Indian Institute of technology, Kanpur

Introduction to various Techniques of Multi-Criteria Decision Making

Anaavi Alok[a], Raghu Nandan Sengupta[a]

Abstract

MCDM represents a collection of techniques with the overall goal to determine a preference ordering among alternative decision options, whose performance is scored against multiple criteria. The MCDM process can support decision-making by helping to structure the problem and providing all involved actors with a common language for discussing and learning about the problem. It has also the potential to improve transparency, auditability, and analytic rigor of decisions.

This paper will focus on four different types of Multi-Criteria Decision Making Methods which are (1) Characteristics Objects Method (2) Conjoint Value hierarchy (3) Choosing By Advantages (4) Data Envelopment Analysis. This paper will discuss methodology, literature survey, pseudo codes of these methods.

Keywords: Fuzzy set theory, characteristic objects method , data envelopment analysis , choosing by advantages ,Conjoint Value hierarchy , rank reversals, conjoint analysis.

1. Introduction

An MCDM problem is characterized by a finite set of m alternatives, denoted as $A = \{A_i \mid i = 1, \ldots, m\}$*, that are evaluated according to n criteria, represented as* $C = \{C_j \mid j = 1, \ldots, n\}$*. The criteria can represent a benefit or a cost. A benefit criterion is desirable to be maximized, i.e., the higher an alternative score in terms of this criterion, the better the alternative is; on the contrary, for cost criteria lower values are preferable. Moreover, each criterion is assigned a weight, expressing its relative importance. These weights can be denoted as* $W = \{w_j \mid j = 1, 2, \ldots, n\}$ *and they are usually normalized, so that their sum is equal to one:* $\sum_{j=1}^{n} wj = 1$

Therefore, the MCDM problem can be concisely expressed in a matrix form, where rows and columns indicate the alternatives and the criteria, respectively. Each element x_{ij} of the decision matrix represents the score of the alternative A_i with respect to the criterion C_j.[1]

	C_1	C_2	...	C_n
	w_1	w_2	...	w_n
A_1	x_{11}	x_{12}	...	x_{1n}
A_2	x_{21}	x_{22}	...	x_{2n}
...	...	...	...	...
A_m	x_{m1}	x_{m2}	...	x_{mn}

Each element x_{ij} represents the score of A_i with respect to C_j.

[1] https://www.frontiersin.org/articles/10.3389/fenvs.2021.635100/full

Different types of decisions will require different types of decision-making methods. Roy (1974) made a classification of decisions types. These can be summarized as follows.

- ***Describing:*** *Description of each alternative and its main consequences.*
- ***Sorting:*** *Sorting all the alternatives into classes or categories.*
- ***Ranking:*** *Constructing a ranking of all admissible alternatives.*
- ***Choosing:*** *Select one and only one alternative (or a combination), the best of all.*

There are also other types of decisions described later by Belton and Stewart (2002).

- ***Selecting a Portfolio:*** *To choose a subset of alternatives from a larger set of possibilities.*
- ***Designing:*** *To research for, identify or create new decision alternatives to meet the goals and aspirations revealed though the decision process.*

Different types of methods can support these types of decisions. Multi-Criteria Decision-Making (MCDM) or Multi-Criteria Decision-Analysis (MCDA) methods help consider multiple criteria in a decision.[2]

The Characteristics Objects method is a new distance-based approach which considers the correlation between components of MCDM functions. An introduction to Fuzzy sets is significant to describe COMET. Comparisons between CO (Characteristics Objects) is easier than comparisons between alternatives. The final ranking of the COMET method is obtained based on the COs and their value of preferences. This ensures that the COMET method is free of rank reversal phenomenon.

CVH is a non-consensual, multi-attribute conjoint measuring system. The mathematical measurement system is built in a two-stage process. The crucial first step is to define the scope of the problem, that is, the precise definition of what is to be measured and what is not.

Choosing By Advantages (CBA) is a decision-making system that supports transparent and collaborative decision-making using comparisons among advantages of alternatives. CBA was developed by Jim Suhr while working in the U.S. Forest Service. The CBA Decision-Making System is based on the idea that decisions should be based on comparative advantages among the alternatives. *This idea can be explained using natural selection where one species survives because it has an advantage versus its prey*.

Data envelopment analysis (DEA) is a “data-oriented” approach for evaluating the performance of a set of peer entities called Decision Making Units (DMU), which convert multiple inputs into multiple outputs.

2.Introduction to Fuzzy sets: -

The characteristic function μ_A of a crisp set $A \subseteq X$; assigns a value either 0 or 1 to each member in X. *A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function which assigns to each object a grade of membership ranging between zero and one.*

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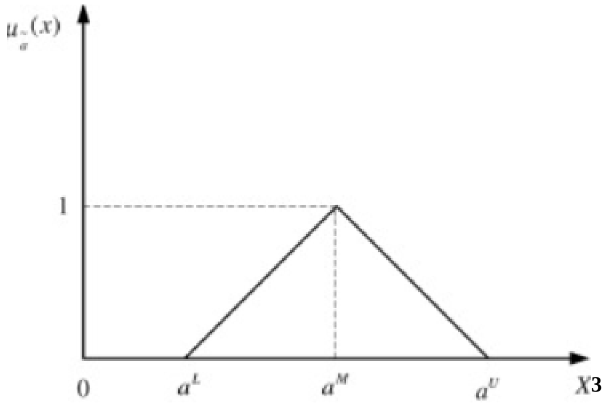
[2] https://paramountdecisions.com/training-guide/

2.1 Triangular Fuzzy Set



$$\mu_{\tilde{A}}(x,a,m,b)=\begin{cases}0, & x\leq a\\ \dfrac{x-a}{m-a}, & a\leq x\leq m\\ 1, & x=m\\ \dfrac{b-x}{b-m}, & m\leq x\leq b\\ 0, & x\geq a\end{cases},$$

2.2 The t-norm operator: product

The t-norm operator is a function T modelling the intersection operation AND of two or more Fuzzy numbers, for example, Ã.

$$\mu_{\tilde{A}}(x) AND \mu_{\tilde{B}}(y)=\mu_{\tilde{A}}(x)\cdot\mu_{\tilde{B}}(y)$$

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3. The Characteristic Objects Method:-

The commonly used rank reversal definition emphasizes that this issue occurs when the rankings for the alternatives are changed with either the addition of or removal of an alternative. COMET is constructed using the Fuzzy model and it is used to calculate preference values of the alternatives, making it a method free of rank reversal(Piegat and Sałabun 2012; Sałabun 2012).

Formal notation of this method can be presented using the following five steps:

1. Define the following space of the problem as follows:

- determine the dimensionality of the problem by selecting number r of criteria, C_1, C_2, ..., C_r;
- select the set of TFNs for each criterion C_i, that is, $\tilde{C}_{i1}$, $\tilde{C}_{i2}$,……,$\tilde{C}_{ici}$.
  The following result can be obtained as follows: -

$$C_1=\left\{\tilde{C}_{11},\tilde{C}_{12},\ldots,\tilde{C}_{1c_1}\right\}$$
$$C_2=\left\{\tilde{C}_{21},\tilde{C}_{22},\ldots,\tilde{C}_{2c_2}\right\}$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$C_r=\left\{\tilde{C}_{r1},\tilde{C}_{r2},\ldots,\tilde{C}_{rc_r}\right\}$$

where c_1, c_2,..., c_r are numbers of the Fuzzy numbers for all criteria.

