CURS 2 - AG

13.0

Determinanți. Teorema Laplace

Fie $A \in \mathcal{M}_n(\mathbb{K})$

a) minor de ordin p

$$\Delta_p = \det(A_{I,J}) = \begin{vmatrix} a_{i_1 j_1} \dots a_{i_1 j_p} \\ a_{i_p j_1} \dots a_{i_p j_p} \end{vmatrix}$$

$I = \{i_1, .., i_p\}$ $\quad 1 \leq i_1 < .. < i_p \leq n$

$J = \{j_1, .., j_p\}$ $\quad 1 \leq j_1 < ... < j_p \leq n$.

b) minor complementar lui Δ_p

$\Delta_c = \det(A_{\bar{I},\bar{J}})$ $\quad \bar{I} = \{1,..,n\} \setminus \{i_1,..,i_p\}$

$\quad \bar{J} = \{1,..,n\} \setminus \{j_1,..,j_p\}$

minorul obținut din A, suprimând liniile $l_{i_1}, .., l_{i_p}$

coloanele $c_{j_1}, .., c_{j_p}$.

c) complement algebric pt Δ_p

$$c = (-1)^{i_1+..+i_p+j_1+..+j_p} \Delta_c = (-1)^{I+J} \det(A_{\bar{I},\bar{J}})$$

Caz particular $p=1$

$\Delta_1 = a_{ij}$, $\Delta_c = \Delta_{ij}$, $c_{ij} = (-1)^{i+j} \Delta_{ij}$ complementul algebric pt a_{ij}

Teorema Laplace

det(A) = suma produselor minorilor de ordinul p cu complemenții algebrici coresp, pt p linii fixate $l_{i_1}, .., l_{i_p}$, resp pt p coloane fixate $c_{j_1}, .., c_{j_p}$

$$\det(A) = \sum_J (-1)^{i_1+...+i_p+j_1+...+j_p} \det(A_{I,J}) \det(A_{\bar{I},\bar{J}})$$

$$= \sum_I (-1)^{i_1+..+i_p+j_1+..+j_p} \det(A_{I,J}) \det(A_{\bar{I},\bar{J}})$$

Dacă $p=1$, at $\Rightarrow$ dezv. unui determinant după o linie sau o coloană

Exemplu

$$A = \begin{pmatrix} 1 & 1 & 2 & 3 \\ 1 & 1 & 3 & 4 \\ 2 & 5 & 1 & -1 \\ -1 & -2 & 2 & 4 \end{pmatrix} \begin{matrix} \leftarrow \\ \leftarrow \\ \\ \\ \end{matrix}$$

$p = 2$, l_1, l_2 fixate.

$$\det A = (-1)^{1+2+1+2} \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} \cdot \begin{vmatrix} 1 & -1 \\ 2 & 4 \end{vmatrix} + (-1)^{1+2+1+3} \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} \begin{vmatrix} 5 & -1 \\ -2 & 4 \end{vmatrix} +$$

$$+ (-1)^{1+2+1+4} \begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} \begin{vmatrix} 5 & 1 \\ -2 & 2 \end{vmatrix} + (-1)^{1+2+2+3} \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} \cdot \begin{vmatrix} 2 & -1 \\ -1 & 4 \end{vmatrix}$$

$$+ (-1)^{1+2+2+4} \begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} \begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix} + (-1)^{1+2+3+4} \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} \begin{vmatrix} 2 & 5 \\ -1 & -2 \end{vmatrix}$$

$$= 0 - 1 \cdot 18 + 1 \cdot 12 + 1 \cdot 7 - 1 \cdot 5 + (-1) \cdot 1$$

$$= -18 + 12 + 7 - 5 - 1 = -5$$

- Algoritmul <u>Gauss-Jordan</u> de inversare a unei matrice.

Fie matricea dublă $(A|I_n) \sim (C|B)$
(prin transformări elementare pe linii)
Dacă A = inversabilă, atunci $C = I_n$, $B = A^{-1}$
(forma eșalon redusă pt A).

