

PRIMITIVAÇÃO IMEDIATA [C. Aplicação]

Calcule as primitivas das seguintes funções:

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|------------------------------------|-----------------------------------|-------------------------------|
| a) $\frac{1}{x^3}$; | b) $(x^3 + 1)^4 x^2$; | c) $\frac{1}{(x+1)^2}$; |
| d) $\sec^2 x \tan x$; | e) $x^{-1} \ln x$; | f) $\sqrt[3]{x^2}$; |
| g) $x \sqrt[3]{x^2}$; | h) $\frac{\arctan(2x)}{1+4x^2}$; | i) e^{5x} ; |
| j) $\frac{10\sqrt{x}}{\sqrt{x}}$; | k) $\frac{x+2}{x^2+4x}$; | l) $\frac{\sin(\ln x)}{x}$; |
| m) $\frac{e^{\frac{1}{x}}}{x^2}$; | n) $\frac{x}{\sqrt{4-x^2}}$; | o) $\frac{1}{x \ln x}$; |
| p) $\frac{\cos x}{1+\sin^2 x}$; | q) $\frac{3x}{\sqrt{1-x^4}}$; | r) $\sin(4x)$; |
| s) $\frac{5x}{1+x^4}$; | t) $e^{\sin x} \cos x$; | u) $\frac{1}{x(1+\ln^2 x)}$. |

Sugestão de resolução:

$$a) \int \frac{1}{x^3} dx = \int \underbrace{x^{-3} \cdot 1}_{R2} dx = \frac{x^{-2}}{-2} + c = -\frac{1}{2x^2} + c, \quad c \in \mathbb{R};$$

$$b) \int (x^3 + 1)^4 x^2 dx = \frac{1}{3} \int \underbrace{(x^3 + 1)^4 \cdot 3x^2}_{R2} dx = \frac{1}{3} \frac{(x^3 + 1)^5}{5} + c = \frac{1}{15} (x^3 + 1)^5 + c, \quad c \in \mathbb{R};$$

$$c) \int \frac{1}{(x+1)^2} dx = \int \underbrace{(x+1)^{-2} \cdot 1}_{R2} dx = \frac{(x+1)^{-1}}{-1} + c = -\frac{1}{x+1} + c, \quad c \in \mathbb{R};$$

$$d) \int \sec^2 x \tan x dx = \int \underbrace{(\tan x)^1 \sec^2 x}_{R2} dx = \frac{(\tan x)^2}{2} + c_1 = \frac{1}{2} \tan^2 x + c_1, \quad c_1 \in \mathbb{R};$$

Alternativa:

$$\int \sec^2 x \tan x dx = \int \underbrace{(\sec x)^1 \sec x \tan x}_{R2} dx = \frac{(\sec x)^2}{2} + c_2 = \frac{1}{2} \sec^2 x + c_2, \quad c_2 \in \mathbb{R};$$

$$e) \int x^{-1} \ln x dx = \int \underbrace{(\ln x)^1 \cdot \frac{1}{x}}_{R2} dx = \frac{(\ln x)^2}{2} + c = \frac{1}{2} \ln^2 x + c, \quad c \in \mathbb{R};$$

$$f) \int \sqrt[3]{x^2} dx = \int \underbrace{x^{\frac{2}{3}} \cdot 1}_{R2} dx = \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + c = \frac{3}{5} \sqrt[3]{x^5} + c, \quad c \in \mathbb{R};$$

$$g) \int x \sqrt[3]{x^2} dx = \int x x^{\frac{2}{3}} dx = \int x^{\frac{5}{3}} dx = \int \underbrace{x^{\frac{5}{3}} \cdot 1}_{R2} dx = \frac{x^{\frac{8}{3}}}{\frac{8}{3}} + c = \frac{3}{8} \sqrt[3]{x^8} + c, \quad c \in \mathbb{R};$$

$$\text{h)} \int \frac{\arctan(2x)}{1+4x^2} dx = \int \arctan(2x) \frac{1}{1+4x^2} dx = \frac{1}{2} \int \underbrace{\arctan(2x) \frac{2}{1+4x^2}}_{R2} dx$$

$$= \frac{1}{2} \frac{\arctan^2(2x)}{2} + c = \frac{1}{4} \arctan^2(2x) + c, \quad c \in \mathbb{R};$$

