MAJLIS ARTS AND SCIENCE COLLEGE
PURAMANNUR

PG DEPARTMENT OF COMPUTER SCIENCE

(Affiliated to the University of Calicut)

OPERATIONS RESEARCH

FIRST MODULE

Second Semester BCA

(Academic Year 2019-20 Onwards and 2017 Admissions)

Questions and Answers based on First Module

Module I

Operation research and LPP: Operation Research and Decision making, Advantages of O R, Approach in decision making, Application of O R, uses and limitations of O R.

(1 Mark Questions)

1. *Operations Research (OR), which is a very powerful tool for ___________.*

   Ans: Decision Making.

2. *Who coined the term Operations Research?*

   Ans:J.F.McCloskey and F.N. Trefethen.

3. *Who defined Operations Research as scientific method of providing executive departments with a quantitative basis for decisions regarding the operations under their control?*

   Ans: Morse and Kimball

4. *Which technique is used in finding a solution for optimizing a given objective, such as profit maximization or cost minimization under certain constraints?*

   Ans: Linear programming

5. *Define Operations Research?*

   Ans: Operations research (OR) is an analytical method of problem-solving and decision-making that is useful in the management of organizations. In operations research, problems are broken down into basic components and then solved in defined steps by mathematical analysis.

(2 Mark Questions)

6. *Cite any two limitations of Operations Research?*

   *Ans: **Costly** : Operations Research (OR) is very costly. This is because OR makes mathematical models for taking decisions and solving problems. The company has to make various models for solving different problems. All this increments the cost.*

   ***Not Realistic** : OR experts make very complex models for solving problems. These models may not be realistic. Hence, they may not be useful for real-life situations.*

Complex : *OR is very complex concept. It is very difficult for an average manager to understand.*

7. Cite the uses of Operations Research?

   Ans :

   - *Scheduling and time management.*
   - *Urban and agricultural planning.*
   - *Enterprise resource planning (ERP) and supply chain management (SCM).*
   - *Inventory management in Organizations.*
   - *Network optimization and engineering.*
   - *Packet routing optimization.*
   - *Risk management*

(4 Mark /Short Essay Questions)

8. What are the advantages of Operations Research?

   Ans:

   - *Enhanced productivity. Operations research helps in improving the productivity of the organizations*
   - *Linear programming. Management is responsible for making important decisions about the organization.*
   - *Improved coordination.*
   - *Lower risks of failure.*
   - *Control on the system.*

9. What are the main characteristics of Operations research techniques?

   Ans:

   Some significant characteristics or features of OR are given below:

   1. ***Decision Making:*** *A major premise of OR is that decision making, irrespective of the situation involved, can be considered as a general systematic process. OR improves the quality of decisions.*
   2. ***Scientific Approach:*** *OR applies scientific methods for the purpose of analysis and solution of the complex problems. It is a formalized process of reasoning. In this approach there is no place for guess work and the person bias of the decision maker.*

3. ***Objective-Oriented Approach:*** *OR attempts to locate the best or optimal solution to the problem under consideration. OR tries to find the best (optimum) decisions relative to largest possible portion of the total organization.*
4. ***Inter-disciplinary team Approach:*** *OR is the inter-disciplinary in nature and requires a team approach to a solution of the problem. Managerial problems have economic, physical, biological and engineering aspects. This requires a blend of people with expertise in the areas of mathematics, statistics, engineering, economics, management, computer science and so on.*
5. ***Imperfectness of solutions:*** *By OR techniques, we cannot obtain perfect answers to our problems but, only quality of the solution is improved from worse to bad answers.*
6. ***Use of Digital Computer:*** *The models of OR need lot of computation and therefore, the use of computers becomes necessary. With the use of computers it is possible to handle complex problems requiring large amount of calculations.*
7. ***Optimize the total output:*** *OR tries to optimize the total output by maximizing the profit and minimizing the cost.*

(8 Mark /Essay Questions)

10. Explain the Phases of OR?

Ans: *OR study generally involves the following major phases*

- ***a) Defining the problem and gathering data***
- ***b) Formulating a mathematical model***
- ***c) Deriving solutions from the model***
- ***d) Testing the model and its solutions***
- ***e) Preparing to apply the model***
- ***f) Implementation***

a) Defining the problem and gathering data

The first task is to study the relevant system and develop a well-defined statement of the problem. This includes determining appropriate objectives, constraints, interrelationships and alternative course of action. The OR team normally works in *an advisory capacity. The team performs a detailed technical analysis of the problem and then presents recommendations to the management. Ascertaining the appropriate objectives is very important aspect of problem definition. Some of the objectives include maintaining stable price, profits, increasing the share in market, improving work morale etc. OR team typically spends huge amount of time in gathering relevant data.·*

*b)**Formulating a mathematical model***

This phase is to reformulate the problem in terms of mathematical symbols and expressions. The mathematical model of a business problem is described as the system of equations and related mathematical expressions.

- ✓ ***Decision variables** (x1, x2 … xn) – 'n' related quantifiable decisions to be made.*
- ✓ ***Objective function** – measure of performance (profit) expressed as mathematical function of decision variables. For example $P=3x_1 +5x_2 + \dots + 4x_n$*
- ✓ ***Constraints** – any restriction on values that can be assigned to decision variables in terms of inequalities or equations. For example $x1 +2x2 \geq 20$*
- ✓ ***Parameters** – the constant in the constraints (right hand side values) The advantages of using mathematical models are Describe the problem more concisely and indicates clearly what additional data are relevant for analysis· Forms a bridge to use mathematical technique in computers to analyze·*

c)Deriving solutions from the model

This phase is to develop a procedure for deriving solutions to the problem. A common theme is to search for an optimal or best solution.

d)Testing the model and its solutions

*After deriving the solution, it is tested as a whole for errors if any. The process of testing and improving a model to increase its validity is commonly referred as **Model validation**. The OR group doing this review should preferably include at least one individual who did not participatein the formulation of model to reveal mistakes.*

e)Preparing to apply the model

After the completion of testing phase, the next step is to install a well-documented system for applying the model. This system will include the model, solution procedure and operating procedures for implementation. The system usually is computer-based. Databases and Management Information System may provide up-to-date input for the model. An interactive computer based system called Decision Support System is installed to help the manager to use data and models to support their decision making as needed. A managerial report interprets output of the model and its implications for applications.

f) Implementation

The last phase of an OR study is to implement the system as prescribed by the management. The success of this phase depends on the support of both top management and operating management. The implementation phase involves several steps.

- *OR team provides a detailed explanation to the operating management*
- *If the solution is satisfied, then operating management will provide the explanation to the personnel, the new course of action.*
- *The OR team monitors the functioning of the new system*
- *Feedback is obtained*
- *Documentation*

11. What are the OR Techniques?

***Probability:** analyze uncertainties and bring out necessary data with reasonable accuracy for the purpose of decision making*

***Linear Programming:** This model is used for resource allocation when the resources are limited and there are number of competing candidates for the use of resources.*

***Decision theory:** OR technique of Decision theory is applied to select best alternative course of action*

***Game theory:** Game theory helps to determine the best course of action for a firm in view of the expected counter moves from the competitors.*

Transportation problem: *The aim of this technique is find out the minimum transportation cost.*

The assignment problem: *This technique is used to assign jobs to efficient and suitable persons at minimum cost.*

Network analysis: *Program evaluation and review technique and critical path method are powerful tools for planning and control of complex jobs involving a large number of complex activities.*

Dynamic programming: *this technique deals with the problems that arise in connection with multi period analysis and decisions.*

Nonlinear programming: *it is that form of programming in which some or all variables are curvy linear.*

Market analysis: *it is a method of analyzing the current movement of the same variable in an effort to predict the future movement of same variable.*

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OPERATIONS RESEARCH

SECOND MODULE

Second Semester BCA

(Academic Year 2019-20 Onwards and 2017 Admissions)

Questions and Answers based on Second Module

Module II

LPP: Introduction, mathematical formulation the problem, canonical and standard forms of LPP. Simplex method, artificial variable technique - Big M and two phase method - problem of degeneracy - concept of duality - dual simplex method.

(1 Mark Questions)

1. *Minimization of objective function in LPP means __________*
   Ans: Least value chosen among the allowable decisions
2. *For maximization LPP, the objective function coefficient for an artificial variable is :*
   Ans: - M
3. *The dual of the Primal maximization LPP having m constraints and n non negative variables Should:*
   Ans: have n constraints and non negative variables.

4. *The variable that we seek to determine in a LPP are called*
   Ans: Decision variables

5. *A basic solution to the system is degenerate if one or more -------vanishing*
   Ans: variable
6. *The basic Solution in which non of the basic variable is zero is called ------*
   Ans: non degenerated
7. *A variable that is added to an inequality to transform it into equality :*
   Ans: Slack variable
8. *Dual of a dual problem of a LPP is is*
   Ans:Primal.

(2 Mark Questions)

9. *State the general linear programming problem in standard form?*

A: Standard form of LPP must have following three characteristics:

- ✓ ***Objective function should be of maximization type***
- ✓ ***All the constraints should of equality type***
- ✓ ***All the decision variables should be nonnegative***

10. *What is Role of pivot element in simplex table?*

A: To determine the leaving variable in from a feasible solution.

11. *What is linear programming problem?*

A: A linear programming problem consists of a linear function to be maximized or minimized subject to certain constraints in the form of linear equations or inequalities.