Exemplu

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ -1 & -1 & 2 \end{pmatrix} \qquad A^{-1} = \frac{1}{\det A} \cdot A^*$$

$$\det A = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 0 & 1 & 5 \end{vmatrix} = 10 - 1 = 9 \neq 0$$

$$A^T = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 2 & -1 \\ 3 & 1 & 2 \end{pmatrix}$$

$A^*_{11} = (-1)^{1+1} \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} = 5$, $A^*_{12} = (-1)^{1+2} \begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} = -7$, $A^*_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} = -4$

$A^*_{21} = (-1)^{2+1} \begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix} = -1$, $A^*_{22} = (-1)^{2+2} \begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} = 5$, $A^*_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix} = -1$

$A^*_{31} = (-1)^{3+1} \begin{vmatrix} 0 & -1 \\ 2 & -1 \end{vmatrix} = 2$; $A^*_{32} = (-1)^{3+2} \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = -1$, $A^*_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 0 \\ 2 & 2 \end{vmatrix} = 2$

$$A^{-1} = \frac{1}{9}\begin{pmatrix} 5 & -7 & -4 \\ -1 & 5 & -1 \\ 2 & -1 & 2 \end{pmatrix}$$

Algoritmul Gauss-Jordan pt A^{-1}

$$(A|I_3) = \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ -1 & -1 & 2 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} \boxed{1} & 2 & 3 & 1 & 0 & 0 \\ 0 & \boxed{1} & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 1 & 5 & 1 & 0 & 1 \end{array}\right)$$

$L_2' = \frac{1}{2}L_2$; $L_3' = L_3 + L_1$

$$\underset{L_3' = L_3 - L_2}{\sim} \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{9}{2} & 1 & -\frac{1}{2} & 1 \end{array}\right)$$

$$\underset{L_3' = \frac{2}{9}L_3}{\sim} \left(\begin{array}{ccc|ccc} \boxed{1} & 2 & 3 & 1 & 0 & 0 \\ 0 & \boxed{1} & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \boxed{1} & \frac{2}{9} & -\frac{1}{9} & \frac{2}{9} \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & \frac{1}{3} & \frac{1}{3} & -\frac{2}{3} \\ 0 & 1 & 0 & -\frac{1}{9} & \frac{5}{9} & -\frac{1}{9} \\ 0 & 0 & 1 & \frac{2}{9} & -\frac{1}{9} & \frac{2}{9} \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{5}{9} & -\frac{7}{9} & -\frac{4}{9} \\ 0 & 1 & 0 & -\frac{1}{9} & \frac{5}{9} & -\frac{1}{9} \\ 0 & 0 & 1 & \frac{2}{9} & -\frac{1}{9} & \frac{2}{9} \end{array}\right)$$

$$\underbrace{}_{A^{-1}}$$

$L_2' = L_2 - \frac{1}{2}L_3$, $L_1' = L_1 - 3L_3$ $L_1' = L_1 - 2L_2$

$\frac{1}{2} + \frac{1}{18} = \frac{10}{18} = \frac{5}{9}$; $1 - \frac{2}{3} = \frac{1}{3}$; $\overset{3}{\frac{1}{3}} + \frac{2}{9} = \frac{5}{9}$

Sisteme de ecuatii algebrice de ordinul 1 cu mai multe necunoscute

$\circledast$ $AX = B$, $A \in \mathcal{M}_{m,n}(\mathbb{R})$, $X = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$, $B = \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix}$

$A = (a_{ij})_{\substack{i=\overline{1,m} \\ j=\overline{1,n}}}$

$\circledast \Leftrightarrow \sum_{j=1}^{n} a_{ij}x_j = b_i$, $i = \overline{1,m}$ (m ecuatii cu n necunoscute)

Interpretare geometrica $\circledast$: $\cap$ a m hiperplane in $\mathbb{R}^n$.

Not $S(A) = \{x = (x_1, .., x_n) \in \mathbb{R}^n \mid AX = B\} \subset \mathbb{R}^n$.

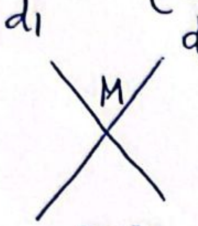
multimea solutiilor sistemului $\circledast$

OBS a) $S(A) \neq \emptyset$ → SCD (sistem compatibil determinat) $\exists!$ soluția /
→ SCN (sist. compatibil nedeterminat) $\exists$ mai multe sol / $\exists$ o inf de soluții.

