Transfer Fucntion Based on Green´s Function Method (TFBGF) to Solve Inverse Heat Conduction problem (IHCP): Manufacturing Process Aplication

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Overview. Currently a lot of interest has been focused on the study of fundamental problems applied to manu-facturing process. The temperature field generated in the cutting process, for example, is subject of extensive research. The studies of these thermal fields in machining are very important for the de-velopment of new technologies aiming to increase the tool lives and to reduce production costs. The fact that these processes are affected and controlled by heat transfer phenomena has motivated a large number of researchers to turn their attention to these kinds of problems. The understanding of these processes involves the skill of modeling and obtaining the temperature field on piece or work piece.

As direct measurements of temperatures at the contact of two moving pieces are very difficult the use of inverse heat conduction techniques represents a good alternative to solve thermal problems due to manufacturing process since this technique takes into account temperatures measured from accessible positions.

Objectives. This course has two distinct modules: i) Module I that deals with heat transfer concepts, fundamentals and Green's functions technique; ii) Module II that deals with the inverse techniques using transfer function based on Green's functions. The inverse technique is based on Green's function and in the equivalence between thermal and dynamic systems. The methodology consists of the study of the solution of the direct problem by means of Green's functions. The aim of the course is present a base to develop analytical and numerical tools to solve thermal problems arising from manufacturing processes as orthogonal machining and welding processes. It means that heat flux generated by friction in orthogonal cutting processes or delivered in a welding process will be estimated.

1 Introduction

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1.1 O método do Gilmar

1.2 Método flash

1.3 Hot disk

2 Fundamentals

The present paper presents a new experimental method developed for obtaining, simultaneously, the thermal conductivity and thermal diffusivity of metallic and nonmetallic materials. The method is based on the use of a new surface probe that has one resistance heater, three termocouples, and one heat flux sensor. The sample is partially heated in a frontal surface while the other face is keeping insolated. Two regions, heated and non-heated front surface are used to estimate the thermal properties.

In this sense, the experimental technique uses two different signal pairs in two different thermal models. The first signal pairs uses two temperature signals measured far from the heated area in order to measure the thermal diffusivity. The second pair uses the heat flux and temperature signals measured on heated area of the sample.

The ratio between the maximum temperature measured in two different point is the key to estimate thermal diffusivity. It is shown in this work that this ratio is only dependent on the thermal difusivity and the shape of the heating. Since the thermal diffusivity has been estimated, thermal conductivity can be estimated minimzing the difference between the experimental values and the values of temperature estimated using a theoretical model.

It can be observed that the main difficulty in simultaneous parameter estimation is the problem of existence and uniqueness of an optimum solution. This may be because multiple solutions of the optimization problem exist or simply because numerical ill conditioning in setting up the problem results in extremely slow convergence of the optimization algorithm.

The method proposed reduces this problem once the thermal diffusivity is estimated independently of the thermal conductivity. It means that the minimization of the objective function defined by the ratio between the maximum temperature measured in two different point is a problem of a single parameter estimation that turns the diffusivity estimation into a better conditioned problem. In addition, the hypothesis used here assumes that the two functions that are minimized independently are unimodal.

That is, the functions have only one minimum in the region of search. Once the convergence was reached, the hypothesis was shown to be a good one. The technique proposed has a great potential for in situ applications once it is suitable for large thickness and does not need a location different from the access surface. É importante observar que a condição de isolamento proposta neste trabalho não é um fator limitador quanto a generalidade desta técnica. Para aplicações in situ, por exemplo, a exposição da superfície a um meio convectivo representa a condiçao de contorno natural. Esta aplicação é apresentada na extensao deste trabalho (Parte 2)

2.1 Direct Problem. A 3D-transient thermal model

The proposed thermal model is a sample initially at uniform temperature, T_0. The sample is then submitted to a heat flux at area $A_1[W/m^2]$ while all other surfaces are kept isolated. Fig. 1 shows the thermal model.

