Aula 5 TP1 05/04

Teste A 2016/2017

Ex 3

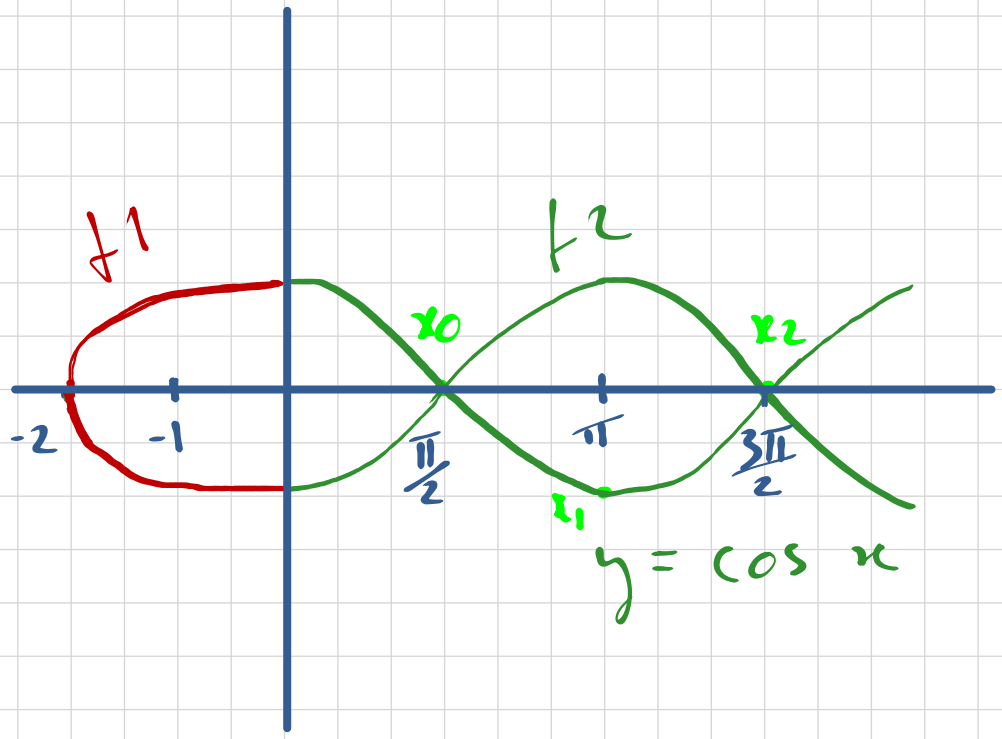
$$f(x) = \begin{cases} \text{Se } -2 \leq x < 0 \\ \text{então } y = \sqrt{1 - x^2/4} \\ \text{se não se } 0 \leq x \leq 2\pi \\ \text{então } y = \cos x \end{cases}$$

$$f(x) \approx P_2(x)$$

$$P_2 = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1)$$

1º Passo

$$f(x) \overset{f1}{=} \begin{cases} \sqrt{1 - \frac{x^2}{4}}, & -2 \leq x < 0 \\ f2 \quad \cos x, & 0 \leq x \leq 2\pi \end{cases}$$



Elipse

$$y = \sqrt{1 - \frac{x^2}{4}} \iff y^2 = 1 - \frac{x^2}{4} \iff \frac{x^2}{4} + y^2 = 1$$

a)

Diferenças Divididas

$a_0 = ?$

$a_1 = ?$

$a_2 = ?$

Tabela:

x_i	$f(x_i)$	D_1	D_2
x_0 $\pi/2$	0	$\frac{-1-0}{\pi - \pi/2} = -\frac{2}{\pi}$	$\frac{\frac{2}{\pi} - \left(-\frac{2}{\pi}\right)}{\frac{3\pi}{2} - \frac{\pi}{2}} = \frac{4}{\pi^2}$
x_1 π	-1	$\frac{0-(-1)}{\frac{3\pi}{2} - \pi} = \frac{2}{\pi}$	
x_2 $\frac{3\pi}{2}$	0		

