Q

1- constract the truth tables of the following formulas.

a) $(Q \wedge (P \to Q)) \to P$

Ans:-

P	Q	$P \to Q$	$Q \wedge (P \to Q)$	$(Q \wedge (P \to Q)) \to P$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

b) $\neg(P \vee (Q \wedge R)) \rightleftarrows ((P \vee Q) \wedge (P \vee R))$

P	Q	R	$P \vee Q$	$Q \wedge R$	$P \vee R$	$P \vee (Q \wedge R)$	$\neg P \vee (Q \wedge R)$
T	T	T	T	T	T	T	F
T	F	F	T	F	T	T	F
T	F	T	T	F	T	T	F
T	T	F	T	F	T	T	F
F	T	T	T	T	T	F	F
F	T	F	F	F	F	F	T
F	F	T	F	F	T	F	T
F	F	F	F	F	F	F	T

$((P \vee Q) \wedge (P \vee R))$	$\neg(P \vee (Q \vee R)) \rightleftarrows ((P \vee Q) \wedge (P \vee R))$
T	F
T	F
T	F
T	F
T	F
F	F
F	F
F	F

(c) $((P \to Q) \wedge (Q \to R)) \to (P \to R)$

P	Q	R	$P \to Q$ (I)	$Q \to R$ (J)	$I \wedge J$ (U)	$P \to R$ (N)	$U \to N$
T	T	T	T	T	T	T	T
T	F	F	F	T	F	F	F
T	F	T	F	T	F	T	T
T	F	F	T	F	F	F	T
F	F	T	T	T	T	T	T
F	F	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

(d) $(P \rightleftarrows Q) \rightleftarrows ((P \wedge Q) \vee (\neg P \wedge \neg Q))$

P	Q	$\neg P$	$\neg Q$	$P \wedge Q$ (L)	$P \rightleftarrows Q$ (I)	$\neg P \wedge \neg Q$ (K)	$L \vee K$	$I \rightleftarrows (L \vee K)$
T	T	F	F	T	T	F	T	T
T	F	F	T	F	F	F	F	T
F	T	T	F	F	F	F	F	T
F	F	T	T	F	T	T	T	T

2) Given the truth values of P and Q as T and those of R and S as F, find truth values of the following.

(a) $(\neg(P \wedge Q) \vee \neg R) \vee ((Q \rightleftarrows \neg P) \to (R \vee \neg S))$

P	Q	R	S	$\neg P$	$\neg R$	$\neg S$	$P \wedge Q$	$\neg(P \wedge Q)$	$\neg(P \wedge Q) \vee \neg P$	$Q \rightleftarrows \neg P$
T	T	F	F	F	T	T	T	F	T	F

$(\neg(P \wedge Q) \vee \neg R) \vee (Q \rightleftarrows \neg P)$	$(R \vee \neg S)$	@
T	T	T

(b)

P	Q	R	S	$\neg Q$	$P \rightleftarrows R$ (I)	$\neg Q \to S$ (J)	$I \wedge J$
T	T	F	F	F	F	T	F

(c) $(P \vee (Q \to (R \wedge \neg P))) \rightleftarrows (Q \vee \neg S)$

P	Q	R	S	$\neg P$	$\neg S$	$(R \wedge \neg P)$	$Q \to (R \wedge \neg P)$	$P \vee (Q \to (R \wedge \neg P))$
T	T	F	F	F	T	F	F	T

$Q \vee \neg S$	$P \vee (Q \to (R \wedge \neg P)))$
T	T

⇒ ~~Conti~~ Contingency

A proposition is neither a tautology nor a contradiction is called contingency.

Exclusive OR

Let p and Q be propositions, the exclusive or of p and Q is denoted by $P \oplus Q$ be the proposition, is true when one of p and Q is true, otherwise false.

P	Q	$P \oplus Q$
T	T	F
T	F	T
F	T	T
F	F	F

~~Converse inverse and contrapositive.~~

* A connective denoted by $\bar{\vee}$ is defined by

P	Q	$P \bar{\vee} Q$
T	T	F
T	F	T
F	T	T
F	F	F

Find a formula using p, Q and the connectives $\wedge$, $\vee$ and $\neg$ whose truth values are identical to the truth values of $P \bar{\vee} Q$.

Soln:- $\neg(P \wedge Q) \wedge (P \vee Q)$

P	Q	$P \wedge Q$	$\neg(P \wedge Q)$	$P \vee Q$	$\neg(P \wedge Q) \wedge (P \vee Q)$
T	T	T	F	T	F
T	F	F	T	T	T
F	T	F	T	T	T
F	F	F	T	F	F

Well-formed formulas

A statement formula is an expression which is a string consisting of variables, parentheses, and connective symbols.

- A well formed formula can be generated by the following ~~formula~~ rules:

1) A stt variable standing alone is a well formed formula
2) If A is a well formed formula, then $\neg A$ is a well formed formula
3) If A and B are well formed formula, then $(A \wedge B)$, $(A \vee B)$, $(A \to B)$ and $(A \rightleftarrows B)$ are well-formed formulas.

Examples for not well-formed formulas

1) $\neg P \wedge Q$
$(\neg P \wedge Q)$ is a well formed formula
2) $(P \to Q) \to (\wedge Q)$ is not a well formed formula because $\wedge Q$ is not

3) $(P \wedge Q) \rightarrow Q)$ – it is not well formed formula

Tautology

A stt formula which is true regardless of the truth values of the stts which replace the variable in it is called a universally valid formula or a tautology or a logical truth

A stt formula which is false regardless of the truth values of the stts which replace the variable in it is called a contradiction.