2. Generate the COs.

- The COs are obtained by using the Cartesian product of TFNs' cores for each criteria as follows:-
  CO = C(C_1) x C(C_2) x C(C_3) x …… x C(C_R)

[3] https://ars.els-cdn.com/content/image/3-s2.0-B9780128155813000117-f11-02-9780128155813.jpg

$$CO_1 = \left\{C\left(\tilde{C}_{11}\right), C\left(\tilde{C}_{21}\right), \ldots, C\left(\tilde{C}_{r1}\right)\right\}$$
$$CO_2 = \left\{C\left(\tilde{C}_{11}\right), C\left(\tilde{C}_{21}\right), \ldots, C\left(\tilde{C}_{r2}\right)\right\}$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$CO_t = \left\{C\left(\tilde{C}_{1c_1}\right), C\left(\tilde{C}_{2c_2}\right), \ldots, C\left(\tilde{C}_{rc_r}\right)\right\}$$

$$t = \prod_{i=1}^{r} c_i$$

where t is a number of CO:

3. Rank and evaluate the COs.

- Determine the matrix of expert judgement (MEJ). This is a result of comparison of the COs by the knowledge of expert. The MEJ structure is as follows:

$$MEJ = \begin{array}{cc} \begin{pmatrix} \alpha_{11} & \alpha_{12} & \ldots & \alpha_{1t} \\ \alpha_{21} & \alpha_{22} & \ldots & \alpha_{2t} \\ \ldots & \ldots & \ldots & \ldots \\ \alpha_{t1} & \alpha_{t2} & \ldots & \alpha_{tt} \end{pmatrix} & \begin{array}{c} CO_1 \\ CO_2 \\ \ldots \\ CO_t \end{array} \\ \begin{array}{cccc} CO_1 & CO_2 & \ldots & CO_t \end{array} & \end{array}$$

where α_{ij} is a result of comparing CO_i and CO_j by the expert. The more preferred CO obtains one point, and the second object obtains null point. If the preferences are balanced, both objects get half point. It depends solely on the knowledge and opinion of the expert and can be presented as equation.

$$\alpha_{ij} = f\left(CO_i, CO_j\right) = \begin{cases} 0.0, f_{\exp}(CO_i) < f_{\exp}\left(CO_j\right) \\ 0.5, f_{\exp}(CO_i) = f_{\exp}\left(CO_j\right) \\ 1.0, f_{\exp}(CO_i) > f_{\exp}\left(CO_j\right) \end{cases}$$

where f_{exp} is an expert judgement function.

$$\alpha_{ii} = f(CO_i, CO_i) = 0.5$$

$$\alpha_{ji} = 1 - \alpha_{ij}$$

- the number of comparisons is reduced from t^2 cases to p cases.
- Afterwards, we obtain a vertical vector of the summed judgements (SJ) as follows:-

$$SJ_i = \sum_{j=1}^{t} \alpha_{ij}$$

- The last step assigns to each CO the approximate value of preference. In the result, we obtain a vertical vector P, where the ith row contains the approximate value of preference for CO_i. This algorithm is presented as a fragment of MATLAB code:

```
1: K = length(unique(SJ));
2: P = zeros(t,1);
3: for i = 1:k
4: ind = find(SJ == max(SJ));
5: P(ind) = (k - i)/(k - 1);
6: SJ(ind) =0;
7: End
```

4. Each CO and value of preference is converted to a Fuzzy rule as follows:-

- IF CO_i THEN P_i:-

$$\text{IF } C\left(\widetilde{C}_{1i}\right) \text{ AND } C\left(\widetilde{C}_{2i}\right) \text{ AND}\dots\text{THEN } P_i$$

- In this way, the complete Fuzzy rule base is obtained, which can be presented as equation:-

$$\begin{array}{llll} IF & CO_1 & THEN & P_1 \\ IF & CO_2 & THEN & P_2 \\ \multicolumn{4}{c}{\dots\dots\dots\dots\dots\dots} \\ IF & CO_t & THEN & P_t \end{array}$$

5. Inference in a Fuzzy model and final ranking.

- Each alternative is a set of crisp number, which corresponding with criteria C_1, C_2, ... ,C_r. It can be presented as follows:-

$A_i = \{a_{1i}, a_{2i}, \dots\dots, a_{ri}\}$ such that:-

$$\begin{array}{l} a_{1i} \in \left[C\left(\widetilde{C}_{11}\right), C\left(\widetilde{C}_{1c_1}\right)\right] \\ a_{1i} \in \left[C\left(\widetilde{C}_{21}\right), C\left(\widetilde{C}_{2c_2}\right)\right] \\ \dots\dots\dots\dots\dots\dots \\ a_{ri} \in \left[C\left(\widetilde{C}_{r1}\right), C\left(\widetilde{C}_{rc_r}\right)\right] \end{array}$$

- Each alternative activates the specified number of Fuzzy rules, where for each one is determined the fulfilment degree of the conjunctive complex premise. Fulfilment degrees of all activated rules sum to one. The preference of alternative is computed as sum of the product of all activated rules, as their fulfilment degrees, and their values of the preference.
- The final ranking of alternatives is obtained by sorting the preference of alternatives.

4. Choosing By Advantages:-

Choosing By Advantages (CBA) is a decision-making system that supports transparent and collaborative decision-making using comparisons among advantages of alternatives[4].

The difference between two alternatives can be viewed as positive or negative—as an advantage of the one alternative, or as a disadvantage of the other alternative. According to Suhr, decisions should be based on the Importance of Advantages, rather than advantages and disadvantages or the quantity of advantages. Traditional methods tend to count both the advantages and the disadvantages of certain alternatives and compare the quantity of advantages to the quantity of disadvantages. However, when they list all the advantages of a specific alternative over another alternative, they essentially list all the disadvantages of the latter compared to the former, and vice versa. So, by also making extra lists for the disadvantages of each alternative, they are double counting the differences. Suhr characterizes double counting as a "critical mistake". Traditional methods tend also to select alternatives that have the most advantages rather than the most important advantages, leading to suboptimal solutions and outcomes.

In addition, sound decisions should be anchored to the relevant facts. Traditional methods tend to base decisions on generalities, without clear and specific meanings. CBA require specific and clear criteria, consistent measurement methods, and correct data to be used. In this way, CBA also postpones the value judgment until those criteria are met.

Also, decision makers need to learn how to use sound methods. According to Suhr (2012), "The CBA skills are as basic and as essential as reading and writing. And like reading and writing skills, they are not acquired naturally." The fourth principle of CBA is based on the concepts of learning and teaching step-by-step, and introducing sound decision-making to the educational systems and to the industry. In addition to learning, the decision makers must skilfully use the sound methods of decision-making. Training and practicing is essential in this matter. According to Koga (2005), "like learning a musical instrument, daily practice is essential".

Finally, Different types of decisions call for different types of methods. Some of the methods in CBA simplify simple decisions by taking fewer steps (e.g. instant CBA), while other simplify complex decisions by take smaller steps (e.g. Tabular Method).

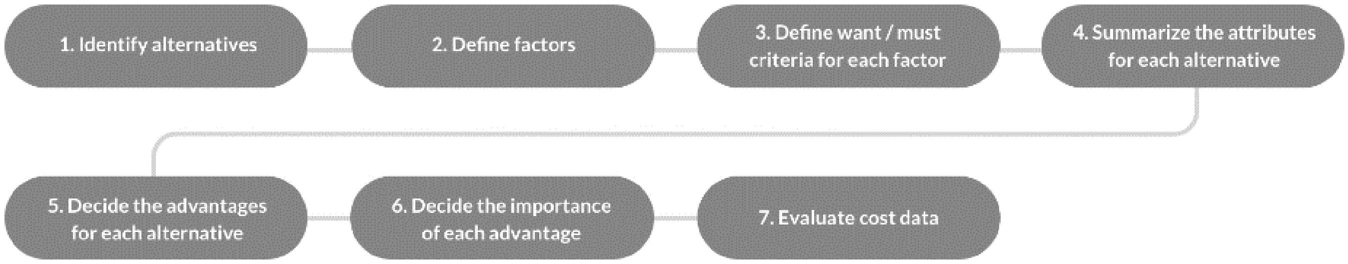
4.1 The CBA Tabular Method: -

CBA simplifies complex decision-making by dividing it into five phases:

- The Stage-Setting Phase
- The Innovation Phase
- The Decision-making Phase
- The Reconsideration Phase
- The Implementation Phase

Each phase is further divided into small steps, and there are CBA methods for each phase of the decision-making process.

[4] https://paramountdecisions.com/training-guide/



CBA Steps (Adopted from Arroyo, Tommelein and Ballard 2013)

In CBA, the alternatives are analysed comparing the advantages of one another and at the end decision makers include the cost analysis with the first and lifecycle cost of the alternatives.

To begin the decision-making process, the stakeholders identify alternatives likely to yield important advantages over other alternatives and that reflect values the team wants to realize in their design solutions (Step 1). They also have to define factors to evaluate attributes of alternatives (Step 2), and agree on the criteria for each factor (Step 3). Criteria can be either a desirable ('want') or a mandatory ('must') decision rule. Alternatives that do not comply with a 'must' criterion are not considered in the following steps.

