2ª Frequencia 20 Julho 2016

1.

X = Rendimentos familiares da primeira região
Y = " " segunda região

$X \sim N(2100, 400)$ $Z = \frac{X - 2100}{400} \sim N(0,1)$

a) $P(2000 < X < 2260) = P\left(\frac{2000-2100}{400} < \frac{X-2100}{400} < \frac{2260-2100}{400}\right) =$

$= P(-0.25 < Z < 0.4) = 0.254$
↳ normalcdf(-0.25, 0.4)

b) $Y \sim N(1400, 225)$ $Z = \frac{Y - 1400}{225} \sim N(0,1)$

$P(Y > X) = P(\underbrace{Y - X}_{} > \emptyset)$ D = Diferença entre Y e X

$D \sim N(E(D), \sigma(D))$

$E(D) = 1400 - 2100 = -700$

$V(D) = V(Y-X) = V(Y) + V(X)$ → o desvio padrão é a raiz quadrada da variância. logo, para chegar à variância de D, elevamos ao quadrado os valores de desvio-padrão de X e Y e depois somamos.

$= 225^2 + 400^2$

$= 210625$

$\sigma(D) = \sqrt{210625}$

$D \sim N(-700, \sqrt{210625})$ $Z = \frac{X-(-700)}{\sqrt{210627}} \sim N(0,1)$

$P(D > 0) = P\left(Z > \frac{0+700}{\sqrt{\ }}\right) = P(Z > 1,525) = 1 - P(Z < 1,525) = 0.636$

225

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Notas: $P(z > 1.525) \Rightarrow$ normalcdf(-99999, -1,525) = 0.636 } calculadora
$P(z > 1.525) \Rightarrow 1 -$ (normalcdf (-99999, 1.525) = 0.636)

2 → X = tempo de vida de um dispositivo

$X \sim E(0.16)$

$\bar{X} = \frac{\sum_{i=1}^{200} x_i}{200} \stackrel{\cdot}{\sim} N(E(\bar{x}), \sigma(\bar{x})) = Z = \frac{\bar{X} - E(\bar{x})}{\sigma(\bar{x})} \sim N(0,1)$

$E(\bar{X}) = \frac{1}{0.16}$ $\sigma(\bar{x}) = \sqrt{\frac{\frac{1}{0.16^2}}{200}} = \sqrt{\frac{1}{5.12}}$

$P(\bar{X} \geq 7) = P\left(\frac{\bar{x} - 1/0.16}{\sqrt{\frac{1}{5.12}}} \geq \frac{7 - 1/0.16}{\sqrt{\frac{1}{5.12}}}\right) =$

$= P(Z \geq 1.69) = 0.045$

↳ normalcdf(-99999, -1.69)

$\left(\frac{\bar{X}-10.18}{\sqrt{\frac{1}{5.12}}} \geq \frac{1-10.18}{\sqrt{\frac{1}{5.12}}}\right) =$

$= P(Z \geq 1.69) = 0.045$

↳ normalcdf(-99999, -1.69)

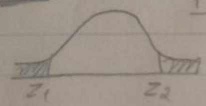
3-

a) Valor médio $= \hat{\mu} = \bar{x} = \sum_{i=1}^{8} x_i \; (13.0+13.5+\ldots+13.7) \; \frac{109.4}{8} = 13.675$

$s_8^2 = \frac{\sum x_i^2}{m-1} - \frac{m\bar{x}^2}{m-1} = \frac{1498.16}{7} - \frac{8\times(13.675)^2}{7} = 0.3021$

$IC_{95\%}(\mu) = ?$ $\quad \left.\begin{array}{l} X \sim N(\mu, \sigma) \\ m < 30 \\ \sigma \text{ conhecido} = 0.5 \end{array}\right\} Z_m = \frac{\bar{X}-\mu}{\sigma}\sqrt{m} \sim N(0,1)$

