Tutorial 5: Impulsive response $X33Y33Z33$ and inverse problem solver.

Ana Paula Fernandes & Gilmar Guimarães

October 24, 2016

Abstract

This tutorial discribe the Impulsive response (or transfer fuction) of $X33Y33Z33$ problem and inverse problem solver.

Equation

$$\begin{aligned}\Theta(x,y,z,t) &= \theta(x,y,z,t) - \int_0^L \int_0^W \int_0^R G(x,y,z,t|x',y',z',0)\theta(x,y,z,0)dx'dy'dz' \\ &= \frac{\alpha}{k}\int_0^t \int_{L_1}^{L_2} \int_{R_1}^{R_2} q(\tau)G(x,y,z,t|x',W,z',\tau)dx'dz'd\tau\end{aligned} \tag{1}$$

$$\Theta(x,y,z,t) = \int_0^t q(\tau)\left(\frac{\alpha}{k}\int_{L_1}^{L_2}\int_{R_1}^{R_2} G(x,y,z,t|x',W,z',\tau)dx'dz'\right)d\tau \tag{2}$$

$$\begin{aligned}h(x,y,z,t) &= \int_0^t \delta(\tau)\left(\frac{\alpha}{k}\int_{L_1}^{L_2}\int_{R_1}^{R_2} G(x,y,z,t|x',W,z',\tau)dx'dz'\right)d\tau \\ &= \delta(t) * \left(\frac{\alpha}{k}\int_{L_1}^{L_2}\int_{R_1}^{R_2} G(x,y,z,t|x',W,z',t-\tau)dx'dz'\right)\end{aligned} \tag{3}$$

$$h(x,y,z,t) = \frac{\alpha}{k}\int_{L_1}^{L_2}\int_{R_1}^{R_2} G(x,y,z,t|x',W,z',\tau)dx'dz' \tag{4}$$

where

$$
\begin{aligned}
&G(x,y,z,t|x',W,z',\tau) = \\
&= \frac{8}{LWR}\sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} e^{-\left(\frac{\alpha_m^2}{L^2}+\frac{\beta_n^2}{W^2}+\frac{\gamma_p^2}{R^2}\right)\alpha t} \\
&\times \frac{\left[\alpha_m \cos\left(\frac{\alpha_m x'}{L}\right) + B_1 \sin\left(\frac{\alpha_m x'}{L}\right)\right]}{(\alpha_m^2 + B_1^2)\left[1 + \frac{B_2}{(\alpha_m^2+B_2^2)}\right] + B_1} \\
&\times \frac{[\beta_n \cos(\beta_n) + B_3 \sin(\beta_n)]}{(\beta_n^2 + B_3^2)\left[1 + \frac{B_4}{(\beta_n^2+B_4^2)}\right] + B_3} \\
&\times \frac{\left[\gamma_p \cos\left(\frac{\gamma_p z'}{R}\right) + B_5 \sin\left(\frac{\gamma_p z'}{R}\right)\right]}{(\gamma_p^2 + B_5^2)\left[1 + \frac{B_6}{(\gamma_p^2+B_6^2)}\right] + B_5} \\
&\times \left[\alpha_m \cos\left(\frac{\alpha_m x}{L}\right) + B_1 \sin\left(\frac{\alpha_m x}{L}\right)\right] \\
&\times \left[\beta_n \cos\left(\frac{\beta_n y}{W}\right) + B_3 \sin\left(\frac{\beta_n y}{W}\right)\right] \\
&\times \left[\gamma_p \cos\left(\frac{\gamma_p z}{R}\right) + B_5 \sin\left(\frac{\gamma_p z}{R}\right)\right]
\end{aligned}
\tag{5}
$$