After identifying the alternatives and defining the factors and criteria, the stakeholders summarize the attributes of each alternative (Step 4). Summarizing attributes consists of listing objective facts or data to describe the attributes of each alternative, corresponding to the factors and criteria shown, thus anchoring the decision-making process to relevant facts. Attributes are inherent to an alternative, so this step avoids subjective judgment. Repeating row by row, for each 'want' criterion, the team then determines which alternative has the least preferred attribute and underlines it.

The next step is to determine the advantages of each alternative (Step 5). This is done by comparing each attribute to the least preferred attribute in each factor. Determining the advantages of each alternative depends on the criteria that pertain to a given factor, but does not require subjective judgment.

After determining the advantages of each alternative, the stakeholders need to decide the importance of each advantage (Step 6). Stakeholders need to explicitly state their preferences for the advantages and assigning a degree of importance to each advantage is therefore the first subjective task that requires stakeholders apply their subjective judgment.[5]

Specific meanings have been given to the terms alternative, factor, criterion, attribute and advantage and their definitions are provided below.

Alternative - Two or more methods, design, systems etc., from which one or a combination of them must be chosen (e.g., choose alternative 1 or alternative 2).

Factor - An element, part, or component of a decision. It is also a container for criteria, attributes, advantages, and other types of data.

Criterion - A decision rule or a guideline (in plural: criteria). It is also any standard on which a judgment is based. It can be indicated as a 'must' criterion (mandatory) or a 'want' criterion (desirable).

Attribute - A characteristic, quality, or consequence of one alternative (not more than one).

[5] https://paramountdecisions.com/training-guide/

Advantage - An improvement, gain, or betterment. Specifically, it is a favourable difference between the attributes of two alternatives (not fewer than two and not more than two).

4.1.1 CBA with an illustration[6] -Part 1: -

1. Define the problem and the alternatives present to choose from. Identify alternatives and define all the alternatives likely to yield important advantages over other alternatives and that reflect values and goals the team wants to realize.
   In this case, the alternatives are :-
   - Toyota Camry Hybrid
   - BMW X5 SUV
   - Mercedes C Class

2. Define factors to evaluate attributes of alternatives and agree on the criterion for each factor. Stakeholders need to identify factors that will help differentiate between alternatives. It is not about which factor is most important. Factors that have an impact on the decision will change depending on the attributes of the alternatives, and the importance's assigned to advantages. Many factors were not considered in the decision-making process since the alternatives have similar attributes for those factors. Criteria can be either a desirable ('want') or a mandatory ('must') decision rule. The factors that team considers are:
   - Fuel economy
   - Trunk space
   - Reliability

Factor	**Alternative 1:** Toyota Camry Hybrid	**Alternative 2:** BMW X5 SUV	**Alternative 3:** Mercedes C Class
Fuel Economy (mpg)	40 mpg	18 mpg	22 mpg
Trunk Space (liters)	5 liters	2 liters	5 liters
Reliability	reliable	Extremely reliable	reliable

3. Decide the advantages of each alternative. This is done by first selecting the least preferred attribute in each factor and then comparing each attribute to the least preferred attribute. After determining the advantages of each alternative, select the alternative that has the bigger advantage for each factor.

- The least preferred attribute for factor fuel economy is 18 mpg. Table 2 shows the least preferred attribute underlined. Therefore, the Toyota Camry Hybrid has an advantage of 22 mpg higher than the BMW X5 SUV, and the Mercedes C Class has an advantage of 4 mpg higher than the BMW X5 SUV.

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[6] https://leanconstructionblog.com/applying-choosing-by-advantages-step-by-step.html

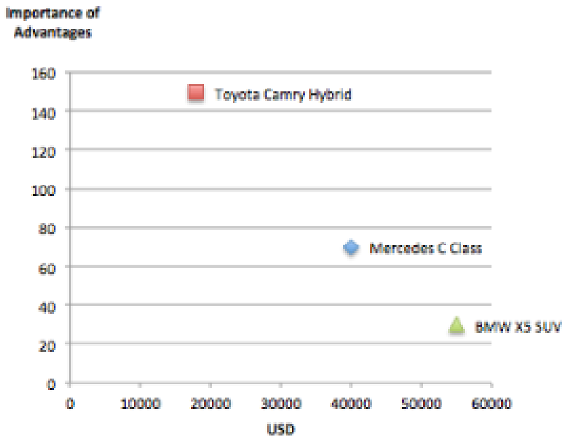
- The least preferred attribute for factor trunk space is 2 litres. Table 2 shows the least preferred attribute underlined. Therefore, both alternatives Toyota Camry Hybrid and Mercedes C Class have an advantage of having 3 more litres compared to the BMW X5 SUV.
- The least preferred attribute for factor Reliability is reliable. Table 2 shows the least preferred attribute underlined. Therefore, the BMW X5 SUV has an advantage of been more reliable than the Toyota Camry Hybrid and Mercedes C Class.

Factor (Criterion)	**Alternative 1:** Toyota Camry Hybrid		**Alternative 2:** BMW X5 SUV		**Alternative 3:** Mercedes C Class	
Fuel Economy (mpg)	Att.: 40 mpg		Att.: 18 mpg		Att.: 22 mpg	
(Higher is better)	Adv.: 22 mpg higher	Imp.:	Adv.:	Imp.:	Adv.: 4 mpg higher	Imp.:
Trunk Space (liters)	Att.: 5 liters		Att.: 2 liters		Att.: 5 liters	
(More is better)	Adv.: 3 more liters	Imp.:	Adv.:	Imp.:	Adv.: 3 more liters	Imp.:
Reliability	Att.: reliable		Att.: extremely reliable		Att.: reliable	
(More is better)	Adv.:	Imp.:	Adv.: more reliable	Imp.:	Adv.:	Imp.:
Total IofAs						

4. Decide the importance of each advantage. Assigning a degree of importance to each advantage is done by first selecting the 'Paramount' advantage —the most important advantage for all factors. The paramount advantage takes the highest spot on the importance scale (100 points).
5. The importance of the most-important advantage in each factor needs to be weighted and compared to the paramount advantage on the established scale. Determining the paramount advantage is done by comparing the most-important advantages in each factor and selecting the most-important of these most-important advantages.
6. The importance of the most-important advantage in each factor needs to be weighted and compared to the paramount advantage. Finally, a degree of importance is assigned to the remaining advantages relative to the paramount advantage. To conclude the process, the team adds up the importance of advantages for each alternative. The design alternative with the greatest total importance of advantages represents the best value solution.

Factor (Criterion)	**Alternative 1:** Toyota Camry Hybrid		**Alternative 2:** BMW X5 SUV		**Alternative 3:** Mercedes C Class	
Fuel Economy (mpg)	Att.: 40 mpg		Att.: 18 mpg		Att.: 22 mpg	
(Higher is better)	Adv.: 22 mpg higher	Imp.: 100	Adv.:	Imp.:	Adv.: 4 mpg higher	Imp.: 20
Trunk Space (liters)	Att.: 5 liters		Att.: 2 liters		Att.: 5 liters	
(More is better)	Adv.: 3 more liters	Imp.: 50	Adv.:	Imp.:	Adv.: 3 more liters	Imp.: 50
Reliability	Att.: reliable		Att.: extremely reliable		Att.: reliable	
(More is better)	Adv.:	Imp.:	Adv.: more reliable	Imp.: 30	Adv.:	Imp.:
Total IofAs		150		30		70

7. Compare the cost of each alternative with the total IofAs and chose the alternative accordingly. In this case the Toyota Camry Hybrid has 150 IofAs, which is more than the BMW X5 SUV (30 IofAs) or the Mercedes C Class (70 IofAs). The Toyota Camry Hybrid cost is US$18.000 approximately, the BMW X5 SUV cost is US$ 55000 approximately, and the Mercedes Class C is US$ 40000 approximately.



Therefore, the Toyota Camry Hybrid is cheaper in the short term and also in the long term due to fuel economy.

