Causally Disentangled Generative Variational AutoEncoder

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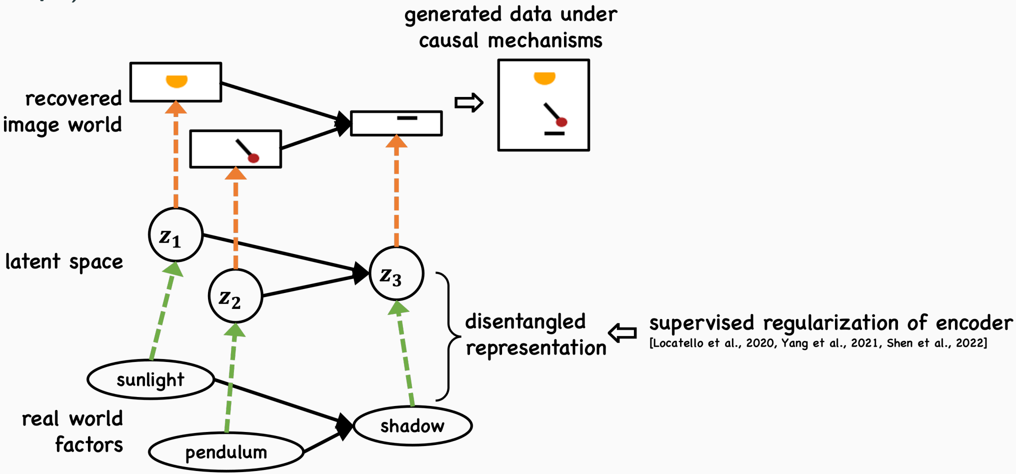
Introduction

Overview

(Example) Pendulum dataset [1]



Goal: **Learning a causally disentangled (causally-aware) generative model**

Contribution:

The disentangled decoder is required to achieve the causally disentangled generative model.

- Assumption (in this presentation): The disentangled representation is already obtained.

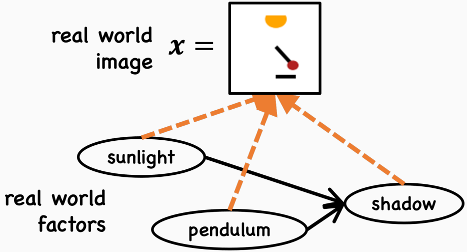
- $\mathbf{z}_1$: sunlight, $\mathbf{z}_2$: pendulum, $\mathbf{z}_3$: shadow

Proposal: CDG-VAE

Assumption 1 (Blocked representation)

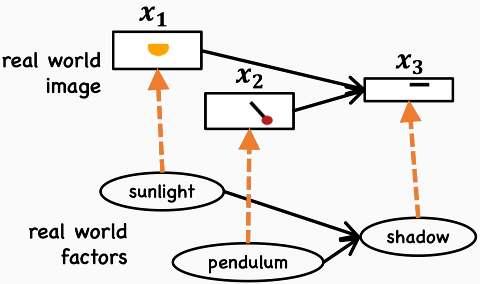
There are (block) causal relationships, $\mathbf{g}_j \to \mathbf{x}_j$, where $\mathbf{g}$ is the ground-truth factors and $j = 1, \cdots, d$.

Conventional representation



$\Longleftrightarrow$

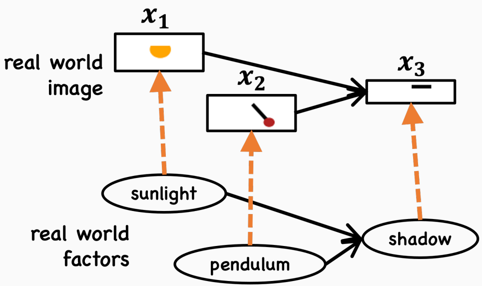
Blocked representation



Definition 1 (Causally Disentangled Generation (CDG))

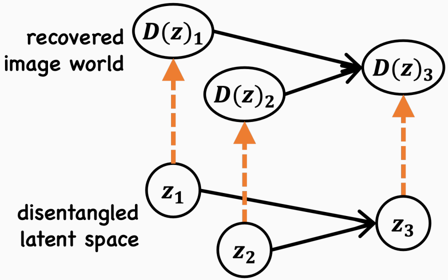
Let $D(\cdot) : \mathbb{R}^d \mapsto \mathbb{R}^p$ be a decoder mean vector of the model where the input is denoted as $\mathbf{z} \in \mathbb{R}^d$. Then the model is causally disentangled generative if, for $i = 1, \cdots, d$, $D(\mathbf{z})_i$ is independent to $\mathbf{z}_s, s \neq i$, given $\mathbf{z}_i$.