Eg:-

P	$\neg P$	$P \vee \neg P$	$P \wedge \neg P$
T	F	T	F
F	T	T	F

1) - From the formulas given below, select those which are well formed according to the definiton in sec. 1-2.7, and indicate which ones are tautologies or contradictions.

a) $(P \rightarrow (P \vee Q))$

Ans:- it is well formed formula

P	Q	$P \vee Q$	$P \rightarrow Q$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

~~it is tautology~~. contigency it is tautology.

b) $((P \rightarrow (\neg P)) \rightarrow \neg P)$

P	$\neg P$	$(P \rightarrow (\neg P))$	$((P \rightarrow (\neg P)) \rightarrow \neg P)$
T	F	F	T
T	F	F	T
F	T	T	T
F	T	T	T

it's tautology

c) $(\neg Q \wedge P) \wedge Q)$

P	Q	$\neg Q$	$\neg Q \wedge P$	$((\neg Q \wedge P) \wedge Q)$
T	T	F	F	F
T	F	T	T	F
F	T	F	F	F
F	F	T	F	F

contradiction

d) $(P \rightarrow (Q \rightarrow R)) \rightarrow (P \rightarrow Q) \rightarrow (P \rightarrow R)))$

P	Q	R	$Q \rightarrow R$	$(P \rightarrow (Q \rightarrow R))$	$(P \rightarrow Q)$	$P \rightarrow R$	$(P \rightarrow Q) \rightarrow (P \rightarrow R)$	(d)
T	T	T	T	T	T	T	T	T
T	F	F	F	F	F	F	T	T
T	F	T	T	T	F	T	T	T
T	T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T	T

e) $((\neg P \rightarrow Q) \rightarrow (Q \rightarrow P))$

p	q	$\neg p$	$\neg p \to q$	$q \to p$	$((\neg p \to q) \to (q \to p))$
T	T	F	T	T	T
T	F	F	T	T	T
F	T	T	T	F	F
F	F	T	F	T	T

f) $((p \land q) \leftrightarrow p)$

p	q	$p \land q$	$(p \land q) \leftrightarrow p$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

contingency

A proposition is neither a tautology nor a contradiction is called contingency

Exclusive or

let p and q be propositions, the exclusive or of p and q is denoted by $p \oplus q$ be the proposition, is true when one of p and q is true, otherwise false

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

converse, inverse and contrapositive

* The converse of the conditional stt $p \to q$ is $q \to p$
* The inverse of the conditional stt $p \to q$ is $\neg p \to \neg q$ (if not p then not q)
* The contrapositive of the conditional stt $p \to q$ is $\neg q \to \neg p$ (if not q then not p)

Eg:- Find the converse and the contrapositive of "If today is Thursday, the i have a test today"

Soln:- converse :- if i have a test today, then today is thursday.

contrapositive:- if i dont have a test today, then today is not thursday.

Logical equivalence.

compound propositions that have the same truth values in all possible cases are called logically equivalent.

From the definition of bi-conditional, $p \leftrightarrow q$ / $p \rightleftarrows q$ is true whenever p and q have the same truth values, so we can say that p and q are logically equivalent if $p \leftrightarrow q$ is a ~~tauto~~ tautology conversely, if $p \leftrightarrow q$ is a tautology, then p & q are equivalent

Notation:- $p \equiv q$ / $p \equiv q$, read as "p is equivalent to q" or $p \Leftrightarrow q$

Eg:- $\neg\neg p$ is equivalent to p

$p \vee p$ is equivalent to p

$(p \wedge \neg p) \vee Q$ is equivalent to Q

$p \vee \neg p$ is equivalent to $Q \vee \neg Q$

Q1 prove $(p \to Q) \Leftrightarrow (\neg p \vee Q)$

p	Q	$\neg p$	$(p \to Q)$	$(\neg p \vee Q)$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Hence, $(p \to Q)$ and $(\neg p \vee Q)$ are identical

so $(p \to Q) \leftrightarrow (\neg p \vee Q)$

2) Show that $\neg(p \oplus Q)$ and $p \leftrightarrow Q$ are logically equivalent

p	Q	$p \oplus Q$	$\neg(p \oplus Q)$	$p \leftrightarrow Q$
T	T	F	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	T	T

So $\neg(p \oplus Q) \Leftrightarrow p \leftrightarrow Q$

3) Show that $\neg(p \vee Q) = \neg p \wedge \neg Q$

p	Q	$\neg p$	$\neg Q$	$p \vee Q$	$\neg(p \vee Q)$	$\neg p \wedge \neg Q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

6th column = 7th column are equal

So, $\neg(p \vee Q) \cdot \neg p \wedge \neg Q$

4) verify that $p \wedge T \equiv p$ and $p \vee F \equiv p$

p	T	$p \wedge T$		p	F	$p \vee F$
T	T	T		T	F	T
F	T	F		F	F	F

$p \wedge T \equiv p$ is true $\quad$ $p \vee F \equiv p$ is true

5) verify that $\neg(\neg p) = p$

p	$\neg p$	$\neg(\neg p)$
T	F	T
F	T	F

Hence $\neg(\neg p) \equiv p$

6) Show the following equivalence

a) $p \to (Q \to p) \Leftrightarrow \neg p \to (p \to Q)$

b) $p \to (Q \vee R) \Leftrightarrow (p \to Q) \vee (p \to R)$

Laws of logic

1) Identity law laws

$p \wedge T \Leftrightarrow p$

$p \vee F \Leftrightarrow p$

$p \vee T \Leftrightarrow T$

$p \wedge F \Leftrightarrow F$

2) Idempotent laws

$p \vee p \Leftrightarrow p$

$p \wedge p \Leftrightarrow p$

3) Double negation law : $\neg(\neg p) \Leftrightarrow p$

4) commatetive law:

