# P8160 - Project 3
# We're Not in Kansas Anymore: An Application of the MCMC Algorithm to Hurricane Trajectory Data

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# Motivation

Climate researchers are interested in modeling hurricane trajectories to forecast wind speed and to predict how destructive a storm might become.

## Data

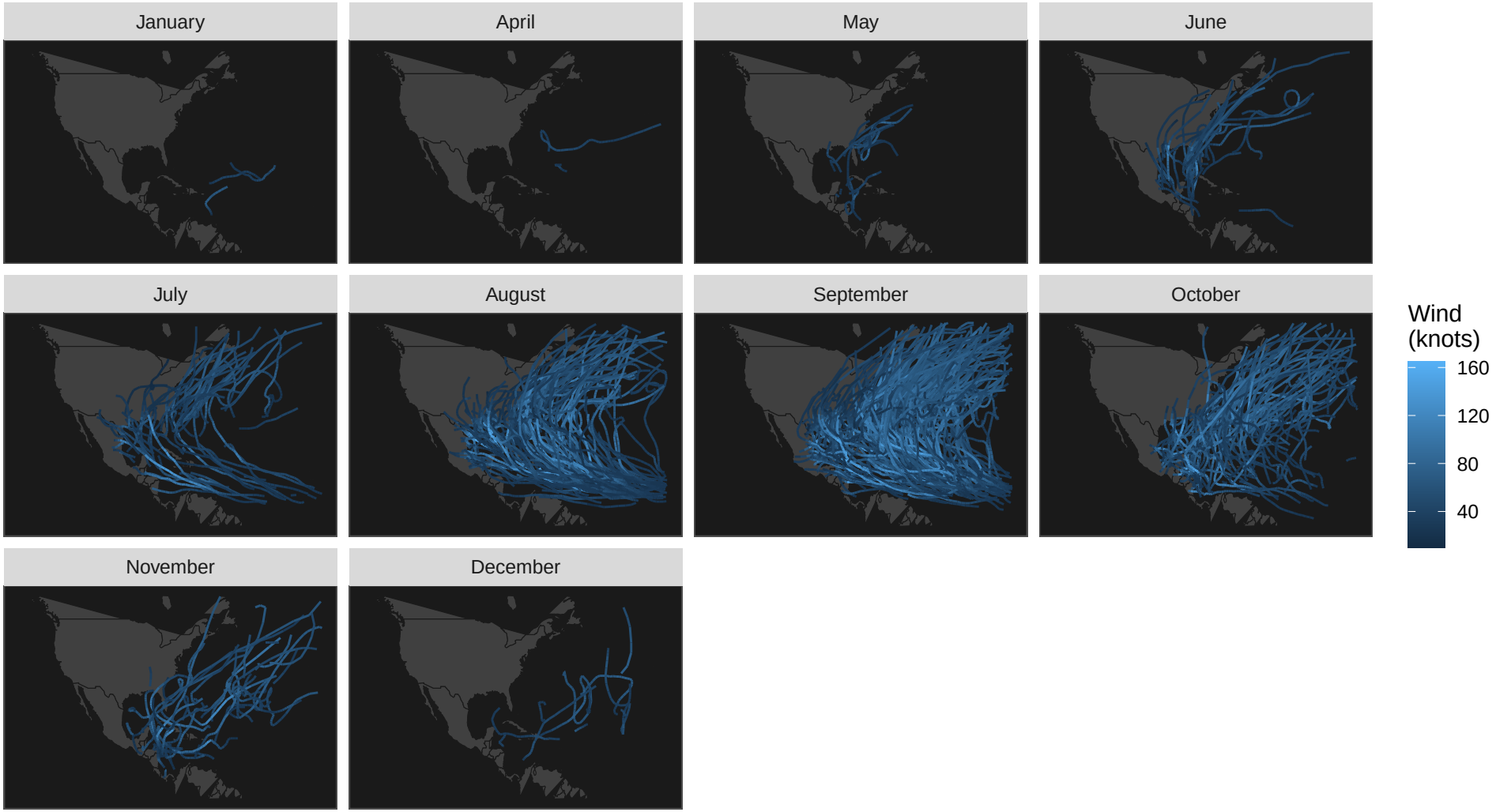
- **ID**: ID of hurricanes
- **Year**: In which the hurricane occurred
- **Month**: In which the hurricane occurred
- **Nature**: Nature of the hurricane
  - ET: Extra Tropical
  - DS: Disturbance
  - NR: Not Rated
  - SS: Sub Tropical
  - TS: Tropical Storm
- **Time**: dates and time of the record
- **Latitude** and **Longitude**: The location of a hurricane check point
- **Wind.kt**: Maximum wind speed (in Knot) at each check point

# Outline

1. Exploration into the data
2. Bayesian modeling of hurricane wind speed
   - Model Equation
   - Posterior Derivation
   - MCMC Algorithm
3. How month, year, and the nature of the hurricane affect wind speed
   - Explore seasonal differences
   - Identify whether average wind speeds are increasing over years
4. How Bayesian estimates are associated with deaths and damage

# Data

Atlantic named Windstorm Trajectories by Month (1950 − 2013)



# Data Cleaning

- We are only concerned about observations that occurred on 6-hour intervals (e.g., hour 0, 6, 12, etc.).
- We will exclude all hurricane IDs that have fewer than 7 observations.
- We use the lag difference ($t-6$ to $t-12$) for latitude, longitude and wind speed to construct $\Delta_{i,1}(t), \Delta_{i,2}(t), \Delta_{i,3}(t)$, and the lag of the wind speed is given by $Y_i(t-6)$
- Through this process, we remove 460 observations, leaving 21578 observations across 681 unique hurricanes.

# Bayesian Model for Hurricane Trajectories

To model the wind speed of the $i^{th}$ hurricane at time $t$ we will use

$$Y_i(t) = \beta_{0,i} + \beta_{1,i} Y_i(t-6) + \beta_{2,i}\Delta_{i,1}(t) + \beta_{3,i}\Delta_{i,2}(t) + \beta_{4,i}\Delta_{i,3}(t) + \epsilon_i(t)$$

Where

- $\Delta_{i,1}(t)$, $\Delta_{i,2}(t)$ and $\Delta_{i,3}(t)$ are changes in latitude longitude and wind speed respectively between $t-12$ and $t-6$
- $\epsilon_i(t) \sim N(0, \sigma^2)$ independent across $t$
- Let $\beta_i = (\beta_{0,i}, \beta_{1,i}, \beta_{2,i}, \beta_{3,i}, \beta_{4,i})^T \sim \mathcal{N}(\mu, \Sigma)$ be multivariate normal distribution where $\mu \in \mathbb{R}^d$ and $\Sigma \in \mathbb{R}^{d \times d}$.