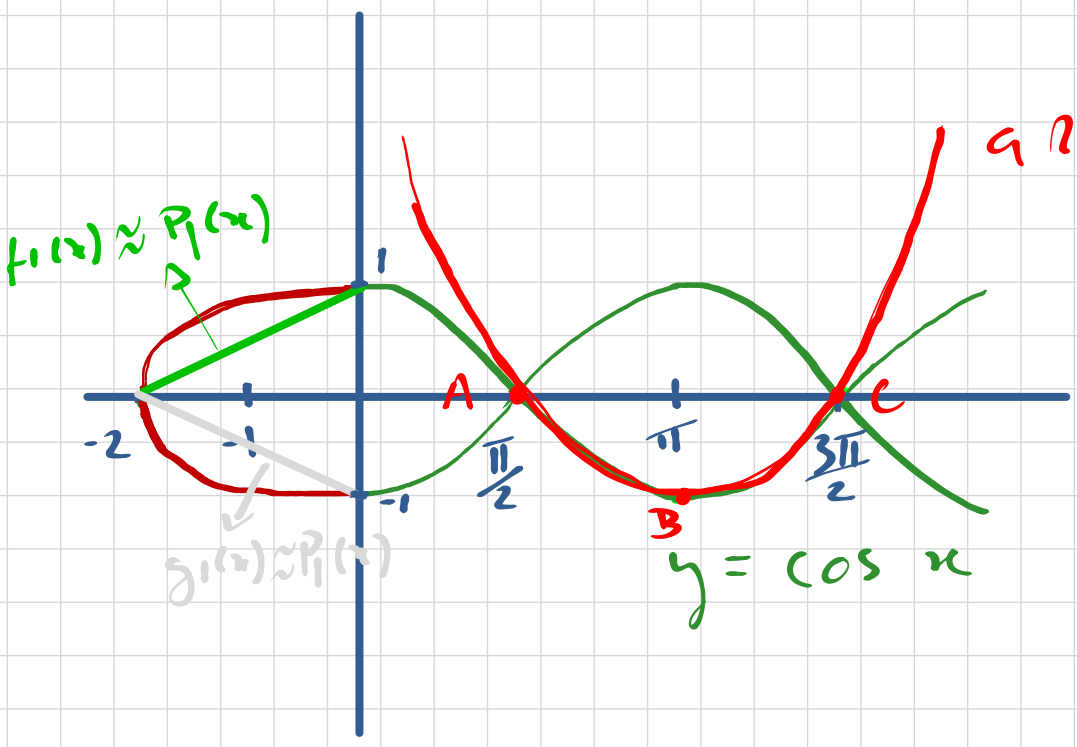
a_0 a_1 a_2

$$P_2(x) = 0 - \frac{2}{\pi}\left(x - \frac{\pi}{2}\right) + \frac{4}{\pi^2}\left(x - \frac{\pi}{2}\right)(x - \pi)$$

$$= -\frac{2}{\pi} - \frac{2}{\pi}\left(x - \frac{\pi}{2}\right) + \frac{4}{\pi^2}\left(x - \frac{\pi}{2}\right)(x - \pi)$$

b) $x \in [-2, 0] \rightarrow f(x) \approx P_1(x)$

$$g(x) \approx P_1(x)$$



arco de parábola

$x \in [\pi/2, 3\pi/2]$

Integração numérica

Objectivo:

$$\int_a^b f(x)\,dx \approx ?$$

AM I

$$F(x) = \int f(x)\,dx$$

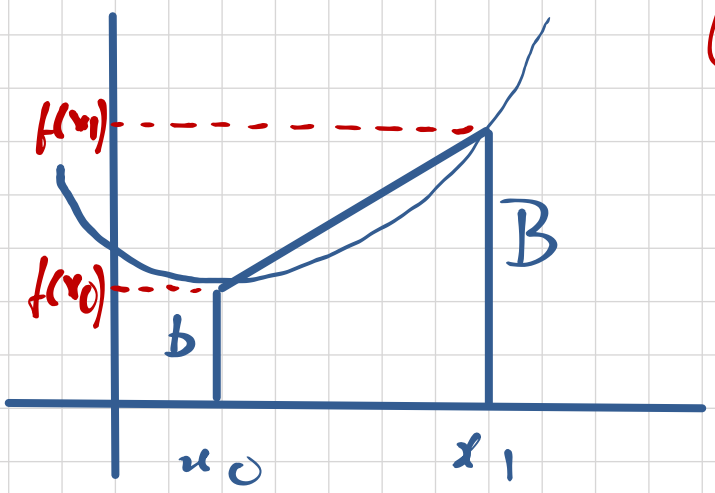
isto é:

$$F'(x) = f(x)$$

$$\int_a^b f(x)\,dx \left[F(x)\right]_a^b$$

$$= F(b) - F(a)$$

Regra dos Trapézios: (simples)

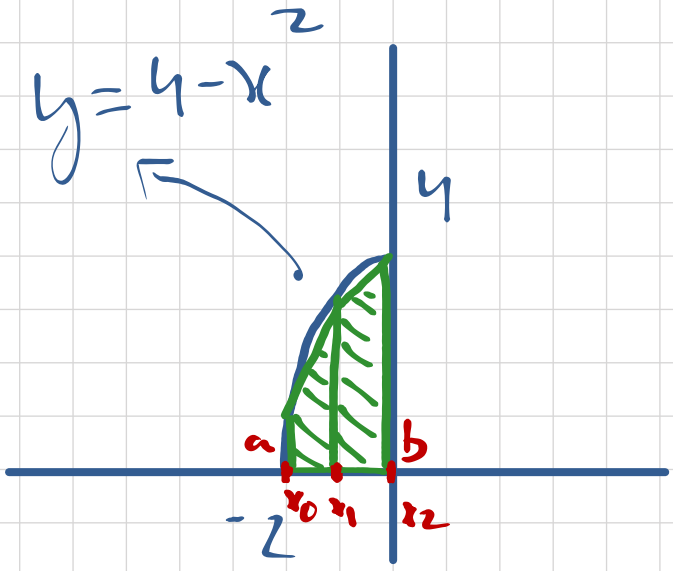


$$A(\square) = \frac{b + B}{2} + h$$

$$A(\square 1) = \frac{f(x_0) + f(x_1)}{2} * h =$$

$$= h/2 \left[f(x_0) + f(x_1)\right] \quad (\text{com } h = x_1 - x_0)$$

Regra dos Trapézios composta:



$$R = \{(x,y) \in \mathbb{R}^2 : -2 \leq x \leq 0 \wedge 0 \leq y \leq 4 - x^2$$

$$\int_{-2}^{2} 4 - x^2 \, dx \approx \frac{h}{2}\left[f(x_0) + 2f(x_1) + f(x_2)\right] \approx$$

$$\approx \frac{1}{2}\left[0 + 2 \times 3 + 4\right] \approx$$

$$\approx 5$$

$$h = \frac{b-a}{n} \Leftrightarrow h = \frac{0-(-2)}{2} = 1$$

n → número de subdivisões do intervalo

(Resultado usando integrais $= \frac{16}{3} \approx 5.33$)

ERRO: 0,33

Quanto maior for n, maior será a precisão do Resultado

Função geogebra:

SomaTrapezoidal(f(x), a, b, n)

- f(x): Função
- a, b: Intervalo
- n: nº de Trapézios