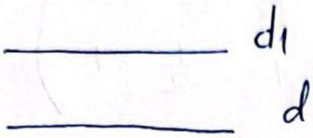
b) $S(A) = \emptyset$ SI (sistem incompatibil) ($\nexists$ soluții).

Cazuri particulare

1) $n = 2$ $\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases}$ $\cap$ 2 drepte în plan.



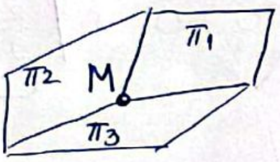
SCD



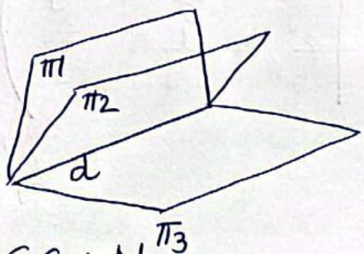
SI

SCN.

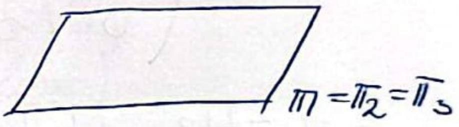
2) $n = 3$ $\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3. \end{cases}$ $\cap$ 3 plane în $\mathbb{R}^3$



SCΔ.

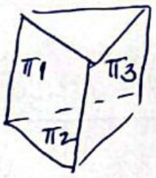


SCΔN.

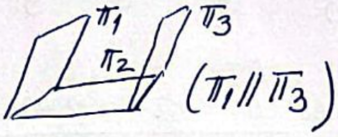


SCdN.

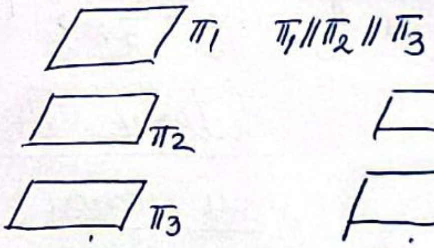
(fețele laterale)



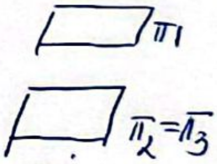
SI



SI



SI



SI

Cazul general

(*) $AX = B$, $\bar{A} = (A|B)$ matricea extinsă, $A \in \mathcal{M}_{m,n}(\mathbb{K})$

Dacă $n = m$ ($A \in \mathcal{M}_n(\mathbb{K})$) și $\Delta = \det A \neq 0 \Rightarrow \exists A^{-1} = \frac{1}{\det A} \cdot A^*$

$$\underbrace{A^{-1} \cdot A}_{I_n} X = A^{-1}B \Rightarrow X = \frac{1}{\det A} A^* B$$

$(x_1, \ldots, x_n) = \left(\frac{\Delta x_1}{\Delta}, \ldots, \frac{\Delta x_n}{\Delta} \right)$; Δ_{x_k} se obțin înlocuind coloana C_k din A cu coloana termenilor liberi, $\forall k = \overline{1,n}$

(sistem de tip Cramer)

Teorema Kronecker - Capelli

⊛ este compatibil $\Longleftrightarrow \text{rg} A = \text{rg} \bar{A}$

Teorema Rouché

⊛ este compatibil $\Longleftrightarrow$ toți minorii caracteristici (dacă $\exists$) sunt nuli

Algoritm

$\text{rg} A = r$, $\Delta_p = \det(A_{I,J})$ (minor principal). $I = \{i_1,.., i_r\}$ $J = \{j_1, .., j_r\}$

Δ_{car} se obtin prin bordare cu coloana t. liberi și adăugarea unei linii l_i, $i \in \{1,..,n\} \setminus I$.

1) Dacă $\exists\ \Delta_{car} \neq 0$, at $\text{rg}\bar{A} = r+1$.

2) Dacă $\text{rg}\bar{A} = r$. $\Rightarrow$ SC.

Fără a restrânge generalitatea (ev. renumerotând indicii) alegem $\Delta_p = \begin{vmatrix} a_{11} & .. & a_{1r} \\ a_{r1} & \dots & a_{rr} \end{vmatrix} \neq 0$.

