BedSAT: Antarctica

Exploring what lies beneath using big data and modelling

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Impact Statement

Antarctica's bed topography data currently has local uncertainties of hundreds of meters in elevation due to sparse and unevenly distributed radar surveys, significantly limiting our ability to predict ice sheet behaviour and sea level rise contributions. Through the BedSAT project, I am developing a novel modelling approach that integrates remote sensing data and airborne-derived estimates, with mathematical and numerical ice flow models to substantially improve bed topography resolution and accuracy. I aim to derive a continent-wide bed topography dataset and conduct sensitivity analyses of dynamic ice loss to different realisations of topographic roughness through 2300CE. My work will quantify how bed topography uncertainties affect ice mass loss projections. This improved understanding can provide more reliable sea level rise predictions, and enable evidence-based policy decisions for climate adaptation strategies. This open-source approach and FAIR data principles will ensure these improvements benefit the broader scientific community and support more effective climate change mitigation planning.

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Antarctica's Landscape

1.1 Climate Impacts and Global Significance

The polar regions are losing ice, and their oceans are changing rapidly [1]. The consequences of this extend to the whole planet and it is crucial for us to understand them to be able to evaluate the costs and benefits of potential mitigation.

Changes in different kinds of polar ice affect many connected systems. Of particular concern is the accelerating loss of continental ice sheets (glacial ice masses on land) in both Greenland and Antarctica, which has become a major contributor to global sea level rise [1]. Impacts extend beyond direct ice loss: as fresh water from melting ice sheets is added into the ocean, it increases ocean stratification disrupting global thermohaline circulation [2]. In addition, cold freshwater can dissolve larger amounts of CO_2 than regular ocean water creating corrosive conditions for marine life [1].

While there is high confidence in current ice loss and retreat observations in many areas, there is more uncertainty about the mechanisms driving these changes and their future progression [3]. Uncertainty increases in regions with variable bed conditions, where characteristics like "slipperiness" and "roughness" are difficult to verify via direct observations. Other problematic areas involve the ice sheet's grounding line (GL), the zone that delineates ice grounded on bedrock from ice shelves floating over the ocean. The retreat rate depends crucially on topographical features like pinning points [3], which lead to increased buttressing by the ice shelf on the upstream ice sheet. Although this mechanism is established, major knowledge gaps persist in mapping bed topography across Antarctic ice sheet margins - with over half of all margin areas having insufficient data within 5 km of the grounding zone [4]. Addressing this data gap through both systematic mapping and improved interpolation utilising auxiliary data streams with more complete coverage would significantly improve both our understanding of current ice dynamics and the accuracy of ice-sheet models projecting future changes.

Topography of Antarctica

Bed topography is one of the most crucial boundary conditions that influences ice flow and loss from the Antarctic Ice Sheet (AIS) [5]. Bed topography datasets are typically generated from airborne radar surveys, which are sparse and unevenly distributed across the Antarctic continent (see figure 2.1). Interpolation schemes to "gap fill" these sparse datasets yield bed topography estimates that have high uncertainties (i.e. multiple hundreds of metres in elevation uncertainty; Morlighem et al., 2020) which propagate through simulations of AIS evolution under climate change [6]. Given the logistical challenges of accessing large parts of the Antarctic continent, there is a crucial need for alternative approaches that integrate diverse and possibly more spatially complete data streams – including satellite data.