Ground-truth DGP



$\Longrightarrow$

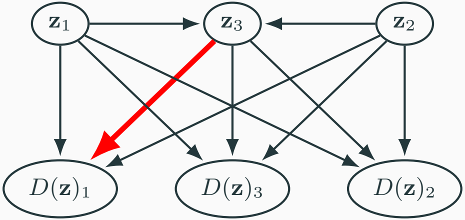
DGP of the decoder



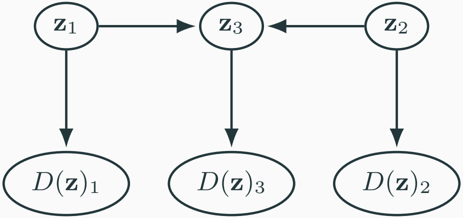
$=$ The disentangled decoder

Why do we need the disentangled decoder?

Entangled decoder $\Rightarrow$ Causally Implausible



Disentangled decoder $\Rightarrow$ Causally Plausible



Definition 2 (Average Causal Effect (ACE))

Suppose that $\mathbf{z}_i, i = 1, \cdots, d$ is intervened with $z_i^{(1)}$ and $z_i^{(2)}$. Then, for $c = 1, \cdots, d$, the average causal effect of $\mathbf{z}_i$ on the annotation $\mathbf{u}_c$ given $z_{ND(i)}$ is defined as

$$ACE(\mathbf{u}_c, \mathbf{z}_i, \mathbf{z}_{ND(i)} = z_{ND(i)}) \quad := \quad \left| \mathbb{E}[\mathbf{u}_c | z_{ND(i)}, do(\mathbf{z}_i := z_i^{(1)})] - \mathbb{E}[\mathbf{u}_c | z_{ND(i)}, do(\mathbf{z}_i := z_i^{(2)})] \right|.$$

$\Rightarrow$ The causal effect is measured by the difference of the annotation vector.

Proposition 1 (Sufficient Condition for CDG)

Let $D(\cdot;\theta) : \mathbb{R}^d \mapsto \mathbb{R}^p$ be a decoder mean vector of the model. If the decoder structure of the model satisfies $D(\mathbf{z};\theta) := \Big(D(\mathbf{z}_1;\theta_1), \cdots, D(\mathbf{z}_d;\theta_d)\Big)$, where $\theta = (\theta_1, \cdots, \theta_d)$, and $D(\cdot;\theta_j)$ is a function parameterized with θ_j for $j = 1, \cdots, d$, then the model satisfies Definition 1.

- a generated image

$$D(\mathbf{z};\theta) = \Big(\quad D(\mathbf{z};\theta)_1 \quad , \quad D(\mathbf{z};\theta)_2 \quad , \quad D(\mathbf{z};\theta)_3 \quad \Big)$$

$$\Downarrow$$

$$D(\mathbf{z};\theta) = \Big(\underbrace{D(\mathbf{z}_1;\theta_1)}_{\text{sunlight image}} , \underbrace{D(\mathbf{z}_2;\theta_2)}_{\text{pendulum image}} , \underbrace{D(\mathbf{z}_3;\theta_3)}_{\text{shadow image}} \Big)$$

What is the property of CDG?

