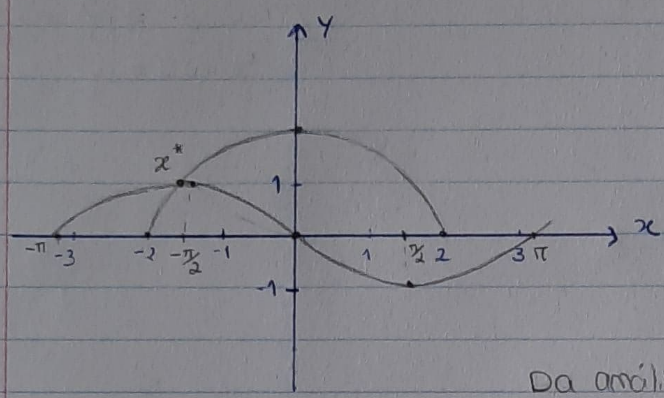
Ana Videira (2015012218) → Exame A+B 2014/2015 (PÁG1)

1.a $\sqrt{4-x^2}+\operatorname{sim}(x)=0 \Leftrightarrow \sqrt{4-x^2}=-\operatorname{sim}(x) \rightarrow y_1=-\operatorname{sim}(x)\,,\ y_2=\sqrt{4-x^2}$

$x^*\in[a;b]\,,\ b-a=2$



Da análise do gráfico conclui-se que:

→ $x^*\in[-2,0]$

→ $D_{\text{função}}=\{x\in\mathbb{R}: 4-x^2\geq 0\}=[-2,2]$

1.b Método de Newton-Raphson:

$f'(x)=(\sqrt{4-x^2}+\operatorname{sim}(x))'=(\sqrt{4-x^2})'+(\operatorname{sim}(x))'=\dfrac{-2x}{2\sqrt{4-x^2}}+\cos(x)=-\dfrac{x}{\sqrt{4-x^2}}-\cos(x)$

$f''(x)=\left(-\dfrac{x}{\sqrt{4-x^2}}\right)'-(\cos(x))'=\dfrac{-\sqrt{4-x^2}+x\left(\frac{-x}{\sqrt{4-x^2}}\right)}{4-x^2}-\operatorname{sim}(x)=\dfrac{-4}{(4-x^2)(\sqrt{4-x^2})}-\operatorname{sim}(x)$

→ assim a função é contínua no seu domínio ①

② $f(-2)=0+\operatorname{sim}(-2)<0$
$f(0)=2+\operatorname{sim}(0)=2>0$
$\Big\}\ f(-2)\times f(0)<0$

③ $f'(x)\neq 0,\ \forall x\in[-2;0]$ porque $\forall x\in[-2,0].\ f'(x)>0$

④ $f(-1)\times f''(-1)=\left(4-(-1^2)+\operatorname{sim}(-1)\right)\left(\dfrac{-4}{(4-(-1)^2)(\sqrt{4-(-1)^2})}-\operatorname{sim}(x)\right)$

$=\underbrace{(\sqrt{3}+\operatorname{sim}(-1))}_{>0}\underbrace{\left(\dfrac{-4}{3\sqrt{3}}-\operatorname{sim}(-1)\right)}_{>0}>0$

→ Assim $x_0=-1$ é uma boa aproximação inicial para o método de Newton-Raphson pois

$x_1=x_0-\dfrac{f(x_0)}{f'(x_0)} \longrightarrow x_1=-1-\dfrac{f(-1)}{f'(-1)}=-1-\dfrac{\sqrt{4-(-1)^2}+\operatorname{sim}(-1)}{-\frac{(-1)}{(4-x^2)(\sqrt{4-x^2})}-\operatorname{sim}(-1)}\simeq 0{,}304$

Ana Videira (2015012218) → Exame A+B 2014/2015 (PÁG 2)

2.a $g(x) \simeq P_2(x) = g(x_0) + g[x_0; x_1](x-x_0) + g[x_0, x_1; x_2](x-x_0)(x-x_1)$

cálculos auxiliares:

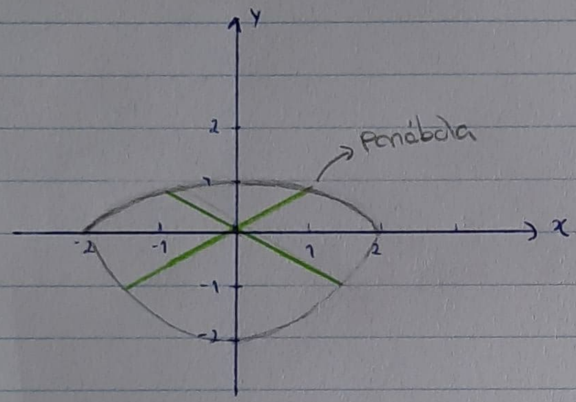
$g[-2;0] = \dfrac{g(0) - g(-2)}{0+2} = \dfrac{-2-0}{2} = -1$

