Deterministic Motion Planning for Redundant Robots along End-Effector Paths

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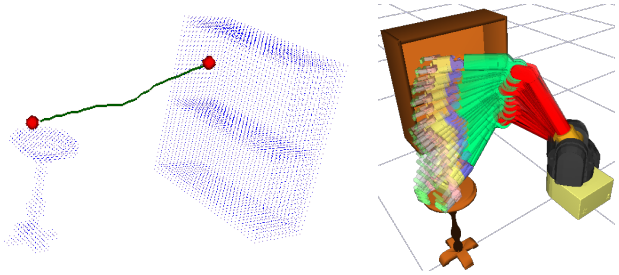
***Abstract*—In this paper we propose a deterministic approach to solve the *Motion Planning along End-Effector Paths problem* (MPEP) for redundant manipulators. Most of the existing approaches are based on local optimization techniques, hence they do not offer global guarantees of finding a path if it exists. Our proposed method is resolution complete. This feature is achieved by discretizing the Jacobian nullspace at each waypoint and selecting the next configuration according to a given heuristic function. To escape from possible local minima, our algorithm implements a backtracking strategy that allows our planner to recover from erroneous previous configuration choices by performing a breadth-first backwards search procedure. We present the results of simulated experiments performed with diverse manipulators and a humanoid robot.**



(a) Workspace Path (b) Execution

Fig. 1. Experiment with a 7-DOF BarretWMA Arm

I. Introduction

The Motion Planning along End-Effector Paths problem (MPEP), as defined by Oriolo in [1], refers to the problem in which a *task-redundant* robot must trace a given end-effector path while avoiding collisions with its environment.

Task redundancy is a desirable feature in robotic manipulators. The additional degrees of freedom allow the robot to not only achieve its primary task, such as end-effector tracing, but it also endows the system with the ability of achieving secondary goals along the way. While our main concern is obstacle avoidance, other redundancy uses in the planning literature include joint velocity control, singularity avoidance and torque minimization.

The MPEP problem implies mapping an end-effector path onto an executable configuration path. Diverse methods have been proposed to solve this problem, most of which are based on a *redundancy resolution approach*. In this area, the majority of the existing solutions are based on local optimization of task-based performance functions [2]. Global methods do exist, but their high computational costs have precluded them from being extensively used.

Our interest in solving the MPEP problem is twofold: First, MPEP is a very common problem in industrial environments, where tasks that require end-effector precision are usually performed (e.g. laser cutting). Our second motivation is related to human-robot interaction. Since service robots (mostly humanoids) will eventually become part of our households, they will be required to navigate through cluttered environments, in which redundancy becomes crucial to successfully achieve manipulation tasks (i.e. grab a bottle). Additionally, MPEP is potentially useful for robots to learn new manipulation tasks from human demonstrations. By considering only the end-effector path as a key feature, it is up to the robot to figure out how to replicate the movement with its own particular kinematic configuration (that may or may not be anthropomorphic).

The rest of the paper is organized as follows: We present background inverse kinematic theory and a brief review of existing approaches related to MPEP in Section II. In Section III we describe our proposed algorithm. Results of simulated experiments are presented in section IV. Finally, Section V concludes this paper.

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II. Background

The forward kinematics of a manipulator is given as follows:

$$x = f(q) \tag{1}$$

and its differential version is expressed as:

$$\dot{x} = J\dot{q} \tag{2}$$

where $x \in R^m$, $q \in R^n$, and m and n are the dimensions of the task and joint space respectively. If $m < n$, the manipulator is *task redundant*.

Manipulator tasks are varied; however, here we briefly describe two of the most common (as it is similary explained in [3]):

- *End-Point Goal Task:* The manipulator is given start and end task space points. The task is to find a sequence of joint space configurations that connect them. This is most known as a *path planning problem*
- *Point-to-Point Task:* The manipulator is given a task space trajectory to be mapped to a joint space trajectory. This is most known as a *tracking problem*, which will be the focus of this paper.

