CONSTRUCTIONS

OVERVIEW

In this lesson, students will learn the techniques for constructing different figures using a variety of tools such as a compass, a straight-edge, paper folding, and an online construction program. Through these constructions, they will gain an understanding of the relationship between geometric figures including equilateral triangles, squares, rhombuses, parallelograms, and inscribed hexagons, particularly in regards to distance and symmetry. Students will be able to not only perform these constructions, but identify and explain each step. The lesson begins with an introduction to construction which is intended to both familiarize the students with working with compasses and provide them with a few fundamentals of construction including angle and perpendicular bisectors as well as how to copy an angle and construct a line that is perpendicular to a point not on the line. From there, students will explore different methods of constructing polygons and through these discoveries will better understand geometric relationships. Each activity has a set of questions associated with it. The instructor may choose to ask the questions in any particular order, either during or after completion of the activity. The purpose of the questions is to not only ensure that students have a complete understanding of the material but also to facilitate critical, higher-order thinking and encourage them to connect the concepts they are learning now and those they have learned in the past or will learn in the future. In order to prepare students for future lessons such as transformations and proofs as well as increase their understanding of these relationships, students will examine constructions in terms of both symmetry and distance. This deeper understanding will additionally allow them to perform constructions of more complex figures not covered in the lesson.

On the second instructional day of this lesson, students will be assigned a project that must be completed by the end of the lesson. In this project, students will be required to combine multiple constructions in order to create a single design. The only restrictions for the project are the inclusion of specific constructions, meaning that students will have the opportunity to showcase their creativity and explore different geometric combinations. Students will be given a variety of resources for completing the project including an online construction software and an instructional foldable. This project must be completed individually and by hand, though students may use any medium and tools they desire so long as their constructions are precise and accurately identified.

After learning each different method of construction, students will collaboratively create a “master list” detailing the steps needed in order to perform that specific construction. Because there are multiple ways to perform certain constructions, there may be multiple lists for each. Students should be exposed to each different construction strategy and should understand and be able to explain why the different strategies yield the same results, though they will only be required to perform one method for each construction. At the end of the unit, students will compile their “master lists” into a foldable which will include diagrams of each process as well as relevant vocabulary. This foldable will be constructed during the lesson review day and will be completed as homework. It is intended to serve as a reference for completing the lesson project as well as for preparing for the unit exam.

The final chapter of this lesson will introduce students to writing construction proofs. Throughout the lesson, they will have been required to provide justifications and explanations for constructions, so they will have already been exposed to the concepts of proofs unknowingly. The focus of this lesson is geared towards showing students how to format their proofs as well as more logically justify certain constructions. Students will be exposed to a variety of different formats for writing proofs including tables and sentence length proofs using math language. The emphasis that was placed on precision of vocabulary throughout the lesson should aid in preparing students to write accurate proofs, however, it is important that the instructor emphasize these definitions and encourage students to be as precise as possible. Students will practice not only writing their own proofs, but examining and interpreting the proofs of other students. This should help them in identifying necessary elements of proofs as well as common mistakes. The project that will be due at the conclusion of this lesson will not directly assess students on proofs, though they will be required to justify and explain certain constructions in their homeworks and tickets out the foor. Students will instead be assessed on proofs at the conclusion of the following lesson in which they will learn about proving congruence and triangle properties.

LESSON CHAPTERS

Chapter 1: Intro to Constructions (two days)
    Project assigned on day 2
Chapter 2: Construct an equilateral triangle (one day)
Chapter 3: Construct a rhombus and a square (one day)
Chapter 4: Construct an inscribed hexagon (one day)
Chapter 5: Introduction to construction proofs (one day)
Lesson Review (one day)

Assessment → project on chapters 1-4

STANDARDS

The standards will be covered in sections referred to as learning objectives throughout the lesson. Each standard is broken up into several learning objectives to not only provide students with a clearer picture of the concepts they need to take away from the class, but also to aid the instructor in the evaluation of student understanding (see assessments below). Precise definitions of vocabulary **(G-CO.A.1)** will be covered throughout the lesson, though they will be reinforced and focused on when students begin to write proofs. Students will have the opportunity to discover formal geometric constructions **(G-CO.D.12)** and utilise them to construct different polygons **(G-CO.D.13).** During constructions and discoveries of constructional methods, students will be asked a series of questions designed to challenge their thinking and introduce them to future concepts such as transformations and reflections **(NC.M2.G-CO.2)** as well as introduce them to theorems they will later be required to prove formally in the following lesson and give them practice explaining and justifying their reasoning **(G-CO.C.9/NC.M2.G-CO.9 and G-CO.C.11)**.

G-CO.A.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
G-CO.D.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
G-CO.D.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
G-CO.C.9/NC.M2.G-CO.9 Prove theorems about lines and angles. Theorems include:

- vertical angles are congruent;
- when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent;
- points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

G-CO.C.11 Prove theorems about parallelograms. Theorems include:

- opposite sides are congruent,

- opposite angles are congruent,
- the diagonals of a parallelogram bisect each other, and conversely,
- rectangles are parallelograms with congruent diagonals.

NC.M2.G-CO.4 Verify experimentally properties of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
NC.M2.G-CO.2 Experiment with transformations in the plane.

- Represent transformations in the plane.
- Compare rigid motions that preserve distance and angle measure (translations, reflections, rotations) to transformations that do not preserve both distance and angle measure (e.g. stretches, dilations).
- Understand that rigid motions produce congruent figures while dilations produce similar figures.

MATERIALS/TECHNOLOGY

For this lesson, students will need to have a compass and straightedge. Materials will be provided in class, but students are encouraged to purchase their own in order to complete homework problems. Students who can not afford the materials will be allowed to complete homework during lunch/office hours as well as rent materials from class. Students who wish to rent must sign and have their parent/guardian sign a contract to ensure the return or replacement of all materials. Additionally, students will be shown alternate ways to create a compass such as using a string that is fixed at one end, so students who do not have access to a formal compass may utilize comparable tools such as this.

Computers will be used in class on the second day of chapter one and may be used throughout the unit and to aid students with the unit project. Using this program: https://sciencevsmagic.net/geo/, students will be able to explore constructions in a quick and easy way, without having to perform them by hand. The instructor should be well versed in the use of the program and ready to answer any questions the student may have. This program is meant to be a supplementary tool to actively engage and challenge students with complex constructions, however, it is important that students be able to explain and perform all constructions without use of the program. Students may not use computers during class time unless otherwise instructed in order to ensure active, hands-on learning and minimize distraction, however, students will have time in class to use the construction software on school-provided computers, though they may continue to utilize it as a tool throughout the lesson. For students who do not have access to a computer outside of school, computers will be available in the classroom and accessible to students during technology explorations, lunch, and office hours. All students will have used the construction program at least once during day two of the introduction chapter and may ask to use it at the end of class (if time permits). If students are having a difficult time grasping certain concepts, it may be helpful for the instructor to demonstrate that concept using the software or to challenge students to perform it themselves. The program will provide an alternate representation of certain constructions and can help students plan their unit project design, though computer access is not necessary for project completion.

ASSESSMENTS

Students will be assessed both informally and formally throughout the course of the lesson. Informal assessments will be conducted through in class discussions, questions from the instructor, answers on daily tickets out the door, and participation in in-class activities. Each day, students must individually complete a “Ticket Out The Door” that consists of one or two problems that address key concepts from that day’s lesson. Formal assessments will be carried out in two separate parts. The first portion consists of assigned homeworks and performance on projects and will be based on proficiency according to learning objectives corresponding to the standards. Each standard will be broken up into learning objectives, and each question/problem/activity will be focused on one of these learning objectives. Because of this method of standards-based grading, students will be scored on their understanding of each standard, rather than receiving one grade for an assessment that encompasses multiple concepts. The goal of this method of assessment is to give students as well as instructors a more detailed picture of how well the students understand the concepts being taught. Proficiency will be measured on a 3 point grading scale (described below): Mastery (3), Developing (2) and Beginner (1), and students who do not earn mastery on the first attempt will have an opportunity to reassess that learning objective and prove mastery. Students who do not attempt to complete the problem or activity will receive a score of 0, which typically may not be reassessed (exceptions can be made in special circumstances at the discretion of the instructor). Reassessments may be done either through individual completion of challenge problems or during lunch/office hours. In order to achieve mastery through reassessment, the student must prove deep understanding of the concepts behind learning objectives by being able to apply them to mathematical problems and justify their application. Students may reassess a learning objective as many times as they wish in order to achieve a level of mastery. The second portion of formal assessments consists of unit tests, unit projects, and final exams. Because these assessments are conducted at the conclusion of the instructional period when the student is expected to have reached mastery on all of the standards, students are graded based on the traditional 100 point scale and the grade may not be reassessed.

Mastery (3)

- My responses demonstrate a deep understanding of the standards and learning objectives.
- I understand not just the “what” but the “How” and “Why” of learning objectives.
- I am proficient at connecting concepts of the learning objectives with other, related indicators.
- I understand the important things about the content/skills.

- I have confidence on how to do the skills on my own most of the time, but I need to continue practicing some parts that still give me problems.
- I may need some help from my group members, but for the most part I am able to complete activities on my own. If given the activity again, I could complete it by myself.
- I actively participate in group activities and provide insightful contributions
- I need my handouts and notes once in a while.

Developing (2)
- I have a general understanding of the content/skills, but I'm also confused about some important parts.
- I need some help from my teacher (one-on-one or small group) to do the skills correctly
- I do not feel confident enough to do the skills on my own
- I need my handouts and notebook most of the time.
- I rely on my group members to guide me through the majority of the activities. I am not sure that I could reproduce the activity on my own.
- I can correctly identify concepts and/or define vocabulary; however I cannot not make connections among ideas and/or independently extend my own learning.
- My responses demonstrate basic understanding of some main ideas, but significant information is missing.

Beginner (1)
- I need lots of help from my teacher (one-on-one).
- I have low confidence on how to do the skills and need more instruction.
- I need my handouts and notebook at all times.
- I do not understand the concept/skills.
- I do not contribute to group activities and rely on them entirely.
- I cannot correctly identify concepts and/or define vocabulary.
- I cannot make connections among ideas or extend the information.
- My responses lack detail necessary to demonstrate basic understanding.