$g[0;2] = \dfrac{g(2) - g(0)}{2-0} = \dfrac{0+2}{2} = 1$

$g[-2;0,2] = \dfrac{g[0;2] - g[-2;0]}{2+2} = \dfrac{1+1}{4} = \dfrac{1}{2}$

assim, $g(x) \simeq P_2(x) = 0 - 1(x+2) + \frac{1}{2}(x+2)x = \frac{1}{2}x^2 - 2$

2.b



2.c Regra de Simpson simples:

$S(x) = f(x) - g(x) = \sqrt{1 - \dfrac{x^2}{2^2}} + \sqrt{4 - x^2} \longrightarrow S(-2) = 0$ $\qquad h = \dfrac{2+2}{2} = 2$

$S(0) = 3$

$S(2) = 0$

assim, $I_S = \frac{2}{3}(0 + 4 \times 3 + 0) = 8 \rightarrow$ área da parábola encontrada por P_2 (sendo P_2 o da alínea anterior)

2.d área da elipse: πab

pelo formulário $\longrightarrow \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 \Leftrightarrow y = \pm b\sqrt{1 - \dfrac{x^2}{a^2}}$

<u>se</u> $f(x) = b\sqrt{1 - \dfrac{x^2}{a^2}}$ <u>e</u> $x \in [-a; a]$; $x_0 = -a$; $x_1 = 0$; $x_2 = a$

R.Trapézios → $A \simeq T = \frac{a}{2}(0 + 2b + 0) = ab$

R.Simpson → $A \simeq S = \frac{a}{3}(0 + 4b + 0) = \frac{4}{3}ab$

Ana Videira (2015012218) → Exame A+B 2014/2015 (PÁG3)

3.a $y' + ty^2 = 0 \iff \frac{dy}{dt} = -ty^2 \iff \frac{1}{y^2}\, dy = -t\, dt$

$$\int \frac{1}{y^2}\, dy = \int y^{-2}\, dy \iff -\frac{1}{y} = -\frac{1}{2}t^2 + C \iff y = \frac{1}{\frac{1}{2}t^2 - C}$$

$$\iff y = \frac{2}{t^2 - 2C} \rightarrow \text{solução geral}$$

$y(1) = 2 \iff \frac{2}{1-2C} = 2 \iff C = 0$

$y(t) = \frac{2}{t^2} \iff y(t) = 2t^{-2} \rightarrow$ solução particular

3.b

i	t_i	$y(t_i)$ exata	y_i Euler	y_i RK2	y_i RK4	erro Euler	erro RK2	erro RK4
0	2	2	2	2	2	0	0	0
1	1,5	0,8889	0	1	0,8590	0,8889	0,1111	0,0299
2	2	0,5	0	0,4062	0,4918	0,5	0,0938	0,0084

$h = \frac{2-1}{2} = 0,5$

$y' + ty^2 = 0 \iff y' = ty^2 \rightarrow f(t, y) = -ty^2$

$y(1) = 2$

$y(1,5) = \frac{2}{(1,5)^2} = 0,8889$

<u>Euler</u>: $y_0 = y(1) = 2$

$y_1 = y_0 + h f(t_0, y_0) = 2 + 0,5 \times (-1 \times 2^2) = 0$

$y_2 = y_1 + h f(t_1, y_1) = 0 + 0,5\,(-1,5 \times 0) = 0$

<u>RK4</u>: $y(t_1) - y_1 \iff 0,02999 = 0,8889 - y_1 \iff y_1 = 0,8885 - 0,0299 = 0,8590$