The inverse kinematics problem consists on finding a mapping such that $q = f^{-1}(x)$. For redundant manipulators there are potentially infinite values q for a given x, which can be obtained by solving (2). A particular solution is given by:

$$\dot{q} = J^{\dagger}\dot{x} \tag{3}$$

where $J^{\dagger}$ is a generalized inverse that can be chosen to minimize a specific criterion. In [4], Whitney used the Moore-Penrose pseudo-inverse to minimize the norm of $\dot{q}$. Whitney further proposed to use a pseudo-inverse Jacobian weighted with the inertia matrix in order to minimize the kinematic energy of the system. A generalization of this is the weighted least-norm method (WLM) presented in [5], which proved to be particularly effective to avoid joint limits. Recent extensions to this approach include the work of Xiang with GWLM [6].

(3) is a particular solution of 2. In general, the set of possible solutions can be expressed as:

$$\dot{q} = J^{\dagger}\dot{x} + (I - J^{\dagger}J)\dot{q}_0 \tag{4}$$

The second right-hand term represents the homogeneous solutions or *self-motions*, that is, motions in the configuration space that do not produce motions in the task space.

Homogeneous solutions are configurations in the Jacobian nullspace. These can be found by projecting an arbitrary vector $\dot{q}_0$ on the nullspace, such as in (4) where the columns of $(I - J^{\dagger}J)$ are the basis of the nullspace of J. Most of the proposed methods differ in their choosing of $\dot{q}_0$. The most widely used method is the *Gradient projection method* (GPM) proposed by Liegeois in [7]. The GPM method consists on defining an optimization function $H(q)$ to be minimized. By defining:

$$\dot{q}_0 = -\alpha\frac{\partial H}{\partial q} \tag{5}$$

it produces solutions that move the manipulator away from undesirable configurations. Diverse H functions have been used in the literature, such as the measure manipulatability in [8], used to avoid singularities. Other applications include obstacle avoidance [9], minimization of torques [10], avoidance of joint limits [7], among others. An excellent overview of these early methods can be found in [11]. For problems with more than one subtask, nested approaches have been proposed [12] [13], based on the relative priority between each subtask. Other methods instead define a weigthed sum of optimization functions, such as in [14].

There are other approaches that tackle the redundancy problem by defining additional constraints such that the problem is no longer redundant. Examples of these are the Extended Jacobian [15] and the Augmented Jacobian [16] [17].

The methods mentioned above solve different kinds of problems; however all of them share the same weakness of being local, which precludes them to fall in local minima. There are a few global approaches in the literature, but these are rarely used in practice due to the intensive computation required (Add citations). In the next section we propose a method that attempts to escape the local minima curse by using backtracking in the discretized nullspace of the manipulator.

III. Proposed Algorithm

A. Basics

We saw in section II that the solution to (2) has the general form:

$$\dot{q} = J^{\dagger}\dot{x} + (I - J^{\dagger}J)\dot{q}_0 \tag{6}$$

where the second right-hand term represents the self-motions in the Jacobian nullspace, which is a subspace of dimensions $(n-m)$. Hence, we can write (6) equivalently as:

$$\dot{q} = J^{\dagger}\dot{x} + \sum_{i=1}^{n-m} w_i\hat{e}_i \tag{7}$$

where:

- $\hat{e}_i$: Normalized basis of the Jacobian nullspace
- w_i: Coefficients of each $\hat{e}_i$

Both representations are equivalent, however (7) is of special interest to us since instead of having to define a vector $q_0 \in R^n$, we define the homogeneous solution in terms of the $(n-m)$ coefficient w_i. The problem, however still has potentially infinite solutions.