4.1.2 CBA with an illustration – part 2:-

1. Ms. Apeksha Athawale a young female executive is planning to buy a car and she has in her mind different attributes (list given below) based on which she wants to make the decision.
   - Purchase cost (in Rs.) which should be minimized
   - Mileage (in km per litre) which should be maximized
   - Maintenance cost per year (in Rs.) which should be minimized
   - Safety features (on a scale of 1 to 10) which should be maximized
   - Overall appeal (on a scale of 1 to 10) which should be maximized
   - Interest rate (in % or Rs.) for car loan, which should be minimized

   Assume the cars being considered by Ms. Athawale are:

   - Ford Figo (FF),
   - Hyundai Elite i20 (HE),
   - Hyundai Grand i10 Nios (HG),
   - Maruti Alto 800 (MA),
   - Maruti Baleno (MB),
   - Maruti Celerio (MC),
   - Maruti Swift (MS),
   - Renault Kwid (RK),
   - Tata Altroz (TA)
   - Tata Tiago (TT).

Thus each available choice of alternatives (i.e., car) can be represented by ordered sextet based on attribute level characteristic, which results in

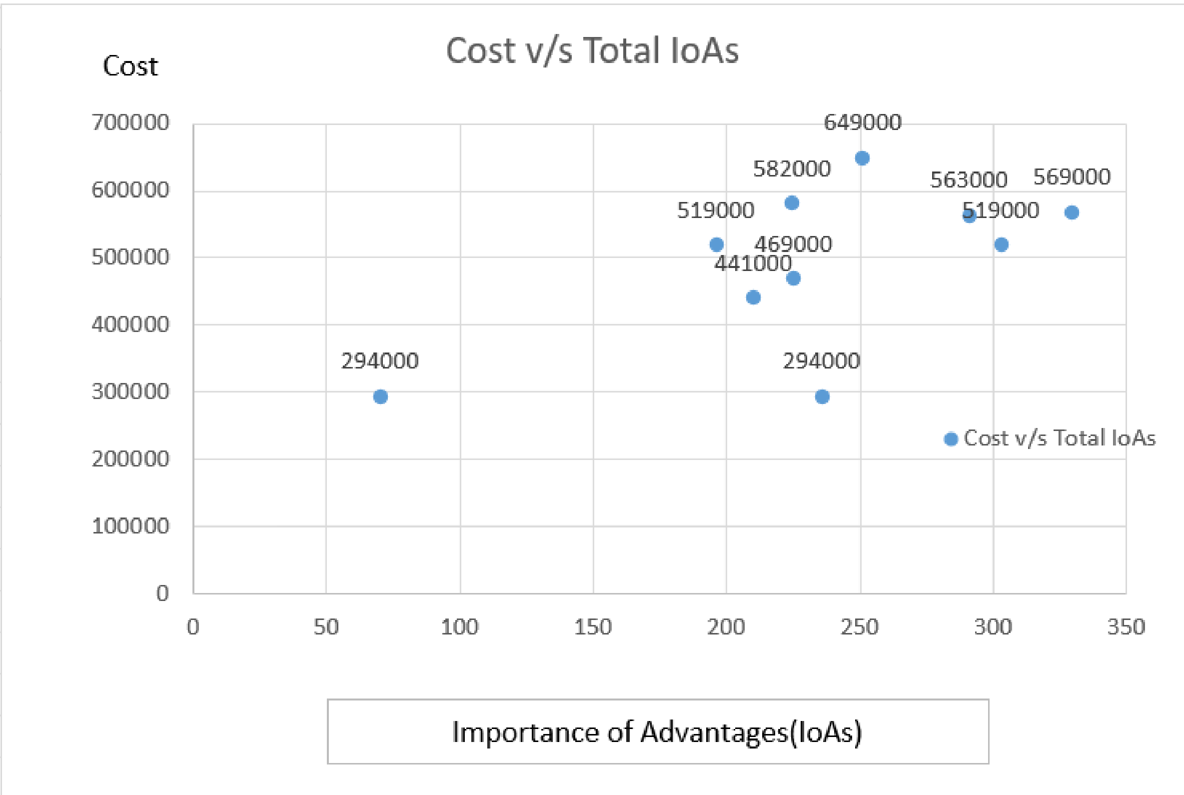
- Ford Figo (FF) = (582000, 24.40, 5831, 5, 5, 7.45)
- Hyundai Elite i20 (HE) = (649000, 22.50, 2825, 5, 3, 7.30)
- Hyundai Grand i10 Nios (HG) = (519000, 24.00, 2752, 5, 3, 7.20)
- Maruti Alto 800 (MA) = (294000, 22.05, 3705, 4, 3, 8.50)
- Maruti Baleno (MB) = (563000, 23.87, 4290, 6, 5, 7.00)
- Maruti Celerio (MC) = (441000, 21.63, 3853, 5, 4, 7.20)
- Maruti Swift (MS) = (519000, 22.00, 4704, 5, 5, 7.30)
- Renault Kwid (RK) = (294000, 22.00, 2125, 4, 3, 7.45)
- Tata Altroz (TA) = (569000, 25.11, 3809, 5, 5, 7.00)
- Tata Tiago (TT) = (469000, 23.84, 3178, 6, 6, 8.50)

Factor (criterion)	FF	HE	HG	MA	MB	MC	MS	RK	TA	TT
Mileage (in km per litre)	24.40	22.50	24.00	22.05	23.87	21.63	22.00	22.00	25.11	23.84
Maintenance cost per year (in Rs.)	5831	2825	2752	3705	4290	3853	4704	2125	3809	3178
Safety features (on a scale of 1 to 10)	5	5	5	4	6	5	5	4	5	6
Overall appeal (on a scale of 1 to 10)	5	3	3	3	5	4	5	3	5	6
Interest rate (in % or Rs.)	7.45	7.30	7.20	8.50	7.00	7.20	7.30	7.45	7.00	8.50

2. The difference in the alternatives can be seen according to the criteria that is defined for each alternative. Now, the importance of advantages for each criterion can be defined.