**Prior Distributions Assumptions**:

- For $\sigma^2$ we assume $\pi(\sigma^2) \propto \frac{1}{\sigma^2}$
- For $\mu$ we assume $\pi(\mu) \propto 1$
- For $\Sigma$ we assume $\pi\left(\Sigma^{-1}\right) \propto |\Sigma|^{-(d+1)} \exp\left\{-\frac{1}{2}\Sigma^{-1}\right\}$

**Goal**: Estimate $\Theta = (B, \mu, \Sigma^{-1}, \sigma^2)$

# Likelihood & Prior Function

**Likelihood** Let $X_i = \left(1, Y_i(t-6), \Delta_{i,1}(t), \Delta_{i,2}(t), \Delta_{i,3}(t)\right)$

$$L(Y \mid \Theta) \propto \prod_{i=1}^{m} \left(\sigma^2\right)^{-\frac{n_i}{2}} \exp\left\{-\frac{1}{2\sigma^2}\left(Y_i - X_i\beta_i\right)^T\left(Y_i - X_i\beta_i\right)\right\}$$

where $m$ is the number of hurricane and $n_i$ is the number of observations for $i^{th}$ hurricane

**Prior** Let $\Theta = (B, \mu, \Sigma^{-1}, \sigma^2)$

$$\pi(\Theta) \propto \left(\sigma^2\right)^{-1} \left|\Sigma^{-1}\right|^{d+1} \exp\left\{-\frac{1}{2}\operatorname{tr}\left(\Sigma^{-1}\right)\right\} \prod_{i=1}^{m} \left|\Sigma^{-1}\right|^{\frac{1}{2}} \exp\left\{-\frac{1}{2}\left(\beta_i - \mu\right)^T \Sigma^{-1}\left(\beta_i - \mu\right)\right\}$$

where $d$ is the dimension of $\mu$

# Posterior Calculation

## Posterior

$$\pi(\Theta \mid Y) \propto \left(\sigma^2\right)^{-\left(1+\frac{\sum_{i=1}^{m} n_i}{2}\right)} \left|\Sigma^{-1}\right|^{d+1+\frac{m}{2}} \exp\left\{-\frac{1}{2}\operatorname{tr}\left(\Sigma^{-1}\right)\right\}$$
$$\times \exp\left\{-\frac{1}{2}\sum_{i=1}^{m}\left(\beta_i-\mu\right)^T\Sigma^{-1}\left(\beta_i-\mu\right)\right\}\exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^{m}\left(Y_i-X_i\beta_i\right)^T\left(Y_i-X_i\beta_i\right)\right\}$$

## Conditional Posterior

$$\beta_i : \pi(\beta_i \mid \Theta_{(-\beta_i)}Y) \propto \exp\left\{-\frac{1}{2}\left(\beta_i-\mu\right)^T\Sigma^{-1}\left(\beta_i-\mu\right)-\frac{1}{2\sigma^2}\left(Y_i-X_i\beta_i\right)^T\left(Y_i-X_i\beta_i\right)\right\}$$

$$\mu : \pi\left(\mu \mid \Theta_{(-\mu)}, Y\right) \sim \mathcal{N}(\bar{\beta}, \Sigma/m), \bar{\beta} = \left(\bar{\beta}_{0,.}, \bar{\beta}_{1,.}, \bar{\beta}_{2,.}, \bar{\beta}_{3,.}, \bar{\beta}_{4,.}\right)^T$$

$$\sigma^2 : \pi\left(\sigma^2 \mid \Theta_{(-\sigma^2)}, Y\right) \propto \left(\sigma^2\right)^{-\left(1+\frac{\sum_{i=1}^{m} n_i}{2}\right)} \times \exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^{m}\left(Y_i-X_i\beta_i\right)^T\left(Y_i-X_i\beta_i\right)\right\}$$

$$\Sigma^{-1} : \pi\left(\Sigma^{-1} \mid \Theta_{(-\Sigma^{-1})}, Y\right) \sim \text{Wishart}\left(3d+3+m, \left(I+\sum_{i=1}^{m}\left(\beta_i-\mu\right)\left(\beta_i-\mu\right)^T\right)^{-1}\right)$$

# MCMC Algorithm

We apply a hybrid algorithm consisting of Metropolis-Hastings steps and Gibbs steps.

Update component wise:

- Sampling proposed $\beta'_{j,i}$, $j = 0, 1...4$, for $i^{th}$ hurricane from proposed distribution $U\left(\beta_{j,i}^{(t)} - a_{j,i}, \beta_{j,i}^{(t)} + a_{j,i}\right)$, where $a_{j,i}$ is the search window for $\beta_{j,i}$. Since the proposed is symmetric, the acceptance probability is the ratio of posterior distribution.
- Then, Gibb step for $\mu$: Sample $\mu^{(t+1)}$ from $\mathcal{N}\left(\bar{\beta}^{(t+1)}, \Sigma^{(t)}/m\right)$, where $\bar{\beta}^{(t+1)}$ is the average of $\beta_i^{(t+1)}$ over all hurricanes.
- Next, Random-walk Metropolis to generate $\sigma^{2'}$ with step size from $U\left(-a_{\sigma^2}, a_{\sigma^2}\right)$ and compare the ratio of posterior distribution with $u \sim U(0, 1)$
- Finally, we sample $\Sigma^{-1^{(t+1)}}$ from

$$\text{Wishart}\left(3d + 3 + m, \left(I + \sum_{i=1}^{m}\left(\beta_i^{(t+1)} - \mu^{(t+1)}\right)\left(\beta_i^{(t+1)} - \mu^{(t+1)}\right)^T\right)^{-1}\right)$$