It can also be observed, in this frontal surface, two arbitrary positions 1 and 2 in which temperatures T_1 e T_2 will be observed (measured and calculated) and used to estimate thermal diffusivity

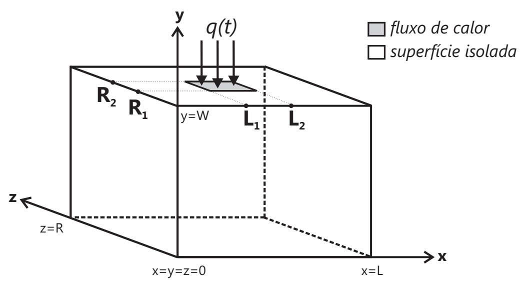

Figure 1: Modelo de experimento

The three-dimensional thermal model can be obtained by the solution of the diffusion equation.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{\partial T}{\partial t} \tag{1}$$

subjected to the boundary conditions

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = \left.\frac{\partial T}{\partial x}\right|_{x=L} = \left.\frac{\partial T}{\partial z}\right|_{z=0} = \left.\frac{\partial T}{\partial z}\right|_{z=R} = \left.\frac{\partial T}{\partial y}\right|_{y=0} = 0 \tag{2}$$

$$k\left.\frac{\partial T}{\partial y}\right|_{y=W} = q_1 \tag{3}$$

and initial condition

$$T(x,y,z,0) = F(x,y,z) = T_0 = T_\infty \tag{4}$$

2.1.1 Analytical solution using Green's fucnion

The solution of the homogeneous problem, described by Equations (**??**)-(**??**) in terms of Green's functions is given by (**??**); (**??**)

$$\theta(x,y,z,t) = \theta_0 + \frac{\alpha}{k}\int_0^t \int_{L_1}^{L_2} \int_{R_1}^{R_2} q(\tau) G(x,y,z,t|x',W,z',\tau) dx'dz'd\tau \quad (5)$$

The Green's function $G(x,y,z,t|x',y',z',\tau)$ is obtained, observing the types of boundary conditions in the directions of x, y and z, as the product of three independent one-dimensional problems as

$$G(x,y,z,t|x',y',z',\tau) = G_{X22}(x,t|x',\tau) \cdot G_{Y22}(y,t|y',\tau) \cdot G_{Z22}(z,t|z',\tau) \quad (6)$$

As the x, y and z directions have the same Green's function, only the variables that characterize the problem in each of the coordinates are changed, obtaining with

$$G_{X22}(x,t|x',\tau) = \frac{1}{L} + \frac{2}{L}\sum_{m=1}^{M} e^{-\left(\frac{m\pi}{L}\right)^2 \alpha(t-\tau)} \cos\left(\frac{m\pi x}{L}\right) \cos\left(\frac{m\pi x'}{L}\right) \quad (7)$$

$$G_{Y22}(y,t|y',\tau) = \frac{1}{W} + \frac{2}{L}\sum_{n=1}^{N} e^{-\left(\frac{n\pi}{L}\right)^2 \alpha(t-\tau)} \cos\left(\frac{n\pi y}{W}\right) \cos\left(\frac{n\pi y'}{W}\right) \quad (8)$$

Therefore

$$G_{Z22}(z,t|z',\tau) = \frac{1}{R} + \frac{2}{R}\sum_{p=1}^{P} e^{-\left(\frac{p\pi}{R}\right)^2 \alpha(t-\tau)} \cos\left(\frac{p\pi W}{R}\right) \cos\left(\frac{p\pi z'}{R}\right) \quad (9)$$

The eigenvalues α_m, β_n e γ_p are obtained by solving the transcendental equation in each direction, where the indices $m = 1, ...M$, $n = 1, ...N$ e $p = 1, ..., P$ define the number of eigenvalues required for the convergence of the series, given the truncation error, ϵ, desired.