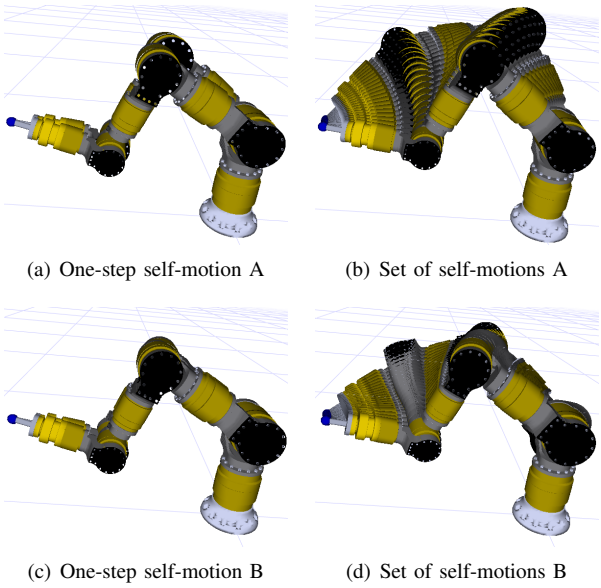
Our approach proposes, instead of finding a q_0 vector (as GPM does), to effectuate a search in the discretized Jacobian nullspace. The discretization is carried out by selecting a range of values for each w_i. Notice that, since we are considering the tracking of a task space *trajectory*, we do only consider homogeneous solutions that are realizable in one time step. To make this clear, please refer to Fig.2. Figures 2.a and 2.c show the one-step self-motions that can be achieved in one time step. Figures 2.b and 2.d show the subset of additional self-motions corresponding to these that would require more than one time-step to be executed, hence they are not considered in the nullspace search explained in this paper[1].

Once the set of self-motions is generated we proceed to eliminate the configurations that are in collision, keeping only the set of *valid* configurations, from which any can be a possible solution for the task space point evaluated. Rather than picking a random configuration from the nullspace set, we choose a configuration such that it maximizes a performance optimization function. Previous work such as [18] suggests a serie of diverse functions to assess the desirability of each configuration. In particular, we have evaluated the so-called Joint Range Availability measurement (JRA):

$$JRA(q) = \sum_{i=1}^{n}\frac{q_i - \bar{q}_i}{q_{iMax} - q_{iMin}} \tag{8}$$

in our experiments. We save the nullspace sets as priority queues ordered w.r.t. the JRA measure of each configuration. The approach explained so far is shown in Algorithm 1

[1]Notice that in the general case of tracking a *path*, *all* the self-motion configurations should also be considered. We are currently working on this topic



(a) One-step self-motion A (b) Set of self-motions A

(c) One-step self-motion B (d) Set of self-motions B

Fig. 2. Illustration of self-motion considered

Algorithm 1: Simple_Track($\mathcal{P}$, q_0, $\mathcal{Q}$)

Input: Workspace trajectory $\mathcal{P}$ and initial joint configuration q_0

Output: Jointspace trajectory $\mathcal{Q}$

1 $q \leftarrow q_0$

2 **for** $i \leftarrow 1$ **to** $\mathcal{P}$.size() **do**

3 if Generate_Sol(q, $\mathcal{P}[i]$, $\mathcal{NS}[i]$) **is** false **then**

4 **return** false

5 **else**

6 $q \leftarrow \mathcal{NS}[i][0]$; // Top priority element

7 $\mathcal{Q}$.push_back(q)

8 **return** true

Algorithm 1 is prone to fall in local minima. This can be easily seen in Line 3 in Algorithm 1, which reports failure if a solution is not found for a workspace point. We know that a solution might exist if previous configurations were chosen differently. Hence, our method improves this by implementing a *backtracking schema* that operates whenever local solutions fail.

B. Backtracking

Algorithm 4 implements a recursive backtracking procedure. It accepts as input the current configuration, the stored nullspace sets of the trajectory mapped so far and the maximum number of backtracking steps allowed. When a failure is detected at time i, our algorithm goes back one step $(i-1)$, pop up the failing configuration and choose a different one from the $\mathcal{NS}[i-1]$. Notice that since $\mathcal{NS}$ stores the nullspace sets as priority queues, the next configuration to be evaluated is the next best, according to our optimization function. In case the backtrack fails again in producing a valid solution, it backtracks to $i-2$ and it will repeat the procedure until it finds a solution or until it reaches the maximum allowed number of backtracking steps.

