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An Examination of the Alleged Role of "Fixation" in the Solution of Several "Insight" Problems

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SUMMARY

A series of experiments were conducted to examine the purported role of fixation in the solution of several insight problems, such as the nine-dot and triangle problems. It has been commonly accepted in psychology that such problems are difficult because subjects are fixated on unwarranted assumptions concerning how the problems are to be solved. In the present studies, subjects were disabused of these allegedly fixating assumptions in order to see whether quick and direct solution of the problem would then follow. For example, subjects working on the nine-dot problem were told that the problem could be solved only by extending their solution lines outside the boundaries of the square formed by the dots, and subjects working on the triangle problem were told to work in three dimensions. The basic finding was that removing the alleged fixation did not result in sudden and direct solution of the problems, indicating that fixation was not a very important factor in making these problems difficult. It was also found that significant facilitation of solution in these problems could be brought about only by giving subjects relatively detailed information about the solution. It was concluded that the terms *fixation* and *insight* are not useful in describing the processes involved in the solution of these problems, and the role of problem-specific knowledge in the solution of insight problems was emphasized. The relationship between the present findings and other recent work on problem solving is also discussed.

The term *insight problems* refers to a heterogeneous group of problems that have received some attention from psychologists over the years (Bourne, Ekstrand, & Dominowski, 1971). Although these problems come from many different subject areas, there are several common characteristics in most of them. First, the solutions that seem most obvious to the naive subject usually do not work. Furthermore, even if the obvious solutions do work, these solutions are not of interest to the experimenter. The experimenter is interested in one particular solution, which is not produced by most subjects. This solution, called the *insightful solution*, is alleged to come about when the subject can achieve insight into the structure of the problem. Furthermore, it is argued that production of the insightful solution is hindered because the subject brings an unwarranted assumption to the problem that is based on past experience with similar problems. According to the Gestalt-oriented theorists who have studied these problems (e.g., Scheerer, 1963), subjects are *fixated* on these unwarranted assumptions, and this fixation interferes with the insight needed to solve the problem. It is also argued that if this source of fixation were removed, solution of such problems should occur, and it should occur quickly and directly, as connoted by the term *insight* (Luchins & Luchins, 1970, p.139).

Some examples of insight problems are shown in Figure 1. In the nine-dot problem, where subjects are to connect the nine dots with four connected straight lines without lifting the pencil from the paper, subjects are assumed to be fixated on the assumption that the lines must conform to the shape of the square; in the triangle problem, in which subjects are to manipulate six matchsticks to construct four equilateral triangles, with one complete matchstick making up one side of each triangle, fixation allegedly is brought

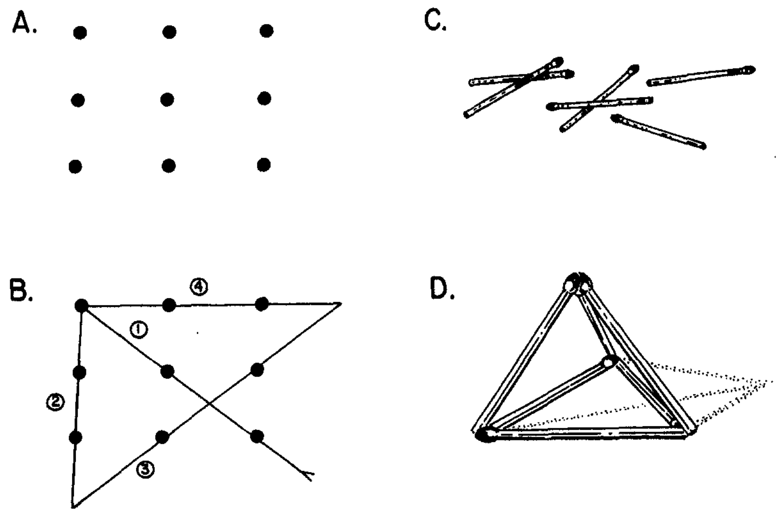

Figure 1. A: The nine-dot problem: Connect the nine dots with four connected straight lines without lifting your pencil from the paper. B: Nine-dot solution. C: The triangle problem: With six matches, construct four equilateral triangles, with one complete match making up one side of each triangle. D: Triangle solution.

about by the assumption that the triangles are to be two-dimensional (Scheerer, 1963).

Insight problems were of interest to Gestalt-oriented psychologists because they wished to demonstrate that past experience is not the central factor in solving problems. According to the Gestalt view, truly productive thought depended first and foremost on insight, and not on the passive use of past experience. Therefore, insight problems, in which past experiences allegedly actually interfered with solutions, were of particular interest. In order to solve an insight problem, Gestalt-oriented psychologists argued that the problem solver had to view the situation in a new way, through "restructuring," which did not depend on past experience.

Although many psychologists do not accept the notions of insight and fixation, they do accept the idea that past experience can often interfere when people try to solve insight problems. For example, Newell and Simon (1972, pp. 90–91), who carried out the pioneering work in the development of information-processing theories of problem solving, discuss the nine-dot problem, arguing that it is a difficult problem solely because subjects represent the shape of the dots as a square, and this representation determines how the subjects attack the problem. Rumelhart (1977), another information-processing theorist, makes the following statement about the nine-dot and triangle problems, among others:

> All of these problems have something in common. They are all very easy problems that are made very difficult by one thing only—our prior expectations about the allowable solutions to these problems. . . . [With the nine-dot problem] the problem comes from the fact that the configuration forms a natural square and we tend to impose the constraint that our lines may not go outside the square. . . . The [triangle] problem causes difficulty because we fail to consider the third dimension . . . and we tend to look only at two dimensions. (p. 262)

Thus, although there is some dispute about the specific theoretical constructs that should be employed to explain the phenomena, there is little disagreement among psychologists of various persuasions as far as the basic phenomena are concerned. As regards the problems in Figure 1, most theorists agree that they are easy problems that are made difficult only because of certain unwarranted assumptions. It is also inter-

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esting to note that in recent years there has been very little research concerning most of the "classic" insight problems. One reason for this may be that the area seems to be closed as far as interesting questions are concerned. Since everyone seems to be in accord on the basic phenomena, and since the important factors influencing performance on these problems seem to be known, there is little more to be learned about insight problems. In summary, one could say that the Gestalt analysis of insight problems has been more or less incorporated into modern discussions of problem solving, although its surface appearance may have been changed somewhat.

However, from one point of view this agreement among theorists is quite remarkable, inasmuch as it is based on no data. Scheerer (1963) seems to have been the popular source of the problems in Figure 1, as well as several other problems, and he presents no data, since his was a review article. Maier (1930) also briefly discussed the nine-dot problem, emphasizing the square shape of the dots, but he presented no data in his article either. Furthermore, no one seems to have published any widely cited data on most insight problems since Scheerer's review. Thus, this area of psychological theorizing is not based on hard data. This lack of data was the first motivating factor behind the present series of experiments.

The second motivating factor was a belief that the nine-dot and triangle problems were in reality very difficult problems, not easy problems whose solutions were hindered by fixation. This belief came out of recent research with a framework for analyzing problem solving that tries to explain how people use their past experiences to solve novel problems (e.g., Weisberg, 1980; Weisberg, DiCamillo, & Phillips, 1978; Weisberg & Suls, 1973; Weisberg, Note 1). This framework has been applied to one insight problem, Duncker's (1945) candle or box problem, with some success, which prompted the investigation of other insight problems. Basically, this viewpoint argues that presentation of a problem serves as a cue to retrieve relevant information from memory. Any information that is retrieved then serves as the basis for solution attempts. In this way, it is assumed that problem solving begins with relevant past experience. However, it is also assumed that subjects do not simply apply old solution methods directly to new problems. Rather, they monitor these solutions, and if they are not working, they try to modify the inadequacy so that the solution is acceptable. In this way, the problem solver is not simply at the mercy of past experience. A truly novel solution can evolve as the problem solver tries to make old knowledge fit the new situation. In this way, this point of view differs from the more traditional associationistic viewpoint (Bourne et al., 1971, pp. 69–70).

Applying this retrieval point of view to the two problems in Figure 1 seems to indicate that these are difficult problems. Consider the nine-dot problem. Recalling past experiences with puzzles and games, people should be inclined simply to go from dot to dot. Because this solution will not work for this problem, subjects should try all the variations of dot-to-dot solutions that they can think of. When they see that such solutions will not work, they must then try some further modification. However, most people have no additional knowledge of how to modify a dot-to-dot solution to solve the problem. Therefore, most people do not solve the nine-dot problem. A similar analysis applies to the triangle problem, presented later. This sort of analysis also implies that people need relatively specific knowledge before they can solve such problems.

Thus, the second factor that motivated the present series of experiments was our interest in the role of task-specific information in solving insight problems. In the present experiments, subjects acquired this information in either of two ways. In some cases, subjects were instructed by the experimenter on the form of the solution to a problem. In other cases, subjects were given prior experience solving similar problems, and the transfer of this information to a given insight problem was then examined.

The third purpose of the present article was to attempt to bring the domain of insight problems into the mainstream of research on problem solving. As indicated above, there has been little recent research concerning insight problems, perhaps because of a belief that we know most of the important factors affecting performance on such problems. Another reason for the lack of research may be the belief that solving insight problems involves processes, such as restructuring and insight, that are different from those studied by most contemporary cognitive psychologists. Therefore, if it could be shown that performance on insight problems can be understood on the basis of principles already available in cognitive theory, this could help to bridge the gap that presently exists between the work on insight problems and other work in problem solving.