(**) sistemul format din primele r ecuații:
(celelalte ec. sunt combinații liniare de $ec_1, .., ec_r$)

a) $m > n$ (nr ec > nr necunoscute)

a_1) $r = \text{rg} A = \text{rg}\bar{A} = n$. SCD. $x_1,.., x_n$ var principale ($\nexists$ var secundare)

a_2) $r = \text{rg} A = \text{rg}\bar{A} < n$

$x_1,.., x_r$ = var principale, $x_{r+1} = \lambda_1, \dots, x_n = \lambda_{n-r} = \lambda_p$ var secundare.

$$(**)\begin{cases} a_{11}x_1 + \dots + a_{1r}x_r = -a_{1r+1}\lambda_1 - \dots - a_{1n}\lambda_p + b_1 \\ \vdots \\ a_{r1}x_1 + \dots + a_{rr}x_r = -a_{rr+1}\lambda_1 - \dots - a_{rn}\lambda_p + b_r \end{cases}$$

Sol este $(\underbrace{x_1,.., x_r}, \lambda_1,.., \lambda_p)$
se exprimă în funcție de $\lambda_1,..., \lambda_p$.

b) $m < n$ SCN, $\text{rg} A = \text{rg}\bar{A} = r \leq m$

- Sistem liniar și omogen: $AX = O_{m,1}$, $A \in \mathcal{M}_{m,n}$

Prop Un SLO este întotdeauna compatibil.

a) $m = n$ $\to$ $\Delta \neq 0$ SCD $\exists!\,(0,..,0)$

$\qquad\quad\ \ \to$ $\Delta = 0$ SCN $\exists$ și sol nenule.

b) $m \neq n$

b₁) $m > n$ (nr ec > nr nec) $\quad$ $rg\,A = r = n$ SCD.

$\qquad\qquad\qquad\qquad\qquad\qquad\quad$ $rg\,A = r < n$ SCN.

b₂) $m < n$ (nr ec < nr nec) $\quad$ SCN.

Aplicații

Ex1 $\begin{cases} ax + y + z = 1 \\ x + ay + z = 1 \\ x + y + az = a \end{cases}$ $\qquad$ Să se rezolve. Discuție.

$$A = \left(\begin{array}{ccc|c} a & 1 & 1 & 1 \\ 1 & a & 1 & 1 \\ 1 & 1 & a & a \end{array}\right)$$

SOL

$$\Delta = \det(A) = \begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{vmatrix} = \begin{vmatrix} a+2 & a+2 & a+2 \\ 1 & a & 1 \\ 1 & 1 & a \end{vmatrix} = (a+2)\begin{vmatrix} 1 & 0 & 0 \\ 1 & a-1 & 0 \\ 1 & 0 & a-1 \end{vmatrix}$$

$$= (a+2)(a-1)^2$$

I $\Delta \neq 0$, $a \in \mathbb{R} \setminus \{-2, 1\}$ SCD ($rg\,A = rg\,\bar{A} = 3$)

$x = \frac{\Delta_x}{\Delta}$, $\Delta_x = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & 1 & a \end{vmatrix} = 0 \Rightarrow x = 0$

$y = \frac{\Delta_y}{\Delta}$, $\Delta_y = \begin{vmatrix} a & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & a \end{vmatrix} = 0 \Rightarrow y = 0$ $\qquad$ $(0, 0, 1)$ Sol unică.

$z = \frac{\Delta_z}{\Delta}$ $\qquad$ $\Delta_z = \Delta$ $\qquad$ $\Rightarrow z = 1$

II $\Delta = 0$ $\quad$ 1) $a = -2$ $\qquad$ $A = \left(\begin{array}{ccc|c} -2 & 1 & 1 & 1 \\ 1 & -2 & 1 & 1 \\ 1 & 1 & -2 & -2 \end{array}\right)$

$\Delta_p = \begin{vmatrix} -2 & 1 \\ 1 & -2 \end{vmatrix} \neq 0 \Rightarrow rg\,A = 2$

$\Delta_c = \begin{vmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{vmatrix} = \Delta = 0 \Rightarrow rg\,\bar{A} = 2$

x, y = var principale

$z = \alpha$ var secundară

$$\begin{cases} -2x + y = 1 - \alpha \\ x - 2y = 1 - \alpha \quad |\cdot 2 \end{cases} \oplus$$

$$-3y = 3 - 3\alpha$$

$y = \alpha - 1$

$x = 1 - \alpha + 2\alpha - 2 = \alpha - 1$

$(x, y, z) \in \{(\alpha - 1, \alpha - 1, \alpha), \alpha \in \mathbb{R}\}$

SCDN.

