The Self-Organization of Insight: Entropy and Power Laws in Problem Solving

Damian G. Stephen[a,b]
James A. Dixon[a,b,c]

Abstract

Explaining emergent structure remains a challenge for all areas of cognitive science, and problem solving is no exception. The modern study of insight has drawn attention to the issue of emergent cognitive structure in problem solving research. We propose that the explanation of insight is beyond the scope of conventional approaches to cognitive science in terms of symbolic representation. Cognition may be better described in terms of an open, nonlinear dynamical system. By this reasoning, insight would be the self-organization of novel structure. Self-organization is a well-studied phenomenon of dynamical systems theory, associated with specific trends in entropy and power-law behavior. We present work using nonlinear dynamics to capture these trends in entropy and power-law behavior and thus to predict the self-organization of novel cognitive structure in a problem-solving task. Future explorations of problem solving will benefit from considerations of the continuous nonlinear interactions among action, cognition, and the environment.

[a]University of Connecticut; [b]Center for the Ecological Study of Perception and Action; [c]Haskins Laboratories

http://dx.doi.org/10.7771/1932-6246.1043

1. Introduction

Human behavior in response to problems is at once richly patterned and flexible. We adapt our behavior so as to anticipate and control outcomes, spontaneously generating novel solutions to problems. The rapid appearance of novel structure is more generally known as emergence. We propose that problem solving is a matter of emergent structure. Our structured yet flexible response to changing environmental demands is a hallmark of cognition. Cognitive science must address both the nature of structure and its emergence. Emergent structures have been identified in a broad range of areas including language acquisition (Hollich, 2006; MacWhinney, 2005; Regier, 2006; Jones & Smith, 2005), categorization (Quinn, 2006; Younger & Johnson, 2006; Yu, 2006), perceptual integration (Burnham & Dodd, 2004; Fingelkurts, Fingelkurts, Krause, Möttönen, & Sams, 2003; Johnson, 2004), mathematical reasoning (Empson & Turner, 2006; Rasmussen, Stephan, & Allen, 2004; Siegler, 2005), and gaze following (Moore, 2006; Triesch, Teuscher, Deák, & Carlson, 2006). In all of these cases, the cognitive system appears to take a sudden leap forward in some important sense (e.g., greater abstraction, generality, or efficiency). Emergent cognitive structures appear to arise abruptly from prior activity of the system. The resulting structures reorganize the cognitive system's interaction with the environment. Thus, the emergence of structure is a radical change in functionality, rather than quantitative improvements. As such, emergence leaves gaps that cognitive science must find a way to bridge.

Emergence is an important phenomenon because it promises to reveal key properties of the system that produces cognitive structure, as well as the nature of cognitive structure itself. What type of system is capable of radically reorganizing itself into a new structure, and how might we understand change within such a system? Problem-solving research is an ideal starting point for understanding emergence. More than any other subfield of cognitive science, problem solving focuses on the ongoing activity of the cognitive system that gives rise to new structure. In this article, we argue that problem solving should be a central concern for cognitive science at large, because it affords careful investigation of emergent structure. First, we review problem-solving research demonstrating the phenomenon of emergence. Next, we explain the profound challenge that emergence poses for the conventional symbolic approach to cognition. Finally, we outline an alternative approach to emergent structure that has deep implications for our understanding of the cognitive system.

2. Emergent Structure in Problem Solving

Examples of emergent cognitive structure can be traced back to early Gestalt research in problem solving. For instance, in some of his most well-known work, Köhler (1925) found that chimpanzees abruptly generated new strategies for reaching food when their initial strategies had proven unsuccessful. Crucially, their initial attempts did not appear to be steps toward the eventual "insightful" solution. For example, after jumping at or reaching for food hanging from a high perch, they spontaneously stacked boxes to make a step ladder to reach the food. In another situation, they fit two short sticks together to fashion a longer stick that could reach food placed outside their cage. As Köhler (1947) pointed out, these innovations are exemplary cases of emergence; the new strategy reflects a reorganization of the system, rather than an incremental improvement in their initial strategy. Köhler argued that insight is a restructuring of the cognitive system.

Modern research on problem solving has continued to wrestle with emergent phenomena, such as insight. In the dominant modern formulation, discontinuous change in problem-solving behavior is assumed to involve a restructuring of an internal representation of the problem. A solver who begins work on a problem using an inappropriate representation may, under some conditions, have the structure of this representation shift abruptly, affording a solution (Bowden, Jung-Beeman, Fleck, & Kounios, 2005; Chronicle, MacGregor, & Ormerod, 2004; Fleck & Weisberg, 2004; Gilhooly & Murphy, 2005; Knoblich, Ohlsson, & Raney, 2001). Insight entails an observable discontinuity in a solver's approach to a problem indicating a restructuring of the solver's representation of the problem (Chronicle et al., 2004; Weisberg, 1996).

Consider the matchstick problem as an example of representational restructuring: a solver is asked to arrange six matchsticks so as to form four equilateral triangles. Because triangles are two-dimensional shapes, the solver will often begin by experimenting with a two-dimensional arrangement of the matchsticks (Weisberg, 1996). This approach reflects a two dimensional representation of the problem. After forming one triangle with three matchsticks and a second triangle with two more matchsticks, the solver is left unsure what to do with the sixth matchstick. So long as the solver represents the problem two dimensionally, every attempt will be a variation of this unsuccessful approach. The solution is to arrange the matchsticks to form a tetrahedron, a three-dimensional shape with six edges (one for each matchstick) and four triangular faces. Thus, only when the solver's representation of the problem changes to allow a three-dimensional construction, can the problem be solved. The important point here is that the tetrahedron solution requires radically restructuring the current representation of the problem. This restructuring results in a new conception of the problem.

Emergent structure is not limited to the initial insight into a problem. Even when an initial strategy is successful, continued experience with a problem can lead to more efficient strategies for arriving at correct solutions. This type of strategy change reflects a restructuring of the representation of the problem, a fundamentally different way of approaching the problem, rather than a quantitative improvement in the existing strategy (Dixon & Kelley, 2006, 2007; Siegler, 2005, 2006; Torbeyns, Arnaud, Lemaire, & Verschaffel, 2004). For example, the solver may suddenly adopt a more abstract representation of the problem or a more efficient route to the solution.

An example of strategy change may be found in the literature on the development of children's arithmetic skills. When children are learning simple addition, they are often asked to sum two addends. To reach a solution, children usually begin with quite simple strategies, but soon discover more complex strategies as they repeatedly solve problems. Siegler (2005, 2006) has made an extensive investigation of these transitions. One of the earliest strategies, the sum strategy, involves putting up the number of fingers specified by each addend and counting them all. For example, given the problem "2 + 4 = ?," a child using the sum strategy will extend two fingers on one hand and four fingers on the other and count her fingers from 1 to 6. The sum strategy reflects the simplest representation of the problem: combining two numbers to produce a third. After using the sum strategy repeatedly, many children spontaneously discover the min strategy. The min strategy involves the child extending as many fingers as the smaller of the two addends and then counting up from the larger addend. For example, given the same problem as above (i.e., 2 + 4 = ?), the child will say "four" and then count 5 and 6, often extending a finger for each count. The min strategy reflects a restructuring of the representation of the problem. For the child, addition is no longer simple combination but is instead the incrementing of a large number by the magnitude of a smaller number. A child who discovers the min strategy is now sensitive to the relative magnitude of the addends and, arguably, has a more abstract grasp of addition (Siegler & Jenkins, 1989), as well as a more efficient strategy. Strategy change is the emergence of a new structure for representing the problem.

3. Symbolic Systems and Emergence

Emergence of new structure is clearly a fundamental property of the cognitive system (Bickhard, 2004). If we are to pursue a deeper understanding of cognition, we will need to consider the nature of emergence and the kinds of structure to which it gives rise. Any description of emergence in cognition requires some characterization of cognitive structure. The dominant approach in cognitive science is to characterize cognitive structure as symbolic representation (Barsalou, 1999; Dietrich & Markman, 2003). A symbol stands for some referent in the environment. Computation over the symbols in the cognitive system occurs via a set of syntactic rules. Newell and Simon were among the original champions of this approach to cognition (Newell & Simon, 1956, 1976). They proposed that information is transmitted through the nervous systems much in the same way that information shuttles through a computer. According to their view, information was encoded into symbols that served to represent the environment. These symbols then provide the basic units of cognitive structure. This formulation was convenient, both for theory construction and for computational simulations of intelligence. Newell and Simon modeled problem-solving behavior as computations over symbolic representations.

Convenience aside, the commitment to symbolic representation turns out to be problematic. Since the rise of symbolic representation as the modal approach to cognition, a number of challenges to this conceptualization have been identified: the homunculus problem (cf. Bickhard & Terveen, 1995), the symbol grounding problem (Glenberg, 1997; Glenberg & Robertson, 2000; Harnad, 1990; Searle, 1980; Zwann & Taylor, 2006), the correspondence error problem (Bickhard, 1993, 1996; Fodor, 1990), and the frame problem (Bickhard, 2001; Dennett, 1984: Haselager, 1997; Pylyshyn, 1987). These problems are all symptomatic of a general program that has been described as *encodingism*, a view that Bickhard and Terveen (1995) present as a major obstacle to reconciling cognitive theory with fundamental properties of biological systems, including emergence.

According to encodingism, the cognitive system assembles bits of information from the environment to form representations. These encodings are simplistic, symbolic pointers to objects in the environment. They indicate central features belonging to the represented objects, and the encodings thus underspecify the actual objects in the environment (Fodor, 2000). Despite their underspecification, these encodings must stand in strong, point-by-point, feature-by-feature correspondence with the represented objects to do work for the cognitive system. Bickhard and Terveen (1995) argued that the emergence of an encoding would be very difficult to explain. Encodingism, they noted, posits that symbols are the smallest unit of meaning. Symbols can be combined together to represent more complex meanings, but there is no way for a genuinely new meaning to enter the system. Symbols do not have the power to create new symbol-referent relations, because those relations are not, by definition, available to the system. New symbols cannot be created and placed in relation to the other symbols in such a system, except by an external intelligent agent. Emergence of novel structure requires a flexibility that would be difficult to achieve with a symbolic representation.