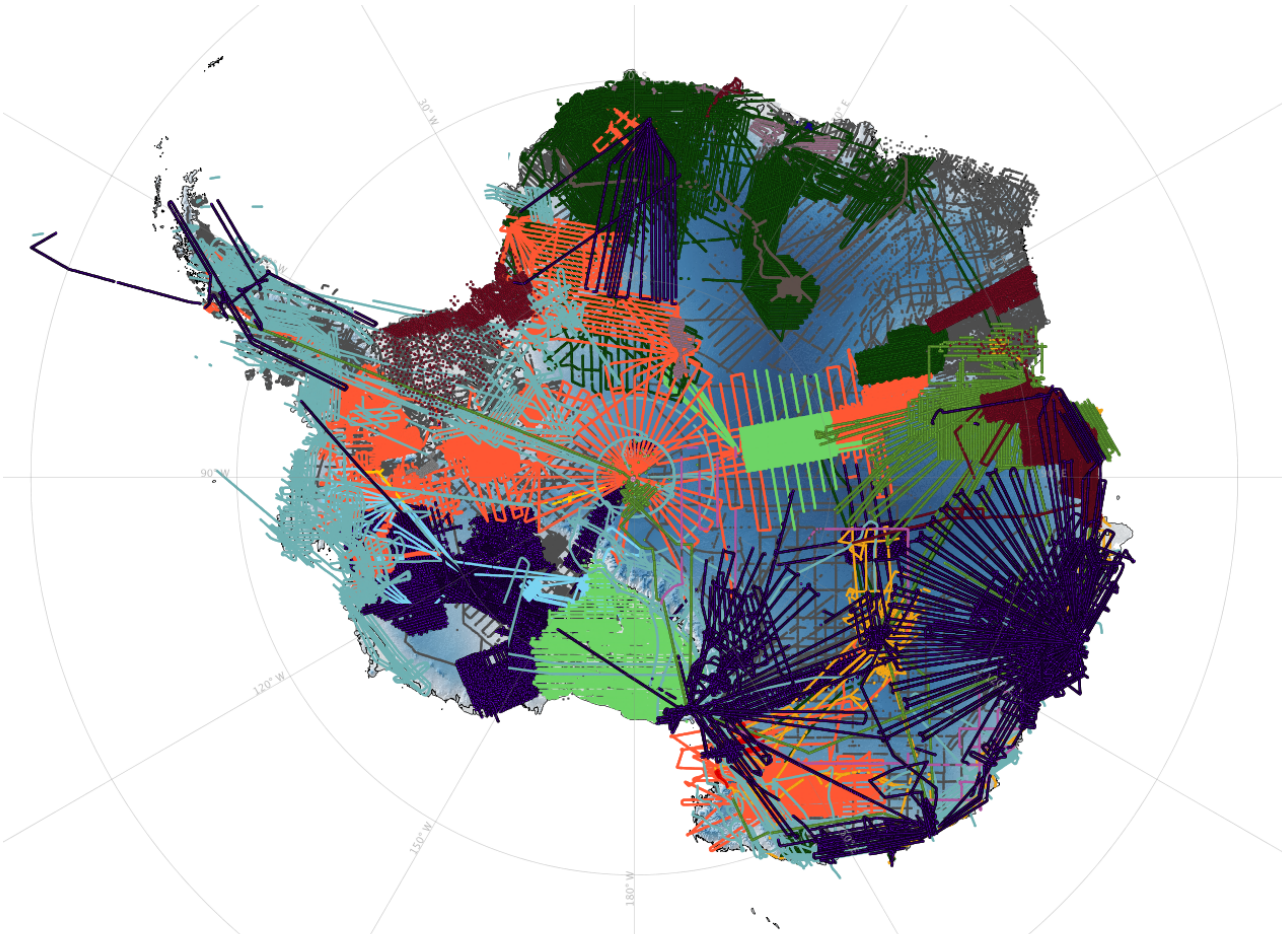

Figure 2.1: Distribution of BedMAP{1,2,3} data tracks (Source: bedmap.scar.org).

2.1 Approaches to Bed Topography Reconstruction

A key objective of this study is to understand objective is to understand the bed topography itself and how it influences ice dynamics. There are two ways to can infer information about this relationship: Through forward modelling, with assumptions of the bed conditions; and through inverse modelling that relies on surface observations.