$p \vee Q \Leftrightarrow Q \vee p$

$p \wedge Q \Leftrightarrow Q \wedge p$

5) Associative law

$(p \vee Q) \vee R \Leftrightarrow p \vee (Q \vee R)$

$(p \wedge Q) \wedge R \Leftrightarrow p \wedge (Q \wedge R)$

6) Distoributive law

$p \wedge (Q \wedge R) \Leftrightarrow (p \vee Q) \wedge (p \vee R)$

$p \wedge (Q \vee R) \Leftrightarrow (p \wedge Q) \vee (p \wedge R)$

7 Demoorgans law

$p \vee (p \wedge Q) \Leftrightarrow p$

$\neg (p \vee Q) \Leftrightarrow \neg p \wedge \neg Q$

8) Absoorption law

$p \vee (p \wedge Q) \Leftrightarrow p$

$p \wedge (p \vee Q) \Leftrightarrow p$

a) $p \vee \neg p \Leftrightarrow T$

$p \wedge \neg p \Leftrightarrow F$

problems

1) S.T $\neg(p \vee (\neg p \wedge Q))$ and $\neg p \wedge \neg Q$ are logically equi-valent by using laws of logic

$\neg(p \vee (\neg p \wedge Q)) \Rightarrow \neg p \wedge \neg(\neg p \wedge Q)$ (by demoorgan law)

$\Rightarrow \neg p \wedge (\neg(\neg p) \vee \neg Q)$ (by ")

$\Rightarrow \neg p \wedge (p \vee \neg Q)$ (by double $\neg$ law)

$\Rightarrow (\neg p \wedge p) \vee (\neg p \wedge \neg Q)$

(by distoributive

$\Rightarrow F \vee (\neg p \wedge \neg Q)$ (by identity law)

$\Rightarrow (\neg p \wedge \neg Q)$

Hence proved.

2) S.T each of the following implication is a tautology by using truth tables

a) $\neg p \wedge (p \vee q) \rightarrow q$

p	q	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$	$\neg p \wedge (p \vee q) \rightarrow q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

Hence the given stt is tautology

b) $(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$(p \rightarrow q) \wedge (q \rightarrow r)$	$p \rightarrow r$	$(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Hence it is a tautology

3- S.T a) $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are not equivalent

p	q	r	$p \to q$	$(p\to q)\to r$	$q \to r$	$p \to (q \to r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	T	T	T	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T
F	F	F	T	F	F	T

The given stt's are not equivalent.

b) $p \to (q \to r)$ are $q \to r$

p	q	r	$q \to r$	$p \to (q \to r)$
T	T	T	T	T
T	F	F	T	T
T	F	T	T	T
T	T	F	F	F
F	T	T	T	T
F	T	F	F	T
F	F	T	T	T
F	F	F	T	T

The given stt's are not equivalent

4) S.T $\neg(p \leftrightarrow q)$ are and $p \leftrightarrow \neg q$ are logically equivalent

p	q	$p \leftrightarrow q$	$\neg(p \leftrightarrow q)$	$\neg q$	$p \leftrightarrow \neg q$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	F	T	F	T
F	F	T	F	T	F

The truth value of $\neg(p \leftrightarrow q)$ & $p \leftrightarrow \neg q$ are identical so they are logically equivalent

Predicate calculus.

consider the two statement

1- Dog is an animal

2 - Cat is an animal

Since both the stts are about two individual, which are animals, so expressing these stts by symbols will require two diff. symbols to denote them. Here we use a new symbol to denote the phrase " is an animal " this part is called a predicate

Suppose the predicate " is an animal" is represented by the predicate letter A, Dog by 'd' and cat by 'c' then stts (1) and (2) can be written as A(d) & A(c) in general, any stt of the type x is S, where S is predicate and x is subject, can be written as $S(x)$

→ A predicate that requires n $(n>0)$ names is known as an n-place predicate

Eg: 3) the predicate A in (1) and (2) is oneplace predicate

4) consider the stt: x is greater than y. represent " greater than " then predicate G represent G is a 2-place predicate since two names are required to express the stt as $G(x,y)$

→ In general, if P is an n-place predicate letter and $x_1, x_2 \dots x_n$ are the names of objects, then $P(x_1, x_2 \dots x_n)$ denote a stt.

Quantifiers

consider the two stts

1- All dogs are animals

2- Every rose is red

we ~~cab~~ can also write the above stts in the following way :

(1') For all x, if x is a dog, then x is an animal

(2') for all x, if x is a rose, then x is red

we use the symbol $(\forall x)$ is (x) to represent the phase "For all x"

Now,

$D(x)$: x is a dog

$A(x)$: x is an animal

$R(x)$: x is red

$C(x)$: x is rose

(1') and (2') can be written as

$1''$: $\forall x\,(D(x) \to A(x))$

$2'$: $\forall x\,(C(x) \to R(x))$

the symbol (x) or $(\forall x)$ are called universal Quantifiers and represent "for all x", "Every x", "for any x".