4. Dynamical interactions and emergence of cognitive structure

Cognitive science has an alternative to the inflexibility of symbolic representation. Developing alongside the symbolic approach has been a competing interpretation of cognitive structure as arising from dynamics. Interestingly, this interpretation comes from the same Gestalt psychologists who also pioneered insight research. According to the Gestalt theory, psychological reality was not a direct, linear mapping of physiological reality, and cognitive structure emerges not from the sum of elemental encodings but from their interactions (Wertheimer, 1938). Wertheimer proposed a theory grounding structure in the dynamics of the cognitive system. In this case, structure is not a symbolic construction to be included or produced by computation, but is instead an organization of physical forces. In more recent times and separate from the Gestalt tradition, Bickhard (2004) has also indicated that dynamical interactions may be a powerful alternative to encodingism. Bickhard proposes that the physics of dynamical interactions are the non-representational foundation from which representation actually emerges.

5. Equilibrium vs. nonequilibrium

In some ways, the early Gestalt psychologists were far ahead of their time, so far that the appropriate physics for emergent structure had yet to be developed. They had a sophisticated theory of cognitive structure, but the conceptual and quantitative tools for pursuing the theoretical and empirical questions raised by this theory were simply not yet available (Epstein, 1988; Shaw & Turvey, 1981). In the days of Wertheimer (1923) and Köhler (1947), the only physics available was an extension of Newtonian mechanics, as epitomized by Boltzmann (1886/1974). Boltzmann is the scientist most often credited with formalizing and drawing together almost three centuries of thermodynamics, a field concerned with the interplay between matter and energy. Energy flow and material structure are intimately bound up in one another, and they behave in lawful, predictable ways.

At the time of Boltzmann (1886/1974), the rules of thermodynamics seemed to be few and simple. The first law was that energy cannot be created or destroyed. Energy was useful to the extent that it could do work. Formally, work is the capacity to move matter, whether this means simple displacement through space or more complex chemical reactions. To the degree that such transformations comprise the gross structure of a physical system, work is the capacity in a system for structure. The second law was that the amount of free energy is always decreasing. Through the many displacements and reactions, a small fraction of the total energy in a physical system was irretrievably lost. So, as time progressed, fewer and fewer structured processes could take place.

The second law of thermodynamics had far reaching implications for all physical systems. It required that all physical systems follow the same trajectory toward a final state, called equilibrium. Equilibrium is a thoroughly disordered regime; one in which there is no free energy and, therefore, no structure (e.g., see Prigogine, 1961; Zumdahl & Zumdahl, 2006 for further discussion of this definition). At equilibrium, all distributions of matter and energy are homogeneous throughout, and no portion of the system is distinguishable from another. The degree of disorder or lost energy was quantified as *entropy*. Entropy is a statistic describing the uncertainty of sampling from a given probability distribution. Entropy, abbreviated as *S*, in its discrete, informational-theoretic form, is expressed as follows:

$$S = -\sum_{i=1}^{n} p_i \log_2 p_i \tag{1}$$

where the given probability distribution has *n* elements and where p_i is the probability of the *i*th value of the distribution (Shannon, 1948). The Boltzmann (1886/1974) worldview is that of physical systems whose structure is continuously dissolving with each transformation, leading to a final endpoint of completely disordered stasis (i.e., maximal entropy). Thus, all systems tend toward greater entropy over time (Boltzmann 1886/1974; Prigogine, 1961; Prigogine & Stengers, 1984; Swenson & Turvey, 1991).

Gestalt theory described physical systems as tending toward equilibrium (see Kohler, 1947). Gestalt laws of organization were intended to organize structure at thermodynamic equilibrium. That is to say, the early Gestalts treated cognitive structure as a natural consequence of converging physical forces. Once these forces had arranged themselves so as to affect equilibrium, the psychological state was achieved. However, as we have just noted, equilibrium is actually the absence of structure. Therefore, thermodynamic equilibrium seemed, ultimately, a poor candidate for explaining the emergence of cognitive structure.

Theorists familiar with these concepts undoubtedly ran headlong into just this quandary, and it is probably no coincidence that physics fell from favor as an explanation of cognitive structure. When Gestalt theory was initially founded, alternatives to Boltzmann (1886/1974) thermodynamics had not been identified, and there were no explanations of self-organizing structure. Self-organization requires new interpretations of physics. Some suggestions have come from irreversible thermodynamics (Iberall, 1977; Prigogine, 1961, 1980, Soodak & Iberall, 1987) and synergetics (Haken, 1983, 2000). These suggestions arose from the recognition of an important distinction, that between closed systems and open systems. The second law of thermodynamics states that entropy will increase and equilibrium will be achieved, but this law holds only for closed systems. Open systems are considerably more flexible in their ability to deal with entropy and their distance from equilibrium, and they call for a physics of self-organization. The physics of self-organization are well matched to Gestalt theory, and we might imagine that a process like self-organization was just what the early Gestalt psychologists had in mind. The early Gestalt psychologists could not have articulated the appropriate physics for the dynamics they envisioned, but now that we know more about self-organization, cognitive science can better realize the Gestalt intuitions (Haken, Stadler, Ditzinger, & Haynes, 2005).

6. The self-organization alternative

Self-organization is a potential property of open systems. The distinction between open and closed systems hinges on interactions between a system and its environment. Closed systems do not exchange any energy with their surrounding environment. In closed systems, dynamics work to maximize entropy. Entropy is unusable energy and, equivalently, a measure of disorder. Increasing entropy dissolves structure and leads a system to greater homogeneity. The cognitive system exhibits progressive complexity of structure, such as would not be possible in a closed system, and is clearly part of a larger system that exchanges matter and energy with its surroundings. Cognition is not a closed system.

An open system exchanges energy with its surrounding environment. In fact, many open systems thrive on a steady flow of energy. As energy enters into the system, some of it is consumed to do work for the system. But energy flow also must produce fluctuations in the system, leading to a more disordered state at the microscopic scale. Thus, the influx of energy produces an increase in entropy. Unlike closed systems, however, open systems are not required to bottle up this entropy. Instead, open systems can self-organize macroscopic structure for the purposes of offloading entropy into the environment. By doing so, they regulate energy flow and promote the emergence of macroscopic structure.

The offloading of entropy is closely tied to the emergence of structure. The example of Rayleigh-Bénard convection from fluid dynamics illustrates the general principles of self-organization (Prigogine & Stengers, 1984). In the Rayleigh-Bénard paradigm, a flame is placed below a dish of fluid producing a temperature gradient within the fluid. A temperature gradient is simply a graded difference in temperature; fluid molecules nearer to the flame have a higher temperature than fluid molecules farther from the flame. Temperature is directly related to the average velocity of the molecules. Thus, imposing a temperature gradient increases the velocity of fluid molecules, producing fluctuations and increasing entropy. In order for the fluid to maintain stability, it must offload this entropy somehow. The ability of the fluid to offload the incoming heat or entropy depends on the structural arrangement of the molecules. As entropy builds up, the fluid molecules spontaneously reconfigure themselves to form a web of hexagonal convection cells. The convecting cells serve to push warmer molecules up, away from the heat source, and send down cooler molecules to replace the warmer ones. The spontaneous restructuring obviously takes place without a governing plan, rather it is a function of the interacting properties of the system.

As the fluid convection example illustrates, entropy and self-organization are intertwined. Self-organization is the means by which a system shifts into a new configuration, allowing the system to offload unwanted entropy. But by the same token, entropy is the stress that provokes self-organization in the first place. Given this tight relationship, changes in entropy provide an important window into self-organization. As energy flows into an open system, entropy increases and continues to increase until it reaches a critical threshold. At this critical threshold, the system must either dissolve under the stress of entropic fluctuations or reorganize itself so as to offload entropy. The increase in entropy leading up to the critical threshold is consistent with reversible thermodynamics (Boltzmann, 1886/1974), but understanding the drop in entropy requires irreversible thermodynamics (Prigogine, 1961, 1980). Irreversible thermodynamics addresses the drop in entropy as a consequence of reorganization of the system, that is, the emergence of new structure. This reduction of entropy has been termed negative entropy, or *negentropy* (Brillouin, 1962; Schrödinger, 1944). When we want to identify self-organization, we should be able to find this brief period of negentropy just before the emergence of new structure.

We suggest that human problem solvers are an open dynamical system, and problems are, of course, part of the environment. Emergent structure is the self-organized result of energy flow between the solver and the problem. Entropy is a crucial indicator of where a system sits on the trajectory to self-organization. What self-organizes is a soft-assembled attractor, a mode of function toward which a dynamical system will gravitate. We join a wide literature (see Thelen & Smith, 1994; Smith, 2005 for reviews) in describing cognitive structure as a self-organizing attractor. In our own problem-solving research, we have attempted to use qualities of dynamical organization to predict the emergence of such an attractor. The implementation of this research program has been facilitated by recent developments in both embodied cognition and nonlinear dynamics. In the next paragraphs, we will describe these developments in greater detail.

7. Interactivity of action and cognition

If we want to assess the entropy of the cognitive system, we need to know where to look. A central challenge for dynamical accounts of cognition has been capturing fine-grained, moment-to-moment changes in mental activity. How can we get measures of the dynamics of the mind? On first consideration, this may appear to be an insurmountable challenge. However, recent work suggests that action and cognition are joined in a massively interconnected and interactive system.

Traditional approaches to cognitive science have presumed that action is a downstream product of cognition. Cognition, on this account, functions as a central executive and the action system simply follows its commands. Current evidence strongly challenges this view, however. For example, Zwaan and Taylor (2006) reported an action-compatibility effect in reading comprehension. They asked participants to make sensibility judgments of sentences while also turning a dial. Comprehension of the text was quicker when the sentences described motions similar to the direction of the dial rotation (e.g., "turn up the volume on the stereo" describes a motion similar to clockwise rotation), and comprehension was slower when the sentences described motions conflicted with the direction of dial rotation (e.g., "turn down the volume on the stereo" would conflict with clockwise rotation).

The tight relationship between action and cognition appears to be a very general phenomenon, not one restricted to dial rotation or other unusual paradigms. For example, Borghi, Glenberg, & Kaschak (2004) showed that simply describing the location of the participant in relation to an imagined object (e.g., "You are driving a car.", "You are fueling a car.") affected the speed with which they could judge whether something was part of the car (e.g., "headlights"). Importantly, the effect of perspective depended on whether the part could be acted upon from that perspective.