The "retrieval" framework outlined earlier is similar to Levine's recent formulations of hypothesis theory (e.g., Levine, 1975; Sweller & Gee, 1978). According to Levine's viewpoint, solutions to problems are conceived of as consisting of groups or "domains" of solutions. Depending on the knowledge and assumptions that the subject brings to the problem, a different solution domain will be sampled. For example, if the subject assumes that a discrimination problem has a simple solution, then a different solution domain will be considered than if the subject assumes that the problem has a complicated solution. Furthermore, if the solution to the problem is not in the domain that the subject is considering, and if this incorrect domain contains many potential solutions, the subject may not exhaust this domain in time to switch to the correct domain and solve the problem.

This viewpoint can easily be extended to the insight problems to be considered here. It can be hypothesized that the difficulty in solving some insight problems comes about for the following reasons: (a) The presentation of the problem, in conjunction with subjects' past experience, suggests one way of attacking the problem (i.e., suggests one domain of solutions). (b) This domain is relatively large, so subjects may never exhaust it. (c) Even if subjects do exhaust the initial domain, the problem may still be difficult because they may not be aware that another domain of solutions exists, due to lack of relevant past experience. (d) Even if subjects do believe that an alternative domain exists, that domain may also be so large that subjects who have to work through the possibilities may still not solve the problem.

This analysis enables us to discuss performance on various insight problems in a straightforward manner, and serves to link the present experiments with other work in problem solving. We shall also see in the experiments that follow that this formulation is not completely accurate as a description of performance on the nine-dot problem. However, it provides a helpful starting point.

Experiment 1

If the nine-dot problem is difficult solely because subjects assume that they must stay within the square, as argued by Gestalt-oriented psychologists, then it should be easy to bring about an efficient solution of this problem simply by telling subjects that the only way to solve the problem is to go outside the boundaries of the square. This was done in the present experiment as follows. The control group attempted to solve the nine-dot problem as presented in Figure 1. An "exhausted" group was told that they had exhausted all the possibilities within the square and that in order to solve the problem, they would have to go outside the square. According to the traditional analysis of the nine-dot problem, the "exhausted" condition should eliminate the fixation and result in easy solution. However, as discussed earlier, it was our expectation that the "exhausted" hint would not be very helpful because the hint only tells the subject to go outside the square; it does not tell the subject what to do once beyond the borders. Therefore, we tested two additional conditions. The Line 1 group was given the "exhausted" hint and was also shown the first line of the solution, as numbered in Figure 1. The Line 1 + 2 group was given Lines 1 and 2 from Figure 1 as part of their hint, in addition to the "exhausted" hint. The Line 1 and Line 1 + 2 groups were given information that could serve to curtail the size of the domain being considered.

To summarize, we tested four conditions: (a) The control group was given the standard nine-dot problem. (b) The "exhausted" group tested the notion that subjects were fixated on the boundaries of the square. (c) The Line

1 group and the Line 1 + 2 group tested whether the problem was really much more difficult than heretofore assumed. (d) In addition, a second control condition (rotated control) received the nine-dot problem rotated 45° so that the configuration of dots looked like a diamond rather than a square, and were given no further information. The rotated control condition was presented to see whether the shape of the diamond pattern might help cue the solution, since the solution lies in the symmetry around one diagonal of the square.

Method

Materials. The problem was presented as a 4 × 4 cm matrix of dots in the center of a sheet of paper. The forms for the Line 1 and the Line 1 + 2 groups were identical except that the corresponding lines from Figure 1 were drawn on the sheet. The rotated control group received the problem rotated 45°.

Procedure. Each subject was tested individually. The control subjects were given the standard form of the problem: Connect the nine dots with four connected straight lines without lifting the pencil from the paper. The subject made a solution attempt on the first sheet, and if it was incorrect, the sheet was taken away and a new sheet was presented for another attempt. All subjects in all groups were urged to do their thinking "on the paper" rather than "in their heads" so that we could obtain a reasonably complete record of their attempts. The control subjects were allowed a maximum of 20 solution attempts of this sort, although they were not informed of how many attempts they could have.

The "exhausted" group was permitted 10 such solution attempts. If they did not solve the problem within these 10 control attempts, they were then told that they had exhausted all the solution possibilities within the square and that in order to solve the problem they would have to go outside the square. The reason the "exhausted" group was first given the 10 control attempts was that we wanted to increase the probability that they would believe it when they were told that they could not solve the problem by staying within the square. It was hoped that the 10 unsuccessful attempts within the square would make the "exhausted" hint maximally effective. The "exhausted" subjects were given 10 additional solution attempts after the hint.

The Line 1 group was also given 10 control attempts, followed by the "exhausted" hint and 10 more solution attempts. However, the latter 10 attempts were carried out on forms with the first line of the solution drawn in, and the subjects were informed that this was the first line of the solution. The Line 1 + 2 group was treated identically, except that their latter 10 attempts were carried out on forms of the problem that contained the first two lines of the solution, and the subjects were so informed.

Subjects. One hundred Temple University undergraduates fulfilling an optional course requirement for introductory psychology served as subjects. They were randomly assigned to conditions, with the restriction that all conditions would ultimately contain 20 subjects.

Results and Discussion

Of the 100 subjects tested, 23 reported that they were familiar with the problem and thus were dropped from the initial analysis. However, inclusion of their data has no effect on the conclusions to be drawn, which will be discussed further below. Also, the performance of the rotated control subjects was identical to that of the standard control group. Therefore, the rotated control group will not be considered further.

The results for the subjects not familiar with the problem are presented in Rows 1–5 in Table 1. The dependent variable is the number of attempts needed to solve the nine-dot problem, and the data indicate that the traditional analysis of this problem is incomplete to say the least. All the groups in Table 1 were treated identically for the first 10 solution attempts, and only one subject solved the problem within these 10 attempts. (This subject was in the Line 1 group and was eliminated from the subsequent analysis.) Thus, the problem was indeed very difficult. Furthermore, none of the 15 control subjects solved the problem within the allotted 20 attempts. Also, none of the 15 control subjects went outside the boundaries of the square in any solution attempt, indicating that "fixation" is almost maximal in this problem.

As can be seen in Table 1, the "exhausted" hint did not have a strong effect on solution, although it did effectively eliminate any fixation brought about by the square's boundaries. All but one subject in the "exhausted" group went outside the square after receiving the hint, but only a small proportion of these subjects eventually solved the problem. Also, those three subjects who did solve the problem took a median of five additional attempts to do it, indicating that breaking the fixation through the "exhausted" hint did not produce any direct insight on how the problem was to be solved. Therefore, although at first glance it might be argued that the modest facilitation brought about by the "ex-

Table 1
Performance on Nine-Dot Problem in Experiment 1

	Condition			
Variable	Control	Exhausted	Line 1	Line 1 + 2
1. Number of subjects (out of 20) not familiar with problem	15	15	14	17
2. Number of naive subjects not solving in first 10 control attempts	15	15	13	17
3. Proportion of subjects in Row 2 solving in Attempts 11–20	.00	.20	.62	1.00
4. Median number of additional attempts needed to solve				
All subjects in Row 2	10[a]	9.66 (3–10)[b]	6.25 (1–10)	1.44 (1–7)
Solvers only	—	5.03 (3–9)	5.0 (1–9)	1.44 (1–7)
5. Proportion of subjects going outside boundaries of square	.00	.93	1.00	1.00
6. Proportion of experienced subjects not solving in first 10 control attempts	1.00	.80	.83	.67
7. Proportion of nonsolvers in Row 6 solving in Attempts 11–20	.00	.00	.60	1.00

[a] Ten additional attempts given after first 10 control attempts.
[b] Ranges are given in parentheses.

hausted" hint supports the notion that fixation is one factor in solution of the nine-dot problem, a closer analysis of the effects of the hint does not support this view. The solutions that occurred were not quick and direct, as the fixation notion would lead one to anticipate.

It is argued above that in order to produce facilitation on this problem, information about the specific form of the solution would have to be given. This expectation was upheld. As can be seen in Table 1, only in the Line 1 + 2 group was impressive facilitation found. In this group, all subjects solved the problem, and a majority did so with two attempts. The Line 1 condition produced more solutions than did the "exhausted" condition, but more than one third of the subjects in the Line 1 condition did not solve the problem, even though all of them went outside the square's boundaries after receiving the hint. Also, the subjects who solved the problem after the Line 1 hint took a median of five additional attempts to do so.

The present results closely parallel those of Burnham and Davis (1969), whose control subjects did not solve the nine-dot problem and who also found that instructions to go outside the square did not greatly facilitate solution.

These conclusions are supported by statistical analysis. The data were analyzed in two ways. First, the number of solution attempts needed to solve the problem was used as the dependent variable, and the difference among the four conditions was tested using the Kruskal-Wallis test. The overall difference was highly significant, $\chi^2(3) = 31.92$, $p < .001$. The separate groups were then tested using one-tailed median tests,[1] and the following pattern was found, with underlining indicating that groups were not different: Control; "exhausted"; Line 1; Line 1 + 2.

The second analysis was based on number of subjects solving, and once again the over-

[1] We tested directional predictions using the transformation $Z = \sqrt{\chi^2(1)}$ (Hays, 1963, p. 362).

all difference was highly significant, χ^2 (3) = 34.43, $p < .001$. Differences between pairs of groups were tested by chi-square tests with $p < .05$, and the following pattern was found: control; "exhausted"; Line 1; Line 1 + 2.