→ The quantifiers which symbolizes the expression such as "there exist some" or "there is atleast one" is called existential Quantifiers and notated it as $(\exists x)$

→ The area of logic that deals with predicate and Quantifiers is called predicate calculus

Eg:- 1) Symbolize the expression "Some roses are red"

solⁿ :- let $C(x)$: x is a rose

$R(x)$: x is red

$(\exists x)\,(C(x) \wedge R(x))$

2) Symbolize the expression "some numbers are irrational"

solⁿ:- let $N(x)$: x is a number

$I(x)$: x is irrational

$(\exists x)\,(N(x) \wedge I(x))$

problems

1) let $P(n)$ be the stt $n+1 > n$, what is the truth value of the quantifiers $\forall n\, P(n)$, were the domain is set of real nos

$n+1 > n$

if $n = 1$

$1+1 > 1$

$2 > 1$

if $n=2$

$2+1 > 2$

$3 > 2$

if $n = 5$

$5+1 > 5$

$6 > 5$

if $n = -2$

$-2+1 > -2$

$-1 > -2$

$P(n)$ is true for all real nos

$\therefore \forall n\, P(n)$ is true

2) let $P(x)$ be the stt $x < 2$, what is the truth.

values of the Quantifier $\forall x P(x)$, were the domain is set of real nos

if $x = 1$

$1 < 2$ is true

if $x = 3$

$3 < 2$ is false

so, $P(x)$ is not true for all real nos

$\therefore \forall x P(x)$ is false

3) let $A(m)$ denotes the stt $m > 3$, what is the truth value of Quantifieds $\exists m\ A(m)$ were the domain is set of real nos

S^n:- if $m = 1$

$1 > 3$ is false

if $m = 5$

$5 > 3$ is true

so $A(m)$ is true for $m = 5$

$\therefore \exists m\ A(m)$ is true

4) let $Q(p)$ denotes the stt $p = p+1$, what is the truth values of the Quantifiers $\exists p\ Q(p)$, were the domain is the set of real nos.

$p = p+1$

if $p = 1$

$1 = 1+1$

if $p = -3$

$-3 = -3+1$

$-3 = -2$ is false

we can't find a real numbers which satisfies the stt $p = p+1$

so $\exists p\ Q(p)$ is false

Set theory

Set is a well defined collection or a class of distinct objects

Eg:- A pair of shoes

A bouquet of flower

Set of vowels

$A = \{2, 4, 6, \dots\}$

The objects of a set are called element or members of the set. If a is an element of set A, then we write $a \in A$, and if a is not an element of A, then we write $a \notin A$, which is equivalent to the negation of the stt a is in A.

ie $\neg(a \in A) \Leftrightarrow a \notin A$

Methods of describing a set

1) Tabular or rosted form, all the elements of a set are listed, the elements are being seperated by common and are enclosed within braces $\{\}$

Eg:- $\{1, 2, 3, 4, 5\}$

2) Set- builder form

In this form all the elements of a set posses a single common property which is not possessed by any elements outside the set.

Eg:- $V = \{x : x \text{ is a vowel in eglish alpha}$

Q1) Write the set of numbers 1, 3, 5, 7, 9 in roster form and set builder form

$A = \{1, 3, 5, 7, 9\}$

$A = \{x : x$ is an odd number and $x < 10\}$

Types of set

1) Finite and infinite set

If the numbers of distinct elements present in a set is finite, then it is called finite set

If the number of distinct elements present in a set is infinite, then it is called infinite set

Eg:- $\{1, 2, 3\}$ - finite set

$\{1, 2, 3, 4, \ldots\}$ infinite set

2) Subset

Let A and B be any two sets. If every element of A is an element of B, then A is called a subset of B or A is said to be included in B, or B included A. Symbolically, $A \subseteq B$ or equivalently $B \supseteq A$ or $A \subseteq B \Leftrightarrow (x)(x \in A \rightarrow x \in B) \Leftrightarrow B \supseteq A$

3) Equal set

Two sets A and B are said to be equal iff $A \subseteq B$ and $B \subseteq A$ or symbolically

$A = B \Leftrightarrow (A \subseteq B \wedge B \subseteq A)$

or Every elements of A is an element in B

4) Proper Subset

A set A is called proper subset of a set B if $A \subseteq B$ and $A \neq B$. Symbolically, it is written as $A \subset B$, so that $A \subset B \Leftrightarrow (A \subseteq B \wedge A \neq B)$

$A \subset B$ is called proper inclusion

5) Universal set

A set is called universal set if it includes every set under discussion. It is denoted by U.

6) Null set / Empty set

A set which does not contain any element is called an empty set or a null set, and it is denoted by $\emptyset$.

Eg:- $\emptyset = \{x / p(x) \wedge \neg p(x)\}$, where $p(x)$ is any predicate

7) For a set A, a collection or family of all subsets of A is called the power set of A and it is denoted by $P(A)$ or 2^A.

Eg:- 1) $S_1 = \{a\}$

$P(S_1) = \{\emptyset, \{a\}\} = \{\emptyset, S_1\}$

2) $S_2 = \{a, b\}$

$P(S_2) = \{\emptyset, \{a\}, \{b\}, S_2\}$

8) Equality of set

If the elements of one set can be put in to one to one correspondance with the element of other sets then the two sets are called equivalent sets. in equivalent sets number of elements are equal.