A growing corpus of evidence from the literature on embodied cognition shows that cognition is intimately connected to the body and its actions. Cognition is not isolated or inaccessible; its activity is in evidence in the spatial and temporal changes in bodily action. Put differently, there is only one highly interactive system at play in generating behavior.

Because the system is massively interactive, changes propagate across the entire system, in much the same way as ripples propagate across a pool. Indeed, it has been shown that in an interactive system the dynamics of any variable can be used to reconstruct the dynamics of the entire system (Kantz & Schreiber, 1997; Sauer, Yorke, & Casdagli, 1991; Takens, 1981). Therefore, action can give us access to cognitive dynamics.

8. Self-organization of cognitive structure: Two key indicators

The issue of reconstructing the dynamics of the cognitive system brings us to the second recent development underlying our approach. Advances in nonlinear dynamics have taken strides toward capturing self-organization in behavior (Abarbanel, 1996). In our work, we have focused on key indicators of self-organization. The first indicator we will discuss is entropy. We will review work that has investigated the role of entropy in predicting the self-organization of a new cognitive structure. The second indicator we will discuss is power-law behavior. Below, we will present further evidence from the work reviewed demonstrating changes in power-law behavior leading up to the self-organization of this cognitive structure. In each case, our purpose is to predict the emergence of a new attractor in the cognitive system using these dynamical indicators.

9. Entropy

As we have noted above, in our discussion of thermodynamics, entropy is closely related to the self-organization of new attractors. Theoretically, the two concepts are bound up in one another. Self-organizing attractors are a tendency of a system toward order, and entropy is the disorder in a system. Nevertheless, a critical level of entropy in a system is precisely the stimulus for self-organization of a new attractor. Entropy and attractors are similarly related in terms of dynamical analyses. Concretely, they constitute the two opposite attributes of the phase-space trajectory, and phase space will be a necessary part of the dynamical tools needed to predict the emergence of cognitive structure. This section will describe both the meaning of phase space and the procedure involved in evaluating phase space for entropy and attractors.

Phase space is essentially the high-dimensional set of points of which a subset composes the trajectory of a dynamical system. Dynamical systems are, by the simplest definition, systems that change. These changes may occur across any of the dynamical system's dimensions. The value of each dimension may be expressed as a number. The value of all dimensions of a system for a given point in time is known as the system's phase; it is a multidimensional point. Phase space is the set of all phases that a system might inhabit. We may find in phase space a multidimensional trajectory describing the entire behavior of the system. For example, Lorenz (1963) modeled Rayleigh-Bénard convection in terms of three dimensions. The first dimension describes the divergence of flow. The second dimension is the slope of the temperature gradient. The third dimension is the nonlinearity of the temperature gradient. A three-dimensional trajectory expressing all of these values simultaneously is the Lorenz model's phase space. If a system's dimensions fluctuate randomly, the system's phase-space trajectory wanders, diverging from earlier portions of the trajectory. Divergence of phase-space trajectory indicates entropy. When a system starts to visit similar phases and repeatedly approximates earlier portions of its trajectory, phase space converges. The convergence of the phase-space trajectory is an attractor.

The primary challenge of capturing phase space has been the problem of dimensionality. The dynamics of cognition have been more elusive than those of Rayleigh-Bénard convection. Unlike the Lorenz (1963) system, biological systems exhibiting cognition are complex and high-dimensional. Because capturing phase space has traditionally required a thorough accounting of dimensions, the phase space in cognition has not been as readily available as in the Lorenz system. Fortunately, nonlinear dynamics has refined Takens's (1981) method for reconstructing phase space (Abarbanel, 1996). This method has proven a fruitful technique for studying a variety of nonlinear systems (Kantz & Schreiber, 1997; Sauer, Yorke, & Casdagli, 1991).[1] Once phase space has been reconstructed, it is possible to assess the dynamical organization and attractor strength using recurrence quantification analysis (RQA; Webber & Zbilut, 1994, 2005).[2] With RQA, it is possible to evaluate attractor strength and to estimate the entropy—or disorder—of the reconstructed phase space. Entropy in RQA applies Eq. 1 to the runs of recurrences in the phase-space trajectory.

[1]In broad terms, Takens' (1981) theorem states that, for an interactive complex system, it is possible to unpack the geometry of the entire system from a single univariate observable from any variable composing the system. The time series of the single univariate observable can be projected into a higher dimensional space. The implication of this theorem is that perfect knowledge of underlying mechanism is not, at the outset, necessary for taking global measures of system dynamics. A single time series can be expanded to reflect the dynamics of the entire system. Takens' (1981) theorem specified a time-delay method for reconstructing the phase space of global dynamics from a single, densely sampled univariate time series (Sauer, Yorke, & Casdagli, 1991). This method involves embedding the original time series in a higher dimensional space. This embedding is done by plotting the original time series against lagged copies of itself. The lag is determined by the timestep of the first local minimum in the autocorrelation of the time series. This lag ensures that the embedding dimensions are maximally orthogonal to the original time series. Use of Takens' theorem comes with the caveat of numerous formal assumptions, rarely met fully in any particular data set. For example, Takens' theorem assumes an unbounded number of measurements and error-free measurement. That said, the technique remains a valuable analysis strategy robust to the inevitable violations of these assumptions, and it has been used throughout the physical and biological sciences to predict emergence of nonlinear, chaotic structure (e.g., Garcia, DeLancey, Almeida, & Chapman, 2007; Hirata, Suzuki, & Aihara, 2008; Martinerie, et al., 1998; Waelbroeck, López-Peña, Morales, & Zertuche, 1994).

[2]RQA (Webber & Zbilut, 1994, 2005) essentially sets up a neighborhood around each point, a neighborhood that is some small percentage of the average distance among all of the points. Remember that the formation of an attractor is a convergence of the phase-space trajectory. When a point A falls within the neighborhood of an earlier point B, point A and point B make what's called a recurrent pair. RQA counts all of the recurrent pairs in phase space. The consecutive recurrent pairs form lines. The lengths of these lines provide a measure of attractor strength. The Shannon (1948) entropy of these lengths provides a measure of disorder in this attractor.

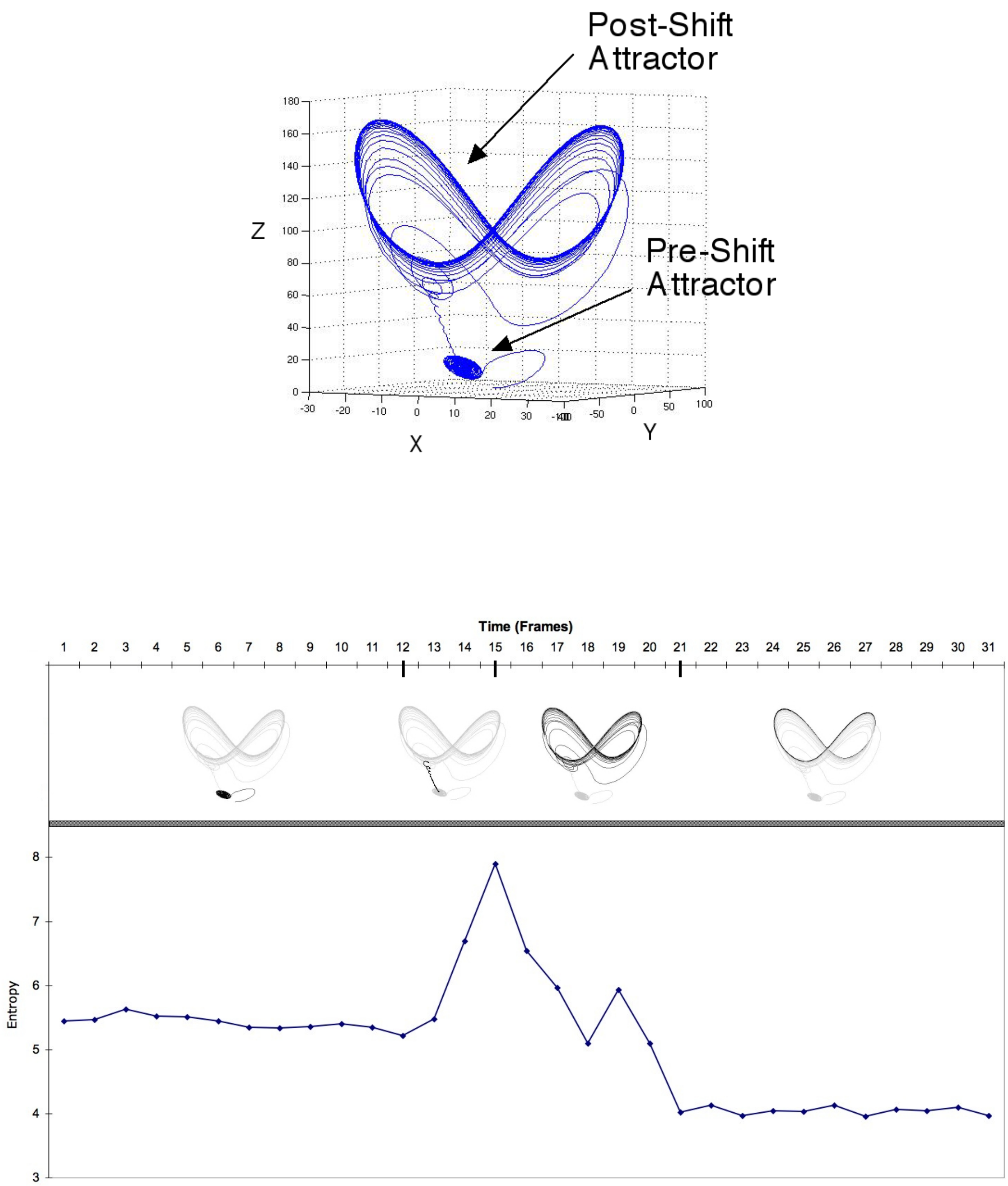


Figure 1. Top panel shows the phase space of the Lorenz system through a phase transition. The system begins in a disk-shaped homoclinic orbit. With an increase in the temperature gradient, the Lorenz system migrates upwards into a transient until it settles into a butterfly-shaped attractor. Bottom panel shows the trend in RQA entropy across four consecutive stages of the Lorenz phase transition, from left to right: disk attractor, departure from disk to transient, transient settling into new attractor, and emergence of butterfly-shaped attractor.

