

coin change :-

n different denominations.

$(K) \Rightarrow$ no of ways to get a change of K

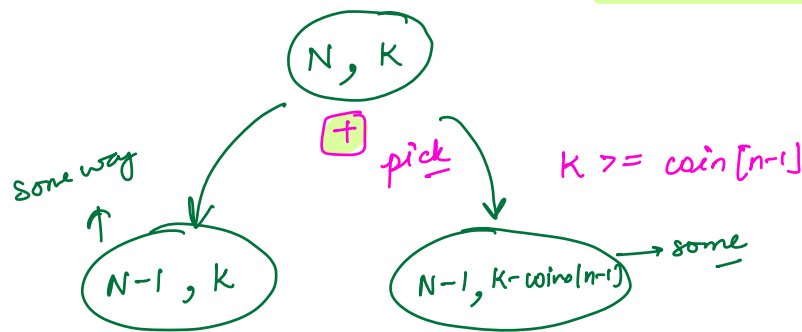
can't include a coin more than once

7 4 9 6 10 13 11 14

$K = 22$

7 4 11
13 9
7 9 6

Q. n , integers - array
count no of
ways in which
subset sum = K



$dp[i][j]$ = no of ways to achieve j change with i available coins

$$dp[i][j] = \begin{cases} dp[i-1][j] \\ + \\ dp[i-1][j - \text{coin}[i-1]] \end{cases}, j \geq \text{coin}[i-1]$$

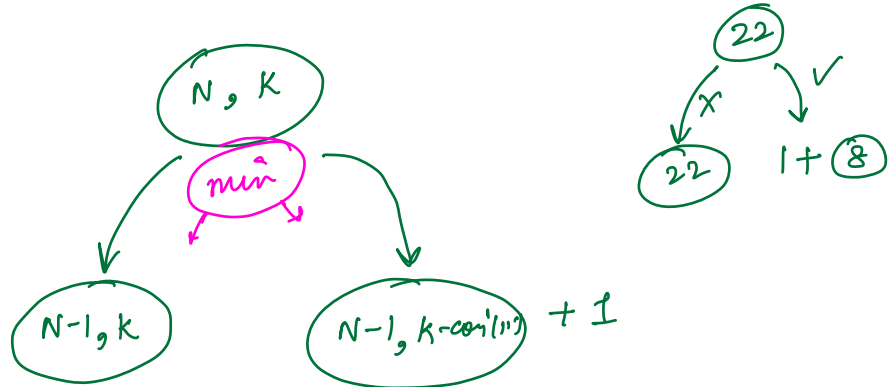
Base case

$j \leq K$
 $j = 1$
 $dp[0][j] = 0$

$i \leq n$
 $i = 0$
 $dp[i][0] = 1$ (required change)
 $dp[0][0] = 1$

min no of coins required for change of k

Q min no of elements to pick such that subset sum = k



Base case:

$$\forall i \leq n \quad \forall dp[i][0] = 0$$

$$\forall j \leq k \quad \forall dp[0][j] = \infty \text{ (INT_MAX)}$$

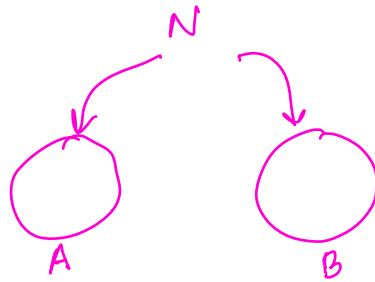
\downarrow
 acc to your cost

\nearrow n+1

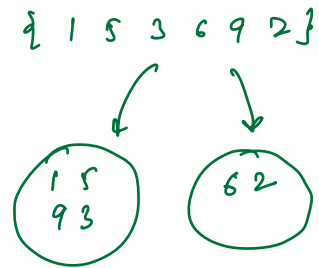
Q

array - N integers. Divide all elements into two subsets.