2) $a = 1$ $\quad A = \overset{-\,7\,-}{\begin{pmatrix} \boxed{1} & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}} \left| \begin{matrix} 1 \\ 1 \\ 1 \end{matrix} \right.$

$rg A = rg \bar{A} = 1$, x = var principală , $y = \alpha$, $z = \beta$ var secundare.

$x = 1 - \alpha - \beta$

$(x, y, z) \in \{(1-\alpha-\beta, \alpha, \beta), \alpha, \beta \in \mathbb{R}\}$ SCdN

Ex2

$$\begin{cases} x - y = 1 \\ 2x + y = 3 \\ ax + \textcircled{2}y = -1 \end{cases} \qquad A = \left(\begin{array}{cc} \boxed{\begin{matrix} 1 & -1 \\ 2 & 1 \end{matrix}} \\ a \quad & 2 \end{array}\right) \left| \begin{matrix} 1 \\ 3 \\ -1 \end{matrix} \right.$$

$\Delta_p = \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} \neq 0$, $rg A = 2$, $\Delta_c = \begin{vmatrix} \boxed{\begin{matrix} 1 & -1 \\ 2 & 1 \end{matrix}} & \begin{matrix} 1 \\ 3 \end{matrix} \\ \begin{matrix} a & 2 \end{matrix} & -1 \end{vmatrix} =$

$= \begin{vmatrix} 0 & 0 & \textcircled{1} \\ -1 & 4 & 3 \\ a+1 & 1 & -1 \end{vmatrix} = \begin{vmatrix} -1 & 4 \\ a+1 & 1 \end{vmatrix} = -1 - 4a - 4 = -4a - 5.$

1) $\Delta_c \neq 0 \Leftrightarrow a \neq -\frac{5}{4}$

$rg A = 2$, $rg \bar{A} = 3$ SI

2) $\Delta_c = 0 \Leftrightarrow a = -\frac{5}{4}$ SCD. ($\nexists$ var secundare)

$$\begin{cases} x - y = 1 \\ 2x + y = 3 \end{cases}$$
$$3x \quad = 4$$

$x = \frac{4}{3}$

$y = 3 - \frac{8}{3} = \frac{1}{3}$

$(x, y) = \left(\frac{4}{3}, \frac{1}{3}\right)$

Ex3. $\begin{cases} x + 2y - z = 0 \\ x + y + z = 0 \end{cases}$ $\quad A = \left(\boxed{\begin{matrix} 1 & 2 \\ 1 & 1 \end{matrix}} \begin{matrix} -1 \\ 1 \end{matrix} \right) \left| \begin{matrix} 0 \\ 0 \end{matrix} \right.$

$rg A = rg \bar{A} = 2$ $\quad x, y$ = var pr. , $z = \alpha$ var sec SCdN.

$$\begin{cases} x + 2y = \alpha \\ x + y = -\alpha \end{cases} \ominus$$
$$/ \quad y = 2\alpha$$

$x = \alpha - 4\alpha = -3\alpha$

$(x, y, z) \in \{(-3\alpha, 2\alpha, \alpha), \alpha \in \mathbb{R}\}$

Def 2 sisteme sn. sisteme echivalente dacă au aceeași multime de solutii

Teoremă Aplicarea transformărilor elementare asupra liniilor matricei $\bar{A}=(A|B)$ conduce la matrice extinse ale unor sisteme echivalente cu $\circledast$.