MOTIVATION

There are several different tactics that may be employed by the instructor in order to encourage students to be motivated and actively engaged in the lessons. The first consists of the distribution of individual achievement points, which may later be exchanged for prizes such as extra credit, an extra day on assignments, or raffle tickets for the chance to win large prizes such as gift cards, stickers, or stationary. Extra credit points may not be awarded on standard-based assessments, however, they may be awarded on grades of completion, participation, or exams, depending on school restrictions. Below is a list of actions for which students may earn points; however, deserving students may be awarded points at any time for any valid reason related to class participation or performance. Points are tracked individually as well as in each class, and at the end of the year the class with the most points per student receives their choice of a pizza or ice cream party. In order to ensure there is no advantage for larger classes, the total number of points received by the students of each class will be divided by the total number of students in that particular period.

Additionally, each month, the teacher will choose one student in each class to be the "Student of the Month". Students may be selected based not only on how well they perform, but also on improvement, significant effort, or any other action that sets them apart from the other students. In order to ensure each student has a chance of being "Student of the Month", students may not win this title more than once, until all students have been named. The names of each month's winners will be displayed on the wall of the classroom for the entire month. This will help to motivate all students to do their best in hopes of winning while giving special recognition to those whose grades might not necessarily reflect their work ethic.

In order to motivate students to put forth their best effort on their unit projects through a little bit of friendly competition, students in each will vote on their favorite design (may be most complex, innovative, colorful, etc) and the winner in each will receive either 5 points of extra credit on the project or 25 achievement points. Students will not be allowed to vote for themselves. Additionally, each student's project will be displayed on the wall of the classroom for others to see.

MISCONCEPTIONS

It is likely that prior to this lesson, the majority of the students will not have worked with constructions in the past, or even with a compass. Because it is such a relatively unfamiliar concept, students are unlikely to have preconceived misconceptions about the topic itself, however, because the shapes that are being constructed are likely to have been used in previous math classes, it is important that the instructor directly address any misconceptions the students may have in regards to precise definitions and encourage students to utilize the precise definitions of terms, even if they may be familiar with the term from previous experiences. Examples of this include the differences between lines, rays, and line segments, the characteristics of certain geometric shapes such as a rhombus or a hexagon, and complete understanding of terms such as bisector and equidistant. A comprehensive list of vocabulary used throughout the unit is provided below. Instructors should ask the students to provide definitions for each term in order to identify any misconceptions related to them as well as provide students with the precise definition of each term as it comes up. Additionally, the instructor should ensure that students can correctly indicate and interpret indications of right angles and congruences within diagrams. This will be done primarily through warm-up activities and in-class discussions, however, the lesson review foldable project will include a tab on vocabulary to serve as a reference for students not only for the unit exam, but for future classes as well.

MULTIPLE REPRESENTATIONS

Throughout this unit, it is essential for students to understand not only the steps needed to perform specific constructions but the reasoning behind these steps. In order to achieve this, students will be exposed to working with constructions on different platforms such as paper folding and online construction programs. Due to the relationship between different geometric shapes, there are multiple ways to construct and represent each shape. This unit will be focused not only on the construction of specific shapes such as equilateral triangles and inscribed hexagons but also on the fundamentals of construction such as copying and bisecting angles. Additionally, students will be taught multiple methods of constructing the same shape, though they will be allowed to use whichever method works best for them. The hope is that armed with these tools, students will be able to form constructions of other shapes not taught through the application of the geometric principles and relationships they will have learned. It is also important for students to be able to recognize and produce different constructions within the cartesian plane and apply them to different linear equations in order to prepare them for future topics such as three dimensional geometric modeling. For this reason, students will be exposed to working with constructions in both formats through homeworks and assessments.

Students will be performing constructions both by hand and on the online construction program. On paper, students will primarily be taught to draw arcs rather than circles (though they may do either), however, this program only allows the user to draw circles and lines with constraints for radius and length based on intersections. These constraints will allow students to more accurately and freely explore the relationships within constructions and will hopefully allow students to visualize the same methods that they are doing on paper in a different format.

Additionally, students will use paper folding tactics to explore these geometric relationships in the context of paper constructions. The paper folding activity takes place in the warm up of the construction of equilateral triangles lesson and is used to aid student discovery when they are asked to construct the triangle themselves. The physical manipulation of the paper will provide a more tangible representation and is aimed primarily at students who may have trouble visualizing, actively focusing, or connecting unrelated concepts, though it will serve as an aid for all students.

Because these different representations will be used in unison rather than taught as separate processes, students will more easily be able to identify and justify the geometric relationships between shapes. In seeing different ways of constructing the same figure and in constructing one figure from another different figure, students will be forced to see the connections between different figures and in manipulating one representation in order to work within another, students will gain a broader understanding of the effects of different constructions than they would when working in one stationary platform.

Students will be taught multiple methods of writing proofs including using tables as well as complete sentences, though they may utilize whichever form they are most comfortable with on homeworks and assessments. Through this, they will be exposed to different mathematical symbols as well as be required to employ precise definitions of vocabulary.

VOCABULARY

Line Segment - the segment between points A and B is the set consisting of A, B, and all points on the line AB between A and B
Equidistant - at an equal distance from
Geometric Construction - a method of constructing complex shapes using the geometric relationship between circles and lines
Interior Angle - one of the angles inside a polygon or one of the internal angles formed when two or more lines are intersected by a transversal
Exterior Angle - An angle formed on the outside of a polygon by extending one of its sides
Figure - a set of points such as lines, polygons, circles and any combination of points, lines, curves, or surfaces.
Circumscribed - Denoting the situation in one geometric figure is drawn outside another. This lesson sees a circle that passes through all six vertices of a given polygon. The circle is said to be circumscribed
Inscribed - Denoting the situation in which one geometric figure is drawn inside another. This is the opposite of circumscribed
Equilateral Triangle - a triangle that has all three sides congruent
Ray - a straight line that extends from a point. Also called a half-line
Radius - A segment from the center of a circle to a point on the circle
Arc - Part of the circumference of a circle, or part of any curve
Circle - Given a point C in the plane and a number $r>0$, the circle with center C and radius r is the set of all points in the plane that are a distance r from the point C
Perpendicular - standing at right angles to the plane of the horizon
Bisector - line that cuts a segment or angle into two parts of equal length or measure
Midpoint - the point on a segment that is equidistant from either endpoint

Chapter 1: Introduction to Constructions

The purpose of this lesson is to familiarize students with working with basic constructions using a compass and a straightedge as well as introduce them to perpendicular and angle bisectors. The goal of splitting up the introduction is to ensure that students are fully comfortable with not only the employment of these two methods but also the mathematical reasoning behind them. Because they make up the building blocks of more complex constructions, the hope is that this introduction will serve as a scaffold that students will later build upon when constructing shapes such as squares and triangles rather than just following what seems like a series of meaningless steps. The homework assigned for this day is intended to help students practice construction strategies as well as work with different problems not covered in class such as bisecting obtuse angles and working within the cartesian plane.

CONCEPTS:

Angle bisector
Perpendicular bisector

LEARNING OBJECTIVES:

Students will gain familiarity working with compass and straightedge.

Students will understand how to construct the perpendicular bisector of a line segment using compass and straightedge, and will understand the definition of a perpendicular bisector.

Students will understand how to bisect a given angle using compass and straightedge, and will understand that the two resultant angles are congruent.

Students will understand the definitions of both perpendicular as well as angle bisector and know how to indicate on a diagram

Students will explore the relationships between circles, lines, and complex geometrical figures.

WARM-UP (5-10 minutes):

This warm up consists of two parts and serves to both ensure students are familiar and comfortable with working with compasses as well as to correct any misconceptions they may have in regards to the tools and vocabulary.

Part 1 - Introduction to tools:

It is likely that students will have compasses that work in different ways and therefore it is essential that the instructor knows how to use different types of compasses prior to this lesson. Additionally, it may be helpful for the instructor to demonstrate how a compass may be fashioned using a string that is fixed at one end. The instructor will begin by asking students the following questions:

Has anyone used a compass before?

NOTE: Students may think you are referring to a compass used in navigation. It is important that the instructor directly address this misconception and draw a distinction between the two. The instructor should specify that this type of compass is used to draw circles of a defined radius. The instructor should gauge student familiarity with the tool as well as have several different types of compasses on hand to show the students.

How do I set the radius of the circle I want to draw using a compass? What about diameter?

The instructor should ask this question for each different type of compass that the students have.

How do I set the center of the circle?

The instructor should then have the students practice using a compass to construct circles of a given radius.

TIP: Depending on the compass, it may be easier to rotate the page and hold the compass still or vice versa.

Part 2 - Introduction to activity and vocabulary:

For the second part of the warm-up, students should be given a chance to discuss in groups before discussing as a class. Students should be encouraged to use math language in order to come up with the most precise definition.

What does it mean for something to be perpendicular? How do I indicate that something is perpendicular on a diagram?

KEY WORDS: intersect(ion), right angle, 90 degrees, square

What does it mean to bisect something?

KEY WORDS: split, divide, equal pieces, two

How do I indicate that the two parts have equal measure? (NOTE: be sure to use words that are not inclusively for lines (i.e. length) in order to avoid confusion and set up the next question)

What can I bisect? Just a line? Can I bisect an angle? What about a ray? (students should be asked to justify their answer)

Rays cannot be bisected because they have only one endpoint

DEMONSTRATION 1: PERPENDICULAR BISECTOR

Because it is unlikely that students will have worked with constructions or similar concepts in the past, a discovery based activity is likely to confuse and discourage students. For this reason, the instructor will act primarily as a facilitator, guiding the students throughout the lesson by providing a demonstration followed by an activity. The demonstration is designed to serve as a more structured form of discovery, and the questions asked to the students throughout are meant to encourage students to think about the logical reasoning behind the placement and radius of the compass.

Students begin with a blank sheet of paper. Instruct students to draw a line segment with labeled endpoints AB. Encourage students to make lines of varying lengths. Ask students if they can figure out a way to find the midpoint of the line segment. One answer could be to fold the paper so that the endpoints line up and the crease is at the midpoint. Ask students if they can figure out another way using a compass.

Draw a line segment AB on the board. Assign each group of students a radius, ensuring that at least one group's radius is less than twice the length of the segment. Instruct the students to take turns drawing on the board first a circle centered at A, then a circle centered at B. Use different colors for each group, if possible.