# Initial Starting Values

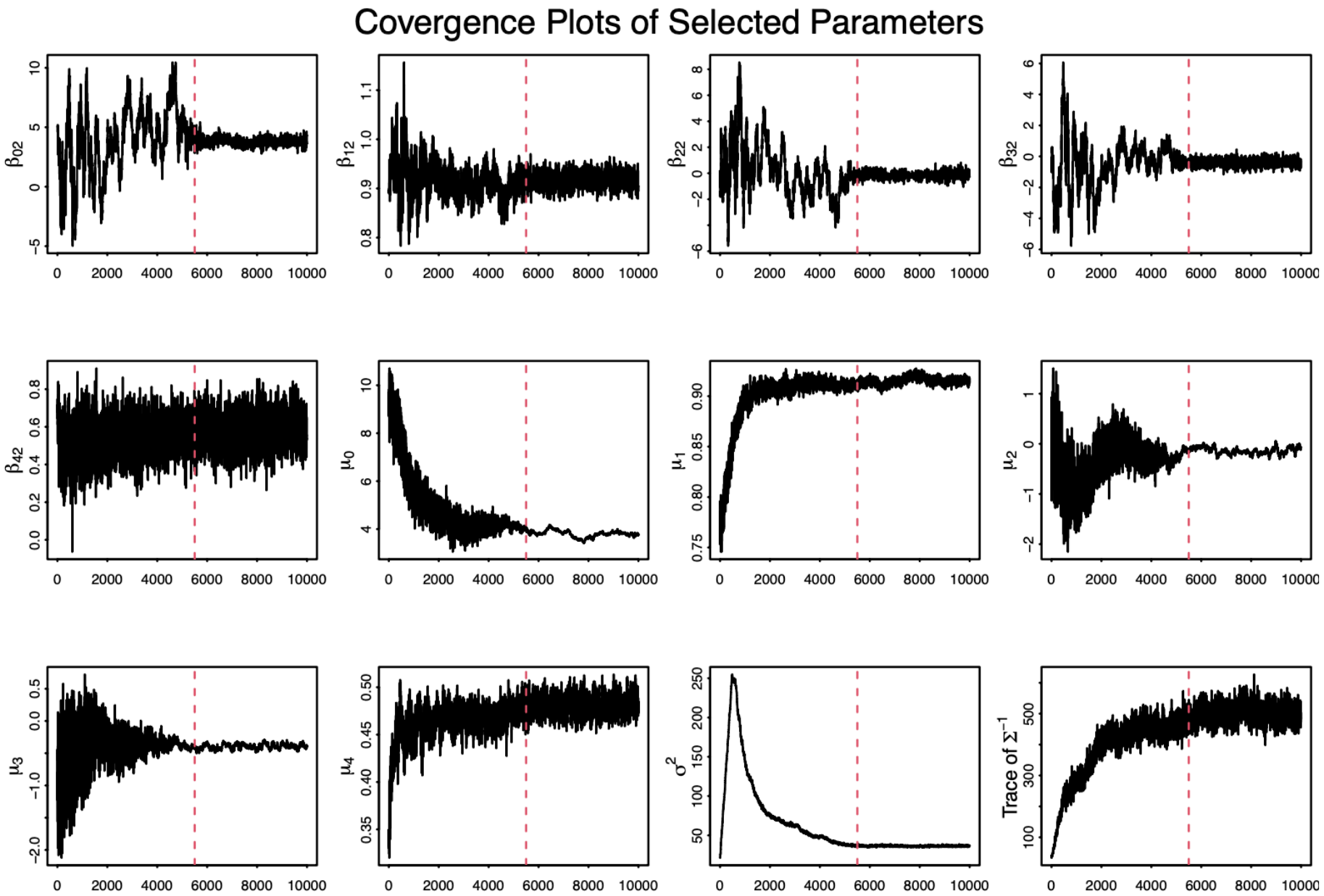
**Initial Values:**

- $\beta_i$: Fit OLS multivariate linear regression (MLR) for $i^{th}$ hurricane and use the coefficients as $\beta_i^{(0)}$
- $\mu$: Average over all $\beta_i^{(0)}$ as $\mu^{(0)}$
- $\sigma^2$: $\hat{\sigma}_i^2$ is the mean square residuals of the OLS model for $i^{th}$ hurricane. Take the mean over all $\hat{\sigma}_i^2$ as $\sigma^{2(0)}$
- $\Sigma^{-1}$: Generate the covariance matrix of $\beta_i^{(0)}$ and take the inverse of the matrix as $\Sigma^{-1(0)}$

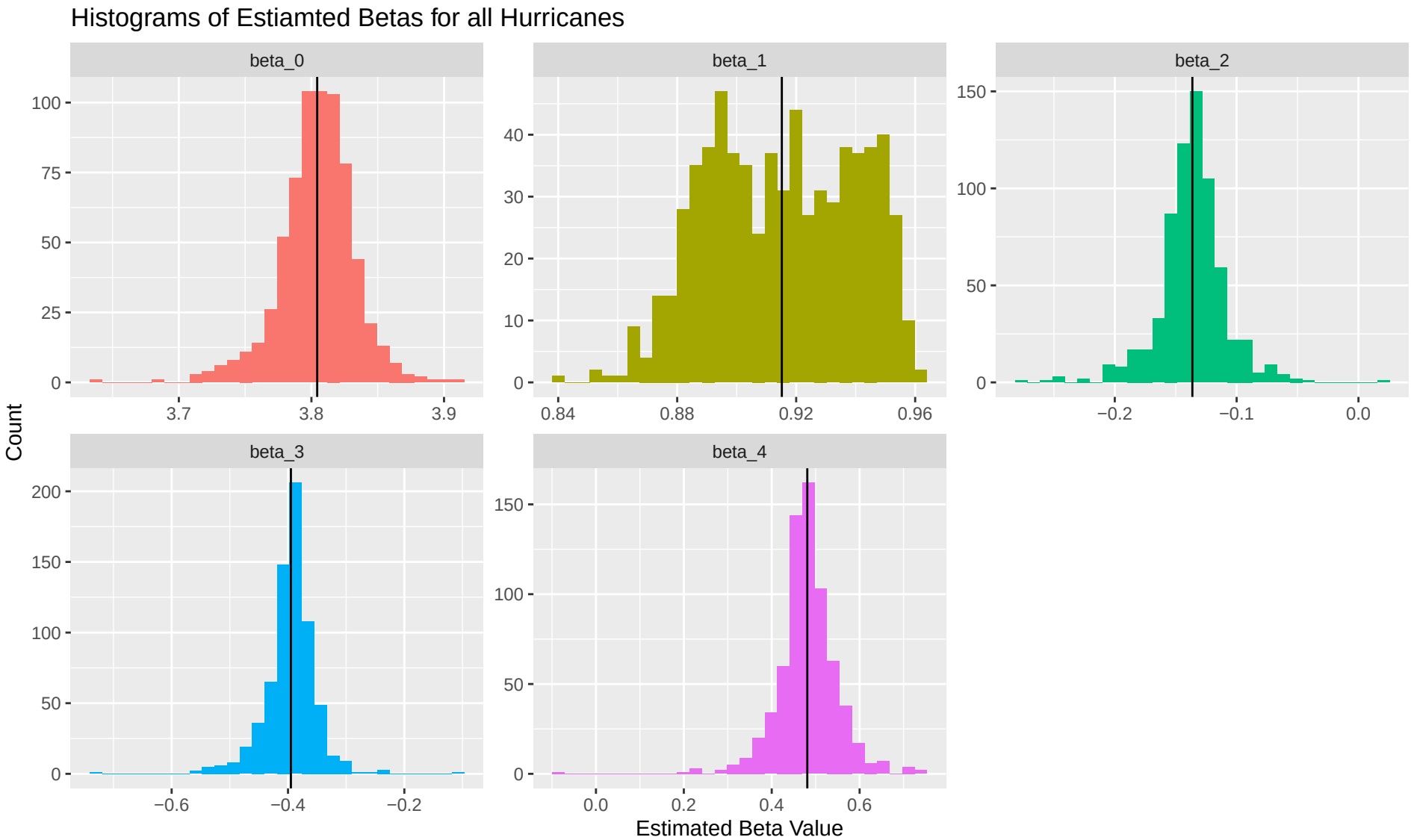
**Table 1:** Range of Search Window and Acceptance Rate for Paraemters Used in the MH Step

| | Search Window | Acceptance Rate (%) |
|---|---|---|
| $\beta_0$ | 1.1 | 45.87 - 51.36 |
| $\beta_1$ | (0.04, 0.1) | 31.67 - 63.68 |
| $\beta_2$ | (0.8, 1.0) | 38.60 - 45.60 |
| $\beta_3$ | (0.5, 0.6) | 33.20 - 61.32 |
| $\beta_4$ | (0.3, 0.4) | 34.95 - 60.45 |
| $\sigma^2$ | 2.0 | 44.83 |