Metoda eliminării Gauss-Jordan

Ex
$$\begin{cases} -x+2y-3z=-2 \\ 2x-6y+9z=3 \\ -3x+2y+2z=-3 \end{cases}$$

$$\bar{A}=(A|B)=\left(\begin{array}{ccc|c} -1 & 2 & -3 & -2 \\ 2 & -6 & 9 & 3 \\ -3 & 2 & 2 & -3 \end{array}\right) \sim \left(\begin{array}{ccc|c} \boxed{1} & -2 & 3 & 2 \\ 0 & -2 & 3 & -1 \\ 0 & \textcircled{-4} & 11 & 3 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & -2 & 3 & 2 \\ 0 & 1 & -\frac{3}{2} & \frac{1}{2} \\ 0 & 0 & 5 & 5 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & 3 & 2 \\ 0 & 1 & -\frac{3}{2} & \frac{1}{2} \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$$\begin{cases} z=1 \\ y=2 \\ x=3 \end{cases} \quad \text{SCD}.$$

$(3,2,1)$ sol unica

Ex
$$\begin{cases} 3x+2y+5z+4t=-1 \\ 2x+y+3z+3t=0 \\ x+2y+3z=-3 \end{cases}$$

$$(A|B) = \left(\begin{array}{cccc|c} 3 & 2 & 5 & 4 & -1 \\ 2 & 1 & 3 & 3 & 0 \\ 1 & 2 & 3 & 0 & -3 \end{array}\right) \sim \left(\begin{array}{cccc|c} \boxed{1} & 2 & 3 & 0 & -3 \\ 2 & 1 & 3 & 3 & 0 \\ 3 & 2 & 5 & 4 & -1 \end{array}\right)$$

$$\sim \left(\begin{array}{cccc|c} 1 & 2 & 3 & 0 & -3 \\ 0 & -3 & -3 & 3 & 6 \\ 0 & -4 & -4 & 4 & 8 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 2 & 3 & 0 & -3 \\ 0 & 1 & 1 & -1 & -2 \\ 0 & 1 & 1 & -1 & -2 \end{array}\right)$$

$$\sim \left(\begin{array}{cccc|c} \boxed{1} & 2 & 3 & 0 & -3 \\ 0 & \boxed{1} & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

x, y = var principale , $z = \alpha$, $t = \beta$ var sec SCdN.

$$\begin{cases} x + 2y = -3 - 3\alpha \\ y = -2 - \alpha + \beta. \end{cases} \Rightarrow x = -3 - 3\alpha + 4 + 2\alpha - 2\beta$$

$$x = 1 - \alpha - 2\beta$$

$$(x, y, z, t) \in \{(1-\alpha-2\beta, -2-\alpha+\beta, \alpha, \beta), \alpha, \beta \in \mathbb{R}\}.$$

Spatii vectoriale

$(\mathbb{K}, +, \cdot)$ corp com, $V \neq \emptyset$

V are structură de spațiu vectorial peste corpul $\mathbb{K}$ dacă

$\exists\ + : V \times V \longrightarrow V$ / aî

$\cdot : \mathbb{K} \times V \longrightarrow V$

sunt satisf. axiomele.

1) $(V, +)$ grup abelian
2) $a \cdot (b \cdot x) = (a \cdot b) \cdot x$
3) $(a+b) \cdot x = a \cdot x + b \cdot x$
4) $a \cdot (x+y) = a \cdot x + a \cdot y$
5) $1_{\mathbb{K}} \cdot x = x$. , $\forall\ a, b \in \mathbb{K}$ (scalari)

$x, y \in V$ (vectori)

$(V, +, \cdot)/_{\mathbb{K}}$

OBS

a) $0_{\mathbb{K}} \cdot x = 0_V$

b) $a \cdot 0_V = 0_V$

c) $(a-b) \cdot x = a \cdot x - b \cdot x$

d) $a \cdot (x-y) = a \cdot x - ay$

Exemple de sp vect

1) $(\mathbb{K},+,\cdot)/_{\mathbb{K}}$ $(\mathbb{R},+,\cdot)/_{\mathbb{R}}$, $(\mathbb{C},+,\cdot)/_{\mathbb{C}}$; $(\mathbb{Q},+,\cdot)/_{\mathbb{Q}}$

$(\mathbb{Z}_p,+,\cdot)/\mathbb{Z}_p$, p = nr prim.

2) $\mathbb{K}' \subset \mathbb{K}$ subcorp $\Rightarrow (\mathbb{K},+,\cdot)/_{\mathbb{K}'}$ sp vect.

$(\mathbb{C},+,\cdot)/_{\mathbb{R}}$, $(\mathbb{C},+,\cdot)/\mathbb{Q}$, $(\mathbb{R},+,\cdot)/\mathbb{Q}$.