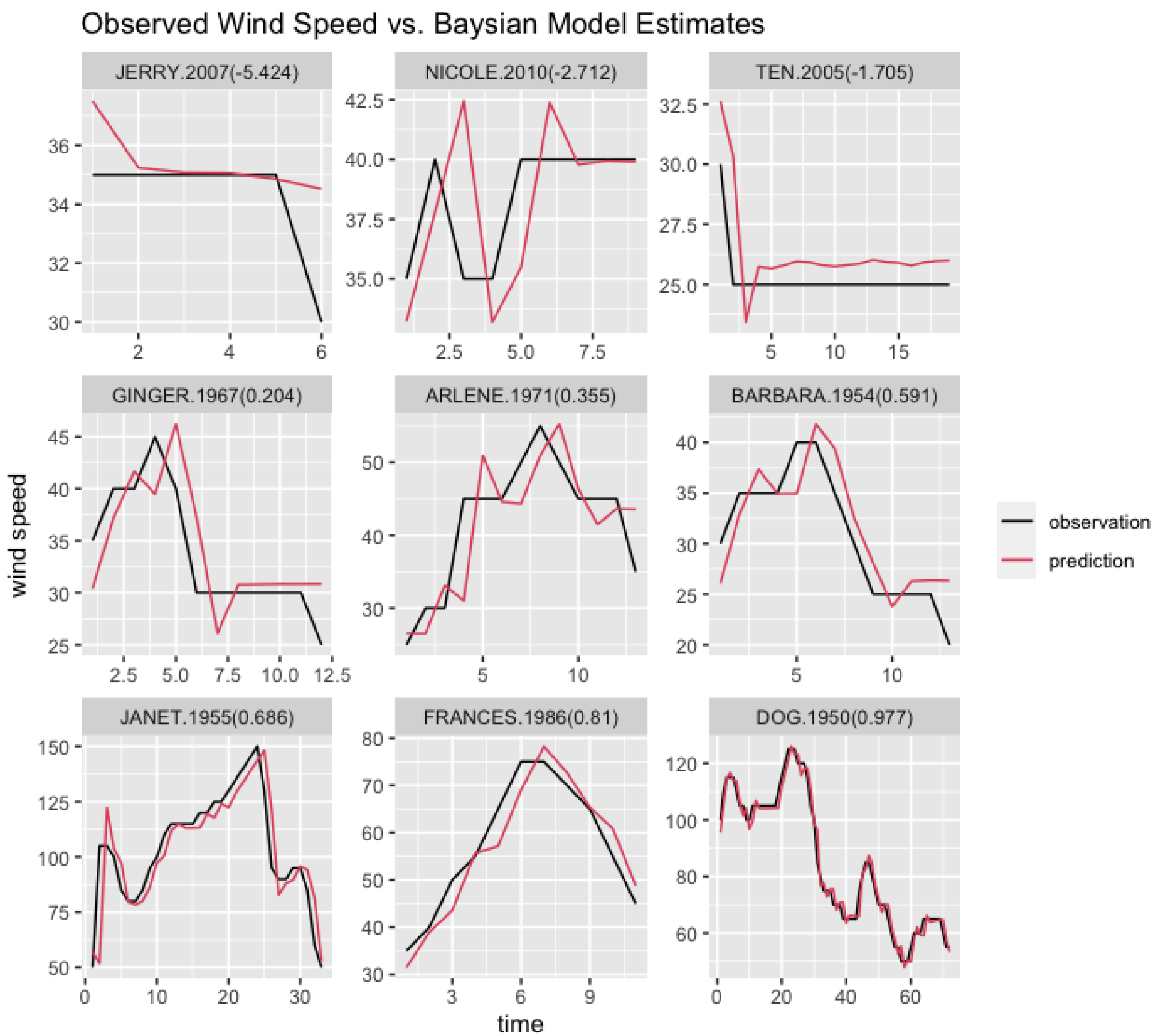

Figure 6: The predicted wind speeds verses the actual observed wind speeds for 9 randomly selected hurricanes.

