

## Set Theory

>Set is the collection of object at any sort  
>That means the collection of object as a common behavior

### Venn diagram

>A Venn diagram uses overlapping circles or other shapes to illustrate the logical relationships between two or more sets of items

>Often, they serve to graphically organize things, highlighting how the items are similar and different  
>Venn diagrams, also called Set diagrams or Logic diagrams, are widely used in mathematics, statistics, logic teaching, linguistics, computer science and business

### Properties of Set

#### Null Set:

>Null set is the set that does not contain any element  
>It is denoted by  $\{\}$  or  $\{\varnothing\}$  (Theeta)

#### Equal Set:

>The set 'x' and 'y' to be equal then its elements are same  
>Example :  $x = \{1, 2, 3\}$  and  $y = \{1, 2, 3\}$  then  $x = y$ .

#### Equivalent Set :

>If the two set said to be equivalent then number of elements in the two sets are equal

>Example :  $x = \{1, 2, 3\}$  and  $y = \{a, b, c\}$   
then  $x = y$

#### Subset :

> Let 'x' and 'y' be the two sets then 'y' is said to be the subset of 'x' then all the elements of y is in x

#### Power Set:

>The Power set (or Powerset) of a Set A is defined as the set of all subsets of the Set A.

>It is denoted by  $P(A)$ .

>Basically, this set is the combination of all subsets including null set, of a given set

>Example : Set  $A = \{a, b, c\}$

The power set  $P(A) = \{\varnothing, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \{a, b, c\}\}$

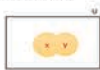
### Operations on a Sets:

#### Union:

>Let 'x' and 'y' be the two sets. Then union of 'x' and 'y' be denoted by  $x \cup y$

>That means the resultant set contains all the elements of 'x' and 'y'

>Example :  $x = \{1, 2, 3, 4\}$  and  $y = \{1, 4, 7, 9\}$   
 $x \cup y = \{1, 2, 3, 4, 7, 9\}$



#### Intersection:

The intersection of two set 'x' and 'y' be denoted by  $x \cap y$

>That means resultant set contain the common elements that are in 'x' and 'y'

>Example :  $x = \{1, 2, 3, 4\}$  and  $y = \{1, 4, 7, 9\}$

$x \cap y = \{1, 4\}$



#### Set Difference:

>The set difference of two set 'x' and 'y' is  $x - y$ .

>That means elements in 'x' not in 'y' and both

>Example :  $x = \{1, 2, 3, 4\}$  and  $y = \{1, 4, 6\}$

then  $x - y = \{2, 3\}$

$y - x = \{6\}$



#### Complement:

>The complement of a set, denoted  $A'$ , is the set of all elements in the given universal set U that are not in A.

>In set-builder notation,  $A' = \{x \in U : x \notin A\}$

>Example :

Let Universal set  $U = \{a, b, c, d, e, f, g, h\}$  and  $A = \{a, c, g\}$

then  $A' = \{b, d, e, f, h\}$



### Theorems in Sets

Let  $x, y, z$  be the subset of set "U", then the following assertions are satisfied

1.  $y \subseteq x$  then  $x \cup y = x$  and  $x \cap y = y$

2. Law of identity:

$x \cup \varnothing = x$   
 $x \cap \varnothing = \varnothing$

3. Law of Independent

$x \cup x = x$   
 $x \cap x = x$

4. Law of Commutative

$x \cup y = y \cup x$   
 $x \cap y = y \cap x$

5. Law of Associative

$(x \cup y) \cup z = x \cup (y \cup z)$   
 $(x \cap y) \cap z = x \cap (y \cap z)$

6. Law of Distributive

$x \cup (y \cap z) = (x \cup y) \cap (x \cup z)$   
 $x \cap (y \cup z) = (x \cap y) \cup (x \cap z)$

7. Law of Absorptivity

$x \cap (x \cup y) = x$   
 $x \cup (x \cap y) = x$

## Mathematical Logic

A Statement or a Preposition is a declarative sentence .

>That means sentence is either true or false, but not both.

>Example :- 4 is an integer – it is true statement

5 is an integer - It is a false statement

• So we can represent the statements as the following way.

P : 4 is an integer

• Where "P" is the statement name and "4 is an integer" is the given statement.

• Example :- Q : 5 is an integer

P : Enjoy the lovely weather.

### Negation:

>Let "P" be the statements, the negation of "P" is "¬P" obtained by negating the statement "P".

>The symbol tilde (¬) is called not.

>If "P" is the statement the its negation is formed by writing it is not

the case of "P".

>It is also represented by  $P'$  ,  $\neg P$  ,  $!P$ .

>Example :- P : 2 is an integer. Negation of this statement is

¬P : 2 is not an integer

### Disjunction:

>Let "P" and "Q" be the statement, then

Disjunction of "P" and "Q" is  $P \cup Q$ .

> That means the statements formed by joining the statements "P" and "Q" using the word OR.

• The symbol  $\cup$  is called OR

### Conjunction:

>Let "P" and "Q" be the statement, then

Conjunction of "P" and "Q" is  $P \cap Q$ .

>That means the statements formed by joining the statements "P"

and "Q" using the word AND.