QUESTIONS:

1. Do you see a pattern between these circles?
2. What do you notice about where the circles intersect?
3. What is significant about this point?
4. What happens when the radius of the circle is less than half of the length of the segment?
5. Why wouldn't we want to choose a radius that is exactly half of the length of the segment?

ACTIVITY 1: PERPENDICULAR BISECTOR

Students begin with a blank sheet of paper. Instruct students to construct a line segment with labeled endpoints AB. Encourage students to construct lines of varying lengths. Have students trade papers with the person next to them. Instruct students to construct a perpendicular bisector of their new line segment. Have students measure to check their work. As a group/table, have students come up with a list of steps to find the perpendicular bisector of a line segment. Discuss as a class and create a “master list”

Master List for Constructing Perpendicular Bisector

1. Start with line segment AB
2. Place the compass on one end of the line segment
3. Set the compass width (radius) to over half the line length
4. Without changing the compass width, draw an arc above and below the line
5. Again, without changing the compass width, place the compass' point on the other end of the line
6. Draw an arc above and below the line so that the arcs cross the first two
7. Using a straightedge, draw a line between the points where the arcs intersect

ACTIVITY 1.5 (5-10 minutes)

This activity is optional and is meant to be used in cases where students are having a difficult time understanding the construction of a perpendicular bisector. It may also be used as an alternate activity for students who are for any reason unable to participate in ACTIVITY 1 (for example, if a student is absent, they may be assigned this activity). Because this activity takes place before the program is officially introduced, it may be necessary to explain how to use the software outside of the given instructions. https://sciencevsmagic.net/geo/

NAME _____________ DATE _______

ACTIVITY 1.5 - Perpendicular Bisector

Using this program, you will learn the steps to constructing the perpendicular bisector of a line segment.

1. You should see two dots. Click on either of them and move your mouse. You will see a circle appear. Move your mouse to the second dot and the circle should become a straight line. When this happens, click your mouse to construct the line. We will be creating a perpendicular bisector of this line.
2. Click either of the dots and drag your mouse until the edge of the circle is touching the other dot. Click your mouse to construct this circle.
3. Repeat this process for the other dot. You should have created a venn diagram.
4. Connecting two dots on this venn diagram will result in the construction of a perpendicular bisector. Locate these two dots and click them to construct a line.
5. Explain using what you know about circles how you know this line is the perpendicular bisector of the original line segment. Explain what it means for something to be a perpendicular bisector

DEMONSTRATION 2: ANGLE BISECTOR (10-15 minutes)

Students begin with a blank sheet of paper. Instruct students to draw an angle with labeled vertex A. Encourage students to make angles of varying measures. Ask students if they can figure out a way to find a line that cuts the angle in half. One answer could be to fold the paper so that the rays that make up the angle line up and the crease is at the bisector. Ask students if they can figure out another way using a compass drawing on what they know from the previous activity. The instructor should give the students enough time to attempt a construction before continuing on to the next step.

Instructor follows steps below on board:

1. Starting with angle, label vertex A
2. Set compass radius to arbitrary measure
3. Draw an arc (or circle) whose center point is on A. Ensure it intersects both rays of the angle. If not, extend the rays. Label intersection with angle rays as arc BC.

4. Set compass radius to arbitrary measure
5. Draw an arc (or circle) whose center point is B. Label circle B
6. Draw an arc (or circle) whose center point is C. Label circle C.
7. Draw a straight line from the vertex A to the intersection of B and C that lie inside the angle

Encourage students to come up to the board to measure angle to verify bisectors.

QUESTIONS:

1. Does the location of my first arc matter? (If students cannot reach a conclusion, have them try bisecting the same angle with different radiuses for step 2)
2. What is significant about the intersection of arc FG and HJ?
3. Does the radius matter for this step?
4. Why do I need to keep my radius constant?
5. Would this work with an obtuse angle?
6. What is significant about the two angles?

ACTIVITY 2: ANGLE BISECTOR (5-10 minutes)

Students begin with a blank sheet of paper. Instruct students to construct angle labeled ABC. Encourage students to construct angles of varying measure. Have students trade papers with the person next to them. Instruct students to construct an angle bisector of their new angle. Have students measure using a protractor to check their work.

As a class, have students come up with a list of steps to find the bisector of an angle. Discuss and create a “master list”.

Master List for Bisecting an Angle

1. Start with angle ABC
2. Place the compass point on the angle's vertex, B
3. Adjust the compass to a medium wide setting. The exact width is not important
4. Without changing the width, draw an arc across each leg of the angle
5. Place the compass on the point where one arc crosses a leg and draw an arc in the interior of the angle
6. Without changing the compass setting, repeat for the other leg so that the two arcs cross. This is the midpoint
7. Using a straightedge, draw a line between the midpoint and the angle's vertex B

During the class discussion, the instructor should focus not only on the construction itself but on the resultant of that construction, particularly in regards to distance and symmetry. The instructor may ask one of the following questions to ensure students have complete understanding of the learning objectives as well as prepare them for writing proofs and manipulating figures using translations and reflections. The instructor should be sure to emphasize that the bisector lies along the line of reflection of the figure.

Higher Order Questions:

1. We know that the point where the perpendicular bisector intersects the line segment is equidistant from the segment's endpoints. What can we say about other points that lie on the perpendicular bisector?
    a. All points that lie on the perpendicular bisector are equidistant from the endpoints. Students will have to prove this on their ticket out the door.
2. How can we use this principle to help us draw a line that is perpendicular to another through a point that is not on the line?
    a. The instructor should point out that the intersection of the arcs that form the perpendicular bisector do not lie of the line. If students are having trouble answering this question, encourage them to think of the problem backwards, where the arc intersections are known, but the endpoints of the line segment are not. Students should understand that because the line does not need to be a bisector, the endpoints of the line are arbitrary. They may then find points on the line that are equidistant from the given point (do this by creating a circle centered at the given point).
3. How can we verify that a figure is bisected without measuring?
    a. When the figure is folded along the bisector, each point on one side of the figure maps to a corresponding point on the other side of the figure.
4. What does this tell us about which figures can or cannot be bisected
    a. A figure can be bisected if and only if there exists a line of reflection along which the figure may be folded. The instructor may demonstrate this by having students place a mirror on the bisector (line of reflection) and observing the resultant mirrored image.
5. Looking at the constructions of both perpendicular and angle bisectors, what do you notice about the procedures in terms of symmetry. (NOTE: this refers to symmetry within the construction on either side of the bisector, not symmetry of the figure itself)
    a. Students should recognize that in addition to the symmetry of the figure along the bisector (line of reflection) the means of construction are also symmetrical. That is to say that the same procedure is performed on both sides of the line/angle, so the constructed line bears the same relationship to each side. This may be used to prove that the bisector is equidistant from the line/legs of the angle as well as the fact that each angle of the resultant angle bisector is half the total angle

DAY 1 TICKET OUT THE DOOR

Jackson and Carol are playing frisbee during recess and are standing 8 meters apart on a line painted on the field. Their friend Raquel is standing in the middle of this line, then walks three meters north of the line. Using what you know about perpendicular bisectors, what can we conclude about the distance between Raquel and Jackson? What about the distance between Raquel and Carol? Use pythagorean theorem to find this distance. What if Raquel had walked only two meters away from the line? How would the distance between her and her friends change? What does this tell us about the distances between the endpoints of a line segment and the points on its perpendicular bisector?

HINT: it may be helpful to draw a diagram of everyone's positions below

Chapter 1 Day 2

The unit project will be introduced on the second day of this lesson, once students have been introduced to the concept of construction and have had practice working with basic constructions such as bisectors. The purpose of the unit project is to give students an opportunity to creatively explore constructions in a simple, user-friendly platform while employing higher level reasoning to construct specific shapes within their design. The project is introduced at the beginning of the unit in the hopes that it will serve as a tool throughout the unit for students to explore the effects of different constructions and constraints (i.e. radii) and to connect together the processes of constructing distinct shapes. The goal is that through the combination of task-based constructions and free exploration, students will understand the relationship between geometric figures rather than just memorize the steps to constructing each shape.

Rather than assign students a specific shape they must construct (i.e. a rose), students have the freedom to construct any design they wish, so long as it includes the elements within the standards. This flexibility not only inhibits cheating, but also allows those students with an interest or aptitude in art and design the opportunity to express themselves creatively, without discouraging the others. Students can make their designs as intricate as they wish, though they will not be graded based on aesthetics. There will be no assigned homework on this day in order to give students a chance to begin working on their projects while the class activity and the program are still fresh in their minds.

CONCEPTS:

Copying an angle
Constructing a line perpendicular to a given line through a point not on the line
Constructional Symmetry

LEARNING OBJECTIVES:

Students will learn how to copy a given angle using a compass and straightedge

Students will learn how to construct a line that is perpendicular to a given line through a point not on the line using a compass and straightedge

Students will explore practicing basic constructions on an online construction program and identify differences between construction methods

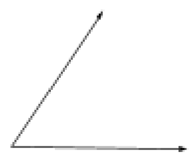
Students will begin to identify elements of symmetry present within not only the geometric figures they construct but also within the constructions themselves

WARM-UP:

The purpose of this warm-up is to refresh and reinforce the concepts of bisectors taught in the previous instructional day as well as to encourage students to think about the demonstration in the context of constructions rather than just random doodles. The instructor will go over the homework from the previous day and answer any questions students may have had. The instructor will then pass out the warm-up sheet.

Day 2 Warm-Up

You will need a compass and a straight edge. Below is a list of steps needed to copy an angle using a compass and straightedge. The steps are not in order. With your group, place the steps in the correct order, then follow the steps to copy the angle below.



1. Draw circle E: center E, radius BA
2. Label either intersection of circle E and circle F as D
3. Use your straightedge to draw ED
4. Label the vertex of the original angle as B
5. Label the intersections of circle B with the sides of the angle as A and C
6. Draw a ray EG as one of the sides of the angle to be drawn.
7. Draw circle F: center F, radius CA
8. Label the intersection of circle E with ray EG as F
9. Draw circle B: center B, any radius

The correct order is: 4,6,9,5,1,8,7,2,3. The instructor should encourage students to test their arrangement before making a final decision on the order of steps. After students have had enough time to complete the warm-up, the instructor should go over the steps and as a class should create a master list detailing the steps needed to copy an angle.