Eg:- $A = \{a, b, c, d, e\}$

$B = \{1, 2, 3, 4, 5\}$

then A and B are equivalent since a corresponds to 1 and b corresponds to 2 and so on

9) Singleton set

A set which contain only one elements is called Singleton / unit set

Eg:- $\{1\}$, $\{2\}$

10) Disjoint set

Two sets A and B are said to be disjoint set, if A and B does not contain any common (same) element

eg:- $A = \{1, 2\}$

$B = \{4, 5\}$

A & B are disjoint

Remark:-

1) Every set is a subset of itself

2) Null set is a subset of every set

3) If a set has n elements, then it has 2^n subsets.

problems

1) Read the given set notation

(a) $A \subseteq B$ - A is subset of B

(b) $A \subset B$ - A is proper set of B

(c) $R \not\subset B$ - R is not a proper set of B

(d) $A \supset B$ - A is super set of B

2) write the subset of $B = \{a, \{b\}, c, d\}$

An:- $B = (a), (\{b\}), (c), (d), (a, \{b\}, c, d), (a, \{b\}), (a, c)$
$\{a, d\}, (\{b\}, a), (\{b\}, c), (\{b\}, d), (c, a), (c, \{b\})$
$(c, d), (d, a), (d, \{b\}), (d, c)$, $\emptyset$

Set operations

1) union of 2 sets

For any two sets A and B, the union of A & B, written as $A \cup B$ is the set of all elements which are members of the set A or the set B or both

Eg:- $\{1, 2, 3\}$ $B = \{a, b, c, d\}$

$A \cup B = \{1, 2, 3, a, b, c, d\}$

2) intersection of two sets.

intersection of two sets A and B is a set of all those element which belongs to both A and B.

ie $A \cap B = \{x / (x \in A) \wedge (x \in B)\}$

Eg:- $A = \{1,2,3\}$, $B = \{3,4,5\}$

$A \cap B = \{3\}$

3) Difference of two sets

Difference of two sets A and B is a set of all elements which belongs to A but does not belongs to B.

$A - B = \{x / x \in A \wedge x \notin B\} = \{x / x \in A \wedge \neg(x \in B)\}$

Eg:- $P = \{1,2,3,4,5\}$, $Q = \{1,2,4,5,6,7\}$

$P - Q = \{3\}$

$Q - P = \{6,7\}$

4) complement of a set

For any set A, the complement of A w.r.t universal set U is U-A

ie $A' = \{x / x \in U \wedge x \notin A\}$

problems

1) Given $A = \{2,5,6\}$ $B = \{3,4,2\}$, $C = \{1,3,4\}$ find A-B, and B-A show that $A-B \neq B-A$ and $A-C = A$

Sol^n $A - B = \{5,6\}$

$B - A = \{3,4\}$

$A - B \neq B - A$

$A - C = \{2,5,6\} = A$

2) Show that $A - B = A \cap B'$ and $A \subseteq B \Leftrightarrow B' \subseteq A'$

Sol^n: $x \in A - B \Leftrightarrow x \in \{x / x \in A \wedge x \notin B\}$

$\Leftrightarrow x \in (A \cap B')$

ie $A - B = A \cap B'$

$A \subseteq B \Leftrightarrow \forall x\,(x \in A \rightarrow x \in B)$

$\Leftrightarrow \forall x\,(\neg(x \in B) \rightarrow \neg(x \in A))$

$\Leftrightarrow \forall x\,(x \notin B \rightarrow x \notin A) \Leftrightarrow (B' \subseteq A')$

3) Given $A = \{x / x$ is an integer and $1 \leq x < 5\}$ $B = \{3,4,5,17\}$ and $C = \{1,2,3 \ldots\}$ find $A \cap B$, $A \cap C$, $A \cup B$, and $A \cup C$

~~4) if S = {~~

4) if $S = \{a,b,c\}$, Find nonempty disjoint sets A_1 & A_2 S $A_1 \cup A_2 = S$

Sol^n

i) $A_1 = \{a\}$

$A_2 = \{b,c\}$

$A_1 \cup A_2 = \{a,b,c\} = S$

ii) $A_1 = \{a,c\}$

$A_2 = \{b\}$

$A_1 \cup A_2 = \{a,b,c\} = S$

iii) $A_1 = \{a,c\}$

$A_2 = \{c\}$

$A_1 \cup A_2 = \{a,b,c\} = S$

5) $U = \{1, 2, \dots 10\}$ And $A = \{3, 4, 5\}$, $B = \{1, 3, 5, 7\}$

Find A', B', and $A' - B'$, $B' - A'$

6) $A = \{1, 3, 5, 7\}$, $B = \{5, 9, 13, 17\}$, $C = \{1, 3, 9, 13\}$

Find, $A \cap B$, $B \cap A$, $A - B$, $(A-B) - C$, $A - (A - B)$

Ans:-

5) $A' = \{1, 2, 6, 7, 8, 9, 10\}$

$B' = \{2, 4, 6, 8, 9, 10\}$

$A' - B' = \{1, 7\}$

$B' - A' = \{4\}$

6) $A \cap B = \{5\}$

$B \cap A = \{5\}$

$A - B = \{1, 3, 7\}$

$(A-B) - C = \{7\}$

$A - (A-B) = \{5\}$

Important laws of set Operation.