At this point, the performance of the Line 1 + 2 group should be mentioned briefly. All the subjects in this group solved the problem, and most did so quickly. This might be dismissed as a trivial result, since these subjects were given so much information. However, from the present point of view, it must be emphasized that much information was necessary in order to make the problem easy. This is further evidence of how difficult the problem really is. We examined the difficulty of the problem further in Experiment 1a.

Finally, it is interesting to consider the results of the 23 subjects who already had some familiarity with the problem. These results are summarized in Rows 6 and 7 of Table 1. As can be seen, the results for these subjects are very similar to those for the naive subjects. Very few of these subjects solved the problem without a hint (3 out of 19; see Row 6 in Table 1), and the effectiveness of the hints was very similar to that found for naive subjects (compare Rows 3 and 7 in Table 1). These results raise questions about forgetting solutions to insight problems. However, because of the small number of subjects involved here, we reexamine the retention of the solution to the nine-dot problem in Experiments 1b and 1c.

Since the present results provide little support for the Gestalt view that the nine-dot problem is difficult because subjects are fixated on the shape of the square, let us now consider these results in terms of the hypothesis-testing framework outlined above. First, we hypothesized that the nine-dot problem might be difficult because the initial incorrect solution domain suggested by the problem is very large, which would result in time running out before the domain was exhausted. This does not seem to be quite correct. It is true that control subjects tried a number of variations on dot-to-dot solutions, but the verbal reports indicate that all of the control subjects quickly became aware that they were repeating the same solutions. Thus, the incorrect domain was effectively exhausted within approximately 10 attempts, but this did not lead the subjects to go outside the square. For most control subjects, then, it seems that the domain containing the correct solution effectively *does not exist*, perhaps because it is in reality a very difficult task to draw a configuration of lines in just the right way to solve the nine-dot problem. That is, perhaps the correct domain exists only when a subject has some relatively specific past experience that can be applied to the problem at hand. "Searching through" this domain would depend on such experience, and if none is available the subject will simply not solve the problem, even though he or she is aware that producing variations on old incorrect solutions is no help whatever. Thus, subjects may appear "fixated" on dot-to-dot solutions because they have no other knowledge to apply to the nine-dot problem and because the experimenter asks them to continue trying to solve the problem.

The group given the "exhausted" hint also gave evidence that the hint was of little help because they possessed no task-specific knowledge that they could apply to the problem. When told to go outside, most of these subjects produced a small number of solution attempts but then gave evidence that they had no idea how they were to proceed.

Thus, the difficulty with the nine-dot problem seems to be that for most subjects the domain containing the correct solution does not exist because subjects do not possess the experience needed to conceptualize the solution. In addition, the outside hint is ineffective for the same reason: Subjects do not know how to "search" the correct domain even when it is pointed out to them. As has been noted elsewhere, a hint is effective only if a subject can use it appropriately, which depends as much on the subject's knowledge as it does on the hint (Maier, 1970).

Experiment 1a

One limitation on the conclusions from Experiment 1 is the relatively small number of solution attempts made by the subjects. Control subjects were allowed only 20 solution attempts; perhaps this was not enough to truly exhaust the domain of dot-to-dot solutions. Therefore, if control subjects had been given considerably more attempts to solve the problem, they might have spontaneously gone outside the dots and solved the problem. Furthermore, it might be argued that perhaps the "exhausted" hint was only minimally effective because the "exhausted" subjects were permitted only 10 attempts before the hint was given. Again, if these subjects had first had a chance to truly exhaust the domain of dot-to-dot solutions, the "exhausted" hint might have then been much more effective. The present experiment examined these questions by giving one group of subjects 100 control attempts to solve the problem before giving them the "exhausted" hint.

Method

Materials and procedure. Materials and procedure were identical to those of the "exhausted" group in Experiment 1, with the following exceptions. Subjects were given 100 solution attempts before the "exhausted" hint was given, and were allowed a maximum of 20 solution attempts following the hint. Also, subjects were tested in small groups rather than individually, as in Experiment 1.

Subjects. Sixteen Temple University undergraduates fulfilling an optional requirement for introductory psychology served as subjects.

Results and Discussion

The results were very similar to those of Experiment 1. Of the 16 subjects, 4 were familiar with the problem, and will be considered later. Of the 12 naive subjects, none solved the problem within the 100 control attempts, although all dot-to-dot possibilities were exhausted by these 100 attempts. Thus, the nine-dot problem is difficult because the domain that contains the correct solution essentially does not exist for most subjects.

Furthermore, even after the initial domain had been completely exhausted, the outside hint was no more effective than it was in Experiment 1. Of the 12 naive subjects who were given the "exhausted" hint, 3 (25%) solved the problem within the 20 attempts that followed. Also, the 3 subjects who solved after the hint needed a mean of 10.7 attempts to do so, which again indicates that the hint produced no sudden insight into the solution to the problem.

The results for the four nonnaive subjects were also similar to those for Experiment 1. One nonnaive subject solved the problem almost immediately, and reported later that he had become familiar with the problem and solution relatively recently. The other three nonnaive subjects did not solve the problem within the 100 control attempts, and only one of them solved the problem after the "exhausted" hint, taking 6 additional attempts to do so.

The results of this experiment further demonstrate the difficulty of the nine-dot problem and the relative ineffectiveness of the "exhausted" hint, contrary to Gestalt theory. The difficulty of the problem seems to involve three factors: the lack of any further possibilities when the original dot-to-dot solution domain is exhausted, the large number of possibilities outside the square if that domain is made available via a hint, and subjects' ignorance of how to "search" this domain.

Experiment 1b

One incidental finding from Experiments 1 and 1a was that subjects who professed familiarity with the nine-dot problem still had great difficulty solving the problem. At first this seems to indicate that solutions to insight problems are not retained particularly well, contrary to the claims of Gestalt theory. However, since we knew nothing about the previous experiences that the subjects in Experiments 1 and 1a might have had with the problem, we carried out a more systematic examination of subjects' memory for the solution to the nine-dot problem.

Method

Procedure. The nine-dot problem was presented as an exercise to a class of psychology majors as part of a class discussion on problem solving in a course on cognitive processes. There were approximately 50 students present, and they were asked to draw the nine dots and were given 5 min. to work on the problem without any hints. No student solved the problem in this time.

The "exhausted" hint was then given, and approximately 2 more minutes were allowed for additional attempts to solve the problem. Once again, no subject solved the problem, so the solution was demonstrated. All the students copied the solution.

The discussion then turned to several other problems and concluded with a critical analysis of Gestalt theory. Nothing further was said about the specific solution to the nine-dot problem. The following semester, 12 of these students were attending a laboratory course in cognitive processes, and during a break in the class they were unexpectedly asked to draw the dots and to try to solve the nine-dot problem. They were given 4 min. to recall the solution. The subjects were then asked to describe as best they could what had happened when they were asked to recall the solution. Approximately 5 months had elapsed between initial presentation of the problem and the subsequent unexpected test.

Results and Discussion

The results supported those found for Experiments 1 and 1a concerning the forgetting of the solution to the nine-dot problem. Of the 12 subjects tested, 5 could not recall the solution within 4 min., and the median solution time to recall the solution for the remaining subjects was 170 sec. It should be noted that someone who knows the nine-dot solution well can reproduce the solution in less than 5 sec. Furthermore, the subjects' descriptions of their recall attempts indicated that they reconstructed their solutions from some aspects of the problem that stood out in memory, in a manner very similar to that which occurs in the recall of prose materials (Weisberg, 1980, chap. 3). Here are two recall descriptions, one from a subject who recalled the solution and one from a subject who did not, which make the reconstructive process clear.

Solver: I remembered that the solution was an arrow going towards one corner. At first I was extending the initial or first line [i.e., Line 1 in Figure1], but not the other lines. Then, after doing that about 5 times, I realized to extend the other lines as well, and then I was able to solve the 9-dot.

Nonsolver: I knew that the solution to the problem involved somehow going outside the lines of the square from solving it before. However, I could not remember exactly how the order went, so I found myself making these long lines outside the square.

Again, these descriptions point to great similarities between long-term recall of the solution to the nine-dot problem and recall in other domains.[2]

Experiment 1c

Memory for the solution to the nine-dot problem over a shorter interval was tested in one further experiment. The subjects were again given a larger number of attempts than were permitted in Experiment 1, in an attempt to provide still further confirmation of the basic difficulty of the problem.[3]

Method

Stimulus materials. The problem was presented as in Experiment 1.

Procedure. The subjects were tested in small groups. They were given the instructions and then worked through 36 solution attempts at their own pace. If a subject did not solve the problem in these 36 attempts, the solution was demonstrated by the experimenter and the subject was excused. One week later the subjects returned for a second session, and they were again asked to solve the problem (i.e., to recall the solution). Thirty-six solution attempts were again provided.

Subjects. Thirty-three undergraduates at Dickinson College fulfilling an optional requirement in introductory psychology served as subjects.

Results and Discussion

The problem again proved to be very difficult. Only 3 of the 33 subjects solved the problem in the first session, and they needed a median of 8 attempts to do so. It should be noted that the groups of subjects in this experiment were working at their own pace, and some of them took approximately 1 hr. to complete their 36 attempts. This is much more time than the individual subjects in Experiment 1a took to complete 100 attempts, indicating that subjects in the pres-

[2] There is one potential difficulty concerning the interpretation of the recall results in Experiments 1b and 1c. Since almost none of the subjects in these experiments solved the problem when it was initially presented, we were testing for recall of a solution *given* to the subjects rather than a solution discovered by the subjects themselves. This might make the present experiments irrelevant as a test of the Gestalt view. In response to this, it should be noted that presenting the solution to the subject still produces a reorganization of the material in the problem, as attested by subjects' comments. If reorganization is the crucial event, the present experiments are relevant as tests of the Gestalt view.

