CURS 11 AG

Geometrie afină euclidiană

Def Fie $(\mathcal{A}, V/_{\mathbb{R}}, \varphi)$ un triplet, unde

1) $\mathcal{A}$ mult nevidă (mulțime de puncte)
2) $V/_{\mathbb{R}}$ spațiu vectorial (spațiu director)
3) $\varphi : \mathcal{A} \times \mathcal{A} \to V$ structură afină care verifică
   a) $\varphi(A,B) + \varphi(B,C) = \varphi(A,C)$, $\forall A,B,C \in \mathcal{A}$
   b) $\exists\, O \in \mathcal{A}$ aî $\varphi_0 : \mathcal{A} \to V$, $\varphi_0(A) = \varphi(O,A)$, $\forall A \in \mathcal{A}$ este bijectivă.

(De fapt $\exists \Rightarrow \forall$)

Not $\varphi(A,B) = \overrightarrow{AB}$

$(\mathcal{A}, V/_{\mathbb{R}}, \varphi)$ s.n. <u>spațiu afin</u>, $\dim \mathcal{A} = \dim V$

<u>Caz particular</u>

$(\mathbb{R}^n, \mathbb{R}^n/_{\mathbb{R}}, \varphi)$, $\varphi : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$, $\varphi(u,v) = v - u$.
str. afină canonică

Def $(\mathbb{R}^n, \mathbb{R}^n/_{\mathbb{R}}, \varphi)$, $M \subset \mathbb{R}^n$ subm. de puncte

$$Af(M) = \left\{ \sum_{i=1}^{n} a_i P_i,\ \sum_{i=1}^{n} a_i = 1,\ a_i \in \mathbb{R},\ P_i \in M,\ i = \overline{1,n} \right\}$$

<u>combinații afine</u> de puncte din M

Def. $\mathcal{A}' \subseteq \mathbb{R}^n = \mathcal{A}$ s.n. <u>varietate liniară</u> sau <u>subspațiu afin</u>

$\Longleftrightarrow [\forall P_1, P_2 \in \mathcal{A}' \Rightarrow Af\{P_1, P_2\} \subseteq \mathcal{A}'$
i.e. $a_1 P_1 + a_2 P_2 \in \mathcal{A}'$, unde $a_1 + a_2 = 1$, $a_1, a_2 \in \mathbb{R}]$

Prop a) $\mathcal{A}' \subseteq \mathbb{R}^n = \mathcal{A}$ subsp afin $\Rightarrow \exists!\, V' \subseteq \mathbb{R}^n = V$ subsp vect. aî $\forall P' \in \mathcal{A}'$, $V' = \{\overrightarrow{P'P}, \forall P \in \mathcal{A}'\}$

b) $\forall P \in \mathbb{R}^n = \mathcal{A}$, $V' \subseteq \mathbb{R}^n = V$ subsp vect
$\exists!\, \mathcal{A}' \subset \mathbb{R}^n$ subsp afin aî $P \in \mathcal{A}'$ și V' = subsp. director

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Exemplu $(\mathbb{R}^n, \mathbb{R}^n/_{\mathbb{R}}, \varphi)$

$\mathcal{A}' = \{x \in \mathbb{R}^n \mid AX = B\} \subset \mathbb{R}^n$ subsp. afin.

$V' = \{x \in \mathbb{R}^n \mid AX = 0\}$ subsp. vect. director

$\forall\, x, y \in \mathcal{A}' \Rightarrow ax + by \in \mathcal{A}'$

$a + b = 1,\ a, b \in \mathbb{R}$

$$\left.\begin{array}{l} AX = B \\ AY = B \end{array}\right\} \overset{?}{\Longrightarrow} \underbrace{A(aX+bY)}_{a\,AX + b\,AY} = B$$

$$a\,AX + b\,AY = aB + bB = \underbrace{(a+b)}_{=1} B = B$$

Caz particular

$(\mathbb{R}^3, \mathbb{R}^3/_{\mathbb{R}}, \varphi)$ $\mathcal{A}' = \left\{x \in \mathbb{R}^3 \;\middle|\; \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_1 + 2x_2 - x_3 = 1 \end{cases}\right\} \subset \mathbb{R}^3$ ssp af.

$V' = \left\{x \in \mathbb{R}^3 \;\middle|\; \begin{cases} x_1 + x_2 - x_3 = 0 \\ x_1 + 2x_2 - x_3 = 0 \end{cases}\right\}$ sp. vect. director.