# MCMC Model Convergence (Burn-In 5500)



# B Estimates


Histograms of Estiamted Betas for all Hurricanes
beta_0
beta_1
beta_2
beta_3
beta_4
Count
Estimated Beta Value

# The $\mu$ and $\sigma^2$ Estiamtes

**Table 2:** Bayesian Estiamtes for $\mu$ and $\sigma^2$

| | $\mu_0$ | $\mu_1$ | $\mu_2$ | $\mu_3$ | $\mu_4$ | $\sigma^2$ | $\sigma^2_{00}$ | $\sigma^2_{11}$ | $\sigma^2_{22}$ | $\sigma^2_{33}$ | $\sigma^2_{44}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Estimates | 3.8 | 0.92 | -0.14 | -0.39 | 0.48 | 36.36 | 0.063 | 0.003 | 0.047 | 0.042 | 0.018 |

$$\hat{\Sigma}^{-1} = \begin{bmatrix} 16.07 & 7.63 & 0.34 & -1.78 & 0.47 \\ 7.63 & 381.32 & 5.39 & 3.96 & -6.32 \\ 0.34 & 5.39 & 21.43 & 1.48 & 1.14 \\ -1.78 & 3.96 & 1.48 & 24.35 & -3.3 \\ 0.47 & -6.32 & 1.14 & -3.3 & 55.12 \end{bmatrix} \quad \hat{\rho} = \begin{bmatrix} 1 & -0.101 & -0.018 & 0.094 & -0.011 \\ -0.101 & 1 & -0.057 & -0.043 & 0.043 \\ -0.018 & -0.057 & 1 & -0.067 & -0.042 \\ 0.094 & -0.043 & -0.067 & 1 & 0.089 \\ -0.011 & 0.043 & -0.042 & 0.089 & 1 \end{bmatrix}$$

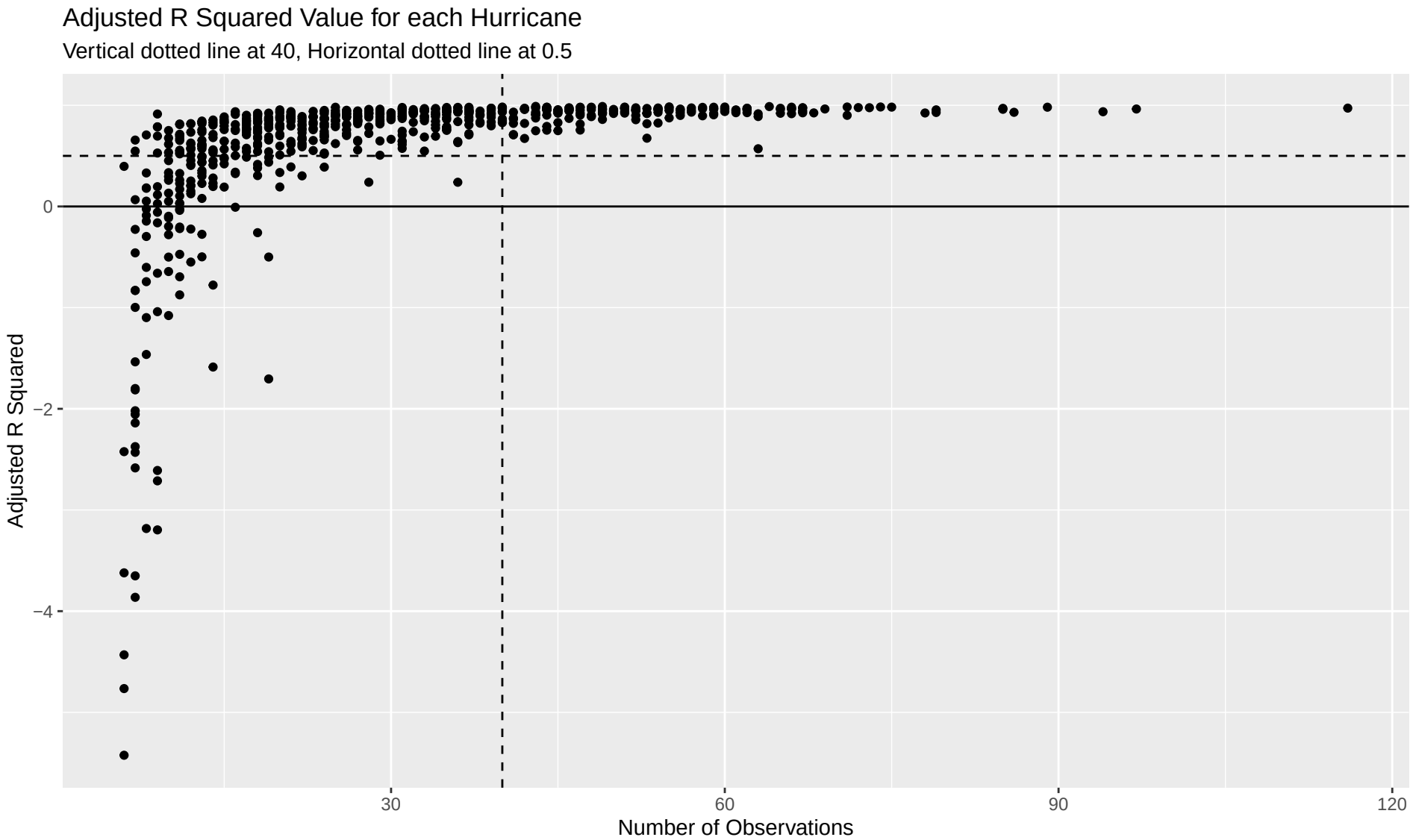
# Model Performance

The overall adjusted $R^2$ of the estimated Bayesian model is 0.9524156.