[3] Thanks are due Walter Chromiak for his assistance in conducting this experiment.

ent experiment in actuality made many more than 36 attempts to solve the problem.

Only 23 of the 33 subjects returned for the second session, and 8 of the 23 were unable to recall the solution, even when given 36 additional attempts. Of the 15 subjects who recalled the solution, 10 did so in one attempt. However, as noted earlier, these subjects worked at their own pace, and the experimenter noted that at least half of the people who solved in one attempt first thought about the problem for a significant amount of time before committing anything to paper. Therefore, the forgetting found in this experiment is even more impressive because of the conservative nature of the results.

Of the 13 returning subjects who did not recall the solution on their first attempt, 6 went outside the square on their first recall attempt, and 5 of these 6 ultimately succeeded in recalling the solution. The remaining 7 subjects did not go outside the square on their first recall attempt, and none of them recalled the solution. Thus, these latter 7 subjects (almost one third of the people tested for recall) forgot the basic insight on which the solution is based, according to Gestalt theory.

To summarize, we found in four experiments that the nine-dot problem was very difficult for naive subjects, and that it was made only slightly easier by telling the subjects to go outside the boundaries of the square. Also, the solution to the problem seemed to have been forgotten in a manner not unlike the manner in which other information is forgotten.

Experiment 2

In Experiment 2, we attempted to give subjects training in solving dot problems by going outside the shape of the dots, thereby providing the subject with some additional knowledge to apply in solving the nine-dot problem. Two sets of training problems were constructed, as shown in Figure 2. The control series (panel A in Figure 2) contained two problems that could be solved by directly following the dots. The "outside" series (panel B in Figure 2) contained two problems that could be solved only by going outside the pattern of dots. It was expected that these "outside" training problems would be solved relatively easily and would lead to an easier solution to the nine-dot problem. Two "outside" groups were tested, with one receiving the standard nine-dot problem and the other receiving the 45° rotation. This latter condition was of interest because the solution from the second training problem could be superimposed directly on the rotated form of the nine-dot problem.

Method

Materials. The training problems were presented on sheets of paper, as was the nine-dot problem.

Procedure. Each subject was tested individually. Subjects were given instructions comparable to those given in the introduction (Figure 1), except that the number of lines needed was changed to correspond to the specific training problem in question. Each subject was allowed a maximum of 10 min. to solve each problem, and two training problems and the nine-dot problem were presented to all subjects, regardless of whether both training problems were solved. Solution time was recorded to the nearest minute.

Subjects. Sixty-one Temple University undergraduates fulfilling an optional requirement in introductory psychology served as subjects.

Results and Discussion

The results are presented in Table 2. The control group performed as expected: They easily solved the two training problems, but none of them solved the nine-dot problem. As in Experiments 1, 1a, 1b, and 1c, solution attempts outside the borders of the square were very rare.

One difference between this experiment and Experiment 1 must be pointed out. In Experiment 1, control subjects were given 20 attempts to solve the problem, whereas in the present experiment they were given 10 min. Our informal observations of the control subjects in Experiment 1 revealed that 20 attempts usually took less than 10 min. to carry out, so the present subjects had at least as much time as those in Experiment 1.

The results of the "outside" training conditions were somewhat surprising. First of all, as in Experiment 1, the orientation of the nine-dot problem made no difference, so the two "outside" groups were combined for further analysis. The unexpected finding was

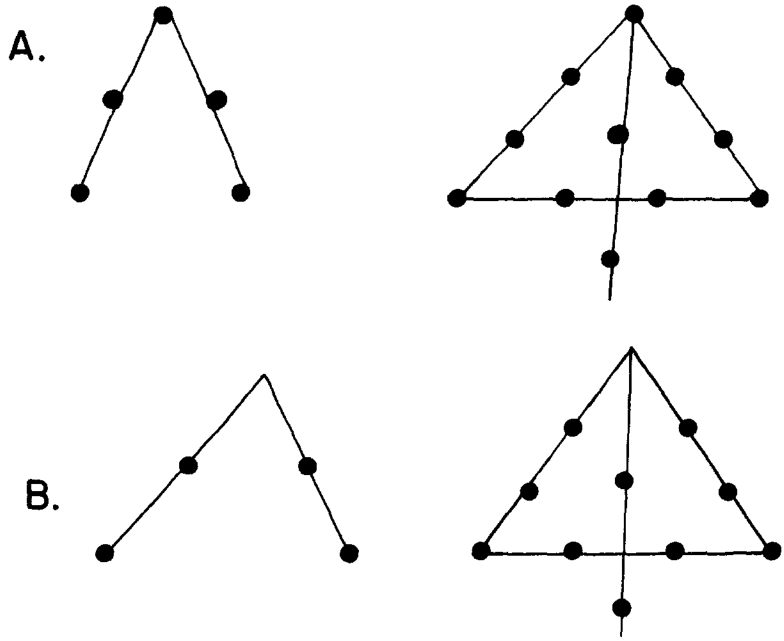

Figure 2. A: Control training series. (The first problem is to be solved with two connected straight lines; the second problem is to be solved with four connected straight lines. Solutions are shown as connected lines.) B: Outside training series: Two- and four-line problems.

that the four-dot "outside" training problem was not easy. In order to obtain 15 subjects in each of the two "outside" groups who had solved both training problems, we had to test a total of 46 subjects. To put it another way, approximately 1 subject in 4 did not solve the four-dot problem, and approximately 1 subject in 3 did not solve both "outside" training problems. The difficulty of the four-dot "outside" problem turned out to be helpful, as will be seen in Experiments 3 and 4. For the moment, let us concentrate on performance on the nine-dot problem.

Of the 30 "outside" training subjects who solved both training problems, 43% solved the nine-dot problem. The difference between the combined "outside" training groups and the control group was significant

Table 2
Influence of Training on Performance on Nine-Dot Problem in Experiment 2

	Median solution time (in min.)			Proportion of subjects solving		
Training condition	Training Problem 1	Training Problem 2	Nine Dot	Training Problem 1	Training Problem 2	Nine Dot
Control[a]	1.0[b] (1–2)[c]	1.5 (1–8)	10	1.0	1.0	.00
Outside[d]	4.5 (1–9)	2.4 (1–9)	8.5 (1–10)	1.0	1.0	.43
Outside[e]	5.7 (1–10)	5.5 (1–10)	10 (1–10)	.74	.63	.28

[a] Fifteen subjects in control group.
[b] Subjects stopped after 10 min. if no solution.
[c] Ranges are given in parentheses.
[d] Thirty subjects who solved both outside training problems.
[e] All 46 subjects given outside training regardless of whether they solved both training problems.

when numbers of solvers of the nine-dot problem were compared, $\chi^2(1) = 9.13$, $p < .002$, one-tailed. However, there is a possible subject-selection artifact here, in that only the more proficient subjects might have solved the "outside" training problems. In order to deal with this difficulty, the data were analyzed again, this time with no subjects eliminated from the "outside" groups. The difference between the control and outside groups was still significant, $\chi^2(1) = 5.40$, $p < .02$, one-tailed. It should also be noted that almost all of the subjects in the "outside" group did indeed go outside the square on the nine-dot problem, but only 28% solved the problem. Thus, we significantly facilitated solution to the nine-dot problem by giving subjects training in solving dot problems by going outside the lines, although the facilitation was not overwhelming.

Of the 30 subjects who solved both "outside" training problems, 10 went outside the square on their very first attempt at the nine-dot problem and 20 did not go outside on their first attempt. Of the former subjects, 70% ultimately solved the problem, whereas only 35% of the latter subjects did. This difference is significant, $Z = 1.81$, $p < .04$, one-tailed, and it indicates that some specific information was acquired by some subjects which they began to apply immediately to the nine-dot problem.

This experiment produced two important results. First, we were able to facilitate solution of the nine-dot problem by giving training in solving similar problems. Second, this facilitation points to a basic similarity between performance on the nine-dot problem and on several other problems discussed in the literature. For example, it has been argued that performance on Duncker's (1945) candle problem depends on the transfer of past experience from similar situations (Weisberg, 1980, chap. 10), and a similar argument has been made concerning concept formation problems (Levine, 1975; Sweller & Gee, 1978; Weisberg, 1980, chap. 11).

Experiment 3

As indicated earlier, the four-dot "outside" training problem was harder than expected. However, this turned out to be helpful, because the four-dot problem seems to be influenced by the same factors that influenced the nine-dot problem, though to a less extreme degree. Subjects initially tried to solve the four-dot problem by going from dot to dot. Most of them ultimately solved the problem by constructing a solution outside the pattern. We would like to be able to specify more precisely why this problem presents difficulty to subjects. The present experiment tried to answer this question by instructing subjects to go outside the dot pattern in order to solve the problem. If this instruction produced quick solutions, it would indicate that most of the difficulty with this problem lay with the initial solution domain or domains suggested by the problem.

Method

Materials. The four-dot problem was presented on a single sheet of paper, as in Experiment 2.

Procedure. Subjects were tested individually. The control group was given standard instructions, whereas the "exhausted" group was instructed at the outset that the only way to solve the problem was to go outside the pattern of dots.