Def $\mathcal{A}', \mathcal{A}'' \subset \mathcal{A}$ subsp. afine

$\mathcal{A}' \parallel \mathcal{A}'' \Leftrightarrow V' \subseteq V''$ sau $V'' \subseteq V'$

Exemplu $\mathcal{A}' = \{x \in \mathbb{R}^3 \mid x_1 - 2x_2 - 2x_3 = 2\}$

$\mathcal{A}'' = \{x \in \mathbb{R}^3 \mid x_1 - 2x_2 - 2x_3 = 1\}$ $\mathcal{A}' \parallel \mathcal{A}''$

$V' = V'' = \{x \in \mathbb{R}^3 \mid x_1 - 2x_2 - 2x_3 = 0\}$ (plane afine paralele)

Def $(\mathcal{E}, (E/_{\mathbb{R}}, \langle\cdot,\cdot\rangle), \varphi)$ s.n. spațiu afin euclidian $\Leftrightarrow$ este un sp. afin în care spațiul director este un sp. vect. euclidian.

Def $(\mathcal{E}, (E, \langle\cdot,\cdot\rangle), \varphi)$ sp. afin euclidian

$\mathcal{E}_1, \mathcal{E}_2 \subseteq \mathcal{E}$ subsp. afine

a) $\mathcal{E}_1, \mathcal{E}_2$ sunt subspații perpendiculare $\Leftrightarrow E_1 \perp E_2$ (sp. vect. directoare)

b) $\mathcal{E}_1, \mathcal{E}_2$ sunt subsp. normale $\Leftrightarrow E_2 = E_1^\perp$

i.e. $E = E_1 \oplus \underbrace{E_1^\perp}_{E_2}$.

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Geometrie analitică euclidiană

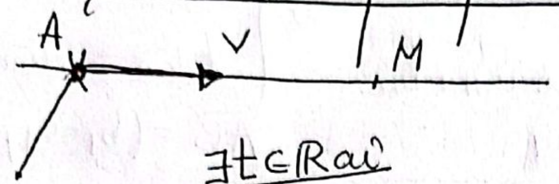
Ecuații ale varietăților liniare

$(\mathbb{R}^n, (\mathbb{R}^n, g_0), \varphi)$

$\mathcal{R} = \{O; e_1, .., e_n\}$ reper cartezian ; $O \in \mathbb{R}^n$, $\{e_1, .., e_n\}$ reper ortonormat

① Ecuația unei drepte afine $\mathcal{D}$

a)



$\mathcal{D}$. $\begin{cases} A \in \mathcal{D} \\ V_{\mathcal{D}} = \langle \{v\} \rangle, v \neq 0 \end{cases}$

$M \in \mathcal{D} \iff \exists t \in \mathbb{R}$ a.î. $\overrightarrow{AM} = t v$

$A, M \in \mathcal{D} \Rightarrow \overrightarrow{AM} \in V_{\mathcal{D}}$.

$\overrightarrow{OA} = \sum_{i=1}^{n} a_i e_i$, $A(a_1, .., a_n)$

$\overrightarrow{OM} = \sum_{i=1}^{n} x_i e_i$, $M(x_1, .., x_n)$

$v = \sum_{i=1}^{n} v_i e_i$

$\overrightarrow{AM} = \overrightarrow{OM} - \overrightarrow{OA} = \sum_{i=1}^{n} (x_i - a_i) e_i$

$\sum_{i=1}^{n} (x_i - a_i) e_i = t \sum_{i=1}^{n} v_i e_i \Rightarrow$

$\mathcal{D}: \; x_i - a_i = t v_i, \; \forall i = \overline{1,n}$, $t \in \mathbb{R}$ ec. parametrice.

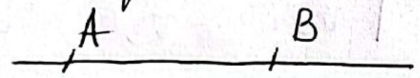
$\mathcal{D}: \; \dfrac{x_1 - a_1}{v_1} = \dots = \dfrac{x_n - a_n}{v_n} = t$

Convenție: Dacă $\exists i_0 \in \{1, .., n\}$ a.î. $v_{i_0} = 0$, at $x_{i_0} - a_{i_0} = 0$

Exemplu: $n = 3$, $A(1,2,3)$, $v = (3,1,1)$

Ec dreptei $\mathcal{D}$: $\dfrac{x_1 - 1}{3} = \dfrac{x_2 - 2}{1} = \dfrac{x_3 - 3}{1} = t \iff \begin{cases} x_1 = 1 + 3t \\ x_2 = 2 + t \\ x_3 = 3 + t, t \in \mathbb{R} \end{cases}$

b)



$\mathcal{D}$, $A, B \in \mathcal{D}$

$v = \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \sum_{i=1}^{n} (b_i - a_i) e_i$, $A(a_1, .., a_n)$, $B(b_1, .., b_n)$

$\mathcal{D}: \; x_i - a_i = t(b_i - a_i), \; \forall i = \overline{1,n}$

$\mathcal{D}: \; \dfrac{x_1 - a_1}{b_1 - a_1} = \dots = \dfrac{x_n - a_n}{b_n - a_n}$.