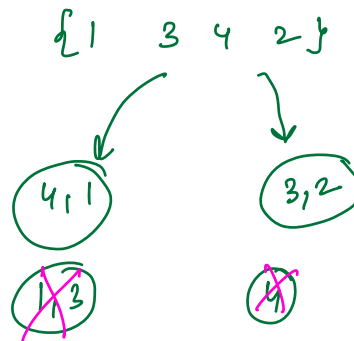
- (i) Find no of ways in which you can divide such that both subsets have equal sum.



$$\text{Sum}(A) = \text{Sum}(B)$$



Every element must belong to one of sets



$$\text{sum}(A) = \text{sum}(B)$$

$$\text{sum}(A) + \text{sum}(B) = \text{total sum}$$

$$\text{sum}(A) + \text{sum}(A) = \text{total sum}$$

$$\text{sum}(A) = \frac{\text{total sum}}{2}$$

total sum is even

(ii) divide

diff of the sum of sets = k

True/False
no of ways

$$\text{sum}(A) - \text{sum}(B) = k$$

$$\text{sum}(A) - \text{sum}(B) = k$$

$$\text{sum}(A) + \text{sum}(B) = \text{totalsum}$$

+

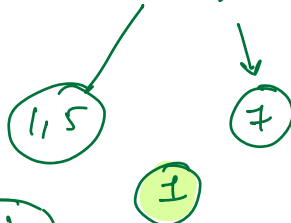
$$2 \text{sum}(A) = k + \text{totalsum}$$

$$\text{sum}(A) = \frac{k + \text{totalsum}}{2}$$

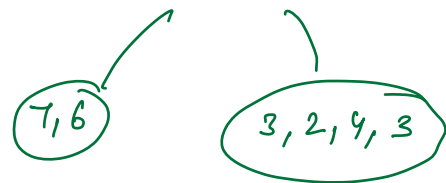
(ii)

minimise the ^{abs} diff of two sets

{ 1, 5, 7 }



{ 3, 2, 4, 7, 6, 3 }



$$dp[N+1][\text{sum}/2+1]$$

totalsum = 100

1

3

SD 51

49 S2

dp[N][50]

dp[N][49]

diff

0

2

4

S1

50

49

48

S2

50

51

52

$$dp[N][\text{sum}/2]$$

$$\rightarrow dp[N][\text{sum}/2 - 1]$$

$$dp[N][50] \rightarrow \begin{matrix} T \\ F \end{matrix}$$

$$dp[N][49] \rightarrow \begin{matrix} T \\ F \end{matrix}$$

$$dp[N][48]$$

prepare dp table using knapsack

$dp[N+1][sum/2+1]$

$K = sum/2$

while ($K \geq 0$)
{

if ($dp[N][K] == true$)
{

$s1 = K;$

$s2 = sum - K;$

return $abs(s1 - s2);$

}

$K--;$

}

#

Knapsack

S.C: $O(n \times w)$ — 2D dp table

$$dp(i, j) = dp(i-1, j)$$

$$dp(i-1, j-w)$$

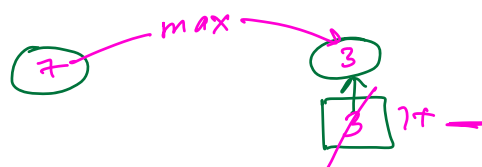
$$n \times w \rightarrow 2 \times w$$

w

0	0	12	15	18	20	22	27	27	29

		0	1	2	3	4	5	6	7	8
w	0	0	0	0	0	0	0	0	0	0
	12	0	0	0	12	12	12	12	12	12
	20	0	0	0	12	12	20	20	20	20
	15	0	0	0	12	12	15	20	20	15+12=27
	6	0	0	6	12	12	18	20	21	27
	10	0	0	6	12	12	18	20	22	27

0	1	2	3	4	5	6	7/2	8
0	0	0	0	0	0	0	0	0 12



dp[w+1]

$\hat{x} - 1$

$$for (i = n + 1; i = 0; i--)$$

1

for ($j = w; j \geq 0; j--$)

$$2 \quad \hat{y}(j) = \text{weight}(i-1)$$

9

1.

