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ROLL NO: -CE21BTECH11008

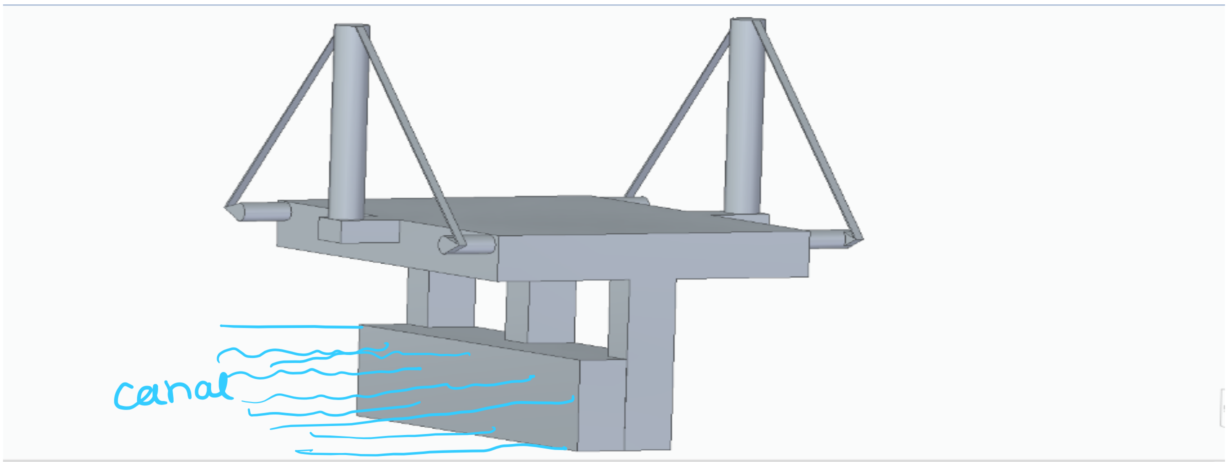
TERM PROJECT

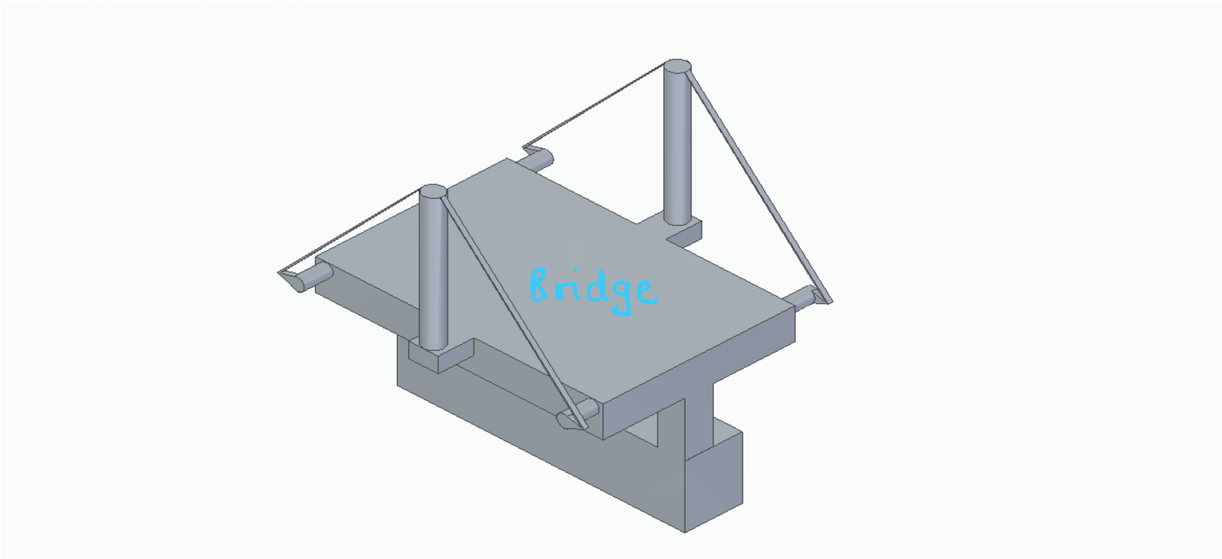
SUBJECT: -MECHANICS OF SOLID

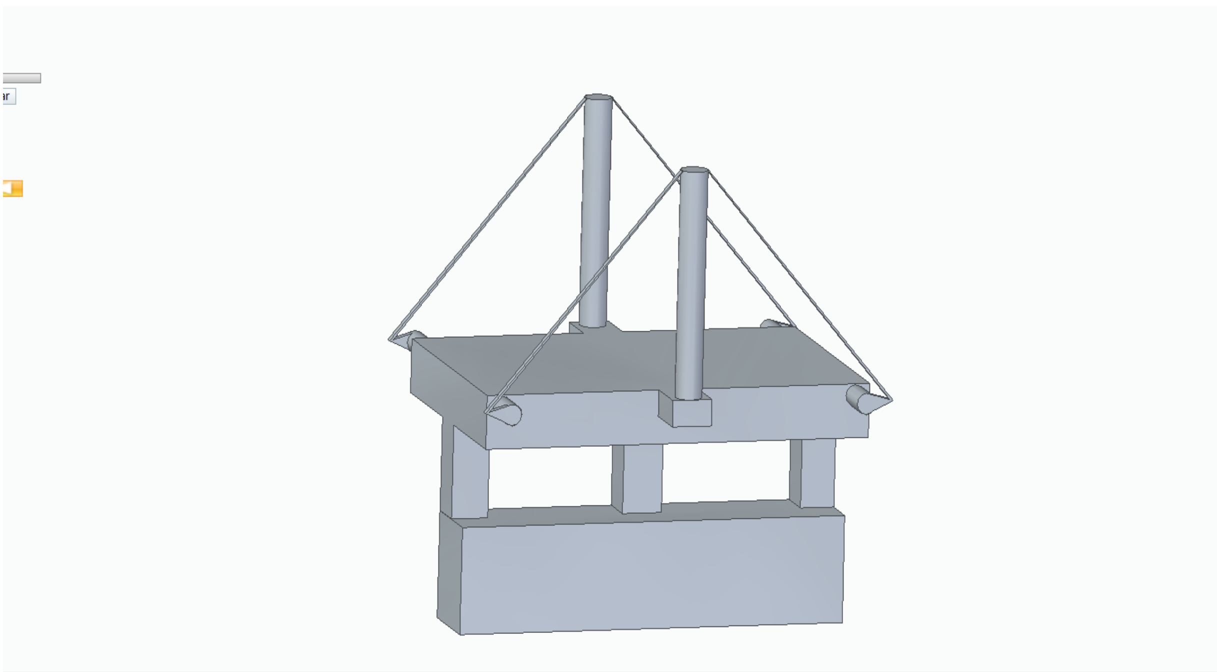
INSTRUCTER: - Dr SURENDRA NADH SOMALA

I have taken a bridge that is made on canal and a small dam is attached to it. I have taken necessary assumption for simplifying the things otherwise it is getting complex. I have analysed its part under different loading condition