To illustrate the utility of the entropy measure for predicting the emergence of new structure, we return to the Rayleigh-Bénard example. Before, we described how a temperature gradient led to the emergence of structure in the convecting fluid. We also saw that this structure could be modeled in terms of an attractor in three dimensions (Lorenz, 1963). To evaluate the ability of RQA entropy to predict the emergence of new structure, we simulated the effect of making the temperature gradient suddenly steeper in the Lorenz model. According to nonequilibrium thermodynamics (Nicolis & Prigogine, 1977; Prigogine, 1961, 1980), increasing the temperature gradient would lead to an increase of entropy, and once entropy had peaked, the system would reorganize into a new structure, reducing entropy and forming a new attractor. The radical change in the phase-space trajectory from one attractor to another is called a phase transition.

We modeled a logistic increase in the temperature gradient using the standard Lorenz system. The top panel of Figure 1 shows the phase-space trajectory of the Lorenz system as it undergoes a phase transition. At lower temperatures, the Lorenz system remained in an orbit. As temperature increased, the Lorenz system spun out of this orbit into a disordered transient. Finally, at higher temperatures, the model fell into a new butterfly-shaped attractor. We ran RQA on overlapping windows of this Lorenz simulation, from start to finish. We plotted entropy across time (see the bottom panel of Figure 1). As is evident, RQA entropy peaks as the Lorenz system leaves the orbit and drops as it settles into the new attractor. The phase transition shown in top panel of Figure 1 is well captured by RQA entropy. The major point here is that entropy provides a theoretically grounded, meaningful measure of system behavior that provides important information about a phase transition in a dynamical system.

Taken together, phase-space reconstruction and RQA provide the means for assessing entropy of the cognitive system from a single, univariate time series. We conjectured that it might be possible to capture the dynamical qualities of the cognitive system with a single univariate time series of motor actions.

10. Gear-system problems: Representational change as a phase transition

We now weave the strands of embodied cognition and nonlinear dynamics back into a problem-solving project, and specifically, we apply these ideas to a cognitive phenomenon: representational change. The remainder of this review deals with the gear-system paradigm (Dixon & Bangert, 2002, 2004; Dixon & Dohn, 2003; Dixon & Kelley, 2006, 2007; Stephen, Dixon, & Isenhower, in press; Trudeau & Dixon, 2007). In the gear-system task, participants are presented with a number of gear systems on a computer screen. Each gear system forms a pathway of interlocking gears beginning with a driving gear that bears an arrow indicating turning direction (Figure 2). The gears never move, but the task of the participant is to correctly predict the turning direction of final gear. The possible

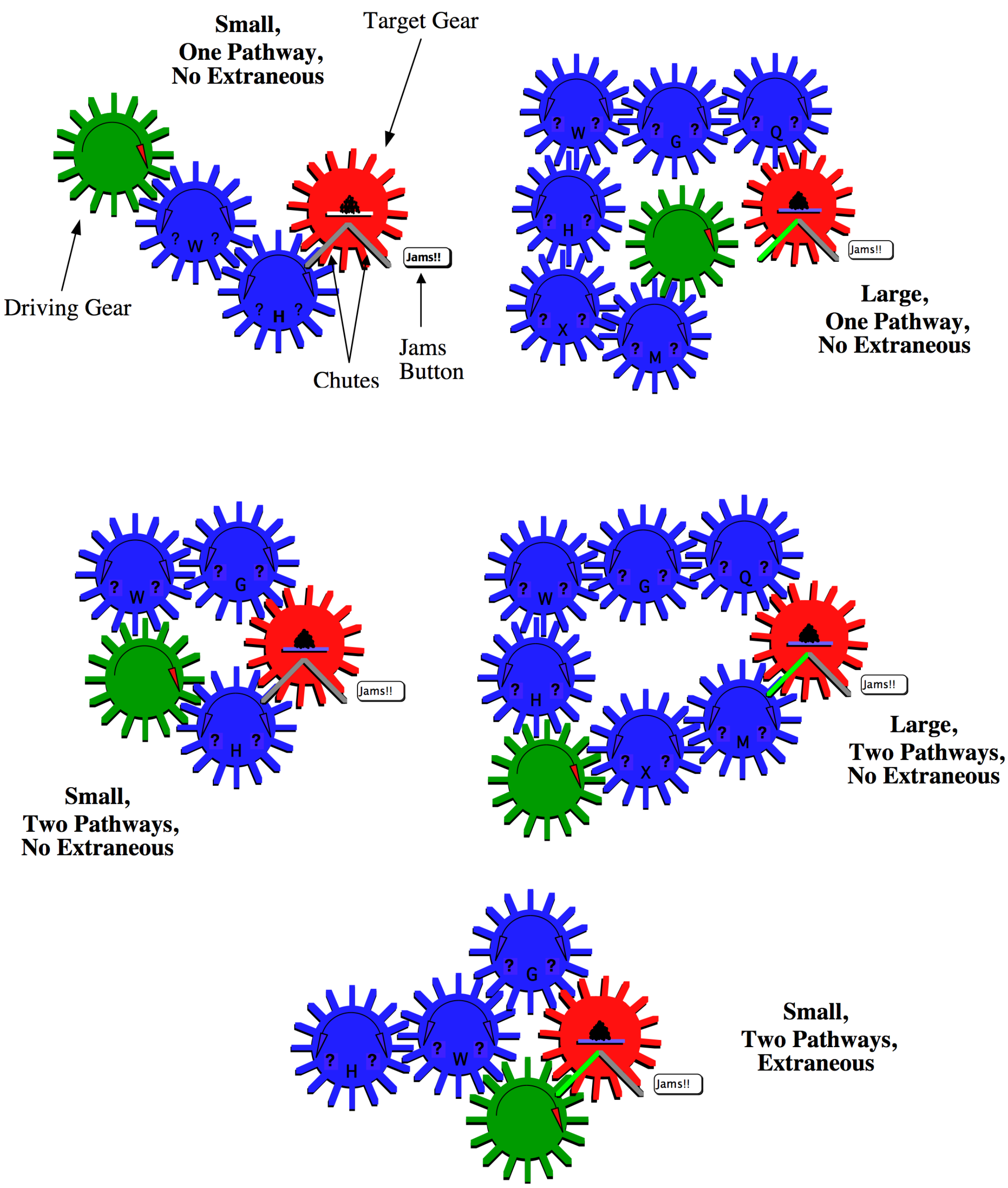


Figure 2. Five examples of gear systems. The gear systems vary on three dimensions: size, number of pathways, and presence of an extraneous gear. Small gear systems (top left, middle left, and bottom) consist of 4 or 5 gears; large gear systems (top right and middle right) consist of 7 or 8 gears. Gear systems have one pathway (top left and top right) or two pathways (middle left, middle right, and bottom). An extraneous gear is one that does not contribute to the turning direction of the final gear (bottom).

responses are "clockwise," "counterclockwise," and, if the final gear is pushed in both ways at once, "jam." The participants receive feedback on their prediction before moving on to the next gear system. Typically, participants complete 36 trials.

By and large, participants are able to correctly predict the turning direction of the final gear, but the gear-system problem serves as an interesting case of representational change (see Dixon & Kelley, 2006; 2007). Participants usually begin solving gear-system problems through use of an approach we call *force-tracing*. The gears never move, as noted above, but were they to move, they would exhibit a transfer of forces, a pushing and pulling among the gear teeth across the pathway of gears. When using this strategy, participants trace the force across the gears with their dominant hand or forefinger. Tracing would begin on the driving gear, in the direction of the arrow indicating its turning direction, and tracing would continue until the final gear. Participants produce their solution based on the motion of their forefinger around the final gear. The force-tracing strategy leads reliably to correct solutions, and participants usually use the force-tracing strategy for quite a few trials.

After several trials of successful use of the force-tracing strategy, an interesting shift occurs (Dixon & Bangert, 2002; Dixon & Kelley, 2006, 2007). Participants suddenly discover that the gears form an alternating sequence. That is, a gear turning clockwise would be followed by a gear turning counter-clockwise, and vice versa. Once participants discover this relation in the gear system, they exhibit the spontaneous emergence of new cognitive structure, the *alternation* strategy. In the alternation strategy, participants simply point or otherwise indicate each successive gear, classifying it as either "clockwise" or "counter-clockwise." The discovery of the alternation strategy indicates a qualitative shift in the participants' representation of the gear-system problem. The representation of the gear system as a series of rotating parts is replaced by the emergent representation of the gear system as an alternating sequence.

The discovery of alternation in the gear-system problem reflects aspects of both embodiment and dynamical systems theory. Dixon and Bangert (2002) explained the phenomenon as a case of embodying relational information. Force-tracing is a strategy that directly involves the bodily action of moving the forefinger along a trajectory available in the visual display of the gear system. Sufficient relational information leads to the emergence of the new representation. Along lines suggested by Thelen and Smith (1994) and Kugler and Turvey (1987), Dixon & Kelley (2006, 2007) began to speculate on the possibility that the shift from one representation of the gear system to another could be a phase transition. By this reasoning, representations of the gear systems are organizations of the cognitive system that afford solution. The force-tracing representation is an organization that specifies manual embodiment of the gear systems. The force-tracing motions themselves serve as the dynamical interactions through which a new structure self-organizes. The new, emergent organization of the cognitive system specifies sequential categorization of the gears as clockwise-turning or counter-clockwise-turning. The explanation on offer was that problem-solving behavior was the dynamically organized embodiment of a task environment.