12. *Define feasible solution ? Optimal solution?*

A:A nonnegative vector of variables that satisfies the constraints of (P) is called a feasible solution to the linear programming problem. A feasible solution that minimizes the objective function is called an optimal solution. A nonnegative vector of variables that satisfies the constraints of (P) is called a feasible solution to the linear programming problem. A feasible solution that minimizes the objective function is called an optimal solution.

13. *Define slack surplus and artificial variables in LPP.*

*A: **Slack variable:** It is used o convert a Less than or equal to (≤) constraint into equality to write standard form. It is ADDED to ≤constraint.*

***Surplus & Artificial variables**: They are used to convert Greater than or equal to (≥) constraint into equality to write standard form.*

14. *What are the advantages of LPP?*

A: LP makes logical thinking and provides better insight into business problems. Manager can select the best solution with the help of LP by evaluating the cost and profit of various alternatives.

- ✓ ***LP provides an information base for optimum allocation of scarce resources.***
- ✓ ***LP assists in making adjustments according to changing conditions.***
- ✓ ***LP helps in solving multi-dimensional problems.***

15. *Write any two applications of LPP?*

  - ✓ ***Financial Management***
  - ✓ ***Inventory Management***

16. *Explain Duality?*

A: The duality in Linear Programming states that every LPP has another LPP related to it and thus can be derived from it. The original LPP is called Primal while the Derived LP is called Dual.

(4 Mark /Short Essay Questions)

17. *What are the Components of linear programming problem?*

*A:**Decision Variables:** The decision variables are the variables which will decide my output. They represent my ultimate solution. To solve any problem, we first need to identify the decision variables. For the above example, the total number of units for A and B denoted by X & Y respectively are my decision variables.*

***Objective Function:** It is defined as the objective of making decisions. In the above example, the company wishes to increase the total profit represented by Z. So, profit is my objective function.*

***Constraints:** The constraints are the restrictions or limitations on the decision variables. They usually limit the value of the decision variables. In the above example, the limit on the availability of resources Milk and Choco are my constraints.*

***Non-negativity restriction:** For all linear programs, the decision variables should always take non-negative values which mean the values for decision variables should be greater than or equal to 0.*

(8 Mark /Essay Questions)

18. Explain the procedure of Simplex Method?

*Ans: **Step1.** Check if the linear programming problem is a standard maximization problem in standard form, i.e., if all the following conditions are satisfied: It's to maximize an objective function. All variables should be non-negative (i.e. ≥ 0). Constraints should all be ≤ a non-negative.*

***Step2.** Create slack variables to convert the inequalities to equations.*

***Step3.** Write the objective function as an equation in the form "left hand side"= 0 where terms involving variables are negative. Example: $Z = 3x + 4y$ becomes $-3x - 4y + Z = 0$.*

***Step4.** Place the system of equations with slack variables, into a matrix. Place the revised objective equation in the bottom row.*

***Step 5.** Select pivot column by finding the most negative indicator. (Indicators are those elements in bottom row except last two elements in that row)*

***Step 6.** Select pivot row. (Divide the last column by pivot column for each corresponding entry except bottom entry and negative entries. Choose the smallest positive result. The corresponding row is the pivot row. In case there is no positive entry in pivot column above dashed line, there is no optimal solutions)*

***Step 7.** Find pivot: Circle the pivot entry at the intersection of the pivot column and the pivot row, and identify entering variable and exit variable at mean time. Divide pivot by itself in that row to obtain 1. Also obtain zeros for all rest entries in pivot column by row operations.*

__Step 8.__ Do we get all nonnegative indicators? If yes, we may stop. Otherwise repeat step 5 to step 7.

__Step 9.__ Read the results: Correspond the last column entries to the variables in front of the first column. The variables not showing are automatically equal to 0.

19. *Explain Two phase method with an example?*

Ans: In the first phase of this method, the sum of the artificial variables is minimized subject to the given constraints (known as auxiliary L.P.P.) to get a basic feasible solution to the original L.P.P. Second phase then optimizes the original objective function starting with the basic feasible solution obtained at the end of Phase 1. The iterative procedure of the algorithm may be summarizing as below:

Step 1:

Write the given L.P.P into its standard form and check whether there exists a starting basic feasible solution.

- *If there is a ready starting basic feasible solution, go to Phase 2.*
- *If there does not exist a ready starting basic feasible solution, go on to the next step.*

PHASE 1

Step 2:

Add the artificial variable to the left side of the each equation that lacks the needed starting basic variables. Construct an auxiliary objective function aimed at minimizing the sum of all artificial variables

Thus, the new objective is to

Minimize $z = A_1 + A_2 + A_3 + \ldots\ldots\ldots\ldots + A_n$

Maximize $z^* = - A_1 - A_2 - A_3 - \ldots\ldots\ldots\ldots - A_n$

where A_i $(i = 1,2,3\ldots m)$ *are the non-negative artificial variables.*

Step 3:

Apply simplex algorithm to the specially constructed L.P.P. The following three cases arise at the least interaction:

- *Max* $z^* < 0$ *and at least one artificial variable is present in the basis with positive value. In such case, the original L.P.P does not possess any feasible solution.*
- *Max* $z^* = 0$ *and at least artificial variable is present in the basis at zero value. In such a case, the original L.P.P possess the feasible solution. In order to get basic*

*feasible solution we may proceed directly to **Phase 2** or else eliminate the artificial basic variable and then proceed to **Phase2**.*

- *Max $z^* = 0$ and no artificial variable is present in the basis. In such a case, a basic feasible solution to the original L.P.P has been found. Go to **Phase 2**.*

PHASE 2

Step 4:

*Consider the optimum basic feasible solution of **Phase 1** as the starting basic feasible solution for the original L.P.P. Assign actual coefficients to the variables in the objective function and a value zero to the artificial variables that appear at zero value in the final simplex table of **Phase 1**.*

Apply usual simplex algorithm to the modified simplex table to get the optimum solution of the original problem.

Example:

Minimize $z = -3x_1 + x_2 - 2x_3$

Subject to

$x_1 + 3x_2 + x_3 \leq 5$
$2x_1 - x_2 + x_3 \geq 2$
$4x_1 + 3x_2 - 2x_3 = 5$

$x_1, x_2, x_3 \geq 0$

Solution:
If the objective function is in minimization form, then convert it into maximization form
Changing the sense of the optimization
Any linear minimization problem can be viewed as an equivalent linear maximization problem, and vice versa. Specifically:

Minimize = Maximize

If z is the optimal value of the left-hand expression, then -z is the optimal value of the right-hand expression.

Maximize $z = 3x_1 - x_2 + 2x_3$

Subject to

$x_1 + 3x_2 + x_3 \leq 5$
$2x_1 - x_2 + x_3 \geq 2$
$4x_1 + 3x_2 - 2x_3 = 5$

Where $x_1, x_2, x_3 \geq 0$

Converting inequalities to equalities

$x_1 + 3x_2 + x_3 + x_4 = 5$

$2x_1 - x_2 + x_3 - x_5 = 2$

$4x_1 + 3x_2 - 2x_3 = 5$

$x_1, x_2, x_3, x_4, x_5 \geq 0$

*Where x_4 is a slack variable, x_5 is a surplus variable. Now, if we let x_1, x_2 and x_3 equal to zero in the initial solution, we will have $x_4 = 5$ and $x_5 = -2$, which is not possible because a surplus variable cannot be **negative**. Therefore, we need **artificial variables**.*

$x_1 + 3x_2 + x_3 + x_4 = 5$

$2x_1 - x_2 + x_3 - x_5 + A_1 = 2$

$4x_1 + 3x_2 - 2x_3 + A_2 = 5$

$x_1, x_2, x_3, x_4, x_5, A_1, A_2 \geq 0$

where A_1 and A_2 are artificial variables.

PHASE 1

In this phase, we remove the artificial variables and find an initial feasible solution of the original problem. Now the objective function can be expressed as

Maximize $0x_1 + 0x_2 + 0x_3 + 0x_4 + 0x_5 + (-A_1) + (-A_2)$

subject to

$x_1 + 3x_2 + x_3 + x_4 = 5$

$2x_1 - x_2 + x_3 - x_5 + A_1 = 2$

$4x_1 + 3x_2 - 2x_3 + A_2 = 5$

$x_1, x_2, x_3, x_4, x_5, A_1, A_2 \geq 0$

Initial basic feasible solution

The intial basic feasible solution is obtained by setting $x_1 = x_2 = x_3 = x_5 = 0$.

Then we shall have $A_1 = 2$, $A_2 = 5$, $x_4 = 5$

Iteration 1:

	c_j	0	0	0	0	0	-1	-1	
c_B	Basic variables B	x_1	x_2	x_3	x_4	x_5	A_1	A_2	Solution values b $(= X_B)$

0	x_4	1	3	1	1	0	0	0	5
-1	A_1	2	-1	1	0	-1	1	0	2
-1	A_2	4	3	-2	0	0	0	1	5
$z_j - c_j$		-6	-2	1	0	1	0	0	

Keycolumn=x_1 column
Minimum(5/1,2/2,5/4)=1
Keyrow = A_1 row
Pivot element = 2
A_1 departs and x_1 enters.