Convenție: Dacă $\exists i_0$ a.î. $b_{i_0} - a_{i_0} = 0$, at $x_{i_0} - a_{i_0} = 0$.

Exemplu: $n = 3$, $A(1,1,3)$, $B(2,5,6)$, $\overrightarrow{AB} = (1,4,3) = v$

$AB: \; \dfrac{x_1 - 1}{1} = \dfrac{x_2 - 1}{4} = \dfrac{x_3 - 3}{3} = t \Rightarrow \begin{cases} x_1 = 1 + t \\ x_2 = 1 + 4t \\ x_3 = 3 + 3t, t \in \mathbb{R} \end{cases}$

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Poziția relativă a 2 drepte afine

$\mathcal{D}_1 : \ x_i - a_i = t v_i, \ \forall i = \overline{1,n}$ $\quad A(a_1, \dots, a_n) \in \mathcal{D}_1$

$\mathcal{D}_2 : \ x_i - b_i = s v_i', \ \forall i = \overline{1,n}$ $\quad B(b_1, \dots, b_n) \in \mathcal{D}_2$

$$\mathcal{D}_1 \cap \mathcal{D}_2 : \begin{cases} t v_1 + a_1 = s v_1' + b_1 \\ \vdots \\ t v_n + a_n = s v_n' + b_n \end{cases} \Rightarrow \begin{cases} t v_1 - s v_1' = b_1 - a_1 \\ \vdots \\ t v_n - s v_n' = b_n - a_n \end{cases}$$

$\overrightarrow{AB} = (b_1 - a_1, \dots, b_n - a_n)$

$$C = \left(\begin{matrix} v_1 & -v_1' \\ \vdots & \vdots \\ v_n & -v_n' \end{matrix}\right) \left| \begin{matrix} b_1 - a_1 \\ \vdots \\ b_n - a_n \end{matrix} \right.$$

① $\operatorname{rg} C = \operatorname{rg} \overline{C} = 2 \Rightarrow \mathcal{D}_1 \cap \mathcal{D}_2 = \{M\}$

② $\operatorname{rg} C = \operatorname{rg} \overline{C} = 1 \Rightarrow \mathcal{D}_1 = \mathcal{D}_2$

③ $\operatorname{rg} C = 1, \ \operatorname{rg} \overline{C} = 2 \Rightarrow$ ~~$\mathcal{D}_1, \mathcal{D}_2$ necoplanare~~ $\mathcal{D}_1 \parallel \mathcal{D}_2$

④ $(\operatorname{rg} C = 2, \operatorname{rg} \overline{C} = 3)$ ~~$\operatorname{rg} C = 1$~~ $\Delta_c \neq 0$ necoplanare.

$\boxed{n=3}$
$$C = \left(\begin{matrix} v_1 & -v_1' \\ v_2 & -v_2' \\ v_3 & -v_3' \end{matrix}\right) \left| \begin{matrix} b_1 - a_1 \\ b_2 - a_2 \\ b_3 - a_3 \end{matrix} \right.$$

$\mathcal{D}_1, \mathcal{D}_2$ necoplanare. $\Delta_c = \begin{vmatrix} v_1 & -v_1' & b_1 - a_1 \\ v_2 & -v_2' & b_2 - a_2 \\ v_3 & -v_3' & b_3 - a_3 \end{vmatrix} \neq 0$

Ex 1) $A(1,1,3)$, $B(2,5,6)$ $\quad \mathcal{D} : \dfrac{x_1}{2} = \dfrac{x_2}{8} = \dfrac{x_3 - 1}{6}$

Precizați poziția dreptelor AB și $\mathcal{D}$.

$\overrightarrow{AB} = (1,4,3)$ $\quad u_{\mathcal{D}} = (2,8,6) = 2(1,4,3)$

$\dfrac{1}{2} \neq \dfrac{1}{8} \neq \dfrac{2}{6}$ $\quad A \notin \mathcal{D}$ $\quad AB \parallel \mathcal{D}$.