Subjects. Thirty Temple University undergraduates fulfilling an optional requirement in introductory psychology served as subjects. They were randomly assigned to conditions, with the restriction that 10 subjects would be in the control group and 20 in the "exhausted" group.

Results and Discussion

The results are presented in Table 3, and they show facilitation brought about by the

Table 3
Performance on Four-Dot Problem in Experiment 3

	Condition	
Variable	Control (n = 10)	"Exhausted" (n = 20)
Mean solution time (in min.)—all subjects	6.5[a] (1–10)[b]	1.0 (1–4)
Median solution time (in min.)—solvers only	3.0 (1–7)	1.0 (1–4)
Proportion solving	.60	1.00

[a] Ten-min. maximum.
[b] Ranges are given in parentheses.

verbal instruction. All of the "exhausted" subjects solved the problem quickly, with over half of them taking less than 2 min. to do so. On the other hand, the control subjects had much more difficulty with the problem. Several did not solve it, and those who did took longer than the "exhausted" subjects. The solution time data for the two groups were compared using a median test, and a significant difference was found, $\chi^2(1) = 5.40$, $p < .02$, one-tailed.

Based on the results from the "exhausted" group, we can draw some tentative conclusions concerning the performance of the control group. It seems that if the control subjects had simply thought of going outside the outlines of the figure, they would probably have solved the problem very quickly, since they would then have been in the same situation as the "exhausted" group, and the latter subjects had little difficulty with the problem. Therefore, the reason it took the control subjects so long to solve the problem was, perhaps, that it took them some time to realize that the answer lay in going outside the figure.

In order for a control subject to realize that the solution to the problem lay outside the boundaries of the figure, several events probably had to occur. First, the subject had to realize that no dot-to-dot solution would solve the problem, and exhausting all the possibilities took some time. Even if the subject realized that the dot-to-dot possibilities were exhausted, however, there were still one or two other factors that could slow down the realization that one had to go outside the figure. Since all subjects initially believed that the problem was a simple connect-the-dots problem, the inadequacy of dot-to-dot solutions would have taken them somewhat by surprise and would have led to reanalysis of the situation. That is, the subjects became aware that there was an unexpected trick in finding the solution to the problem, that the problem was not what it seemed to be, and they then tried to determine what the trick was. Observation of our subjects indicated that, at this point, some subjects tried to solve the problem by changing their interpretation of some aspect of the problem as presented. For example, some subjects tried to trace over a line that they had already drawn to avoid lifting their pencils from the paper, they used slightly curved lines, and so on. These subjects assumed that the trick lay in the way the instructions were to be interpreted. When they were told that these variations were not acceptable, most of them then got to the solution. It should be emphasized that it is not necessary to assume that some additional process, such as "spontaneous insight," is required to explain these results. It is simply that the problem is designed so that there are several solution domains that are sampled before the subject gets around to the correct one.

Experiment 4

On the basis of Experiment 3, it was concluded that most of the difficulty with the four-dot problem came about because subjects tried to apply their past experiences with dot puzzles to the problem. Most people's past experiences with dot puzzles are such that they interfere with solutions to the present problem. If subjects have had relevant past experiences, solution of the four-dot problem should be facilitated.

The present experiment was undertaken to facilitate solution of the four-dot problem by first giving subjects a problem using the same solution shape. The training condition is shown in Figure 3. The training problem involves simply drawing two lines to connect all the dots. The subject need only follow the line of dots in order to solve the problem. The overall shape of the solution is the same for the training problem and the four-dot problem, and the configuration of dots is also similar for the two problems. It was expected that these factors would produce significant facilitation relative to a control group that saw only the four-dot problem.

Method

Materials. The problems were presented as in the foregoing experiments.

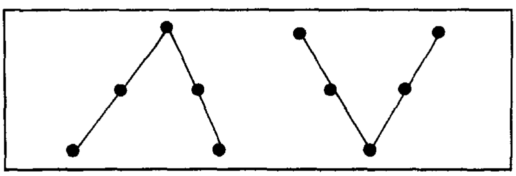

Figure 3. Two training conditions for Experiment 4.

Table 4
Influence of Training on Four-Dot Problem in Experiment 4

	Condition	
Variable	Control (n = 20)	Shape training (n = 35)
Median solution time (in min.)	2.5[a] (1–10)[b]	1.0 (1–10)
Proportion solving	.80	.91

[a] Ten-min. maximum.
[b] Ranges are given in parentheses.

Procedure. Subjects were tested individually, as in earlier experiments. Assignment to conditions was random, with the restriction that approximately twice as many subjects would be in the training group as in the control group. Instructions for the training group indicated that the training problem could be solved with two connected lines. The control group was given no training problem. The training and control groups were given the same standard instructions for the four-dot problem.

Subjects. Fifty-five Temple University undergraduates fulfilling a requirement in introductory psychology served as subjects.

Results and Discussion

The training problem was solved in less than 1 min. by all subjects. The results for the four-dot problem are presented in Table 4, and they indicate that training produced a significant degree of facilitation on the four-dot problem. The problem was solved significantly faster by subjects who had previous experience solving an easy problem with a similar configuration of dots and a similar-shaped solution. Of the 35 subjects in the training group, 23 solved the four-dot problem within 2 min. The solution time data were analyzed by a median test, and the training subjects solved the four-dot problem significantly faster than the control subjects did, as predicted, $\chi^2(1) = 4.42$, $p < .02$, one-tailed.

The training problem, which required the subjects to stay within the outline of dots, did not further interfere with solution of the four-dot problem, in which subjects must go outside the dots. This indicates that the difficulty in this problem may not simply be the result of fixation on the shape of the configuration, since the training subjects left the shape more quickly after a training problem that required them to follow a shape. The reason subjects stayed within the borders of the figure in the nine- and four-dot problems may be that their past experiences directed them to go from dot to dot, which resulted in their staying within the figure. That is, this may not be a perceptual effect at all in the sense that the shape of the dots did not "hold" the subject within it. Rather, the reason subjects stayed within the square on the nine-dot problem is simply that they went from dot to dot. Again, these results are important because they emphasize the role of transfer of past experience from an old problem to a new one, which has been emphasized recently in discussions of other problems (Levine, 1975; Sweller & Gee, 1978; Weisberg, 1980).

Experiment 5

In Experiment 4, solution of the four-dot problem was facilitated by providing subjects with relevant past experience in the form of the shape of the solution. The present experiment attempted to do the opposite, that is, to slow down solution of the four-dot problem. This was done by giving subjects a training problem in which a dot-to-dot solution solved the problem. However, because of the shape of the training problem, the solution cannot be transferred to the four-dot problem. The training problem is presented in Figure 4. It was expected that the experience with the training problem would set subjects to work within the domain of dot-to-dot solutions on the four-dot problem, thereby slowing down solution to the problem.

Method

Materials. The problems were presented as in Experiment 4.

Procedure. The procedure was identical to that in Experiment 4, with the training group first solving the

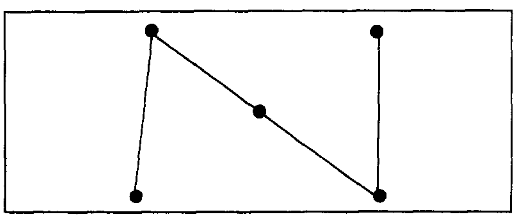

Figure 4. Negative training condition for Experiment 5: Connect the dots with three straight lines.

problem in Figure 4. As in Experiment 4, the control group did not receive a training problem.

Subjects. Fifty Temple University undergraduates fulfilling an optional course requirement in introductory psychology served as subjects.

Results and Discussion

As expected, the dot-to-dot training problem was trivially easy, with all subjects solving it in less than 1 min. The results for the four-dot problem are presented in Table 5, and they indicate that negative transfer did occur. The dot-to-dot group took longer to solve the problem, and more subjects in this group did not solve the problem within the allotted time. Statistical analysis indicated that the difference in number of solvers was significant, $\chi^2(1) = 4.37, p < .02$, one-tailed. The difference in solution times just missed being significant, $Z = 1.57$, $p = .058$, one-tailed, by the Mann-Whitney U test. It seems reasonable to conclude, therefore, that interference was produced by past experience with a dot-to-dot solution that had a shape that could not be transferred to the four-dot problem.

Experiments 4 and 5 demonstrate that a dot-to-dot solution from an earlier problem can sometimes facilitate solution of the four-dot problem, but it can also sometimes interfere with it. The crucial factor seems to be whether the shape of the specific solution can be transferred to the new problem. This indicates that subjects may transfer relatively specific information from their past experiences to a new problem, as was expected from the framework that stimulated these experiments.

Conclusions Concerning the Nine-Dot Problem

Let us now return to the problem with which we began, the nine-dot problem, and summarize our conclusions. It will be helpful to organize the discussion around the points raised earlier in considering the traditional description of the problem.

1. *The nine-dot problem is an easy problem that is made difficult by the subject's unwarranted assumption.* This statement is incorrect on several grounds. First, eliminating the assumption makes the problem only slightly easier. Second, the subject's assumption is not unwarranted. The subject does not know that this problem cannot be solved in a connect-the-dots manner, so the subject cannot know that past experience will be irrelevant until it has been used. Again, it is important to point out the similarity of this reasoning to that applied to other sorts of problems in the literature (e.g., Levine, 1975; Sweller & Gee, 1978). This indicates that "insight" problems may not be the unique situations they are sometimes assumed to be.