The impulsive response is given by (Fernandes et al., 2015),

$$
\begin{aligned}
&h(x,y,z,t) = \frac{\alpha}{k}\frac{8}{LWR}\sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} e^{-\left(\frac{\alpha_m^2}{L^2}+\frac{\beta_n^2}{W^2}+\frac{\gamma_p^2}{R^2}\right)\alpha t} \\
&\times \frac{\left[L\left(\sin\left(\frac{\alpha_m L_2}{L}\right) - \sin\left(\frac{\alpha_m L_1}{L}\right)\right) - \frac{B_1 L}{\alpha_m}\left(\cos\left(\frac{\alpha_m L_2}{L}\right) + \cos\left(\frac{\alpha_m L_1}{L}\right)\right)\right]}{(\alpha_m^2 + B_1^2)\left[1 + \frac{B_2}{(\alpha_m^2+B_2^2)}\right] + B_1} \\
&\times \frac{[\beta_n \cos(\beta_n) + B_3 \sin(\beta_n)]}{(\beta_n^2 + B_3^2)\left[1 + \frac{B_4}{(\beta_n^2+B_4^2)}\right] + B_3} \\
&\times \frac{\left[R\left(\sin\left(\frac{\gamma_p R_2}{R}\right) - \sin\left(\frac{\gamma_p R_1}{R}\right)\right) - \frac{B_5 R}{\gamma_p}\left(\cos\left(\frac{\gamma_p R_2}{R}\right) + \cos\left(\frac{\gamma_p R_1}{R}\right)\right)\right]}{(\gamma_p^2 + B_5^2)\left[1 + \frac{B_6}{(\gamma_p^2+B_6^2)}\right] + B_5} \\
&\times \left[\alpha_m \cos\left(\frac{\alpha_m x}{L}\right) + B_1 \sin\left(\frac{\alpha_m x}{L}\right)\right] \\
&\times \left[\beta_n \cos\left(\frac{\beta_n y}{W}\right) + B_3 \sin\left(\frac{\beta_n y}{W}\right)\right] \\
&\times \left[\gamma_p \cos\left(\frac{\gamma_p z}{R}\right) + B_5 \sin\left(\frac{\gamma_p z}{R}\right)\right]
\end{aligned}
\tag{6}
$$

Matlab Code Snippet to impulsive response

```
for s=2:length(t)
 somatorio=0;
 for k=1:p
  for j=1:n
   for i=1:m
    Amnp =  ( ( alfam(i)/L )^2 + ( betan(j)/W )^2 + ( gamap(k)/R )^2 ) * alfa;
    parcela = ...
    exp( - Amnp * t(s) ) ...
    * ( ( ( alfam(i) * cos( alfam(i) * x/L ) + B1 * sin( alfam(i) * x/L ) )  / ( ( alfam(i)^2 + B1^2 ) * ( 1 + ( B2 / ( alfam(i)^2 + B2^2 ) ) ) + B1 ) ) ) ...
    * ( ( ( betan(j) * cos( betan(j) * y/W ) + B3 * sin( betan(j) * y/W ) )  / ( ( betan(j)^2 + B3^2 ) * ( 1 + ( B4 / ( betan(j)^2 + B4^2 ) ) ) + B3 ) ) ) ...
    * ( ( betan(j) * cos( betan(j) ) + B3 * sin( betan(j) ) ) ) ...
    * ( ( ( gamap(k) * cos( gamap(k) * z/R ) + B5 * sin( gamap(k) * z/R  ) ) / ( ( gamap(k)^2 + B5^2 ) * ( 1 + ( B6 / ( gamap(k)^2 + B6^2 ) ) ) + B5 ) ) ) ...
    * (  sin( alfam(i) * L2/L )  -  sin( alfam(i) * L1/L )  -  ( B1 / alfam(i) ) * ( cos( alfam(i) * L2/L ) ) +  ( B1 / alfam(i) ) * ( cos( alfam(i) * L1/L ) ) )  ...
    * (  sin( gamap(k) * R2/R )  -  sin( gamap(k) * R1/R )  -  ( B5 / gamap(k) ) * ( cos( gamap(k) * R2/R ) ) +  ( B5 / gamap(k) ) * ( cos( gamap(k) * R1/R ) ) ) ;
    somatorio = somatorio + parcela;
   end
  end
 end
 H(s) = (alfa/kCOND)*(8/W) *  somatorio;
end
```

Matlab Code Snippet to IHCP solver

```
NR=2^20;
Hfreq=fft(H,NR);
Tfreq=fft(T,NR);
qfreq=(Tfreq./Hfreq);
qtime=(ifft(qfreq)/(dt));
```

References

A. P. Fernandes, M. B. dos Santos, and G. Guimarães. An analytical transfer function method to solve inverse heat conduction problems. *Applied Mathematical Modelling*, 2015. ISSN 0307-904X. doi: http://dx.doi.org/10.1016/j.apm.2015.02.012.