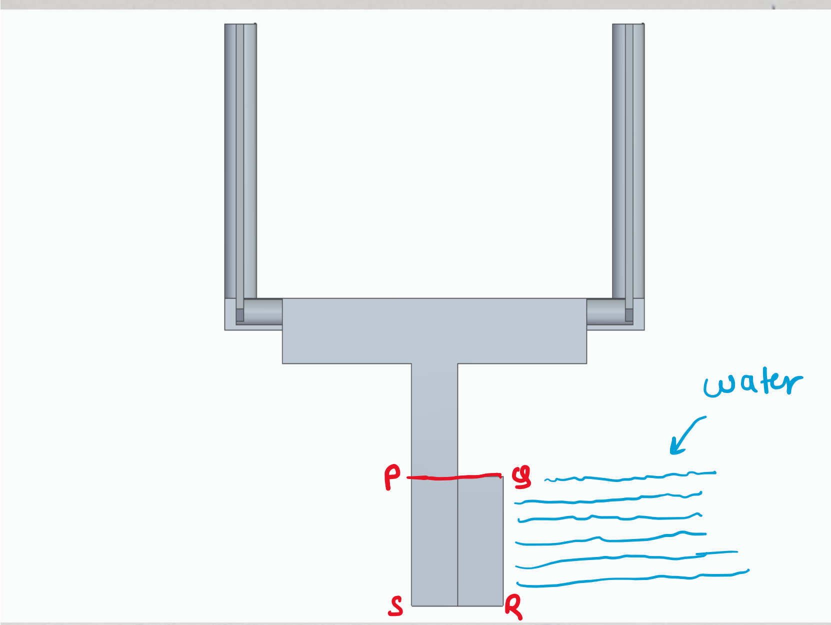
This is the CAD model of project



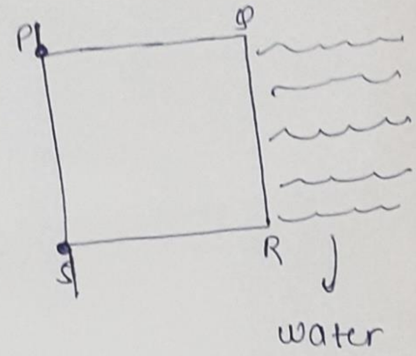




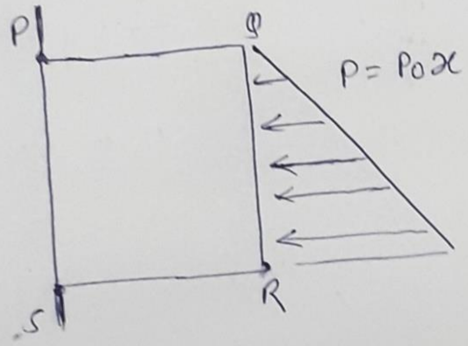
Part-i PQRS



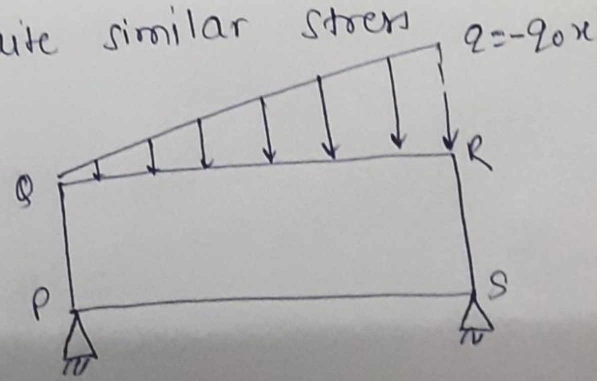
Assuming section PQRS and analysing simply in the presence of water besides this



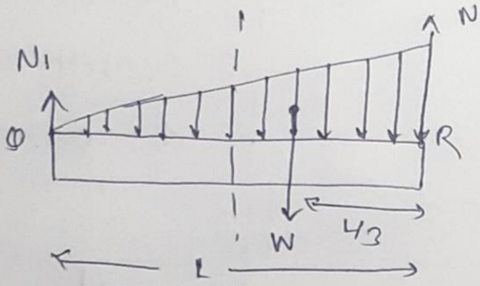
we know pressure due to water increses with depth linearly.



This give same or quite similar stress trajectory of



First let's find shearforce & Moment



$$N_1 + N_2 = \frac{1}{2} \times L \times q_0 L = W$$

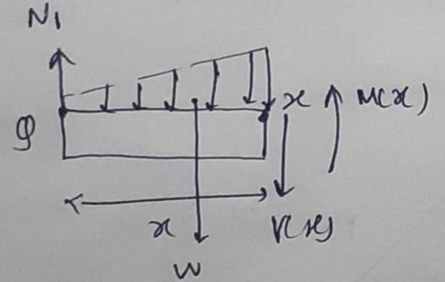
$$= \frac{q_0 L^2}{2}$$

making moment around R = 0

$$W \cdot \frac{L}{3} = N_1 L$$

$$\frac{q_0 L^2 \cdot \cancel{L}}{2 \cdot 3} = N_1 \cancel{L}$$

$$\boxed{N_1 = \frac{q_0 L^2}{6}}$$



At equilibrium

$$\Sigma F_y = 0$$

$$N_1 - \frac{1}{2} \times x \times q_0 x - V(x) = 0$$

$$V(x) = \frac{q_0 L^2}{6} - \frac{q_0 x^2}{2}$$

NOW,

making moment around x = 0

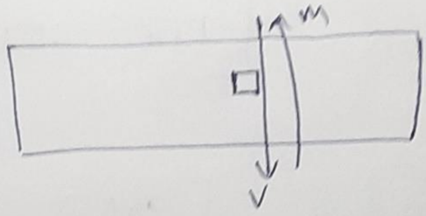
$$M(x) = N_1 x - W \cdot \frac{x}{3}$$

$$= \frac{q_0 L^2}{6} x - \frac{q_0 x^2 \cdot x}{6} = \frac{q_0 L^2 x}{6} - \frac{q_0 x^3}{6}$$

We know, At any elements in the square crosssection

$$\sigma = \frac{My}{I}$$

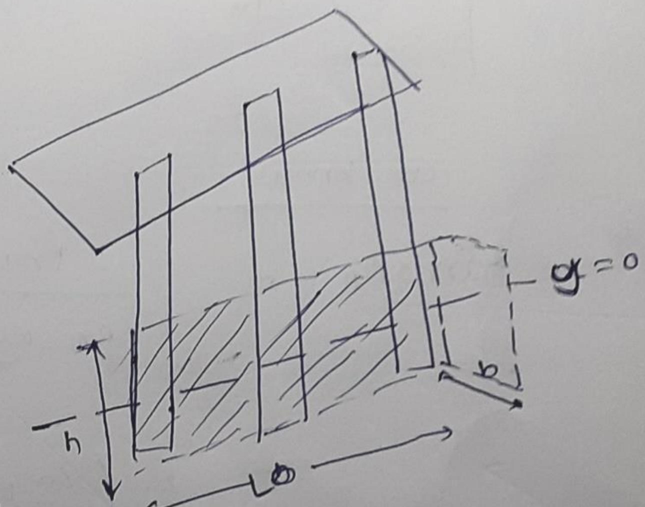
$$\tau = \frac{VQ}{Ib}$$



Here, $\sigma_{yy} \neq 0$ as there is load of bridges are transferring by peers only (3 in number)

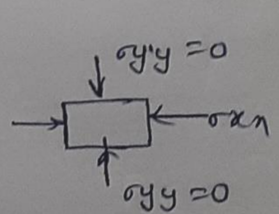
Assuming $\sigma_{yy} = 0$

on most of part



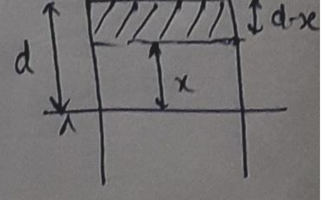
$$\sigma_{xx} = \frac{My}{I}$$

$$\sigma_{xx} = \frac{\left(\frac{q_0 L^2 x}{6} - \frac{q_0 x^3}{6}\right) y}{\frac{1}{12} \times b \times h^3}$$



$h = 2d$

similarly,



$$\tau = \frac{VQ}{Ib} = \frac{\left(\frac{q_0}{6} L^2 - \frac{q_0 x^2}{2}\right) \times \left(\frac{d^2 - y^2}{2}\right)}{\frac{1}{12} \times b \times h^3 \times b}$$

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & \sigma_{yy} \end{bmatrix} = \text{2d Stress tensor}$$

$$\begin{bmatrix} \dfrac{\left(\dfrac{q_0 L^2 x}{6} - \dfrac{q_0 x^3}{6}\right) y}{\frac{1}{12} b h^3} & \tau_{xy} \\ \tau_{yx} & 0 \end{bmatrix}$$