2. *When the fixation is broken, solution should come quickly and easily, as in a flash of insight.* This statement is also incorrect. Even when fixation is broken and all subjects go outside the square, only a minority of subjects solve the nine-dot problem. Also, those subjects who do solve the problem under these conditions do so relatively slowly, by working out the solution one step at a time. Thus, we do not have the spontaneous falling into place of a solution, as the term *insight* suggests. In the present experiments, it seems more accurate to say that the outside hint simply opens up a large domain of possible solutions, which subjects work through on a step-by-step basis, based on their experience with similar situations and/or on the logic of the situation. Furthermore, a large majority of subjects have basically no idea how to "search through" this domain, probably due to a lack of relevant past experience. This again points to the basic difficulty of the nine-dot problem.

3. *Solution of such problems is relatively independent of task-specific knowledge.* Once again, this statement is incorrect. The only way to produce anything approaching

Table 5
Performance on Four-Dot Problem in Experiment 5

Variable	Condition: Control ($n = 25$)	Condition: Dot to dot ($n = 25$)
Median solution time (in min.)	5.75[a] (1–10)[b]	8.50 (1–10)
Proportion solving	.80	.52

[a] Ten-min. maximum.
[b] Ranges are given in parentheses.

an insightful solution to the nine-dot problem is to give the subject detailed information about the solution. This detailed information cuts down the domain of solutions through which the subject must then work.

4. *Solutions to "insight" problems will not be forgotten.* Again, the present experiments have demonstrated that solutions to "insight" problems can be forgotten, and have also demonstrated that recall of the solution to the nine-dot problem involves processes similar to those involved in the recall of other sorts of material.

Experiment 6

Based on the analysis of dot problems, it seems that the crucial factors that make an "insight" problem difficult are the following: (a) The problem and instructions direct the subject to work within a domain that does not contain the correct solution. (b) The subject cannot tell that this domain does not contain the solution. (c) If and when this domain is exhausted, there may be no other domain that is available to the subject.

Given this analysis, consider the following hypothetical solution. A subject is given a problem that seems to have a straightforward solution, but when the subject tries the solution, it does no work. The correct solution actually requires a "reorganization" of the problem. Thus, we have a typical insight problem. However, assume further that the structure of this hypothetical problem is such that the subject can immediately see that the seemingly obvious solution will not work and that there are very few if any variations on the obvious solution that are possible. That is, the domain of incorrect solutions called forth by the problem is very small and easily exhausted. Finally, assume that the design of the problem makes obvious the general form of the correct solution once the subject realizes that the seemingly obvious solution is incorrect. That is, the alternative domain is obvious and also limited. According to our analysis, such a problem should be very easy to solve. Subjects should first produce the obvious but incorrect solution and should then immediately modify it in the correct way to solve the problem. This hypothesis was tested by accident using another of Scheerer's (1963) problems, the horses-and-riders problem, shown in Figure 5.

Scheerer reports that this problem is very difficult because subjects are fixated on putting each rider on one of the horses in the picture. However, as shown in Figure 5, this solution will not work. According to Scheerer, his subjects kept on trying to get the riders on the existing horses. As with the other problems he discusses, Scheerer presents no data. Initially, we assumed that the problem would also be difficult for our subjects, so two conditions were prepared. The control group was presented with the problem as in Figure 5, panel A. A second group was given instructions designed to overcome the expected fixation, as was done with the dot problems. These unfixated subjects were told: "With these two pictures it is possible to construct a new picture which contains two horses, with a rider on each." It was expected, based on Scheerer's (1963) discussion, that the control subjects would have trouble with the problem, so the question was whether the special instructions would be facilitative.

Method

Materials. Drawings A and B in Figure 5, panel A, were mounted on cardboard. Drawing A was 15 cm square.

Procedure. The subjects were tested individually. Each subject was given the problem as shown in Figure 5, panel A, with the appropriate instructions. The subjects simply manipulated the two drawings, and time to solution was recorded.

Subjects. Twelve Temple University undergraduates fulfilling a requirement in introductory psychology served as subjects.

Results and Discussion

Our initial expectations were totally incorrect. The problem was very easy for the control subjects, with 5 out of the first 6 solving in less than 2 min. The unfixated subjects performed equivalently. The overall median solution time for all 12 subjects was less than 1 min. Initially we expected to test 20 subjects in each group, but it was obvious that the problem was very easy, so we stopped early.

It is unclear why Scheerer's subjects found the problem so difficult but ours found it so easy. In retrospect, it seems reasonable that this should be so. First, subjects place picture

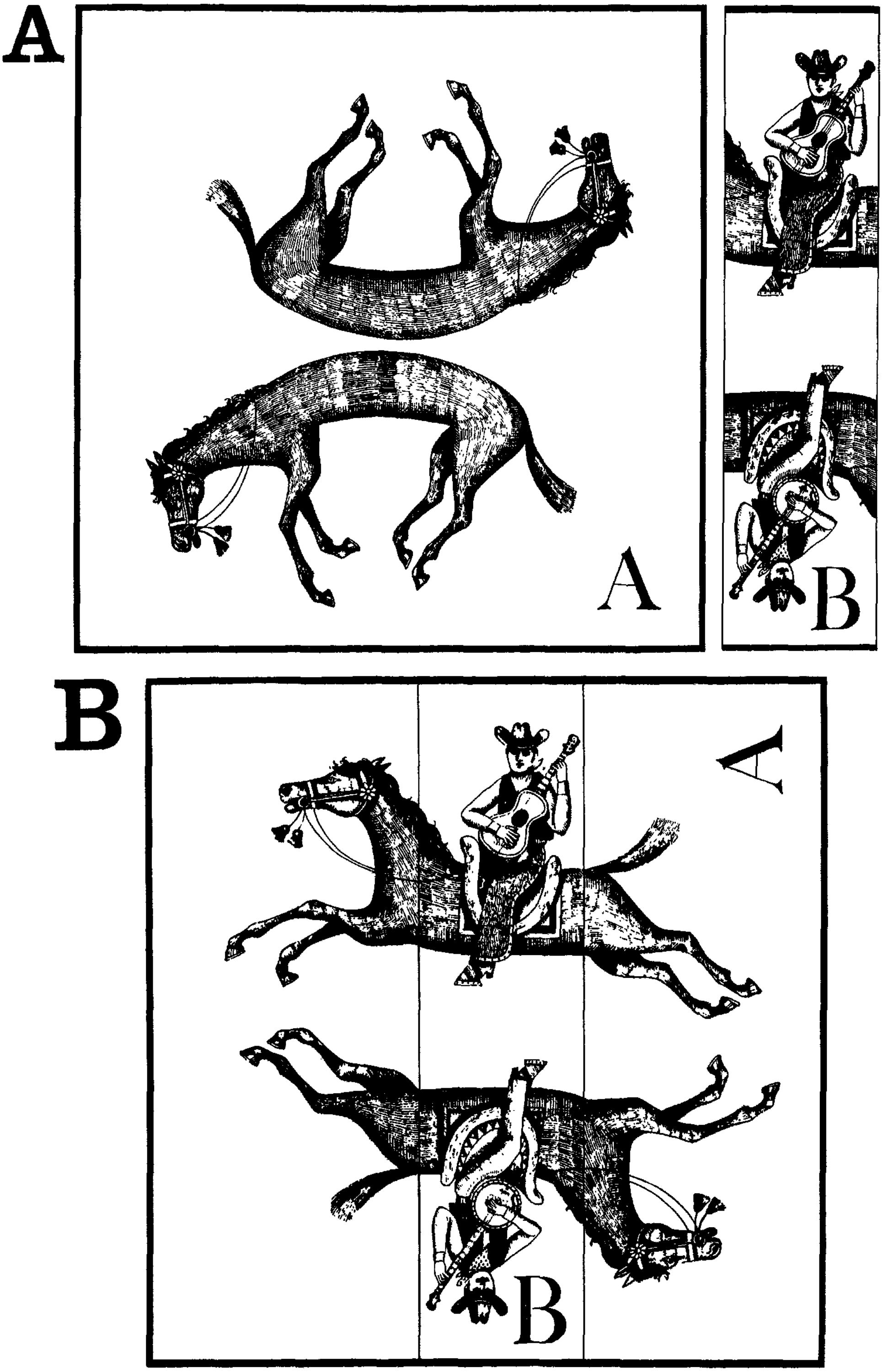

Figure 5. A: Horse-and-rider problem. B: Solution. (From "Problem Solving" by M. Scheerer, *Scientific American,* 1963, *208*(4), 118–128. Copyright 1963 by Scientific American, Inc. Reprinted by permission.)

B on picture A so that each rider ought to be on a horse, but this does not work. This exhausts the domain of initial incorrect solutions. The alternative domains that are then available are extremely limited because the instructions say, "Place picture B on picture A." That is, they solve the problem by rearranging B on A until they hit on the solution by trial and error.

The present results indicate that when a problem is unambiguous and the subject's options are clear, there is no fixation on an initially chosen solution. The fact that a subject is initially "misdirected" does not mean that he or she will stay misdirected.