Iteration 2:

	c_j	**0**	**0**	**0**	**0**	**0**	**-1**	
c_B	**Basic variables B**	x_1	x_2	x_3	x_4	x_5	A_2	**Solution values b (= X_B)**
0	x_4	0	7/2	1/2	1	1/2	0	4
0	x_1	1	-1/2	1/2	0	-1/2	0	1
-1	A_2	0	5	-4	0	2	1	1
$z_j - c_j$		0	-5	4	0	-2	0	

A_2 departs and x_2 enters.Here, Phase 1 terminates because both the artificial variables have been removed from the basis.

PHASE 2

*The basic feasible solution at the end of **Phase 1** computation is used as the initial basic feasible solution of the problem. The original objective function is introduced in **Phase 2** computation and the usual simplex procedure is used to solve the problem.*

Iteration 3:

	c_j	3	-1	2	0	0	
c_B	Basic variables B	x_1	x_2	x_3	x_4	x_5	Solution values b (= X_B)
0	x_4	0	0	33/10	1	-9/10	33/10
3	x_1	1	0	1/10	0	-3/10	11/10
-1	x_2	0	1	-4/5	0	2/5	1/5
z_j-c_j		0	0	-9/10	0	-13/10	

Iteration 4:

	c_j	3	-1	2	0	0	
c_B	Basic variables B	x_1	x_2	x_3	x_4	x_5	Solution values b (= X_B)
0	x_4	0	9/4	3/2	1	0	15/4
3	x_1	1	3/4	-1/2	0	0	5/4
0	x_5	0	5/2	-2	0	1	1/2
z_j-c_j		0	13/4	-7/2	0	0	

Iteration 5:

	c_j	3	-1	2	0	0	

c_B	Basic variables B	x_1	x_2	x_3	x_4	x_5	Solution values b (= X_B)
2	x_3	0	3/2	1	2/3	0	5/2
3	x_1	1	3/2	0	1/3	0	5/2
0	x_5	0	11/2	0	4/3	1	11/2
z_j-c_j		0	17/2	0	7/3	0	

Result:

An optimal policy is $x_1 = 5/2$, $x_2 = 0$, $x_3 = 5/2$. The associated optimal value of the objective function is

Max z = 3 x (5/2) – 0 + 2 x (5/2) = 25/2.

20. *Briefly explain the Big M method?*

Ans: The Big-M-Method is an alternative method of solving a linear programming problem involving artificial variables. To solve a L.P.P by simplex method, we have to start with the initial basic feasible solution and construct the initial simplex table. In the previous problems we see that the slack variables readily provided the initial basic feasible solution. However, in some problems, the slack variables cannot provide the initial basic feasible solution. In these problems atleast one of the constraint is of = or ≥ type. "Big-M-Method is used to solve such L.P.P.
The big M-method or the method of penalties consists of the following basic steps :

Step 1:

Express the linear programming problem in the standard form by introducing slack and/or surplus variables, if any.

Step 2:

Introduce non-negative variables to the left hand side of all the constraints of (> or =) type. These variables are called artificial variables. The purpose of introducing artificial variables is just to obtain an initial basic feasible solution. However, addition of these artificial variables causes violation of the corresponding constraints. Therefore we would like to get rid of these variables and would not allow them to appear in the optimum simplex table. To achieve this, we assign a very large penalty ' — M ' to these artificial variables in the objective function, for maximization objective function.

Step 3:

Solve the modified linear programming problem by simplex method.

At any iteration of the usual simplex method there can arise any one of the following three cases :

(a) There is no vector corresponding to some artificial variable, in the basis y_b.

In such a, case, we proceed to step 4.

(b) There is at least one vector corresponding to some artificial variable, in the basis y_B, at the zero level. That is, the corresponding entry in X_B is zero. Also, the co-efficient of M in each net evaluation Z_j- C_j(j = 1, 2, n) is non- negative.

In such a case, the current basic feasible solution is a degenerate one. This is a case when an optimum solution to the given L.P.P. includes an artificial basic variable and an optimum basic feasible solution still exists.

(c) At least one artificial vector is in the basis y_B, but not at the zero level. That is, the corresponding entry in X_B is non-zero. Also coefficient of M in each net evaluation Z_j - C_j is non-negative,

In this case, the given L.P.P. does not possess any feasible solution.

Step 4:

Application of simplex method is continued until either an optimum basic feasible solution is obtained or there is an indication of the existence of an unbounded solution to the given L.P.P.

Note. *While applying simplex method, whenever a vector corresponding to some artificial variable happens to leave the basis, we drop that vector and omit all the entries corresponding to its column from the simplex table.*

Example:

Maximize $z = x_1 + 5x_2$

Subject to

$3x_1 + 4x_2 \leq 6$
$x_1 + 3x_2 \leq 2$

$x_1, x_2 \geq 0$

Solution:

Converting inequalities to equalities

By introducing surplus variables, slack variables and artificial variables, the standard form of LPP becomes

Maximize $x_1 + 5x_2 + 0x_3 + 0x_4 - MA_1$

Subject to

$3x_1 + 4x_2 + x_3 = 6$

$x_1 + 3x_2 - x_4 + A_1 = 2$

$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0,\ A_1 \geq 0$

where,

x_3 *is a slack variable*

x_4 *is a surplus variable*

A_1 *is an artificial variable.*

Initial basic feasible solution

$$x_1 = x_2 = x_4 = 0,\ A_1 = 2,\ x_3 = 6$$

Iteration 1:

	c_j	1	5	0	0	$-M$	
c_B	**Basic variables B**	x_1	x_2	x_3	x_4	A_1	**Solution values b ($= X_B$)**
0	x_3	3	4	1	0	0	6
$-M$	A_1	1	3	0	-1	1	2
z_j-c_j		$-M-1$	$-3M-5$	0	M	0	

Calculating values for index row ($z_j - c_j$)

$z_1 - c_1 = 0 \times 3 + (-M) \times 1 - 1 = -M - 1$

$z_2 - c_2 = 0 \times 4 + (-M) \times 3 - 5 = -3M - 5$

$z_3 - c_3 = 0 \times 1 + (-M) \times 0 - 0 = 0$

$z_4 - c_4 = 0 \times 0 + (-M) \times (-1) - 0 = M$

$z_5 - c_5 = 0 \times 0 + (-M) \times 1 - (-M) = 0$

As M is a large positive number, the coefficient of M in the z_j-c_j row would decide the incoming basic variable.

As -3M < -M, x_2 becomes a basic variable in the next iteration.
Key column = x_2 column.
Minimum (6/4, 2/3) =2/3
Key row = A1 row
Pivot element =3.
A1 departs and x_2 enters.

Note: *The iteration just completed, artificial variable A_1 was eliminated from the basis. The new solution is shown in the following table.*

Iteration 2:

	c_j	1	5	0	0	
c_B	**Basic variables B**	x_1	x_2	x_3	x_4	**Solution values b (= X_B)**
0	x_3	5/3	0	1	4/3	10/3
5	x_2	1/3	1	0	−1/3	2/3
$z_j - c_j$		2/3	0	0	−5/3	

Iteration 3:

	c_j	1	5	0	0	
c_B	**Basic variables B**	x_1	x_2	x_3	x_4	**Solution values b (= X_B)**
0	x_4	5/4	0	3/4	1	5/2
5	x_2	3/4	1	1/4	0	3/2
$z_j - c_j$		11/4	0	5/4	0	

Result:

The optimal solution is $x_1 = 0$, $x_2 = 3/2$

Max $z = 0 + 5 \times 3/2 = 15/2$

Additional Problems

21.

Solve the following LPP :

Minimize $Z = 3X_1 + 8X_2$

subject to constraints,

$$X_1 + X_2 = 200$$
$$X_1 \le 80$$
$$X_2 \ge 60$$
$$X_1, X_2 \ge 0.$$

Ans:

	C	3	8	0	0	M	M		
	Var	X1	X2	S2	S3	A1	A3		
M	A1	1	1	0	0	1	0	200	0
0	S2	1	0	1	0	0	0	80	0
M	A3	0	1	0	-1	0	1	60	0
	Z	M	2M	0	-M	M	M	Z =	0
	C-Z	3-M	8-2M	0	M	0	0		

Iteration:

	C	3	8	0	0	M	M		
	Var	X1	X2	S2	S3	A1	A3		
M	A1	1	1	0	0	1	0	200	200
0	S2	1	0	1	0	0	0	80	0
M	A3	0	1	0	-1	0	1	60	60
	Z	M	2M	0	-M	M	M	Z =	260M
	C-Z	3-M	8-2M	0	M	0	0		

key column 4

key row 5

key element 1.0

Iteration:

	C	3	8	0	0	M	M		
	Var	X1	X2	S2	S3	A1	A3		
M	A1	1	0	0	1	1	-1	140	140
0	S2	1	0	1	0	0	0	80	80
8	X2	0	1	0	-1	0	1	60	60
	Z	M	8	0	-8+M	M	8-M	Z =	480+140M
	C-Z	3-M	0	0	8-M	0	-8+2M		

key column 3

key row 4

key element 1.0

Iteration:

	C	3	8	0	0	M	M		
	Var	X1	X2	S2	S3	A1	A3		
M	A1	0	0	-1	1	1	-1	60	60
3	X1	1	0	1	0	0	0	80	80
8	X2	0	1	0	-1	0	1	60	-60
	Z	3	8	3-M	-8+M	M	8-M	Z =	720+60M
	C-Z	0	0	-3+M	8-M	0	-8+2M		