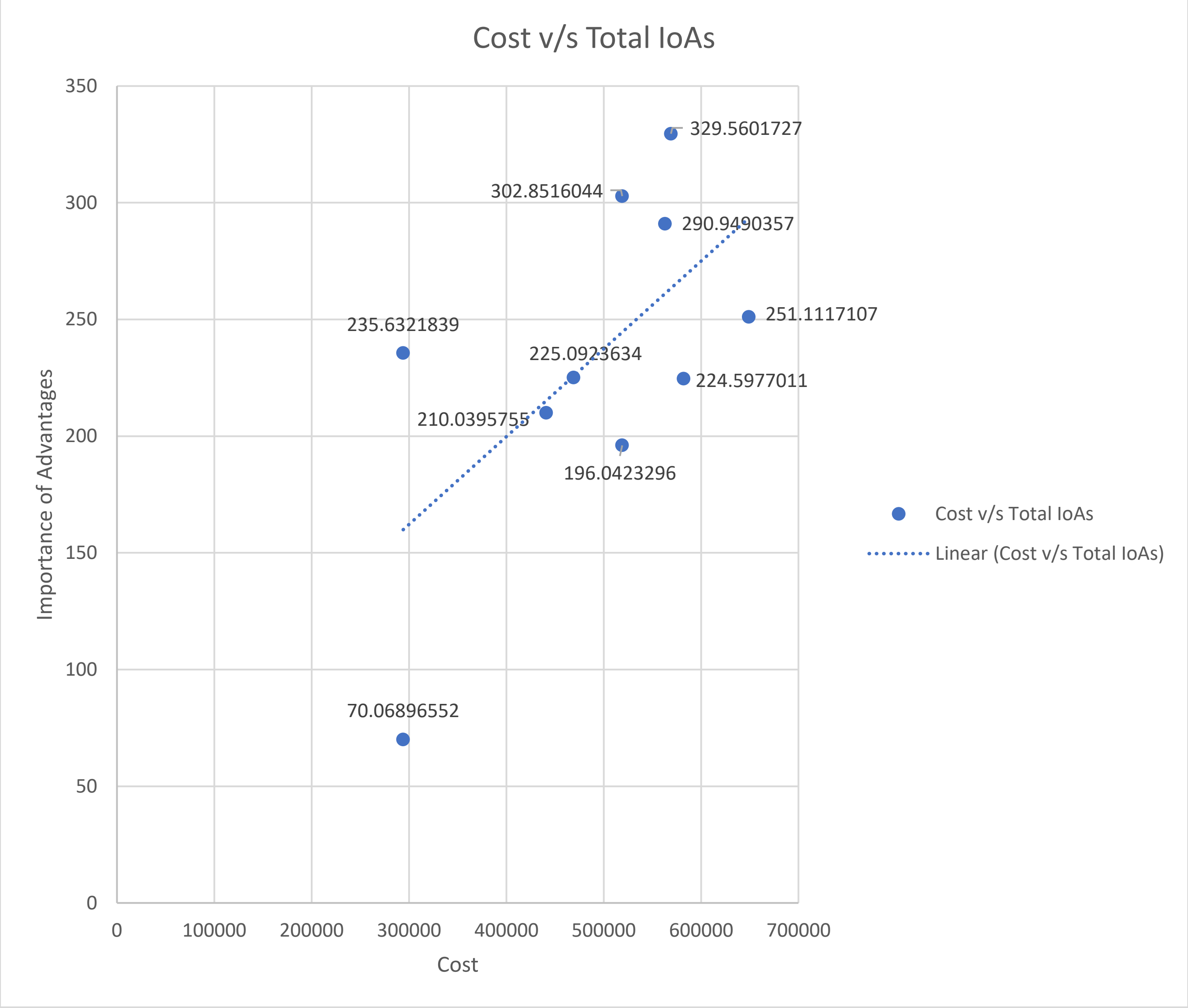
Factor(criteria)	FF	HE	HG	MA	MB	MC	MS	RK	TA	TT
Mileage (in km per litre)	24.4	22.5	24	22.05	23.87	21.63	22	22	25.11	23.84
(higher is better)	2.77	0.87	2.37	0.42	2.24	0	0.37	0.37	3.48	2.21
(IoAs)	79.59770115	25	68.10344828	12.06896552	64.367816	0	10.632184	10.6321839	100	63.50574713
Maintenance cost per year (in Rs.)	5831	2825	2752	3705	4290	3853	4704	2125	3809	3178
(lower is better)	0	3006	3079	2126	1541	1978	1127	3706	2022	2653
(IoAs)	0	81.1117107	83.08148948	57.36643281	41.58122	53.3729088	30.410146	100	54.5601727	71.5866163
Safety features (on a scale of 1 to 10)	5	5	5	4	6	5	5	4	5	6
(IoAs) (higher is better)	50	50	50	40	60	50	50	40	50	60
Overall appeal (on a scale of 1 to 10)	5	3	3	3	5	4	5	3	5	6
(IoAs)(higher is better)	25	15	15	15	25	20	25	15	25	30
Interest rate (in % or Rs.)	7.45	7.3	7.2	8.5	7	7.2	7.3	7.45	7	8.5
(lower is better)	1.05	1.2	1.3	0	1.5	1.3	1.2	1.05	1.5	0
(IoAs)	70	80	86.66666667	0	100	86.6666667	80	70	100	0
Total IoAs	224.5977011	251.111711	302.8516044	70.06896552	290.94904	210.039575	196.04233	235.632184	329.560173	225.0923634
Cost	582000	649000	519000	294000	563000	441000	519000	294000	569000	469000

3. Now the graph comparing the cost and the importance of advantages of the different alternatives based on the criterion can be made and the decision maker can choose an alternative depending upon what suits her the most.



4. Decide which alternative the decision maker wants to select based on how much she is willing to spend in order to obtain a certain level of importance of advantages.

5. The total Importance of Advantages (the importance of advantages for each alternative added up) vs. Cost chart help her evaluate the best way of spending her resources. The decision maker would evaluate 'increments' between alternatives—an increase or decrease in cost from one alternative to another — against the value (Importance of advantages) it provides, and make a decision accordingly.



5.Data Envelopment Analysis:-

Data envelopment analysis (DEA), occasionally called frontier analysis, was first put forward by Charnes, Cooper and Rhodes in 1978. It is a performance measurement technique which, as we shall see, can be used for evaluating the *relative efficiency* of *decision-making units* (*DMU's*) in organisations. Here a DMU is a distinct unit within an organisation that has flexibility with respect to some of the decisions it makes, but not necessarily complete freedom with respect to these decisions.

Since the technique was first proposed much theoretical and empirical work has been done. Many studies have been published dealing with applying DEA in real-world situations.

Building on the ideas of Farrell, the 1978 work "Measuring the efficiency of decision making units" by Charnes, Cooper & Rhodes applied linear programming to estimate, for the first time, an empirical, production-technology frontier. In Germany, the procedure had earlier been used to estimate the marginal productivity of R&D and other factors of production. Since then, there have been a large number of books and journal articles written on DEA or about applying DEA in various sets of problems.

Starting with the CCR model, named after Charnes, Cooper, and Rhodes, many extensions to DEA have been proposed in the literature. They range from adapting implicit model assumption such as input and output orientation, distinguishing technical and allocative efficiency, adding limited disposability of inputs/outputs or varying returns-to-scale to techniques that utilize DEA results and extend them for more sophisticated analyses, such as stochastic DEA or cross-efficiency analysis.

5.1 Why data envelopment analysis?

DEA sharpens the evaluative features of mathematical programming to assess actual accomplishments and deliver superior management information. DEA is an analytical system acclaimed to be "a leading-edge method of performance measurement that supports benchmarking, continuous improvement, and strategic analysis." One can only agree with Bouysou (1999) that "DEA can safely be considered as one of the recent success stories" in operations research.

For the purpose of DEA, observations are collected for a finite number of entities called decision-making units (DMUs), such as private firms, hospitals, or policy-making units (PMUs) (as in our case) entire countries, cities, regions, etc. The construction of a typical DEA model will then involve the following steps:

•determining the input and output variables.

•choice of optimization orientation—input minimization or output maximization (in the present volume output maximization is usually assumed).

•possible weight restriction (see further below).

•the use of cross-sectional data or longitudinal data.

The efficiency score of DEA is 1.00 for frontier points; the virtual output then equals virtual input. It is less than one for sub efficient points located behind the frontier (virtual output falling short of virtual input). The frontier (envelope) itself consists of the efficient observations (with scores equal to one) and the surface of the convex hull enveloping them. Frontier points are referred to as "best practice" or "benchmarks."

5.2 Methodology: -

In a one-input, one-output scenario, efficiency is merely the ratio of output over input that can be produced, while comparing several entities/DMUs based on it is trivial. However, when adding more inputs or outputs the efficiency computation becomes more complex. Charnes, Cooper, and Rhodes (1978) in their basic DEA model (the CCR) define the objective function to
find DMU_j's efficiency (θj) as:

$$\max \quad \theta_j = \frac{\sum_{m=1}^{M} y_m^j u_m^j}{\sum_{n=1}^{N} x_n^j v_n^j},$$

where the DMUj's known M outputs $y_j1,\dots,y_jm$ are multiplied by their respective weights $u_j1,\dots,u_jm$ and are divided by the N inputs $x_j1,\dots,x_jn$ multiplied by their respective weights $v_j1,\dots,v_jn$.

The efficiency score θj is to be maximized, under the constraints that using those weights on each DMU_k

for k =1,… , K, no efficiency scores exceeds one:

$$\frac{\sum_{m=1}^{M} y_m^k u_m^j}{\sum_{n=1}^{N} x_n^k v_n^j} \leq 1 \qquad k = 1, \dots, K,$$

and all inputs, outputs and weights have to be non-negative. To allow for linear optimization, one typically constrains either the sum of outputs or sum of inputs to equal a fixed value (typically 1).