Of course, backtracking can be computationally intensive if the depth is too big. In order to avoid backtracking as much as possible, we must use a reasonable heuristic function such that backtrack is only used in extreme cases. In our experiments we only used the JRA function, which seemed to produce adequate results.

Our proposed method is shown in Algorithm 3. The recursive backtracking routine (which actually implements a breadth-first search) is presented in Algorithm 4. Notice that the latter algorithm depends on the ForwardSearch routine(Algorithm 5) which searches for a solution from the backtracked configuration $(i-step)$. Also note that each time a backtrack is executed, the nullspace sets generated previously from $(i-step-1)$ to i are cleared and updated according to the new configuration found in $(i-step)$.

Algorithm 2: Generate_Sol(q, p, ns)

Input: Current joint configuration q, next workspace point p

Output: Nullspace queue ns corresponding to q

1 $ds \leftarrow (p - \text{ToTaskSpace}(q))$

2 $q_p \leftarrow J^{\dagger}(q) \cdot ds$; // Least-norm sol

3 $\hat{e} = J(q)$.kernel() ; // J nullspace basis

4 **forall** *combinations of* w_j **do**

5 $q_{ns} \leftarrow q + q_p + \sum_{j=1}^{m-n} w_j \hat{e}_j$

6 **if** $\| p$ - ToTaskSpace(q_{ns})$\| <$ *thresh* **then**

7 **if** InCollision(q_{ns}) **is** false **and** InLimits(q_{ns}) **is** true **then**

8 ns.Push_Queue(q_{ns}, JRA(q_{ns}))

9 **if** ns.size() > 0 **then**

10 **return** true

11 **else**

12 **return** false

IV. Experiments and Results

In this section we present a series of simulated experiments with diverse manipulators. The simulations were made using the integrated GRIP + DART[2] software platform. For collision detection we used the VCOLLIDE package [19]. All the experiments were executed on a Intel 2 Core Duo (2.80GHz).

A. Preliminaries

Before presenting the experiments, some details should be mentioned:

- All the experiments presented are *end-effector position tracking* problems. We chose to focus this paper on

[2]GRIP (https://github.com/golems/grip) is a robotics simulator based on the physical engine DART (https://github.com/golems/dart). Both packages are OpenSource projects currently developed at Georgia Tech

Algorithm 3: Complete_Track($\mathcal{P}$, q_0, $\mathcal{Q}$, *maxStep*)

Input: Workspace trajectory $\mathcal{P}$ and initial joint configuration q_0, maximum backtrack allowed *maxStep*

Output: Jointspace trajectory $\mathcal{Q}$

```
q ← q_0
for i ← 1 to P.size() do
    if Generate_Sol(q, P[i], NS[i]) is false then
        step ← 1
        b = Backtrack(i, step, maxStep, NS, P)
        if b is false then
            return false
        else
            Update(Q, i, step, NS)
    else
        q ← NS[i][0]
        Q.push_back(q)
return true
```

Algorithm 4: Backtrack(*i*, *step*, *maxStep*, $\mathcal{NS}$, $\mathcal{P}$)

Input: current index i, current backtrack *step*, maximum backtrack *maxStep*, nullspace sets $\mathcal{NS}$, workspace trajectory $\mathcal{P}$

Output: Updated $\mathcal{NS}$

```
if step < maxStep then
    if ForwardSearch(i, step, maxStep, NS, P) is false then
        return Backtrack(i, step++, maxStep, NS, P)
    else
        return true
else
    return ForwardSearch(i, step, maxStep, NS, P)
return true
```

Algorithm 5: ForwardSearch(*i*, *step*, *maxStep*, $\mathcal{NS}$, $\mathcal{P}$)

Input: current index i, current backtrack *step*, maximum backtrack *maxStep*, nullspace sets $\mathcal{NS}$

Output: Updated $\mathcal{NS}$

```
ForwardClear_NS(i, step, NS)
j ← i - step
found ← false
while NS[j].size() > 0 and found is false do
    b ← Generate_Sol(NS[j][0], P[j+1], NS[j+1])
    if b is true then
        if step > 1 then
            found ← ForwardSearch(i, step-1, maxStep, NS, P)
        else
            found ← true
    else
        NS[j].Pop_Queue()
if found is false then
    if step is maxStep then
        return false
    else
        NS[j+1].Pop_Queue()
else
    return true
```