Iteration:

	C	3	8	0	0	M	M		
	Var	X1	X2	S2	S3	A1	A3		
0	S3	0	0	-1	1	1	-1	60	0
3	X1	1	0	1	0	0	0	80	0
8	X2	0	1	-1	0	1	0	120	0
	Z	3	8	-5	0	8	0	Z =	1200
	C-Z	0	0	5	0	-8+M	M		

Solution:
X1 = 80.0
X2 = 120.0
Z = 1200.0

Solve by Big M method :

Maximize $Z = 6x + 4y$

Subject to : $2x_1 + 3y \leq 30$

$3x + 2y \leq 24$

$x + y \geq 3$

$x, y \geq 0$

Ans:

	C	6	4	0	0	0	-M		
	Var	X1	X2	S1	S2	S3	A3		
0	S1	2	3	1	0	0	0	30	0
0	S2	3	2	0	1	0	0	24	0
-M	A3	1	1	0	0	-1	1	3	0
	Z	-M	-M	0	0	M	-M	Z =	0
	C-Z	6+M	4+M	0	0	-M	0		

Iteration:

	C	6	4	0	0	0	-M		
	Var	X1	X2	S1	S2	S3	A3		
0	S1	2	3	1	0	0	0	30	15
0	S2	3	2	0	1	0	0	24	8
-M	A3	1	1	0	0	-1	1	3	3
	Z	-M	-M	0	0	M	-M	Z =	-3M
	C-Z	6+M	4+M	0	0	-M	0		

key column 3

key row 5

key element 1.0

Iteration:

	C	6	4	0	0	0	-M		
	Var	X1	X2	S1	S2	S3	A3		
0	S1	0	1	1	0	2	-2	24	12
0	S2	0	-1	0	1	3	-3	15	5
6	X1	1	1	0	0	-1	1	3	-3
	Z	6	6	0	0	-6	6	Z =	18
	C-Z	0	-2	0	0	6	-6-M		

key column 7

key row 4

key element 3.0

Iteration:

	C	6	4	0	0	0	-M		
	Var	X1	X2	S1	S2	S3	A3		
0	S1	0	5/3	1	-2/3	0	0	14	0
0	S3	0	-1/3	0	1/3	1	-1	5	0
6	X1	1	2/3	0	1/3	0	0	8	0
	Z	6	4	0	2	0	0	Z =	48
	C-Z	0	0	0	-2	0	-M		

Solution:
X1 = 8.0
Z = 48.0

23. *Write the dual of following LPP*

NiniNize z $= 20x_1 + 40x_2$

$36\,x_1 + 6x_2 \geq 108,$

$3x_1 + 12x_2 \geq 36,$

$20\,x_1 + 10x_2 \geq 100$ and $x_1 \geq 0,\ x_2 \geq 0.$

Ans: first we have to write the given LPP in standard form.
Introducing surplus variables $s_1, s_2, s_3 \geq 0$, the primal problem in standard form can be written as

NiniNize z $= 20x_1 + 40x_2 + 0.\ s_1 + 0.\ s_2 + 0.\ s_3$

$36\,x_1 + 6x_2 - s_1 = 108,$

$3x_1 + 12x_2 - s_2 = 36,$

$20\,x_1 + 10x_2 - s_3 = 100$

$x_1, x_2, s_1, s_2, s_3 \geq 0.$

***Associated dual** is*

NaxiNize z $* = 108w_1 + 36w_2 + 100w_3$

$36w_1 + 3w_2 + 20w_3 \leq 20,$

$6w_1 + 12w_2 + 10w_3 \leq 40,$

$-w_1 \leq 0,\ -w_2 \leq 0,\ -w_3 \leq 0$

where w_1, w_2, w_3 are unrestricted in sign dominated by $w_1 \geq 0,\ w_2 \geq 0,\ w_3 \geq 0$

MAJLIS ARTS AND SCIENCE COLLEGE PURAMANNUR

PG DEPARTMENT OF COMPUTER SCIENCE

(Affiliated to the University of Calicut)

OPERATIONS RESEARCH

THIRD MODULE

Second Semester BCA

(Academic Year 2019-20 Onwards and 2017 Admissions)

Questions and Answers based on Third Module

Module III

Transportation Model: North West Corner rule, Least Cost method, Vogel's approximation Method, Loops in transportation table, Degeneracy in transportation table, transshipment problem.

(1 Mark Questions)

1. *While Solving a transportation problem the occurrence of degeneracy means that*

   *Ans: the **solution** so obtained is not feasible.*
2. *A transportation problem is said to be balance if*

   Ans: total demand=total supply
3. *A feasible solution of a transportation problem involves exactly m+n-1 possible variables is known as*

   Ans: Basic Feasible Solution.

(2 Mark Questions)

4. *What is an unbalanced transportation problem?*

   Ans: The problem of transporting a product from several factories (supply origins) to a number of warehouses (demand destinations) is generally considered as a very good application area for linear programming technique. When the total number of units available at the supply origins is equal to the total number of items available at the demand destinations, it is termed a balanced transportation problem. If these two values are not equal, it is termed an unbalanced problem.
5. *Explain transportation problem? Show that it can be considered as LPP.*

   *Ans: The **Transportation Method** of linear programming is applied to the problems related to the study of the efficient transportation routes i.e. how efficiently the product from different sources of production is transported to the different destinations, such as the total transportation cost is minimum.*

(4 Mark /Short Essay Questions)

6. *Explain transportation problem? Give its Mathematical Formulation?*

   Ans: Let there be m origins and n destinations. Let the amount of supply at the i th origin is a_i. Let the demand at j th destination is b_j.The cost of transporting one unit of an item from origin i to destination j is c_{ij} and is known for all combinations (i,j). Quantity transported from origin i to destination j be x_{ij}The objective is to determine the quantity x_{ij} to be transported over all routes (i,j) so as to minimize the total transportation cost. The supply limits at the origins and the demand requirements at the destinations must be satisfied. The above transportation *problem can be written in the following tabular form:*

		Destinations					
		1	2	3	…	n	supply
Origins	1	(x_{11}) C_{11}	(x_{12}) C_{12}	(x_{13}) C_{13}	…	(x_{1n}) C_{1n}	a_1
	2	(x_{21}) C_{21}	(x_{22}) C_{22}	(x_{23}) C_{23}	…	(x_{2n}) C_{2n}	a_2
	⋮		⋮	⋮	⋮	⋮	⋮
	m	(x_{m1}) C_{m1}	(x_{m2}) C_{m2}	(x_{m3}) C_{m3}	…	(x_{mn}) C_{mn}	a_m
	demand	b_1	b_2	b_3	…	b_n	

Table-10.1

Now the linear programming model representing the transportation problem is given by

The objective function is Minimize $Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$ Subject to the constraints

$\sum_{j=1}^{n} x_{ij} = a_i$, i=1,2,…….m (supply constraints)

$\sum_{i=1}^{m} x_{ij} = b_j$, j=1,2,…….$n$ (demand constraints)

$x_{ij} \geq 0$ for all i,j. (non-negative restrictions)

7. *Define degeneracy in transportation problem? How it is resolved?*

*Ans: The objective of transportation problem is to determine the amount to be transported from each origin to each destination such that the total transportation cost is minimized. In a transportation problem, if a basic feasible solution with m origins and n destinations has less than m+ n -1 positive Xij i.e. occupied cells, then the problem is said to be a **degenerate transportation problem**. The degeneracy problem does not cause any serious difficulty, but it can cause computational problem wile determining the optimal minimum solution. There fore it is important to identify a degenerate problem as early as beginning and take the necessary action to avoid any computational difficulty. The degeneracy can be identified through the following results:"In a transportation problem, a degenerate basic feasible solution exists if and only if some partial sum of supply (row) is equal to a partial sum of demand (column).*

8. *Explain MODI method to find optimal solution in transportation problem?*

Ans: The Modified Distribution Method, also known as MODI method or u-v method, which provides a minimum cost solution (optimal solution) to the transportation problem. The following are the steps involved in this method.

Step 1: *Find out the basic feasible solution of the transportation problem using any one of the three methods discussed in the previous section.*

Step 2: *Introduce dual variables corresponding to the row constraints and the column constraints. If there are m origins and n destinations then there will be m+n dual variables. The dual variables corresponding to the row constraints are represented by ui, i=1,2,….m where as the dual variables corresponding to the column constraints* are represented by v_j, $j=1,2,\ldots..n$. The values of the dual variables are calculated from the equation given below $u_i + v_j = c_{ij}$ if $x_{ij} > 0$

Step 3: Any basic feasible solution has $m + n - 1$ $x_{ij} > 0$. Thus, there will be $m + n - 1$ equation to determine $m + n$ dual variables. One of the dual variables can be chosen arbitrarily. It is also to be noted that as the primal constraints are equations, the dual variables are unrestricted in sign.

Step 4: If $x_{ij}=0$, the dual variables calculated in Step 3 are compared with the c_{ij} values of this allocation as $c_{ij} - u_i - v_j$. If al $c_{ij} - u_i - v_j \geq 0$, then by the theorem of complementary slackness it can be shown that the corresponding solution of the transportation problem is optimum. If one or more $c_{ij} - u_i - v_j < 0$, we select the cell with the least value of $c_{ij} - u_i - v_j$ and allocate as much as possible subject to the row and column constraints. The allocations of the number of adjacent cell are adjusted so that a basic variable becomes non-basic.