$$\text{i)} \int e^{5x} dx = \frac{1}{5} \int \underbrace{e^{5x} 5}_{R3} dx = \frac{1}{5} e^{5x} + c, \quad c \in \mathbb{R};$$

$$\text{j)} \int \frac{10^{\sqrt{x}}}{\sqrt{x}} dx = \int 10^{\sqrt{x}} \frac{1}{\sqrt{x}} dx = 2 \int \underbrace{10^{\sqrt{x}} \frac{1}{2\sqrt{x}}}_{R4} dx = 2 \frac{10^{\sqrt{x}}}{\ln 10} + c = \frac{2}{\ln 10} 10^{\sqrt{x}} + c, \quad c \in \mathbb{R};$$

$$\text{k)} \int \frac{x+2}{x^2+4x} dx = \frac{1}{2} \int \frac{2(x+2)}{x^2+4x} dx = \frac{1}{2} \int \frac{2x+4}{\underbrace{x^2+4x}_{R5}} dx = \frac{1}{2} \ln|x^2+4x| + c, \quad c \in \mathbb{R};$$

$$\text{l)} \int \frac{\sin(\ln x)}{x} dx = \int \frac{1}{x} \sin(\underbrace{\ln x}_{R7}) dx = -\cos(\ln x) + c, \quad c \in \mathbb{R};$$

$$\text{m)} \int \frac{e^{\frac{1}{x}}}{x^2} dx = \int e^{\frac{1}{x}} \frac{1}{x^2} dx = - \int \underbrace{e^{\frac{1}{x}} \left(-\frac{1}{x^2}\right)}_{R3} dx = -e^{\frac{1}{x}} + c, \quad c \in \mathbb{R};$$

$$\text{n)} \int \frac{x}{\sqrt{4-x^2}} dx = \int (4-x^2)^{-\frac{1}{2}} x dx = -\frac{1}{2} \int \underbrace{(4-x^2)^{-\frac{1}{2}} (-2x)}_{R2} dx = -\frac{1}{2} \frac{(4-x^2)^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= -\sqrt{4-x^2} + c, \quad c \in \mathbb{R};$$

$$\text{o)} \int \frac{1}{x \ln x} dx = \int \frac{1}{x} \frac{1}{\ln x} dx = \int \frac{\frac{1}{x}}{\underbrace{\ln x}_{R5}} dx = \ln|\ln x| + c, \quad c \in \mathbb{R};$$

$$\text{p)} \int \frac{\cos x}{\underbrace{1+\sin^2 x}_{R19}} dx = \arctan(\sin x) + c, \quad c \in \mathbb{R};$$

$$\text{q)} \int \frac{3x}{\sqrt{1-x^4}} dx = \int \frac{3x}{\sqrt{1-(x^2)^2}} dx = \frac{3}{2} \int \frac{\frac{2x}{2}}{\underbrace{\sqrt{1-(x^2)^2}}_{R18}} dx = \frac{3}{2} \arcsin(x^2) + c, \quad c \in \mathbb{R};$$

$$\text{r)} \int \sin(4x) dx = \frac{1}{4} \int \underbrace{4 \sin(4x)}_{R7} dx = \frac{1}{4} (-\cos(4x)) + c = -\frac{1}{4} \cos(4x) + c, \quad c \in \mathbb{R};$$

$$\text{s)} \int \frac{5x}{1+x^4} dx = 5 \int \frac{x}{1+(x^2)^2} dx = \frac{5}{2} \int \frac{\frac{2x}{2}}{\underbrace{1+(x^2)^2}_{R19}} dx = \frac{5}{2} \arctan(x^2) + c, \quad c \in \mathbb{R};$$

$$\text{t)} \int \underbrace{e^{\sin x} \cos x}_{R3} dx = e^{\sin x} + c, \quad c \in \mathbb{R};$$

$$\text{s)} \int \frac{1}{x(1+\ln^2 x)} dx = \int \frac{1}{x} \frac{1}{1+\ln^2 x} dx = \int \frac{\frac{1}{x}}{\underbrace{1+\ln^2 x}_{R19}} dx = \arctan(\ln x) + c, \quad c \in \mathbb{R}.$$