Seminar 10

Spații vectoriale euclidiene. Repere ortonormate

(1) $g: \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ $g(x,y) = a x_1 y_1 + b x_1 y_2 + b x_2 y_1 + c x_2 y_2$

a) $g \in L^s(\mathbb{R}^2, \mathbb{R}^2, \mathbb{R})$

b) g produs scalar $\Leftrightarrow \begin{cases} a > 0 \\ ac - b^2 > 0 \end{cases}$

Sol: ? $(\mathbb{R}^3, g)$ spațiu euclidian real (este g produs scalar

$\Leftrightarrow$ 1. $g \in L^s(\mathbb{R}^2, \mathbb{R}^2, \mathbb{R})$

2. g poz. def (Q forma pătratică are signatura $= (3,0)$)

$G = G^T \Rightarrow g \in L^s(\mathbb{R}^3, \mathbb{R}^3, \mathbb{R})$

$Q: \mathbb{R}^3 \to \mathbb{R}$ forma pătratică asoc

$$Q(x) = 3x_1^2 + 2x_2^2 + x_3^2 + 4x_1x_2 + 4x_2x_3$$

Met. Jacobi: $\Delta_1 = 3$, $\Delta_2 = 2$, $\Delta_3 = \begin{vmatrix} 3 & 2 & 0 \\ 2 & 2 & 2 \\ 0 & 2 & 1 \end{vmatrix} = -10 \neq 0$

$\exists R$ a.î. $Q(x) = \frac{1}{3} {x_1'}^2 + \frac{3}{2} {x_2'}^2 + \frac{-10}{2} {x_3'}^2$

$\Rightarrow (2,1) =$ signatura $\Rightarrow (\mathbb{R}^3, g)$ nu este sp. vect. euclidian

EXAMEN

(3) $(\mathbb{R}^3, g_0)$, $g_0: \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$, $g_0(x,y) = x_1y_1 + x_2y_2 + x_3y_3$ produs scalar canonic

$U = \{x \in \mathbb{R}^3 \mid x_1 + x_2 - x_3 = 0\}$

a) $U^\perp$

b) Să se det. un reper ortonormat $R = R_1 \cup R_2$ în $\mathbb{R}^3$ unde $R_1 =$ reper ortonormat în U, $R_2 =$ —"— $U^\perp$

$U^{\perp} = \{x \in \mathbb{R}^3 \mid g_0(x,y) = 0 \mid \forall y \in U\}$

$U = \{(x_1, x_2, x_1 + x_2 \mid x_1, x_2 \in \mathbb{R}\} = \{(x_1, 0, x_1) + (0, x_2, x_2)\} =$

$= <\{\underset{f_1}{(1,0,1)}, \underset{f_2}{(0,1,1)}\}$

$\dim U = 2 \Rightarrow \{f_1, f_2\}$ reper în U

$x_1 + x_2 - x_3 = 0 \qquad g_0(x, (1,1,-1)) = 0$

$U^{\perp} = <\{\underset{f_3}{(1,1,-1)}\}>$

Ortonormăm Procedeul Gram - Schmidt:

$\underset{\text{reper arbitrar}}{\{f_1, f_2\}} \longrightarrow \underset{\text{reper ortogonal}}{\{e_1 e_2\}} \longrightarrow \underset{\text{reper ortonormat}}{\{e_1', e_2'\}}$

$e_1 = f_1 = (1,0,1) \Rightarrow e_1' = \frac{e_1}{\|e_1\|} = \frac{1}{\sqrt{2}}(1,0,1)$

$e_2 = f_2 = \frac{g_0(f_2, e_1)e_1}{g_0(e_1, e_1)} = (0,1,1) - \frac{1}{2}(1,0,1)$

$= \left(-\frac{1}{2}, 1, \frac{1}{2}\right) - \frac{1}{2}(-1,2,1) \Rightarrow e_2' = \frac{1}{\sqrt{6}}(-1,2,1)$

Exemplu: $u = \alpha u', \ \alpha > 0$

versor $\rightarrow \frac{u}{\|u\|} = \frac{u'}{\|u\|}$

$R_1 = \{e_1', e_2'\}$ reper ortonormat în U

$\mathbb{R}^3 = \underset{\dim = 2}{U} \oplus \underset{\dim = 1}{U^{\perp}}$

$R_2 = \left\{ \frac{1}{\sqrt{3}} \cdot (1,1,-1) \right\}$ reper ortonormat în $U^{\perp}$

$\rightarrow R = R_1 \cup R_2$ reper ortonormat în $\mathbb{R}^3$

5) $(\mathbb{R}^3, g_0)$, $R = \{ f_1 = (1,2,3),\ f_2 = (0,1,1),\ f_3 = (1,2,5) \}$

a) R reper în $\mathbb{R}^3$. Să se ortonormeze

b) $f_1 \times f_2$

c) $f_1 \wedge f_2 \wedge f_3$

$$f_1 \times f_2 = \begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & 2 & 3 \\ 0 & 1 & 1 \end{vmatrix} = e_1 \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} - e_2 \begin{vmatrix} 1 & 3 \\ 0 & 1 \end{vmatrix} +$$

$$+ e_3 \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = (-1,-1,1)$$

$$f_1 \wedge f_2 \wedge f_3 = \langle f_1, f_2 \times f_3 \rangle = \langle f_3, f_1 \times f_2 \rangle =$$

$$= \begin{vmatrix} 1 & 2 & 5 \\ 1 & 2 & 3 \\ 0 & 1 & 1 \end{vmatrix}$$

$$= g_0\left((1,2,5), (-1,-1,1) \right)$$

$$= -1-2+5 = 2$$

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