Step 5: A fresh set of dual variables are calculated and repeat the entire procedure from Step 1 to Step 5.

9. Explain some of the areas where transportation techniques are employed?

Ans: **The following are some of areas where transportation techniques are employed.**

- ✓ **Scheduling Airlines**
- ✓ **Deciding appropriate place to site new facilities such as warehouse, factory etc.**
- ✓ **Telecommunication Industry**
- ✓ **Health Services**

(8 Mark /Essay Questions)

10.

Solve the following transportation problem :

To

From	A	B	C	D	Available
I	21	16	25	13	11
II	17	18	14	23	13
III	32	27	18	41	19
Requirement	6	10	12	15	43

INITIAL SOLUTION

Finding Initial basic solution using Vogel method

Calculating row and column penalties. Finding maximimun penalty. Selecting cell with minimum cost in maximum penalty row/column and allocating it. Here, allocating Cell C14

Pena lty	4	1	4	10	
3	× 21	× 16	× 25	11 13	0
3	17	17	14	23	13
9	32	27	18	41	19
	6	10	12	4	

Calculating row and column penalties. Finding maximimun penalty. Selecting cell with minimum cost in maximum penalty row/column and allocating it. Here, allocating Cell C24

Pena lty	15	10	4	18	
0	× 21	× 16	× 25	11 13	0
3	17	17	14	4 23	9
9	32	27	18	× 41	19
	6	10	12	0	

Calculating row and column penalties. Finding maximimun penalty. Selecting cell with minimum cost in maximum penalty row/column and allocating it. Here, allocating Cell C21

Pena lty	15	10	4	0	
0	× 21	× 16	× 25	11 13	0
3	6 17	17	14	4 23	3
9	× 32	27	18	× 41	19
	0	10	12	0	

Calculating row and column penalties. Finding maximimun penalty. Selecting cell with minimum cost in maximum penalty row/column and allocating it. Here, allocating Cell C22

Pena lty	0	10	4	0	
0	× 21	× 16	× 25	11 13	0
3	6 17	3 17	× 14	4 23	0
9	× 32	27	18	× 41	19
	0	7	12	0	

Calculating row and column penalties. Finding maximimun penalty. Selecting cell with minimum cost in maximum penalty row/column and allocating it. Here, allocating Cell C33

Pena lty	0	0	0	0	
0	× 21	× 16	× 25	11 13	0
0	6 17	3 17	× 14	4 23	0
9	× 32	27	12 18	× 41	7
	0	7	0	0	

Calculating row and column penalties. Finding maximimun penalty. Selecting cell with minimun cost in maximum penalty row/column and allocating it. Here, allocating Cell C32

Pena lty	0	0	0	0	
0	× 21	× 16	× 25	11 13	0
0	6 17	3 17	× 14	4 23	0
0	× 32	7 27	12 18	× 41	0
	0	0	0	0	

Now all remaining supply and demand is 0. So, initial basic solution is achieved.

Total Cost: 793

OPTIMAL SOLUTION CALCULATION

Now, Finding Optimal solution using UV method

NEW ITERATION

Calculating U and V for all Rows and Columns and calculating penalty for unallocated cells

UV	7	7	-2	13	
0	-14 21	-9 16	-27 25	11 0 13	0
10	6 0 17	3 0 17	-6 14	4 0 23	0
20	-5 32	7 0 27	12 0 18	-8 41	0
	0	0	0	0	

Here all penalties is less then zero. So, it is optimum solution. Solution is as follows:

UV	7	7	-2	13	
0	-14 21	-9 16	-27 25	11 0 13	0
10	6 0 17	3 0 17	-6 14	4 0 23	0
20	-5 32	7 0 27	12 0 18	-8 41	0
	0	0	0	0	

SOLUTION

X14 -> 11
X21 -> 6
X22 -> 3
X24 -> 4
X32 -> 7
X33 -> 12

Cost = 11 * 13 + 6 * 17 + 3 * 17 + 4 * 23 + 7 * 27 + 12 * 18 = 793

Total Cost: 793

11. Explain various methods to obtain Initial Basic Feasible Solution in transportation problem?

Ans:

North West Corner Method:

The method starts at the North West (upper left) corner cell of the tableau (variable x11).

Step -1: *Allocate as much as possible to the selected cell, and adjust the associated amounts of capacity (supply) and requirement (demand) by subtracting the allocated amount.*

Step -2: *Cross out the row (column) with zero supply or demand to indicate that no further assignments can be made in that row (column). If both the row and column becomes zero simultaneously, cross out one of them only, and leave a zero supply or demand in the uncrossed out row (column).*

Step -3: *If exactly one row (column) is left uncrossed out, then stop. Otherwise, move to the cell to the right if a column has just been crossed or the one below if a row has been crossed out. Go to step -1.*

Least Cost Method

The least cost method is also known as matrix minimum method in the sense we look for the row and the column corresponding to which Cij is minimum. This method finds a better initial basic feasible solution by concentrating on the cheapest routes. Instead of starting the allocation with the northwest cell as in the North West Corner Method, we start by allocating as much as possible to the cell with the smallest unit cost.

If there are two or more minimum costs then we should select the row and the column corresponding to the lower numbered row. If they appear in the same row we should select the lower numbered column. We then cross out the satisfied row or column, and adjust the amounts of capacity and requirement accordingly. If both a row and a column is satisfied simultaneously, only one is crossed out. Next, we look for the uncrossed-out cell with the smallest unit cost and repeat the process until we are left at the end with exactly one uncrossed-out row or column.

Vogel Approximation Method (VAM):

VAM is an improved version of the least cost method that generally produces better solutions. The steps involved in this method are:

Step 1: *For each row (column) with strictly positive capacity (requirement), determine a penalty by subtracting the smallest unit cost element in the row (column) from the next smallest unit cost element in the same row (column).*

Step 2: *Identify the row or column with the largest penalty among all the rows and columns. If the penalties corresponding to two or more rows or columns are equal we select the topmost row and the extreme left column.*

Step 3: We select Xij as a basic variable if Cij is the minimum cost in the row or column with largest penalty. We choose the numerical value of Xij as high as possible subject to the row and the column constraints. Depending upon whether ai or bj is the smaller of the two ith row or jth column is crossed out.

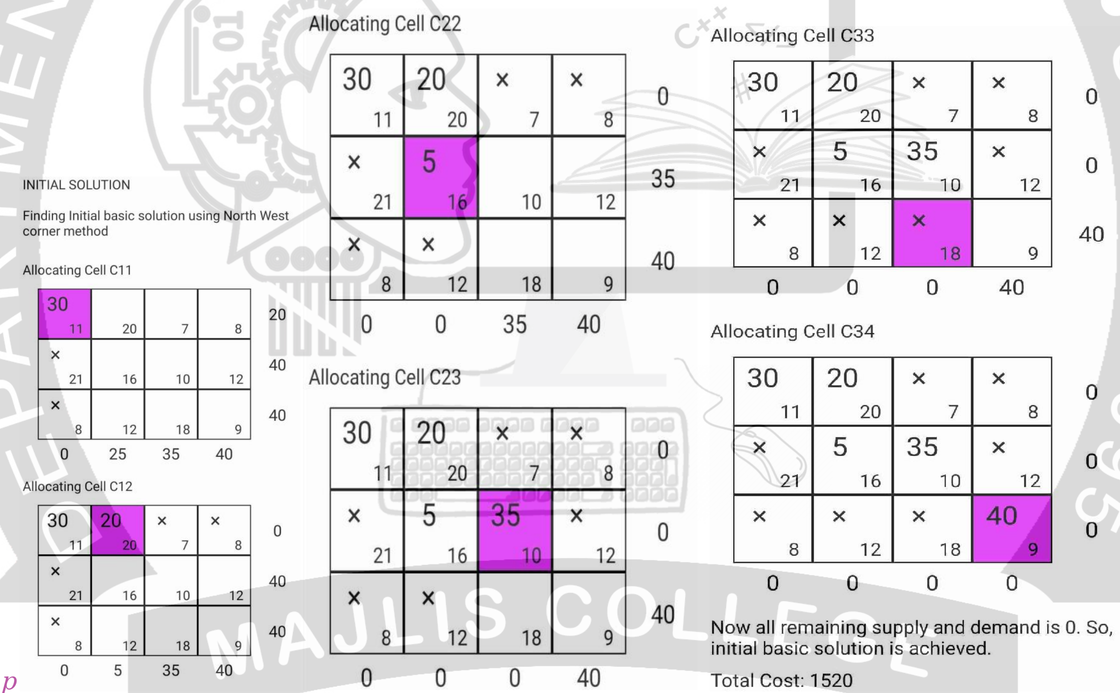
Step 4: The Step 2 is now performed on the uncrossed-out rows and columns until all the basic variables have been satisfied.

