Q Given 2 strings, find the length of **longest common subsequence**

lowercase

S1: a b b c d g f
S2: b a c d e g f

a c d g f
ans = 5

S1: k l a g r i p
S2: l g i g k m

l g i
ans = 3

a b c d e
g f c e

a b c d
b c e f

a & f can't coexist in the soln

LCS S1[0-n-1] & S2[0-m-1]

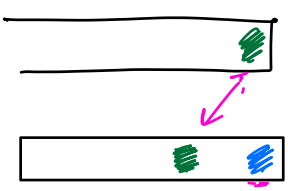
S1[n-1] == S2[m-1]

1 + LCS((0-n-2), (0-m-2))

S1[n-1] != S2[m-1]

max

LCS(0-n-2, 0-m-1) LCS(0-n-1, 0-m-2)



a b c d
a b c e

overlapping subproblems

a b c
a b c e

a b
a b c e

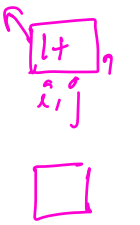
a b c
a b c

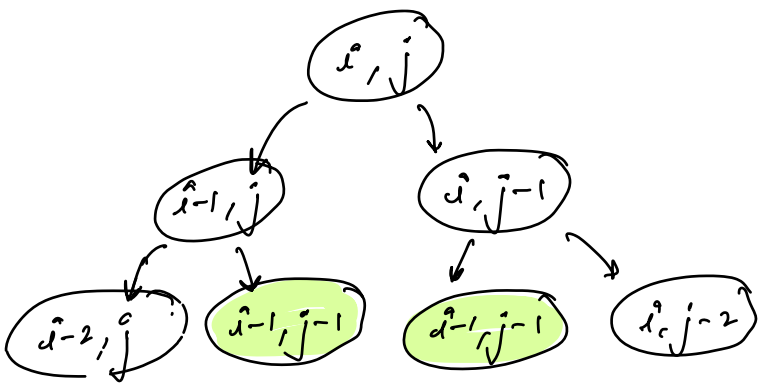
a b c d
a b c

a b c
a b c

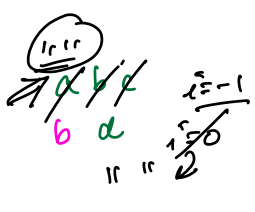
a b c d
a b

$$LCS(i,j) = \begin{cases} 1 + LCS(i-1, j-1); & \text{if } s1[i] == s2[j] \\ \max(LCS(i-1, j), LCS(i, j-1)) & \text{else} \end{cases}$$



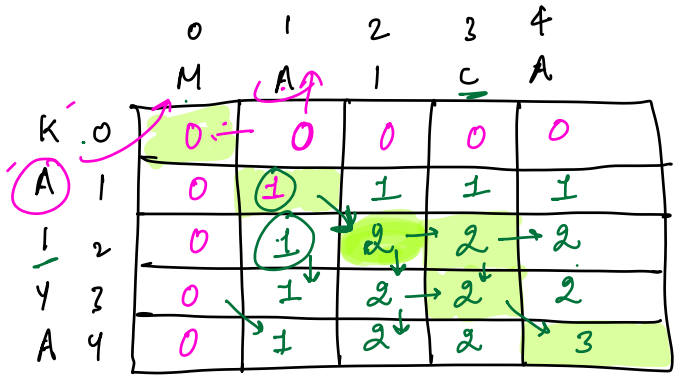



```
int lcs[n][m];          // -1

findlcs( s1, s2, i, j)        → indices      0-i, 0-j
{
    if( i==-1 || j==-1) return 0;

    if( lcs[i][j] != -1) return lcs[i][j];

    if (s1[i] == s2[j])
    {
        lcs[i][j] = 1 + findlcs( i-1, j-1); }
    else
        lcs[i][j] = max( findlcs( i-1, j) , findlcs(i, j-1))
    } ——→ return lcs[i][j];
}
```



s1 = KAIYA
s2 = MAICA

dp[5][5]



length

i = n-1, j = m-1

ans = 'AIA'

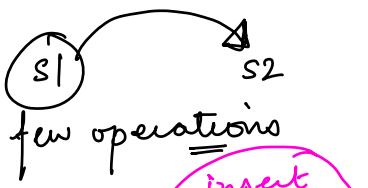
final ans = reverse(ans);

T.C: O(n*m)

S.C: O(n*m)

better optimized ≈ O(m)