3.3 Seasonal Differences and Yearly Trends Results

3.5 Death and Damages Results

4. Discussion

4.1. Summary of Findings

4.2. Limitations

4.3 Future Work

4.4. Group Contributions

References

[1] one

[2] two

Appendices

Appendix A

Calculating the conditional Priors for each paramater.

$$\boldsymbol{\beta}_i : \pi(\boldsymbol{\beta}_i \mid \boldsymbol{\Theta}_{(-\boldsymbol{\beta}_i)}\boldsymbol{Y}) \propto \exp\left\{-\frac{1}{2}\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}\right)^T \Sigma^{-1}\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}\right) - \frac{1}{2\sigma^2}\left(\boldsymbol{Y}_i - \boldsymbol{X}_i\boldsymbol{\beta}_i\right)^T\left(\boldsymbol{Y}_i - \boldsymbol{X}_i\boldsymbol{\beta}_i\right)\right\}$$

$$\boldsymbol{\mu} : \pi\left(\boldsymbol{\mu} \mid \boldsymbol{\Theta}_{(-\boldsymbol{\mu})}, \boldsymbol{Y}\right) \propto \exp\left\{-\frac{1}{2}\sum_{i=1}^{m}\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}\right)^T \Sigma^{-1}\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}\right)\right\}$$

$$\begin{aligned}
(\boldsymbol{\beta}_i - \boldsymbol{\mu})^T\Sigma^{-1}(\boldsymbol{\beta}_i - \boldsymbol{\mu}) &= \operatorname{tr}\left(\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}\right)^T \Sigma^{-1}\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}\right)\right) \\
&= \operatorname{tr}\left(\Sigma^{-1}\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}\right)\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}\right)^T\right) \\
\pi\left(\boldsymbol{\mu} \mid \boldsymbol{\Theta}_{(-\boldsymbol{\mu})}, Y\right) &\propto \exp\left\{-\frac{1}{2}\operatorname{tr}\left(\Sigma^{-1}\sum_{i=1}^{m}\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}\right)\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}\right)^T\right)\right\} \\
&= \exp\left\{-\frac{1}{2}\operatorname{tr}\left(\Sigma^{-1}m\left(\boldsymbol{\mu} - \bar{\boldsymbol{\beta}}\right)\left(\boldsymbol{\mu} - \bar{\boldsymbol{\beta}}\right)^T\right)\right\} \\
&= \exp\left\{-\frac{1}{2}(\boldsymbol{\mu} - \bar{\boldsymbol{\beta}})^T\Sigma^{-1}m(\boldsymbol{\mu} - \bar{\boldsymbol{\beta}})\right\} \\
&\Rightarrow \boldsymbol{\mu} \mid \boldsymbol{\Theta}_{(-\boldsymbol{\mu})}, \boldsymbol{Y} \sim N(\bar{\boldsymbol{\beta}}, \Sigma/m)
\end{aligned}$$

$$\sigma^2 : \pi\left(\sigma^2 \mid \boldsymbol{\Theta}_{(-\sigma^2)}, \boldsymbol{Y}\right) \propto \left(\sigma^2\right)^{-\left(1+\frac{\sum_{i=1}^{m} n_i}{2}\right)} \times \exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^{m}\left(\boldsymbol{Y}_i - \boldsymbol{X}_i\boldsymbol{\beta}_i\right)^T\left(\boldsymbol{Y}_i - \boldsymbol{X}_i\boldsymbol{\beta}_i\right)\right\}$$

$$\begin{aligned}
\Sigma^{-1} : \pi\left(\Sigma^{-1} \mid \boldsymbol{\Theta}_{(-\Sigma^{-1})}, \boldsymbol{Y}\right) &\propto \left|\Sigma^{-1}\right|^{d+1+\frac{m}{2}} \exp\left\{-\frac{1}{2}\operatorname{tr}\left(\Sigma^{-1}\right)\right\}\exp\left\{-\frac{1}{2}\sum_{i=1}^{m}\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}\right)^T\Sigma^{-1}\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}\right)\right\} \\
&= \left|\Sigma^{-1}\right|^{d+1+\frac{m}{2}} \exp\left\{-\frac{1}{2}\operatorname{tr}\left(\Sigma^{-1}\right) - \frac{1}{2}\operatorname{tr}\left(\Sigma^{-1}\sum_{i=1}^{m}\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}\right)\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}\right)^T\right)\right\} \\
&= \left|\Sigma^{-1}\right|^{d+1+\frac{m}{2}} \exp\left\{-\frac{1}{2}\operatorname{tr}\left(\Sigma^{-1}\left(I + \sum_{i=1}^{m}\left(\beta_i - \mu\right)\left(\beta_1 - \mu\right)^T\right)\right)\right\} \\
&\Rightarrow \Sigma^{-1} \mid \boldsymbol{\Theta}_{(-\Sigma^{-1})}, \boldsymbol{Y} \sim \text{Wishart}\left(3d + 3 + m, \left(\boldsymbol{I} + \sum_{i=1}^{m}\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}\right)\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}\right)^T\right)^{-1}\right)
\end{aligned}$$