Tutorial 3: Response impulsive of the $X22$ problem and inverse problem solver.

Ana Paula Fernandes & Gilmar Guimarães

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Abstract

Tutorial 3 is about the impulsive response (transfer function) of the $X22$ problem and inverse problem solver.

Impulsive response equation

$$h(x,t) = \frac{\alpha}{k} G_{X22}(x,t|0,t) = \frac{\alpha}{kL} + \frac{2\alpha}{kL} \sum_{m=1}^{M} e^{-\left(\frac{m\pi}{L}\right)^2 \alpha t} \cos\left(\frac{m\pi x}{L}\right) \tag{1}$$

Matlab Code Snippet

```
for  c=2:length(t)
 for a=1:length(x)
  somaH=0;
  for j=1:m
   parcelaH = (cos(j*pi*x(a)/L)) *  exp(-(j*pi/L)^2*alfa*t(c));
   somaH = somaH + parcelaH;
   H(a,c) = alfa/(k*L)+(alfa*2)/(k*L)*somaH;
  end
 end
end
```

Inverse problem solver

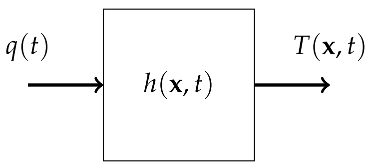

Figure 1: dynamic system: direct problem

$$T(\mathbf{x},t) = h(\mathbf{x},t) * q(t). \tag{2}$$

$$T(\mathbf{x}, s) = H(\mathbf{x}, s).q(s), \quad \text{where } H(\mathbf{x}, s) = \mathcal{L}\{h(\mathbf{x}, t)\}. \tag{3}$$

$$T(\mathbf{x}, t) = \mathcal{L}^{-1}\{H(\mathbf{x}, s).q(s)\} = h(\mathbf{x}, t) * q(t) = \int_0^t h(\mathbf{x}, t - \tau) q(\tau) d\tau. \tag{4}$$

The inverse problem (Fernandes et al., 2015),

$$q(s) = T(\mathbf{x}, s) / H(\mathbf{x}, s). \tag{5}$$

$$q(t) = \mathcal{L}^{-1}\{q(s)\}. \tag{6}$$

so

$$q(t) = T(\mathbf{x}, t) * \mathcal{L}^{-1}\{1/H(\mathbf{x}, s)\}. \tag{7}$$

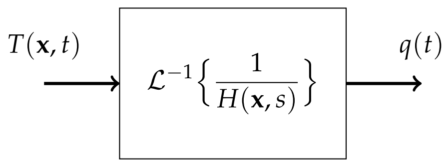

Figure 2: dynamic system: inverse problem

Matlab Code Snippet

The line 4 is equivalent to Eq. (5) and line 5 correspond to Eq. (7). Laplace Transform are replaced for Fourier Fast Transform.

```
1 NR=2^20;
2 Hfreq=fft(H,NR);
3 Tfreq=fft(T,NR);
4 qfreq=(Tfreq./Hfreq);
5 qtime=(ifft(qfreq)/(dt));
```

References

A. P. Fernandes, M. B. dos Santos, and G. Guimarães. An analytical transfer function method to solve inverse heat conduction problems. *Applied Mathematical Modelling*, 2015. ISSN 0307-904X. doi: http://dx.doi.org/10.1016/j.apm.2015.02.012.