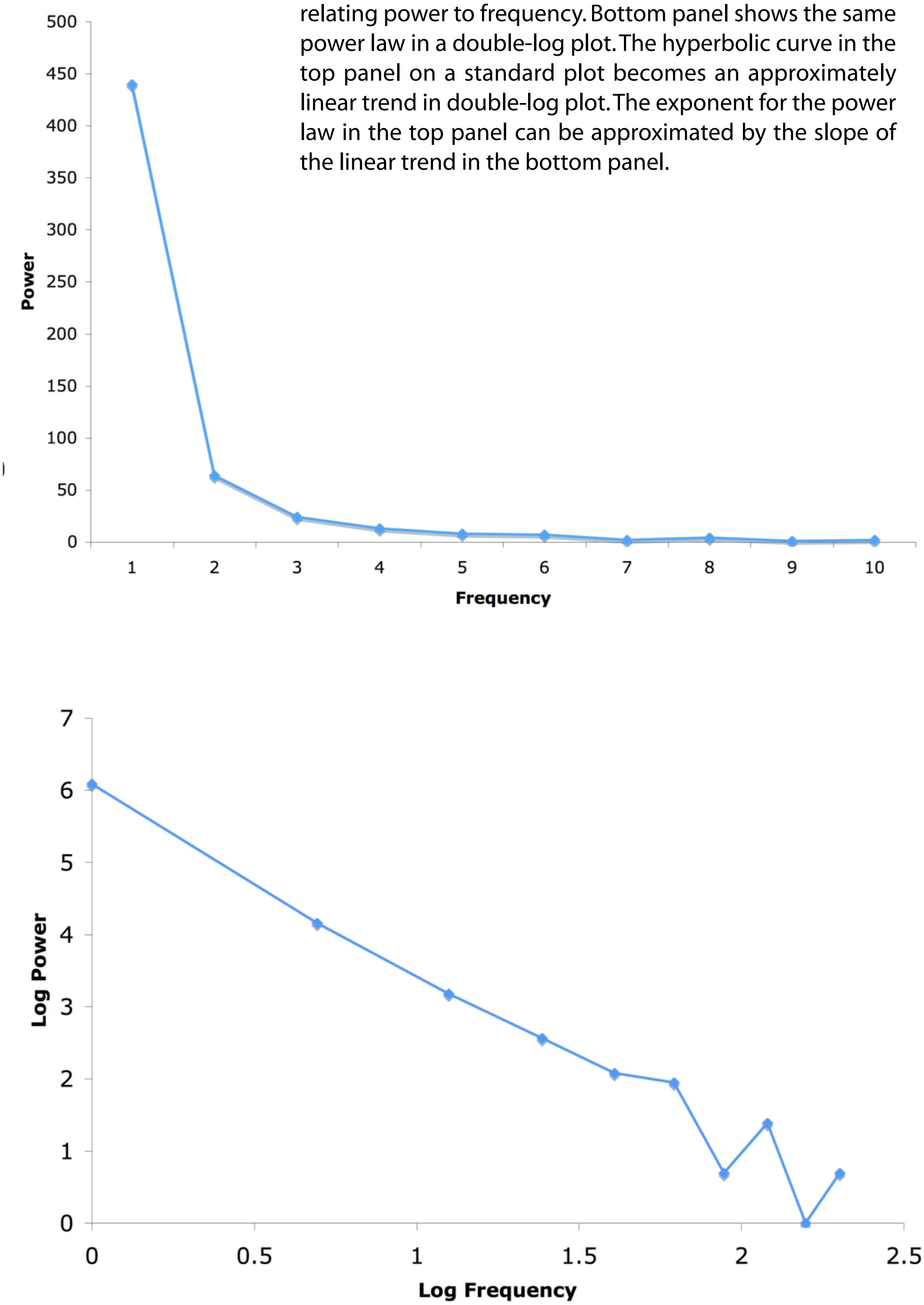
We elaborated on this explanation of strategy change with predictions based solely on the entropy of action (Stephen et al., in press). In doing so, we sought to test the broad predictions from the dynamical systems theory of cognition (Kugler & Turvey, 1987; Thelen & Smith, 1994). According to this theory, a cognitive organism is an open dynamical system. This dynamical system is embedded in a structured environment with which it interacts continuously. These interactions occur through action as well as through all those biological functions supporting action. The metabolism of energy in the environment leads to fluctuations within the organism. These fluctuations constitute the same influx of entropy arising from the metabolism of energy as in any other open dynamical system. Cognitive structures emerge as a natural function of an open dynamical system's tendency to offload entropy. The emergence of a new representation should be marked by a sudden increase of entropy, reflecting a critical instability, followed by negentropy, that is, a rapid decrease in entropy (Brillouin, 1962; Schrödinger, 1944). Negentropy would be indicative of the self-organization of a new representation.

A combined use of motion tracking and nonlinear dynamics allowed us to predict the emergence of the alternation representation (Stephen et al., in press). In all respects, we replicated the procedure for the gear-system task used previously by Dixon and colleagues (Dixon & Bangert, 2002, 2004; Dixon & Dohn, 2003). The only addition was the use of motion tracking on the forefinger of each participant's dominant hand. For each trial, we calculated the angular velocity time series for the participant's force-tracing. Using the angular velocity time series, we applied the methods described above[1, 2] to reconstruct the phase space of the force-tracing motions and to capture the dynamical quality of these motions, separately for each trial. Specifically, we used RQA entropy to model the discovery of alternation. We reconstructed phase space and computed RQA for each trial for participants who had not yet discovered alternation. We modeled the discovery of alternation with event-history analysis, using RQA entropy to predict to the event of discovery. Just before discovery, there was a peak in entropy and a subsequent drop in entropy. This finding served as support for a dynamical systems interpretation of strategy change.

11. Power-law behavior

The second key indicator of self-organization that we employed was power-law behavior. Power-law behavior is a particular type of statistical relationship in which the activity of the system is interrelated across multiple levels or scales. In a power-law relationship the frequency and the magnitude of the behavior can be described as a power function:

Figure 3. Top panel illustrates an idealization of a power law relating power to frequency. Bottom panel shows the same power law in a double-log plot. The hyperbolic curve in the top panel on a standard plot becomes an approximately linear trend in double-log plot. The exponent for the power law in the top panel can be approximated by the slope of the linear trend in the bottom panel.



$$P(f) = kf^{-a} \tag{2}$$

$$\log P(f) = \log kf^{-a} \tag{3}$$

$$\log P(f) = \log k + f^{-a} \tag{4}$$

$$\log P(f) = \log k - \alpha \log f, \tag{5}$$

where P is power as a function of frequency f, involving a positive constant k and a scaling exponent α. The top panel of Figure 3 shows an example of a power-law relationship.

One way to understand power-law behavior is in terms of nesting. As observation of a system takes on progressively finer grain, progressively more detail is uncovered. Each element of the system is made up of smaller elements, which are themselves made up of yet smaller elements, and so on. There are no fundamental, indivisible units in such nested structure. What remain fundamental are the relationships among elements. At all scales of observation, the dynamics are the same. In this case, structure is driven by interactions rather than by separable components (Jensen, 1998).

The link between self-organization and power-law behavior is embedded in the details of how self-organization occurs. Because self-organizing systems reform themselves into new structures (in the absence of a controlling external agent), the current configuration of micro-elements (i.e., microscopic constituent parts) must become flexible. At all scales, the constraints among micro-elements must break or loosen to some degree before the system can change. This breaking apart of micro-elements brings about the nested structure that generates power-law behavior. It allows the interactions to dominate the behavior of the system. That is, the micro-elements can now explore different structural relationships with each other, at every scale. When the micro-elements arrive at a new configuration, then the system exhibits different structure (Bak, 1996; Jensen, 1998).

The degree to which the system exhibits power-law behavior changes as it undergoes reorganization. The breaking of constraints increases the degree of power-law behavior (i.e., the nested behavior increases). As the system reconfigures, the micro-elements are again constrained and power-law behavior decreases. Figure 4 shows a schematic example of the system's elements breaking apart, and thereby increasing the degree of nesting or power-law behavior. Figure 4 illustrates the breaking of constraints across three size scales. In reality, this process continues *ad infinitum* across the size scales (Bak, 1996; Jensen, 1998).

One standard approach to evaluating the degree of power-law behavior is to run spectral analysis on a time series. Spectral analysis decomposes a time series into its constituent oscillations (i.e., sine waves), from very low frequency to very high frequency. It describes the dynamics of micro-elements at all time scales; each frequency represents a different time scale. The top panel of Figure 3 shows a power-law relationship between frequency and power. The bottom panel shows the same relationship plotted on axes of