12. *Obtain the initial basic feasible solution for the following transportation problem by various methods?*

	W_1	W_2	W_3	W4	Availability
F_1	11	20	7	8	50
F_2	21	16	10	12	40
F_3	8	12	18	9	40
Requirements	30	25	35	40	

Ans:

North West Corner Method



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Least Cost Method

INITIAL SOLUTION

Finding Initial basic solution using Least Cost method

Allocating Cell C13

11	20	35 (7)	8	15
21	16	× 10	12	40
8	12	× 18	9	40
30	25	0	40	

Allocating Cell C21

× 11	20	35 (7)	8	15
30 (21)	16	× 10	12	10
× 8	12	× 18	9	40
0	25	0	40	

Allocating Cell C14

× 11	× 20	35 (7)	15 (8)	0
30 (21)	16	× 10	12	10
× 8	12	× 18	9	40
0	25	0	25	

Allocating Cell C34

× 11	× 20	35 (7)	15 (8)	0
30 (21)	16	× 10	× 12	10
× 8	12	× 18	25 (9)	15
0	25	0	0	

Allocating Cell C32

× 11	× 20	35 (7)	15 (8)	0
30 (21)	16	× 10	× 12	10
× 8	15 (12)	× 18	25 (9)	0
0	10	0	0	

Allocating Cell C22

× 11	× 20	35 (7)	15 (8)	0
30 (21)	10 (16)	× 10	× 12	0
× 8	15 (12)	× 18	25 (9)	0
0	0	0	0	

Now all remaining supply and demand is 0. So, initial basic solution is achieved.

Total Cost: 1560

Vogels Approximation Method

INITIAL SOLUTION

Finding Initial basic solution using Vogel method

Calculating row and column penalties. Finding maximimun penalty. Selecting cell with minimum cost in maximum penalty row/column and allocating it. Here, allocating Cell C32

Pena lty	3	4	3	1	
1	11	× 20	7	8	50
2	21	× 16	10	12	40
1	8	25 (12)	18	9	15
	30	0	35	40	

Calculating row and column penalties. Finding maximimun penalty. Selecting cell with minimum cost in maximum penalty row/column and allocating it. Here, allocating Cell C31

Pena lty	3	0	3	1	
1	11	× 20	7	8	50
2	21	× 16	10	12	40
1	15 (8)	25 (12)	× 18	× 9	0
	15	0	35	40	

Calculating row and column penalties. Finding maximimun penalty. Selecting cell with minimum cost in maximum penalty row/column and allocating it. Here, allocating Cell C11

Pena lty	10	0	3	4	
1	15 (11)	× 20	7	8	35
2	× 21	× 16	10	12	40
0	15 (8)	25 (12)	× 18	× 9	0
	0	0	35	40	

Calculating row and column penalties. Finding maximimun penalty. Selecting cell with minimum cost in maximum penalty row/column and allocating it. Here, allocating Cell C14

Penalty	0	0	3	4	
1	15 (11)	× (20)	× (7)	**35** (8)	0
2	× (21)	× (16)	(10)	(12)	40
0	15 (8)	25 (12)	× (18)	× (9)	0
	0	0	35	5	

Calculating row and column penalties. Finding maximimun penalty. Selecting cell with minimum cost in maximum penalty row/column and allocating it. Here, allocating Cell C23

Penalty	0	0	0	0	
0	15 (11)	× (20)	× (7)	35 (8)	0
2	× (21)	× (16)	**35** (10)	(12)	5
0	15 (8)	25 (12)	× (18)	× (9)	0
	0	0	0	5	

Calculating row and column penalties. Finding maximimun penalty. Selecting cell with minimum cost in maximum penalty row/column and allocating it. Here, allocating Cell C24

Penalty	0	0	0	0	
0	15 (11)	× (20)	× (7)	35 (8)	0
0	× (21)	× (16)	35 (10)	**5** (12)	0
0	15 (8)	25 (12)	× (18)	× (9)	0
	0	0	0	0	

Now all remaining supply and demand is 0. So, initial basic solution is achieved.

Total Cost: 1275

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OPERATIONS RESEARCH

FOURTH MODULE

Second Semester BCA

(Academic Year 2019-20 Onwards and 2017 Admissions)

Questions and Answers based on Fourth Module

Module IV

Assignment model: Mathematical formulation of the problem - assignment algorithm-impossible algorithms - travelling salesman problem

Short answers and Essay Questions

1. ***Define Assignment problem***

Assignment problem is a special LPP which deals with assignment of workers to machines, clerks to various checkout counters, salesman to different sales areas, service crews to different districts and so on. Thus in an assignment problem, the question is how the assignments should be made in order that the total cost involved is minimized.

2. ***Explain the method to solve assignment problem***

This method is specially designed to handle assignment problems in an efficient way. HAM is based on the concept of opportunity cost. For a typical balanced assignment problem involving a certain number of people and an equal number of jobs, with an objective function of minimization type, the method is applied as listed in the following steps.

Step 1: *Locate the smallest cost element in each row of the cost table and subtract it from each element in that row. As a result there shall be at least one zero in each row of this new table, called the reduced cost table.*

Step 2: *In the reduced cost table obtained, consider each column and locate the smallest element in it. Subtract the smallest value from every other entry in the column. As a result of this , there would be at least one zero in each of the rows and columns of the second reduced cost table.*

Step 3: *Draw the minimum number of horizontal and vertical lines (not diagonal lines) that are required to cover all 0 elements. If the number of lines drawn is equal to n (i.e. the number of rows or columns) the solution is optimal and proceeds to step 6. If the number of lines drawn is less than n, go to step 4.*

Step 4: *Select the smallest uncovered cost element. Subtract this element from all uncovered elements including itself and add this element to each value located at the intersection of any two lines. The cost elements through which only one line passes remain unaltered.*

Step 5: *Repeat step 3 and 4 until an optimal solution is obtained.*

Step 6: *Given the optimal solution, make the job assignments as indicated by the zero elements. This can be done as follows:*

a) *Locate a row which contains only one 'zero' element. Assign the job corresponding to this element to its corresponding person. Cross out the zeros, if any, in the column corresponding to the element, which is indicative of the fact that the particular job and person are no more available.*
b) *Repeat (a) for each of such rows, which contain only one zero. Similarly, perform the same operation in respect of each column containing only one zero element, crossing out the zero(s), if any, in the row in which the element lies.*
c) *If there is no row or column with only a single 0 element left, then select a row or column arbitrarily and choose one of the jobs (or persons) and make the assignment. Now cross the remaining zeros in the column and row in respect of which the assignment is made.*

d) Repeat steps (a) through (c) until all assignments are made.

e) Determine the total cost with reference to the original cost table.

3. Problems

A production supervisor is considering how he should assign the four jobs that are to be performed, to four of the workers. He wants to assign the jobs to the workers such that the aggregate time to perform the jobs is the least. Based on previous experience, he has the information on the time taken by the four workers in performing these jobs, as given in the table:

Worker	**Job**			
	A	B	C	D
1	45	40	51	67
2	57	42	63	55
3	49	52	48	64
4	41	45	60	55

Table 1: Time taken (in minutes) by 4 Workers

Solution: The solution to this problem has been discussed in a step wise manner:

Step 1: The minimum elements of each row is subtracted from all elements in the row as shown in the following table known as the reduced cost table or opportunity cost table:

Worker	**Job**			
	A	B	C	D
1	5	0	11	27
2	15	0	21	13
3	1	4	0	16
4	0	4	19	14

Step 2: For each column of the above table the minimum value is subtracted from all other values giving us another reduced cost table:

Worker	**Job**			
	A	B	C	D
1	5	0	11	14
2	15	0	21	0
3	1	4	0	3
4	0	4	19	1

Step 3: Draw the minimum number of lines covering all zeros. We will first cover those rows/columns which contain larger number of zeros such that we obtain the following reduced cost table.

Worker	**Job**			
	A	B	C	D
1	5	0	11	14
2	15	0	21	0
3	1	4	0	3
4	0	4	19	1

Step 4: Since the number of lines drawn=4(=n), the optimal solution is obtained. The assignments are made after scanning the rows and columns for unit zeros. Assignments made are shown with squares as depicted in the table.

Worker	Job			
	A	B	C	D
1	5	$\boxed{0}$	11	14
2	15	0	21	$\boxed{0}$
3	1	4	$\boxed{0}$	3
4	$\boxed{0}$	4	19	1

Assignments are made in the following order. Since row 1, 3 and 4 contain 1 zero each, we assign 1-B, 3-C and 4-A. Since worker 1 has been assigned the job B, we cross the zero in the second column of the second row. After making these assignments, only worker 2 and job D are left for assignment. The final pattern of assignments is 1-B, 2-D, 3-C and 4-A involving a total time of 40+55+48+41=184 minutes (as taken from the original table).

4. ***What is an unbalanced assignment problem?***

The Hungarian method of solving assignment problems require that the number of columns should be equal to the number of rows in which case the problem is known as balanced problem and when the rows and columns are unequal, it is called an unbalanced assignment problem. In case of such class of problems, one to one match is not possible.
In such situations, dummy columns / rows, whichever is smaller in number are inserted with zeros as the cost elements. For example, in case of 4x5 cost matrix, a dummy row is added. In each column in respect of this row, a zero would be placed. After this process of adding dummy rows/ columns, the problem is solved in a usual manner.

5. ***Problem***
You are given information about cost of performing different jobs by different persons. The job-person marking x indicates that the individual involved cannot perform the particular job. Using this information, state (a) the optimal assignment of jobs (b) the cost of such assignment.