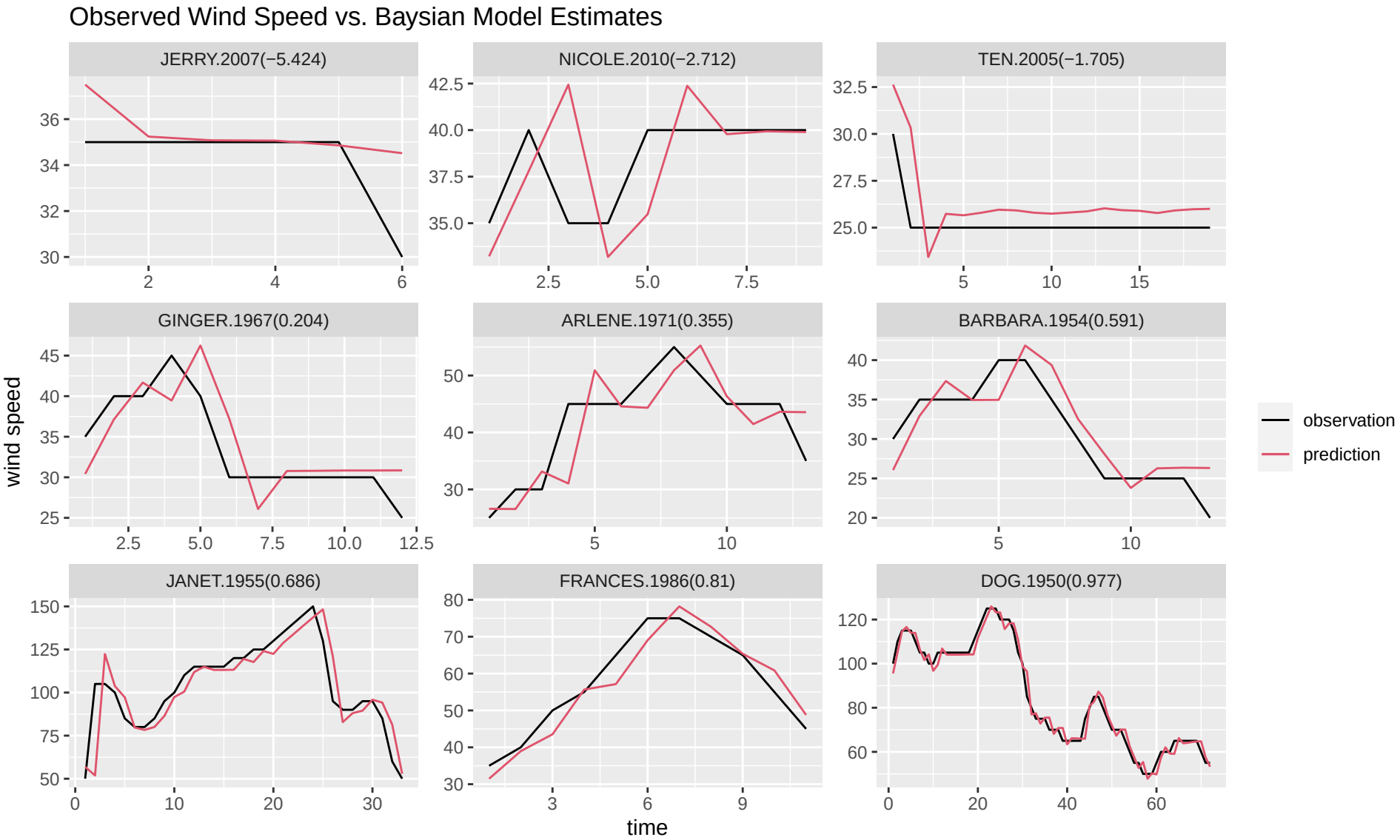
**Table 3:** $R^2_{adj}$ for each hurricane

| $R^2_{adj}$ | Count of Hurricanes | Percentage(%) |
|---|---|---|
| 0.6-1 | 522 | 76.7 |
| 0.2-0.6 | 79 | 11.6 |
| $< 0.2$ | 80 | 11.7 |