We know

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$= \frac{2 \cdot \left(\frac{q_0}{6} L^2 - \frac{q_0 x^2}{2}\right) \times \left(\frac{d^2 - y^2}{2}\right) \times \cancel{2}}{\cancel{2} \cdot \left(\frac{q_0}{6} L^2 x - \frac{q_0 x^3}{6}\right) \times y}$$

$$= \frac{\left(\frac{L^2}{3} - x^2\right)\left(\frac{d^2 - y^2}{2}\right)}{\frac{1}{6}\left(L^2 x - x^3\right) \times y}$$

$$\tan 2\theta_p = \frac{(L^2 - 3x^2)(d^2 - y^2)}{(L^2 x - x^3)\, y}$$

Now,

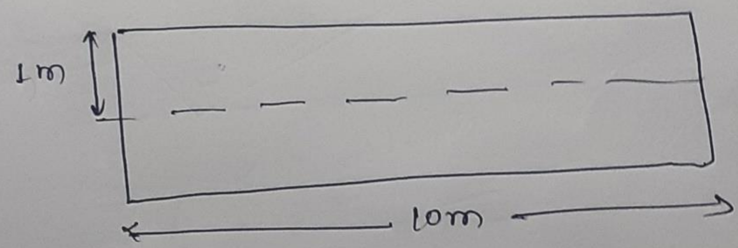
for simplification

let $L = 10\,m$

& $d = 1\,m$ as it will give trajectory similar to other

$$\tan 2\theta_p = \frac{(100 - 3x^2)(1-y^2)}{(100x - x^3)\cdot y}$$

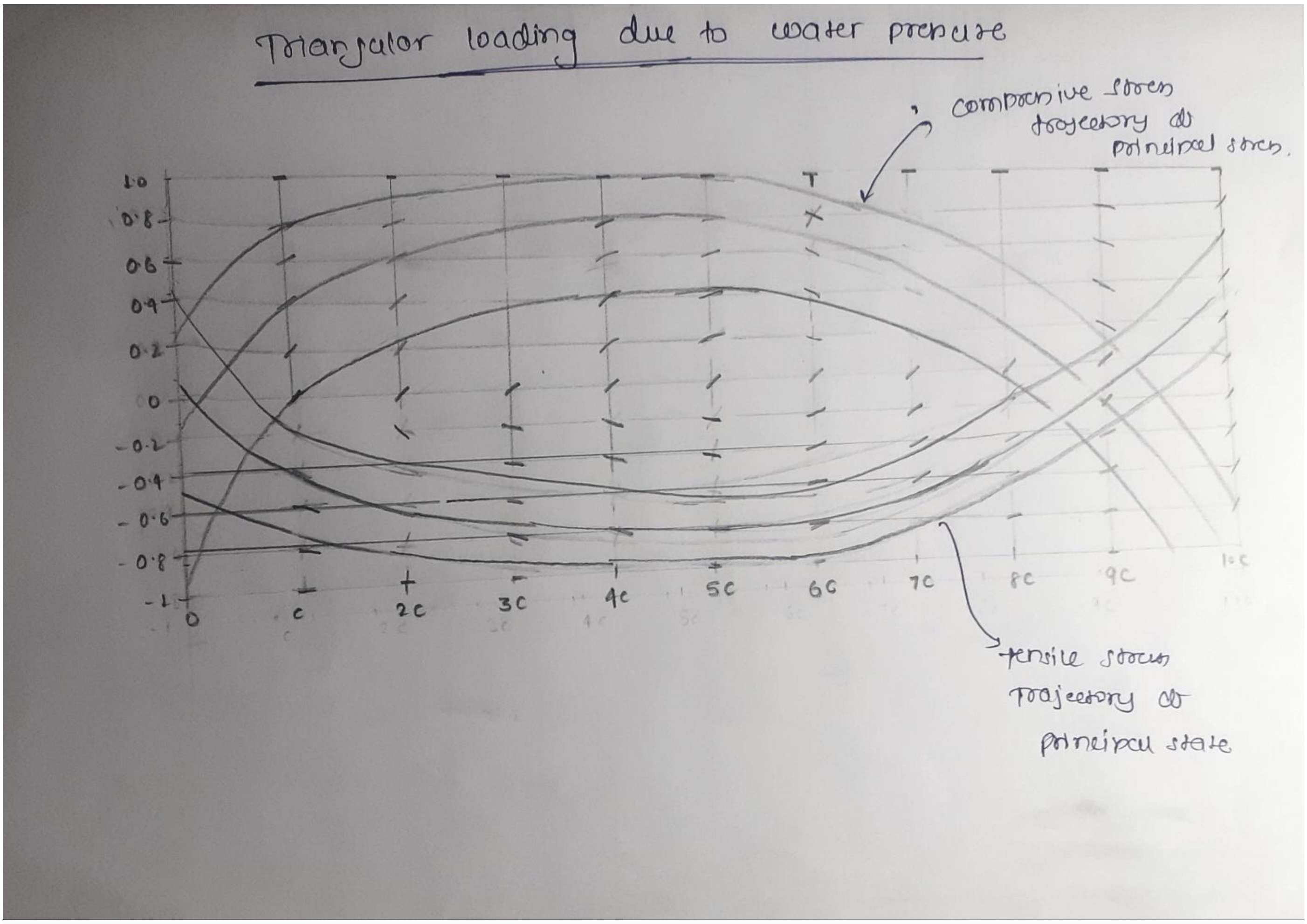
$$\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{(100-3x^2)(1-y^2)}{(100x-x^3)\,y}\right)$$