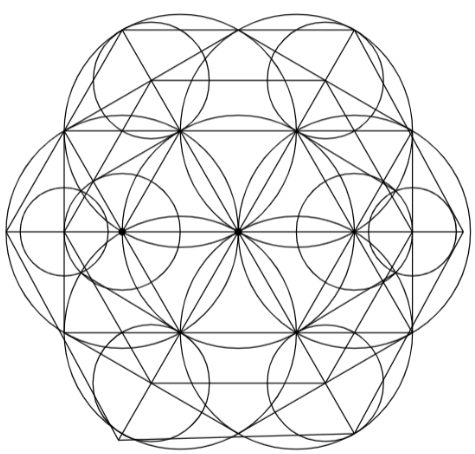
Master List for Copying an Angle

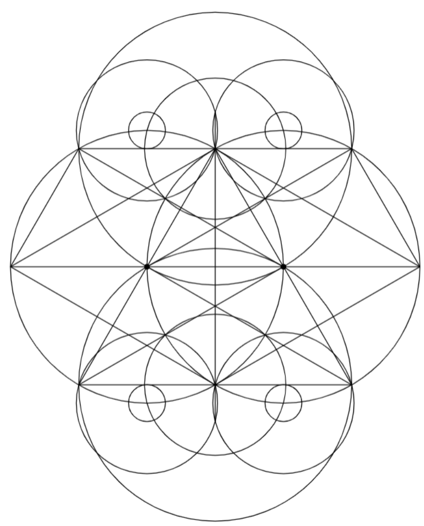
1. Draw a ray. This will be the side of the new angle.
2. Sweep an arc on the original angle. Wider compass setting is better, though the exact setting does not matter
3. Without changing the compass width, draw this arc from one of the endpoint of your ray.
4. Measure the distance where the arcs intersect the two sides of the original angle. Set your compass to this width
5. Place your compass on the intersection of your ray and the arc and sweep an arc from this point so that it intersects the arc from step 3
6. Using a straightedge, connect the intersection of these arcs with the endpoint you chose in step 3

DEMONSTRATION 3: INTRO TO ONLINE CONSTRUCTIONS

Prior to the demonstration, the instructor will introduce and assign the unit project (handout below). The purpose of the demonstration is to introduce students to the software and show them the real-world applications of constructions within art and design. The instructor may either create their own design or use the examples below, while explaining how to use the program both within the context of the project as well as within the context of constructions in general. The instructor should encourage the students to use the program as a tool throughout the unit to help them better understand the effects of different constructions. It is important, however, that the teacher emphasize the fact that the program is not essential for understanding of the concepts or completion of the project and is merely meant to be a supplementary tool where students can have the freedom to easily explore different constructions. The demonstration should last between five and fifteen minutes to ensure that students have enough time to use the software and ask any logistical questions they may have.

- You will always begin with two dots.
- You can either make a circle or a line whose radius and length are the distance between any two dots respectively
- Each intersection will create a new dot, from which more constructions may be made





ACTIVITY 3: ONLINE CONSTRUCTIONS

Each student should be given a computer and a link to the program https://sciencevsmagic.net/geo/. Students should complete the following hand out before the end of the period. Students should use the rest of the time to practice using the program and brainstorm designs for their project.

ACTIVITY 3 - ONLINE CONSTRUCTIONS

Go to the following website:

You should see two dots. Clicking on either of these dots will allow you to make either a line connecting the two or a circle whose radius is equal to the distance between the dots and whose center is at the dot that was selected. Each intersection will create a new dot from which more circles or lines may be constructed.

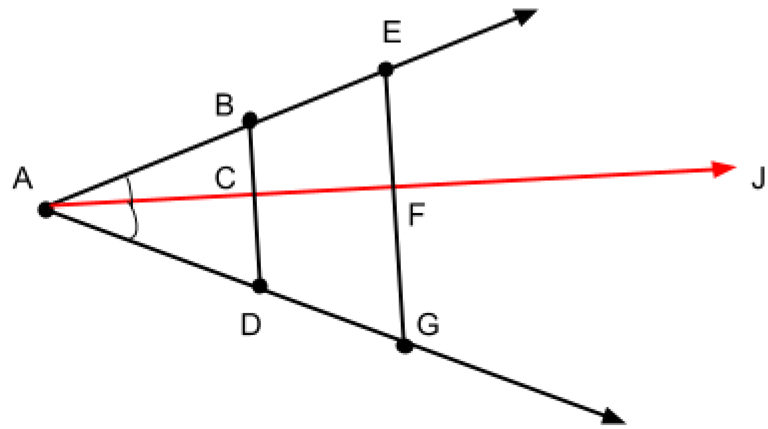
1. Draw a line connecting the two given dots. Your task is to use what you have learned to construct a perpendicular bisector of this line segment on the program. Copy the steps you took below.

2. How is this method of constructing a perpendicular bisector different from the one you learned in class? How is it similar?

DAY 2 TICKET OUT THE DOOR

1. Looking at the figure below, prove that
   a. Segments EF and FG are congruent
   b. Segments BC and CD are congruent

2. What is significant about the line AJ?



Chapter 2: Constructing an Equilateral Triangle

CONCEPTS:

Compass and straightedge construction of an equilateral triangle
Equilateral triangle construction given a line segment (leg)
Equilateral triangle construction given a circle

LEARNING OBJECTIVES:

Students will be able to construct an equilateral triangle using only a straightedge and a compass given either a line segment or a circle

Students will explore the effects of perpendicularly and angularly bisecting triangle sides and vertices respectively

WARM-UP (10-15 mins):

The purpose of the warm-up is to get students thinking about the relationships between geometric shapes, and using information from the previous lesson, will allow them to more accurately predict and later justify the steps of constructing an equilateral triangle using a compass and straightedge. This hands-on activity will keep the students actively engaged and introduce topics in a fun, unconventional way. After having completed the warm up, students should be able to determine the necessary steps for creating an equilateral triangle in the activity portion of this lesson, however, they may need a bit of guidance. Encourage students to use what they know about triangles and distances along with what they learned about bisectors.

Students begin with a cut out circular sheet of paper.

1. Fold the circle in half to create a semi circle
2. Fold the circle in half again to create a quarter circle
3. Unfold steps 1 and 2 to get a full circle
4. Take any point on the circle and fold it to the center, ensuring that the arc touches the center point of the circle (where the folds from steps 1 and 2 meet)
5. Holding your finger on the top edge of this fold and using it as a guide, fold another point to the center of the circle
6. Fold the remaining arc to this center point (TRIANGLE)
7. Take any vertex of the triangle and fold it down to the base of the triangle (TRAPEZOID)
8. Take either of the remaining vertices of the triangle and fold it up to meet the top of the trapezoid (RHOMBUS)
9. Fold the rhombus in half (TRIANGLE AGAIN!!)
10. (FUN STEP - OPTIONAL) unfold the three small triangles and connect their vertices (TRIANGULAR PRISM)

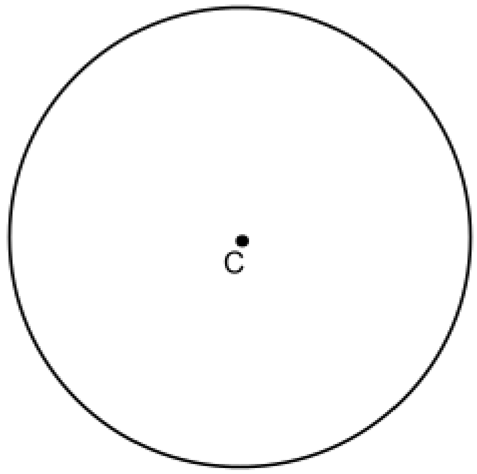

center point


ACTIVITY 4: EQUILATERAL TRIANGLES

In this activity, students will build on the skills they learned in the previous lessons such as perpendicular and angle bisectors to investigate and discover how to construct an equilateral triangle. Using the folded triangle they made during the warm-up activity as a reference, students will be able to identify the geometric relationships between circles and equilateral triangles and use them to solve the problem. Because there are multiple ways to construct an equilateral triangle, the instructor should encourage students to explore different methods before referring to the answer sheet below, as it is just one way an equilateral triangle may be constructed. If students are having difficulties constructing the triangle, there are several hints below that the instructor may use to guide the students. However, these hints should be given one at a time and students should have enough time in between to try out different methods and strategies. At the conclusion of the activity, the instructor should encourage students to discuss the different methods they used, and as a class should investigate and explain why all of these different methods work. Additionally, students should create a “master list” for each method. Examples of master lists are shown below.

ACTIVITY 4 - EQUILATERAL TRIANGLES

You and your group have been charged with setting up the arena for this year's Hunger Games. Unfortunately, due to uprisings in the district, only three districts will be competing with only one tribute representing them. In order to make sure they all have an equal chance of getting to the cornucopia (point C), your task is to determine where to place each of the three competitors so that they are located the same distance away from it and from each other. Supplies are low in the Capital so the only tools you are provided are a map of the starting area, a compass, and a straight edge. Using what you know about constructions to find the position of each tribute.

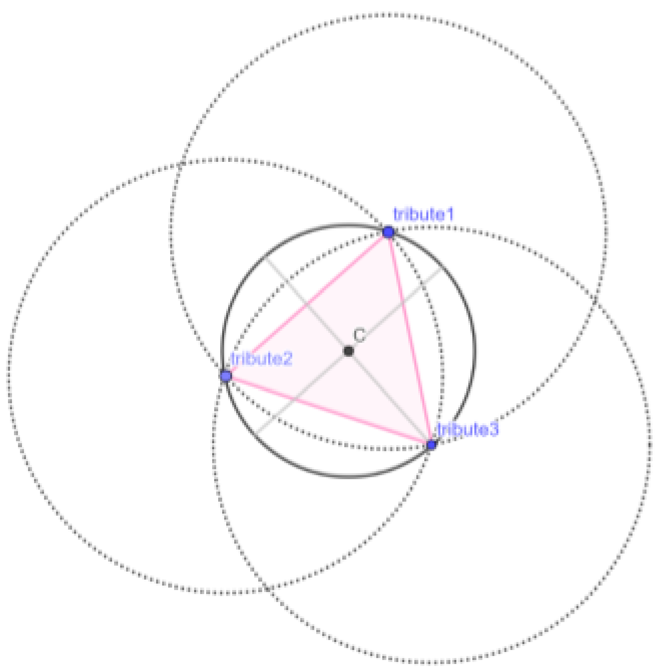


HINT: if you get stuck, use the foldable from the warm-up as a reference

EXTRA HINTS/GUIDING QUESTIONS:

1. You are trying to make the first triangle from the warm up (the big one), where each vertice is the location of a tribute.
2. Start by choosing any point on the circle. This will be the first vertex.
   a. Why does it not matter where on the circle we put the first tribute? (all points on the circle are equal distance from center C)
3. The points on the circle that touch the center from the warm up are not the vertex of the triangle
4. Draw a line connecting this tribute to the center of the circle and extend it to the other side of the triangle.
   a. What is significant about the point where this line intersects the circle? (it is the same distance from the center)
   b. How do we know that this is the position of the second tribute?
   c. Why can't we just use another random point on the circle if all points are the same distance from the center? (they also have to be the same distance from each other)
5. How could a bisector help us find the location of the last tribute?
   a. Would we want to use an angle bisector or a perpendicular bisector? Why?