2) $\mathcal{D}_1 : \dfrac{x_1 - 1}{1} = \dfrac{x_2 - 2}{2} = \dfrac{x_3}{-1} = t \Rightarrow \begin{cases} x_1 = 1 + t \\ x_2 = 2 + 2t \\ x_3 = -t \end{cases}$

$\mathcal{D}_2 : \dfrac{x_1 - 3}{1} = \dfrac{x_2 - 6}{1} = \dfrac{x_3 + 2}{3} = s$ $\quad \begin{cases} x_1 = 3 + s \\ x_2 = 6 + s \\ x_3 = -2 + 3s. \end{cases}$

$$\mathcal{D}_1 \cap \mathcal{D}_2 : \begin{cases} 1+t = 3+s \\ 2+2t = 6+s \\ -t = -2+3s \end{cases} \Rightarrow \begin{cases} t-s = 2 \\ 2t-s = 4 \\ -t-3s = -2 \end{cases} \quad C = \left(\begin{array}{cc|c} 1 & -1 & 2 \\ 2 & -1 & 4 \\ -1 & -3 & -2 \end{array} \right)$$

$u_1 = (1,2,-1)$, $A(1,2,0)$ $\qquad \overrightarrow{AB} = (2,4,-2)$

$u_2 = (1,1,3)$, $B(3,6,-2)$

$$\Delta_p = \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} \neq 0, \quad \Delta_c = \begin{vmatrix} 1 & -1 & 2 \\ 2 & -1 & 4 \\ -1 & -3 & -2 \end{vmatrix} = 0$$

$rg\, C = rg\, \bar{C} = 2$ SCD $\qquad \mathcal{D}_1 \cap \mathcal{D}_2 = \{P\}$ (concurente)

$$\begin{cases} t-s = 2 \\ 2t-s = 4 \end{cases} \ominus$$
$$t \quad / = 2$$

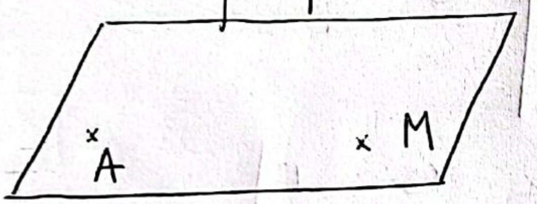
$t = 2 \Rightarrow P(1+2, 2+2\cdot 2, -2)$

$P(3,6,-2)$

$P = B$

② Ecuatia unui plan afin.
(subsp. afin 2-dimensional)

a)



$A \in \pi$, $V_\pi = \langle \{u, v\} \rangle$

$\{u,v\}$ SLI

π

$\forall M \in \pi \Rightarrow \exists\, t,s \in \mathbb{R}$ ai $\overrightarrow{AM} = tu + sv$ $\qquad u = \sum_{i=1}^{n} u_i e_i$

$\overrightarrow{AM} = \overrightarrow{OM} - \overrightarrow{OA}$ $\qquad v = \sum_{i=1}^{n} v_i e_i$

$= \sum_{i=1}^{n} (x_i - a_i) e_i$

$\pi : x_i - a_i = t u_i + s v_i, \forall i = \overline{1,n}$

Exemplu $n = 3$ $\quad A(1,-1,2)$, $\quad u = (2,3,1)$, $v = (4,1,3)$

$$\pi \begin{cases} x_1 - 1 = 2t + 4s \\ x_2 + 1 = 3t + s \\ x_3 - 2 = t + 3s \end{cases} \Leftrightarrow \pi : \begin{vmatrix} x_1 - 1 & 2 & 4 \\ x_2 + 1 & 3 & 1 \\ x_3 - 2 & 1 & 3 \end{vmatrix} = 0$$

$\pi : (x_1 - 1) \cdot 8 - (x_2+1) 2 + (x_3 - 2)(-10) = 0 \quad |:2$

$\pi : 4(x_1 - 1) - (x_2 + 1) - 5(x_3 - 2) = 0 \Rightarrow 4x_1 - x_2 - 5x_3 + 5 = 0$

ec. generală.

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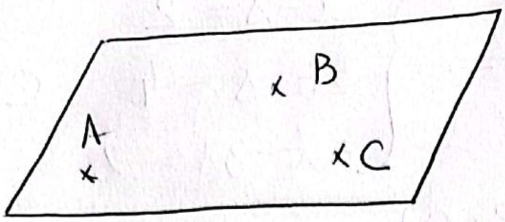
b)



$A, B, C \in \pi$ (necoliniare)

$\{\overrightarrow{AB}, \overrightarrow{AC}\}$ SLI

$V_\pi = \langle \{\overrightarrow{AB}, \overrightarrow{AC}\} \rangle$

$\pi: x_i - a_i = t(b_i - a_i) + s(c_i - a_i) \quad \forall i = \overline{1,n}$ $\qquad A(a_1,..,a_n), B(b_1,..,b_n), C(c_1,..,c_n)$

Exemplu $n = 3 \quad A(1,1,1), B(-1,1,1), C(2,0,0)$.