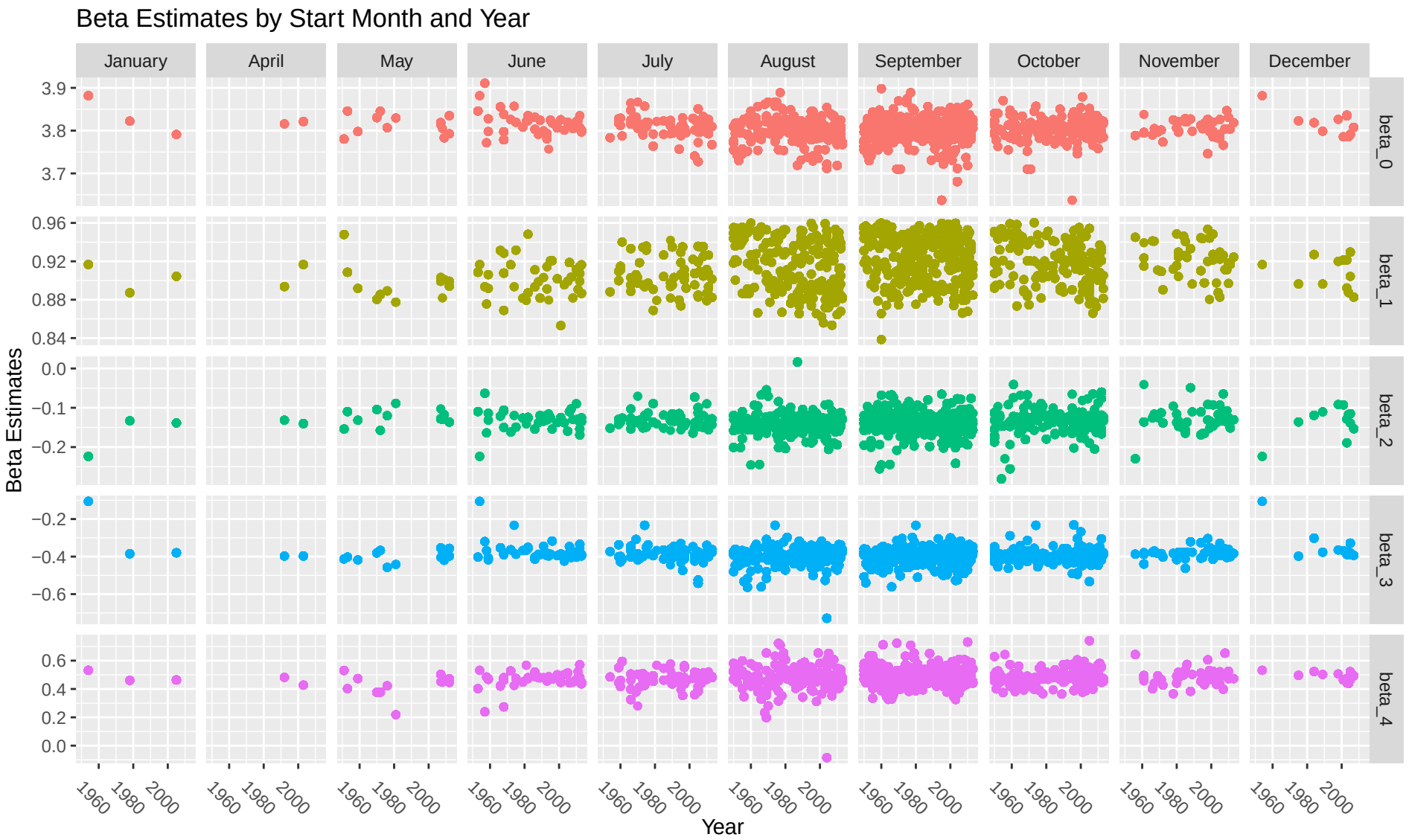
# Model Performance



# Model Performance

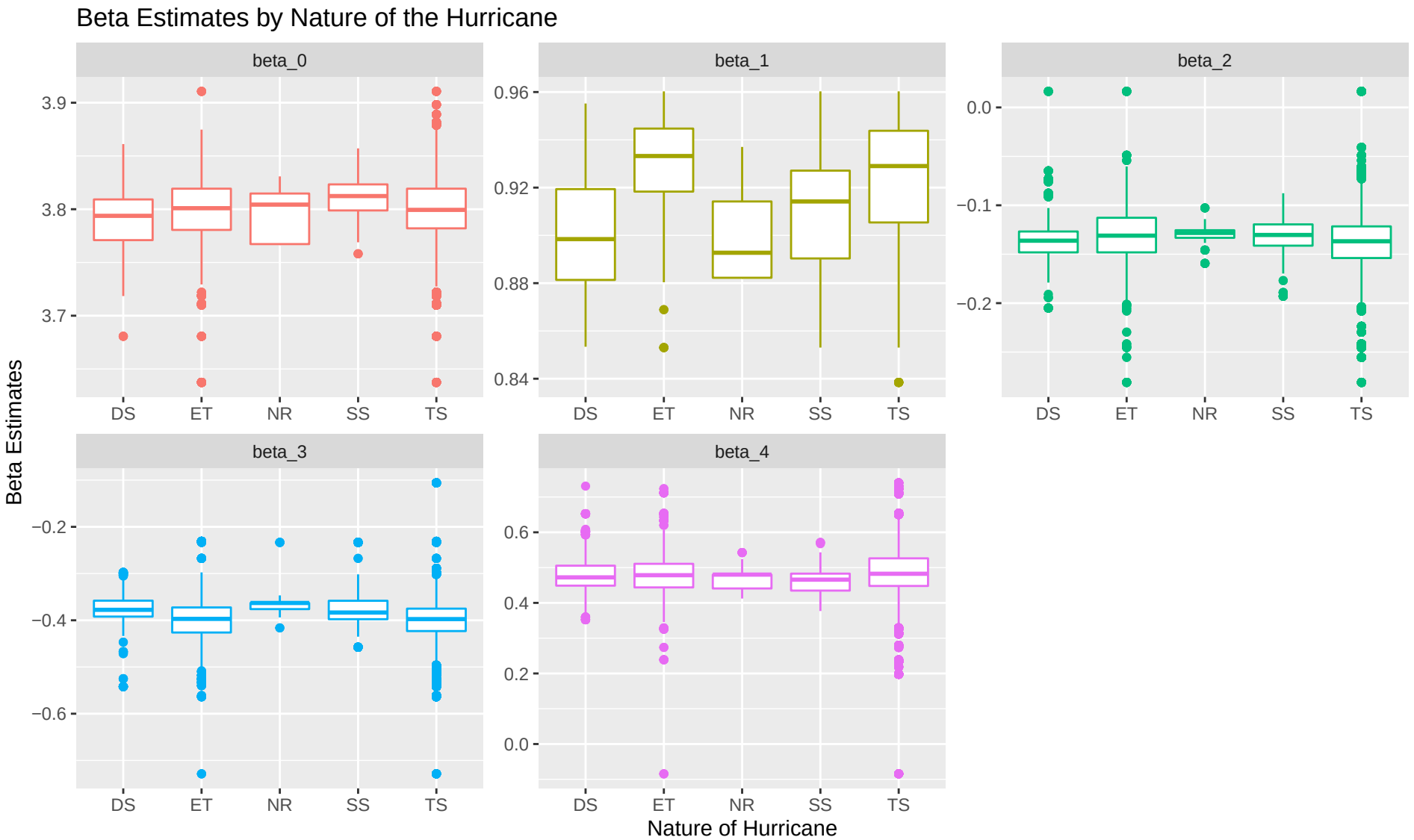


# Understanding Seasonal Differences




Typical Hurricane season is June to November

# Understanding Nature of Hurricane Differences



# Modeling Seasonal, and Nature Difference

**Model:**

For each $\beta$ value we fit a different linear model.

$$\begin{aligned} Y_{ij} &= \alpha_{0j} + \alpha_{1j} \times \text{Decade}_i \\ &\quad + \alpha_{(k+1)j} I(\text{Nature} = k)_i \\ &\quad + \alpha_{(l+5)j} I(\text{Month} = l)_i + \epsilon_{ij} \end{aligned}$$

Where $i$ is the hurricane, $j$ is the Beta model, $k \in (ET,\ NR,\ SS,\ TS)$ making DS the reference group. Let $l \in$(April - December).

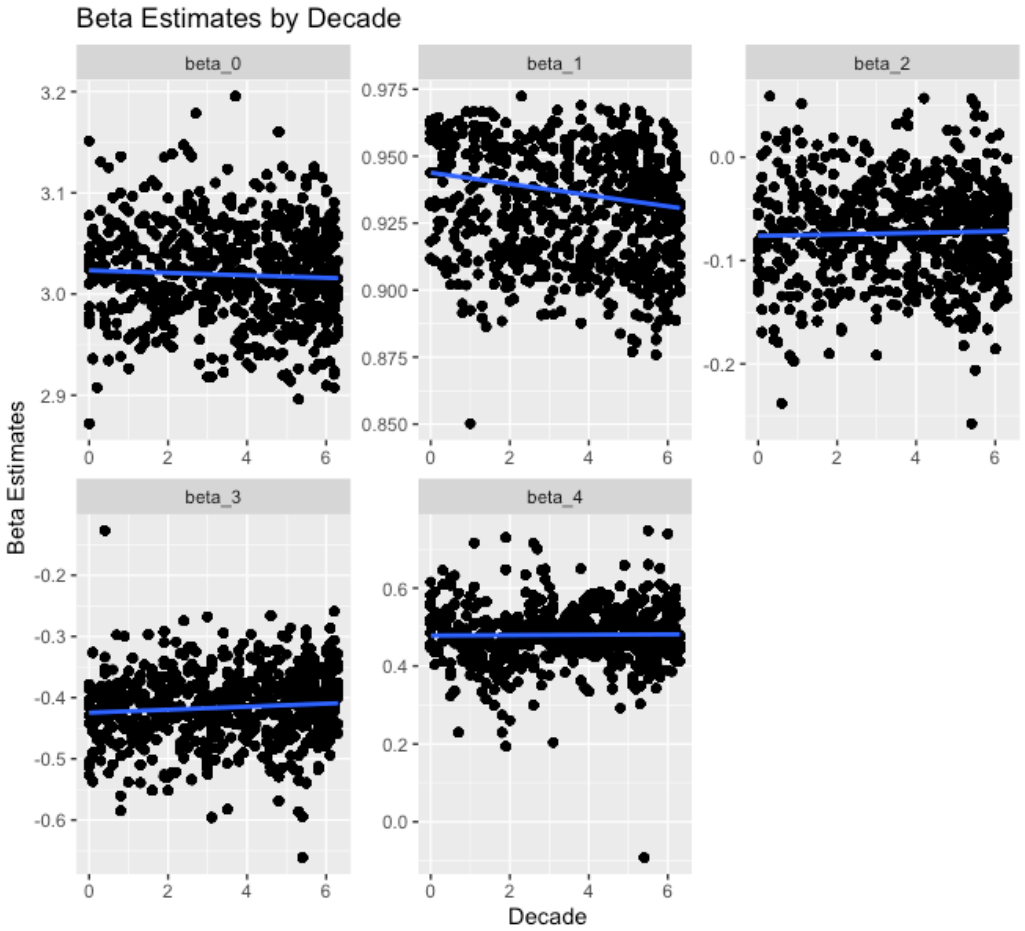
**Results:**

- No nature indicators were significant
- No month indicators were significant
- For the $\beta_1$ model, the decade estimated is significant with point estimate -0.003 (-0.002, -0.004)

That is, month and nature do not exhibit a linear association with each beta value.

