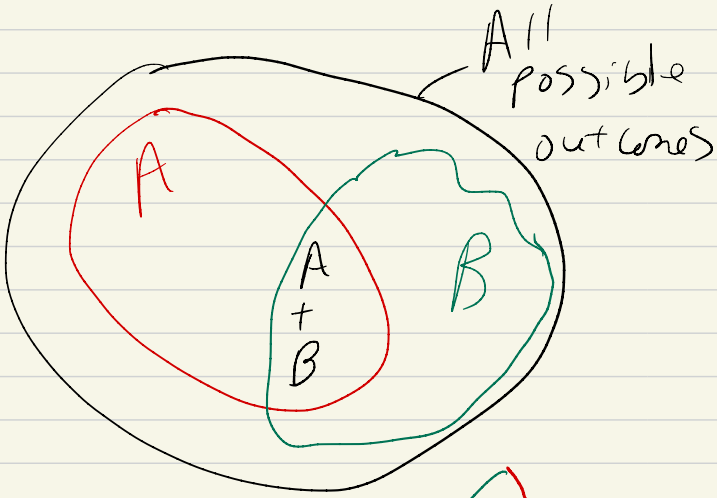
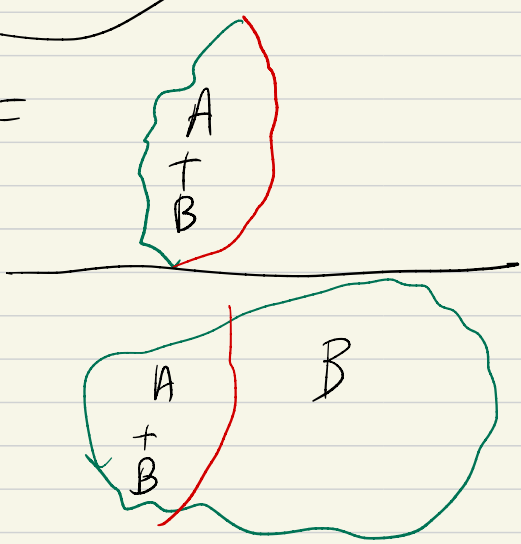
Conditional Probability

We call Pr(A|B) the probability of A conditional on B.



$$Pr(A|B) =$$

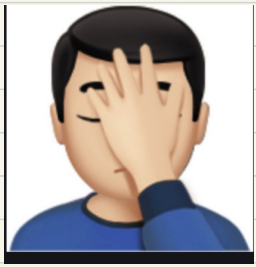


Bayes's Rule

Formal Definition $$Pr(A|B) = \frac{Pr(B|A) \cdot Pr(A)}{Pr(B)}$$

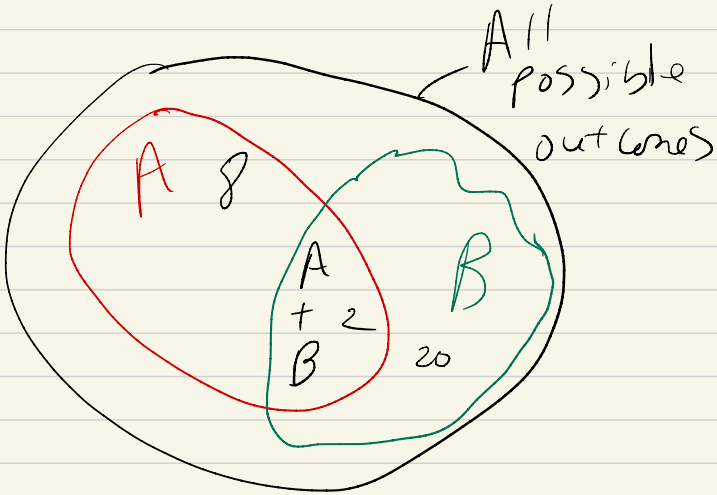
so...

If you know the things on the right you can figure out the things on the left.