$\overrightarrow{AB} = (-2,0,0)$

$\overrightarrow{AC} = (1,-1,-1)$

$$\pi: \begin{vmatrix} x_1 - 1 & -2 & 1 \\ x_2 - 1 & 0 & -1 \\ x_3 - 1 & 0 & -1 \end{vmatrix} = 0$$

$$2 \begin{vmatrix} x_2 - 1 & -1 \\ x_3 - 1 & -1 \end{vmatrix} = 0 \Rightarrow -x_2 + 1 + x_3 - 1 = 0 \Rightarrow -x_2 + x_3 = 0.$$

$$\pi: \begin{vmatrix} x_1 & x_2 & x_3 & 1 \\ 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ 2 & 0 & 0 & 1 \end{vmatrix} = 0$$

π: ~~$x - x_1^A$, $x - x_2^A$~~

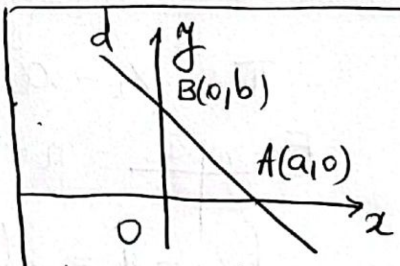
$$\pi: \begin{vmatrix} x_1 & x_2 & x_3 & 1 \\ x_1^A & x_2^A & x_3^A & 1 \\ x_1^B & x_2^B & x_3^B & 1 \\ x_1^C & x_2^C & x_3^C & 1 \end{vmatrix} = 0$$

$\boxed{\text{OBS}}$ $n = 3$

1) $\pi: ax_1 + bx_2 + cx_3 + d = 0, \quad a^2 + b^2 + c^2 > 0$

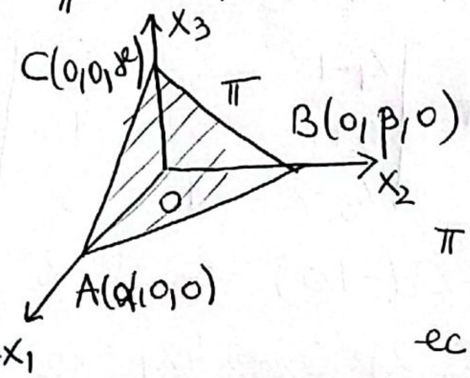
ec. generală

$N_\pi = (a, b, c)$ normala a planului.



$d: \frac{x}{a} + \frac{y}{b} = 1$

2)



$$\pi: \frac{x_1}{\alpha} + \frac{x_2}{\beta} + \frac{x_3}{\gamma} = 1$$

ec prin tăieturi a planului π.

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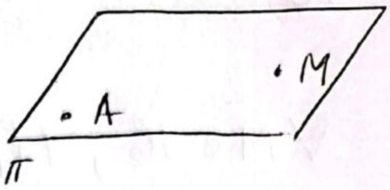
3) π $\quad A(a_1, a_2, a_3) \in \pi \quad , V_\pi = \langle\{u, v\}\rangle$

$$N = u \times v = \begin{vmatrix} e_1 & e_2 & e_3 \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$



$M \in \pi \Rightarrow \langle \overrightarrow{AM}, N \rangle = 0$

Exemplu $\quad A(1, 2, 3) \in \pi \quad u = (0, 1, 3)$

$v = (4, 5, 0)$

$\pi = ?$

$$N = u \times v = \begin{vmatrix} e_1 & e_2 & e_3 \\ 0 & 1 & 3 \\ 4 & 5 & 0 \end{vmatrix} = e_1(-15) - e_2(-12) + e_3(-4)$$

$$= (-15, 12, -4)$$

$\pi : -15x_1 + 12x_2 - 4x_3 + d = 0$

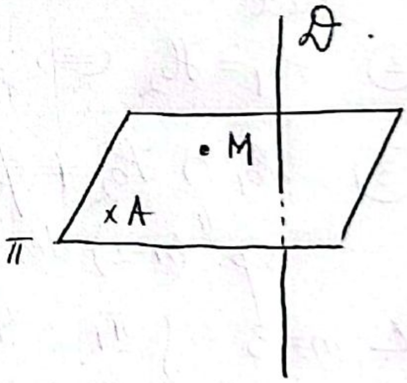
$A(1, 2, 3) \in \pi \Rightarrow -15 + 24 - 12 + d = 0 \Rightarrow d = 15 - 12 = 3$

SAU $\langle \overrightarrow{AM}, N \rangle = 0 \Rightarrow \langle (x_1 - 1, x_2 - 2, x_3 - 3), (-15, 12, -4) \rangle = 0$