1) commutative law

$A \cup B = B \cup A$

$A \cap B = B \cap A$

2) Associative law

$A \cup (B \cup C) = (A \cup B) \cup C$

$A \cap (B \cap C) = (A \cap B) \cap C$

3) Distributive law

~~$A \cup (B \cup C)$~~

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

4) Demorgans law

$(A \cup B)' = A' \cap B'$

$(A \cap B)' = A' \cup B'$

problems

1) using the following sets verify that

$A \cup (B \cup C) = (A \cup B) \cup C$

$A = \{1, 2, 3\}$, $B = \{2, 4, 6\}$, $C = \{3, 4, 5\}$

$B \cup C = \{2, 4, 6, 3, 5\}$

$A \cup (B \cup C) = \{1, 2, 3, 4, 5, 6\}$ — ①

$A \cup B = \{1, 2, 3, 4, 6\}$

$(A \cup B) \cup C = \{1, 2, 3, 4, 6, 5\}$ — ②

From ① & ② we get $A \cup (B \cup C) = (A \cup B) \cup C$

② If $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$, $C = \{1, 3, 5\}$

P.T, $A - (B \cup C) = (A - B) \cap (A - C)$

$B \cup C = \{1, 3, 4, 5\}$

$A - (B \cup C) = \{2\}$ — ①

$A - B = \{1, 2\}$

$A - C = \{2\}$

$(A - B) \cap (A - C) = \{2\}$ — ②

From ① & ② we get $A \cup (B \cup C) = (A \cup B) \cup C$

2) ~~If $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$, $C = \{1, 3, 5\}$~~

~~P.T. $A - (B \cup C) = (A - B) \cap (A - C)$~~

~~$B \cup C = \{$~~

3) If $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $A = \{1, 2, 3\}$, $B = \{2, 4, 5\}$

$C = \{2, 4, 6\}$ verify that

1) $(A \cup B)' = A' \cap B'$

2) $(A \cap B)' = A' \cup B'$

Solⁿ :- 1) $A \cup B = \{1, 2, 3, 4, 5\}$

$(A \cup B)' = \{6, 7, 8\}$ — ①

$A' = \{4, 5, 6, 7, 8\}$

$B' = \{1, 3, 6, 7, 8\}$

$A' \cap B' = \{6, 7, 8\}$ — ②

From ① & ② we get $(A \cup B)' = A' \cap B'$

② $A \cap B = \{2\}$

$(A \cap B)' = \{1, 3, 4, 5, 6, 7, 8\}$ — ③

$A' \cup B' = \{1, 3, 4, 5, 6, 7, 8\}$ — ④

From ③ & ④ we get $(A \cap B)' = A' \cup B'$

ordered pair

the elements x & y listed in a given fixed order, form an ordered pair denoted by (x, y). In the ordered pair x is called the first component and y is called the IInd component. $(a, b) = (c, d)$ if and only if

$a = c$ and $b = d$

Q-1) Find a and b, if $(2a - 3, 4) = (4a, b + 5)$

Solⁿ :- $2a - 3 = 4a$ $\qquad$ $b + 5 = 4$

$-3 = 4a - 2a$ $\qquad$ $b = 4 - 5$

$-3 = 2a$ $\qquad$ $= -1$

$\frac{-3}{2} = a$

cartesion product of sets

let A and B be any 2 non-empty sets, then the set of all ordered pair (x, y) where $x \in A$ and $y \in B$ is called the cartesion product of A and B. and denoted by $A \times B$ also known as cross product

$$A \times B = \{(x, y) : x \in A \wedge y \in B\}$$

Eg :- Let $A = \{1, 2\}$, $B = \{3, 2\}$

$A \times B = \{(1, 3), (1, 2), (2, 3), (2, 2)\}$

$B \times A = \{(3, 1), (3, 2), (2, 1), (2, 2)\}$

Note

1) If A & B are two different non empty sets. then $A \times B \neq B \times A$

2) if either A or B is empty, then $A \times B = \emptyset$

3) Number of elements in $A \times B$ is $n(A) \times n(B)$

4) If one of the 2 sets $A \& B$ is infinite and the other set is non-empty, then $A \times B$ is an infinite set.

RELATION

Let A and B be any two non emp the relation from A to B is the of $A \times B$ if R is a relation f and $(a,b) \in R$, then we write related to b) and if $(a,b) \notin R$ write $a \not R b$ (a not related t

Eg:- Consider the set $A = \{2, 3,$
$B = \{3, 4$

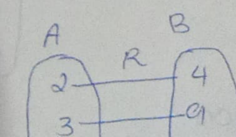
the relation from A to B is
$2R_4, 2R_6 \quad 3R_3, 3R_6$
$R = \{(2,4), (2,6), (3,3)$

Note

1) if A contain m elements and n elements then $A \times B$ contain

2) if each subset of $A \times B$ is a A to B, then the numbers of relation from A to B is

Eg_2:-



RELATION

let A and B be any two non empty sets then the relation from A to B is the subset of $A \times B$ if R is a relation from A to B and $(a,b) \in R$, then we write aRb (a is related to b). and if $(a,b) \notin R$ then we write $a \not R b$ (a not related to b)

Eg:- consider the set $A = \{2,3,4\}$
$B = \{3,4,5,6\}$

the relation from A to B is

$2R4, 2R6, 3R3, 3R6, 4R4$

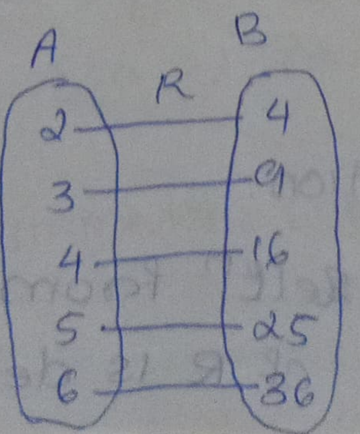
$R = \{(2,4), (2,6), (3,3), (3,6), (4,4)\}$