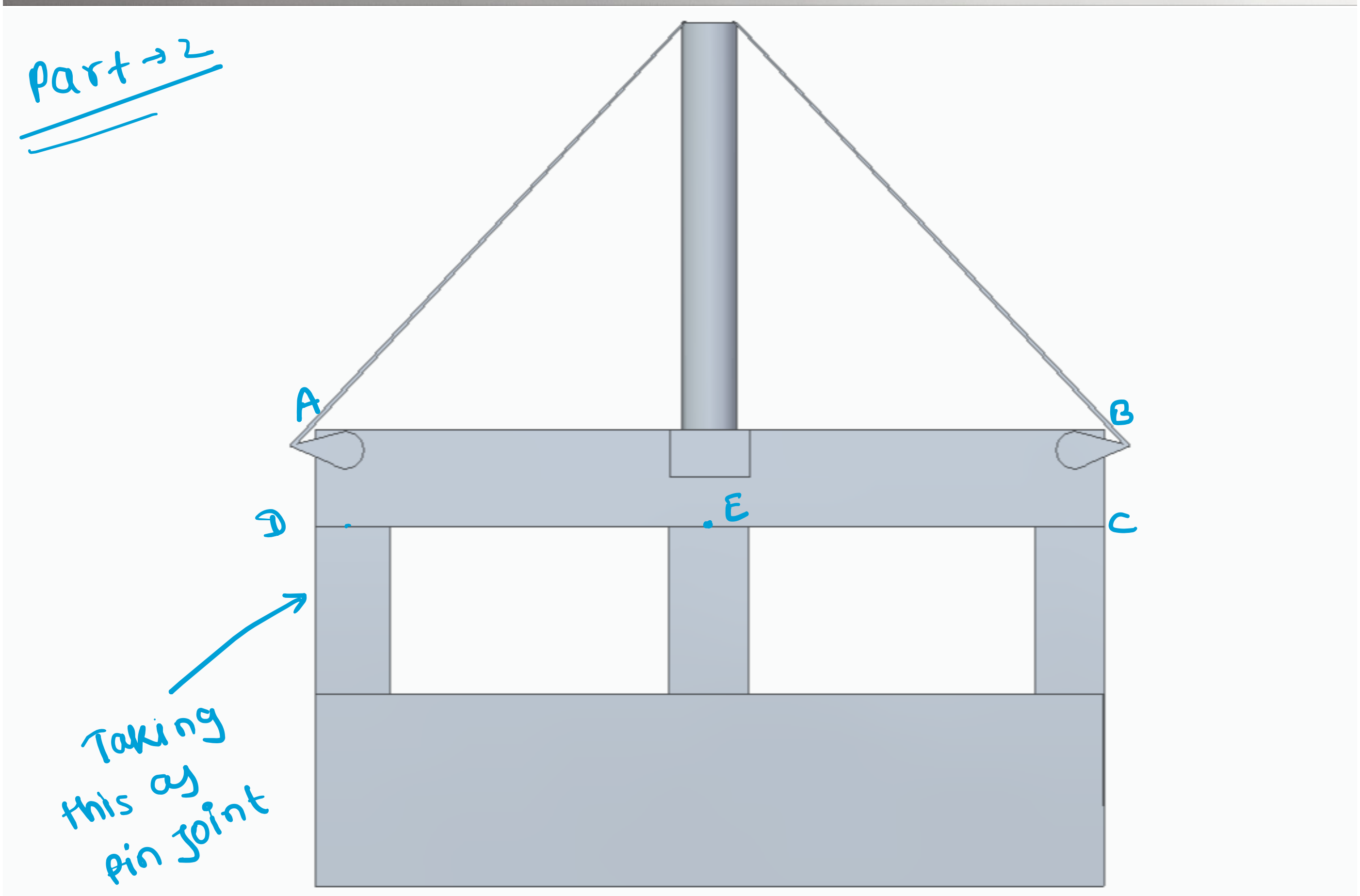


X	Y	θp	θt(in compression	X	Y	θp	θp(compre
1	1	0	0	3	1	0	0
1	0.8	11.89656	11.89656	3	0.8	3.430697	3.430697
1	0.6	23.13191	23.13191	3	0.6	7.959762	7.959762
1	0.4	32.03992	32.03992	3	0.4	14.65795	14.65795
1	0.2	38.99802	38.99802	3	0.2	26.03876	26.03876
1	0	45	45	3	5.55E-17	45.00001	45.00001
1	-0.2	-38.998	51.00198	3	-0.2	-26.0388	63.96125
1	-0.4	-32.0399	57.96008	3	-0.4	-14.6579	75.34206
1	-0.6	-23.1319	66.86809	3	-0.6	-7.95976	82.04024
1	-0.8	-11.8966	78.10344	3	-0.8	-3.4307	86.5693
1	-1	0	90	3	-1	0	90
2	1	0	0	4	1	0	0
2	0.8	5.82692	5.82692	4	0.8	1.991905	1.991905
2	0.6	13.02675	13.02675	4	0.6	4.686907	4.686907
2	0.4	21.95265	21.95265	4	0.4	9.002083	9.002083
2	0.2	32.77802	32.77802	4	0.2	18.30354	18.30354
2	0	45	45	4	0	45	45
2	-0.2	-32.778	57.22198	4	-0.2	-18.3035	71.69646
2	-0.4	-21.9527	68.04735	4	-0.4	-9.00208	80.99792
2	-0.6	-13.0267	76.97325	4	-0.6	-4.68691	85.31309
2	-0.8	-5.82692	84.17308	4	-0.8	-1.99191	88.00809
2	-1	0	90	4	-1	0	90
5	1	0	0	6	1	0	90
5	0.8	0.859179	0.859179	6	0.8	-0.26857	89.73143
5	0.6	2.03376	2.03376	6	0.6	-0.63652	89.36348
5	0.4	3.984806	3.984806	6	0.4	-1.25255	88.74745
5	0.2	8.872338	8.872338	6	0.2	-2.8553	87.1447
5	0	45	45	6	0	45	45
5	-0.2	-8.87234	81.12766	6	-0.2	2.855297	2.855297
5	-0.4	-3.98481	86.01519	6	-0.4	1.252547	1.252547
5	-0.6	-2.03376	87.96624	6	-0.6	0.636515	0.636515
5	-0.8	-0.85918	89.14082	6	-0.8	0.268566	0.268566
5	-1	0	90	6	-1	0	0
7	1	0	90	8	1	0	90
7	0.8	-1.69523	88.30477	8	0.8	-4.09012	85.90988
7	0.6	-3.99687	86.00313	8	0.6	-9.40804	80.59196
7	0.4	-7.72728	82.27272	8	0.4	-16.9275	73.07248
7	0.2	-16.1451	73.85493	8	0.2	-28.4443	61.55567
7	0	45	45	8	0	45	45
7	-0.2	16.14507	16.14507	8	-0.2	28.44433	28.44433
7	-0.4	7.72728	7.72728	8	-0.4	16.92752	16.92752

7	-0.6	3.996873	3.996873	8	-0.6	9.408036	9.408036
7	-0.8	1.695226	1.695226	8	-0.8	4.090116	4.090116
7	-1	0	0	8	-1	0	0

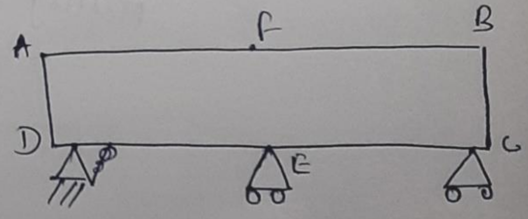
9	1	0	90	10	1		90
9	0.8	-10.3111	79.68894	10	0.8		45
9	0.6	-20.8666	69.13339	10	0.6		45
9	0.4	-30.1708	59.82925	10	0.4		45
9	0.2	-38.0055	51.99454	10	0.2		45
9	0	45	45	10	0		45
9	-0.2	38.00546	38.00546	10	-0.2		45
9	-0.4	30.17075	30.17075	10	-0.4		45
9	-0.6	20.86661	20.86661	10	-0.6		45
9	-0.8	10.31106	10.31106	10	-0.8		45
9	-1	0	0	10	-1		45


Triangular loading due to water pressure
compressive stress trajectory of principal stress.
1.0
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
c
2c
3c
4c
5c
6c
7c
8c
9c
10c
tensile stress trajectory of principal state

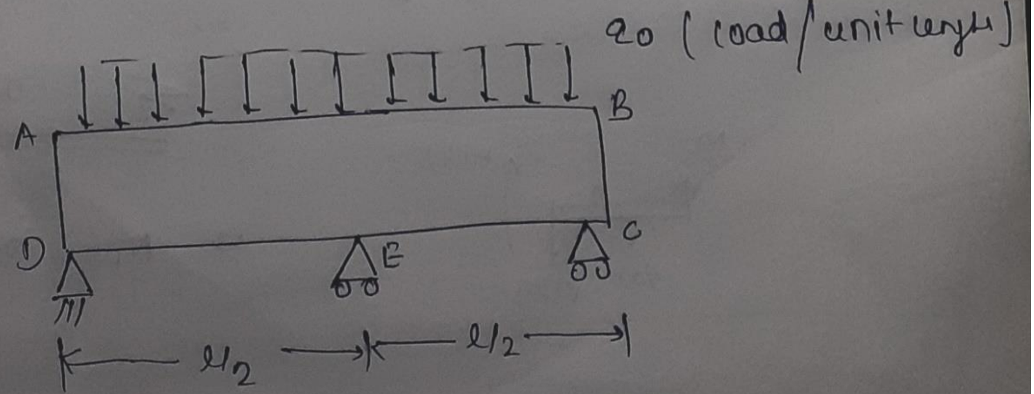

Part → 2
A
B
D
E
C
Taking this as pin joint

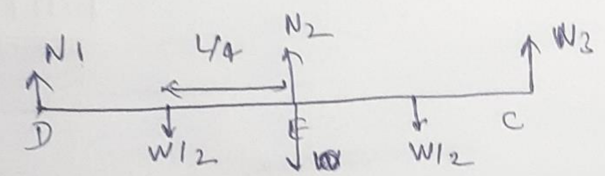
part → 2

Taking out section ABCD, This have quite similar stress trajectory to



To make things simple we are assuming uniform load and neglecting load by peers above it initially.





$W = q_0 L$

due to symmetry $N_1 = N_3$

$N_1 + N_2 + N_3 = q_0 L$

$2N_1 + N_2 = q_0 L$ — (1)

And writing moment equation about E.

$N_1 \cdot \cancel{2}L + N_2 \cdot 0 - W \cdot \frac{L}{4} = 0$

$N_1 \cancel{L} = W \cdot \frac{\cancel{L}}{4}$

$N_1 = \frac{W}{4}$ — (11) $= \frac{q_0 L}{4}$

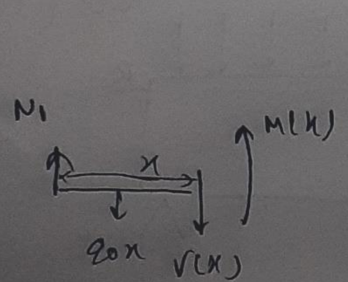
Putting in equation (1)

$\Rightarrow N_2 = q_0 L - 2N_1$

$N_2 = W - 2 \cdot \frac{W}{4} = \frac{W}{2} = \frac{q_0 L}{2}$

Now, finding shear force & Bending moment in section DE.