SAMPLE ANSWER



MASTER LIST FOR CONSTRUCTING EQUILATERAL TRIANGLE GIVEN A CIRCLE

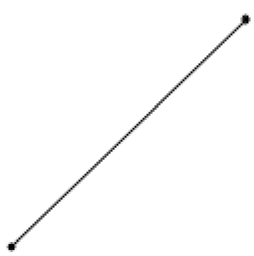
1. Draw a circle and place a point on the circle. Do not change the width of the compass.
2. Draw a circle centered at this point whose radius is the same as the original circle. It should look like a venn diagram.
3. Without changing the compass width, draw a circle centered at the point where the other two intersect.
4. Using your straightedge, connect every other point where an arc intersects the original circle to form the equilateral triangle

MASTER LIST FOR CONSTRUCTING EQUILATERAL TRIANGLE FROM A LINE SEGMENT (LEG)

1. Begin with line segment AB. This will be one leg of your triangle.
2. Use your compass to measure the length of AB. Set your compass to this width.
3. Draw a circle centered at A whose radius is AB
4. Draw a circle centered at B whose radius is AB
5. Using your straightedge, connect the intersection of these two circles to each of the endpoints of AB to form an equilateral triangle

DAY 3 TICKET OUT THE DOOR

In today's activity you learned how to construct an equilateral triangle from a circle. Your task is to now construct an equilateral triangle given one of its legs on the diagram below. Write each step you took to perform the construction.



CONSTRUCTION STEPS:

Chapter 3: Constructing Parallelograms

CONCEPTS:

Compass and straightedge construction of rhombus
Compass and straightedge construction of a square
Compass and straightedge construction of a parallelogram

LEARNING OBJECTIVES:

Students will be able to construct a perfect rhombus using only a straightedge and a compass

Students will be able to construct a perfect square using only a straightedge and a compass

Students will be able to construct any parallelogram using only a straightedge and a compass

Students will be able to define the characteristics of different types of parallelograms

Students will be able to identify relationships between parallelograms and triangles and use them within their constructions

WARM-UP:

Before beginning the warm-up the instructor should go over the homework from the previous day and answer any questions students may have had. They should encourage students who completed the challenge problem to share their responses. If no student was able to complete the challenge problem, they will have a chance to complete it as a reassessment.

The purpose of this warm up is to give students an understanding of how constructions are helpful for creating precise shapes as well as review the characteristics of squares and rhombuses with the goal of not only addressing any misconceptions they may have about the shapes but also to help them understand the reasons for certain constructions. For example, if students know that a square has four sides that meet at right angles, they can infer that they will need to construct perpendicular lines using the methods they have learned in the previous chapters. The second part of this warm-up is meant to show students the relationships between equilateral triangles, which they learned to construct in the previous chapter, and rhombuses. The hope is that they will be able to use these connections to determine the steps needed to construct a rhombus during the activity.

Part 1: On a blank sheet of paper (unlined), have students try to draw a rhombus and a square free-handed. While students are drawing, review the characteristics of both shapes. Have students measure using a ruler/protractor to see if they constructed a perfect rhombus/square.

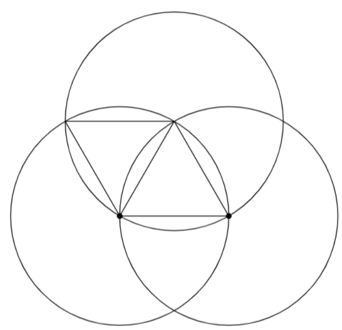
QUESTIONS:

1. How many sides does a rhombus/square have?
2. How is a square different from a rhombus?
3. How are the sides of a rhombus related? A square?
4. Is a trapezoid a type of rhombus? Why not? What about a square? A rectangle?
5. In the context of constructions, how can we make sure we have these characteristics?

Part 2:

Student Instructions: Using what you have learned, construct two equilateral triangles that share a common side. List the steps needed to accomplish this construction. When you are finished, discuss these steps with your groups. Take notice of what you each did differently.

Sample Solution:



The instructor should have students discuss as a class the different steps they took to construct the two triangles. The instructor should then ask the students to identify any other shapes that are present within this diagram. The students should notice that the two equilateral triangles form a rhombus.

ACTIVITY 5: RHOMBUS FROM A LINE SEGMENT

From the warm-up, the student should have identified that a rhombus can be constructed from two equilateral triangles. This activity will encourage them to use this relationship to discover how to construct a rhombus from a given line segment. It is likely that from the warm-up students will deduce the objective of this activity, so in order to challenge students a little bit more and get them thinking, they will be asked to draw a line segment on a piece of paper (they will think they are going to be constructing a rhombus from this line segment so it is likely that they will be preparing to begin the construction), and in asking them to switch with a neighbor, they will be pushed a little bit farther out of their comfort zone, the problem will require more cognitive attention, and as a result, students are likely to be more active when learning.

On a blank sheet of paper, have students draw a line segment with labeled endpoints PQ. Encourage them to draw lines of varying lengths. Have them trade papers with the person next to them. Have students construct a rhombus whose sides are equal to the length of the given line segment. If the page is too small to fit the rhombus, encourage students to place a page under the sheet to increase total drawing area.

As a group/table, have students create a detailed list of steps for constructing a rhombus. Then, discuss as a class and create a “master list”

GUIDING QUESTIONS

1. How did I make sure that all of my sides were the same length?
2. Does it matter where I choose to place point S?
3. How would my shape change if I had decreased the radius of my compass before finding the last vertex? What if I had increased it?
4. How would I construct a rhombus if I was given a side length that was less than my line segment?

MASTER LIST FOR CONSTRUCTING A RHOMBUS:

1. Create a line segment PQ
2. Set compass radius to length of PQ (or desired side length)
3. Draw circle P, center at P and radius PQ
4. Chose any point on circle P. Label point S
5. Draw line segment SP
6. Without changing compass radius, bisect angle SPQ. Label arc intersection point R
7. Connect vertices to make rhombus PQRS

DEMONSTRATION 4: CONSTRUCTING A SQUARE

The purpose of this demonstration is to guide students towards constructing a square using a compass and a straightedge. Because constructing a square is significantly different from the other shapes students have learned, the instructor will act as a guide, and the class will discover together. While the instructor is provided with a list of guiding questions, they should first ask the students to see if they can work together to figure out the steps before a guiding question is asked. The instructor should take student suggestions and try them out on the board. If they produce the desired result, the students should discuss as a class why this is the case and ask if this will work in all scenarios. If they do not produce the desired results the instructor should facilitate a discussion with the goal of modifying and determining alternate steps or an alternate order that will create a square. Before performing each step, the instructor should ask students to explain why they believe that step will help to construct a square and see if any students disagree or have a different idea. The instructor should continue this process until a square has been constructed, providing gentle guidance whenever necessary, while ensuring students have enough time to think, explore, and experiment with different possible methods.

Instructor draws a line segment with labeled endpoints AB on board.

What are the characteristics of a square?

How can we make sure all of our sides are the same length?

How can we make sure all of our sides are perpendicular? How have we made perpendicular lines in the past?

Instructor draws perpendicular bisector of line segment AB on board, making sure to extend the line past the intersections of each arc.

How can we make sure all of our sides are the same length? (Draw a circle whose center is at the midpoint of AB and radius is equal to ½ AB)

Instructor draws a circle whose diameter is equal to AB centered at the midpoint of AB. Label intersections of this circle and perpendicular bisector as C and D. Connect all four vertices to make square ACBD.

Encourage students to measure using ruler/protractor to verify accuracy.

QUESTIONS:

1. How could I use these principles to create a square whose sides are equal to my line segment AB?
2. How is this similar to constructing a rhombus? How is it different?
3. How do I know, without measuring that my sides are the same length?
4. How do I know, without measuring, that my sides intersect at right angles?
5. How could I use these principles to create a rectangle?
6. What do you notice about the diagonals of my square? How is this similar to those of a rhombus? How are they different? Will there be any relationship between the diagonals of a rectangle? Explain and justify.

ACTIVITY 6: SQUARE CONSTRUCTION CHALLENGE

The purpose of this activity is to not only emphasize the concepts learned in the demonstration, but introduce a challenge where the sides must be twice the length of the given line segment. In order to solve this problem, students must think not only about the steps but about the effect of each step and use critical thinking to modify these steps to produce a square that meets the criteria.

On a blank sheet of paper, have students draw a line segment with labeled endpoints AB. Encourage them to draw lines of varying lengths. Have them trade papers with the person next to them. Have students construct a square whose sides are twice the length of the given line segment. If the page is too small to fit the square, encourage students to place a page under the sheet to increase total drawing area.

As a group/table, have students create a detailed list of steps for constructing a square. Then, discuss as a class and create a “master list”

MASTER LIST FOR CONSTRUCTING A SQUARE

1. Construct line segment AB
2. Create a perpendicular bisector of line segment AB. Label midpoint point O
3. Create circle whose center is O with radius equal to AO and/or BO (equal length)
4. Label intersections of circle O with perpendicular bisector as C and D
5. Connect all points to make square ACBD

DAY 4 TICKET OUT THE DOOR

Using any methods you have learned throughout this unit, construct a rectangle in the space below using only a straightedge and a compass. Write out the steps you took to constructing your rectangle and compare them to the steps needed to construct a rhombus or a square. How are they similar? How are they different? When you have finished, discuss with a partner the steps you each took. Try constructing a rectangle by following your partners steps below.

