Instituto Superior de Engenharia de Coimbra
DEPARTAMENTO DE FÍSICA E MATEMÁTICA

Análise Matemática I - **Engenharia Informática**

TPC nº3 Data limite de entrega: 20/Out/2015 (23h59m)

Regras de derivação [**D. Análise**]

3. Faça a correspondência entre as derivadas e a respectiva função:

Derivada $f(x)$	Função $F(x)$
a) $f(x) = \dfrac{-\sin\left(\sqrt{2x}\right)}{\sqrt{2x}}$	1) $F(x) = \arccos(e^x)$
b) $f(x) = \dfrac{\sin x \cos x}{\sqrt{\sin^2(x)+1}}$	2) $F(x) = \ln x$
c) $f(x) = \dfrac{1}{\sqrt{4-x^2}}$	3) $F(x) = \arctan\left(\dfrac{x}{3}\right)$
d) $f(x) = \dfrac{1}{x}$	4) $F(x) = \ln\left(x^2\right)$
e) $f(x) = 3x^2\cos\left(x^3+1\right)$	5) $F(x) = \cos\left(e^x\right)$
f) $f(x) = \dfrac{2}{x}$	6) $F(x) = e^{\arctan x}$
g) $f(x) = \dfrac{-3\,e^{3x}}{2\sqrt{1-e^{3x}}}$	7) $F(x) = \cos\left(\sqrt{2x}\right)$
h) $f(x) = \dfrac{3}{9+x^2}$	8) $F(x) = \sqrt{\sin^2(x)+1}$
i) $f(x) = -e^x\sin\left(e^x\right)$	9) $F(x) = \arcsin\left(\dfrac{x}{2}\right)$
j) $f(x) = \dfrac{-1}{(x+1)^2}$	10) $F(x) = \sqrt{1-e^{3x}}$
k) $f(x) = \cot x$	11) $F(x) = \dfrac{1}{\sqrt{x}}$
l) $f(x) = \dfrac{\cos x}{2-\cos^2 x}$	12) $F(x) = \ln\left(\tan x\right)$
m) $f(x) = \dfrac{e^{\arctan x}}{1+x^2}$	13) $F(x) = \sqrt{x^2+1}$
n) $f(x) = \dfrac{2\sin x}{(\cos(x)+1)^2}$	14) $F(x) = \ln(\sin x)$
o) $f(x) = \dfrac{-e^x}{\sqrt{1-e^{2x}}}$	15) $F(x) = \cot(\ln x)$
p) $f(x) = \dfrac{e^x}{e^x+2}$	16) $F(x) = \dfrac{2}{\cos(x)+1}$
q) $f(x) = \dfrac{x}{\sqrt{x^2+1}}$	17) $F(x) = \dfrac{1}{x+1}$
r) $f(x) = \dfrac{2}{\sin(2x)}$	18) $F(x) = \ln\left(e^x+2\right)$
s) $f(x) = \dfrac{-1}{x\sin^2(\ln x)}$	19) $F(x) = \sin\left(x^3+1\right)$
t) $f(x) = \dfrac{-1}{2\sqrt{x^3}}$	20) $F(x) = \arctan(\sin x)$

Sugestão de resolução:

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1) $$F'(x) = \underbrace{\Big(\arccos(e^x)\Big)'}_{R20} = \frac{-\overbrace{(e^x)'}^{R9}}{\sqrt{1-(e^x)^2}} = \frac{-e^x\,\overbrace{(x)'}^{R2}}{\sqrt{1-e^{2x}}} = \frac{-e^x}{\sqrt{1-e^{2x}}} \quad \text{(o)};$$

2) $$F'(x) = \underbrace{(\ln x)'}_{R12} = \frac{\overbrace{(x)'}^{R2}}{x} = \frac{1}{x} \quad \text{(d)};$$