Substituting the Green's function for the thermal problem of interest (Eq. (**??**)) in Equation (5) yields the analytical expression in terms of the variable θ.

que também pode ser escrita como

$$\theta(x,y,z,t) = \tag{10}$$

$$\frac{\alpha}{k} \times \left[A_1 \int_0^t q(\tau) d\tau + \sum_{m=1}^{\infty} A_{2n} \int_0^t \left[q(\tau) e^{\frac{n^2}{W^2}\pi^2\alpha(t-\tau)} \right] d\tau \right. \tag{11}$$

$$+ \sum_{p=1}^{\infty} B_p \int_0^t \left[q(\tau) e^{\frac{p^2}{R^2}\pi^2\alpha(t-\tau)} \right] d\tau + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{mn} \times \left[q(\tau) e^{\left(\frac{m^2}{L^2}+\frac{n^2}{W^2}\right)\pi^2\alpha(t-\tau)} \right] d\tau \tag{12}$$

$$\sum_{m=1}^{\infty}\sum_{p=1}^{\infty} B_{np} \int_0^t \left[q(\tau) e^{\left(\frac{m^2}{L^2}+\frac{p^2}{R^2}\right)\pi^2\alpha\tau} \right] d\tau \tag{13}$$

$$+ \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty} C_{mn} \times \int_0^t \left[q(\tau) e^{\left(\frac{m^2}{L^2}+\frac{n^2}{W^2}+\frac{p^2}{R^2}\right)\pi^2\alpha\tau} \right] d\tau] \tag{14}$$

onde as varáveis$A_1, A_{2n}, B_n, C_p, AB_{nm}, BC_{np}, ABC_{mnp}$ são termos que dependem somente das autofunções, autovalores e dos parâmetros geométricos L, W e R e são obtidas como

$$A_1 = \frac{1}{LWR}[L_2 - L_1][R_2 - R_1]$$
$$A_2m = \frac{2}{WR}[R_2 - R_1] \cos\left(\frac{m\pi x}{L}\right) \left[\sin\left(\frac{m\pi L_2}{L}\right) - \sin\left(\frac{m\pi L_1}{L}\right)\right]$$
$$B_1n = \frac{2}{LWR}[L_2 - L_1][R_2 - R_1] \cos\left(\frac{n\pi y}{W}\right) \cos(n\pi)$$
$$C_1p = \frac{2}{LW}[L_2 - L_1] \cos\left(\frac{p\pi z}{R}\right) \left[\sin\left(\frac{p\pi R_2}{R}\right) - \sin\left(\frac{p\pi R_1}{R}\right)\right] \times \frac{1}{p\pi}$$
$$AB_{1mn} = \frac{4}{WR}[R_2 - R_1] \cos\left(\frac{m\pi x}{L}\right) \left[\sin\left(\frac{m\pi L_2}{L}\right) - \sin\left(\frac{m\pi L_1}{L}\right)\right] \times \frac{1}{m\pi} \cos\left(\frac{n\pi y}{W}\right) \cos(n\pi)$$
$$AB_{2mn} = +\frac{\alpha}{k}\frac{4}{W} \cos\left(\frac{m\pi x}{L}\right) \left[\sin\left(\frac{m\pi L_2}{L}\right) - \sin\left(\frac{m\pi L_1}{L}\right)\right] \frac{1}{m\pi}$$
$$B_{2n} = \times \cos\left(\frac{p\pi z}{R}\right) \left[\sin\left(\frac{p\pi R_2}{R}\right) - \sin\left(\frac{p\pi R_1}{R}\right)\right]$$
$$AC_pn = \frac{4}{LW}[L_2 - L_1] \cos\left(\frac{n\pi y}{W}\right) \cos(n\pi) \cos\left(\frac{p\pi z}{R}\right) \times \left[\sin\left(\frac{p\pi R_2}{R}\right) - \sin\left(\frac{p\pi R_1}{R}\right)\right]$$
$$A_{6m} = \frac{\alpha}{k}\frac{8}{W} \cos\left(\frac{m\pi x}{L}\right) \left[\sin\left(\frac{m\pi L_2}{L}\right) - \sin\left(\frac{m\pi L_1}{L}\right)\right] \frac{1}{m\pi}$$
$$AC_{pn} = \times \cos\left(\frac{n\pi y}{W}\right) \cos(n\pi) \cos\left(\frac{p\pi z}{R}\right) \left[\sin\left(\frac{p\pi R_2}{R}\right) - \sin\left(\frac{p\pi R_1}{R}\right)\right] \frac{1}{p\pi}$$

The solution in terms of the original variable T is given by $T = \theta + T_\infty$.