Experiment 7

The last problem to be considered in this paper is the triangle problem, shown in Figure 1. This problem is similar in structure to the nine-dot problem, and has been interpreted in a similar way. Supposedly, subjects are fixated on triangles in two dimensions and never come to the insight that constructing a tetrahedron would solve the problem. If this reasoning is correct, instructing subjects to work in three dimensions ought to produce insight and greatly facilitate solution. However, if we once again consider the problem from the subject's point of view, this problem also should be a very difficult one. First, the domain of possible two-dimensional combinations of the six sticks is very large. Therefore, the subject might never realize that two-dimensional solutions will not work. Second, even if such a realization is made, it does not tell the subject what to do; again, an alternative domain may not exist for some subjects. Therefore, the subject ought to have to work out the three-dimensional solution after being told about it, rather than having it be immediately obvious. The only way a subject should solve this problem relatively quickly, from this point of view, is if he or she is familiar with the geometry of tetrahedrons. If so, such information could be cued, either by the goal of the problem or by a hint, and would then serve to direct further solution attempts. The situation parallels that with the nine-dot problem.

This reasoning was tested by instructing some subjects (the 3-D group) that the triangle problem was solvable only by working in three dimensions. It was expected that these instructions would not bring about a flash of insight in most subjects. In addition to the 3-D and control groups, a third group (the pyramid group) was instructed that the way to solve the problem was to construct a pyramid with a triangle as its base. This group was included to determine whether the 3-D instructions were maximally facilitative or whether additional specific information was needed, as we hypothesized.

Method

Materials. Six small sticks, each measuring 18 cm in length, were used.

Procedure. Each subject was tested individually. Assignment to conditions was random. The instructions to the control group were identical to those in Figure 1. The 3-D group was also told: "You will have to go into three dimensions. In other words, you will have to make a three-dimensional figure." The pyramid group's instructions were the same as the 3-D group's except that the last phrase was "make a pyramid with three sides and a base." Solution time was recorded to the nearest minute. A total of 10 min. was allowed, but subjects were not informed of this.

Subjects. Forty-five Temple University undergraduates fulfilling a course requirement served as subjects.

Results and Discussion

The results are presented in Table 6, and once again questions can be raised about the fixation explanation. The 3-D instructions facilitated solutions relative to control instructions, but they were not as facilitative as the pyramid instructions. (Also two subjects in the 3-D condition did not solve after receiving the hint.) This indicates that subjects either need to have specific knowledge about tetrahedrons available or have to construct a tetrahedron relatively slowly. This construction begins with trial-and-error arrangements of the sticks, although our subjects' familiarity with pyramids probably was helpful as they worked out the solution.

The data in Table 6 were analyzed in two ways. First, frequency of solution was considered, and a significant difference was found among the three groups, $\chi^2(2) < 20.0$, $p < .001$. The two experimental groups both differed from the control group, $p < .005$, by chi square, but did not differ from each other. Second, the solution time data were analyzed using a median test, and once again a significant difference was found among the three groups, $\chi^2(2) < 20.0$, $p < .001$. Furthermore, the pyramid instructions produced significantly faster solutions than the 3-D instructions, as predicted, $\chi^2(1) = 5.06$, $p < .02$, one-tailed.

These results indicate that subjects are not simply fixated in two dimensions, because even when they are disabused of this "assumption," quick and direct solution of the problem does not occur. The reason that solution is relatively slow with 3-D instructions may be related to the reason that solution is infrequent with control instructions: Even if subjects feel that they have exhausted the possibilities within two dimensions, they do not see any purpose in going to three dimensions. For example, on receiving the 3-D instructions, some subjects stood one stick on the table, thereby going into three dimensions, and then said, "Now what do I do?" They would then try to put the other sticks together with this one until something that looked reasonable was produced. The whole procedure was very inefficient and smacked of trial and error. This pattern was seen in all of the subjects in the 3-D condition; although some subjects were quicker than others, all of them worked out the solution one step at a time, with revisions along the way. Thus, the 3-D instructions did not produce sudden insight into the problem; they simply opened up a new domain to be worked through. These results and conclusions parallel those from the nine-dot problem.

Table 6
Influence of Hints on Performance on Triangle Problem in Experiment 7

	Condition		
Variable	Control (n = 15)	3-D (n = 15)	Pyramid (n = 15)
Median solution time (in min.)	10[a]	3.0 (1–10)[b]	1.17 (1–3)
Proportion solving	.00	.87	1.00

[a] Ten-min. maximum.
[b] Ranges are given in parentheses.

Experiment 7a

It might be objected that our subjects were not given enough time in Experiment 7 for spontaneous solutions to occur. That is, perhaps 10 min. was not enough time for subjects to exhaust the incorrect solution domain. According to this viewpoint, if subjects had been given enough time to exhaust the two-dimensional domain, there might have been a significant number of spontaneous solutions, and the 3-D instructions would thus have been much more effective. In order to make a stronger claim concerning the difficulty of this problem, an additional group of subjects was first given 20 min. to solve the problem before the 3-D hint was given.

Method

Procedure. The procedure was identical to that in Experiment 7, except that subjects were given 20 min. to solve the problem before the 3-D hint was presented. The subjects were then given 10 more minutes to work on the problem.

Subjects. Ten Temple University undergraduates fulfilling an optional requirement in introductory psychology served as subjects.

Results and Discussion

Three of the 10 subjects solved the problem within 20 min. One such solver needed only 1 min. to solve the problem, and he solved the problem relatively directly, without trial and error. Postexperimental questioning revealed no familiarity with the problem. The other two spontaneous solvers needed 14 and 16 min., respectively. In addition, these latter two subjects exhibited the pattern of trial and error described in the discussion of Experiment 7. In sum, extending the time before the hint to 20 min. produced some additional solutions, but the facilitation was not universal. It should be emphasized that 20 min. is an extremely long time to work on the triangle problem, and the present subjects produced many incorrect solution attempts within that time. Thus, even though it cannot be flatly stated that still more time would have been of no further help, it seems reasonable to conclude that for many subjects the correct solution domain does not exist when the problem is presented with no hints.

Of the seven subjects who received the 3-D hint, three did not solve the problem in the 10 min. following the hint. The other four subjects required between 1 and 2 min.

to solve the problem, but once again all of them exhibited the trial-and-error pattern described above.

General Discussion

On Doing Away With "Insight" and "Fixation"

It has become accepted in psychology that certain problems are solved through insight and that fixation, in the form of unwarranted assumptions, interferes with this insight. An example of this was given above, in the quote from Rumelhart's (1977) introductory text on cognitive processes. The same acceptance of this point of view can be found in texts on introductory psychology (e.g., Krech, Crutchfield, & Livson, 1970). However, one important conclusion to come from the present experiments is that it would be better if we did not use the terms *fixation* and *insight* to describe the behavior of subjects on the "insight problems" that we have studied.

There are several objections that can be raised to these terms, on both logical and empirical grounds. First, as has been noted by others (e.g., Kintsch, 1977, chap. 7), insight and fixation are not really explanations, but are merely descriptions. In addition, there may be circularity involved in the use of the terms. For example, the only way we know that a subject was fixated was that the subject did not solve the problem. However, the reason that the problem was not solved was that the subject was fixated, which puts us in a circle. A parallel circularity concerns the use of insight.

Even if we ignore the logical difficulties with these terms, there are also empirical problems, as the present studies indicate. First, if fixation were based on unwarranted assumptions, removing these assumptions should result in easy solution. However, in both the nine-dot and the triangle problems, removing the alleged fixation does not result in all of the subjects easily solving the problem. Second, insightful solutions should come about quickly and directly, but in most cases they do not.

Also, one criterion of an insightful solution is that it will not be easily forgotten. However, in Experiments 1b and 1c, subjects with previous knowledge of the solution of the nine-dot problem had difficulty recalling it. In sum, several testable hypotheses that can be derived from the insight point of view were not supported in the present experiments.

Finally, there is one specific aspect of the use of the terms *insight* and *fixation* that deserves discussion. The term *fixation* has a negative connotation: One is fixated when one keeps doing something that one ought to stop doing. However, in order to go beyond the incorrect solutions based on one's past experience, one would first have to know that such solutions will not work. Thus, if a person is working on obvious solutions, *and if that person does not know that these solutions will not work*, that person is not fixated. As we have seen, one of the difficult aspects of these problems may be that it is not clear that the obvious solutions will not work, and it also may not be clear what else might work. In such cases, it is not surprising that subjects keep trying incorrect solutions.

It should be emphasized that there is more than simply a semantic quibble involved here. If one accepts "fixation" as an explanation for the lack of solution to a problem, it leads one to certain expectations about how such solutions can be brought about. An example of this comes from a recent discussion of insight in problem solving.

> This book is a careful selection of problems that seem difficult, and indeed are difficult if you go about trying to solve them in traditional ways. But if you can free your mind from standard problem solving techniques, you may be receptive to an aha! reaction that leads immediately to a solution. . . . We have tried to surround each puzzle with a pleasant, amusing story line intended to put you in a playful mood. Our hope is that this mood will help you break away from standard problem solving routines. (Gardner, 1978, p. vii)

Gardner thus feels that some general sort of flexibility of thought, which can be acquired relatively independently of the problem in question, can lead to solution. On the other hand, the present viewpoint argues that solution to a given problem depends first and foremost on a body of knowledge that can be brought to the problem. Therefore, the present viewpoint would predict that there should not be very general transfer from one "insight" problem to another if they are from very different domains. These differential predictions indicate that more than semantics is involved here.

An Alternative Viewpoint

In recent years, there has not been very much research in the area of insight problems. One reason for this is discussed in the introduction. After Scheerer's (1963) review article, it may have seemed to many psychologists that we knew all the important things about these problems. A second reason for this dearth of research may be that it was felt that exotic processes such as "fixation," "reorganization," and "insight" were involved in the solutions to insight problems. However, the present studies seem to indicate that the solution behavior for at least two of these problems can be understood in a straightforward manner: People apply their knowledge to new problems, and if their knowledge is not directly useful, they try to produce something new that will solve the problem through a straightforward extension of what they know. No exotic processes, such as sudden insight, are involved.