Proposition 2 (Necessary Conditions for CDG)

For $i = 1, \cdots, d$, assume that arbitrary x and $z_{ND(i)}$ are given. $z^{(j)}_{(i, z_{ND(i)}, x)}$ denotes the value of $\mathbf{z}$ under intervention $do(\mathbf{z}_i := z_i^{(j)})$ given x and $z_{ND(i)}$, for $j = 1, 2$. For $c = 1, \cdots, d$, under the faithfulness assumption, if the model satisfies CDG (Definition 1) and

1. $c \in ND(i)$, then

$$ACE(\mathbf{u}_c, \mathbf{z}_i, z_{ND(i)}) = 0.$$

$\Rightarrow$ *Non-descendants are NOT affected by the intervention.*

2. there is a directed path from $\mathbf{z}_i$ to $\mathbf{z}_c$ where $c \in Des(i)$, then

$$0 < ACE(\mathbf{u}_c, \mathbf{z}_i, z_{ND(i)}) \leq \mathbb{E}_{p(\mathbf{x})} \left| \mathbb{E}[\mathbf{u}_c | z^{(1)}_{(i, z_{ND(s)}, \mathbf{x})}] - \mathbb{E}[\mathbf{u}_c | z^{(2)}_{(i, z_{ND(s)}, \mathbf{x})}] \right|,$$

where $p(\mathbf{x})$ is the probability density function of $\mathbf{x}$.

$\Rightarrow$ *Descendants are affected by the intervention.*

Definition 3 (Causal Disentanglement Metric (CDM))

For $c, i = 1, \cdots, d$, the causal disentanglement metric (CDM) is defined as

$$CDM(c, i) := \mathbb{E}[ACE(\mathbf{u}_c, \mathbf{z}_i, \mathbf{z}_{ND(i)})],$$

where $\mathbb{E}$ indicates the expectation with respect to $\mathbf{z}_{ND(i)}$.

$\Rightarrow$ Expected value of the average causal effect.

1. **interventional robustness** [Suter et al., 2019]
2. **counterfactual generativeness** [Reddy et al., 2022]

Experiments

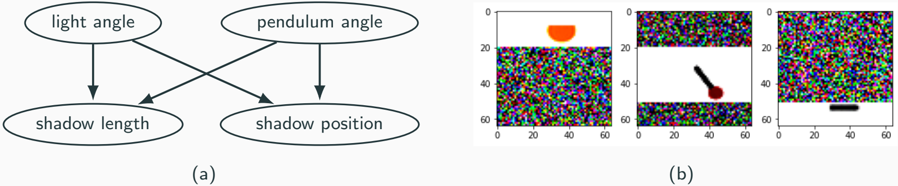
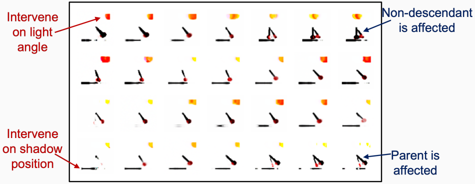
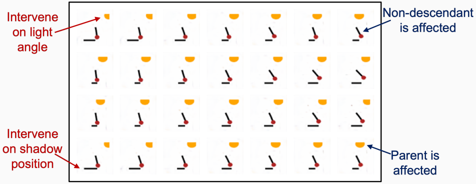


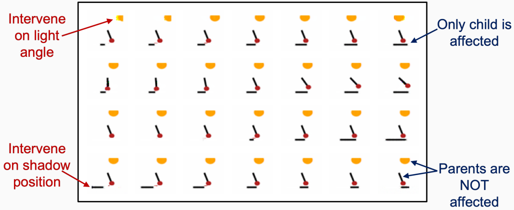
Figure 1: (a) DAG of the ground-truth factors: $\mathbf{g}_1$(light angle), $\mathbf{g}_2$(pendulum angle), $\mathbf{g}_3$(shadow length), and $\mathbf{g}_4$(shadow position). (b) From left to right, $\mathbf{x}_1$ (light), $\mathbf{x}_2$ (pendulum), and $\mathbf{x}_3$ (shadow).



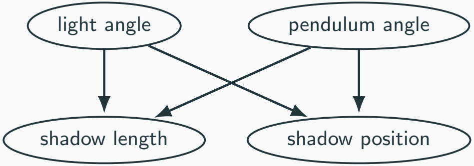
(a) CausalVAE [Yang et al., 2021]



(b) DEAR [Shen et al., 2022]



(c) CDG-VAE



(d) DAG of the ground-truth factors

$\Rightarrow$ CDG-VAE under Proposition 1 enables CDG!

