Geometrie analitică euclidiană

1) $(\mathbb{R}^3, (\mathbb{R}^3, g_0), \varphi)$ - sp afin euclidian cu structura afină canonică

$\varphi : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}^3$

$\varphi(u, v) = v - u$ - struct afină

$\varphi(A, B) \overset{not}{=} \overrightarrow{AB}$

$A(3, -1, 3)$ $\overrightarrow{OA} = 3e_1 - e_2 + 3e_3$

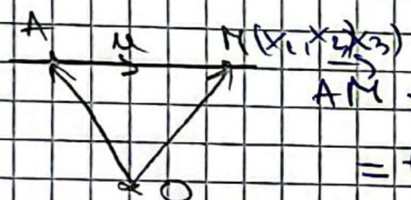
$R = \{O; e_1, e_2, e_3\}$

$B(5, 1, -1)$

$u = (-3, 5, -6)$

a) ec lui $\mathcal{D}$ a.î. $A \in \mathcal{D}$, $V_{\mathcal{D}} = \langle \{u\} \rangle$

OBS: a)



$\overrightarrow{AM} = (x_1 - 3)e_1 + (x_2 + 1)e_2 + (x_3 - 3)e_3$

$= t u = t(-3e_1 + 5e_2 - 6e_3)$

$\mathcal{D}: \frac{x_1 - 3}{-3} = \frac{x_2 + 1}{5} = \frac{x_3 - 3}{-6} = t$ $\begin{cases} x_1 - 3 = -3t \\ x_2 + 1 = 5t \\ x_3 - 3 = -6t \end{cases}$

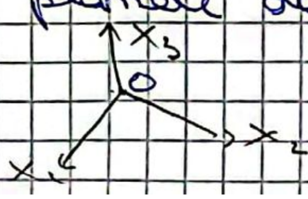
b) ec dreptei AB

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (5 - 3)e_1 + [1 - (-1)]e_2 + (-1 - 3)e_3$

$= (2, 2, -4) = 2(1, 1, -2)$

$AB: \frac{x_1 - 3}{1} = \frac{x_2 + 1}{1} = \frac{x_3 - 3}{-2} = t \Rightarrow \begin{cases} x_1 = t + 3 \\ x_2 = t - 1 \\ x_3 = 3 - 2t \end{cases}$ $t \in \mathbb{R}$

c) Să se afle pct de int. ale dreptei $\mathcal{D}$ cu planele de coordonate



$\mathcal{D} \cap Ox_1x_2 = \{M\}$

$Ox_1x_2 : x_3 = 0$

$\mathcal{D}: x_1 = 3 - 3t; x_2 = 5t - 1; x_3 = 3 - 6t$

$\Rightarrow 3-6t=0$

$M\left(\frac{3}{2}, \frac{3}{2}, 0\right)$

$\mathcal{D} \cap Ox_2x_3 = \{N\}$

$Ox_2x_3 : x_1 = 0$

$3-3t=0 \Rightarrow t=1$

$N(0, 4, -3$

$\mathcal{D} \cap Ox_1x_3 = \{P\}$

$Ox_1x_3 : x_2 = 0$

$-1+5t=0 \Rightarrow t=\frac{1}{5}$

$P\left(\frac{12}{5}, 0, \frac{9}{5}\right)$

$x_3 = 3 - 6t = 3 - \frac{6}{5} = \frac{9}{5}$

2 Scrieți ec. dr. $\mathcal{D}$ a.î. $A(2,-5,3) \in \mathcal{D}$ și $\mathcal{D} \parallel \mathcal{D}'$, unde

$$\mathcal{D}': \begin{cases} 2x_1 - x_2 + 3x_3 + 1 = 0 \\ 5x_1 + 4x_2 - x_3 + 1 = 0 \end{cases}$$

<u>Metoda I</u>: $x_3 = t \Rightarrow \begin{cases} 2x_1 - x_2 = -3t - 1 \;/\cdot 4 \\ 5x_1 + 4x_2 = t - 1 \end{cases} \Rightarrow \begin{cases} 8x_1 - 4x_2 = -12t - 4 \\ 5x_1 + 4x_2 = t - 1 \end{cases}$

$\Rightarrow 13x_1 = -11t - 5 \Rightarrow x_1 = -\frac{5}{13} - \frac{11}{13}t$

$\Rightarrow \frac{-10}{13} - \frac{22}{13}t - x_2 = -3t - 1 \Rightarrow x_2 = \frac{-10}{13} - \frac{22}{13}t + \frac{13}{13}\cdot 3t + \frac{13}{13}$