Procedure Update($\mathcal{Q}$, *i*, *step*, $\mathcal{NS}$)

```
for j ← 1 to step do
    // Update the changed joint trajectory points
    Q[i - j] ← NS[i - j][0]
end
```

Procedure ForwardClearNS(*i*, *step*, $\mathcal{NS}$)

```
if step > 1 then
    for j ← 1 to step do
        NS[i - j].clear()
    end
end
```

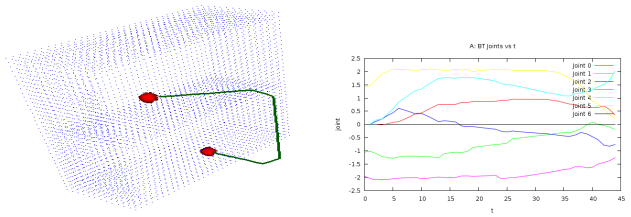
position (rather than pose) tracking since the manipulator has more redundant degrees of freedom with respect to this kind of tasks. Having less constraints poses a more challenging scenario to test our algorithm.

- The workspace trajectories used in the experiments were generated with a low-level workspace planner described in [20]
- All the kinematics evaluations were handled by the DART dynamics package. Additional linear algebra calculations (such as the nullspace basis or the Jacobian pseudo-inverse) were performed using the Eigen[3] C++ library.
- All the experiments used a discretization of 10 values per each w_i. The range of each w_i was $[-10, 10]$. The nullspace basis $\hat{e}_i$ were normalized to have the same relative norm w.r.t each other.

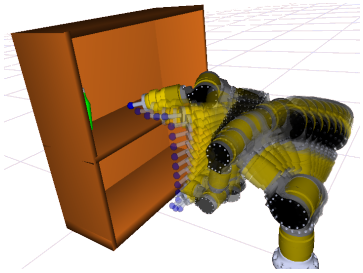
[3]http://eigen.tuxfamily.org

B. Experiment 1: LWA3

Our first series of experiments were done with a 7 DOF LWA3 arm, as shown in Fig.3. The initial configuration of the robot has its end effector below the cabinet. The workspace trajectory to follow is shown in Fig.3(b). Notice that the end-effector trajectory is very close to the cabinet, hence the tracking is harder since there are more chances of the manipulator links colliding with the object. The resulting



(a) Workspace trajectory (b) Joint trajectory



(c) Execution

Fig. 3. Experiment with a 7-DOF LWA3 Schunk arm

execution is shown in Fig.3(c).

C. Experiment 2: BarretWMA

Our second experiment was done with a 7 DOF BarretWMA manipulator. The scenary kept the same as in the experiment with the LWA3 but the initial configuration of the arm changed as it is shown in Fig.4. The arm has its end effector located on top of the cabinet and its target location is resting on top of the table. The resulting trajectory is shown in Fig.4.(c). In this execution, two backtrack calls were needed (at time step 3 and 31 out of the total trajectory of 35 points) both of them requiring a depth of 2. Notice that a simple search such as GPM is not able to solve this problem for this particular workspace to follow.

D. Experiment 3: Mitsubishi

1) Case 1: Straight Line:

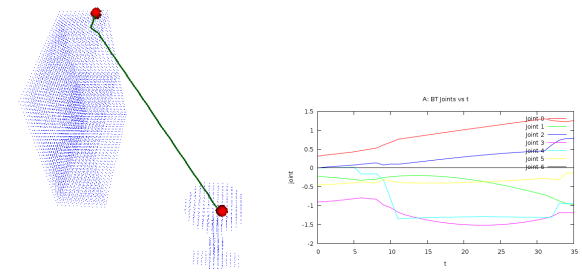
2) Case 2: Straight Line:

