Worksheet 1 - Math 001 Solutions

Solve these questions:

1) $\dfrac{4x-4}{x} + \dfrac{16x-16}{7x^2}$

Solution:

$$\frac{4x-4}{x} + \frac{16x-16}{7x^2} = \frac{4(x-1)}{x} + \frac{16(x-1)}{7x^2}$$

$$= \frac{4(x-1)\cdot 7x}{7x^2} + \frac{16(x-1)}{7x^2}$$

$$= \frac{28x(x-1)+16(x-1)}{7x^2}$$

$$= \frac{(x-1)(28x+16)}{7x^2}$$

$$= \frac{4(x-1)(7x+4)}{7x^2}$$

2) Find the distance between $P(2,2)$ and $Q(-2,3)$

Solution:

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$= \sqrt{(-2-2)^2 + (3-2)^2}$$

$$= \sqrt{(-4)^2 + (1)^2}$$

$$= \sqrt{16+1}$$

$$= \sqrt{17}$$

3) $(3x - \sqrt{2y})(3x + \sqrt{2y})$

Solution:

$$(3x - \sqrt{2y})(3x + \sqrt{2y}) = (3x)^2 - (\sqrt{2y})^2$$

$$= 9x^2 - 2y$$

4) $(2\sqrt{x})(3\sqrt[3]{x})$

Solution:

$$(2\sqrt{x})(3\sqrt[3]{x}) = 6 \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{3}}$$
$$= 6x^{\frac{1}{2}+\frac{1}{3}}$$
$$= 6x^{\frac{3}{6}+\frac{2}{6}}$$
$$= 6x^{\frac{5}{6}}$$

5) $(2x-5)^2$

Solution:

$$(2x-5)^2 = (2x)^2 - 2(2x)(5) + (5)^2$$
$$= 4x^2 - 20x + 25$$

6) $x^2 + 6x + 9 = 0$

Solution:

$$x^2 + 6x + 9 = 0$$
$$(x+3)^2 = 0$$
$$x + 3 = 0$$
$$x = -3$$

7) If the midpoint M of the line segment joining $(2, -1)$ and $(2x, 3)$ is $(8, 1)$ Find the value of x.

Solution:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$(8, 1) = \left(\frac{2 + 2x}{2}, \frac{-1 + 3}{2}\right)$$
$$(8, 1) = \left(\frac{2 + 2x}{2}, 1\right)$$

From x-coordinate:

$$\frac{2 + 2x}{2} = 8$$
$$2 + 2x = 16$$
$$2x = 14$$
$$x = 7$$

8) The equation of the circle with center at $(-3, 4)$ and radius 5 is

Solution:

$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - (-3))^2 + (y - 4)^2 = 5^2$$
$$(x + 3)^2 + (y - 4)^2 = 25$$

9) $\dfrac{x^2 + 5x + 6}{x^2 + 8x + 16}$

Solution:

$$\frac{x^2 + 5x + 6}{x^2 + 8x + 16} = \frac{(x + 2)(x + 3)}{(x + 4)^2}$$

10) $2 - 2^2 \left[\sqrt{4} + 2(-2)\right]$

Solution:

$$2 - 2^2 \left[\sqrt{4} + 2(-2)\right] = 2 - 4\left[2 + (-4)\right]$$
$$= 2 - 4\left[-2\right]$$
$$= 2 - (-8)$$
$$= 2 + 8$$
$$= 10$$

11) $(x^3 - x^2 + 5x + 1) - (x^3 - 2x^2 - 3x - 5)$

Solution:

$$(x^3 - x^2 + 5x + 1) - (x^3 - 2x^2 - 3x - 5)$$
$$= x^3 - x^2 + 5x + 1 - x^3 + 2x^2 + 3x + 5$$
$$= (-x^2 + 2x^2) + (5x + 3x) + (1 + 5)$$
$$= x^2 + 8x + 6$$

12) What is the scientific notation of $\dfrac{140000}{0.02}$?

Solution:

$$\frac{140000}{0.02} = \frac{1.4 \times 10^5}{2 \times 10^{-2}}$$
$$= 0.7 \times 10^7$$
$$= 7 \times 10^6$$

13) Solve $\left[\dfrac{2x^{-3}y^4}{3x^2y^{-1}}\right]^{-2}$

Solution:

$$\left[\frac{2x^{-3}y^4}{3x^2y^{-1}}\right]^{-2} = \left[\frac{2}{3} \cdot x^{-3-2} \cdot y^{4-(-1)}\right]^{-2}$$

$$= \left[\frac{2}{3} \cdot x^{-5} \cdot y^5\right]^{-2}$$

$$= \left(\frac{2}{3}\right)^{-2} \cdot x^{10} \cdot y^{-10}$$

$$= \left(\frac{3}{2}\right)^2 \cdot \frac{x^{10}}{y^{10}}$$

$$= \frac{9}{4} \cdot \frac{x^{10}}{y^{10}}$$

14) Find the intercepts of $y^2 + x^2 = 9$

Solution:
x-intercepts (set $y = 0$):

$$0 + x^2 = 9$$

$$x^2 = 9$$

$$x = \pm 3 \quad \Rightarrow \quad (3,0), (-3,0)$$

y-intercepts (set $x = 0$):

$$y^2 + 0 = 9$$

$$y^2 = 9$$

$$y = \pm 3 \quad \Rightarrow \quad (0,3), (0,-3)$$

15) Solve the equation $\dfrac{5}{6}x - 4 = \dfrac{1}{3}x$

Solution:

$$\frac{5}{6}x - 4 = \frac{1}{3}x$$

$$\frac{5}{6}x - \frac{1}{3}x = 4$$

$$\frac{5}{6}x - \frac{2}{6}x = 4$$

$$\frac{3}{6}x = 4$$

$$\frac{1}{2}x = 4$$

$$x = 8$$

16) Factor the polynomial $x^3 + x^2 + 4x + 4$

Solution:

$$\begin{aligned}
x^3 + x^2 + 4x + 4 &= (x^3 + x^2) + (4x + 4) \\
&= x^2(x + 1) + 4(x + 1) \\
&= (x + 1)(x^2 + 4)
\end{aligned}$$

17) Which are rational numbers in the list $\{-1, 0.12, -3.555, \sqrt{16}, -\pi^2, \sqrt{7}\}$?

Solution:

Rational numbers: $-1, 0.12, -3.555, \sqrt{16}$

Irrational numbers: $-\pi^2, \sqrt{7}$

18) Let $A = \{2, 3, 6, 7, 8\}$, $B = \{x : x$ is a natural number less than or equal 8? find $A \cap B$?

Solution:

$$B = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$A \cap B = \{2, 3, 6, 7, 8\}$$

19) True or false: $-7 \in \{1, 7\}$?

Solution:

False. The set $\{1, 7\}$ contains only the elements 1 and 7, not -7.