Tutorial 2: Comparison $X22$ versus $X20$.

Ana Paula Fernandes & Gilmar Guimarães

October 24, 2016

Abstract

Tutorial 2 is the comparison between $X22B0T0$ versus $X20B0T0$. Where $X20B0T0$ problem is described in tutorial 1.

Equation

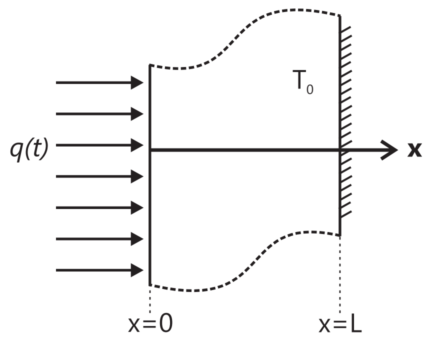

Figure 1: $X22$ Problem

The temperature distribuition is discribed by the differencial equation

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{1a}$$

with boundary conditions

$$-k\frac{\partial T}{\partial x}\bigg|_{x=0} = q(t); \qquad \frac{\partial T}{\partial x}\bigg|_{x=L} = 0 \tag{1b}$$

and initial temperature

$$T(x,0) = F(x) = T_0 \tag{1c}$$

$$
\begin{aligned}
T(x,t) = &\int_{x'=0}^{L} G(x,t|x',0)F(x')dx' \\
&+ \alpha \int_{\tau=0}^{t} \int_{x'=0}^{L} G(x,t|x',\tau)\frac{g(x',\tau)}{k} dx' d\tau \\
&+ \alpha \int_{\tau=0}^{t} G(x,t|0,\tau)\frac{f_1(\tau)}{k_1} d\tau \\
&+ \alpha \int_{\tau=0}^{t} G(x,t|L,\tau)\frac{f_2(\tau)}{k_2} d\tau
\end{aligned}
\tag{2}
$$

Considering the conditions

$$
F(x) = T_0; \qquad g(x,t) = 0; \qquad f_1(t) = q(t); \qquad \text{e} \qquad f_2(t) = 0 \tag{3}
$$

$$
T(x,t) = T_0 + \alpha \int_{0}^{\tau} G_{X22}(x,t|x',t-\tau)\left.\frac{q(\tau)}{k}\right|_{x'=0} d\tau \tag{4}
$$

Where $G_{X22}(x,t|x',t-\tau)$ is given by (Cole et al., 2010)

$$
G_{X22}(x,t) = \frac{1}{L} + \frac{2}{L}\sum_{m=1}^{M} e^{-\left(\frac{m\pi}{L}\right)^2 \alpha (t-\tau)} \cos\left(\frac{m\pi x}{L}\right)\cos\left(\frac{m\pi x'}{L}\right) \tag{5}
$$

Matlab Code Snippet

```
for j=1:m
 Bn=(j*pi/L)^2 * alfa ;
 parcelaTX22 =  cos(j*pi*x(a)/L) * (exp(-Bn*t(c)))/j^2;
 somaTX22 = somaTX22 + parcelaTX22;
 parcelaHX22 =  cos(j*pi*x(a)/L) * (exp(-Bn*t(c)));
 somaHX22 = somaHX22 + parcelaHX22;
end
% Temperature
TX22(a,c) = (q0*L/k)*( (alfa*t(c)/L^2) + ( (1/2)*(x(a)/L)^2 )
    - (x(a)/L) + (1/3) - ( (2/pi^2)*somaTX22 ) );
```

References

K. Cole, J. Beck, A. Haji-Sheikh, and B. Litkouhi. *Heat Conduction Using Green?s Functions, 2nd Edition*. Series in Computational Methods and Physical Processes in Mechanics and Thermal Sciences. CRC Press, 2010. ISBN 9781439895214.