Tutorial 1: Fundamental solution $X20$.

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Abstract

Tutorial 1 is about to the fundamental solution of the $X20$ heat conduction problem. Indeed this problem is referred as $X20B\_T0$ by Cole et al. (2010) to describe a semi-infinite body with specified surface heat flux.

Equation

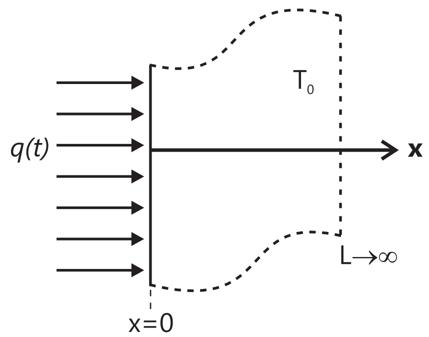

Figure 1: $X20B_{-}T0$ Problem

The temperature distribuition in a semi-finite body is discribed by the differencial equation

$$\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T(x,t)}{\partial t} \quad x > 0;\ t > 0 \tag{1a}$$

with bondary conditions

$$-\frac{k\partial T(x,t)}{\partial x}\bigg|_{x=0} = q(t); \qquad T(x,t) \to 0 \quad \text{quando} \quad x \to \infty \tag{1b}$$

and initial temperature

$$T(x,0) = 0 \tag{1c}$$

Solution by Green Function, $q(t) = q_0$

The Green Function Soluion Equation fot the tempeature takes the form

$$T(x,t) = \alpha \int_{\tau=0}^{t} \frac{f(\tau)}{k} G_{X20}(x,t|0,\tau) d\tau \tag{2}$$

The heat flux at the boundary is $f(t)$ with units of W/m^2. Where $GX20$ is given in Tables of Beck by

$$G_{X20}(x,t|x',\tau) = \frac{1}{\sqrt{4\pi\alpha(t-\tau)}} \left\{ \exp\left[-\frac{(x-x')^2}{4\alpha(t-\tau)} \right] + \exp\left[-\frac{(x+x')^2}{4\alpha(t-\tau)} \right] \right\} \tag{3}$$

In the case where $f(t) = q_0$, that is, constant heat flux, thus, the problem is denoted by $X20B1T0$ and the tempeature is given by

$$T(x,t) = \frac{q_0}{k} (4\alpha t)^{1/2} \,\mathrm{ierfc}\left[\frac{x}{(4\alpha t)^{1/2}} \right] \tag{4}$$

$$\mathrm{ierfc}(x) = \pi^{-1/2} e^{-x^2} - x\,\mathrm{erfc}(x)$$

Matlab Code Snippet

```
for c=1:length(t)
 % X20 (pag. 188 + ierfc def. pag 501 - Cole et al. (2010))
   for a=1:length(x)
    TX20(a,c) = (q0/k)*sqrt(4*alfa*t(c))* ((1/sqrt(pi))
     *exp(-(x(a)/sqrt(4*alfa*t(c)))^2)-((x(a)/sqrt(4*alfa*t(c))))
     *erfc((x(a)/sqrt(4*alfa*t(c)))));
   end
end
```

References

K. Cole, J. Beck, A. Haji-Sheikh, and B. Litkouhi. *Heat Conduction Using Green?s Functions, 2nd Edition*. Series in Computational Methods and Physical Processes in Mechanics and Thermal Sciences. CRC Press, 2010. ISBN 9781439895214.