- **Forward models**
  The aim of forward models is to see how bed properties impact ice dynamics. A key example is using a large ensemble of bed topographies to investigate how bed uncertainties impact simulated ice mass loss. In this example geostatistical methods can be used to generate bed topographies that either preserve elevation or texture:
  - **Geostatistics** Statistical methods specialized for analyzing spatially correlated data. In glaciology, this approach is used to interpolate between sparse measurements and characterise spatial patterns in bed properties, often employing techniques like kriging [7].
- **Inversion models**
  The aim of these models is to understand bed properties through knowledge of surface or other variables. A key example is the retrieval of bed topography or basal slipperiness from surface elevation and velocities.
  - **Control method inversion**: A variational approach that minimizes mismatches between observed and simulated fields through a cost function approach. Remote sensing data and theoretical ice flow models are used to obtain basal conditions [8]. Often needs regularization terms to prevent non-physical features or over-fitting [9].
  - **4dvar**: Four-dimensional variational data assimilation - Similar to the control method inversion algorithm, but adds a time dimension. Used to optimize model parameters and initial conditions [9]. Can handle time-varying data and evolving glacier states, making it more suitable for dynamic systems unlike control methods, this makes them more computationally demanding [9].
  - **Mass conservation**: Used to constrain inversion models and fill data gaps by employing physical conservation laws, particularly effective for reconstructing bed topography where direct measurements are sparse [5, 10]. Requires (contemporary) measurements of ice thickness at the inflow boundary to properly constrain the system [9].
  - **Markov Chain Monte Carlo (MCMC)**: A probabilistic method that generates sample distributions to quantify uncertainties in ice sheet parameters and models [9]. While powerful for uncertainty quantification, these methods remain computationally intensive for continental-scale ice sheet models [9].
  - **EnKF** Ensemble Kalman Filter. A sequential data assimilation method that uses an ensemble of model states to estimate uncertainty and update model parameters based on observations [9].

This study aims to develop an integrated method combining forward and inverse modeling to improve bed topography estimates by leveraging high-resolution satellite surface data in regions where radar data is sparse. Despite revolutionary advances in satellite technology that provide unprecedented surface detail, a key challenge in glaciology remains: how to fully utilize this wealth of information in regions where our understanding of subglacial conditions is limited. Our approach will integrate more comprehensive dynamical ice models with modern computational capabilities to develop better bed topography.

2.2 Theoretical Frameworks

Understanding how bed features manifest in surface observations requires a theoretical framework that connects these two domains. The modelling approach used in this project relies on two different theoretical frameworks that relate bed topography and surface features. Using synthetic data, observations and these modelling frameworks, my goal is understanding the limitations of each approach and how they can be improved.

The first framework was originally developed by Budd [11]. This model relates ice flow over bedrock perturbations to surface expressions using a two-dimensional bi-harmonic stress equation. The modelling carried out in [11] determined ice-sliding velocities for wide ranges of roughness, normal stress, and shear stress relevant to real glaciers [11]. Budd's framework fits in my project as a means of verifying the validity of the physics in my ice-sheet model, since it describes important effects of bedrock disturbances on the transient evolution of the transferred basal disturbances onto the surface. The theory makes several key predictions that have been confirmed through spectral analysis of real ice cap profiles:

1. A wavelength of minimum damping occurs at approximately 3.3 times the ice thickness,

2. surface undulations exhibit a $\pi/2$ phase lag relative to bedrock features with steepest surface slopes occurring over the highest bedrock points, and

3. the amplitude reduction depends systematically on ice speed, viscosity, thickness, and wavelength.

Importantly, Budd's framework demonstrates that energy dissipation and basal stress patterns are maximized for bedrock irregularities with wavelengths several times the ice thickness, while smaller-scale bedrock variations decay exponentially with distance into the ice and have minimal impact on overall ice motion. This selective filtering of bedrock signals provides crucial insights for understanding which scales of bed topography most significantly influence ice dynamics.