# Exploring wind speeds over the years

Model: $Y_i = \alpha_{0i} + \alpha_{1i} \times \text{Decade}$ where $Y_i$ is each $\beta_i$ and $i \in (0, \ldots, 4)$.




**Linear Model Output for Each $\beta$**

| Characteristic | Beta | 95% CI[1] | p-value |
|---|---|---|---|
| beta0 | | | |
| decade | -0.001 | -0.002, 0.000 | 0.2 |
| beta1 | | | |
| decade | -0.003 | -0.004, -0.002 | **<0.001** |
| beta2 | | | |
| decade | 0.000 | -0.001, 0.001 | 0.6 |
| beta3 | | | |
| decade | 0.002 | 0.000, 0.004 | **0.041** |
| beta4 | | | |
| decade | 0.001 | -0.002, 0.004 | 0.5 |

[1] CI = Confidence Interval

- $\beta_1$: Indicates a decrease in the change of wind speed over years
- $\beta_3$: Indicates an increase in the impact of change in longitude over years

# Deaths and Damages - Data Exploration

Distribution of Continuous Variables (Standardized)

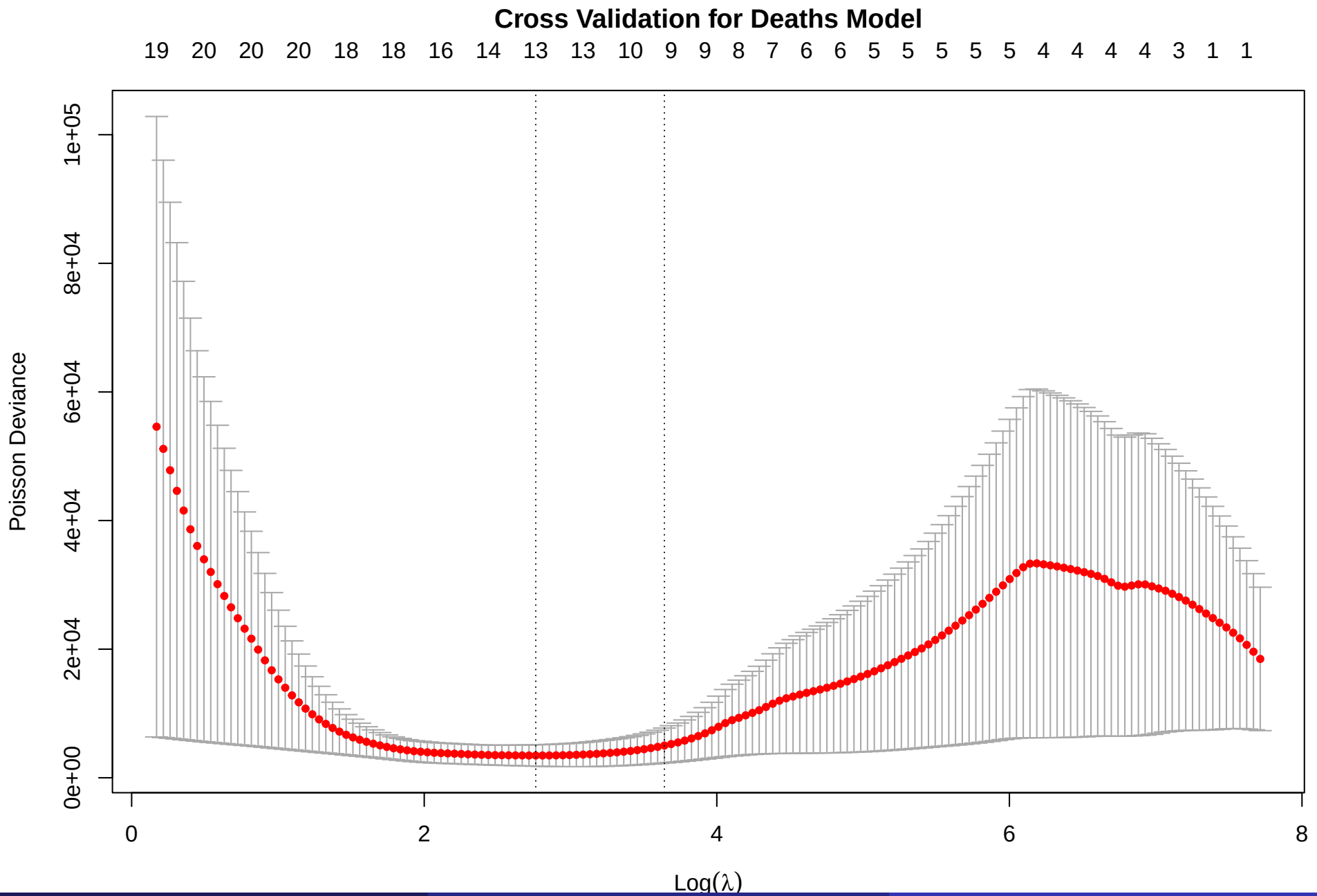
| **Characteristic** | **N = 43** |
|---|---|
| damage | (-0.46, -0.43, -0.33, -0.09, 5.19) |
| deaths | (-0.29, -0.28, -0.27, -0.22, 5.81) |
| maxspeed | (-1.94, -0.81, 0.14, 0.89, 1.64) |
| meanspeed | (-1.67, -0.82, -0.02, 0.74, 2.18) |
| maxpressure | (-3.88, -0.57, 0.10, 0.32, 2.09) |
| meanpressure | (-3.94, 0.24, 0.31, 0.36, 0.41) |
| hours | (-1.87, -0.85, 0.00, 0.74, 2.50) |
| total_pop | (-0.98, -0.69, -0.37, 0.13, 2.74) |
| percent_poor | (-0.30, -0.30, -0.30, -0.29, 3.64) |
| percent_usa | (-1.22, -1.22, 0.14, 0.96, 1.14) |

Note: For each characteristic, values denote the minimum, 25th percentile, median, 75th percentile, and maximum, respectively.