$\underline{\underline{\Sigma F_y = 0}}$



$\Rightarrow N_1 = q_0 x + V(x)$

$V(x) = N_1 - q_0 x$

$= \frac{q_0 L}{4} - q_0 x$

$\boxed{V(x) = q_0 \left(\frac{L}{4} - x\right)}$

Finding moment,

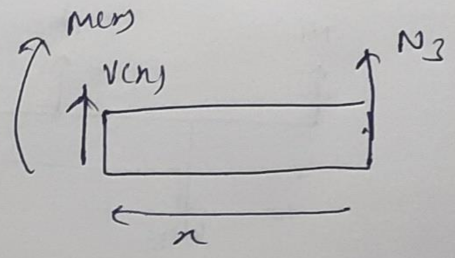
$$M(x) = N_1 x - q_0 x \cdot \frac{x}{2}$$

$$\boxed{M(x) = \frac{q_0}{4} L x - \frac{q_0 x^2}{2}}$$

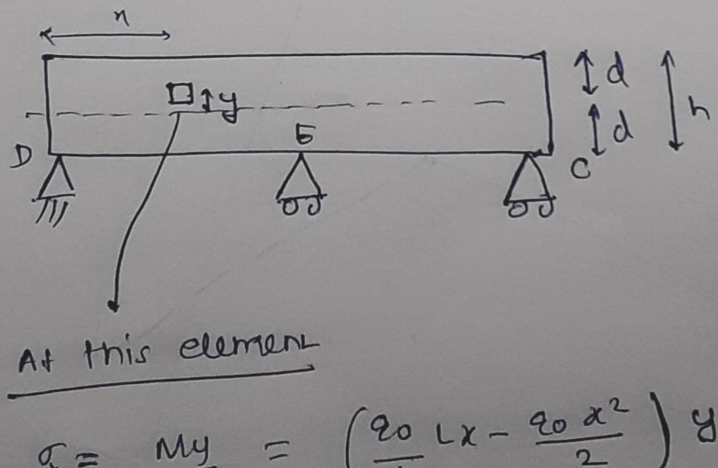
As part DE & EC are symmetric to

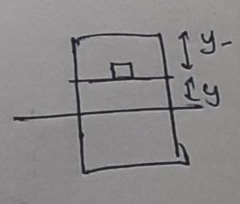
so, value of moment & V(x) will come

provided x is taken from right side



Now,





At this element

$$\sigma_x = \frac{My}{I} = \frac{\left(\frac{q_0}{4} L x - \frac{q_0 x^2}{2}\right) y}{I}$$

$$\tau = \frac{VQ}{Ib} = \frac{q_0 \left(\frac{L}{4} - x\right) \cdot \frac{(d-y)(d+y)}{2}}{I}$$

$$= \frac{q_0 \left(\frac{L}{4} - x\right) (d^2 - y^2)}{2I}$$

Stress tensor $\begin{bmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & \sigma_{yy} \end{bmatrix}$

$\underline{\sigma_{yy} = 0}$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_{xx} - \sigma_{yy}} = \frac{\cancel{2} \cdot \cancel{q_0}\left(\frac{L}{4} - x\right)(d^2 - y^2) \cdot \cancel{I}}{\cancel{2I} \cdot \cancel{q_0}\left(\frac{Lx}{4} - \frac{x^2}{2}\right) y}$$

$$\tan 2\theta_p = \frac{\left(\frac{L}{4} - x\right)(d^2 - y^2)}{\left(\frac{L}{4} - \frac{x}{2}\right)(xy)}$$

Now, for making graph let $L = 10$ unit
$d = 1$ unit

$$\tan 2\theta_p = \frac{\left(\frac{10}{4} - x\right)(1 - y^2)}{\left(\frac{10}{4} - \frac{x}{2}\right)(xy)} = \frac{(10 - 4x)(1 - y^2)}{(10 - 2x)\, xy}$$

and setting $x = 1, 2, 3, 4, 5$ unit

$y = 0, 0.2, 0.4, 0.6, 0.8, 1$
$-0.2, -0.4, -0.6, -0.8$

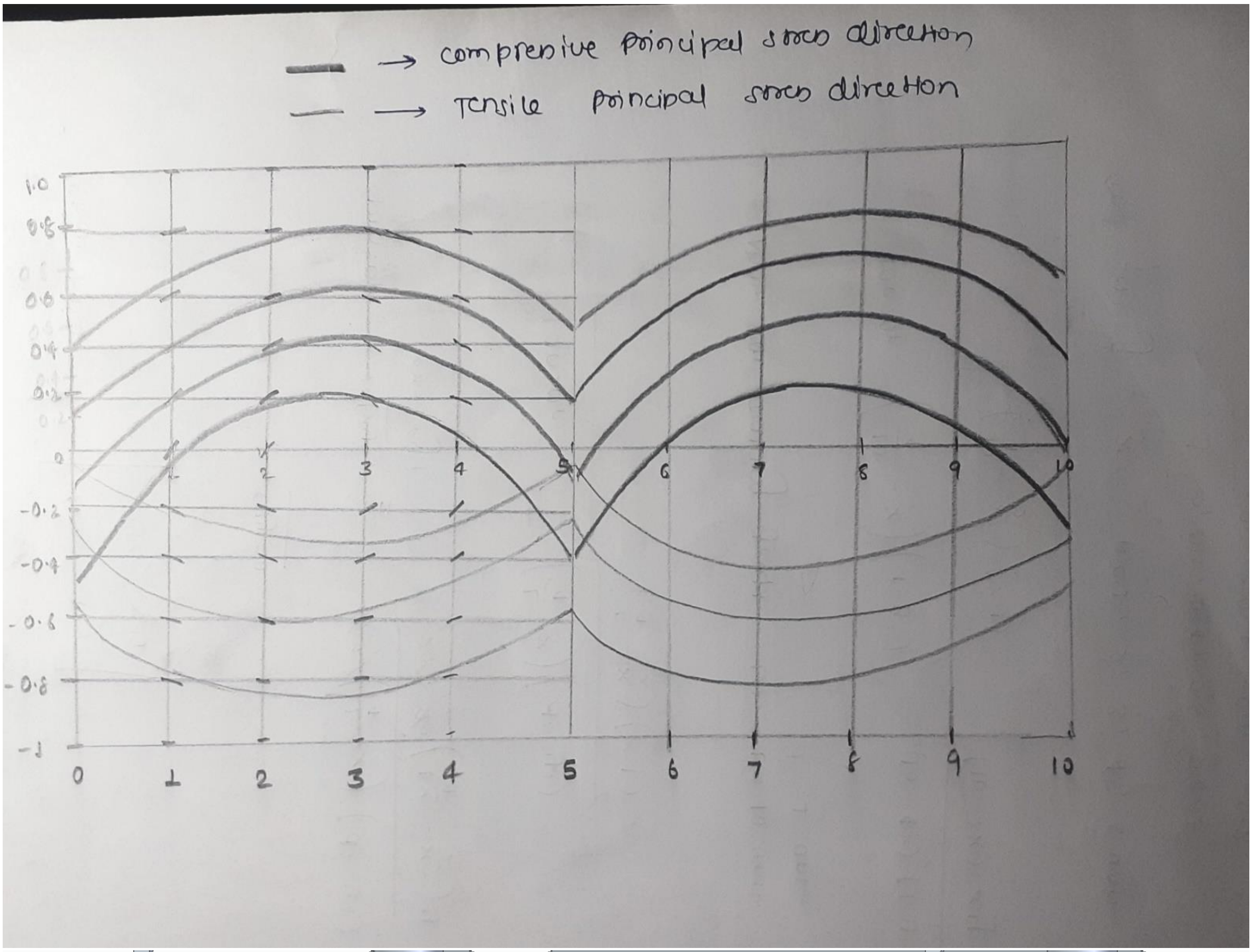
x	y	θp
1	1	0
1	0.8	9.324771
1	0.6	19.32991
1	0.4	28.79385
1	0.2	37.23795
1	0	45
1	-0.2	-37.238
1	-0.4	-28.7939
1	-0.6	-19.3299
1	-0.8	-9.32477
1	-1	0