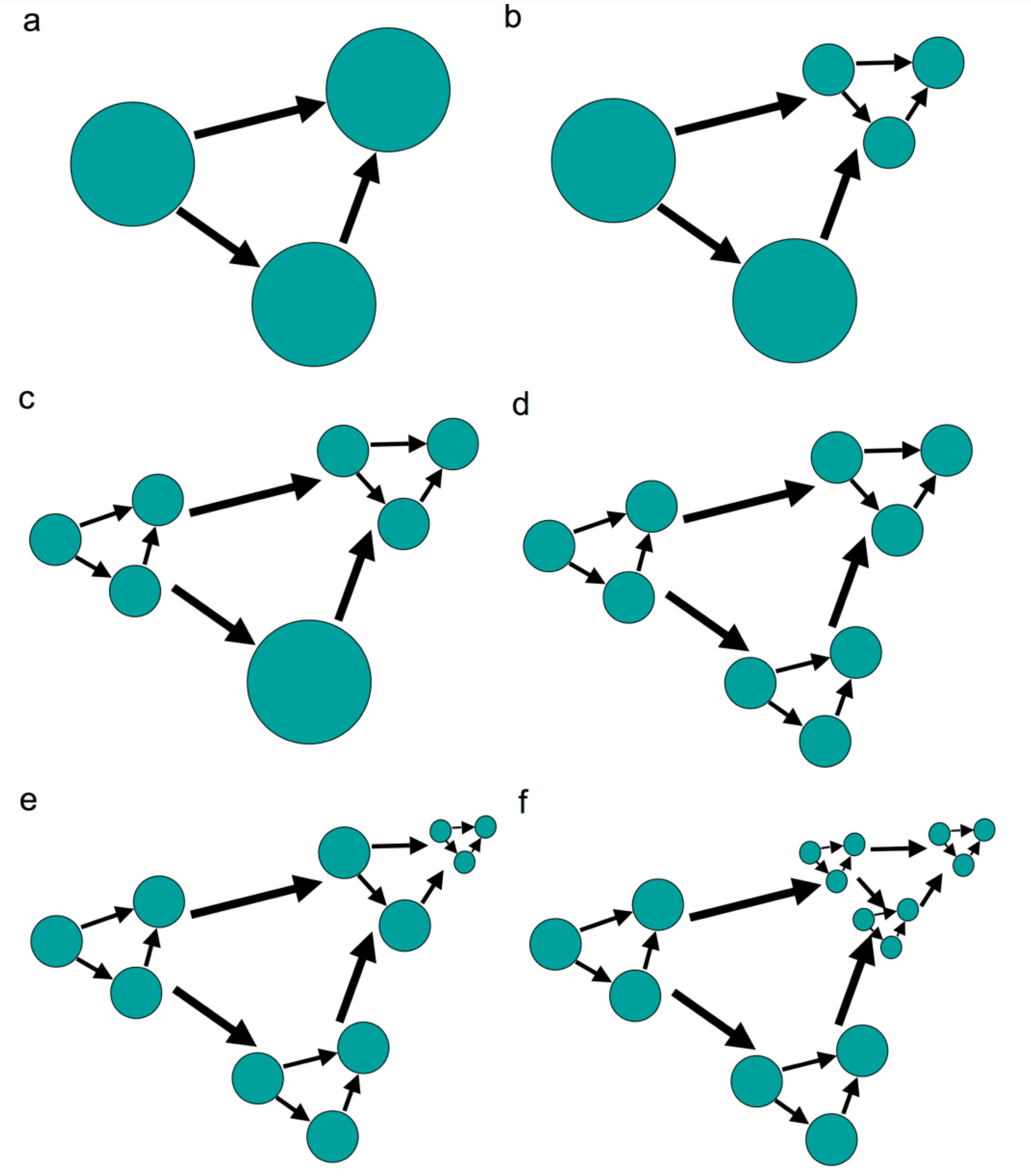


Figure 4. Schematic illustrating the nesting that generates power-law behavior. The macroscopic components apparent in (a) break down into smaller and smaller divisions from (b) to (f) into progressively smaller elements. The tripartite organization at the macroscopic scale is found within each of the three elements, and so on down to the more microscopic scales. The arrows among the parts indicate the interactions between the elements, that is, the dynamics among elements that remain invariant across scales. Large elements populate the system in (a), smaller elements first appear in (b) and overtake the entire system in (d), and even smaller elements appear in the rightmost portion of (e). These smallest elements spread further in (f).

log-power against log-frequency (i.e., a double-log plot). The slope of a linear regression of this power spectrum is an estimate of the power-law exponent (see Eqs. 2-5). For a power spectrum with slope $-\alpha$, it is more convenient simply to describe the power-law exponent in terms of α. The power-law exponent of a system can vary over time and can be an indicator of criticality. As a system approaches a phase transition, its power-law exponent will approach a critical value. The critical value of this exponent is the threshold value beyond which the system will be at risk for a phase transition. In short, steepness of the power-law function is related to the likelihood that a self-organizing system will take on a new structure (Van Orden et al., 2003).

As we noted above, the power-law exponent increases toward its critical value, progressively more constraints within the system are broken, allowing the incipient phase transition. In the case of the gear-system problems, we might expect a specific trajectory in the power-law behavior of the force-tracing motions. As participants continued force-tracing, they would be more likely to discover the alternation strategy and thus at greater risk for the phase transition that this discovery entails. So, power-law exponents should increase as participants continue force-tracing. As the new representation of the gear system emerges, power-law behavior decreases, and the constraints of the new structure should return. This return of constraints would entail a decrease in power-law exponent just before discovery. A power-law exponent of $\alpha = 1$ has been taken as the standard for self-organized criticality and pure interaction-dominant dynamics because it reflects a perfect balance between incremental logarithmic power with incremental logarithmic frequency. At such a balance, the system is optimally uncommitted to any particular configuration (see Bak, 1996; Gilden, 2001; Jensen, 1998; Van Orden et al., 2003). We wish to discuss interaction dominance as occurring along a continuum, where α indicates the degree to which the system is approaching a phase transition.

We now present new findings from our work with the gear-system paradigm (Stephen et al., 2007, in press). We compiled the angular velocity time series from both studies and ran spectral analyses on each trial for each participant, computing the slope to estimate power-law exponents. In keeping with our discussion above, we expected that power-law behavior would first exhibit an increase in power-law exponent (i.e., an extending of power-law behavior to longer time-scales) as participants completed more trials of the gear-system task without discovering alternation and then a decrease in power-law exponent (i.e., a weakening of power-law behavior) as participants approached discovery.

We captured these effects in a simple growth-curve model. The dependent measure, α, is modeled as a function of trial number, pre-discovery trial, a dichotomous variable distinguishing discoverers from non-discoverers (coded as "everdisc," since it indexed whether participants *ever disc*overed), and an interaction between everdisc and trial number. Growth-curve modeling is a maximum-likelihood, regression technique developed for over-time data analysis. Like ordinary-least-squares multiple regression, it assigns B

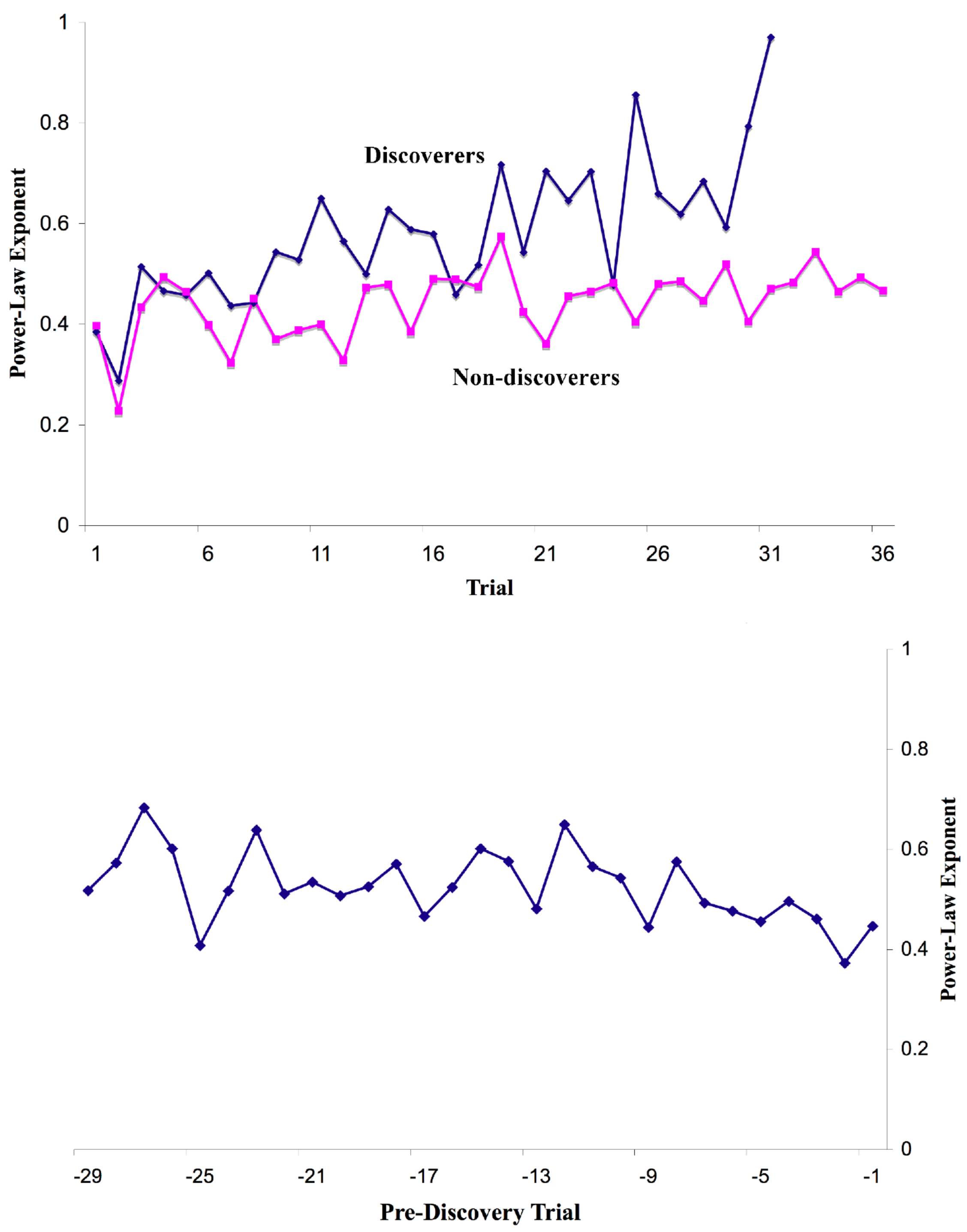


Figure 5. Power-law exponents of force-tracing motions by trial. Top panel shows a plot of power-law exponent by trial, with separate curves for discoverers and non-discoverers. Bottom panel shows a plot of power-law exponent by pre-discovery trial, indicating power-law behavior on the approach to discovery.

coefficients to main effects and interactions. The improvement of model fit with each new B can be assessed with a single-degree-of-freedom chi-square statistic. This chi-square statistic tests the amount of improvement in the deviance of the model, often expressed as -2LL, where LL stands for log-likelihood (Singer & Willett, 2003). Figure 5 shows the mean power-law exponent for participants who have not yet discovered alternation. As can be seen in the figure, the power-law exponent increased over trials as predicted. Interestingly, the power-law exponent increased more rapidly for participants who discovered relative to non-discovers, $B = .01$, change in -2 LL $\chi^2(1) = 4.40$, $p < .05$. The top panel of Figure 5 shows a separate curve for discoverers and non-discoverers. Consistent with the predictions described above, the power-law exponent decreased just prior to discovery, $B = -.01$, change in -2 LL $\chi^2(1) = 4.40$, $p < .05$. This effect is illustrated in the bottom panel of Figure 5 in which each individual's trials are aligned on discovery.

As predicted, the phase transition underlying the discovery of alternation exhibited a period of relatively long-range power-law behavior as old structural constraints dissolved and as new structural constraints emerged. The implication of this finding is that the cognitive system moves through a continuum of interaction-dominance in order to take on new structure. At one end of this spectrum, there would be a regime of complete component-dominance in which structure might justifiably be decomposed into separable parts. This regime of complete component-dominance is characterized by maximal constraint. The other end of the spectrum would be a regime of complete interaction-dominance in which there are no separable components but only nested interactions. Complementary to the regime of complete component-dominance, the regime of complete interaction-dominance is characterized by fluctuation.