$1 - \underline{0.025} = 0.975$ $\qquad \frac{1-0.95}{2} = \underline{0.025}$



Z_1 e Z_2 : $P(Z_1 < Z_m < Z_2) = 0.95$

$Z_1 : P(Z_m < Z_1) = 0.025$

$Z_2 : P(Z_m < Z_2) = 0.975$ $\quad$ invT(0.975, 7) = 2.3646

$-2.3646 < \frac{\bar{X}-\mu}{\sigma}\sqrt{m} < 2.3646$ $\quad | \quad IAC_{95\%} = \left]\bar{X} - 2.3646\frac{\sigma}{\sqrt{m}}, \bar{X}+2.3646\frac{\sigma}{\sqrt{m}}\right[$

σ conhecido $= 0.5$

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$\Leftrightarrow -2.3646\frac{\sigma}{\sqrt{m}} - \bar{X} < -\mu < 2.3646\frac{\sigma}{\sqrt{m}} - \bar{X}$ $\quad | \quad IC(\mu) = \left]13.675 - 2.3646\times\frac{0.5}{\sqrt{8}}, 13.675 + 2.3646\times\frac{0.5}{\sqrt{8}}\right[$

$\Leftrightarrow \underbrace{\bar{X} + 2.3646\frac{\sigma}{\sqrt{m}}}_{\text{Lim. superior}} > \mu > \underbrace{\bar{X} - 2.3646\frac{\sigma}{\sqrt{m}}}_{\text{Lim. inferior}}$ $\quad | \quad IC(\mu) =]13,25 , 14.09[$

$Z_1: P(Z_m < Z_1) = 0.025$

$Z_2: P(Z_m < Z_2) = 0.975$ imvT(0.

$$-2.3646 < \frac{\bar{X} - \mu}{\sigma}\sqrt{m} < 2.3646$$

$$-2.3646\frac{\sigma}{\sqrt{m}} - \bar{X} < -\mu < 2.3646\frac{\sigma}{\sqrt{m}} - \bar{X}$$

$$\underbrace{\bar{X} + 2.3646\frac{\sigma}{\sqrt{m}}}_{\text{Lim. superior}} > \mu > \underbrace{\bar{X} - 2.3646\frac{\sigma}{\sqrt{m}}}_{\text{Lim. inferior}}$$

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$Z_1 \quad Z_2$

T(0.975, 7) = 2.3646

$$IAC_{95\%} = \left]\bar{X} - 2.3646\frac{\sigma}{\sqrt{m}}, \bar{X} + 2.3646\frac{\sigma}{\sqrt{m}}\right[$$

$$IC(\mu) = \left]13.675 - 2.3646 \times \frac{0.5}{\sqrt{8}}, 13.675 + 2.3646 \times \frac{0.5}{\sqrt{8}}\right[$$

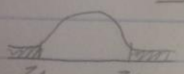
$$IC(\mu) = \left]13{,}25, 14.09\right[$$

3 →

b) Uma vez que fala em 90% dos casos temos de calcular um $IC_{90\%}(\mu)$ utilizando cálculos da alínea anterior

$1-0.05=0.95$

$\frac{1-0.90}{2}=0.05$



Z_1 e Z_2: $P(Z_1<Z_n<Z_2)=0.90$

Z_1: $P(Z_n<Z_1)=0.05$

Z_2: $P(Z_n<Z_2)=0.95$ $invT(0.95,7)=1.8945$

$1.8945\times\frac{0.5}{\sqrt{n}}\leq 0.1$

$\frac{1.8945\times0.5}{0.1}\leq\sqrt{n}\iff\left(\frac{1.8945\times0.5}{0.1}\right)^2\leq n\iff 89.72\leq n\implies n\geq 90$

Deve incluir 90 caixas

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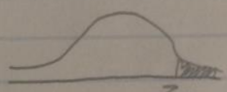
c) hipóteses do teste: $H_0: \sigma=0.50$ $H_1: \sigma>0.5$

μ é desconhecido

nível de significância: 0.05

$Z=\frac{(n-1)\,S^2_n}{\sigma^2}$ $\chi^2_{n-1=7}$



$P(Z\leq Z_c)=1-0.05=0.95$ $(n=7, P=0.95)$ pág. 12 $RC=[14.07,+\infty[$

Zobs: $\frac{7\times0.3021}{0.5^2}=8.4588$ não pertence à região crítica

não rejeitamos H_0