Distributions from flexsurv:

"gengamma"	Generalized gamma (stable)	mu	AFT	
"gengamma.orig"	Generalized gamma (original)	scale	AFT	
"genf"	Generalized F (stable)	mu	AFT	
"genf.orig"	Generalized F (original)	mu	AFT	
"weibull"	Weibull	scale	AFT	✓
"gamma"	Gamma	rate	AFT	✓
"exp"	Exponential	rate	PH	
"llogis"	Log-logistic	scale	AFT	✓
"lnorm"	Log-normal	meanlog	AFT	✓
"gompertz"	Gompertz	rate	PH	✓

"exponential" and "lognormal" can be used as aliases for "exp" and "lnorm", for compatibility with survreg.

MRL formulas from paper

GAMMA:

$$m(x) \;=\; \frac{x^{\alpha} exp\left[-\frac{x}{\lambda}\right]}{\lambda^{\alpha-1}\Gamma(\alpha) S_{\mathbf{X}}(x)} + \lambda\alpha - x$$

Gompertz with Shape/scale $\alpha, \lambda > 0$:

$$\Rightarrow m(x) \;=\; \frac{e^{\lambda/\alpha}\left(\frac{1}{\alpha}\right)\Gamma_{inc}(0, z(x))}{exp\left[\frac{\lambda}{\alpha}\left(1-e^{\alpha x}\right)\right]} = e^{z(x)}\left(\frac{1}{\alpha}\right)\Gamma_{inc}(0, z(x))$$

$$\left(\text{where } \; z(x) = \frac{\lambda}{\alpha}e^{\alpha x}\right) \quad \text{where } \Gamma_{inc}(a, x) = \int_x^{\infty} t^{a-1}e^{-t}dt \text{ where } x, a \geq 0$$

Log-logistic with Shape/Scale $\alpha, \lambda > 0$:

$$z(t) = z = \frac{\left(\frac{t}{\lambda}\right)^{\alpha}}{1+\left(\frac{t}{\lambda}\right)^{\alpha}}.$$

$$\left(\frac{\lambda}{\alpha}\right)\Gamma\left(1-\frac{1}{\alpha}\right)\Gamma\left(\frac{1}{\alpha}\right)\underbrace{\int_{z(x)}^{1}\frac{\Gamma\left(1-\frac{1}{\alpha}+\frac{1}{\alpha}\right)}{\Gamma\left(1-\frac{1}{\alpha}\right)\Gamma\left(\frac{1}{\alpha}\right)}(1-z)^{\left(1-\frac{1}{\alpha}\right)-1}z^{\frac{1}{\alpha}-1}dz}_{\text{survival function of a beta}}$$

$$= \left(\frac{\lambda}{\alpha}\right)\Gamma\left(1-\frac{1}{\alpha}\right)\Gamma\left(\frac{1}{\alpha}\right)S_{\mathbf{Z}}\left(z(x); 1-\frac{1}{\alpha}, \frac{1}{\alpha}\right)\left(1+\left(\frac{x}{\lambda}\right)^{\alpha}\right)$$

log-Normal

$$m(x) \;=\; \frac{e^{\left(\mu+\frac{\sigma^2}{2}\right)}\left[1-\Phi\left(\frac{ln(x)-(\mu+\sigma^2)}{\sigma}\right)\right]}{1-\Phi\left(\frac{ln(x)-\mu}{\sigma}\right)} - x$$

Truncated Normal

$S_{\mathbf{Y}}(y) = 1 - \Phi\left(\frac{y-\mu}{\sigma}\right)$

$$m_{\mathbf{X}}(x) = \frac{\frac{\sigma}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}^2\right)} + \mu S_{\mathbf{Y}}(z(x))}{S_{\mathbf{Y}}(x)} - x$$

Weibull with shape/scale $\alpha, \lambda > 0$

$S(x) = exp\left[-\left(\frac{x}{\lambda}\right)^{\alpha}\right]$. Let $z(t) = z = t^{\alpha}$,

$$\frac{1}{\alpha}\int_{z(x)}^{\infty} z^{\frac{1}{\alpha}-1} e^{-\frac{z}{\lambda^{\alpha}}} dz = \frac{1}{\alpha}(\lambda^{\alpha})^{\frac{1}{\alpha}} \Gamma\left(\frac{1}{\alpha}\right) \int_{z(x)}^{\infty} \frac{z^{\frac{1}{\alpha}-1} e^{-\frac{z}{\lambda^{\alpha}}}}{(\lambda^{\alpha})^{\frac{1}{\alpha}} \Gamma\left(\frac{1}{\alpha}\right)}$$

$Z \sim \Gamma\left(\text{shape} = \frac{1}{\alpha}, \text{scale} = \lambda^{\alpha}\right)$.

$$m(x) = \frac{\left(\frac{\lambda}{\alpha}\right) \Gamma\left(\frac{1}{\alpha}\right) S_{\mathbf{Z}}(z(x))}{S_{\mathbf{X}}(x)}$$

Summary:

Distribution	Hazard Rate	Mean Residual Life
Gamma(α, λ) shape parameter $\alpha > 0$ scale parameter $\lambda > 0$	$\alpha < 1$ DCR $\alpha = 1$ constant $(1/\lambda)$ $\alpha > 1$ INC	$\alpha < 1$ INC $\alpha = 1$ constant(λ) $\alpha > 1$ DCR
Gompertz(α, λ) shape parameter $\alpha > 0$ scale parameter $\lambda > 0$	$\forall\alpha$ INC	$\forall\alpha$ DCR
Loglogistic(α, λ) shape parameter $\alpha > 0$ scale parameter $\lambda > 0$	$\alpha \leq 1$ DCR $\alpha > 1$ UBT	$\alpha \leq 1$ undefined $\alpha > 1$ BT
Lognormal(μ, σ) mean $\mu \in \mathbf{R}$ variance $\sigma^2 > 0$	UBT	BT
Truncated normal(μ, σ) mean $\mu \in \mathbf{R}$ variance $\sigma^2 > 0$	INC	DCR
Weibull(α, λ) shape parameter $\alpha > 0$ scale parameter $\lambda > 0$	$\alpha < 1$ DCR $\alpha = 1$ constant $(1/\lambda)$ $\alpha > 1$ INC	$\alpha < 1$ INC $\alpha = 1$ constant(λ) $\alpha > 1$ DCR