Chapter 4: Constructing an Inscribed Hexagon

CONCEPTS:

Constructing an inscribed hexagon using only a compass and a straightedge
Exploring the use of triangles to form other more complex shapes

LEARNING OBJECTIVES:

Students will learn to construct an inscribed hexagon using only a compass and straightedge

Students will explore the geometric relationship between triangles and other shapes and use this relationship to form other more complex polygons

WARM-UP:

Before beginning the warm-up the instructor should go over the homework from the previous day and answer any questions students may have had. They should encourage students who completed the challenge problem to share their responses. If no student was able to successfully complete the challenge problem, they will have a chance to complete it as a reassessment. There will be no homework assigned for this day's lesson. Instead, students will have a chance to work on their project before they begin learning about proofs.

The warm-up is intended to refresh students on the concepts covered in previous chapters, with special emphasis on the construction of equilateral triangles with the goal of allowing students to discover how to construct an inscribed hexagon by constructing multiple adjacent triangles. Additionally, the warm-up will serve to reintroduce and define useful vocabulary such as what it means to be inscribed as well as review the characteristics of a hexagon.

Give each student at least six cut out congruent equilateral triangles. Encourage the students to rearrange them to make different shapes. Have students create a list of all of the different shapes they made. Then, as a class, discuss the different shapes made and how this could be applied to constructions.

ACTIVITY 7: INSCRIBED HEXAGON

In table groups, students should work together to construct a hexagon that is inscribed in a circle. Students should be encouraged to explore multiple different methods of construction with the goal of finding the simplest one. For each method of construction, students should write the steps they took. After 15 minutes, have one student from each table go to another table and discuss different methods they each used. Repeat the process every 5 minutes until each student has heard all of the different methods. As a class, decide on the most efficient method and create a "master list"

MASTER LIST FOR CONSTRUCTING INSCRIBED HEXAGON

1. Draw a circle and place a point on the circle. Do not change the width of the compass.
2. Draw a circle centered at this point whose radius is the same as the original circle. It should look like a venn diagram.
3. Without changing the compass width, draw a circle centered at the point where the other two intersect.
4. Using your straightedge, connect every point where an arc intersects the original circle to form the hexagon inscribed in the original circle

DAY 5 TICKET OUT THE DOOR

Use what you have learned to construct a polygon with 5 sides or more than 6 sides. Explain each step you took and predict the steps needed to construct a polygon of n number of sides. Justify your prediction and compare with your classmates

Chapter 5: Introduction to Construction Proofs

CONCEPTS:

Examining why constructions produce the desired results
Using precise language
Two-column proofs
Paragraph proofs

LEARNING OBJECTIVES:

Students will more thoroughly examine why compass and straightedge constructions produce the desired results

Students will practice using precise mathematical language and gain an understanding as to the importance of precise definitions

Students will begin working with different forms of proofs including two-columns proofs and paragraph proofs.

WARM-UP:

The first part of the warm-up is intended to introduce students to one format of a mathematical proof as well as show them the effects of employing imprecise definitions when trying to prove or justify a specific theorem. The second part of the warm-up is to introduce students to different methods of proofs in order to give them a general sense of how certain proofs are structured and how one proof can differ from another. The instructor should give the students a brief overview of what a proof is making sure to emphasize that observations, experiments, and measurements are not proof.

Part 1: instruct the students to read the proof below. They should discuss the proof in groups and try to find ways to disprove the argument that my dog pooped on the floor (ex was there anyone else home?) Students should then write a proof to prove anything they would like (the statement can be true or false) and should swap with a partner. The proofs may be in two column format like the one shown below or written in paragraph form. Every statement must have a reason. It’s best to start by listing your givens. The partner should try and find ways to disprove the proof.

WARM-UP PART 1 Proof that my dog pooped on the floor

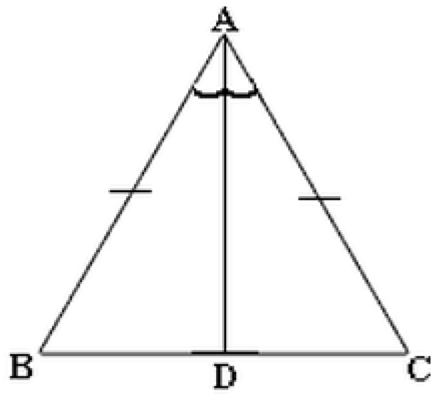
Statement	Reason
1. There is poop on the floor	1. Given
2. There was not poop on The floor an hour ago	2. Given
3. I am toilet trained	3. Given
4. I did not poop on the Floor	4. Definition of toilet trained
5. My dog poops every Now and then	5. Everybody poops postulate
6. Dog's can't use toilets	6. Toilets are made for Humans
7. My dog pooped on the floor	7. My dog had nowhere Else to poop

Part 2: Below is a list of some basic properties. Instruct students to keep the list in their notes as it will come in handy in the future. Go through each property and answer any questions students may have about them. Give examples if necessary.

Property	Meaning	Geometry Example
Reflexive Property	A quantity is equal to itself.	$AB = AB$
Transitive Property	If two quantities are equal to the same quantity, then they are equal to each other.	If $AB = BC$ and $BC = EF$, then $AB = EF$.
Symmetric Property	If a quantity is equal to a second quantity, then the second quantity is equal to the first.	If $OA = AB$, then $AB = OA$.
Addition Property of Equality	If equal quantities are added to equal quantities, then the sums are equal.	If $AB = DF$ and $BC = CD$, then $AB + BC = DF + CD$.
Subtraction Property of Equality	If equal quantities are subtracted from equal quantities, the differences are equal.	If $AB + BC = CD + DE$ and $BC = DE$, then $AB = CD$.
Multiplication Property of Equality	If equal quantities are multiplied by equal quantities, then the products are equal.	If $\mathrm{m}\angle ABC = \mathrm{m}\angle XYZ$, then $2(\mathrm{m}\angle ABC) = 2(\mathrm{m}\angle XYZ)$.
Division Property of Equality	If equal quantities are divided by equal quantities, then the quotients are equal.	If $AB = XY$, then $\frac{AB}{2} = \frac{XY}{2}$.
Substitution Property of Equality	A quantity may be substituted for its equal.	If $DE + CD = CE$ and $CD = AB$, then $DE + AB = CE$.
Partition Property (includes "Angle Addition Postulate," "Segments add," "Betweenness of Points," etc.)	A whole is equal to the sum of its parts.	If point C is on $\overline{AB}$, then $AC + CB = AB$.

ACTIVITY 8: PROOFS

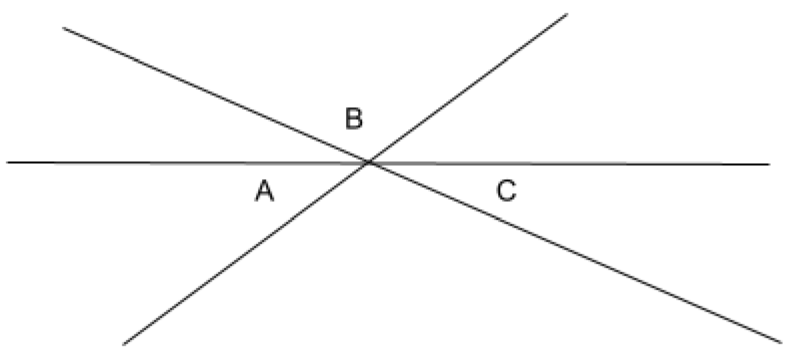
The purpose of this activity is to introduce students to geometric proofs and expose them to working with different types of proofs. Students will work in teams of two to complete this activity. Using the proof given below, they must determine first, what is being proven and fill in the blank space. Then, they must rewrite the proof using complete sentences.



Statements	Reasons
1. $\overline{AB} \cong \overline{AC}$	1. Given
2. $\angle BAD \cong \angle DAC$	2. Given AD is the angle bisector
3. $\overline{AD} \cong \overline{AD}$	3. Common side.
4. $\Delta ABE \cong \Delta DCE$	4.?

DAY 6 TICKET OUT THE DOOR

Prove that the sum of the labeled angles in the diagram below is 180 degrees. You may use either a two column proof or write in complete sentences. You may refer to the chart you got during the warm-up if you need help. Remember, every statement must have a reason