Note that the solution of the direct problem of heat conduction $X22Y22Z22$ is determined since the heat flux, $q(t)$ is known.

In this sense, Eq.( ) is used to calculate temperature at positions 1 and 2.

2.2 Inverse Problem

2.2.1 Thermal difusvityy estimation

As ever mentioned, in a previous work Guimaraes et al, (1995) have observed that the delay between the experimental and theoretical phase factor of a generalized the impedance of a 1D dynamic system is an exclusive function of the thermal diffusivity α.

This input/output dynamical system is shown in Figure xxx

In this system, the input, X(t), and output, Y(t), data are defined, respectively as: $X(t) = q_1(t) + q_2(t)$ and $Y(t) = T_1(t) - T_2(t)$ and the impedance, $G_H(f)$, are defined as

$$H(f) = \frac{Y(f)}{X(f)} = \frac{T_1(f) - T_2(f)}{q_1(f) + q_2(f)} \tag{15}$$

The pair Temperature and heat flux are measured and calculated at two surface of the 1D thermal model.

It should be observed that the transformed impedance, $H(f)$ in the f–x plane is a complex variable which in a polar form can be written by

$$H(f) = \frac{Y(f)}{X(f)} \tag{16}$$

Guimaraes et al, (1995) have derived the analytical expression for the 1D transient generalized impedance phase factor and prove that this function has only thermal diffusiity dependence. it means,

$$\phi(f) = arctan(H(f)) = \frac{Re(H(f))}{Im(H(f))} = function(\alpha) \tag{17}$$

where Re(H) and Im(H) are, respectively, the real and imaginary parts of H. In other work, Borges et al (2006) extended this idea for a 3D transient model.The input/output dynamical system is shown in Figure xxx and data are defined, respectively as: $X(t) = q_1(t)$ and $Y(t) = T_1(t) - T_2(t)$, In this case, the impedance that has an exclusive dependence on thermal diffusivity, $H(f)$, is defined as

$$H(f) = \frac{Y(f)}{X(f)} = \frac{T_1(f) - T_2(f)}{q_1(f)} = function(\alpha) \tag{18}$$

It can be observed in both equation Eq(x) and Eq(xx) that if heat flux is an external force, the thermal diffusivity dependence is basically due to the difference between the temperature in two different positions. This fact can be observed indistintely in time or frequency domain, as well as in one or three dimensional model.

For example, Eq.(xx) is valid for any heat flux value, including a constant or step heat flux.

This idea is extended here to the aspect ratio of the maximum difference betwwen two temperature measured in two different positions.

it means, here in this work, it is proposed a impedance based on only in the temperature diference as follows

Aplicando-se a Transformada de Fourier em ambos os lados da Eq.(11) e usando as propriedades de convolução obtem-se

$$\theta(x,y,z,s) = q(s)\frac{\alpha}{k} \times \Bigg[A_1 + \sum_{m=1}^{\infty} A_{2n}\frac{\gamma_n}{\alpha} + \sum_{p=1}^{\infty} B_p\frac{\gamma_p}{\alpha} +$$
$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{mn}\frac{\gamma_{mn}}{\alpha} + \sum_{m=1}^{\infty}\sum_{p=1}^{\infty} B_{np}\frac{\gamma_{np}}{\alpha} + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty} C_{mn}\frac{\gamma_{mn}}{\alpha}]$$

ou ainda

$$\theta(x,y,z,s) = q(s)\frac{\alpha}{k} \times B_i(x_i,y_i,z_i,s,\alpha)$$

se

$$B_i = \Bigg[A_1 + \sum_{m=1}^{\infty} A_{2n}\frac{\gamma_n}{\alpha} + \sum_{p=1}^{\infty} B_p\frac{\gamma_p}{\alpha} +$$
$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{mn}\frac{\gamma_{mn}}{\alpha} + \sum_{m=1}^{\infty}\sum_{p=1}^{\infty} B_{np}\frac{\gamma_{np}}{\alpha} + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty} C_{mn}\frac{\gamma_{mn}}{\alpha}]$$

Usando a Eq.( ) para o cálculo das temperaturas em duas posições distintas T_1 e T_2 e calculando a razão destas temperaturas, obtem-se

$$\frac{\theta(x_1,y_1,z_1,s)}{\theta(x_2,y_2,z_2,s)} = \frac{B_1(x_1,y_1,z_1,s,\alpha)}{B_2(x_2,y_2,z_2,s,\alpha)} = Z_\theta = function(\alpha)$$

Observa-se que Z_θ independe tanto do fluxo de calor imposto quanto da condutividade térmica. Esta razão é então esolhida como a variável a ser medida e calculada para a obtenção da difusividade térmica.

