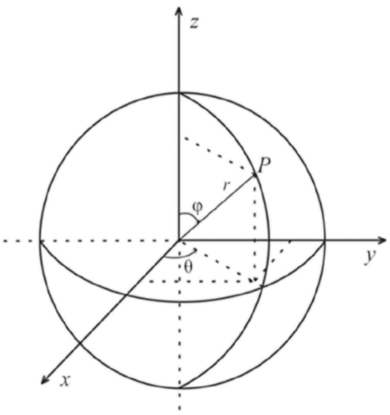
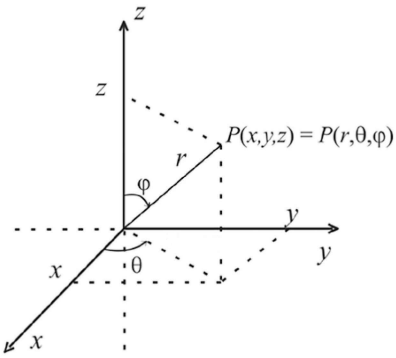
Coordenadas Esféricas

Provar que o Volume de uma esfera de raio r é igual a $\frac{4}{3}\pi r^3$**.**

Coordenadas Cartesianas: $x^2 + y^2 + z^2 = r^2$

Coordenadas Esféricas: R = r





S=$\left\{(R, \theta, \varphi) \in IR^3 = 0 \leqslant R \leqslant r \wedge 0 \leqslant \theta \leqslant 2\pi \wedge 0 \leqslant \varphi \leqslant \pi\right\}$

Volume:

V = $\iiint 1\, dr\, dy\, dz$ = $\iiint 1. | -R^2. sin(\varphi)|\, d\varphi\, d\theta\, dR$

$\Leftrightarrow \int_0^r \int_0^{2\pi} \int_0^{\pi} R^2. sin(\varphi)\, d\varphi\, d\theta\, dR$

$\Leftrightarrow \int_0^r \int_0^{2\pi} R^2. \left[-cos(\varphi) \right]_0^{\pi} d\theta\, dR$ = $\int_0^r \int_0^{2\pi} 2.R^2\, d\theta\, dR$

$\Leftrightarrow 2\int_0^r R^2. \left[\theta \right]_0^{2\pi} dR$ = $4\pi\int_0^r R^2\, dR$

$\Leftrightarrow 4\pi \left[\frac{R^3}{3} \right]_0^r$ = $4\pi. \left(\frac{r^3}{3} - 0 \right)$ = $\frac{4}{3}. \pi^3$