$y(t_2) - y_1 \iff 0,5 - 0,4916 = 0,0084$

<u>RK2</u>: $|y(t_1) - y_1| = |0,8889 - 1| = 0,1111$

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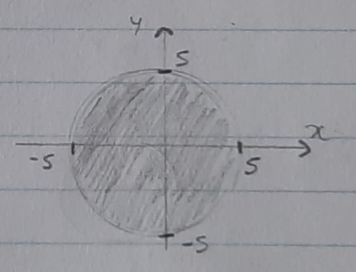
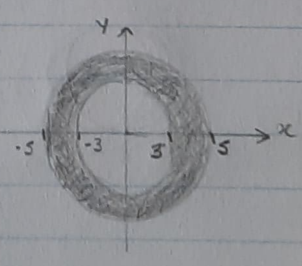
3.c
```
function y = NEuler(f,a,b,m,y0)
h=(b-a)/m;
t=a:h:b;
y=zeros(1,m+1);
for i=1:m
    y(i+1) = y(i)+h*f(t(i),y(i));
end
```

```
function y = RK2(f,a,b,m,y0)
h=(b-a)/m;
t=a:h:b;
y=zeros(1,m+1);
y(1)=
for i=1:m
   k1 = h* f(t(i),y(i));
   k2= h* f(t(i+1), y(i)+k);
   y(i+1)= y(i) + (k1+k2)/2;
end
```

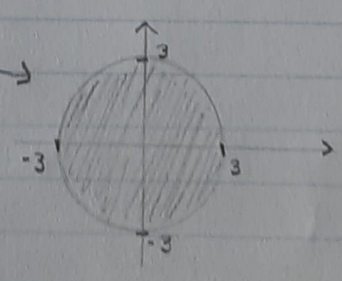
4.a $D_g = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 \leq 25\}$ →



$D_h = \{(x,y) \in \mathbb{R}^2 : 9 \leq x^2+y^2 \leq 25 \land -f(x,y) \geq 0\}$
$= \{(x,y) \in \mathbb{R}^2 : 9 \leq x^2+y^2 \leq 25\}$



$D_j = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 \leq 9 \land f(x,y)+25 \geq 0\}$
$= \{(x,y) \in \mathbb{R}^2 : 0 \leq x^2+y^2 \leq 9\}$



$D_f = D_h \cup D_j$
$= \{(x,y) \in \mathbb{R}^2 : 9 \leq x^2+y^2 \leq 25 \lor 0 \leq x^2+y^2 \leq 9\}$
$= \{(x,y) \in \mathbb{R}^2 : 0 \leq x^2+y^2 \leq 25\}$

Ana Videira (201501 2218) $\rightarrow$ Exame A+B 2014/2015 (PÁGS)

$D_\ell = \{ (x,y) \in \mathbb{R}^2 : 0 \leq x^2+y^2 \leq 25 \}$

$\ell(x,y) :=$ se $9 < x^2+y^2 \leq 25$

então $z \leftarrow \sqrt{-f(x,y)} = \sqrt{25-x^2-y^2}$

senão se $x^2+y^2 \leq 9$

então $z \leftarrow \frac{4}{3}\sqrt{f(x,y)+25} = \frac{4}{3}\sqrt{x^2+y^2}$

interseção com o plano XoY : $z=0$

$\sqrt{25-x^2-y^2} = 0 \Leftrightarrow x^2+y^2 = 25 \rightarrow \frac{4}{3}\sqrt{x^2+y^2} = 0 \Leftrightarrow x^2+y^2 = 0$

interseção com o plano XoZ : $y=0$

$\sqrt{25-x^2-0} = z \Leftrightarrow z^2+x^2 = 25 \rightarrow \frac{4}{3}\sqrt{x^2+0} = z \Leftrightarrow z = \frac{4}{3}|x|$

interseção com o plano YoZ : $x=0$

$\sqrt{25-0^2-y^2} = z \Leftrightarrow z^2+y^2 = 25 \rightarrow \frac{4}{3}\sqrt{0+y^2} = z \Leftrightarrow z = \frac{4}{3}|y|$

4.C i) $P(0,0)$ é ponto de acumulação

$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} j(x,y)$

$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} (x^2+y^2-25) = -25$

$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{4}{3}\sqrt{x^2+y^2} = \frac{4}{3}\times 0 = 0$ $\rightarrow$ assim a afirmação é falsa.

iii) ℓ contínua em $C = \{ (x,y) \in \mathbb{R}^2 : x^2+y^2 = 9 \}$ $\qquad x_0^2+y_0^2 = 9$

$\ell(x_0,y_0) = \frac{4}{3}\sqrt{x_0^2+y_0^2} = \frac{4}{3}\sqrt{9} = 4$

$\lim_{(x,y)\to(x_0,y_0)} \sqrt{-x^2-y^2+25} = \sqrt{-x_0^2-y_0^2+25} = \sqrt{-(x_0^2+y_0^2)+25} = \sqrt{-9+25} = 4$

Como $\lim_{(x,y)\to(x_0,y_0)} \ell(x,y) = \ell(x_0,y_0)$ $\rightarrow$ assim a função é contínua