Handout 3: Bernoulli and Poisson processes

Due date : February 24^{th}

Brief recap

1.

Exercise 2. Let X_i $(i = 1, 2, \dots)$ be i.i.d. random variables with mean 0 and variance 2; Y_i $(i = 1, 2, \dots)$ be i.i.d. random variables with mean 2. Assume that all variables X_i, Y_j are independent. Consider the following statements:

(A) $\frac{X_1+\cdots+X_n}{n}$ converges to 0 in probability as $n \to \infty$.

(B) $\frac{X_1^2+\cdots+X_n^2}{n}$ converges to 2 in probability as $n \to \infty$.

(C) $\frac{X_1Y_1+\cdots+X_nY_n}{n}$ converges to 0 in probability as $n \to \infty$.

Which of these statements are always true? Write `True` or `False` in each of the boxes below.

A.	**B.**	**C.**

Exercise 3. Problem 21.4.
In the fair Gambler's Ruin game with initial stake of n dollars and target of T dollars, let e_n be the number of \$1 bets the gambler makes until the game ends (because he reaches his target or goes broke).

1. Describe constants a, b, c such that

$$e_n = ae_{n-1} + be_{n-2} + c \tag{1}$$

for $1 < n < T$.

2. Let e_n be defined by (1) for all $n > 1$, where $e_0 = 0$ and $e_1 = d$ for some constant d. Derive a closed form (involving d) for the generating function $E(x) = \sum_{n=0}^{\infty} e_n x^n$.

3. Find a closed form (involving d) for e_n.

4. Use part (c) to solve for d.

5. Prove that $e_n = n(T - n)$.