E. Experiment 4: Dual-Armed robot

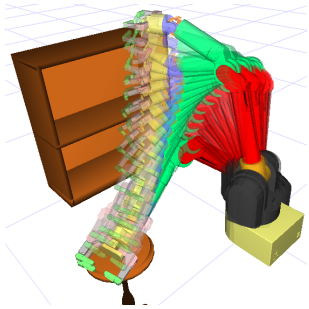
V. Conclusions and Future Work

We have presented an approach to solve the trajectory tracking problem for redundant manipulators based on the discretization of the Jacobian nullspace and a backtracking strategy to prevent local minima traps. In contrast with previous approaches, we exploit the nullspace by considering more than one possible configuration and allow our method to be more flexible. In contrast with previous work in which a set of joints were arbitrarily selected as non-redundant, our approach uses the nullspace basis of the Jacobian to perform the search with the minimum required values.

As future work, we intend to devise more simulated experiments with more challenging scenarios and further test the presented method in a physical manipulator. Additionally, we plan to test our approach with *pose tracking* problems to verify its effectiveness.

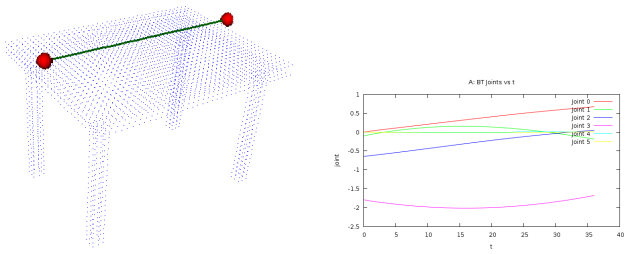


(a) Workspace path (x, y, z) (b) Tracking

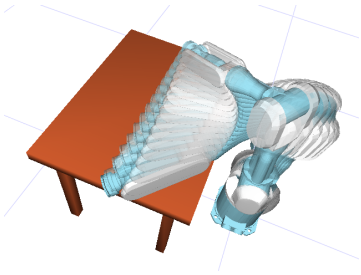


(c) Execution

Fig. 4. Experiment with a 7-DOF BarretWMA



(a) Workspace trajectory (b) Tracking



(c) Execution

Fig. 5. Experiment with a 6-DOF manipulator

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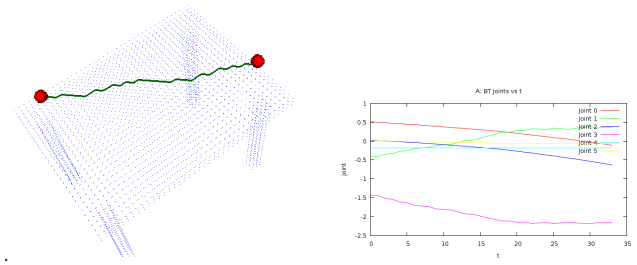
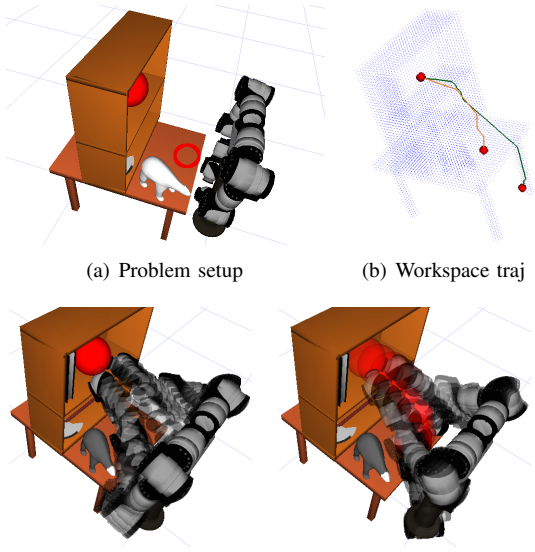


(a) Workspace trajectory (b) Tracking

(c) Execution

Fig. 6. Experiment with a 6-DOF manipulator



(a) Problem setup (b) Workspace traj

(c) Reach execution (d) Move execution

Fig. 7. Experiment with a Dual-Armed robot

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