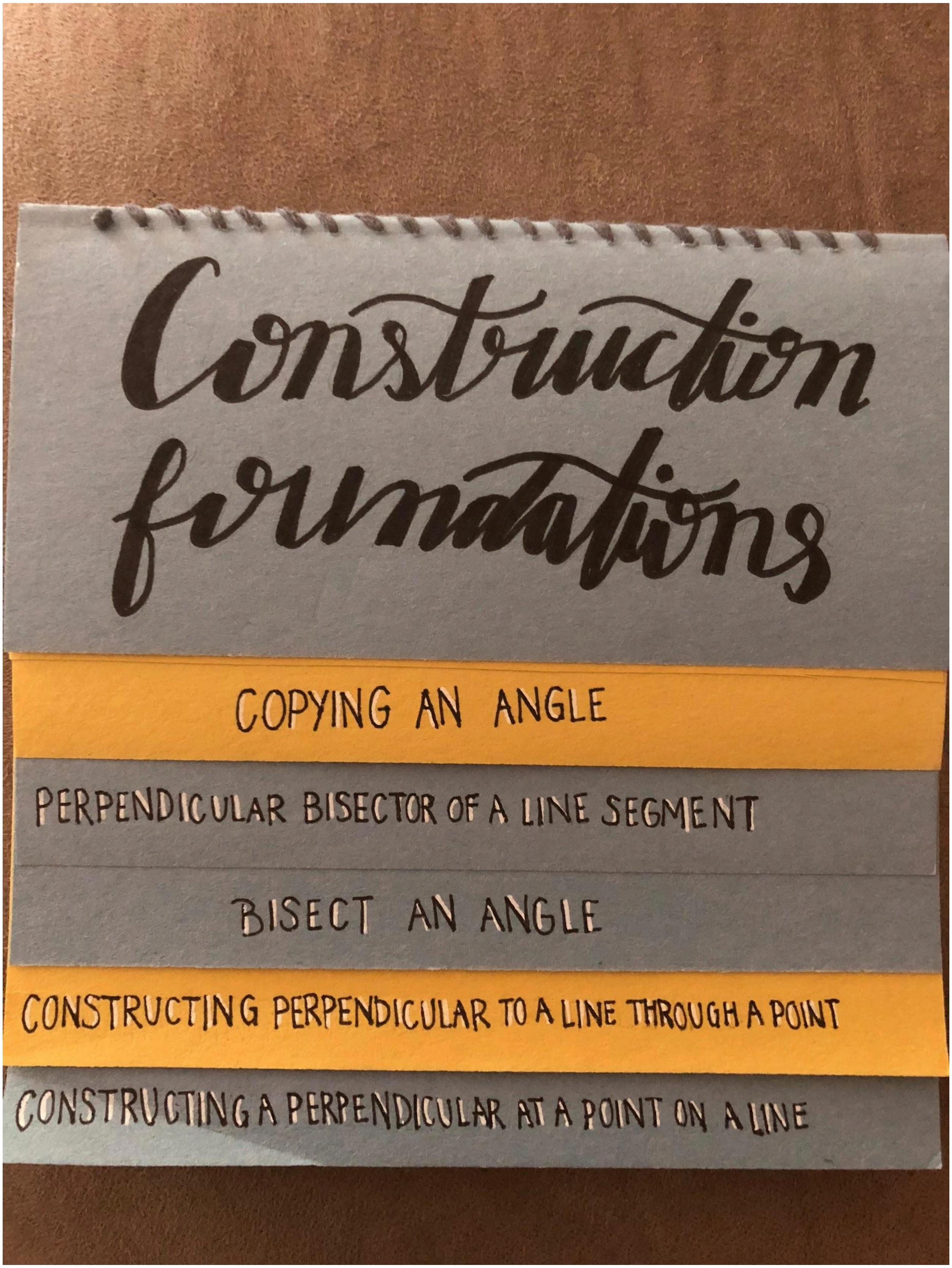


Lesson Review

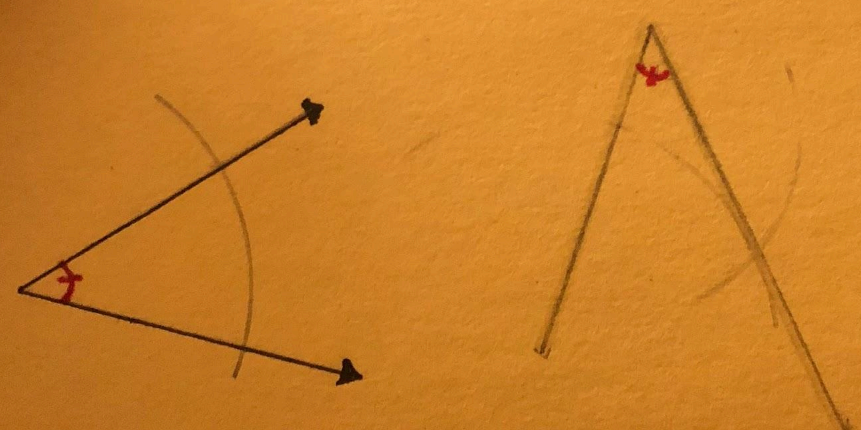
This day will be used to review the lesson materials and prepare for the unit exam as well as correct any final misconceptions the students may have. Students will review my creating a foldable. The instructor should provide construction paper, scissors, glue, markers, crayons, and any other materials for decorating or designing. Students will be allowed to work on their foldables together, though they should each create their own. Additionally, students will not receive a grade for completion of the foldable, however, the instructor should not direct this fact to the attention of the students to avoid a loss of interest and motivation, unless explicitly asked. The instructor should emphasize to the students that their foldable will serve as a resource not only for the upcoming and exam which will be administered after the conclusion of the entire congruence, constructions, and proofs unit, but for any future scenarios in which they will be required to perform precise constrictions and that the more work they put into their foldable, the more useful of a resource it will be. The instructor should give some examples such as in the fields of art, architecture, and engineering. Additionally, the instructor should emphasize to the students that the content within the foldable is the most important element and encourage students to prioritize content over design; they can work on that part at home but once they leave they will not be able to ask any questions about the content.

The foldable is a double-sided ten-tab booklet (five tabs on each side). To construct it, students will need at least three sheets of paper, which they should cut vertically (hot-dog). They should now have six sheets of paper. To construct the booklet, lay three sheets of paper on top of each other and offset them vertically by approximately half an inch (exact size does not matter). Once they are offset, fold the three sheets in half horizontally (hamburger) so that there are a total of six tabs offset approximately half an inch from each other. The top tab should be significantly larger than the rest. This will be the title tab. Repeat this process using the other three sheets to create the second half of the foldable. Secure the two halves together using any method you'd like (i.e. staples, glue, string, etc).

One half of the foldable will focus of the fundamentals of constructions; bisecting a line segment and an angle, copying an angle, and more. Students will designate one tab to each of the five foundational construction methods and write out the steps (either from the master list or they may create their own if they wish) as well as include a diagram of each construction. The other side will be structured similarly, though it will be focused on the applicational constructions of geometric shapes such as squares and triangles. There is an extra tab on this side that will be dedicated to vocabulary and any extra notes the students may want to include. Students may choose which vocabulary words from the list they wish to omit, however, the instructor should emphasize the importance of precision when it comes to definitions, particularly when constructing proofs and when being asked to justify a response. An example foldable is shown in the pictures below.


Construction foundations
COPYING AN ANGLE
PERPENDICULAR BISECTOR OF A LINE SEGMENT
BISECT AN ANGLE
CONSTRUCTING PERPENDICULAR TO A LINE THROUGH A POINT
CONSTRUCTING A PERPENDICULAR AT A POINT ON A LINE

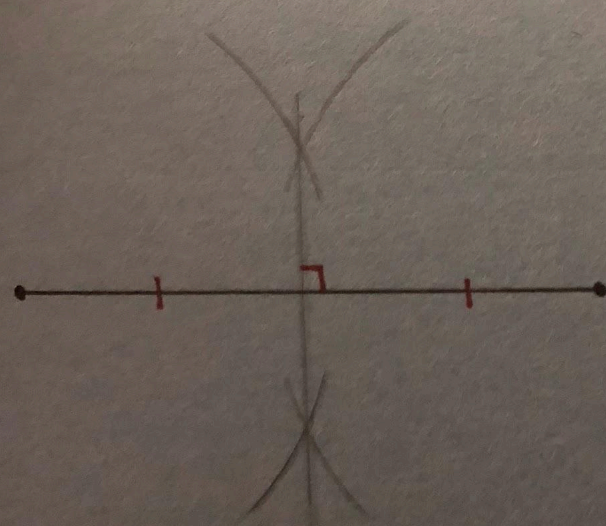
1. Draw a ray. This will be the side of the new angle.
2. Sweep an arc on the original angle. Wider compass is better.
3. Keep your compass the same width and draw this arc from the endpoints of your ray.
4. Measure the distance where the arc intersects the two sides of the original angle. Set your compass to this width.
5. Place your compass on the intersection of your ray and the arc, and sweep an



COPYING AN ANGLE

PERPENDICULAR BISECTOR OF A LINE SEGMENT

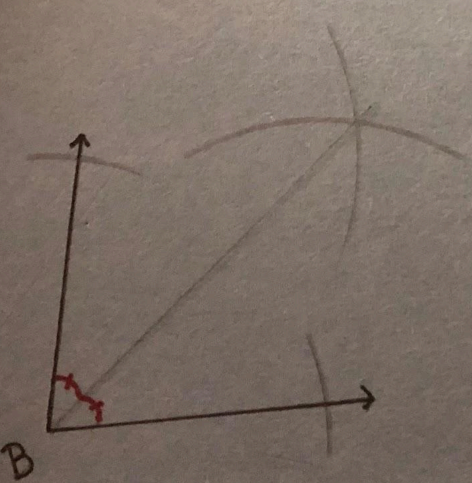
1. Place the compass on one end of the line segment
2. Set the compasses' width to over half the line length
3. Without changing the compasses' width, draw an arc above and below the line.
4. Without changing the compasses' width, place the compasses' point on the other end of the line.
5. Draw an arc above and below the line so that the arcs cross the first two
6. [illegible]ing a straightedge, draw a line between the points where [illegible] arcs intersect.



PERPENDICULAR BISECTOR OF A LINE SEGMENT

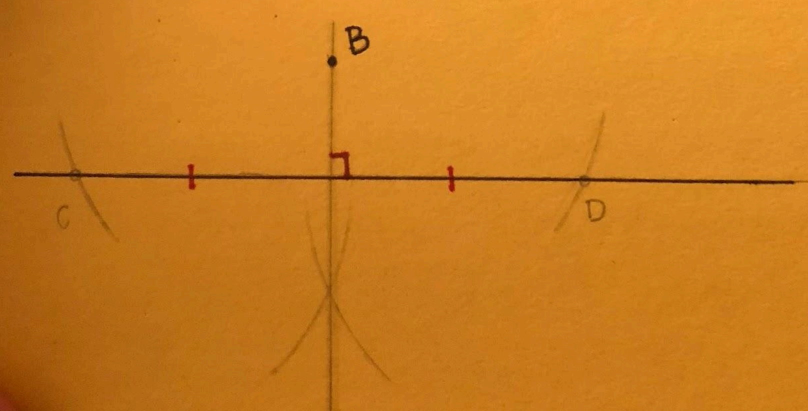
1. Place the compasses' point on the angle's vertex, B
2. Adjust the compass to a medium wide setting. The exact width is not important.
3. Without changing the width, draw an arc across each leg of the angle.
4. Place the compass on the point where one arc crosses a leg and draw an arc in the interior of the angle
5. Without changing the compass setting, repeat for the other leg so that the two arcs cross. This is the midpoint