A critical aspect of Budd's theoretical framework is understanding how ice rheology affects the bed-to-surface transfer relationships. Glen's flow law typically employs a stress exponent $n \approx 3$ for ice under most natural conditions, reflecting the strongly nonlinear relationship between stress and strain rate. Budd's analysis revealed that under certain low-stress conditions, ice deformation can behave more linearly $n \approx 1$ than conventional wisdom suggests. This rheological distinction has profound implications for bed-to-surface transfer functions: linear rheology ($n = 1$) produces different amplitude dampening and phase relationships compared to nonlinear rheology ($n = 3$), particularly for wavelengths around the critical 3.3 times ice thickness scale. My current experimental framework systematically explores this in 1D by generating forward models for multiple synthetic bedrock profiles across four scenarios combining rheological assumptions ($n = 1$ vs $n = 3$) with basal boundary conditions (no-slip vs sliding), enabling direct comparison of how these physical assumptions affect the detectability and reconstruction of bed features from surface observations. Understanding these differences is essential for developing robust inversion methods, as the choice of rheological model fundamentally determines the mathematical relationship between observable surface expressions and the underlying bed topography I seek to reconstruct.

Crucially, Budd's work established the concept of frequency-dependent transfer functions that act as "filters" between bed and surface topography. This transfer function approach, expressed as $\psi(\omega) = \frac{\text{surface amplitude}}{\text{bed amplitude}}$ for wavelength $\lambda = 2\pi/\omega$, provides a direct mathematical framework for inversion. By inverting these transfer functions, one can theoretically reconstruct bed topography from surface observations, particularly for wavelengths where the damping factor is minimal and the signal-to-noise ratio is optimal.

The second framework in my analysis builds upon these foundational concepts through recent work by Ockenden et al., which demonstrates how shape and mechanical properties of the ice bed significantly influence ice flow, with changes at the bed potentially leading to large differences in predicted ice loss rates [12]. Recent work by Ockenden et al. demonstrates both the capabilities and limitations of current inversion approaches in addressing this problem.

The core principle of the method by Ockenden et al. (2022) relies on the fact that variations in basal topography, slipperiness, and roughness cause measurable disturbances to the surface flow of the ice. Through linear perturbation analysis, they establish a systematic relationship between surface observations and bed conditions. This relationship can be expressed as $y = f(x)$, where y represents surface measurements (velocity and topography), x represents bed properties (topography and slipperiness), and f is the forward model [13]. The inversion process, $x = f^{-1}(y)$, estimates bed conditions from surface observations.

The method works best when analyzing perturbations that are small relative to mean properties, under specific conditions including:

1. A linear viscous medium ($n = 1$)
2. Non-linear sliding law ($m > 0$)
3. Steady-state conditions
4. Spatially constant zero-order solutions

Using high-resolution datasets (REMA surface elevation at 8m resolution and NASA ITS_LIVE velocity at 120m resolution), their approach performs well for:

- Areas with moderate topographic gradients
- Features not aligned with ice flow direction
- Medium-wavelength (5-50km) bedrock features

Nevertheless, significant limitations emerge when:

- Dealing with steep topography where the shallow-ice-stream approximation breaks down
- Handling variable slipperiness parameters
- Attempting to validate slipperiness predictions due to lack of ground-truth data

2.2.1 Bridging Classical and Modern Approaches

While both frameworks address the fundamental bed-to-surface relationship, they operate at different levels of complexity and make different assumptions. Budd's approach provides the fundamental physical understanding of how specific wavelengths propagate through ice, establishing theoretical limits on what bed features can be detected from surface observations. Ockenden's method extends this to practical applications using real satellite data but relies on linearised assumptions that may break down under certain conditions. My research aims to bridge these approaches by combining Budd's rigorous transfer function analysis with comprehensive forward modeling that relaxes some of the restrictive assumptions inherent in linear perturbation methods. By systematically exploring how different rheological models ($n = 1$ vs $n = 3$) and basal conditions affect the bed-to-surface transfer functions, this work aims to develop a robust inversion method that can better handle the nonlinear physics of ice flow while maintaining the theoretical rigor established by Budd's foundational analysis.