>The symbol  $\cap$  is called AND..

### Theorem - 2:

Every tree has either one or two Centre's.

> Proof :

>Given Tree "T", degree of vertices  $u \& v$  is  $d(u, v)$ , that

is the maximum distance in the Tree "T".

>Where  $u \& v$  are the two end points. Finding the

Centre of a given tree "T", remove all pendant vertex

repeatedly form the each step we get a new tree.

>Repeat this process until one vertex or one edge (with pair of vertices) has been left.

>So that if one vertex is left, that vertex is Centre of a tree.

> If one edge is left, then the pair of vertices associated with that edge is center

### Theorem - 3

>In any tree with two or more vertices there are at least two pendant vertices.

>Proof :

>Every tree has "n" vertices and "(n-1)" edges.

>Then total degree of tree =  $2e = 2(n-1)$

>We assume that in a tree, no vertex has degree one and every vertex have degree at least two.

>Then the total degree of a tree =  $2n$ .

>Which contradict the total degree of a tree.

>In another way assume that given tree, one

vertex has degree one and remaining (n-1)

vertices has degree at least two.

>So total degree =  $1 + 2(n-1)$

$= 1 + 2n - 2$

$= 2n - 1$

>Which also contradict the total degree of a tree.

### Theorem - 1

• A graph is a tree, then it is minimally connected.

• Proof :

>Tree is connected graph but it is circuit-less. A

tree has "n" vertices and (n-1) edges means the

minimum number of edges can be used to form tree. So we called tree is a minimally connected

### Quantifiers

Let "P" be the a predictive even though 'P' is not a

statement.

Then it can turned into a statement through a

process is

called Quantification.

• There are two quantifiers

• Universal Quantifiers

• Existential QuantifiersQuantifiers

• Universal Quantifiers :

• Let 'P' be the predictive statement and 'P' be the

domain of all

numbers.

• Then universal quantifiers of 'P' is statement

for all values of 'x', statement 'P' is true

$\forall x P(x)$  is true

• The symbol  $\forall$  used for all/for every.Quantifiers

• Existential Quantifiers :

• Let 'P' be the predictive statement and 'P' be the

domain of all

numbers.

• Then existential quantifiers of 'P' is statement

some values of 'x', statement 'P' is true

$\exists x P(x)$  is true

• The symbol  $\exists$  used for some values

Uniqueness Quantifiers :

• Let 'P' be the predictive statement .

• Then uniqueness quantifiers of 'P' is statement

for one values of 'x', statement 'P' is true

$\exists! x P(x)$  is true

• The symbol  $\exists!$  used for some values.

• Example  $\exists! x (x-2 = 0)$

## Converse :

• The statement  $Q \rightarrow P$  is called Converse of the

implication

statement  $P \rightarrow Q$  .

• Inverse :

• The statement " $P \rightarrow \neg Q$  is called the Inverse of the

implication

statement  $P \rightarrow Q$  .

• Contrapositive :

• The statement " $Q \rightarrow \neg P$  is called the Contrapositive of the

implication statement  $P \rightarrow Q$  .

## Contradiction

• A statement formula A is said to be a "Contradiction", if the truth value of statement A is False(F) for any assignment of any truth values "T" and "F" to the statement variable.

• Example :  $\neg P \wedge P$

## Tautology

• A statement formula A is said to be a "Tautology", if the truth value of statement A is True(T) for any assignment of any truth values "T" and "F" to the statement variable.

• Example :  $\neg P \rightarrow (P \rightarrow Q)$

## Contingency

• A statement formula A is said to be a "Contingency", if the truth value of statement A is True(T) and False(F) for any assignment of any truth values "T" and "F" to the statement variable.

• Example :  $(P \rightarrow Q) \rightarrow P$

**Hamiltonian graph** – A connected graph G is called Hamiltonian graph if there is a cycle which includes every vertex of G and the cycle is called Hamiltonian cycle. Hamiltonian walk in graph G is a walk that passes through each vertex exactly once.

### Travelling Salesman Problem

Suppose a salesman wants to visit a certain number of cities allotted to him. He knows the distance of the journey between every pair of cities. His problem is to select a route the starts from his home city, passes through each city exactly once and return to his home city the shortest possible distance. This problem is closely related to finding a Hamiltonian circuit of minimum length. If we represent the cities by vertices and road connecting two cities edges we get a weighted graph where, with every edge  $e_i$  a number  $w_i$ (weight) is associated.

A physical interpretation of the above abstract is: consider a graph G as a map of n cities where  $w(i, j)$  is the distance between cities i and j. A salesman wants to have the tour of the cities which starts and ends at the same city includes visiting each of the remaining a cities once and only once.

In the graph, if we have n vertices (cities), then there is (n-1)! Edges (routes) and the total number of Hamiltonian circuits in a complete graph of n vertices will be Travelling Salesman Problem.

### What is Prims algorithm explain with example?

Prim's algorithm finds the subset of edges that includes every vertex of the graph such that the sum of the weights of the edges can be minimized. Prim's algorithm starts with the single node and explore all the adjacent nodes with all the connecting edges at every step

P	¬P
T	F
F	T