A similar analysis has recently been made of another insight problem, Duncker's (1945) candle problem (Weisberg, DiCamillo, & Phillips, 1978; Weisberg & Suls, 1973; Weisberg, Note 1). In the candle problem, the subject is to attach a candle to a wall: the available objects include a box of tacks. Duncker was particularly interested in the use of the tack box as a candleholder because he felt that such a solution required the subject to overcome fixation on the box's function as a container for the tacks. Not all subjects think of using the box as a candleholder, and those subjects who do produce a candleholder often do so as a late solution. According to Duncker, failure to use the box as a candleholder is the result of fixation on the box's function. However, an alternative explanation is that subjects begin by contemplating tacking the candle to the wall because such an operation would directly meet the requirements of the problem. Only if such a solution is unsuccessful will subjects attempt to modify it, which is where the box comes in. Thus, the candle problem is also a problem that is designed so that subjects try certain solutions but must reject these solutions before the "insightful" solution can be produced. Once again, we may not need to refer to special processes, such as "fixation" and "insight," to understand why some solutions are attempted before others in these problems.

Other investigators have also argued that other alleged "insight" problems are also solved through the relatively straightforward application of past experience and that fixation and insight are not involved (e.g., Saugstad & Raaheim, 1957; Weaver & Madden, 1949).

Furthermore, we have seen that the present results can be understood, at least in general terms, using a theoretical framework that has had widespread acceptance among cognitive psychologists: hypothesis theory. Thus, there seems to be a way to bring the study of insight problems into the mainstream of cognitive psychology—and perhaps bring about a unity that has been lacking in the area of problem solving.

Generality of the Present Conclusions

In the present article, we have referred to "the Gestalt viewpoint" without explicitly discussing the views of any theorists other than Scheerer. However, the importance of fixation brought about by past experience was also emphasized by other major Gestalt theorists concerned with problem solving. Let us consider the more modern theorists first. Maier (1930) briefly discussed both the nine-dot and the triangle problems in a manner almost identical to that of Scheerer. Also, in one of his last written works on problem solving, Maier (1970, p.5) distinguished between a situation in which past experience can be applied to a new situation (which Maier considers a learning situation) and a true problem-solving situation, in which past experience must be "spontaneously reorganized." Finally, in many places Maier (e.g., 1930) talks of "habitual directions," which can interfere with the ultimate solution to a problem.

Duncker (1945) discussed the triangle

problem in terms very similar to those of Scheerer. In addition, Duncker spent some time studying situations in which past experience allegedly interferes with productive thinking. Duncker's notion of functional fixation has been explicitly criticized elsewhere (Weisberg, 1980, chap. 10; Weisberg & Suls, 1973) on grounds similar to those raised in the present paper against Scheerer's notion of fixation.

Luchins (e.g., Luchins & Luchins, 1959) tested many thousands of subjects on water jar problems in order to demonstrate the blinding effects of past experience on problem solving. This work is explicitly cited by Scheerer as an example of how past experience can make subjects so "blind" to a simple solution to a problem that they behave mechanically, without thinking. Although we have not examined water jar problems here, if the present negative analysis of insight and fixation is correct, it should be possible to explain the behavior of subjects on water jar problems without recourse to these notions. Such an interpretation has been proposed elsewhere (Weisberg, 1980, chap. 12).

Among the earlier Gestalt theorists, precursors of Scheerer's theorizing can be clearly seen. In his discussion of productive thinking, Wertheimer (1959, p. 73) talks of the "structural reorganization" required for solution to many problems. He also plays down the positive role of past experience, and at one point he states: "[I]t is my impression . . . that children can be able to find out, by actual reasonable working at a problem, just what is needed—without special previous experience. They find the needed experiences for themselves, in a reasonable way" (p. 91).

The present results do not support this assertion with regard to the performance of undergraduates on the nine-dot and triangle problems. Wertheimer (1959, p. 84) also argued that an environmental atmosphere fostering freedom, friendliness, and optimism helped in the solution of problems requiring insight, which is echoed in the quote from Gardner (1978) presented earlier. Finally, Wertheimer cites the work of such younger psychologists as Maier, Duncker, and Luchins as supporting his viewpoint. In sum, Scheerer's theorizing seems to be an extension of Wertheimer's, and therefore the present criticism of Scheerer would also be relevant to Wertheimer. In addition, the work of Köhler (e.g., 1969, chap. 4) also seems to be a direct precursor of Scheerer, and the present criticisms are applicable to him also. In many places Köhler (e.g., 1969, chap. 4) discussed the interfering effects of past experience as an explanation for the lack of a solution to a problem. Köhler also emphasized the spontaneous reorganization of a problem independent of experience (1969, chap. 4).

In conclusion, there seems to be a reasonably unified body of work that could be called the Gestalt view, and therefore the present analysis has broader relevance than solely to the theorizing of Scheerer.

Solving the Nine-Dot Problem

As a final point, it would be of value to try to relate the present work to other recent work in solving "move" problems, such as the missionaries-and-cannibals problem and the towers-of-Hanoi problem (e.g., Greeno, 1974; Jeffries, Polson, Razran, & Atwood, 1977). In the missionaries-and-cannibals problem, for example, the subject is told that three missionaries and three cannibals are on the bank of a river that they wish to cross, and they have a boat that can carry no more than two people at any time. The problem is to get the missionaries and cannibals across the river using the boat as transportation without at any time having the cannibals outnumber the missionaries on either bank of the river. Thus, the problem is to get everyone ferried across the river without losing any missionaries.

One can describe this problem and other "move" problems by specifying the initial state and the various "paths" that can be constructed by carrying out legal combinations of moves from the initial state. Some of these paths lead to the goal state, and thus are the solution paths. Recent attempts to explain the behavior of subjects on "move" problems have argued that at each state the subject chooses the next move by evaluating the moves that are possible and picking that move which produces the state closest to the goal. In order for this to occur, it is assumed that subjects are able to rank problem states on the basis of similarity to the goal state. The subjects can then mentally carry out the various possible moves at any given time and choose the one that gets them closest to the solution.

Let us now apply this sort of analysis to the nine-dot problem. First of all, the nine-dot problem is complex because there are many more moves possible at any time than in the missionaries-and-cannibals problem. In the initial state of the nine-dot problem, for example, one can "make a move" by starting at any dot, which means that there are at least nine possible moves from the initial state of the nine-dot problem and that the subject is making only one move from each dot. This complexity may make the nine-dot problem similar to problems like chess, which are so complex that they require strategies based on detailed knowledge that enable subjects to reduce the complexity to manageable limits (Newell & Simon, 1972).

There is one further difference between the nine-dot problem and the missionaries-and-cannibals problem that makes the nine-dot problem more difficult. In the missionaries-and-cannibals problem, the subject knows the specific characteristics of the solution, and the subject also knows the specific moves that can be used to solve the problem. In the nine-dot problem, both the goal and the moves are defined only in general, not specific, terms. Therefore, in order to begin to carry out some specific solution attempt, the subject must make some assumptions about the type of problem involved. We have argued that subjects assume that the problem is a connect-the-dots puzzle and begin to produce specific solutions from there.

Based on this analysis, the nine-dot problem is "ill-defined" (in Hayes's, 1978, chap. 11, terms) because the problem solver's knowledge plays a role in defining the problem itself. This is much less true in the case of the missionaries-and-cannibals problem, if it is true at all. (See Greeno, 1974, for a discussion of the point at which the subject's knowledge might play a role in the missionaries and cannibals problem, although Greeno's conclusion is disputed by Jeffries et al., 1977.) It should also be noted that the triangle problem is ill defined in the same way that the nine-dot problem is: The solution and the legal moves are specified in very general terms, and the subject must fill in the rest, as discussed earlier. Given these differences between the insight problems studied here and the "move" problems studied recently by other investigators, it is questionable whether the same sort of analysis can be applied to all of them.

Conclusions

There are four conclusions to be drawn from the present series of experiments, one negative and three positive. The negative conclusion is that the terms *insight* and *fixation* should not be used to describe the behavior of subjects on the problems we have examined.

The first positive conclusion is that these problems are difficult for several reasons. The presentation of the problem initially implies that a certain domain of solutions should be considered. If this domain is a large one, and if it does not contain the correct solution, subjects may not exhaust this domain in the time given them to solve the problem. Furthermore, even if the subject does exhaust the incorrect domain, the domain containing the correct solution is also very large, and again subjects may not have time to work their way through it. Finally, in some cases the domain containing the correct solution may not exist for a subject because of the lack of problem-specific past experience.

The second positive conclusion is that the performance of subjects on some insight problems can be understood by applying a version of hypothesis theory to those situations, as in the preceding paragraph. This indicates that the area of insight problems is not an isolated area, but may involve processes of the sorts studied by cognitive psychologists in other areas.

The final conclusion concerns the importance of problem-specific experience in problem solving. The present results seem to indicate that experience from a relatively similar problem is needed to solve a new insight problem. This is in opposition to the view that solution of such problems will come about if a naive subject can simply be encouraged to think freely about the problem.

Reference Note

1. Weisberg, R. *Solutions to a problem as indirect expressions of knowledge.* Manuscript submitted for publication, 1981.

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