$-15(x_1 - 1) + 12(x_2 - 2) - 4(x_3 - 3) = 0$

$-15x_1 + 12x_2 - 4x_3 + \underbrace{15 - 24 + 12}_{3} = 0$

c) $A(a_1, .., a_n) \in \pi \quad , \pi \perp \mathcal{D}$



$\mathcal{D} : \dfrac{x_1 - x_1^0}{u_1} = \dots = \dfrac{x_n - x_n^0}{u_n}$

$N_\pi = u_\mathcal{D}$

$\pi : \langle \overrightarrow{AM}, u_\mathcal{D} \rangle = 0$

$\pi : (x_1 - a_1)u_1 + \dots + (x_n - a_n)u_n = 0$

Exemplu $\quad A(1, 0, 3) \in \pi \quad , \mathcal{D} \perp \pi, \ \mathcal{D} : \dfrac{x_1 - 1}{2} = \dfrac{x_2 - 1}{1} = \dfrac{x_3}{2}$

$N_\pi = u_\mathcal{D} = (2, 1, 2)$

$\pi : 2x_1 + x_2 + 2x_3 + d = 0$

$A(1, 0, 3) \in \pi \Rightarrow 2 + 0 + 6 + d = 0 \Rightarrow d = -8$

SAU $\langle (x_1 - 1, x_2 - 0, x_3 - 3), (2, 1, 2) \rangle = 0 \Rightarrow 2(x_1 - 1) + 1(x_2 - 0) + 2(x_3 - 3) = 0$

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① Fie $f:\mathbb{R}^3\to\mathbb{R}^3$, $f(x_1,x_2,x_3)=(-3x_1+2x_2,-5x_1+4x \ldots -x_3)$

③ Ec unui hiperplan afin / ssp $(n-1)$-dimensional)

$\mathcal{H}$: $A(a_1,\ldots,a_n)\in\mathcal{H}$, $V_{\mathcal{H}}=\langle\{u_1,\ldots,u_{n-1}\}\rangle$

$\{u_1,\ldots,u_{n-1}\}$ SLI

$\forall M\in\mathcal{H}$, $\exists t_1,\ldots,t_{n-1}\in\mathbb{R}$ a.î. $\overrightarrow{AM}=t_1u_1+\ldots+t_{n-1}u_{n-1}$.

$\mathcal{H}: x_i-a_i=t_1u_1^i+\ldots+t_{n-1}u_{n-1}^i,\ i=\overline{1,n}$

$$\mathcal{H}:\begin{vmatrix} x_1-a_1 & u_1^1 & \ldots & u_{n-1}^1\\ \vdots & \vdots & & \vdots\\ x_n-a_n & u_1^n & & u_{n-1}^n\end{vmatrix}=0$$

$\mathcal{H}: A_1x_1+\ldots+A_nx_n+A_0=0$, $A_1^2+\ldots+A_n^2>0$

$N_{\mathcal{H}}=(A_1,\ldots,A_n)$ normală la $\mathcal{H}$.

Poz. relativă a 2 hiperplane

$\mathcal{H}_1: A_1x_1+\ldots+A_nx_n+A_0=0$

$\mathcal{H}_2: A_1'x_1+\ldots+A_n'x_n+A_0'=0$

① $\mathcal{H}_1\parallel\mathcal{H}_2$ dist $\iff \dfrac{A_1}{A_1'}=\ldots=\dfrac{A_n}{A_n'}\neq\dfrac{A_0}{A_0'}$

② $\mathcal{H}_1=\mathcal{H}_2 \iff \dfrac{A_1}{A_1'}=\ldots=\dfrac{A_n}{A_n'}=\dfrac{A_0}{A_0'}$

③ $\mathcal{H}_1\cap\mathcal{H}_2\neq\emptyset$

$\quad$ '' ssp. $(n-2)$ dim.

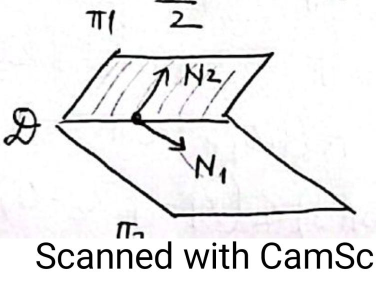
EX $n=3$ $\begin{cases}\pi_1: x_1+x_2+x_3=1\\ \pi_2: 2x_1-x_3=0.\end{cases}$ $\left(\begin{array}{ccc|c}1&1&1&1\\2&0&-1&0\end{array}\right)$

$\pi_1\cap\pi_2=?$

$x_3=t\Rightarrow \begin{cases}x_1+x_2=1-t\\ 2x_1=t\end{cases}$ $\Rightarrow x_2=1-t-\dfrac{t}{2}=1-\dfrac{3t}{2}$, $\Rightarrow x_1=\dfrac{t}{2}$

$\mathcal{D}: \begin{cases}x_1=\frac{1}{2}\cdot t\\ x_2=1-\frac{3}{2}t\\ x_3=t\end{cases}$ $\quad\mathcal{D}: \dfrac{x_1}{\frac{1}{2}}=\dfrac{x_2-1}{-\frac{3}{2}}=\dfrac{x_3}{1}$