12. General Discussion

Problem-solving research is the study of cognition as it unfolds in action. Two additional facts imbue this seemingly uncontroversial statement with deep implications. First, cognition and action are intertwined in a highly interactive system. While it may seem trivial that cognition should have consequences for action, we emphasize the slightly more unintuitive converse. Action is not simply a downstream effect of commands from cognition. Action exerts its own influence on cognition. Here the story gets interesting because action entails a continuous exchange with the environment as it must bring about a fit between a cognitive system and external constraints. The ability for a cognitive system to move adaptively through its environment relies on the system's openness. Therefore, cognition, action, and the environment are involved in a tightly knit process. In this light, problem solving can be understood as an open, nonlinear system, and our explanations will require the concepts and methods normally brought to bear on such systems.

We propose that the theory of open, nonlinear systems (see Ebeling & Sokolov, 2005; Hilborn, 1994; Klimontovich, 1991 for further discussion of this theory) is of utmost relevance to problem solving. It provides a compelling account of the unfolding of cognition in action and the phenomenon of insight. The exchange between an open, nonlinear system and its environment leads to complex interactions and energy flows. These interactions and flows amount to perturbations with which the system must cope. The solution to such perturbations is the self-organization of new steady states. Action is the complex interface at which the cognitive system and environment meet. The self-organizing steady states are cognitive structures that emerge as a result of exchanges between the cognitive system and environment. Insight is thus emergent structure forged amidst the nonlinear interactions of cognition, action, and the environment.

As controversial as this proposal may seem at first, the core ideas are not new. Köhler (1947) suspected long ago that something along these lines was at play. According to Gestalt theory, dynamics of the cognitive system in its environment conspire to produce insight. All that was missing from the Gestalt enterprise was the correct physics; not enough was yet known about the kind of dynamics needed to make the Gestalt theory concrete.

Although the ideas are old, the tractability of putting Köhler's suspicions to the test is new. The recent developments in nonlinear dynamics suggest exciting ways to revisit the old interpretations of psychological issues (Haken et al., 2005). In our research, we have placed problem solving in dynamical terms, only applying newer aspects of what dynamical system theory has to offer.

13. The gear-system paradigm: Insights from problem solving

In our work with the gear-system paradigm (Dixon & Bangert, 2002, 2004 ; Dixon & Dohn, 2003; Dixon & Kelley, 2006, 2007; Stephen et al., in press), we have focused on the interaction of the solver with the environment. We propose that these interactions may carry subtle structure that has not yet been fully explored and that action captures the fine-grained details of these interactions. Whereas action has often been treated merely as a downstream consequence of existing cognitive structure, we propose that action is crucial for understanding the emergence of new cognitive structure. Thus, our work has been an attempt to describe cognitive structure as an emergent property of the interactions between solver and task environment.

The interaction of a problem-solving system with its environment finds a strong parallel with the notion of an open, nonlinear dynamical system. This framework rests solely on the assumption that a cognitive system behaves according to the laws of physics, specifically those of irreversible thermodynamics (Iberall, 1977; Prigogine, 1961, 1980; Soodak & Iberall, 1987). Following this framework, we set out to test the hypothesis that cognitive structure is a self-organizing consequence of action. Our findings provided preliminary support for this prediction.

The work reviewed above strikes a contrast with more conventional approaches to problem solving that involve representational systems. These representational systems often rely on symbols or insular mental processes. On our account, a representation is dynamical organization specifying the behavior of a biological system in a given task environment. In the conventional interpretations of cognition as a symbolic or mental phenomenon, computations play a major role in structuring impoverished sensory inputs and structuring actions around preexisting representations (Fodor, 2000). We propose that, on the contrary, the continuous, dynamic interactions between an organism and its environment are sufficient to generate new cognitive structure. Indeed, we recognize that the term "representation" may no longer apply to our description of cognitive structure. A representation would entail an intermediary cognition structure between organism and environment and an indirect relationship between the two. The dynamical organization does not represent information to any mental faculty but, rather, directly embodies the environmental structure to which it refers (Thelen & Smith, 1994).

Our approach to problem solving deviates dramatically from the traditional approaches taken more frequently in cognitive science. We propose that it is both possible and instructive to consider cognitive structure in non-representational terms. Clearly, problem-solving behavior reflects structure in the cognitive system. This structure appears sufficiently stable in the short term to have merited a symbolic description. However, if we wish to tackle the issue of emergent structure that lies at the heart of problem solving, the symbolic description will benefit from a thoroughly nonsymbolic account from nonlinear dynamics. Taking a step in this direction means reconciling symbolic description with the morphology and dynamics of embodiment. In this view, structure in problem-solving behavior is a by-product of self-organization. Improvement of performance and optimization remain the important factors, but our approach simply phrases these factors in the subsymbolic terms of nonlinear dynamics.

In this formulation, cognition is the perpetual breaking and reforming of constraints to produce progressively complex order. There is no mental agent exploiting rules and manipulating sensory data. Instead, the cognitive system is softly assembled organism and in direct contact with a world rich with environmental structure. The soft assembly of the cognitive system and its tendency toward interaction-dominance leaves it open to reorganization and sensitive to what new structure the environment might afford. Environmental structure is the groundwork upon which the reorganization occurs, giving rise to emergent cognitive structure. Throughout, there is the ongoing sequence noted above, namely, the breaking and reforming of constraints. Changes in entropy and power-law behavior each indicate this ongoing sequence, and we have shown these changes to hold for the emergence of cognitive structure in the gear-system task. We anticipate that concepts from nonlinear dynamics, such as entropy and power-law behavior, will soon be central to our understanding of cognition.

References

Abarbanel, H. D. I. (1996). *Analysis of observed chaotic data.* New York: Springer.

Bak, P. (1996). *How nature works: The science of self-organized criticality*. New York: Springer.

Barsalou, L. W. (1999). Perceptual symbol systems. *Behavioral and Brain Sciences, 22,* 577-609.

Bickhard, M. H. (1993). Representational content in humans and machines. *Journal of Experimental and Theoretical Artificial Intelligence, 5,* 285-333.

Bickhard, M. H. (1996). Troubles with computationalism. In W. O' Donohue & R. F. Kitchener (Eds.), *The philosophy of psychology* (pp. 173-183). London: Sage.

Bickhard, M. H. (2001). Why children don't have to solve the frame problems: Cognitive representations are not encodings. *Developmental Review, 21,* 224-262.

Bickhard, M. H. (2004). The dynamic emergence of representation. In H. Clapin, P. Staines, & P. Slezak (Eds.), *Representation in mind: New approaches to mental representation* (pp. 71-90). New York: Elsevier.

Bickhard, M. H., & Terveen, L. (1995). *Foundational issues in artificial intelligence and cognitive science: Impasse and solution*. Amsterdam: Elsevier Science.

Boltzmann, L. (1974). The second law of thermodynamics. Address to a formal meeting of the Imperial Academy of Science (S. G. Brush, Trans.). In B. McGuinness (Ed.), *Theoretical physics and psychological problems* (pp. 13-32). Boston: Reidel. (Original work published in 1886).

Borghi, A. M., Glenberg, A. M., & Kaschak, M. P. (2004). Putting words in perspective. *Memory and Cognition, 32,* 863-873.

Bowden, E. M., Jung-Beeman, M., Fleck, J., & Kounios, J. (2005). New approaches to demystifying insight. *Trends in Cognitive Sciences, 9,* 322-328.

Brillouin, L. (1962). *Science and information theory.* New York: Academic Press.

Burnham, D., & Dodd, B. (2004). Auditory-visual speech integration by prelinguistic infants: Perception of an emergent consonant in the McGurk effect. *Developmental Psychobiology, 45,* 204-220.

Chronicle, E. P., MacGregor, J. N., & Ormerod, T. C. (2004). What makes an insight problem? The roles of heuristics, goal conception, and solution recoding in knowledge-lean problems. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 30,* 14-27.

Dennett, D. (1984). Cognitive wheels: The frame problem of AI. In C. Hookway (Ed.), *Minds, machines and evolution: Philosophical studies* (pp. 129-151). Cambridge: Cambridge University Press.

Dietrich, E., & Markman, A. B. (2003). Discrete thoughts: Why cognition must use discrete representations. *Mind and Language, 18,* 95-119.

Dixon, J. A., & Bangert, A. S. (2002). The prehistory of discovery: Precursors of representational change in solving gear-system problems. *Developmental Psychology, 38,* 918-933.

Dixon, J. A., & Bangert, A. S. (2004). On the spontaneous discovery of a mathematical relation during problem solving. *Cognitive Science, 28,* 433-449.

Dixon, J. A., & Dohn, M. C. (2003). Redescription disembeds relations: Evidence from relational transfer and use in problem solving. *Memory and Cognition, 31,* 1082-1093.

Dixon, J. A., & Kelley, E. (2006). The probabilistic epigenesis of knowledge. In R. Kail (Ed.) *Advances in child development and behavior, vol. 34* (pp. 323-361). New York: Academic Press.

Dixon, J. A., & Kelley, E. (2007). Theory revision and redescription: Complementary processes in knowledge acquisition. *Current Directions in Psychological Science, 16,* 111-115.

Ebeling, W., & Sokolov, I. M. (2005). *Statistical thermodynamics and stochastic theory of nonlinear systems far from equilibrium*. Hackensack, NJ: World Scientific Publishing Company.

Empson, S. B., & Turner, E. E. (2006). The emergence of multiplicative thinking in children's solutions to paper folding tasks. *Journal of Mathematical Behavior, 25,* 46-56.

Epstein, W. (1988). Has the time come to rehabilitate Gestalt theory? *Psychological Research, 50,* 2-6.

Fingelkurts, A. A., Fingelkurts, A., Krause, C. M., Möttönen, R., & Sams, M. (2003). Cortical operational synchrony during audio-visual speech integration. *Brain and Language, 85,* 297-312.

Fleck, J. I., & Weisberg, R. W. (2004). The use of verbal protocols as data: An analysis of insight in the candle problem. *Memory and Cognition, 32,* 990-1006.

Fodor, J. A. (1990). *A theory of content.* Cambridge, MA: MIT Press.

Fodor, J. A. (2000). *The mind doesn't work that way: The scope and limits of computational psychology*. Cambridge, MA: MIT Press.

Garcia, S. P., DeLancey, L. B., Almeida, J. S., & Chapman, J. W. (2007). Ecoforecasting in real time for commercial fisheries: The Atlantic white shrimp as a case study. *Marine Biology, 152,* 15-24.

Gilden, D. L. (2001). Cognitive emissions of 1/*f* noise. *Psychological Review, 108,* 33-56.

Gilhooly, K. J., & Murphy, P. (2005). Differentiating insight from non-insight problems. *Thinking and Reasoning, 11,* 279-302.