<u>Note</u>

1) if A contain m elements and B contain n elements then $A \times B$ contains mn elements

2) if each Subset of $A \times B$ is a relⁿ from A to B, then the numbers of different relation from A to B is 2^{mn} elements

Eg₂:-



Relation on A

Let A be a non-empty set, Then a relt^n from A to A ie a subset of $A \times A$ is called a relt^n of A

Domain and Range of a Relation

Let A and B be the two sets and R be a reln from A to B, ie $R \subseteq A \times B$, Then domain of R is defined as the set of all the first entry of the ordered pair which belongs to R. The range of R is defined as the set of all second entry of the ordered pair which belongs to R. Thus

$$\text{Domain } (R) = \{a : (a,b) \in R \ \& \ a \in A\}$$

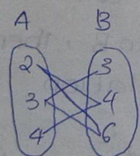
$$\text{Range } (R) = \{b : (a,b) \in R \ \& \ b \in B\}$$

Eg:- Let $A = \{2,3,4\}$, $B = \{3,4,5,6\}$

$$R = \{(2,4), (2,6), (3,3), (3,6), (4,4)\}$$

Domain$(R) = \{2,3,4\}$

Range $(R) = \{4,6,3\}$



Inverse relation.

let R be any relt^n from set A to set B the inverse of R is denoted by R^{-1} is a is a relation from B to A

ie $R^{-1} = \{(b,a), bRa\}$

Eg:- consider $A = \{1,3,5\}$, $B = \{2,4,6,8\}$

$$R = \{(1,4), (3,6), (5,8), (1,8)\}$$

$$R^{-1} = \{(4,1), (6,3), (8,5), (8,1)\}$$

Types of relation

1] Reflexive relation

let A be a non empty set and R be a relt^n on A then R is said to be reflexive relt^n, if aRa, $(a,a) \in R$, $\forall a \in A$

ie every element of A is related to itself

Eg:- The set of parallel lines on a plane is a reflexive relation

2] Symmetric relation

Let A be a non-empty set and R be a rel^n on A, The R is said to be Symmetric, if $aRb \Rightarrow bRa \ \forall (a,b) \in A$

ie $(a,b) \in R \Rightarrow (b,a) \in R$

Eg:- let $A = \{a,b,c\}$

$$R = \{(a,a), (a,b), (a,c), (b,a), (c,a)\}$$

Result

If R is symmetric, then $R = R^{-1}$

3] Transitive relation

A relation R on a non empty set A is said to be transitive, if a is related to b and b is related to c, then a is related to c

ie if aRb, bRc the aRc. $\forall$ a, b, c $\in$ R

Eg :- Let A = {a, b, c}

R = {(a, b), (b, c), (a, c)}

R is transitive, because (a, b) $\in$ R, (b, c) $\in$ R, and (a, c) $\in$ R

4] Antisymmetric relation

Let a relation R on a non empty set A is said to be antisymmetric, if aRb and bRa $\Rightarrow$ a = b $\forall$ a, b $\in$ A

ie (a, b) $\in$ R & (b, a) $\in$ R $\Rightarrow$ a = b $\forall$ a, b $\in$ A

---

problems

1) consider the following five relations on the set

A = {1, 2, 3, 4}

R_1 = {(1, 1), (1, 2), (2, 3), (1, 3), (4, 4)}

R_2 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)}

R_3 = {(1, 3), (2, 1)}

R_4 = $\emptyset$, the empty relation

R_5 = AXA, the universal relation.

Determine which of the relations are (a) ~~symme~~ reflexive, (b) Symmetric, (c) Antisymmetric (d) transitive.

Ans:- R_1 = ~~reflexive~~ antisymmetric, transitive

R_2 = reflexive, symmetric, transitive

R_3 = transitive

R_4 =

R_5 =

2- let x = {1, 2, 3, 4} and R = {(1, 1), (1, 4) (4, 1) (4, 4), (2, 2), (2, 3), (3, 2), (3, 3)}

Determine this relation is equivalent or not

a) Reflexive :- Here (1, 1), (2, 2), (3, 3), (4, 4) $\in$ R

so, R is reflexive

b) Symmetric :- if (1, 4) $\in$ R then (4, 1) $\in$ R

if (2, 3) $\in$ R then (3, 2) $\in$ R

so R is Symmetric

c) Transitive

$\underbrace{1 - 4 - 1}$ (1, 1) $\in$ R

$\underbrace{2 - 3 - 2}$ (2, 2) $\in$ R

so R is transitive

$\therefore$ R is an equivalent relation

3) Let x = {1, 2, 3, ... 7} and R = {(x, y) / x - y is devisible by 3}

Show that R is an equivalence relation

a) For any $a \in X$, $a-a$ is divisible by 3

ie $1 \in X$, $1-1 = 0/3 = 0$

hence aRa or R is reflexive

b) For any $a, b \in X$, if $a-b$ is devisible by 3,

then $b-a$ is divisible by 3

$3, 6 \in X$, if $3-6 = -3/3 = -1$ then

$6-3 = 3/3 = 1$

Hence R is symmetric

c) For any $a, b, c \in X$, if aRb and bRc then

both $a-b$ and $b-c$ are divisible by 3, so

that $a-c = (a-b)+(b-c)$ is also divisible

by 3.