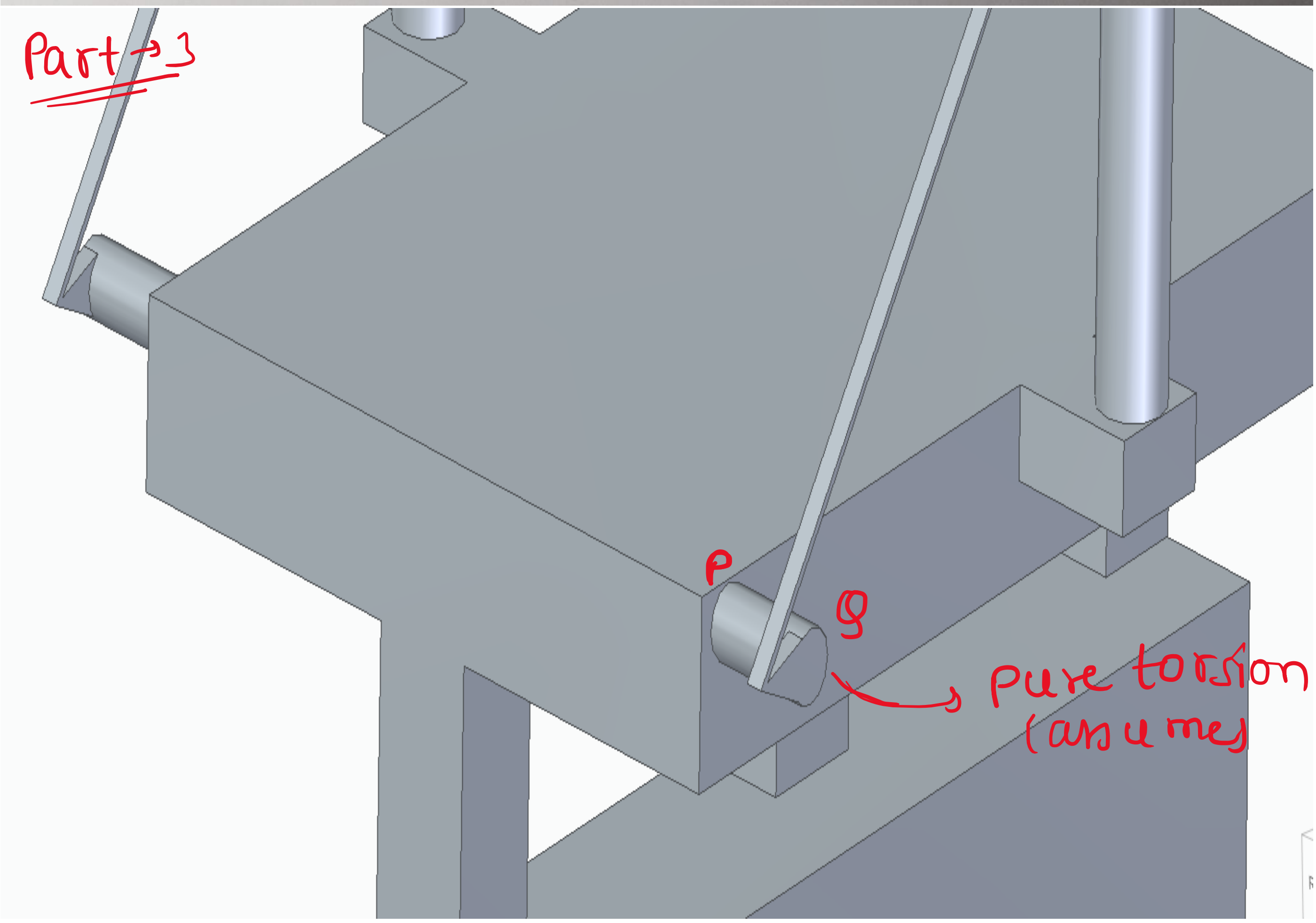
x	y	θp
2	1	0
2	0.8	2.144577
2	0.6	5.0403
2	0.4	9.645025
2	0.2	19.32991
2	0	45
2	-0.2	-19.3299
2	-0.4	-9.64503
2	-0.6	-5.0403
2	-0.8	-2.14458
2	-1	0

x	y	θp
3	1	0
3	0.8	-2.14458
3	0.6	-5.0403
3	0.4	-9.64503
3	0.2	-19.3299
3	0	45
3	-0.2	19.32991
3	-0.4	9.645025
3	-0.6	5.0403
3	-0.8	2.144577
3	-1	0

x	y	θp
4	1	0
4	0.8	-9.32477
4	0.6	-19.3299
4	0.4	-28.7939
4	0.2	-37.238
4	0	45
4	-0.2	37.23795
4	-0.4	28.79385
4	-0.6	19.32991
4	-0.8	9.324771
4	-1	0

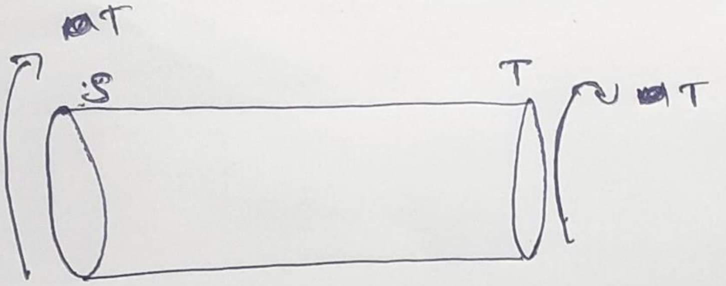
x	y	θp
5	1	0
5	0.8	45
5	0.6	45
5	0.4	45
5	0.2	45
5	0	45
5	-0.2	45
5	-0.4	45
5	-0.6	45
5	-0.8	45
5	-1	45


→ compressive principal stress direction
→ Tensile principal stress direction
1.0
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
1
2
3
4
5
6
7
8
9
10


Part → 3
P
Q
Pure torsion (assume)

Part → 3 ST

Assuming massless



Assuming the section ST in pure torsion

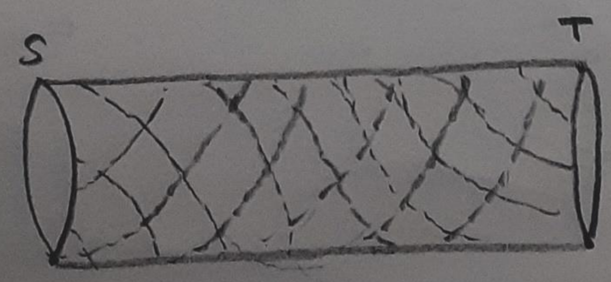
At the surface of this cylindrical section

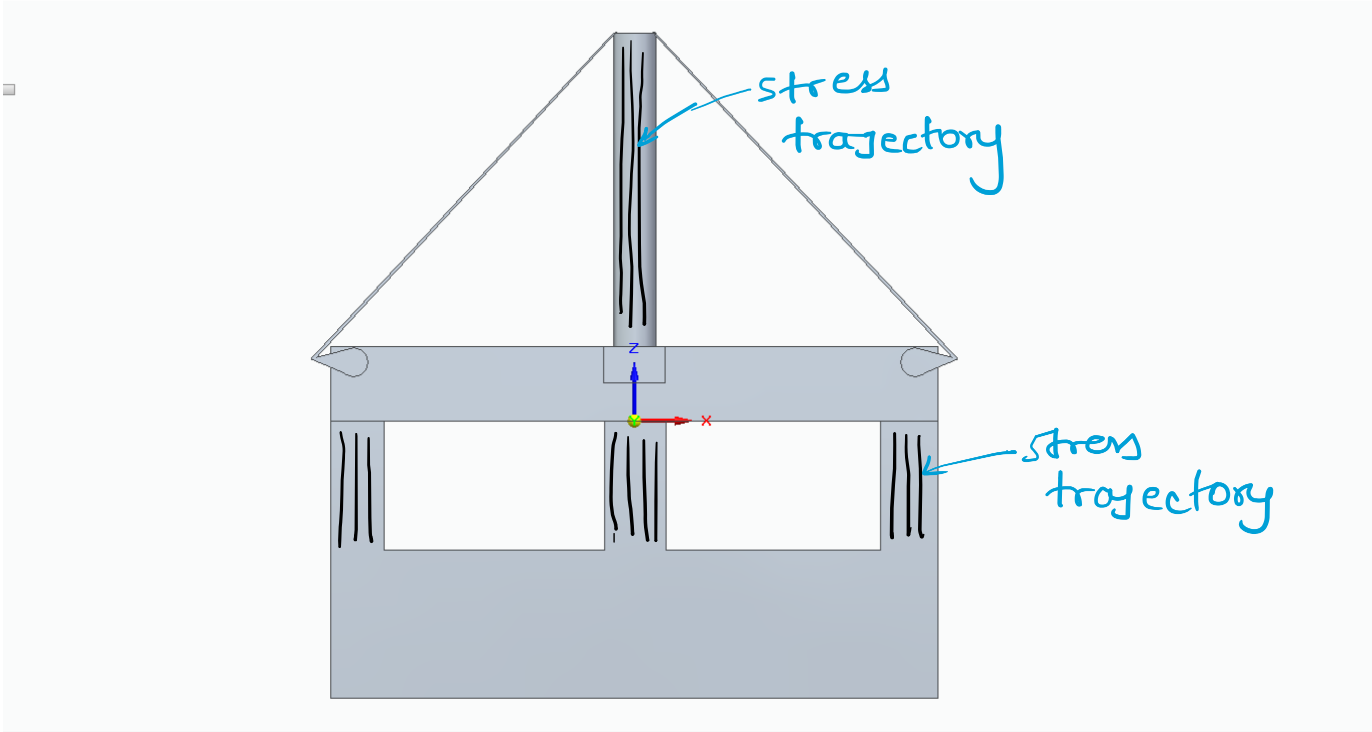
$$\tau = \frac{Tr}{J}$$

and $\sigma_{xx} = 0$ & $\sigma_{yy} = 0$ as this is only in pure torsion

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_{xx} - \sigma_{yy}} = \frac{Tr}{J(0-0)} = \infty$$

