Instituto Superior de Engenharia de Coimbra
DEPARTAMENTO DE FÍSICA E MATEMÁTICA

Análise Matemática I - **Engenharia Informática**

TPC nº9 Data limite de entrega: 1/Dez/2015 (23h59m)

Primitivação por partes

C. Aplicação

Resolva as seguintes primitivas, utilizando a técnica de primitivação por partes.

6) $\displaystyle\int \frac{x}{\sqrt[3]{x+1}}\,dx\,;$

11) $\displaystyle\int \frac{x^3}{\sqrt[3]{1-x^2}}\,dx\,;$

7) $\displaystyle\int \sin x \ln(\cos^2 x)\,dx\,;$

extra 1) $\displaystyle\int \arcsin x\,dx\,;$

extra 2) $\displaystyle\int \arctan x\,dx\,.$

Sugestão de resolução:

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6) Tem-se

$$\int \frac{x}{\sqrt[3]{x+1}}\,dx = \int \underbrace{x}_{d}\,\underbrace{(x+1)^{-\frac{1}{3}}}_{p}\,dx$$

cálculos auxiliares:

$$\boxed{\begin{array}{l} \bullet\ \int \underbrace{(x+1)^{-\frac{1}{3}}\cdot 1}_{R2}\,dx = \frac{(x+1)^{\frac{2}{3}}}{\frac{2}{3}} = \frac{3}{2}(x+1)^{\frac{2}{3}} \\ \bullet\ (x)' = 1 \end{array}}$$

$$\begin{aligned}
&\overset{PP}{=} x\,\frac{3}{2}(x+1)^{\frac{2}{3}} - \int 1\,\frac{3}{2}(x+1)^{\frac{2}{3}}\,dx \\
&= \frac{3}{2}\,x\,\sqrt[3]{(x+1)^2} - \frac{3}{2}\int \underbrace{(x+1)^{\frac{2}{3}}\cdot 1}_{R2}\,dx \\
&= \frac{3}{2}\,x\,\sqrt[3]{(x+1)^2} - \frac{3}{2}\,\frac{(x+1)^{\frac{5}{3}}}{\frac{5}{3}} + c \\
&= \frac{3}{2}\,x\,\sqrt[3]{(x+1)^2} - \frac{9}{10}\,\sqrt[3]{(x+1)^5} + c\,,\ \ c \in \mathbb{R}\,.
\end{aligned}$$

11) Tem-se

$$\begin{aligned}
\int \frac{x^3}{\sqrt[3]{1-x^2}}\,dx &= \int x^3\,(1-x^2)^{-\frac{1}{3}}\,dx \\
&= \int \underbrace{x^2}_{d}\ \underbrace{x\,(1-x^2)^{-\frac{1}{3}}}_{p}\,dx
\end{aligned}$$

cálculos auxiliares:

- $\int x(1-x^2)^{-\frac{1}{3}}\,dx = -\frac{1}{2}\int \underbrace{-2x(1-x^2)^{-\frac{1}{3}}}_{R2}\,dx = -\frac{1}{2}\,\frac{(1-x^2)^{\frac{2}{3}}}{\frac{2}{3}} = -\frac{3}{4}\,(1-x^2)^{\frac{2}{3}}$
- $(x^2)' = 2x$

$$\begin{aligned}
&\overset{PP}{=} x^2\Big(-\frac{3}{4}\,(1-x^2)^{\frac{2}{3}}\Big) - \int 2x\,\Big(-\frac{3}{4}\,(1-x^2)^{\frac{2}{3}}\Big)\,dx \\
&= -\frac{3}{4}\,x^2\,\sqrt[3]{1-x^2} - \frac{3}{4}\int \underbrace{-2x\,(1-x^2)^{\frac{2}{3}}}_{R2}\,dx \\
&= -\frac{3}{4}\,x^2\,\sqrt[3]{1-x^2} - \frac{3}{4}\,\frac{(1-x^2)^{\frac{5}{3}}}{\frac{5}{3}} + c \\
&= -\frac{3}{4}\,x^2\,\sqrt[3]{1-x^2} - \frac{9}{20}\,\sqrt[3]{(1-x^2)^5} + c\,,\ \ c \in \mathbb{R}\,.
\end{aligned}$$

7) Tem-se

$$\int \sin x \ln(\cos^2 x)\,dx = \int \underbrace{\sin x}_{p}\underbrace{\ln(\cos^2 x)}_{d}\,dx\,dx$$

cálculos auxiliares:

- $\int \underbrace{1\sin x}_{R7}\,dx = -\cos x$
- $\big(\ln(\cos^2 x)\big)' = \frac{(\cos^2 x)'}{\cos^2 x} = \frac{2\cos x(-\sin x)}{\cos^2 x} = \frac{-2\sin x}{\cos x}$

$$\begin{aligned}
&\overset{PP}{=} \sin x \ln(\cos^2 x) - \int \cos x\,\frac{-2\sin x}{\cos x}\,dx \\
&= \sin x \ln(\cos^2 x) + 2\int \underbrace{1\sin x}_{R7}\,dx \\
&= \sin x \ln(\cos^2 x) + 2\,(-\cos x) + c \\
&= \sin x \ln(\cos^2 x) - 2\cos x + c\,,\ \ c \in \mathbb{R}\,.
\end{aligned}$$

extra 1) Tem-se

$$\int \arcsin x\,dx = \int \underbrace{1}_{p}\ \underbrace{\arcsin x}_{d}\,dx\,dx$$

cálculos auxiliares:

- $\int \underbrace{1}_{R1}\,dx = x$
- $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$

$$\begin{aligned}
&\overset{PP}{=} x\arcsin x - \int x\,\frac{1}{\sqrt{1-x^2}}\,dx \\
&= x\arcsin x - \int x\,(1-x^2)^{-\frac{1}{2}}\,dx \\
&= x\arcsin x - \Big(-\frac{1}{2}\Big)\int \underbrace{-2x\,(1-x^2)^{-\frac{1}{2}}}_{R2}\,dx \\
&= x\arcsin x + \frac{1}{2}\,\frac{(1-x^2)^{\frac{1}{2}}}{\frac{1}{2}} + c \\
&= x\arcsin x + \sqrt{1-x^2} + c\,,\ \ c \in \mathbb{R}\,.
\end{aligned}$$

extra 2) Tem-se

$$\int \arctan x \, dx = \int \underbrace{1}_{p} \underbrace{\arctan x}_{d} \, dx \, dx$$

cálculos auxiliares:

$$\boxed{\begin{array}{l} \bullet \int \underbrace{1}_{R1} \, dx = x \\ \bullet \ (\arctan x)' = \frac{1}{1+x^2} \end{array}}$$

$$\begin{aligned} &\overset{PP}{=} x \arctan x - \int x \, \frac{1}{1+x^2} \, dx \\ &= x \arctan x - \frac{1}{2} \int \underbrace{\frac{2x}{1+x^2}}_{R5} \, dx \\ &= x \arctan x - \frac{1}{2} \ln(1+x^2) + c \,, \ c \in \mathbb{R} \,. \end{aligned}$$