Glenberg, A. M. (1997). What memory is for. *Behavioral and Brain Sciences, 20,* 1-55.

Glenberg, A. M., & Robertson, D. A. (2000). Symbol grounding and meaning: A comparison of high-dimensional and embodied theories of meaning. *Journal of Memory and Language, 43,* 379-401.

Haken, H. (1983). *Synergetics: An introduction.* New York: Springer.

Haken, H. (2000). *Information and self-organization: A macroscopic approach to complex systems*. New York: Springer.

Haken, H., Stadler, M., Ditzinger, T., & Haynes, J. (2005). Synergetics of perception and consciousness. *Gestalt Theory, 27,* 8-28.

Harnad, S. (1990). The symbol grounding problem. *Physica D, 42,* 335-346.

Haselager, W. F. G. (1997). *Cognitive science and folk psychology: The right frame of mind.* London: Sage.

Hilborn, R. C. (1994). *Chaos and nonlinear dynamics: An introduction for scientists and engineers*. New York: Oxford University Press.

Hirata, Y., Suzuki, H., & Aihara, K. (2008). Wind modeling and its possible application to control of wind farms. In D. Mandic, M. Golz, A. Kuh, D. Obradovic, & T. Tanaka (Eds.), *Signal processing techniques for knowledge extraction and information* (pp. 23-36). New York: Springer.

Hollich, G. (2006). Combining techniques to reveal emergent effects in infants' segmentation, word learning, and grammar. *Language and Speech, 49,* 3-19.

Iberall, A. S. (1977). A field and circuit thermodynamics for integrative physiology, I: Introduction to the general notions. *American Journal of Physiology, 233,* 171-180.

Jensen, H. J. (1998). *Self-organized criticality: Emergent complex behavior in physical and biological systems*. Cambridge: Cambridge University Press.

Johnson, S. P. (2004). Development of perceptual completion in infancy. *Psychological Science, 15,* 769-775.

Jones, S. S., & Smith, L. B. (2005). Object name learning and object perception: A deficit in late talkers. *Journal of Child Language, 32,* 223-240.

Kantz, H., & Schreiber, T. (1997). *Nonlinear time series analysis*. Cambridge: Cambridge University Press.

Klimontovich, Y. L. (1991). *Turbulent motion and the structure of chaos: The new approach to the statistical theory of open systems*. New York: Springer.

Knoblich, G., Ohlsson, S., & Raney, G. E. (2001). An eye movement study of insight problem solving. *Memory and Cognition, 29,* 1000-1009.

Köhler, W. (1925). *The mentality of apes.* New York: Harcourt Brace Jovanovich.

Köhler, W. (1947). *Gestalt psychology: An introduction to new concepts in modern psychology*. New York: Liveright.

Kugler, P. N., & Turvey, M. T. (1987). *Information, natural law, and the self-assembly of rhythmic movement*. Hillsdale, NJ: Erlbaum.

Lorenz, E. (1963). Deterministic nonperiodic flow. *Journal of Atmospheric Sciences, 20,* 130-141.

MacWhinney, B. (2005). The emergence of linguistic forming time. *Connection Science, 17,* 191-211.

Martinerie, J., Adam, C., Le Van Quyen, M., Baulac, M., Clemenceau, S., Renault, B., & Varela, F. J. (1998). *Nature Medicine, 4,* 1173-1176.

Moore, C. (2004). Modeling the development of gaze following needs attention to space. *Developmental Science, 9,* 149-150.

Newell, A. (1990). *Unified theories of cognition*. Cambridge, MA: Harvard University Press.

Newell, A., & Simon, H. A. (1956). The logic theory machine. *IRE Transactions on Information Theory, IT-2,* 61-79.

Newell, A., & Simon, H. A. (1976). Computer science as empirical enquiry: Symbols and Search. *Communications of the Association for Computing Machinery, 19,* 113-126.

Nicolis, G., & Prigogine, I. (1977). *Self-organization in nonequilibrium systems: From dissipative structures to order through fluctuations*. New York: Wiley.

Prigogine, I. (1961). *Introduction to the thermodynamics of irreversible processes*. New York: Interscience.

Prigogine, I. (1980). *From being to becoming*. San Francisco: W. H. Freeman.

Prigogine, I., & Stengers, I. (1984). *Order out of chaos: Man's new dialogue with nature.* New York: Basic Books.

Pylyshyn, Z. W. (Ed.). (1987). *The robot's dilemma: The frame problem in artificial intelligence*. Norwood, NJ: Ablex.

Quinn, P. C. (2006). On the emergence of perceptual organization and categorization in young infants: Roles for perceptual process and knowledge access. In L. A. Balter & C. S. Tamis-LeMonda (Eds.), *Child psychology: A handbook of contemporary issues* (pp. 109-131). New York: Psychology Press.

Rasmussen, C., Stephan, M., & Allen, K. (2004). Classroom mathematical practices and gesturing. *Journal of Mathematical Behavior, 23,* 301-323.

Regier, T. (2003). Emergent constraints on word-learning: A computational perspective. *Trends in Cognitive Sciences, 7,* 263-268.

Sauer, T., Yorke, J., & Casdagli, M. (1991). Embedology. *Journal of Statistical Physics, 65,* 579-616.

Schrödinger, E. (1944). *What is life?: The physical aspect of the living cell*. Cambridge: Cambridge University Press.

Searle, J. R. (1980). Minds, brains, and programs. *Behavioral and Brain Sciences, 3,* 417-457.

Shannon, C. E. (1948). A mathematical theory of communication. *Bell System Technical Journal, 27,* 379-423, 623-656.

Shaw, R. E., & Turvey, M. T. (1981). Coalitions as models for ecosystems: A realist perspective on perceptual organization. In M. Kubovy & J. Pomerantz (Eds.), *Perceptual organization* (pp. 343-415). Hillsdale, NJ: Erlbaum.

Siegler, R. S. (2005). Children's learning. *American Psychologist, 60,* 769-778.

Siegler, R. S. (2006). Microgenetic analyses of learning. In W. Damon & R. M. Lerner (Series Eds.) & D. Kuhn & R. S. Siegler (Vol. Eds.), *Handbook of child psychology, vol. 2: Cognition, perception, and language* (6th ed., pp. 464-510). Hoboken, NJ: Wiley.

Siegler, R. S., & Jenkins, E. A. (1989). *How children discover new strategies.* Hillsdale, NJ: Erlbaum.

Simon, H. A. (1969). *The sciences of the artificial*. Cambridge, MA: MIT Press.

Singer, J. D., & Willett, J. B. (2003). *Applied longitudinal data analysis: Modeling change and event occurrence*. New York: Oxford University Press.

Smith, L. B. (2005). Cognition as a dynamic system: Principles from embodiment. *Developmental Review, 25,* 278-298.

Soodak, H., & Iberall, A. S. (1987). Thermodynamics and complex systems. In F. E. Yates (Ed.), *Self-organizing systems: The emergence of order* (pp. 459-469). New York, NY: Plenum Press.

Stephen, D. G., Dixon, J. A., & Isenhower, R. W. (in press). Dynamics of representational change: Entropy, action, and cognition.

Swenson, R., & Turvey, M. T. (1991). Thermodynamics reasons for perception-action cycles. *Ecological Psychology, 3,* 317-348.

Takens, F. (1981). Detecting strange attractors in turbulence. In D. Rand & L. S. Young (Eds.), *Lecture notes in mathematics, vol. 898: Dynamical systems and turbulence* (pp. 366-381). Berlin: Springer.

Thelen, E., & Smith, L. (1994). *A dynamic systems approach to the development of cognition and action*. Cambridge, MA: MIT Press.

Torbeyns, J., Arnaud, L., Lemaire, P., & Verschaffel, L. (2004). Cognitive change as strategy change. In A. Demetriou & A. Raftopoulos (Eds.), *Cognitive developmental change: Theories, models and measurement* (pp. 186-216). Cambridge: Cambridge University Press.

Triesch, J., Teuscher, C., Deák, G. O., & Carlson, E. (2006). Gaze following: Why (not) learn it? *Developmental Science, 9,* 125-147.

Trudeau, J., & Dixon, J. A. (2007). Embodiment and abstraction: Actions create relational representations. *Psychonomic Bulletin & Review, 14,* 994-1000.

Van Orden, G. C., Holden, J. G., & Turvey, M. T. (2003). Self-organization of cognitive performance. *Journal of Experimental Psychology: General, 132,* 331-350.

Waelbroeck, H., López-Peña, R., Morales, T., & Zertuche, F. (1994). Prediction of tropic rainfall by local phase space reconstruction. *Journal of Atmospheric Sciences, 51,* 3360-3364.

Webber, C. L., & Zbilut, J. P. (1994). Dynamical assessment of physiological systems and states using recurrence plot strategies. Journal of Applied Physiology, 76, 965-973.

Webber, C. L., & Zbilut, J. P. (2005). Recurrence quantification analysis of nonlinear dynamical systems. In M. A. Riley & G. Van Orden (Eds.) Tutorials in contemporary nonlinear methods for the behavioral sciences (pp. 26-94). Retrieved February 3, 2006. http://www.nsf.gov/sbe/bcs/pac/nmbs/nmbs.pdf

Weisberg, R. W. (1996). Prolegomena to theories of insight in problem-solving: A taxonomy of problems. In R. J. Sternberg & J. E. Davidson (Eds.), The nature of insight (pp. 157-196). Cambridge, MA: MIT Press.

Wertheimer, M. (1938). Laws of organization in perceptual forms. London: Harcourt Brace Jovanovich.

Younger, B. A., & Johnson, K. E. (2006). Infants' developing appreciation of similarities between model objects and their real-world referents. Child Development, 77, 1680-1697.

Yu, C. (2005). The emergence of links between lexical acquisition and object categorization: A computational study. Connection Science, 17, 381-397.

Zumdahl, S. S., & Zumdahl, S. A. (2006). Chemistry. Houghton Mifflin: Boston.

Zwaan, R. A., & Taylor, L. J. (2006). Seeing, acting, understanding: Motor resonance in language comprehension. Journal of Experimental Psychology: General, 135, 1-11.

Acknowledgment

Partial support was provided by NSF grant BCS-0643271.

Paper submitted on December 5, 2007.

The final version accepted on December 18, 2008.