Table 1: Numbers in parentheses are lower and upper bounds of CDM. 'L' and 'NL' denote the model with linear and nonlinear f, and '*' denotes the semi-supervised learned model. Mean and standard deviation values are obtained from 10 repeated experiments. 'pos' denotes shadow position. ↑ denotes higher is better and ↓ denotes lower is better.

	Interventional Robustness ↓		Counterfactual Generativeness ↑	
Model	$CDM(light, length)$	$CDM(angle, pos)$	$CDM(length, angle)$	$CDM(pos, pos)$
VAE(L)	$(0.44, 0.44)_{\pm(0.35,0.35)}$	$(0.28, 0.28)_{\pm(0.30,0.31)}$	$(0.31, 0.32)_{\pm(0.16,0.15)}$	$(0.27, 0.28)_{\pm(0.25,0.24)}$
VAE(NL)	$(0.38, 0.40)_{\pm(0.28,0.27)}$	$(0.27, 0.33)_{\pm(0.25,0.24)}$	$(0.33, 0.34)_{\pm(0.12,0.12)}$	$(0.31, 0.34)_{\pm(0.21,0.20)}$
InfoMax(L)	$(0.42, 0.43)_{\pm(0.39,0.38)}$	$(0.38, 0.38)_{\pm(0.34,0.34)}$	$(0.40, 0.40)_{\pm(0.26,0.25)}$	$(0.29, 0.31)_{\pm(0.22,0.20)}$
InfoMax(NL)	$(0.37, 0.39)_{\pm(0.32,0.30)}$	$(0.26, 0.33)_{\pm(0.28,0.25)}$	$\mathbf{(0.44, 0.44)}_{\pm(0.21,0.21)}$	$(0.31, 0.34)_{\pm(0.19,0.16)}$
CausalVAE	$(0.28, 0.28)_{\pm(0.11,0.10)}$	$(0.17, 0.17)_{\pm(0.09,0.08)}$	$(0.10, 0.10)_{\pm(0.04,0.04)}$	$(0.29, 0.29)_{\pm(0.09,0.09)}$
DEAR	$(0.21, 0.23)_{\pm(0.16,0.15)}$	$(0.26, 0.29)_{\pm(0.25,0.24)}$	$(0.23, 0.25)_{\pm(0.23,0.23)}$	$(0.16, 0.20)_{\pm(0.18,0.16)}$
CDG-VAE(L)	$\mathbf{(0.00, 0.00)}_{\pm(0.00,0.00)}$	$\mathbf{(0.00, 0.00)}_{\pm(0.00,0.00)}$	$(0.24, 0.25)_{\pm(0.10,0.09)}$	$(0.69, 0.69)_{\pm(0.25,0.25)}$
CDG-VAE(NL)	$\mathbf{(0.00, 0.00)}_{\pm(0.00,0.00)}$	$\mathbf{(0.00, 0.00)}_{\pm(0.00,0.00)}$	$(0.35, 0.36)_{\pm(0.16,0.15)}$	$\mathbf{(0.78, 0.78)}_{\pm(0.24,0.24)}$
CDG-VAE(L)*	$\mathbf{(0.00, 0.00)}_{\pm(0.00,0.00)}$	$\mathbf{(0.00, 0.00)}_{\pm(0.00,0.00)}$	$(0.21, 0.22)_{\pm(0.09,0.07)}$	$(0.66, 0.66)_{\pm(0.22,0.22)}$
CDG-VAE(NL)*	$\mathbf{(0.00, 0.00)}_{\pm(0.00,0.00)}$	$\mathbf{(0.00, 0.00)}_{\pm(0.00,0.00)}$	$(0.29, 0.30)_{\pm(0.12,0.11)}$	$\mathbf{(0.79, 0.79)}_{\pm(0.21,0.21)}$

References

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Thank you!

Figure 2: GitHub repository link of CDG-VAE.