Using a straight edge, draw a line between the midpoint and the angle's vertex B



BISECT AN ANGLE

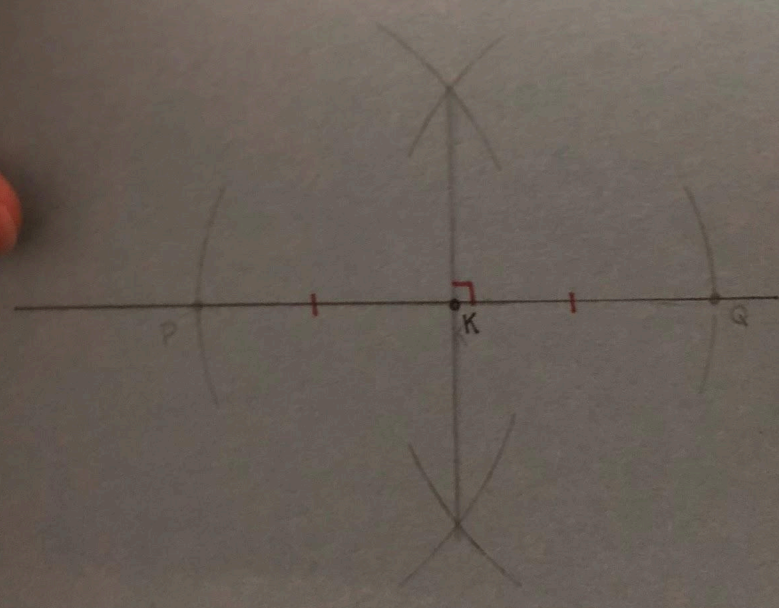
1. place the compass on the given external point B
2. set the compass width to approximately 50% more than the distance to the line. The exact width does not matter.
3. Draw an arc across the line on each side of B, making sure not to adjust the compass in between.
4. Label these points C and D. NOTE: you may have to extend the line
5. Keep the compass at the same width. For each point C, D, draw an arc below the lines so that the arcs cross.
6. place a straightedge between B and the point where the arcs intersect, then draw the perpendicular line from B to the line



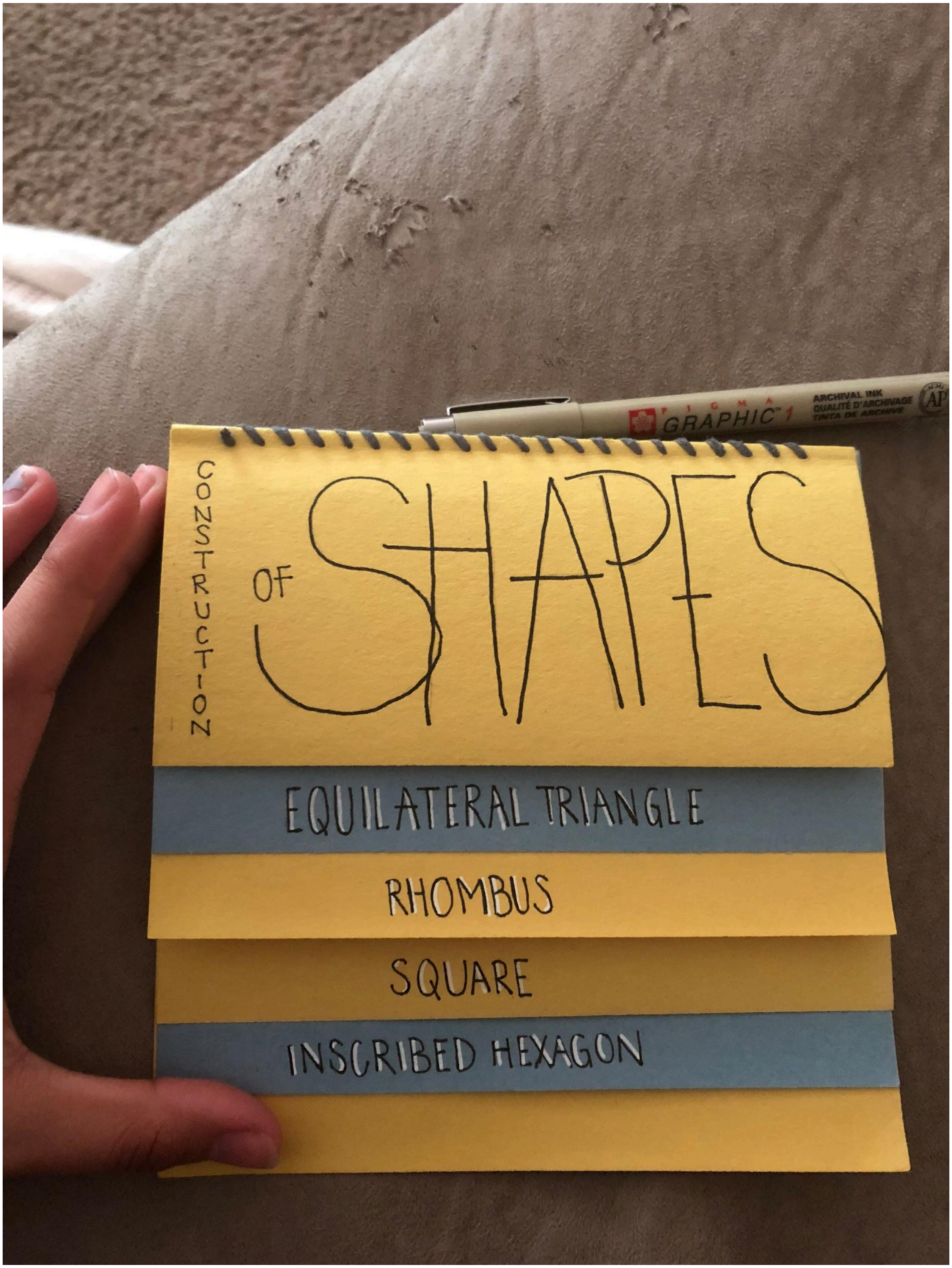
CONSTRUCTING PERPENDICULAR TO A LINE THROUGH A POINT

1. Set the compass width to a medium setting. The actual width does not matter
2. Without changing the compass's width, mark a short arc on the line at each side of the point K, forming the points P, Q. These two points are thus equidistant from K.

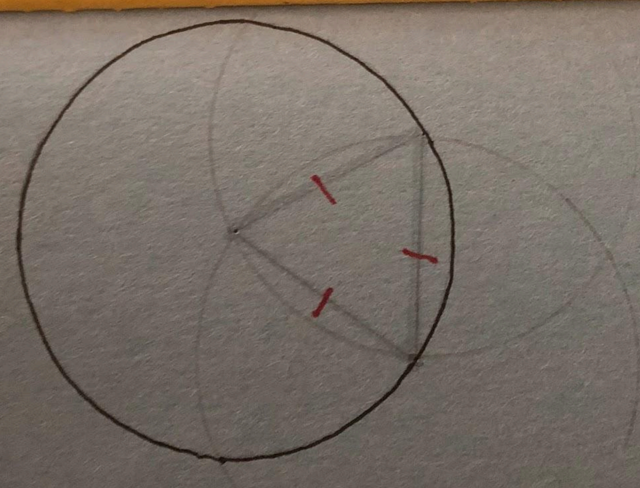
   NOTE: You may have to extend your line
3. Increase the compass to almost double the width. (The exact width is not important)
4. From P, mark off a short arc above K
5. Without changing the compasse's width, repeat from point Q so that the two arcs cross each other, creating point R.
6. Using the straightedge, draw a line from K to where the arcs cross.



CONSTRUCTING A PERPENDICULAR AT A POINT ON A LINE


CONSTRUCTION
OF SHAPES
EQUILATERAL TRIANGLE
RHOMBUS
SQUARE
INSCRIBED HEXAGON

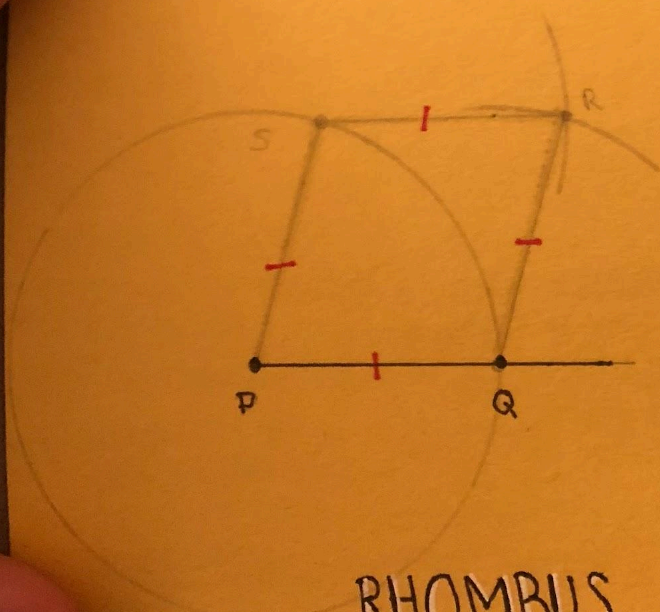
1. draw a circle and place a point anywhere on the circle.
2. without changing the width of the compass, draw a circle centered at this point. it should look like a venn diagram.
3. without changing the compass width, draw a circle centered at the point where these two circles intersect.
4. using a straightedge, connect every other point where an arc intersects the original circle to form an equilateral triangle.



EQUILATERAL TRIANGLE

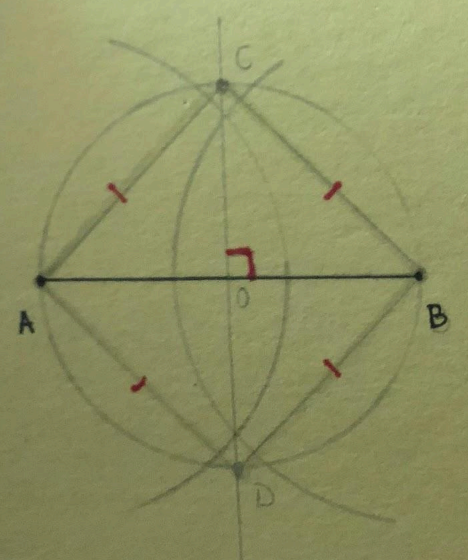
RHOMBUS

1. create a line segment $\overline{PQ}$.
2. set compass width to length of $\overline{PQ}$. (or desired side length)
3. Draw circle P, center at P, with radius PQ
4. chose any point on circle P. Label it S.
5. draw line segment SP.
6. without changing compass radius, bisect angle $\angle SPQ$. label arc intersection point R.
7. connect vertecies to form rhombus PQRS



RHOMBUS

1. construct line segment $\overline{AB}$.
2. construct the perpendicular bisector of $\overline{AB}$. label intersection point O.
3. draw circle O: centered at O, radius $\overline{AO}$ or $\overline{BO}$
4. label intersections of circle O with perpendicular bisectors as C and D.
5. using your straightedge, connect all points to make square ACBD.



SQUARE