Conditional Probability

We call Pr(A|B) the probability of A conditional on B.



$$Pr(A|B) = \frac{A \text{ and } B}{A \text{ or } B} = \frac{2}{22} = \frac{1}{11}$$

$$Pr(A) = \frac{10}{30} = \frac{1}{3}$$

$$Pr(B|A) = \frac{2}{10} = \frac{1}{5}$$

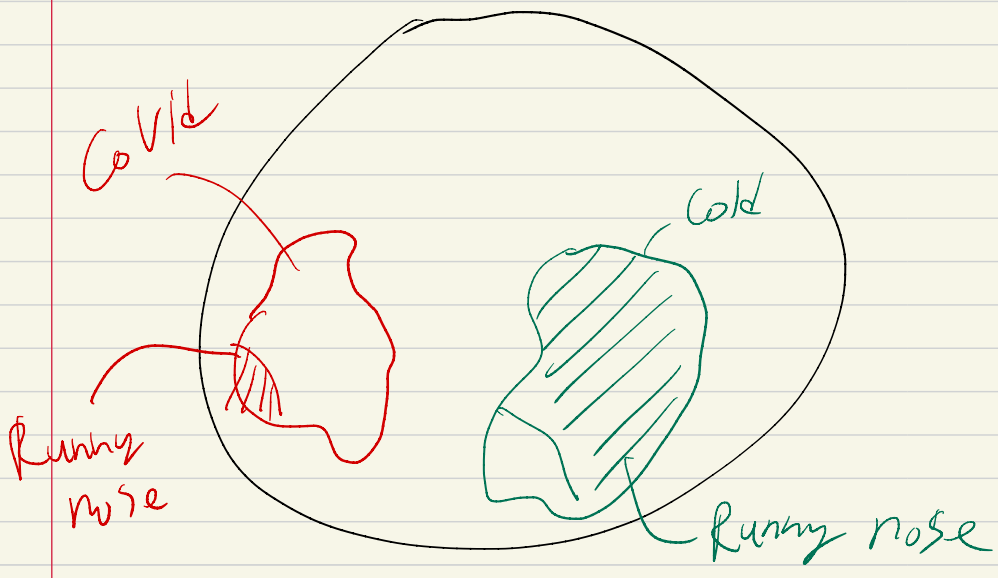
$$Pr(A) \cdot Pr(B|A) = \frac{1}{15}$$

$$Pr(B) = \frac{22}{30} = \frac{11}{15}$$

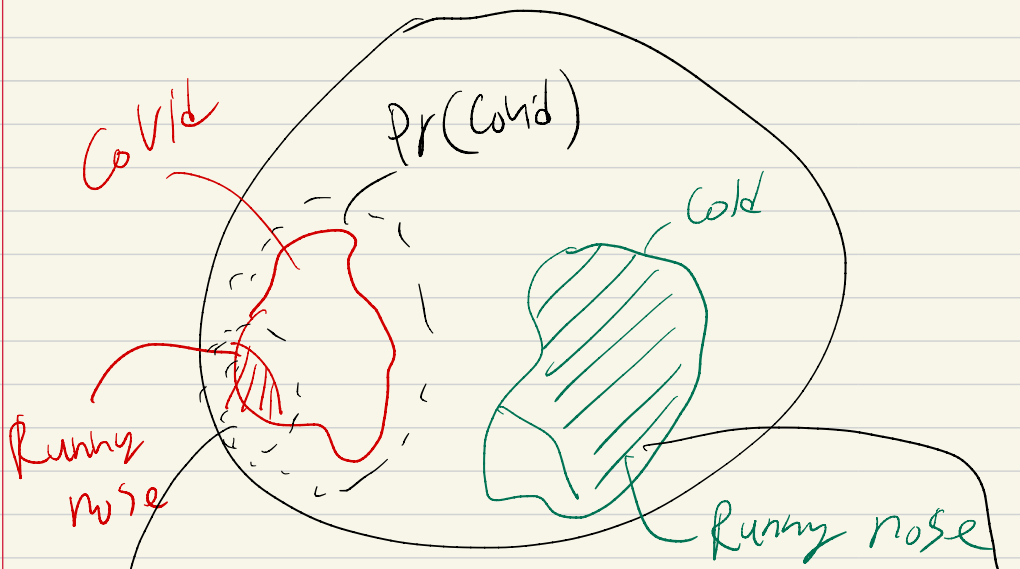
$$\frac{Pr(A) \cdot Pr(B|A)}{Pr(B)} = \frac{\frac{1}{15}}{\frac{11}{15}} = \frac{1}{11}$$