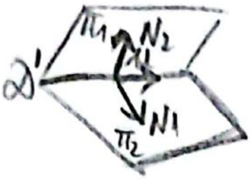
$\Rightarrow x_2 = \frac{3}{13} + \frac{17}{13}t$

$$\mathcal{D}': \begin{cases} x_1 = -\frac{5}{13} - \frac{11}{13}t \\ x_2 = \frac{3}{13} + \frac{17}{13}t \\ x_3 = t \end{cases} \Rightarrow \mathcal{D}': \frac{x_1 + \frac{5}{13}}{\frac{-11}{13}} = \frac{x_2 - \frac{3}{13}}{\frac{17}{13}} = \frac{x_3}{1} = t$$

$$\Rightarrow \mathcal{D}: \frac{x_1 - 2}{-11} = \frac{x_2 + 5}{17} = \frac{x_3 - 3}{13} = t \quad (*)$$

$$\Pi : a x_1 + b x_2 + c x_3 + d = 0$$

$$N_{\Pi} = (a, b, c)$$



Metoda II :

$$\Pi_1 : 2x_1 - x_2 + 3x_3 + 1 = 0$$

$$N_1 = (2, -1, 3)$$

$$\Pi_2 : 5x_1 + 4x_2 - x_3 + 1 = 0$$

$$N_2 = (5, 4, -1)$$

$$\mu_{\mathcal{D}'} = N_1 \times N_2 = \begin{vmatrix} e_1 & e_2 & e_3 \\ 2 & -1 & 3 \\ 5 & 4 & -1 \end{vmatrix} = \begin{vmatrix} -1 & 3 \\ 4 & -1 \end{vmatrix} e_1 - \begin{vmatrix} 2 & 3 \\ 5 & -1 \end{vmatrix} e_2 + \begin{vmatrix} 2 & -1 \\ 5 & 4 \end{vmatrix} e_3 =$$

$$\mu_{\mathcal{D}'} = -11e_1 + 17e_2 + 13e_3 = (-11, 17, 13) \Rightarrow (*)$$

(3) $\Pi: x_1 + x_2 + x_3 = 1$

$M(1, 2, -1)$

$D: \frac{x_1 - 1}{2} = \frac{x_2 - 1}{-1} = \frac{x_3}{3} = t$

a) ec de D' a.î. $M \in D'$ și $D' \perp \Pi$

b) ec planului Π' a.î. $M \in \Pi'$ și $\Pi' \perp D$

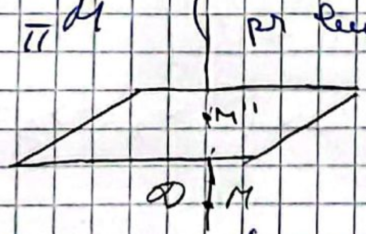
c) — " — Π'' a.î. $M \in \Pi''$ și $D \subset \Pi''$

d) pr D "

e) $pr_{\Pi} M$ (pr lui M pe Π) **OBS**: $\Pi: ax_1 + bx_2 + cx_3 + d = 0$

$N_{\Pi} = (a, b, c)$

a)



$M'' = pr_{\Pi} M$

$N_{\Pi} = u_{D'} = (1, 1, 1)$

$D': \frac{x_1 - 1}{1} = \frac{x_2 - 2}{1} = \frac{x_3 + 1}{1} = t$

$$\begin{cases} x_1 = 1 + t \\ x_2 = 2 + t \\ x_3 = t - 1 \end{cases} \quad t \in \mathbb{R}$$

b)

$M' = pr_D M$

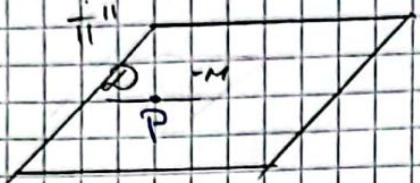
$N_{\pi'} = u_D = (2,-1,3)$

$\left.\begin{array}{l} \pi' = 2x_1 - x_2 + 3x_3 + d = 0 \\ M(1,2,-1) \in \pi' \end{array}\right\} \Rightarrow 2-2-3+d$

$d = 3$

$\pi' : 2x_1 - x_2 + 3x_3 + 3 = 0$

c)