$2\theta = 90^\circ$ $\quad \theta_p = \frac{90^\circ}{2} = 45^\circ$ So, stress trajectory are at 45° on the surface



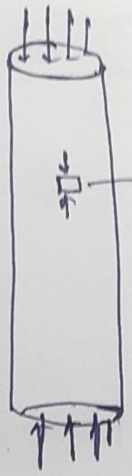

stress trajectory
Z
X
stress trajectory

Part 4 for rectangular & cylindrical Beam

Now, If we talk about simple axial loading

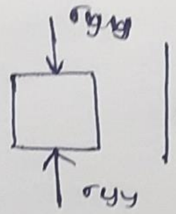
we are neglecting the mass to make thing simple.

also, mass will be very less in comparision to load applied



So, shear force will not affect that much ~0.

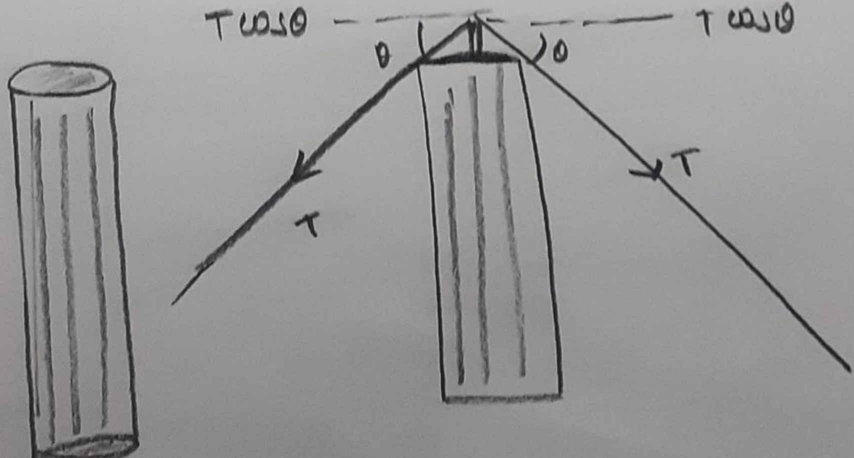
principal stress it self as only axial force is there.

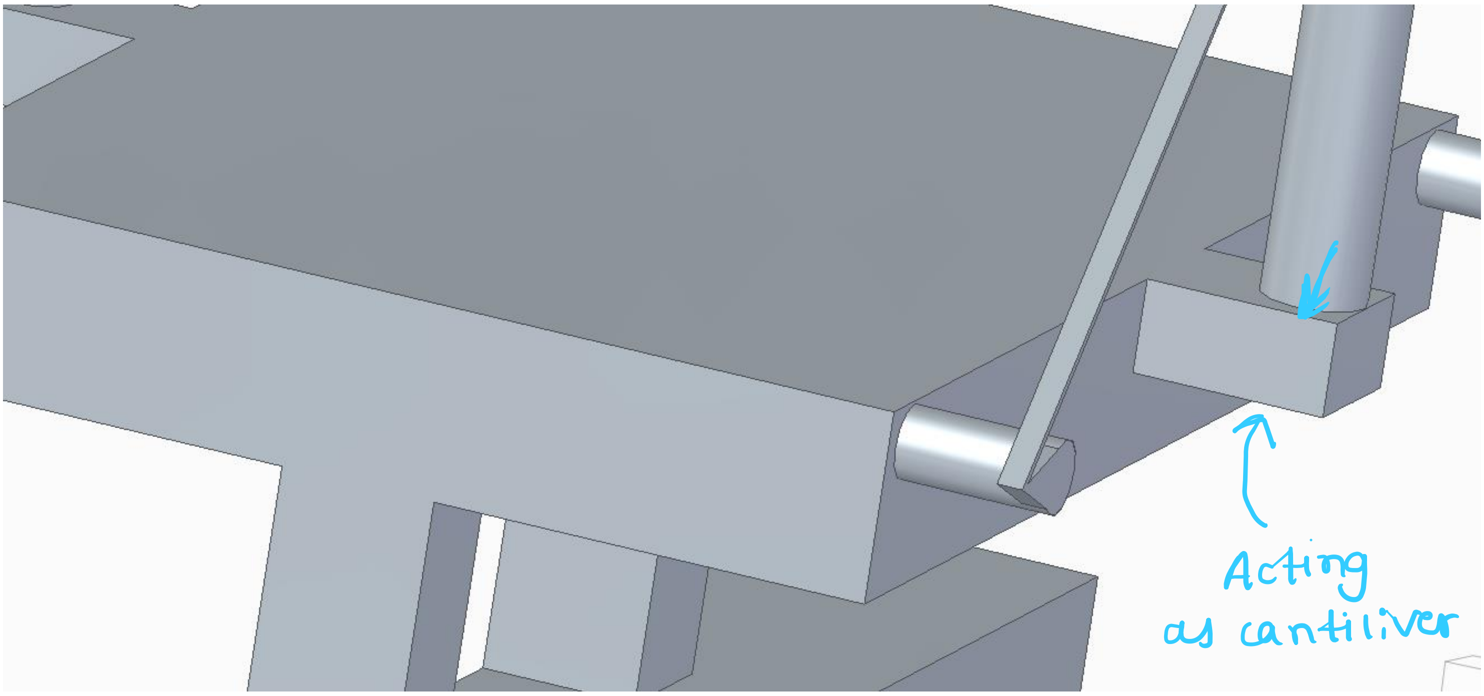


direction of stress trajector

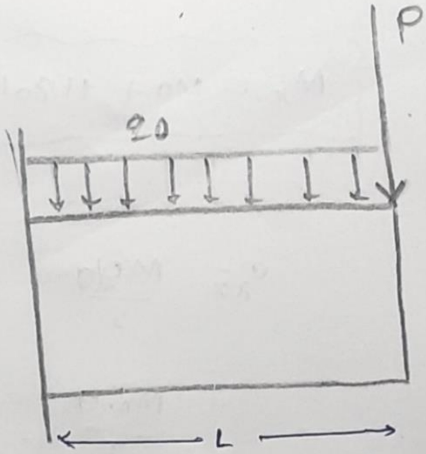
$\theta = 0$

So, this stress trajectory will be simple straight line



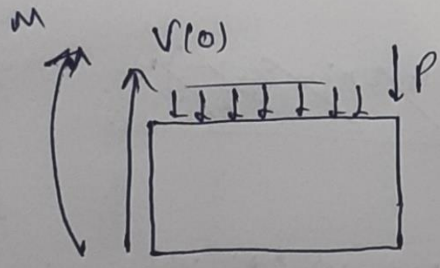

Acting
as cantiliver

part → 5 (cantilever type)



for making things simpler let $P = 10 q_0 L$.