# Deaths and Damages - Prediction and Inference

- Model Selection
  - Poisson regression with total population as offset
    - $y_i \sim \text{Poisson}(\theta_i)$
    - $\log(\theta_i) = \mathbf{x}_i' \gamma$
  - penalized regression via lasso
  - optimal $\lambda$ selected via leave-one-out cross validation
    - feasible with small $n$
    - non-random
- Post-selection Inference
  - bootstrap smoothing as proposed by Efron (2014)
    - fit full model via Poisson-family GLM to obtain $\widehat{\theta}$
    - for single bootstrap $b$, draw $\mathbf{y}_b^* \sim \text{Poisson}(\widehat{\theta})$
    - execute lasso with $\mathbf{y}_b^*$ as output to obtain $\hat{\gamma}_b^*$
    - compute empirical standard error of $(\hat{\gamma}_b^*)$

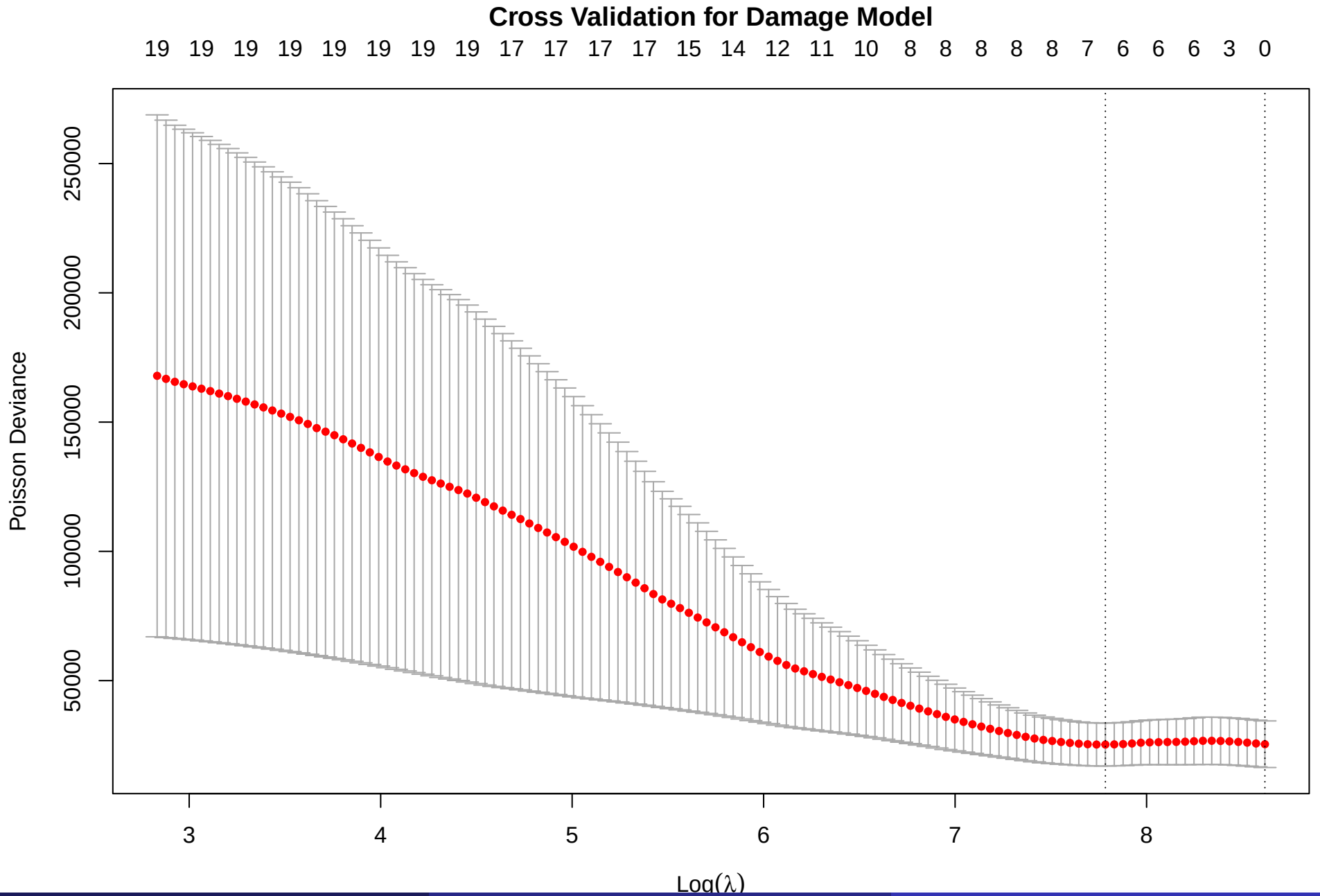
# Hurricane Deaths - Model Selection



# Hurricane Deaths - Inference

| Covariate | Estimate | SE | p-value | Left CI | Right CI |
|---|---|---|---|---|---|
| season | 0.0385 | 0.0013 | 0.0000 | 0.0359 | 0.0410 |
| monthJuly | -3.7255 | 0.1095 | 0.0000 | -3.9401 | -3.5108 |
| monthOctober | 0.8073 | 0.0484 | 0.0000 | 0.7124 | 0.9022 |
| natureNR | 3.6402 | 0.1214 | 0.0000 | 3.4022 | 3.8782 |
| natureTS | 3.1078 | 0.0774 | 0.0000 | 2.9560 | 3.2596 |
| maxpressure | -0.0379 | 0.0049 | 0.0000 | -0.0474 | -0.0283 |
| hours | 0.0054 | 0.0002 | 0.0000 | 0.0050 | 0.0057 |
| percent_poor | 0.0568 | 0.0005 | 0.0000 | 0.0558 | 0.0578 |
| percent_usa | -0.0007 | 0.0005 | 0.1436 | -0.0016 | 0.0002 |
| beta_0 | 23.0045 | 0.5643 | 0.0000 | 21.8985 | 24.1104 |
| beta_1 | -10.9904 | 2.0393 | 0.0000 | -14.9873 | -6.9935 |
| beta_2 | 1.1669 | 0.7284 | 0.1092 | -0.2608 | 2.5945 |
| beta_3 | 17.3277 | 0.3498 | 0.0000 | 16.6421 | 18.0132 |

# Hurricane Damages - Model Selection



# Hurricane Damages - Inference

| Covariate | Estimate | SE | p-value | Left CI | Right CI |
|---|---|---|---|---|---|
| season | 0.0399 | 0.0004 | 0.0000 | 0.0392 | 0.0406 |
| monthJuly | -0.5762 | 0.0192 | 0.0000 | -0.6137 | -0.5386 |
| monthOctober | 0.4680 | 0.0074 | 0.0000 | 0.4536 | 0.4824 |
| percent_usa | 0.0001 | 0.0001 | 0.1052 | 0.0000 | 0.0003 |
| beta_1 | 10.5907 | 0.2533 | 0.0000 | 10.0943 | 11.0871 |
| beta_2 | -3.3668 | 0.1255 | 0.0000 | -3.6128 | -3.1208 |
| beta_3 | 2.4291 | 0.0669 | 0.0000 | 2.2979 | 2.5603 |

# Conclusions

- Based on $\hat{\mu}$, on average, increases in the current wind speed and acceleration are associated with increases in the future wind speed, whereas the future wind will slow when the hurricane moves north and east
- Hurricanes with more observations enjoy better MCMC estimates
- With such a high-dimensional parameter space,
  - burn-in takes longer than expected, and
  - convergence is very sensitive to starting values
- There does not seem to be a nature or month effect on wind speeds
- Estimated $\beta$ coefficients generally prove useful in predicting damage and deaths
  - particularly, $(\beta_1, \beta_2, \beta_3)$ are always selected
  - however, $\beta_2$ is only inferentially significant in the damage model