Baye's Rule can help us out when we have counts of observations.

Example: You know how frequently some types of symptoms occur if you have a cold virus and if you have coronavirus. Given that you have a runny nose, what is the probability you have COVID?



$$Pr(\text{covid} \mid \text{Runny nose}) = \frac{\text{▨}}{\text{▨} + \text{▨}}$$



$$\#\ \text{Covid w/RN} = Pr(RN \mid \text{Covid}) \cdot Pr(\text{Covid})$$

$$\#\ \text{Cold w/RN} = Pr(RN \mid \text{Cold}) \cdot Pr(\text{Cold})$$

$$Pr(\text{Covid} \mid \text{Runny nose}) = \frac{Pr(RN \mid \text{Covid})\, Pr(\text{Covid})}{Pr(RN \mid \text{Covid}) \cdot Pr(\text{Covid}) + Pr(RN \mid \text{Cold})\, Pr(\text{Cold})}$$

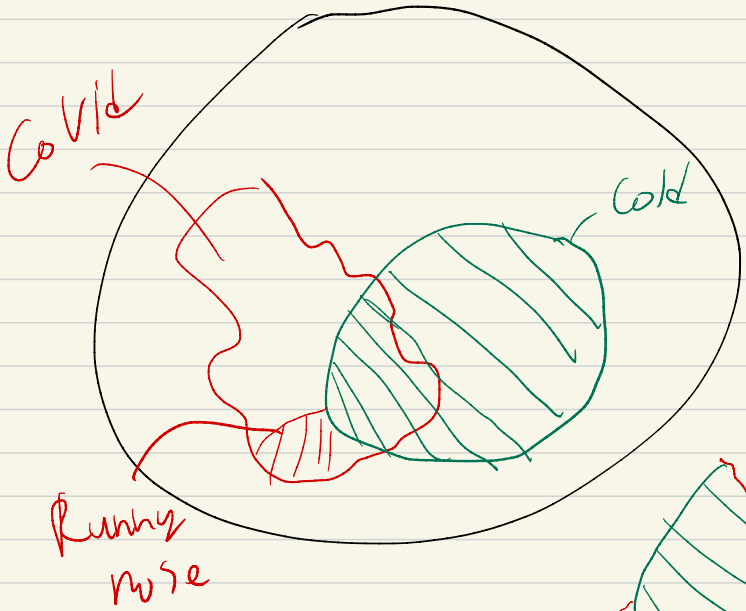
$Pr(RN) =$

e.g. $\frac{x}{x+y}$

So if $y \uparrow$ the value goes down.

To the Break out Rooms!!

Redo with possibility that you have both COVID and a cold, but always have symptoms with a cold. Assume chance of getting each is independent.



$$Pr(\text{Covid} \mid \text{Runny Nose}) = \frac{\text{Pr(Covid)} \cdot \left(\text{Pr(Cold)} \cdot 1 + (1 - \text{Pr(Cold)}) \cdot \text{Pr(RN} \mid \text{Cold)}\right)}{\text{Pr(Covid)} \cdot \left(\text{Pr(Cold)} \cdot 1 + (1 - \text{Pr(Cold)}) \cdot \text{Pr(RN} \mid \text{Cold)}\right) + \text{Pr(Cold)}\left(1 - \text{Pr(Covid)}\right)}$$