$\boxed{\text{OBS}}$ $u_{\mathcal{D}}=N_1\times N_2=\begin{vmatrix}e_1&e_2&e_3\\1&1&1\\2&0&-1\end{vmatrix}$ $=(-1,3,-2)$



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Intersectia unei drepte cu un hiperplan:

$$\mathcal{D}: \frac{x_1-a_1}{u_1} = \dots = \frac{x_n-a_n}{u_n} = t \Rightarrow \begin{cases} x_1 = a_1 + tu_1 \\ \vdots \\ x_n = a_n + tu_n \end{cases}$$

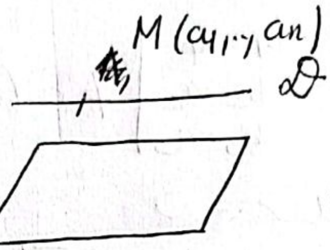
$$\mathcal{H}: A_1x_1 + \dots + A_nx_n + A_0 = 0$$

$$\mathcal{D}\cap\mathcal{H}: (a_1+tu_1)A_1 + \dots + (a_n+tu_n)A_n + A_0 = 0$$

$$t(u_1A_1 + \dots + u_nA_n) + a_1A_1 + \dots + a_nA_n + A_0 = 0$$

$$u = (u_1, \dots, u_n) = u_{\mathcal{D}}$$

$$N = (A_1, \dots, A_n) = N_{\mathcal{H}}$$



1) $\langle u, N\rangle = 0$

$u_1A_1 + \dots + u_nA_n = 0$

$A_1a_1 + \dots + A_na_n + A_0 \neq 0$ $\mathcal{H}$

$\mathcal{D}\parallel\mathcal{H}$

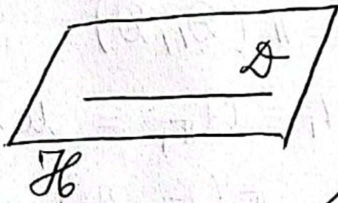
$\mathcal{D}\not\subset\mathcal{H}$

$M\notin\mathcal{H}$.

$\mathcal{D}\cap\mathcal{H} = \emptyset$

2) $u_1A_1 + \dots + u_nA_n = 0$

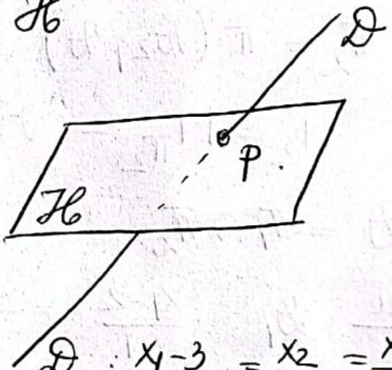
$A_1a_1 + \dots + A_na_n + A_0 = 0$



$\mathcal{D}\subset\mathcal{H}$.

$\mathcal{D}\cap\mathcal{H}$ = o inf pcte.

3) $\mathcal{D}\cap\mathcal{H} = \{P\}$!



$$t = -\frac{a_1A_1 + \dots + a_nA_n + A_0}{u_1A_1 + \dots + u_nA_n}.$$

Exemplu

$\pi: x_1 + x_2 + x_3 - 2 = 0$, $\mathcal{D}: \frac{x_1-3}{1} = \frac{x_2}{3} = \frac{x_3}{1} = t$

$\mathcal{D}\cap\pi$

$$\mathcal{D}: \begin{cases} x_1 = 3+t \\ x_2 = 3t \\ x_3 = t \end{cases} \qquad 3+t+3t+t-2=0 \Rightarrow$$

$$5t = -1 \Rightarrow t = -\frac{1}{5}$$

$$P\left(3-\frac{1}{5}, 3\cdot\left(-\frac{1}{5}\right), -\frac{1}{5}\right) \Rightarrow P\left(\frac{14}{5}, -\frac{3}{5}, -\frac{1}{5}\right)$$