Objective function minimization, Z_θ

Embora a prova de que a função Z_θ seja independente do fluxo de calor e da condutividade térmica tenha sido apresentada no domínio da frequencia, é imediata a conclusão de que essa independência tambem se estende ao domínio do tempo, bastando aplicar a transformada inversa de Fourier na Eq.( ).

Assim a função objetivo definida como o desvio quadrático entre a razão das temperaturas T_1/T_2 chamada aqui de impedancia Z_θ é escolhida como função a ser minimizada para a obtenção da difusividade térmica. The values of α will be supposed to be those that minimize equation ( ). In this work, this minimization is done by using the golden section method with polynomial approximation [ ].

$$S_z = \sum_{n=1}^{\infty} (Z_e(t) - Z_t(t))^2 \tag{18}$$

Uma alternativa ainda mais simples e eficiente é usar a impedancia calculada apenas para o valor máximo de temperatura para uma determinada posiçao, que chamaremos de t_1 para descriçao do método. Nesse caso, o valor da difusividade térmica seria obtida pela minimizaçao do mínimo quadrado apenas relativa a uma temperatura (T_1).

$$S_{Zmax} = (Z_{e,max}(t) - Z_{t,max}(t))^2 \tag{19}$$

onde

$$Z_{e,max}(t) = \frac{T_{1,max}(t_1)}{T_2(t_1)} \tag{20}$$

sendo t_1 o tempo correspondente à temperatura máxima de $T_1(t)$

2.2.2 Thermal conductivity estimation

Since the thermal diffusivity value is obtained, an objective function based on least square temperature error can be used to estimate the thermal conductivity. In this case, there is no identification problem as just one variable is being estimated. Therefore, the variable k will be supposed to be the parameter that minimizes the least square function, Smq, based on the difference between the calculated and experimental temperature at any single or more position defined by

$$S_q = \sum_{n=1}^{\infty} (Y(t) - T(t))^2 \tag{21}$$

where Y_i and T(t) are the experimental and theoretical temperature, measured and calculated at the position i, respectively. n is the total number of time measurements.

The optimization technique used to obtain l is also the golden section method with polynomial approximation [].

theoretical temperature are calculated using Eq.(2.2.1) as shown in previous section using Green's function method (**??**)

3 Experimental setup

It can be observed that the boundary conditions of the thermal problem (1) must be guaranteed. This means that the isolated condition (in all the surfaces where the sample is not in contact with the heater) must be obtained for the success of the technique. An efficient form of obtaining experimental isolation is to expose these surfaces to an evacuated atmosphere. The sample/heater assembly is inserted inside the vacuum chamber as shown schematically in Figure xxx. Figure xxx shows the vacuum chamber. This work investigates the thermal properties of an AISI304 stainless steel sample with a thickness of 10mm and lateral dimensions of 139 x 65 mm. The sample, initially in thermal equilibrium at T0, is then submitted to an uniform heat flux. The heat is supplied by a 318 ohm electrical resistance heater, covered with silicone rubber, with lateral dimensions of 50 x 50mm and a thickness of 0.3 mm. The temperatures are measured using surface thermocouples (type K). Two type K thermocouples are attached to the frontal surface of the test-plate (AISI 304) (Figure xx) . Collection and storage of the data from the thermocouples is achieved using a microcomputer-based data acquisition system (HP 75000 B E1326B), abbreviated to DAS. The DAS, with a control software, sampled (multiplexed) each thermocouple at intervals of 0.27 s (totalling 2048 points for each thermocouple). The DAS was also used to acquire the voltage and current signals of the heater. Figure xxx shows a typical signal of the power generated by the resistive element heater on the conductive sample. Typical signal values of temperature are shown in Figure xxx. Dimensionless heat flux for a typical experiment is shown in Figure xxx, in the next section.