1

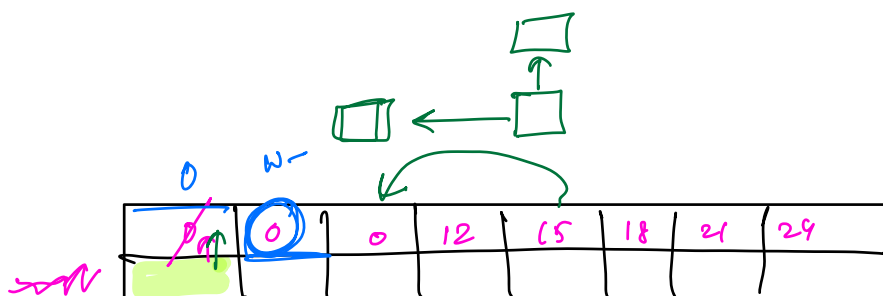
2

$$dp[i][j] = \max(dp[j], \text{value}[i-1] + dp[j - \text{weight}[i-1]])$$

$[0 - \infty]$ Knapsack

 $dp[i][j]$
$$dp[i-1][j]$$

dp[i][j] = -1



```
for (int i=0; i<=n; i++)
```

```
{
```

```
    for (j=0; j<=w; j++)
```

l to R

```
    {
```

```
        if (j < weight[i-1])
```

```
        {
```

```
            dp[j] = max(
```

dp[j],
value[i] + dp[j - weight[i-1]])

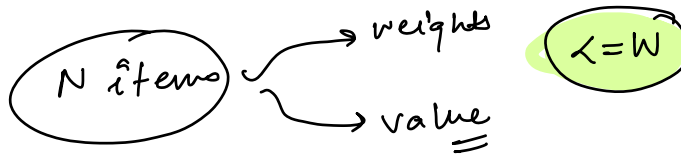
```
        }
```

```
    }
```

```
}
```

0 to w

#



$\leq W$

max value

$1 \leq N \leq 500$
 $1 \leq \text{value}[i] \leq 50$
 $1 \leq w[i] \leq 10^9$
 $1 \leq W \leq 10^9$

N, W - max value

N, value min weight

max possible value =

$500 * 50$
 $= 25000$

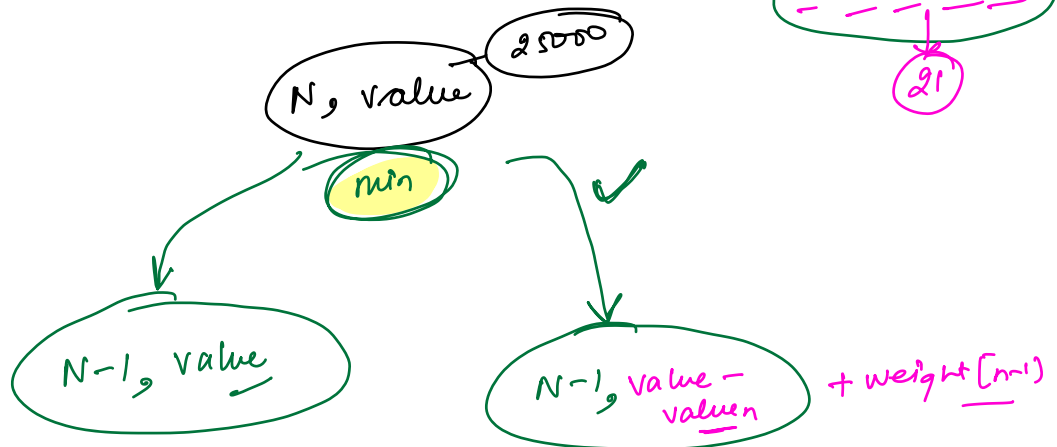
10

15 kg

0	1	2	3	4	5	6	7	8	9	10
↓	↓	↓	↓	↓						
2	8	12	16	20						

1 9 6 3 2

 21



$N=3$
 $W=7$

value \equiv 2 1 3
weights \equiv 3 2 4

0 \rightarrow 6
 $2+1+3$

	0	1	2	3	4	5	6
0	0	∞	∞	∞	∞	∞	∞
1	0	∞	$3+0$	∞	∞	∞	∞
2	0	$2+0$	3	5	∞	∞	∞
3	0	2	3	4	6	7	9

$N=7$

$dp[N][j] \leq W$
ans = max j

$dp[n+1][\text{max value} + 1]$

Sum of all values

$N \times \text{max value}[i]$

$$dp[i][j] = \min(dp[i-1][j], \text{weight}[i-1] + dp[i-1][j - \text{value}[i-1]])$$