Person	Job				
	J_1	J_2	J_3	J_4	J_5
P_1	27	18	X	20	21
P_2	31	24	21	12	17
P_3	20	17	20	X	16
P_4	22	28	20	16	27

Solution: Balancing the problem and assigning a high cost to the pairings P_1-J_3 and P_3-J_4, we obtain the following cost table:

Person	Job				
	J_1	J_2	J_3	J_4	J_5
P_1	27	18	M	20	21
P_2	31	24	21	12	17

***P*₃**	*20*	*17*	*20*	*M*	*16*
***P*₄**	*22*	*28*	*20*	*16*	*27*
***P*₅(dummy)**	*0*	*0*	*0*	*0*	*0*

From the above table a reduced cost table is derived as depicted below. However the cells with prohibited assignments continue to be shown with the cost element M, as M is defined to be extremely large such that subtraction or addition of a value does not practically affect it.To test optimality, lines are drawn to cover all zeros.

Person	**Job**				
	J_1	J_2	J_3	J_4	J_5
P_1	9	0	M	2	3
P_2	19	12	9	0	5
P_3	4	1	4	M	0
P_4	6	12	4	0	11
P_5(dummy)	0	0	0	0	0

Since the number of lines covering zeros is less than n, we select the lowest uncovered cell which equals
4. With this value we obtain the revised reduced cost table as depicted below:

Person	**Job**				
	J_1	J_2	J_3	J_4	J_5
P_1	9	$\boxed{0}$	M	6	3
P_2	15	8	5	$\boxed{0}$	1
P_3	4	1	4	M	$\boxed{0}$
P_4	2	8	$\boxed{0}$	0	7
P_5(dummy)	$\boxed{0}$	0	0	4	0

Since in the above table, the number of lines covering zeros equals 5(=n), an optimal assignment can be made which is : P_1-J_2, P_2-J_4, P_3-J_5, P_4-J_3 while job J_1 would remain unassigned. The assignment pattern would cost 18+12+16+20=66 in aggregate.

6. ***What is the Maximization Case of Assignment Problem?***

In some situations the assignment problem may call for maximization of profit, revenue etc. as the objective. For dealing with such problems, we first change it into an equivalent minimization problem. This is achieved by subtracting each of the elements of the given pay-off matrix from a constant value (say K). Usually the largest of all values in the given matrix is located and then each one of the values is subtracted from it. Then the problem is solved the same way as the minimization problem.

7. ***Problem***

   A company plans to assign 5 salesmen to 5 districts in which it operates. Estimates of sales revenue in thousands of rupees for each salesman in different districts are given in the following table. In your opinion, what should be the placement of the salesmen if the objective is to maximize the expected sales revenue?

Salesman	District				
	D_1	D_2	D_3	D_4	D_5
S_1	40	46	48	36	48
S_2	48	32	36	29	44
S_3	49	35	41	38	45
S_4	30	46	49	44	44
S_5	37	41	48	43	47

Solution:

Since this is a maximization problem, we first subtract each of the entries in the table from the largest one (i.e. 49) to obtain the following opportunity loss matrix:

Salesman	District				
	D_1	D_2	D_3	D_4	D_5
S_1	9	3	1	13	1
S_2	1	17	13	20	5
S_3	0	14	8	11	4
S_4	19	3	0	5	5
S_5	12	8	1	6	2

Now we can proceed the same way as in case of minimization problems by following the below mentioned steps:
Step 1: Subtract minimum value in each row from every value in the row so as to obtain the following reduced cost table:

Salesman	District				
	D_1	D_2	D_3	D_4	D_5
S_1	8	2	0	12	0
S_2	0	16	12	19	4
S_3	0	14	8	11	4
S_4	19	3	0	5	5
S_5	11	7	0	5	1

Step 2:Now subtract minimum value in each column from each value in that column in the above reduced cost table to obtain the following table. Test for optimality by drawing lines to cover zeros.

Salesman	District				
	D_1	D_2	D_3	D_4	D_5
S_1	8	0	0	7	0
S_2	0	14	12	14	4
S_3	0	12	8	6	4
S_4	9	1	0	0	5
S_5	1	5	0	0	1

Since the number of lines covering all zeros is fewer than n, we select the least uncovered cell value, which equals 4 and obtain a modified table as shown below:

Salesman	District				
	D_1	D_2	D_3	D_4	D_5

S_1	12	0	0	7	0
S_2	0	10	8	10	0
S_3	0	8	4	2	0
S_4	23	1	0	0	5
S_5	15	5	0	0	1

There are more than one optimal assignments possible in this case because of existence of multiple zeros in different rows and columns. The following assignments are possible:

S_1-D_2,S_2-D_1,S_3-D_5,S_4-D_3,S_5-D_4

or S_1-D_2,S_2-D_5,S_3-D_1,S_4-D_3,S_5-D_4 *or* S_1-D_2,S_2-D_5,S_3-D_1,S_4-D_4,S_5-D_3 *or*

S_1-D_2,S_2-D_1,S_3-D_5,S_4-D_4,S_5-D_3

Each of these assignment patters would lead to an expected aggregate sales equal to 231 thousand rupees.

9. ***Which method is used for solving assignment problem?***

   Ans:-Hungarian Method

10. ***What is travelling salesman problem?***

    *The **traveling salesman problem** consists of a **salesman** and a set of cities. The **salesman** has to visit each one of the cities starting from a certain one (**e.g.** the hometown) and returning to the same city. The **challenge** of the **problem** is that the **traveling salesman** wants to minimize the total length of the trip.*

11.

Solve the following assignment problem :

Man \ Job	J_1	J_2	J_3	J_4	J_5
M_1	11	17	8	16	20
M_2	9	7	12	6	15
M_3	13	16	15	12	16
M_4	21	24	17	28	26
M_5	14	10	12	11	13

Solution:

Original matrix:

11	17	8	16	20
9	7	12	6	15
13	16	15	12	16
21	24	17	28	26
14	10	12	11	13

Step 1:
Row deduction

3	9	0	8	12
3	1	6	0	9
1	4	3	0	4
4	7	0	11	9
4	0	2	1	3

Step 2:
Column deduction

2	9	0	8	9
2	1	6	0	6
0	4	3	0	1
3	7	0	11	6
3	0	2	1	0

Step 3:
Assign zeros

2	9	$\boxed{0}$	8	9
2	1	6	$\boxed{0}$	6
$\boxed{0}$	4	3	$\cancel{0}$	1
3	7	$\cancel{0}$	11	6
3	$\boxed{0}$	2	1	$\cancel{0}$

Step 4:
Row deduction

$$\begin{matrix} 0 & 7 & 0 & 6 & 7 \\ 2 & 1 & 8 & 0 & 6 \\ 0 & 4 & 5 & 0 & 1 \\ 1 & 5 & 0 & 9 & 4 \\ 3 & 0 & 4 & 1 & 0 \end{matrix}$$

Step 5:
Column deduction

$$\begin{matrix} 0 & 7 & 0 & 6 & 7 \\ 2 & 1 & 8 & 0 & 6 \\ 0 & 4 & 5 & 0 & 1 \\ 1 & 5 & 0 & 9 & 4 \\ 3 & 0 & 4 & 1 & 0 \end{matrix}$$

Step 6:
Assign zeros

$$\begin{matrix} \times\!\!\!\!\!0 & 7 & \times\!\!\!\!\!0 & 6 & 7 \\ 2 & 1 & 8 & \boxed{0} & 6 \\ \boxed{0} & 4 & 5 & \times\!\!\!\!\!0 & 1 \\ 1 & 5 & \boxed{0} & 9 & 4 \\ 3 & \boxed{0} & 4 & 1 & \times\!\!\!\!\!0 \end{matrix}$$

Step 6:
Do tick marking and Modify matrix

After marking & drawing lines:

$$\begin{matrix} \times\!\!\!\!\!0 & 7 & \times\!\!\!\!\!0 & 6 & 7 \\ 2 & 1 & 8 & \boxed{0} & 6 \\ \boxed{0} & 4 & 5 & \times\!\!\!\!\!0 & 1 \\ 1 & 5 & \boxed{0} & 9 & 4 \\ 3 & \boxed{0} & 4 & 1 & \times\!\!\!\!\!0 \end{matrix}$$

Modified matrix :

0	6	0	6	6
2	0	8	0	5
0	3	5	0	0
1	4	0	9	3
4	0	5	2	0

Step 7:
Row deduction

0	6	0	6	6
2	0	8	0	5
0	3	5	0	0
1	4	0	9	3
4	0	5	2	0

12. Problem solution

Solve the following minimal assignment problem :

15	13	14	17
11	12	15	13
13	12	10	11
15	17	14	16

Original matrix:

15	13	14	17
11	12	15	13
13	12	10	11
15	17	14	16

Step 1:
Row deduction

2	0	1	4
0	1	4	2
3	2	0	1
1	3	0	2

Step 2:
Column deduction

2	0	1	3
0	1	4	1
3	2	0	0
1	3	0	1

Step 3:
Assign zeros

2	$\boxed{0}$	1	3
$\boxed{0}$	1	4	1
3	2	$\cancel{0}$	$\boxed{0}$
1	3	$\boxed{0}$	1

Step 4:
Solution

x1->y2, x2->y1, x3->y4, x4->y3, Value: 49

MAJLIS ARTS AND SCIENCE COLLEGE
PURAMANNUR

PG DEPARTMENT OF COMPUTER SCIENCE

(Affiliated to the University of Calicut)

OPERATIONS RESEARCH

FIFTH MODULE

Second Semester BCA

(Academic Year 2019-20 Onwards and 2017 Admissions)

Questions and Answers based on Fifth Module

Module 5

Network Scheduling: Concept of network, basic components, PERT and CPM, Rules of network construction, maximal flow problem, project scheduling critical path calculations, advantages of network (PERT/CPM).