3.1 Thermal diffusivity estimation: 3D thermal model

A Figura xxx apresenta a aplicação de um fluxo de calor típico. As Figuras xxx apresentam, respectivamente a evoluçao das temperaturas nas posiçoes de 1 a 6 como consequencia da imposiçao desse fluxo . As posiçoes exatas de cada sensor bem como o valor máximo alcançado por cada temperatura é apresentado na Tabela xxx.

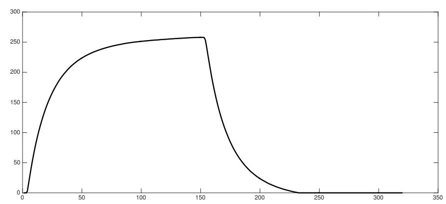

Figure 2: Fluxo de calor

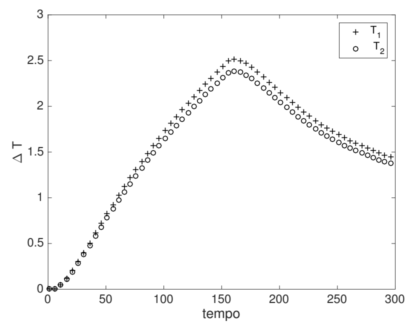

Figure 3: Comportamento de $T-1$ e T_2 experimental.

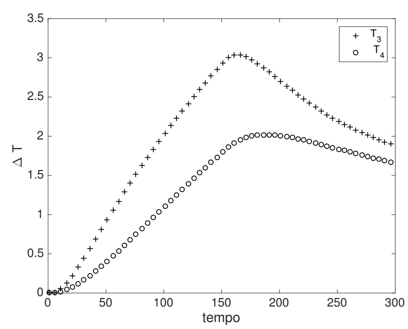

Figure 4: Comportamento de $T-3$ e T_4 experimental.

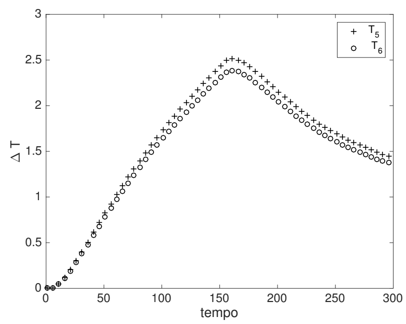

Figure 5: Comportamento de T_5 e T_6 experimental.

Table 1: Posição, temperatura máxima e tempo para o modelo teórico X22Y22Z22.

Posição	$x[m]$	$y[m]$	$z[m]$	ΔT_{max}	tempo[s]
T_1	17,67e-2	W	R/2	2,5114	161
T_2	18,07e-2	W	R/2	2,3837	161
T_3	L/2	W-0,17e-2	R/2	3,0415	163
T_4	L/2	W-0,47e-2	R/2	2,0174	189
T_5	L/2	W	17,67e-2	2,5114	161
T_6	L/2	W	18,07e-2	2,3837	161

O valor estimado para a difusividade térmica usando os resultados mostrados na tabela 1minimizando a função erro **??**, são mostrados na tabela 2

Table 2: Posição, temperatura máxima e tempo para o modelo teórico X22Y22Z22.

Posição	Valor	α teórico	α estimado	Erro %
$\frac{T_2}{T_1}$	0,9492	1.1708e-07	1.1708e-07	0
$\frac{T_4}{T_3}$	0,6633	1.1708e-07	1.1708e-07	0
$\frac{T_6}{T_5}$	0,9492	1.1708e-07	1.1708e-07	0