Some advantages of DEA approach are:

- *no need to explicitly specify a mathematical form for the production function*
- *capable of handling multiple inputs and outputs*
- *capable of being used with any input-output measurement, although ordinal variables remain tricky*
- *the sources of inefficiency can be analysed and quantified for every evaluated unit*
- *using the dual of the optimization problem identifies which DMUs is evaluating itself against which other DMUs*

Some of the disadvantages of DEA are:

- *results are sensitive to the selection of inputs and outputs*
- *high efficiency values can be obtained by being truly efficient or having a niche combination of inputs/outputs*
- *the number of efficient firms on the frontier increases with the number of inputs and output variables*
- *a DMU's efficiency scores may be obtained by using non-unique combinations of weights on the input and/or output factors*[7]

5.3.1DEA with an illustration: -1

Assume that we have the following data:

- Unit 1 produces 100 items per day, and the inputs per item are 10 dollars for materials and 2 labour-hours
- Unit 2 produces 80 items per day, and the inputs are 8 dollars for materials and 4 labour-hours
- Unit 3 produces 120 items per day, and the inputs are 12 dollars for materials and 1.5 labour-hours

To calculate the efficiency of unit 1, we define the objective function (OF) as:-

- Max efficiency: - $(100v_1)/(10u_1+2u_2)$

which is subject to (ST) all efficiency of other units (efficiency cannot be larger than 1):

- Efficiency of unit 1: $(100v_1)/(10u_1+2u_2) \leq 1$
- Efficiency of unit 2: $(80v_1)/(8u_1+4u_2) \leq 1$

[7] https://en.wikipedia.org/wiki/Data_envelopment_analysis

- Efficiency of unit 3: $(120v_1)/(12u_1+1.5u_2) \leq 1$
  and non-negativity: u, v >0

A fraction with decision variables in the numerator and denominator is nonlinear. Since we are using a linear programming technique, we need to linearize the formulation, such that the denominator of the objective function is constant (in this case 1), then maximize the numerator.

- The new formulation would be:
- Max efficiency: - $(100v_1)$

which is subject to (ST) all efficiency of other units (efficiency cannot be larger than 1):

- Efficiency of unit 1: $(100v_1)$ $-(10u_1+2u_2) \leq 0$
- Efficiency of unit 2: $(80v_1)$ $-(8u_1+4u_2) \leq 0$
- Efficiency of unit 3: $(120v_1)$ $-(12u_1+1.5u_2) \leq 0$
  and non-negativity: u, v >0
    - $10u_1+2u_2=1$

5.3.2 DEA with an illustration :- 2

Example

Consider a number of bank branches. For each branch we have a single output measure (number of personal transactions completed) and a single input measure (number of staff).

The data we have is as follows:

Bank Branches	Personal transactions (‘000s)	Number of staff
A	125	18
B	44	16
C	80	17
D	23	11

For example, for the branch B in one year, there were 44,000 transactions relating to personal accounts and 16 staff were employed.

How can then these branches and their performances be compared using these data?

Ratios

A commonly used method is *ratios*. Typically we take some output measure and divide it by some input measure. Note the terminology here, we view branches as taking *inputs* and converting them (with varying degrees of efficiency, as we shall see below) into *outputs*.

For our bank branch example we have a single input measure, the number of staff, and a single output measure, the number of personal transactions. Hence we have:

Bank Branches	Personal transactions (‘000s)	Number of staff	Ratio = output/input Personal transactions per staff member (‘000s)
A	125	18	6.94
B	44	16	2.75
C	80	17	4.71
D	23	11	2.09

Here we can see that A has the highest ratio of personal transactions per staff member, whereas D has the lowest ratio of personal transactions per staff member .As A has the highest ratio of 6.94 we can compare all other branches to it and calculate their *relative efficiency* with respect to A. To do this we divide the ratio for any branch by 6.94 (the value for A) and multiply by 100 to convert to a percentage. This gives:

Bank Branches	Personal transactions (‘000s)	Number of staff	Ratio = output/input Personal transactions per staff member (‘000s)	Relative efficiency
A	125	18	6.94	100%
B	44	16	2.75	40%
C	80	17	4.71	68%
D	23	11	2.09	30%

The other branches do not compare well with A, so are presumably performing less well. That is, they are relatively less efficient at using their given input resource (staff members) to produce output (number of personal transactions).

- Typically, there are more than one output and input. For the bank branch example suppose now that there are two output measures (number of personal transactions completed and number of business transactions completed) and the same single input measure (number of staff) as before.

Bank Branches	Personal transactions (‘000s)	Business transactions (‘000s)	Number of staff
A	125	50	18
B	44	20	16
C	80	55	17
D	23	12	11

For example, for the B branch in one year, there were 44,000 transactions relating to personal accounts, 20,000 transactions relating to business accounts and 16 staff were employed.

As before, a commonly used method is *ratios*, just as in the case considered before of a single output and a single input. Typically, each of the output measures are taken and divided by the input measure.

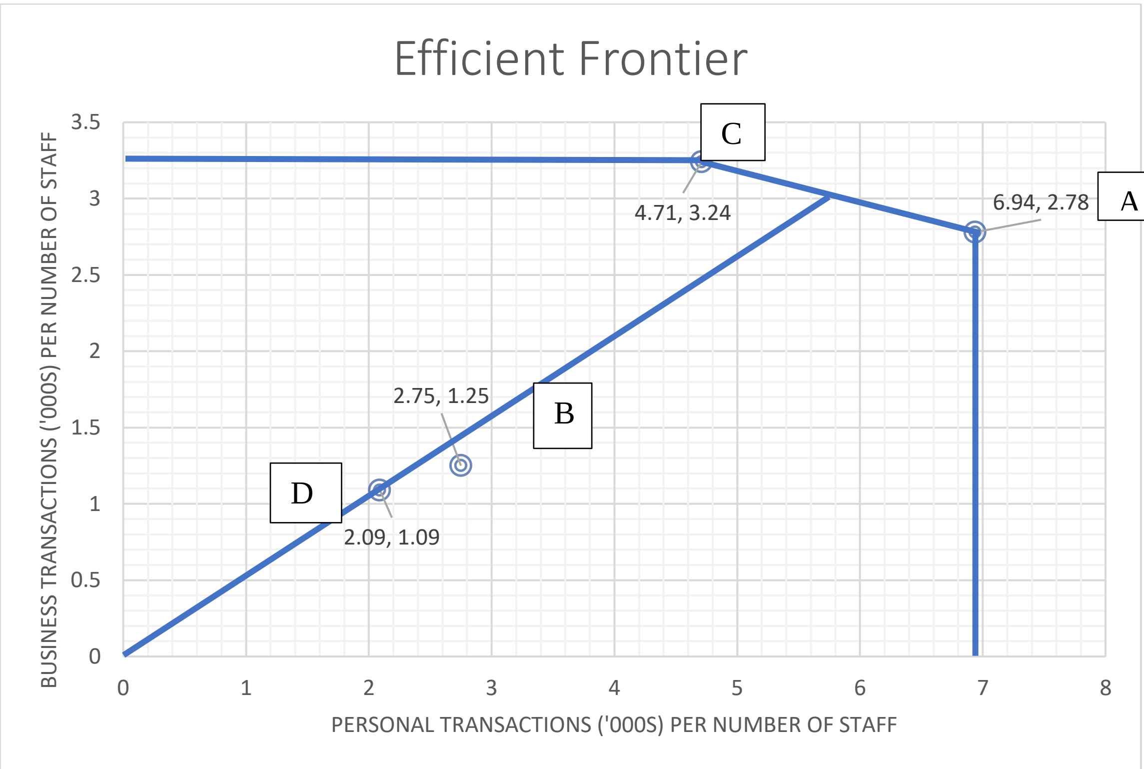
For our bank branch example, the input measure is plainly the number of staff (as before) and the two output measures are number of personal transactions and number of business transactions. So ,there are 2 ratios.

Bank Branches	Personal transactions (‘000s) per number of staff	Business transactions(‘000s) per number of staff
A	6.94	2.78
B	2.75	1.25
C	4.71	3.24
D	2.09	1.09

- Here we can see that A has the highest ratio of personal transactions per staff member, whereas C has the highest ratio of business transactions per staff member.
- B and D do not compare so well with A and C, so are presumably performing less well. That is, they are relatively less efficient at using their given input resource (staff members) to produce outputs (personal and business transactions).
- One problem with comparison via ratios is that different ratios can give a different picture and it is difficult to combine the entire set of ratios into a single numeric judgement.
- For example, consider B and D - B is (2.75/2.09) = 1.32 times as efficient as D at personal transactions but only (1.25/1.09) = 1.15 times as efficient at business transactions. These figures needs to be combined into a single judgement.
- This predicament of different ratios giving a different picture would increase when the number of branches are increased.