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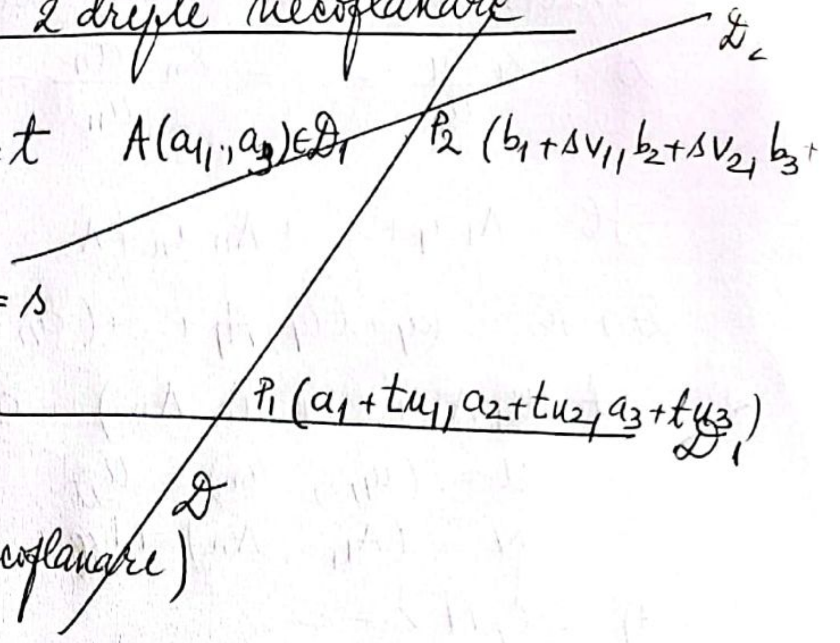
Perpendiculara comună a 2 drepte necoplanare

$n = 3$

$\mathcal{D}_1 : \dfrac{x_1 - a_1}{u_1} = \dots = \dfrac{x_3 - a_3}{u_3} = t \qquad A(a_1, .., a_3) \in \mathcal{D}_1$

$\mathcal{D}_2 : \dfrac{x_1 - b_1}{v_1} = \dots = \dfrac{x_3 - b_3}{v_3} = s$

$B(b_1, .., b_3) \in \mathcal{D}_2$



$$\begin{vmatrix} u_1 & v_1 & b_1 - a_1 \\ u_2 & v_2 & b_2 - a_2 \\ u_3 & v_3 & b_3 - a_3 \end{vmatrix} \neq 0 \text{ (necoplanare)}$$

$$\begin{cases} \langle \overrightarrow{P_1P_2}, u \rangle = 0 \\ \langle \overrightarrow{P_1P_2}, v \rangle = 0 \end{cases} \Rightarrow s, t \Rightarrow P_1, P_2$$

OBS

$\pi_1 = \pi(\mathcal{D}_1, \mathcal{D}) \qquad N = u \times v = u_{\mathcal{D}}$.

$N_1 = N_{\pi_1} = u \times N, \quad A \in \mathcal{D}_1 \Rightarrow A \in \pi_1$

$\pi_2 = \pi(\mathcal{D}_2, \mathcal{D})$

$N_2 = N_{\pi_2} = v \times N, \quad B \in \mathcal{D}_2, \ B \in \pi_2$

$\mathcal{D} = \pi_1 \cap \pi_2$

Exemplu

$\mathcal{D}_1 : \dfrac{x_1 - 2}{1} = \dfrac{x_2}{2} = \dfrac{x_3 - 3}{1} = t \Rightarrow \begin{cases} x_1 = 2 + t \\ x_2 = 2t \\ x_3 = 3 + t \end{cases} \quad u = (1, 2, 1), \ A(2, 0, 3)$

$\mathcal{D}_2 : \dfrac{x_1 - 1}{2} = \dfrac{x_2 - 3}{1} = \dfrac{x_3}{1} = s \Rightarrow \begin{cases} x_1 = 1 + 2s \\ x_2 = 3 + s \\ x_3 = s \end{cases} \quad v = (2, 1, 1), \ B(1, 3, 0)$

$\overrightarrow{AB} = (-1, 3, -3)$

$$\begin{vmatrix} 1 & 2 & -1 \\ 2 & 1 & 3 \\ 1 & 1 & -3 \end{vmatrix} \neq 0 \text{ (necoplanare)}$$

$P_1(2 + t, 2t, 3 + t) \in \mathcal{D}_1, \quad P_2(1 + 2s, 3 + s, s) \in \mathcal{D}_2$

$\overrightarrow{P_1P_2} = (2s - t - 1, \ s - 2t + 3, \ s - t - 3)$

$$\begin{cases} \langle \overrightarrow{P_1P_2}, u \rangle = 0 \\ \langle \overrightarrow{P_1P_2}, v \rangle = 0 \end{cases} \Rightarrow \begin{cases} 2s - t - 1 + 2(s - 2t + 3) + s - t - 3 = 0 \\ 2(2s - t - 1) + 1(s - 2t + 3) + 1(s - t - 3) = 0 \end{cases}$$

$\Rightarrow t = s = 2 \qquad P_1(4, 4, 5), \ P_2(5, 5, 2), \ \overrightarrow{P_1P_2} = (1, 1, -3)$

$P_1P_2 : \dfrac{x_1 - 4}{1} = \dfrac{x_2 - 4}{1} = \dfrac{x_3 - 5}{-3}$; $\text{dist}(d_1, d_2) = \|\overrightarrow{P_1P_2}\| = \sqrt{11}$

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