For any $1, 4, 7 \in X$, if $1-4 = -3/3$ and

$4-7 = -3/3$

So that $1-7 = (1-4)+(4-7)$

$= 3+3$

$= 6/3 = 3$

Hence R is transitive

$\therefore$ R is an equivalence relation.

4) Let R denoted a relation on the set of ordered pairs of positive intiger such that

$(x,y) R (u,v)$ iff $xv = yu$ Show that R is an equivalence relation

S^n:- a) since $(x,y) R (x,y)$ $\forall (x,y) \in A$,

$xy = yx$

so R is reflexive

b) $(x,y) R (u,v)$

$\Rightarrow xv = yu$

$\Rightarrow yu = xv$

and so $(u,v) R (x,y)$

$\therefore$ R is symmetric

$(1,4) R (2,4)$

$1 \times 4 = 2 \times 2$

$4 = 4$

$2 \times 2 = 1 \times 4$

$4 = 4$

$(2,4) R (1,2)$

c) $(x,y) R (u,v)$ and $(u,v) R (a,b)$

$\Rightarrow xv = yu$ and $ub = va$ $\Rightarrow \frac{b}{v} = \frac{a}{u}$

$\Rightarrow x\cancel{v} \times \frac{b}{\cancel{v}} = y\cancel{u} \times \frac{a}{\cancel{u}}$

$xb = ya$

$(x,y) R (a,b)$

$\therefore$ R is transitive

5) check whether the relation R defined in the set $\{1,2,3,4,5,6\}$ as $R = \{(a,b) : b = a+1\}$ is symmetric or transitive

Sol^n : $R = \{(a,b) : b = a+1\}$

$= \{(1,2),(2,3),(3,4),(4,5),(5,6)\}$

a) R is not reflexive, bcz $(1,1),(2,2),(3,3)$
$(4,4) \notin R$

b) R is not symmetric bcz, $(1,2) \in R$, $(2,1) \notin R$
$(2,3) \in R$, $(3,2) \notin R$

c) R is not transitive
$(1,2), (2,3) \in R$
but $(1,3) \notin R$
$(3,4), (4,5) \in R$
but $(3,5) \notin R$

partial order relation

consider a non-empty set A, a relⁿ R on S is called partial ordering of S or partial order, if it has the following properties

1) Reflexive
2) Antisymmetric
3) Transitive.

Eg:- Let R be the set of real nos, consider the relation $\leq$ on R

1) Reflexive, for any $a \in R$, $a \leq a$ $(a = a)$
2) Antisymmetric for any $a, b \in R$
$a \leq b$, $b \leq a$
$\Rightarrow a = b$

3) Transitive, for any $a, b, c \in R$
$a \leq b$, $b \leq c$
$\Rightarrow a \leq c$

Partial ordering

→ A binary relation R in a set P is called a partial order relation or a partial ordering in P iff R is reflexive, antisymmetric, and transitive.
we usually denote partial ordering using the symbol "$\leq$" if $\leq$ is a partial ordering on P, then the ordered pair $(P, \leq)$ is called a partially ordered set

→ Let $(P, \leq)$ be a partially ordered set. If for every $x, y \in P$ we've either $x \leq y \vee y \leq x$, then $\leq$ is called a simple ordering or linear ordering on P, and $(P, \leq)$ is called a totally ordered or simply ordered set or a chain.

6) prove that the relation "congruence modulo m" given by $= \{(x,y) / x-y$ is divisible by $m\}$

solⁿ:- If R be the relation, then
$xRy \Leftrightarrow x-y$ is divisible by m.
xRx because $x-x=0$ is divisible by m
So R is reflexive
if $xRy \Rightarrow x-y$ is devisible by m

$y - x = -(x-y)$ is divisible by m

$\Rightarrow y R x$, $\therefore$ R is symmetric

let $x R y$ and $y R z$

$\Rightarrow x - y = k_1 m$, $y - z = k_2 m$

$\therefore x - z = (x-y) + (y-z)$

$= k_1 m + k_2 m = (k_1 + k_2) m$

so R is transitive

$\therefore$ R is reflexive, symmetric and transitive

$\therefore$ R is an equivalence relation

7) Relative R in the set N of natural numbers defined as $R = \{(x,y) : y = x+5$ and $x < 4\}$, prove the equivalence relation

Soln: $R = \{(1,6),(2,7),(3,8)\}$ $\quad A = \{1,2,3\}$

R is not reflexive

R is not symmetric

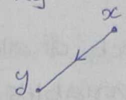
R is transitive, there is no pair (x,y) & $(y,z) \in R$

then (x,z) cannot belong to R

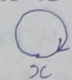
$\therefore$ R is not equivalance relⁿ

Graphs of Relation

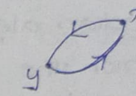
→ xRy



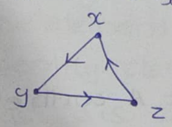
xRx



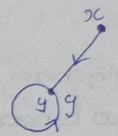
$xRy \wedge yRx$



$xRy \wedge yRz \wedge zRx$



$xRy \wedge yRy$



eg:- let $X = \{1,2,3,4\}$ and $R = \{(x,y) / x > y\}$

Draw the graph of R.

Soln $\{(2,1),(3,1),(3,2),(4,1),(4,2),(4,3)\}$