3) $$F'(x) = \underbrace{\left(\arctan\left(\frac{x}{3}\right)\right)'}_{R21} = \frac{\overbrace{\left(\frac{x}{3}\right)'}^{R3}}{1+\left(\frac{x}{3}\right)^2} = \frac{\frac{1}{3}\,\overbrace{(x)'}^{R2}}{1+\frac{x^2}{9}} = \frac{\frac{1}{3}}{\frac{9}{9}+\frac{x^2}{9}} = \frac{\frac{1}{3}}{\frac{9+x^2}{9}} = \frac{1}{3}\,\frac{9}{9+x^2} = \frac{3}{9+x^2} \quad \text{(h)};$$

4) $$F'(x) = \underbrace{\left(\ln\left(x^2\right)\right)'}_{R12} = \frac{\overbrace{(x^2)'}^{R7}}{x^2} = \frac{2x\,\overbrace{(x)'}^{R2}}{x^2} = \frac{2x}{x^2} = \frac{2}{x} \quad \text{(f)};$$

5) $$F'(x) = \underbrace{\left(\cos\left(e^x\right)\right)'}_{R14} = -\underbrace{(e^x)'}_{R9}\sin\left(e^x\right) = -e^x\,\underbrace{(x)'}_{R2}\sin\left(e^x\right) = -e^x\sin\left(e^x\right) \quad \text{(i)};$$

6) $$F'(x) = \underbrace{\left(e^{\arctan x}\right)'}_{R9} = e^{\arctan x}\,\underbrace{(\arctan x)'}_{R21} = e^{\arctan x}\,\frac{\overbrace{(x)'}^{R2}}{1+x^2} = e^{\arctan x}\,\frac{1}{1+x^2} = \frac{e^{\arctan x}}{1+x^2} \quad \text{(m)};$$

7) $$F'(x) = \underbrace{\left(\cos\left(\sqrt{2x}\,\right)\right)'}_{R14} = -\left(\sqrt{2x}\,\right)'\sin\left(\sqrt{2x}\,\right) = -\underbrace{\left(\sqrt{2}\,\sqrt{x}\,\right)'}_{R3}\sin\left(\sqrt{2x}\,\right) = -\sqrt{2}\,\underbrace{\left(x^{\frac{1}{2}}\right)'}_{R7}\sin\left(\sqrt{2x}\,\right)$$

$$= -\sqrt{2}\,\frac{1}{2}\,x^{-\frac{1}{2}}\,\underbrace{(x)'}_{R2}\sin\left(\sqrt{2x}\,\right) = -\frac{\sqrt{2}}{(\sqrt{2}\,)^2}\,\frac{1}{x^{\frac{1}{2}}}\sin\left(\sqrt{2x}\,\right) = -\frac{1}{\sqrt{2}}\,\frac{1}{\sqrt{x}}\sin\left(\sqrt{2x}\,\right) = -\frac{\sin\left(\sqrt{2x}\,\right)}{\sqrt{2x}} \quad \text{(a)};$$

8) $$F'(x) = \left(\sqrt{\sin^2(x)+1}\right)' = \underbrace{\left(\left(\sin^2(x)+1\right)^{\frac{1}{2}}\right)'}_{R7} = \frac{1}{2}\left(\sin^2(x)+1\right)^{-\frac{1}{2}}\underbrace{\left(\sin^2(x)+1\right)'}_{R4}$$

$$= \frac{1}{2}\,\frac{1}{\left(\sin^2(x)+1\right)^{\frac{1}{2}}}\left(\underbrace{\left(\sin^2(x)\right)'}_{R7}+\underbrace{(1)'}_{R1}\right) = \frac{1}{2}\,\frac{1}{\sqrt{\sin^2(x)+1}}\,2\sin x\,\underbrace{(\sin x)'}_{R13}$$

$$= \frac{1}{2}\,\frac{1}{\sqrt{\sin^2(x)+1}}\,2\sin x\,\underbrace{(x)'}_{R2}\cos x = \frac{\sin x\cos x}{\sqrt{\sin^2(x)+1}} \quad \text{(b)};$$