#2 Edit distance



can't change s2

insert
delete
replace

find minimum cost

cost associated

Insert ✓
delet ✓
replace

char

s1 = a c (b) a b c
s2 = a b c

cost i = 2
cost d = 2
cost r = 3

= (2)

s1 = a b c d ✗ (replace e) — delete (2)
s2 = a b e

⇒ (5)

s1 = a c (6) d (3) x y ✗ — delete (2)
s2 = a b c g x

(7)

a b c d
a e d

mincost (0-n-1 , 0-m-1)

s1[n-1] == s2[m-1]

mincost (n-2, m-2)

a b c d g d
a b d

s1[n-1] != s2[m-1]

min

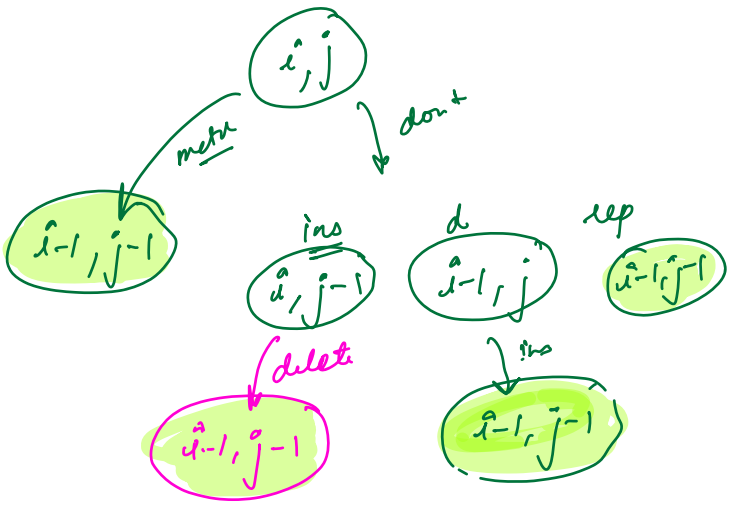
insert → cost i + mincost (n-1, m-2)

d → cost d + minCost (n-2, m-1)

a b c d ✗
a b d

replace → cost r + mincost(n-2, m-2)

a b c d g
a b d



s1 = "ab"
s2 = ""

s1 = ""
s2 = "ab"

```
if ( i == -1 && j == -1) return 0;
else if ( i == -1) return cost d * (i+1);
else j == -1 return cost i * (j+1);
```

$$dp[i][j] = \begin{cases} dp[i-1][j-1] & \text{if } s1[i] == s2[j] \\ \min(\text{cost } i + dp[i][j-1], & \times \\ \quad \text{cost } d + dp[i-1][j], \\ \quad \text{cost } r + dp[i-1][j-1]). \end{cases}$$

[n+1][m+1]

	""	a	b	c	f
""	0	1*cost i	2*cost i	3*cost i	4*cost i
a	1*cost d				
b	2*cost d				
c	3				
f	4				

#3 Wildcard pattern match

Regex expression

s: a b a c d
p: a b a c d

'?' → replace with exactly one character

p: a ? a ? d
s: a b b a d

pattern → ? & *

s: a b b a c
p: a * c

'*' ⇒ replace with sequence of 0 or more characters (any)

p: x * z * *
s: x b b z z c

if string matches pattern or not ?

*
a
ab
abc
abbc

abcde
a?cde

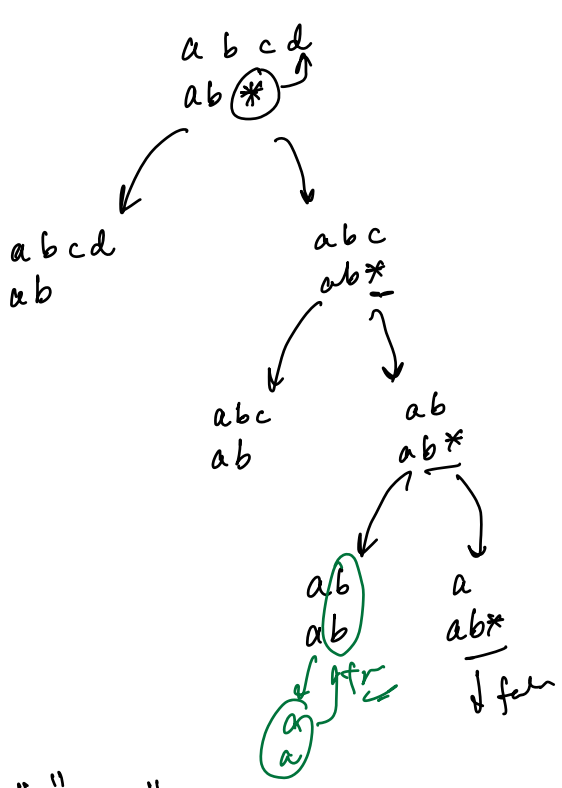
a b c e
a ? c ?

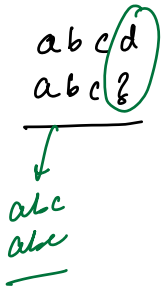
check(n-1, m-1) ③ s[n-1] != pat[m-1] return false

① (s[n-1] == pat[m-1] || pat[m-1] == '?')
check(n-2, m-2)

② pat[m-1] == '*'
replace 0 char: check(n-1, m-2)
replace: check(n-2, m-1)
0 / 1

abcde
ab?d*





$s = ""$
$p = "****"$

```
if ( i == -1 && j == -1) return true;
else if ( i != -1 && j == -1) return false;
else if ( j != -1 && i == -1) // if only * are in pattern return true
                              else return false;
```

$$dp[i][j] = \begin{cases} dp[i-1][j-1] & \text{if } (s[i]==p[j] \;||\; p[j]=='?') \\ dp[i][j-1] \;||\; dp[i-1][j] & \text{else if } (p[j]=='*') \\ \text{false.} & \text{else} \end{cases}$$