5.3.3 Graphical Analysis of the illustration above:-

One way around the problem of interpreting different ratios, at least for problems involving just two outputs and a single input, is a simple graphical analysis. On plotting the ratios of the personal transactions and business transactions for each of the branches :-



The positions on the graph represented by A and C demonstrate a level of performance which is superior to all other branches. A horizontal line can be drawn, from the y-axis to C, from C to A, and a vertical line from A to the x-axis. This line is called the *efficient frontier* (sometimes also referred to as the *efficiency frontier*). Mathematically the efficient frontier is the convex hull of the data.

The efficient frontier, derived from the examples of best practice contained in the data, represents a standard of performance that the branches not on the efficient frontier could try to achieve.

It can now be considered how the name *data envelopment analysis* arises - the efficient frontier envelopes (encloses) all the data we have.

Whilst a picture is all very well a number is often easier to interpret. Any branches on the efficient frontier are 100% efficient (have an efficiency of 100%). Hence, in this example, A and C have efficiencies of 100%.

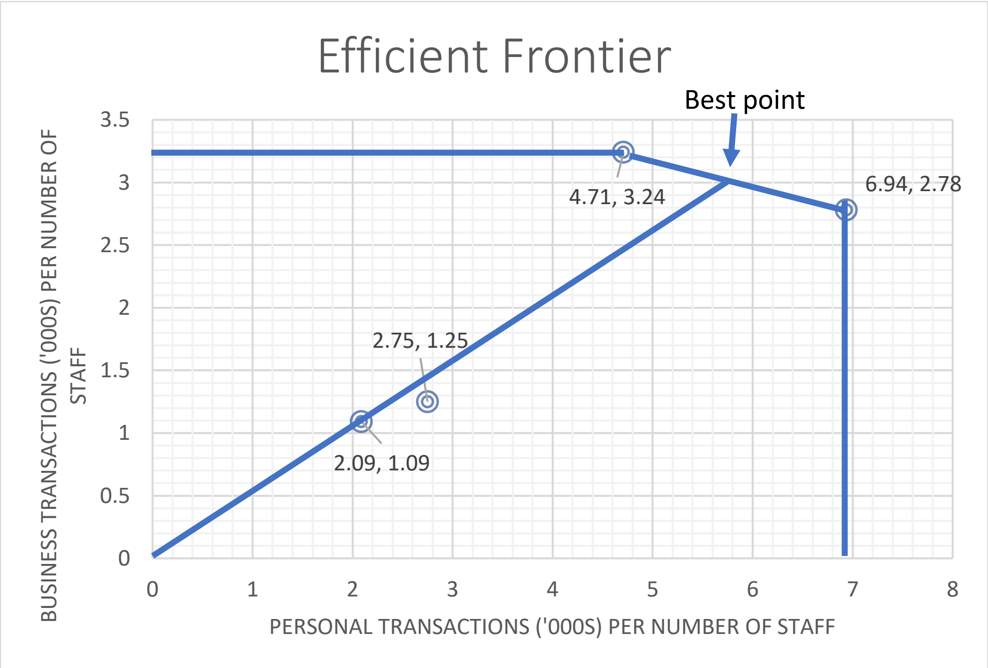
This is not to say that the performance of A and/or C could not be improved. It may, or may not, be possible to do that. However, we can say that, *on the evidence (data) available*, we have no idea of the extent to which their performance can be improved.

It is important to note here that:

- **DEA only gives out *relative efficiencies* - efficiencies relative to the data considered. It does not, and cannot, give absolute efficiencies.**
- **No new information was used to execute the graphical analysis of the example being considered.**

Quantifying the efficiency scores for the inefficient DMU's:-

- The efficient DMU's are the ones which lie on the efficient frontier graph .To increase the efficiency of the DMU's ,either the number of transactions has to be increased keeping the number of staff the same or the number of staff has to be decreased keeping the number of transactions the same.
- For D to become more efficient and also maintaining the same ratio of number of personal transactions to number of business transactions ; the number of staff employed in the Bank Branch D can be reduced in such a way that it lies on the efficient frontier graph.



5.3.3 DEA with an illustration:-

Example:-

Rank the IIMs on the basis of 3 input values that are :-

- Number of students enrolled
- Number of faculty members
- Financial resources

And 2 output values that are:-

- Project details
- Median salary of students graduating in the next year

IIMs	IIM ahmedabad	IIM Bangalore	IIM Calcutta	IIM Lucknow	IIM Kozhikode	IIM Indore
Input						
number of students	986	946	1096	953	794	1683
number of faculty members	128	113	95	87	75	100
financial resources (in Rs)	2359162165	885926011	1733460113	1173997552	387085247	698010577
Output						
median salary (in Rs)	2148548	2000000	2000000	1780000	1765000	1550000
project details/annual earnings(in Rs)	713896482	246831367	368629720	116429720	211295411	351082685

- Using python on google colab, each of the inputs and outputs were given a weight of 1, the following results are obtained .

```
Unit IIM A theta: 1.0000

Unit IIM B theta: 0.9514

Unit IIM C theta: 1.0000

Unit IIM L theta: 0.8694

Unit IIM K theta: 1.0000

Unit IIM I theta: 1.0000
```

Though the results are not that accurate primarily because different weights might have been assigned to the input and output values by NIRF.

6. Conjoint value hierarchy:-

6.1 Conjoint Analysis:-

- For understanding Conjoint value hierarchy, few concepts of conjoint analysis is implemented .

Conjoint Analysis, short for "consider jointly" is a marketing insight technique that provides consumers with combinations, pairs or groups of products that are a combination of various features and ask them what they prefer.

The product is described by several attributes and each attribute has several levels. The example discussed in this article is a full profile study which is ideal for a small set of attributes (around 4 to 5). One of the greatest strengths of Conjoint Analysis is its ability to develop market simulation models that can predict consumer behaviour to changes in the product.

Conjoint Analysis can be applied to a variety of difficult aspects of the Market research such as product development, competitive positioning, pricing, product line analysis, segmentation, and resource allocation.

6.2 Full Profile Conjoint Analysis: -

Full-profile Conjoint Analysis is one of the most fundamental approaches for measuring attribute utilities. In a full-profile conjoint task, different product descriptions are developed, ranked, and presented to the consumer for preference evaluations.

Each product profile is designed as part of a full factorial or fractional factorial experimental design that evenly matches the occurrence of each attribute with all other attributes.

By controlling the attribute pairings in a fractional factorial design, the researcher can estimate the respondent's utility for each level of each attribute tested using a reduced set of profiles.

Conjoint measurement focuses on buyers, the demand side of markets. It is a survey-based statistical technique used in market research that helps determine how people value different attributes (feature, function, benefits) that make up an individual product or service.

The objective of conjoint analysis is to determine what combination of a limited number of attributes is most influential on respondent choice or decision making. A controlled set of potential products or services is shown to survey respondents and by examining how they make choices among these products, the implicit valuation of the individual elements making up the product or service can be determined. Since we look at the product choice along with the components that make up that choice, considered jointly, the analysis was named Conjoint analysis.

Conjoint analysis techniques may also be referred to as multi attribute compositional modelling, discrete choice modelling, or stated preference research, and are part of a broader set of trade-off analysis tools used for systematic analysis of decisions.

6.2.1 Methodology

Conjoint experiment design involves four different steps:

- Determine the type of study
- Select customer segment
- Identify the relevant attributes
- Specify the attributes' levels
- Design questionnaire
- Collate results

6.2.2 Introduction to Conjoint Value Hierarchy:-

Conjoint measurement is a set of tools and results first developed in Economics and Psychology in the beginning of the '60s. Its, ambitious, aim is to provide measurement techniques that would be adapted to the needs of the Social Sciences in which, most often, multiple dimensions have to be taken into account.

Soon after its development, people working in decision analysis realized that the techniques of conjoint measurement could also be used as tools to structure preferences.

6.2.3 Conjoint Measurement Tools in decision theory :-

The starting point of most works in decision theory is a binary relation $\gtrsim$ on a set A of objects. This binary relation is usually interpreted as an "at least as good as" relation between alternative courses of action gathered in A.

Manipulating a binary relation can be quite cumbersome as soon as the set of objects is large. Therefore, it is not surprising that many works have looked for a *numerical representation* of the binary relation $\gtrsim$. The most obvious numerical representation amounts to associate a real number *V(a)* to each object $a \in A$ in such a way that the comparison between these numbers faithfully reflects the original relation $\gtrsim$.