9) $$F'(x) = \underbrace{\left(\arcsin\left(\frac{x}{2}\right)\right)'}_{R19} = \frac{\overbrace{\left(\frac{x}{2}\right)'}^{R3}}{\sqrt{1-\left(\frac{x}{2}\right)^2}} = \frac{\frac{1}{2}\,\overbrace{(x)'}^{R2}}{\sqrt{1-\frac{x^2}{4}}} = \frac{\frac{1}{2}}{\sqrt{\frac{4}{4}-\frac{x^2}{4}}} = \frac{\frac{1}{2}}{\sqrt{\frac{4-x^2}{4}}} = \frac{\frac{1}{2}}{\frac{\sqrt{4-x^2}}{\sqrt{4}}}$$

$$= \frac{1}{2}\,\frac{2}{\sqrt{4-x^2}} = \frac{1}{\sqrt{4-x^2}} \quad \text{(c)};$$

10) $$F'(x) = \left(\sqrt{1-e^{3x}}\right)' = \underbrace{\left(\left(1-e^{3x}\right)^{\frac{1}{2}}\right)'}_{R7} = \frac{1}{2}\left(1-e^{3x}\right)^{-\frac{1}{2}}\underbrace{\left(1-e^{3x}\right)'}_{R4+R3} = \frac{1}{2}\,\frac{1}{\left(1-e^{3x}\right)^{\frac{1}{2}}}\left(\underbrace{(1)'}_{R1}-\underbrace{\left(e^{3x}\right)'}_{R9}\right)$$

$$= \frac{1}{2}\,\frac{1}{\sqrt{1-e^{3x}}}\,\big(-e^{3x}\big)\,\underbrace{(3x)'}_{R3} = \frac{1}{2}\,\frac{1}{\sqrt{1-e^{3x}}}\,\big(-e^{3x}\big)\,3\,\underbrace{(x)'}_{R2} = \frac{-3e^{3x}}{2\sqrt{1-e^{3x}}} \quad \text{(g)};$$

11) $$F'(x) = \left(\frac{1}{\sqrt{x}}\right)' = \left(\frac{1}{x^{\frac{1}{2}}}\right)' = \underbrace{\left(x^{-\frac{1}{2}}\right)'}_{R7} = -\frac{1}{2}\,x^{-\frac{3}{2}}\,\underbrace{(x)'}_{R2} = -\frac{1}{2}\,\frac{1}{x^{\frac{3}{2}}} = -\frac{1}{2}\,\frac{1}{\sqrt{x^3}} = -\frac{1}{2\sqrt{x^3}} \quad \text{(t)};$$

12) $$F'(x) = \underbrace{\Big(\ln\big(\tan x\big)\Big)'}_{R12} = \frac{\overbrace{(\tan x)'}^{R15}}{\tan x} = \frac{\overbrace{(x)'}^{R2}\sec^2 x}{\tan x} = \frac{\sec^2 x}{\tan x} = \frac{\frac{1}{\cos^2 x}}{\frac{\sin x}{\cos x}} = \frac{1}{\cos^2 x}\,\frac{\cos x}{\sin x} = \frac{1}{\cos x \sin x}$$

$$= \frac{2}{\underbrace{2\cos x\sin x}_{\text{Tab pág 1 (7)}}} = \frac{2}{\sin(2x)} \quad \text{(r)};$$

13) $$F'(x) = \left(\sqrt{x^2+1}\right)' = \underbrace{\left((x^2+1)^{\frac{1}{2}}\right)'}_{R7} = \frac{1}{2}\,(x^2+1)^{-\frac{1}{2}}\,\underbrace{(x^2+1)'}_{R4} = \frac{1}{2}\,\frac{1}{(x^2+1)^{\frac{1}{2}}}\Big(\underbrace{(x^2)'}_{R7}+\underbrace{(1)'}_{R1}\Big)$$

$$= \frac{1}{2}\,\frac{1}{\sqrt{x^2+1}}\,2x\,\underbrace{(x)'}_{R1} = \frac{x}{\sqrt{x^2+1}} \quad \text{(q)};$$

