1) show $m(a+bX) = a + bm(X)$

$$m(a+bX) = \frac{1}{N}\sum_{i=1}^{N}(a+bx_i)$$

$$= \frac{1}{N}\left(\sum_{i=1}^{N}a + b\sum_{i=1}^{N}x_i\right)$$

$$= \frac{1}{N}(Na) + b\left(\frac{1}{N}\sum_{i=1}^{N}x_i\right)$$

$$= a + bm(X)$$

2) show $\mathrm{cov}(X,X) = S^2$

$$\mathrm{cov}(X,X) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(x_i - m(X))$$

$$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))^2 = S^2$$

3) show $\mathrm{Cov}(X, a+bY) = b\,\mathrm{cov}(X,Y)$

☆ $m(a+bY) = a + bm(Y)$

$$\mathrm{cov}(X, a+bY) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))((a+by_i) - m(a+bY))$$

$$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(a + by_i - (a + bm(Y)))$$

$$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))\cdot b(y_i - m(Y))$$

$$= b\left(\frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(y_i - m(Y))\right)$$

$$= b\,\text{cov}(X, Y)$$

4) show that $\text{cov}(a + bX, a + bY) = b^2 \text{cov}(X, Y)$

since

$$(a + bx_i) - m(a + bX) = b(x_i - m(X))$$

$$(a + by_i) - m(a + bY) = b(y_i - m(Y))$$

so

$$\text{cov}(a + bX, a + bY) = \frac{1}{N}\sum_{i=1}^{N} b(x_i - m(X)) \cdot b(y_i - m(Y))$$

$$= b^2 \text{cov}(X, Y)$$

$$\text{cov}(bX, bX) = b^2 \text{cov}(X, X) = b^2 s^2$$

5)

Yes, if $b>0$, $a+bX$ is increasing, so it preserves order. The median will shift accordingly.

$$\text{med}(a+bX) = a + b\,\text{med}(X)$$

The quartiles will shift the same way.

$$IQR(a+bX) = (Q3_{a+bX} - Q1_{a+bX})$$
$$= (a + bQ3_X) - (a + bQ1_X)$$
$$= b\,IQR(X)$$

6) Let $P(X=0) = 0.5$, $P(X=4) = 0.5$

$$m(X) = 0 + 2 = 2$$
$$(m(X))^2 = 2^2 = 4$$
$$m(X^2) = E[X^2] = 0 + 8 = 8$$
$$8 \neq 4 = (m(X))^2$$

$$\sqrt{m(X)} = \sqrt{2}$$
$$m(\sqrt{X}) = E[\sqrt{X}] = 0 + 1 = 1$$

$$m(\sqrt{X}) = 1 \neq \sqrt{2} = \sqrt{m(X)}$$