3.2 Thermal diffusivity estimation: 1D semi-inifinite transient model

Observa-se, da secção anterior que a difusividade térmica foi obtida usando-se apenas informações de duas temperaturas superficiais, e ainda, sem a necessidade de informaçao do fluxo de calor. Esse fato se deve à difusividade térmica ser responsável pelo atrazo (ou defasagem) entre estas temperaturas. Assim, o mesmo conceito físico pode ser aplicado às temperaturas $T_1 e_2$ porem considerando as suas evoluções a partir de um modelo unidimensional transiente. Observa-se na figura xxx que o calor responsavel pelo evolucao dessas temperaturas é a componente tangencial do calor aplicado, portanto uma intensidade de fluxo menor, chamado aqui de q_{0t} que apesar de desconhecida não é necessária para o cálculo da difusividade térmica, como descrito na secao xxxx.. A vantagem do uso de um modelo unidimensional é a simplicidade de seu uso como solucao analítica. Uma vez que a influência dos contornos é bastante reduzida ao se considerar a impedancia Z_θ adota-se o modelo de fluxo de calor imposto em um meio semi-infinito, ou seja,

The one-dimensional thermal model can be obtained by the solution of the diffusion equation 22a

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{22a}$$

sujeito às condições de contorno

$$-k\frac{\partial T}{\partial x}\bigg|_{x=0} = q_1 \tag{22b}$$

e à condição inicial

$$\theta(x,0) = T(x,0) - T_\infty \tag{22c}$$

A equação-solução, Eq.22a, pode assim ser aplicada para a solução do problema térmico em termos de funções de Green

$$\theta_{Y20}(y,t) = \frac{2\alpha}{\sqrt{\pi}k}\int_0^t \frac{q(\tau)}{(4\alpha(t-\tau))^{\frac{1}{2}}} e^{-\frac{y^2}{4\alpha(t-\tau)}}\, d\tau \tag{23}$$

Considerando fluxo de calor constante obtém-se

$$\theta_{Y20}(y,t) = \frac{q_0}{k\sqrt{4\alpha t}}\ \mathrm{ierfc}\left(\frac{y}{\sqrt{4\alpha t}}\right) \tag{24}$$

Analogamente ao modelo 3D o mesmo procedimento anterior é usado para a estimativa da difusividade térmica no modelo 1D semi-infinito. A única mudança é relativa às temperaturas calculadas. Como o fluxo de calor nesse modelo é desconhecido, as temperaturas nas posicoes 1 e 2 não são calculadas diretamente, mas sim a impedância Z_θ que independe desse fluxo, ou seja,

A relação entre θ_1 e θ_2 é dada por

$$Z_\theta = \frac{\theta_{Y20}(y_1, t)}{\theta_{Y20}(y_2, t)} = \frac{\frac{q_0}{k\sqrt{4\alpha t}} \text{ ierfc}\left(\frac{y_1}{\sqrt{4\alpha t}}\right)}{\frac{q_0}{k\sqrt{4\alpha t}} \text{ ierfc}\left(\frac{y_2}{\sqrt{4\alpha t}}\right)} = \frac{\text{ierfc}\left(\frac{y_1}{\sqrt{4\alpha t}}\right)}{\text{ierfc}\left(\frac{y_2}{\sqrt{4\alpha t}}\right)} \tag{25}$$

Nesse caso, o valor da difusividade térmica seria obtida pela minimizaçao do mínimo quadrado de $Z_{\theta,m\,ax}$, ou seja, analogamente à Eq.( ).

$$S_{Z\theta,m\,ax} = (Z_{e,max}(t) - Z_{t,max}(t))^2 \tag{26}$$

onde

$$Z_{e,max}(t) = \frac{\theta_1(t_{1max})}{\theta_2(t_{1max})} \tag{27}$$

sendo t_{1max} o tempo correspondente à temperatura máxima de $T_2(t)$

As mesmas temperaturas medidas usadas nas posiçoes 1, 2, 3,4,5 e 6 são novamente usadas para o cálculo da impedância experimental. Entretanto, a impedância teórica Z_θ é nesse caso calculada usando a Eq. ( ) do modelo 1D. A Tabela xxx apresenta os valores obtidos para a estimativa da difusividade térmica para os vários pares de temperatura.

Table 3: Posição, temperatura máxima e tempo para o modelo teórico X20.

Posição	Valor	α teórico	α estimado	Erro %
$\frac{T_2}{T_1}$	0,6682	1.1708e-07	1.1708e-07	0