Handout 3: Poisson processes

Due date :

Exercise 1. Passengers arrive at a bus stop according to a Poisson process with rate λ. The arrivals of buses are exactly t minutes apart.

1. Show that on average, the sum of the waiting times of the riders on one of the buses is $\frac{1}{2}\lambda t^2$.

Exercise 2. Suppose we have a couple of Poisson processes with rates λ_1, λ_2 respectively. For an interval of length δ small enough, give an approximation of $P((k_1, k_2); \delta)$ for $k_1, k_2 \in \{1, 2\}$. A student receives phone calls according to a Poisson process with rate λ. Unfortunately she has lost her cell phone charger. The battery's remaining life is a random variable T with mean μ and variance σ^2. Let $N(T)$ be the number of phone calls she receives before the battery dies.

1. Find E[N(T)] and Var[$N(T)$].

Exercise 3. On a whatsApp question-ans-answer group, $N \sim \text{Pois}(\lambda_1)$ questions will be posted tomorrow, with λ_1 measured in questions/day. Given N , the post times are i.i.d. and uniformly distributed over the day (a day begins and ends at midnight). When a question is posted, it takes an $\text{Exp}(\lambda_2)$ amount of time (in days) for an answer to be posted, independently of what happens with other questions.

1. Find the probability that a question posted at a uniformly random time tomorrow will not yet have been answered by the end of that day.

Exercise 4. In an endless football match, goals are scored according to a Poisson process with rate λ. Each goal is made by team A with probability p and team B with probability 1?p. For $j > 1$, we say that the jth goal is a turnaround if it is made by a different team than the (j?1)st goal. For example, in the sequence AABBA, the 3rd and 5th goals are turnarounds.

1. In n goals, what is the expected number of turnarounds?

2. What is the expected time between turnarounds, in continuous time?

References and acknowledgments: Introduction to probability (Blitzstein and Huang)