14) $$F'(x) = \underbrace{\Big(\ln(\sin x)\Big)'}_{R12} = \frac{\overbrace{(\sin x)'}^{R13}}{\sin x} = \frac{\overbrace{(x)'}^{R2}\cos x}{\sin x} = \frac{\cos x}{\sin x} = \cot x \quad \text{(k)};$$

15) $$F'(x) = \underbrace{\Big(\cot(\ln x)\Big)'}_{R16} = -\underbrace{(\ln x)'}_{R12}\csc^2(\ln x) = -\frac{\overbrace{(x)'}^{R2}}{x}\csc^2(\ln x) = -\frac{1}{x}\csc^2(\ln x) = -\frac{1}{x\sin^2(\ln x)} \quad \text{(s)};$$

16) $$F'(x) = \left(\frac{2}{\cos(x)+1}\right)' = \underbrace{\left(2\big(\cos(x)+1\big)^{-1}\right)'}_{R3} = 2\underbrace{\left(\big(\cos(x)+1\big)^{-1}\right)'}_{R7} = 2(-1)\big(\cos(x)+1\big)^{-2}\underbrace{\big(\cos(x)+1\big)'}_{R4}$$

$$= -2\,\frac{1}{\big(\cos(x)+1\big)^2}\Big(\underbrace{(\cos x)'}_{R14}+\underbrace{(1)'}_{R1}\Big) = -2\,\frac{1}{\big(\cos(x)+1\big)^2}\,(-1)\,\underbrace{(x)'}_{R2}\sin x = \frac{2\sin x}{\big(\cos(x)+1\big)^2} \quad \text{(n)};$$

17) $$F'(x) = \left(\frac{1}{x+1}\right)' = \underbrace{\left((x+1)^{-1}\right)'}_{R7} = -(x+1)^{-2}\,\underbrace{(x+1)'}_{R4} = -\frac{1}{(x+1)^2}\Big(\underbrace{(x)'}_{R2}+\underbrace{(1)'}_{R1}\Big) = -\frac{1}{(x+1)^2} \quad \text{(j)};$$

18) $$F'(x) = \underbrace{\Big(\ln\big(e^x+2\big)\Big)'}_{R12} = \frac{\overbrace{(e^x+2)'}^{R4}}{e^x+2} = \frac{\overbrace{(e^x)}^{R9}+\overbrace{(2)'}^{R1}}{e^x+2} = \frac{e^x\,\overbrace{(x)'}^{R2}}{e^x+2} = \frac{e^x}{e^x+2} \quad \text{(p)};$$

19) $$F'(x) = \underbrace{\Big(\sin\big(x^3+1\big)\Big)'}_{R13} = \underbrace{(x^3+1)'}_{R4}\cos\big(x^3+1\big) = \Big(\underbrace{(x^3)'}_{R7}+\underbrace{(1)'}_{R1}\Big)\cos\big(x^3+1\big)$$

$$= 3x^2\,\underbrace{(x)'}_{R2}\cos\big(x^3+1\big) = 3x^2\cos\big(x^3+1\big) \quad \text{(e)};$$

20) $$F'(x) = \underbrace{\Big(\arctan(\sin x)\Big)'}_{R21} = \frac{\overbrace{(\sin x)'}^{R13}}{1+(\sin x)^2} = \frac{\overbrace{(x)'}^{R2}\cos x}{1+\sin^2 x} = \frac{\cos x}{1+\underbrace{\sin^2 x}_{\text{Tab. pag 1 (1)}}} = \frac{\cos x}{1+1-\cos^2 x}$$

$$= \frac{\cos x}{2-\cos^2 x} \quad \text{(l)}.$$