$P \in D \quad P(1,1,0) \quad (t=0)$

$M(1,2,-1)$

$u_D = (2,-1,3)$

$\vec{PM} = (1-1)e_1 + (-1+1)e_2 + (3-0)e_3$ un vector din plan π' calculez

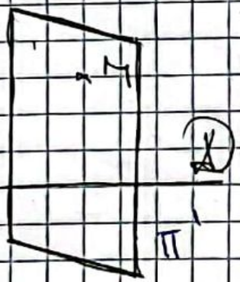
$M \quad \vec{PM}(0,1,-1)$

avem nevoie de 2 vectori

$$\pi'' : \begin{vmatrix} x_1 - 1 & 2 & 0 \\ x_2 - 2 & -1 & 1 \\ x_3 - (-1) & 3 & -1 \end{vmatrix} = 0$$

se rez $\Rightarrow \pi'' : x_1 - x_2 - x_3 = 0$

d)



$$D : \begin{cases} x_1 = 1 + 2t \\ x_2 = 1 - t \\ x_3 = 3t \end{cases}$$

$D \cap \pi' = \{M'\}$

$\pi' : 2x_1 - x_2 + 3x_3 = 0$

$2 + 4t - 1 + t + 9t + 3 = 0$

$14t + 4 = 0 \Rightarrow t = -\frac{4}{14} = \frac{-2}{7}$

$M'\left(\frac{3}{7}, \frac{9}{7}, \frac{-6}{7}\right)$

d) $D \cap$ ''

$t+1+t-2+t-1=1$

$3t=3 \Rightarrow t=1$

$M''(2,3,0)$

• Cum verific dacă două drepte sunt sau nu coplanare?

④ $D_1: \begin{cases} x_1+x_3=0 \\ x_2-x_3-1=0 \end{cases}$; $D_2: \begin{cases} x_2=0 \\ x_3=0 \end{cases}$

a) Dem D_1, D_2 necoplanare

b) E_c ⊥ comune dr D_1, D_2

c) $d(D_1, D_2)$

a) $x_3=t \Rightarrow D_1: \begin{cases} x_1=-t \\ x_2=t+1 \\ x_3=t \end{cases}, t \in \mathbb{R}$

$u_1=(-1,1,1)$

$M_1(0,1,0)\ (t=0)$

$x_1=s \Rightarrow D_2: \begin{cases} x_1=s \\ x_2=0 \\ x_3=0 \end{cases}, s \in \mathbb{R}$

$u_2=(1,0,0)$

$M_2(0,0,0)$ pt $(s=0)$

D_1 și D_2 necoplanare $\Leftrightarrow$ ~~de~~ $\begin{array}{c} u_1 \quad u_2 \quad \vec{M_1M_2} \\ \begin{vmatrix} -1 & 1 & 0 \\ 1 & 0 & -1 \\ 1 & 0 & 0 \end{vmatrix} \end{array} \neq 0$

$\vec{M_1M_2}=(0,-1,0)$

$D_1 \cap D_2 = \{\emptyset\}$

$\begin{cases} -t=s \\ t+1=0 \\ t=0 \end{cases}$

$-t+s=0$
$t+1=0$
$t=0$

$\left(\begin{matrix} -1 & 1 \\ 1 & 0 \\ 1 & 0 \end{matrix}\right) \begin{matrix} 0 \\ -1 \\ 0 \end{matrix}$

egalăm x_1, x_2, x_3

b) $\mathcal{D} \perp$ comună

$\mathcal{D}\cap\mathcal{D}_1=\{P_1\}$, $P_1(-t, t+1, t)$

$\mathcal{D}\cap\mathcal{D}_2=\{P_2\}$, $P_2(s, 0, 0)$

$\overrightarrow{P_1P_2}=(s+t, -t-1, -t)$

$$\left.\begin{array}{l}\langle \overrightarrow{P_1P_2}, u\rangle = 0\\ \langle \overrightarrow{P_1P_2}, u_2\rangle = 0\end{array}\right\} \Rightarrow \begin{array}{l}-s-t-t-1-t=0\\ s+t=0\end{array}$$

$$\Rightarrow s=\frac{1}{2}, \quad t=-\frac{1}{2}$$

$\Rightarrow P_1\left(\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right)$

$P_2\left(\frac{1}{2}, 0, 0\right)$

$\overrightarrow{P_1P_2}=\left(0, -\frac{1}{2}, \frac{1}{2}\right)=\frac{1}{2}(0, -1, 1)$

$\mathcal{D}: \dfrac{x-\frac{1}{2}}{0}=\dfrac{x_2}{-1}=\dfrac{x_3}{1}$ ← am ales P_2

înseamnă că $\begin{cases} x_1-\frac{1}{2}=0\\ \dfrac{x_2}{-1}=\dfrac{x_3}{1}\end{cases}$

c) $d(\mathcal{D}_1, \mathcal{D}_2)=\|\overrightarrow{P_1P_2}\|=\sqrt{\frac{2}{4}}=\frac{\sqrt{2}}{2}$

norma $\quad \sqrt{\text{suma componentelor}}$