Sequencing models: processing n jobs through two machines, n jobs through three machines, two jobs through m machines.

1. Sequencing problem involving processing of two jobs on n machines:

   Ans: Have a condition that the processing of two jobs must be in the same order.
2. In critical path analysis, CPM is:

   Ans: event oriented
3. In PERT analysis, the shortest possible time to perform an activity, assuming that everything goes well is called:

   Ans: most likely
4. In sequencing problem, the time interval between starting the first job and completing the last job including the idle time in the particular order by the given set of machines is called

   Ans: Total elapsed time
5. Total float related to critical activity is

   Ans: zero
6. An activity which started immediately after the completion of one or more activities is known as .... activity.

   Ans: Successor activity
7. The time for which a machine does not have a job to process is called...

   Ans: idle time
8. Define Optimistic Time Estimates.

   Ans: Optimistic time is a concept in PERT to represent the shortest estimated time period within which a task is likely to be completed, and is used in project planning.
9. Define slack time and total float in the context of network model.

   Ans: Total float is the difference between the finish date of the last activity on the critical path and the project completion time. Any delay in an activity on the critical path would reduce the amount of total float available on the project.

Slack time can be defined as the amount of time a task can be delayed without causing another task to be delayed or impacting the completion date of your project.

10. Describe an event in Network Scheduling.

Ans: An event is a specific instant of time, which makes the start or end of an activity. Event consumes neither time nor resources.

11. Define Sequencing problem.

Ans. It is the selection of an appropriate order in which large number of jobs can be assigned to a finite number of machines to optimize the outputs in terms of time, cost or profit.

12. Explain Network Analysis.

Ans: Network Analysis is used to plan, monitor and control the projects to minimize cost, project time, optimum utilization of resources, avoiding delays.

13. What is no passing rule in sequencing problem?

Ans: No passing rule means that the passing is not allowed, i.e., the same order of jobs is maintained over each machine. If n jobs are to be processed through two machines A and B in the order AB, then this means that each job will go to machine A first and then to B.

14. Define critical activity

Ans: Critical Activity that is referred to a specific schedule activity that is occurred to be an element of a critical path that happens to be placed within a project schedule. Critical activities are single-minded that helps in the process of implementation and utilization of the critical path method.

15. Explain the procedure of problems with n jobs and two machines.

Ans:

Step 1: find the shortest processing time among jobs not yet scheduled.

Step 2: if the shortest processing time is on machine 1, assign the job as early as possible.

Otherwise assign it as late as possible (machine 2).

Step 3: eliminate the last job scheduled.

Step 4: repeat steps 1,2 until all jobs scheduled.

16. Distinguish between PERT and CPM analysis.

Ans:

Project Evaluation and Review Technique (PERT):

PERT is appropriate technique which is used for the projects where the time required or needed to complete different activities are not known. PERT is majorly applied for scheduling, organization and integration of different tasks within a project. It provides the blueprint of project and is efficient technique for project evaluation.

Critical Path Method (CPM):

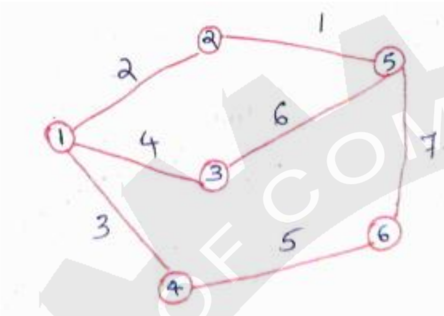
CPM is a technique which is used for the projects where the time needed for completion of project is already known. It is majorly used for determining the approximate time within which a project can be completed. Critical path is the largest path in project management which always provide minimum time taken for completion of project.

PERT	CPM
PERT is that technique of project management which is used to manage uncertain (i.e., time is not known) activities of any project.	CPM is that technique of project management which is used to manage only certain (i.e., time is known) activities of any project.
It is event-oriented technique which means that network is constructed on the basis of event.	It is activity-oriented technique which means that network is constructed on the basis of activities.
It is a probability model.	It is a deterministic model.
It majorly focuses on time as meeting time target or estimation of percent completion is more important.	It majorly focuses on Time-cost trade off as minimizing cost is more important.
It is appropriate for high precision time estimation.	It is appropriate for reasonable time estimation.
It has Non-repetitive nature of job.	It has repetitive nature of job.
There is no chance of crashing as there is no certainty of time.	There may be crashing because of certain time boundation.
It doesn't use any dummy activities	It uses dummy activities for representing sequence of activities.
It is suitable for projects which required research and development.	It is suitable for construction projects.

17.

Draw the network diagram to the following activities :

Activity (i, j) :	1 - 2	1 - 3	1 - 4	2 - 5	3 - 5	4 - 6	5 - 6
Time duration :	2	4	3	1	6	5	7



18.

A small maintenance project consists of the following 10 jobs. Draw network diagram (arrow diagram). Calculate : (a) T_E and T_L values of all events ; (b) EST, LST, EFT, LFT of all activities ; and (c) Floats of all the activities. Also obtain : (i) Critical activities ; and (b) Project duration :

Activity :	1 - 2	2 - 3	2 - 4	3 - 5	3 - 6	4 - 6	4 - 7	5 - 8	6 - 8	7 - 8
Duration :	4	6	10	8	2	12	4	15	14	8

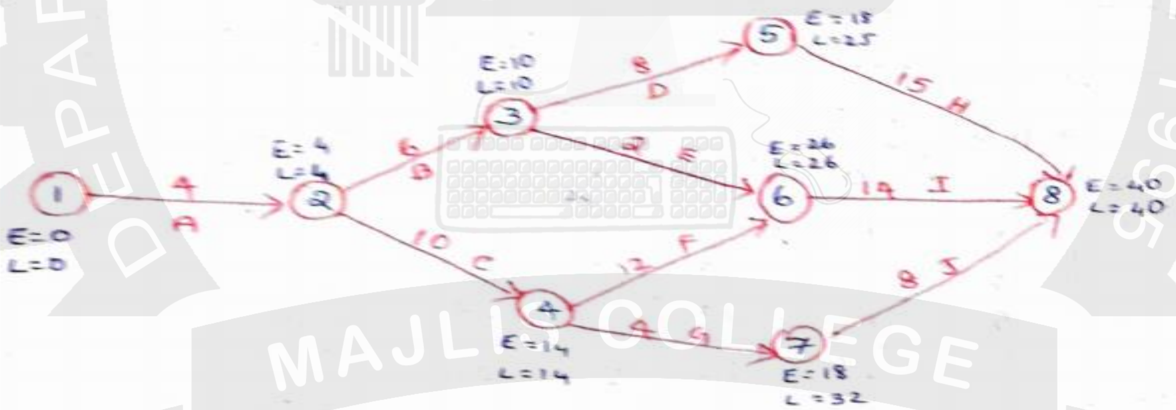
Activity		Duration	EST	EFT	LST	LFT	Total Float
1 – 2	A	4	0	4	0	4	0
2 – 3	B	6	4	10	4	10	0
2 – 4	C	10	4	14	4	14	0
3 – 5	D	8	10	18	17	25	7
3 – 6	E	2	10	12	24	26	14
4 – 6	F	12	14	26	14	26	0
4 – 7	G	4	14	18	28	32	14
5 – 8	H	15	18	33	25	40	7
6 – 8	I	14	26	40	26	40	0
7 – 8	J	8	18	26	32	40	14

EST = E ; LFT = L

EFT = EST + Activity duration

LST = LFT – Activity duration

Total float = (LFT – EFT) or (LST – EST)



Critical activities : A, B, C, F, I

: 1–2, 2–3, 2–4, 4–6, 6–8

Project duration : 4 + 10 + 12 + 14 = <u>40</u>.

19.

We have five jobs each of which must go through the two machines A and B in the order A, B. Processing times in hours are given in the table below :

Job	:	1	2	3	4	5
Time on Machine A	:	5	1	9	3	10
Time on Machine B	:	2	6	7	8	4

Determine a sequence for the five jobs that will minimize the elapsed time. Also find : 4

(a) Total minimum elapsed time ; and (b) Idle time for machine. (i) A ; and (ii) B.

sequencing Problem

	1	2	3	4	5
A	5	$\boxed{1}$	9	3	10
B	2	6	7	8	4

seq: _ _ _ _ _

→

	1	2	3	4	5
A	5		9	3	10
B	$\boxed{2}$		7	8	4

seq: 2 _ _ _ _

	1	2	3	4	5
A			9	$\boxed{3}$	10
B			7	8	4.

seq: 2 _ _ _ 1

	1	2	3	4	5
A			9		10
B			7		$\boxed{4}$

seq: 2 4 _ _ 1

	1	2	3	4	5
A			9		
B			$\boxed{7}$		

seq: 2 4 _ 5 1

seq: 2 4 3 5 1

	1	2	3	4	5
A	5	$\boxed{1}$	9	$\boxed{3}$	10
B	$\boxed{2}$	6	$\boxed{7}$	8	$\boxed{4}$.