2.3 Current Opportunities

Current Antarctic bed topography reconstruction methods fail to utilize the wealth of presently available satellite-derived surface data. While mathematical models linking bed to surface through ice dynamics (such as those by Ockenden and Budd) provide a foundation for inferring bed topography from satellite data, they have significant limitations. My approach with BedSAT builds upon theoretical foundations and recent inversion methods to better understand how bed conditions—including slipperiness, roughness, and pinning points—affect both grounding line retreat rates and their surface expressions. BedSAT will connect surface observations with bed topography using more realistic rheological and geometric assumptions through an iterative process: initially inverted bed topography will feed into ice dynamics models with these improved assumptions, allowing comparison between model predictions and established datasets like NASA's ITS_LIVE. I expect to utilise Machine learning methods to systematize this process, enhancing the analytical capabilities for the project's final phase.

1927 words in this section.

Methods

3.1 Aims

I will address the following research questions:

1. How does the bed topography manifest on the ice surface?

2. To what extent do interpolation uncertainties in bed topography datasets affect the accuracy of Antarctic Ice Sheet evolution simulations under different climate change scenarios?

3. What is the impact of variable bed conditions and topography influence the rate of GL retreat in continental ice sheets?

Underpinning these research questions are the following objectives:

1. Develop an ice sheet modelling approach to assimilate satellite remote sensing datasets to improve knowledge of the bed (BedSAT) informed by mathematical models of ice flow over topography;

2. Derive a new bed topography for Antarctica using BedSAT;

3. Evaluate the impact of the improved bed topography on projections of ice mass loss from Antarctica under climate warming through sensitivity analyses.

3.2 Research plan methodology

Objective 1 is to derive the BedSAT method. I will use a regional catchment in Antarctica for which relatively more radar data are available and has an indicative range of topography features, e.g. the Aurora Subglacial Basin, East Antarctica, extensively surveyed by Collaborative Exploration of the Cryosphere through Airborne Profiling (ICECAP) [14]. In developing my inversion approach, I will consider important factors like sliding variability, to distinguish between topographical signatures and sliding-induced features in surface expressions. I will implement a realistic sliding-law in the Ice-sheet and Sea-level System Model (ISSM) that incorporates Budd's determinations of ice-sliding velocities across varying roughness, normal stress, and shear stress conditions. By calibrating these advanced models against known radar-surveyed regions with varying sliding conditions and validating with other observational data, I can better isolate true bed topographical features from friction-related artifacts and create a more robust inversion framework.

The second phase of the project (objective 2) will apply the methodology developed in objective 1 to the whole Antarctic continent, deriving a continent-wide bed topography dataset. Using covariance properties from existing radar surveys, I will generate a number of realisations of bed topography with unique high-resolution, and statistically-consistent topographic roughness. In the third phase of the project will use the new bed topography datasets to conduct a sensitivity analysis of ice sheet model projections to 2300 CE, investigating the impact of the new topography and different realisations of roughness on ice and subglacial hydrological flow and ice mass loss from Antarctica.

Objective 1

1. Develop an ice sheet modelling approach to assimilate satellite remote sensing datasets to improve knowledge of the bed (BedSAT) informed by mathematical models of ice flow over topography;

Key Investigation Areas for Objective 1

1. **Model Development Strategy**
   - I will maintain invariant (but spatially variable) bed traction throughout the modeling timeframe to isolate topographical effects in the inversion approach.
   - My model will incorporate available thermal distribution, velocity field, and ice thickness data, as these parameters be constrained using observations.
   - Through spectral analysis of surface expressions, I will identify the topographical features that most strongly influence surface patterns.
   - To account for variations in ice behavior with thickness, I will simulate scenarios ranging from thick ice with slippery base to thin ice with sticky base, using observed velocity patterns as constraints.
2. **Transfer Functions**
   - I will develop efficient transfer function methods for rapid bed topography inversion, validating against known bed configurations from radar data.
   - My transfer functions will be tested across various ice thickness and flow conditions.
   - Cross-validation against radargrams will provide direct verification of my inversion results.
   - Spatial covariance analysis of existing radar data will inform my statistical framework and error propagation through the inversion process.
   - I will account for friction roughness and high-amplitude variations, using observed surface velocity patterns as constraints.
3. **Model Validation**
   - I will apply quantitative error reduction metrics and compare systematically against existing bed topography products.
   - Model limitations and breaking points will be identified through systematic testing across extreme scenarios, constrained by physical principles.