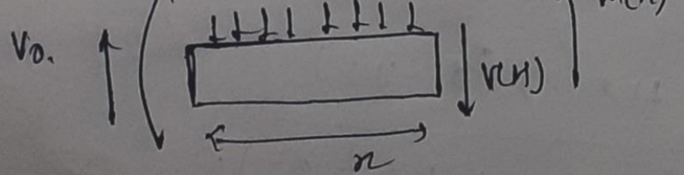
Let's find shear force & bending moment



$V(0) = q_0 L + P$

$= q_0 L + 10 q_0 L$

$= 11 q_0 L$



$M(0) = q_0 L \cdot \frac{L}{2} + P \cdot L$

$= \frac{q_0 L^2}{2} + 10 q_0 L^2$

$M_0 = q_0 L^2 \cdot \frac{21}{2}$

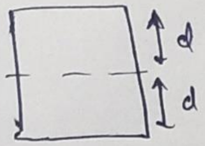
$V(x) = V_0 - q_0 x$

$= 11 q_0 L - q_0 x = q_0 (11L - x)$

$\Sigma M_x = 0$

$$M_x = M_0 + V_0 x - q_0 x \cdot \frac{x}{2}$$

$$\boxed{M_x = M_0 + 11 q_0 L x - \frac{q_0 x^2}{2}}$$



$$\sigma_{xx} = \frac{M y}{I} \qquad \sigma_{yy} \approx 0$$

$$= \frac{M_x \cdot y}{I} \qquad \tau = \frac{VQ}{Ib} = \frac{q_0 (11l - x) \cdot (d^2 - y^2)}{2 \cdot I}$$

$$\tan 2\theta = \frac{2 \tau_{xy}}{\sigma_{xx} - \sigma_{yy}}$$

$$= \cancel{\frac{2\left(\left(M_0 + 11 q_0 L x - \frac{q_0 x^2}{2}\right) y\right.}{}}$$

$$= \frac{\cancel{2} \cdot q_0 (11l - x)(d^2 - y^2) : I}{\cancel{2} I \cdot \left(q_0 \cdot \frac{21}{2} L^2 + 11 q_0 L x - \frac{q_0 x^2}{2}\right) y}$$

$$\tan 2\theta_p = \frac{(11l - x)(d^2 - y^2)}{\left(\frac{21}{2} L^2 + 11 L x - \frac{x^2}{2}\right) y}$$

x	y	θp
1	1	0
1	0.8	1.211162
1	0.6	2.863043
1	0.4	5.583662
1	0.2	12.14314
1	0	45
1	-0.2	-12.1431
1	-0.4	-5.58366
1	-0.6	-2.86304
1	-0.8	-1.21116
1	-1	0

x	y	θp
2	1	0
2	0.8	1.097482
2	0.6	2.595585
2	0.4	5.070469
2	0.2	11.11818
2	0	45
2	-0.2	-11.1182
2	-0.4	-5.07047
2	-0.6	-2.59559
2	-0.8	-1.09748
2	-1	0

x	y	θp
3	1	0
3	0.8	1.002423
3	0.6	2.371652
3	0.4	4.63891
3	0.2	10.2376
3	0	45
3	-0.2	-10.2376
3	-0.4	-4.63891
3	-0.6	-2.37165
3	-0.8	-1.00242
3	-1	0

x	y	θp
4	1	0
4	0.8	0.92175
4	0.6	2.181417
4	0.4	4.271055
4	0.2	9.474183
4	0	45
4	-0.2	-9.47418
4	-0.4	-4.27106
4	-0.6	-2.18142
4	-0.8	-0.92175
4	-1	0

x	y	θp
5	1	0
5	0.8	0.852418
5	0.6	2.017799
5	0.4	3.953828
5	0.2	8.806811
5	0	45
5	-0.2	-8.80681
5	-0.4	-3.95383
5	-0.6	-2.0178
5	-0.8	-0.85242
5	-1	0

x	y	θp
6	1	0
6	0.8	0.792187
6	0.6	1.87557
6	0.4	3.67748
6	0.2	8.218966
6	0	45
6	-0.2	-8.21897
6	-0.4	-3.67748
6	-0.6	-1.87557
6	-0.8	-0.79219
6	-1	0

x	y	θp	x	y	θp
7	1	0	8	1	0
7	0.8	0.739368	8	0.8	0.692667
7	0.6	1.750782	8	0.6	1.640402
7	0.4	3.434599	8	0.4	3.219455
7	0.2	7.697583	8	0.2	7.232227
7	0	45	8	0	45
7	-0.2	-7.69758	8	-0.2	-7.23223
7	-0.4	-3.4346	8	-0.4	-3.21945
7	-0.6	-1.75078	8	-0.6	-1.6404
7	-0.8	-0.73937	8	-0.8	-0.69267
7	-1	0	8	-1	0

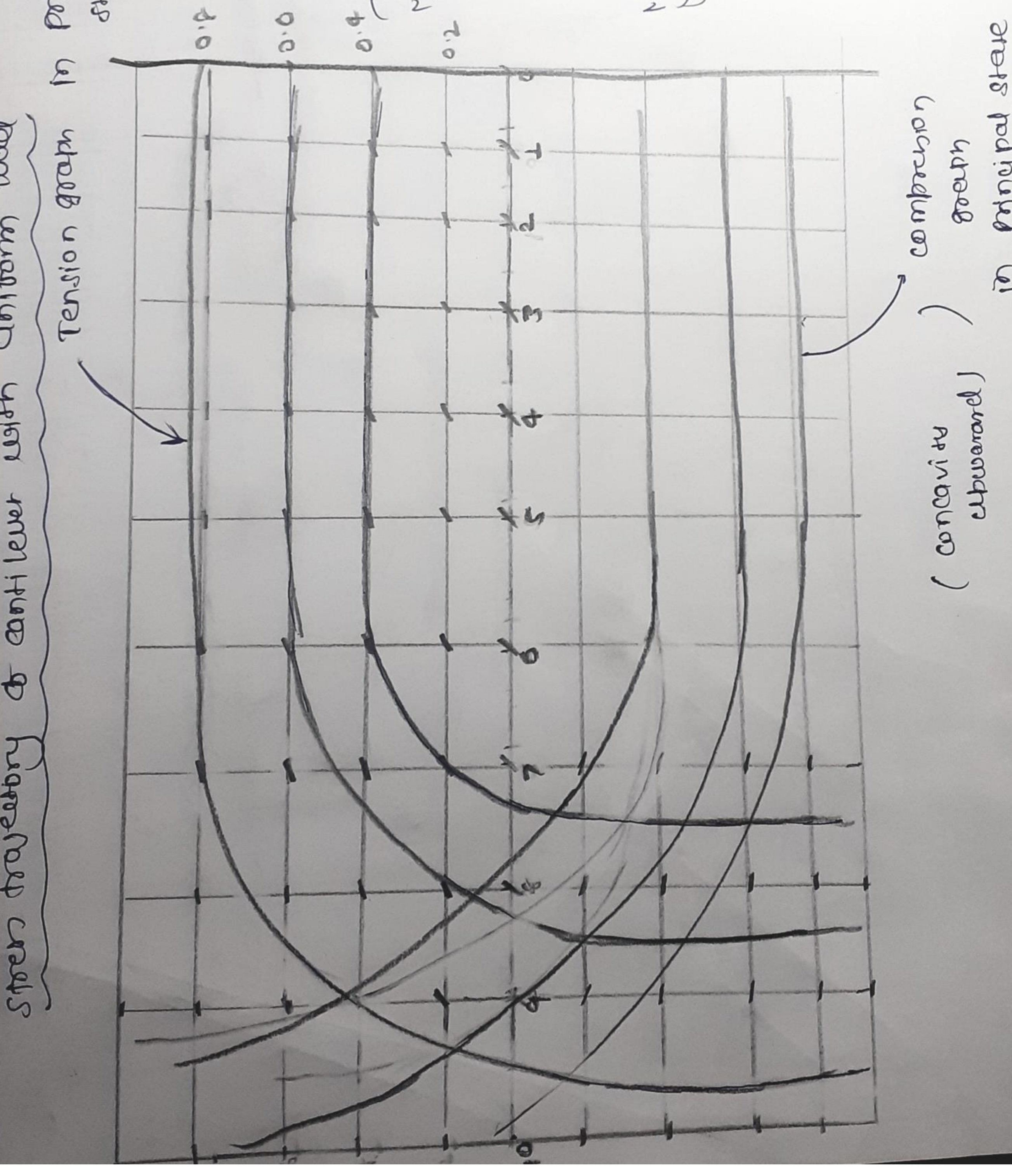
x	y	θp	x	y	θp
9	1	0	10	1	0
9	0.8	0.651074	10	0.8	0.61379
9	0.6	1.542061	10	0.6	1.453882
9	0.4	3.027547	10	0.4	2.855297
9	0.2	6.814483	10	0.2	6.437502
9	0	45	10	0	45
9	-0.2	-6.81448	10	-0.2	-6.4375
9	-0.4	-3.02755	10	-0.4	-2.8553
9	-0.6	-1.54206	10	-0.6	-1.45388
9	-0.8	-0.65107	10	-0.8	-0.61379
9	-1	0	10	-1	0

Now, take assumption

$d = 1$ unit

$l = 10$ unit

$$\tan 2\theta_p = \frac{(110 - x)(1 - y^2)}{\left(\frac{21}{2} \times 100 + 110x - \frac{x^2}{2}\right) y} = \frac{(220 - 2x)(1 - y^2)}{(2100 + 220x - x^2) y}$$



THANK YOU