This leads to defining a real-valued function V on A, such that:

$$a \gtrsim b \Leftrightarrow V(a) \geq V(b),$$

When such a numerical representation is possible, one can use V instead of $\gtrsim$ and, e.g., apply classical optimization techniques to find the most preferred elements in A given $\gtrsim$. We shall call such a function V a *value function*.

There are many situations in decision theory which call for the study of binary relations defined on product sets. Among them let us mention:

- Multiple criteria decision making using a preference relation comparing alternatives evaluated on several attributes
- Decision under uncertainty using a preference relation comparing alternatives evaluated on several states of nature
- Consumer theory manipulating preference relations for bundles of several goods
- Intertemporal decision making using a preference relation between alternatives evaluated at several moments in time
- Inequality measurement comparing distributions of wealth across several individuals .

6.2.3 Example:-

(EVEN SWAPS TECHNIQUE) A consultant considers renting a new office. Five different locations have been identified after a careful consideration of many possibilities, rejecting all those that do not meet several requirements.

His feeling is that five distinct characteristics, we shall say five attributes, of the possible locations should enter into his decision: his daily commute time (expressed in minutes), the ease of access for his clients (expressed as the percentage of his present clients living close to the office), the level of services offered by the new office (expressed on an ad hoc scale with three levels:

A (all facilities available), B (telephone and fax), C (no facilities)), the size of the office expressed in square feet, and the monthly cost expressed in dollars.

The evaluation of the five offices is given in the table. The consultant has well-defined preferences on each of these attributes, independently of what is happening on the other attributes. His preference increases with the level of access for his clients, the level of services of the office and its size. It decreases with commute time and cost. This gives a first easy way to compare alternatives through the use of dominance.

	a	*b*	*c*	*d*	*e*
Commute	45	25	20	25	30
Clients	50	80	70	85	75
Services	A	B	C	A	C
Size	800	700	500	950	700
Cost	1850	1700	1500	1900	1750

- An alternative y is dominated by an alternative x if x is at least as good as y on all attributes while being strictly better for at least one attribute. Clearly dominated alternatives are not candidate for the final choice and may, thus, be dropped from consideration.
- The reader will easily check that, on this example, alternative b dominates alternative e: b and e have similar size, but b is less expensive, involves a shorter commute time, an easier access to clients and a better level of services.
- We may therefore forget about alternative e. This is the only case of "pure dominance" in our table. It is however easy to see that d is "close" to dominating a. the only difference in favour of a being on the cost attribute (50 $ per month).
- This is felt more than compensated by the differences in favour of d on all other attributes: commute time (20 minutes), client access (35 %) and size (150 sq. feet).

Dropping all alternatives that are not candidate for choice, this initial investigation allows to reduce the problem to:

	b	*c*	*d*
Commute	*25*	*20*	*25*
Clients	*80*	*70*	*85*
Services	*B*	*C*	*A*
Size	*700*	*500*	*950*
Cost	*1700*	*1500*	*1900*

- A natural way to proceed is then to assess trade offs. Observe that all alternatives but c have a common evaluation on commute time. We may therefore ask the consultant, starting with office c what gain on client access would compensate a loss of 5 minutes on commute time. We are looking for an alternative c’ that would be evaluated as follows: and judged indifferent to c Although this is not an easy question, it is clearly crucial in order to structure preferences.

	c	c'
Commute	*20*	**25**
Clients	*70*	$\mathbf{70+\delta}$
Services	*C*	*C*
Size	*500*	*500*
Cost	*1500*	*1500*

- Since trade-offs questions may be difficult, it is wise to start with an attribute requiring few assessments (in the example, all alternatives but one have a common evaluation on commute time). Clearly this attribute should be traded against one with an underlying “continuous” structure (cost, in the example).
- Suppose that the answer is that for **δ** = 8, it is reasonable to assume that and would be indifferent. This means that the decision table can be reformulated as follows:

	b	c'	d
Commute	*25*	*25*	*25*
Clients	*80*	*78*	*85*
Services	*B*	*C*	*A*
Size	*700*	*500*	*950*
Cost	*1700*	*1500*	*1900*

- It is then apparent that all alternatives have a similar evaluation on the first attribute which, therefore, is not useful to discriminate between alternatives and may be forgotten. The reduced decision table is as follows:

	b	c'	d
Clients	*80*	*78*	*85*
Services	*B*	*C*	*A*
Size	*700*	*500*	*950*
Cost	*1700*	*1500*	*1900*

- There is no case of dominance in this reduced table. Therefore further simplification calls for the assessment of new trade-offs. Using cost as the reference attribute, we then proceed to “neutralize” the service attribute. Starting with office c', this means asking for the increase in monthly cost that the consultant would just be prepared to pay to go from level “C” of service to level “B”. Suppose that this increase is roughly 250 $. This defines alternative c''. Similarly, starting with office we ask for the reduction of cost that would exactly compensate a reduction of services from “A” to “B”. Suppose that the answeris 100 $ a month, which defines alternative d'. The decision table is reshaped as:

	b	c''	d'
Clients	*80*	*78*	*85*
Services	*B*	**B**	**B**
Size	*700*	*500*	*950*
Cost	*1700*	***1750***	***1800***

- We may forget about the second attribute which does not discriminate any morebetween alternatives. When this is done, it is apparent that c'' is dominated by *b* and can be suppressed. Therefore the decision table at the stage looks like the following:

	b	d'
Clients	*80*	*85*
Size	*700*	*950*
Cost	*1700*	*1800*

- Unfortunately, this table reveals no case of dominance. New trade-offs have to be assessed. We may now ask, starting with office b, what additional cost the consultant would be ready to incur to increase its size by 250 square feet. Suppose that the rough answer is 250 $ a month, which defines b'. We are now facing the following table.

	b'	d'
Clients	*80*	*85*
Size	**950**	*950*
Cost	**1950**	*1800*

- Attribute size may now be dropped from consideration. But, when this is done,it is clear that d' dominates b'. Hence it seems obvious to recommend office *d* as the final choice.
- The above process is simple and looks quite obvious. If this works, why be interested at all in "measurement" if the idea is to help someone to come up with a decision?
- First observe that in the above example, the set of alternatives was relatively small. In many practical situations, the set of objects to compare is much larger than the set of alternatives in our example. Using the even swaps technique could then require a considerable number of difficult tradeoff questions.
- Furthermore, as the output of the technique is not a preference model but just the recommendation of an alternative in a given set, the appearance of new alternatives (e.g. because a new office is for rent) would require starting a new round of questions. This is likely to be highly frustrating. Finally, the informal even swaps technique may not be well adapted to the, many, situations, in which the decision under study takes place in a complex organizational environment.
- In such situations, having a formal model to be able to communicate and to convince is an invaluable asset. Such a model will furthermore allow to conduct extensive sensitivity analysis and, hence, to deal with imprecision both in the evaluations of the objects to compare and in the answers to difficult questions concerning tradeoffs.
- This clearly leaves room for a more formal approach to structure preferences. But where can "measurement" be involved in the process? It should be observed that, beyond surface, there are many analogies between the even swaps processand the measurement of length considered above.
- Conjoint measurement techniques may roughly be seen as a formalization of the even swaps technique that leads to building a numerical model of preferences much in the same way that we built a numerical model for length. This will require assessment procedures that will rest on the same principles asthe standard sequence technique used for length. This process of "measuring preferences" is not an easy one. It will however lead to a numerical model of preference that will not only allow us to make a choice within a limited number of alternatives but that can serve as an input of computerized optimization algorithms that will be able to deal with much more complex cases.

7.Conclusion

In their daily lives, people usually weigh multiple criteria implicitly and may be comfortable with the consequences of such decisions that are made based on only intuition. On the other hand, when stakes are high, it is important to properly structure the problem and explicitly evaluate multiple criteria. In making the decision of whether to build a nuclear power plant or not, and where to build it, there are not only very complex issues involving multiple criteria, but there are also multiple parties who are deeply affected by the consequences.

Structuring complex problems well and considering multiple criteria explicitly leads to more informed and better decisions. There have been important advances in this field since the start of the modern multiple-criteria decision-making discipline in the early 1960s.

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