- Grid independence testing will ensure solution robustness across different spatial resolutions.
- Sensitivity analysis will examine the impact of my model assumptions.

Note: Detailed methodological outlines for Objectives 2 and 3 will be developed following the completion and refinement of the BedSAT method in Objective 1.

655 words in this section.

Resources

I require high performance computing resources (including compute and storage) from the National Computing Infrastructure (NCI). These resources are already available via a Flagship between NCI and the Monash-led Australian Research Council project Securing Antarctica's Environmental Future (SAEF).

Currently available data and Framework

The project will make use of a number of new remote sensing datasets and software tools.

1. **Reference Elevation Model of Antarctica (REMA)**
   Constructed from hundreds of thousands of Digital Elevation Models (DEMs) derived from high-resolution Maxar satellite imagery, REMA is calibrated with Cryosat-2 and ICESat altimetry [15].

2. **ITS_LIVE Antarctic surface velocities and elevation**
   The NASA-administered ITS_LIVE website provides automated, high-resolution datasets of Antarctic surface velocities and ice surface elevation change, derived from satellite observations [16].

3. **BedMAP**
   A suite of gridded products describing surface elevation, ice-thickness and the seafloor and subglacial bed elevation of Antarctica, based on a compilation of data collected by a large number of researchers using a variety of techniques, with the aim of representing a snap-shot of understanding of the Antarctic region [17]. BEDMAP lacks detailed information on bedrock type, sediment layers, or geothermal heat flux, all of which affect ice dynamics intruducing model uncertainty.

4. **BedMachine Antarctica**
   A high-resolution map of Antarctic subglacial bed topography that provides unprecedented detail of basal features. The dataset combines multiple ice thickness measurements with mass conservation principles, satellite-derived ice flow velocities, and surface mass balance from regional atmospheric models [5]. Similarly to BedMAP, BedMachine does not explicitly model basal properties.

5. **Collaborative Exploration of the Cryosphere through Airborne Profiling (ICECAP)**
   Since 2012, the project has obtained extensive data on ice thickness mapping and surface elevation in regions of the East Antarctic grounding zone, also comprehensive gravity mapping in areas beneath the Totten Glacier cavity [18].

6. **Ice-sheet and Sea-level System Model (ISSM)**
   ISSM is a finite-element numerical ice sheet model. It has been used to simulate the Antarctic Ice Sheet's response to various climate scenarios and assess future mass loss contributions to sea level rise [5, 8, 19].

Data management and archiving

Data will be published adhering to FAIR principles (Findable, Accessible, Interoperable, Reusable), ensuring transparency and accessibility. The final bed topography datasets will be published at the Australian Antarctic Data Centre (AADC) under an open source licence. All production model outputs will be published with unique DOIs at repositories aligned with the corresponding journal articles. Model outputs will be archived to tape at NCI using existing SAEF resources, as well as backed up to storage available through Monash MASSIVE M3 account aligned with project supervisor Dr McCormack. All journal articles published through this project will be open source, and tier 1 journals will be targeted.

Risk

The project is highly feasible and low risk, given that it is a desk-based modelling and data assimilation project. All the data to be used in this project are freely available for download, and project supervisors are experts in ice sheet modelling using ISSM.

Fieldwork

Fieldwork is not necessary to achieve the objectives of the project; however, there may be the opportunity to participate in fieldwork through the ICECAP airborne geophysics project (led CI of ICECAP is project supervisor Dr Jason Roberts, Australian Antarctic Division), which will be instrumental in training of geophysical instruments and in developing broader expertise in the field.

Career Development

I have secured a position in the highly competitive Karthaus Glaciology Summer School taking place in northern Italy in May 2025. This prestigious program will provide comprehensive training across all aspects of glaciology directly relevant to my research, offering not only technical knowledge but also valuable networking opportunities with leading researchers in the field.

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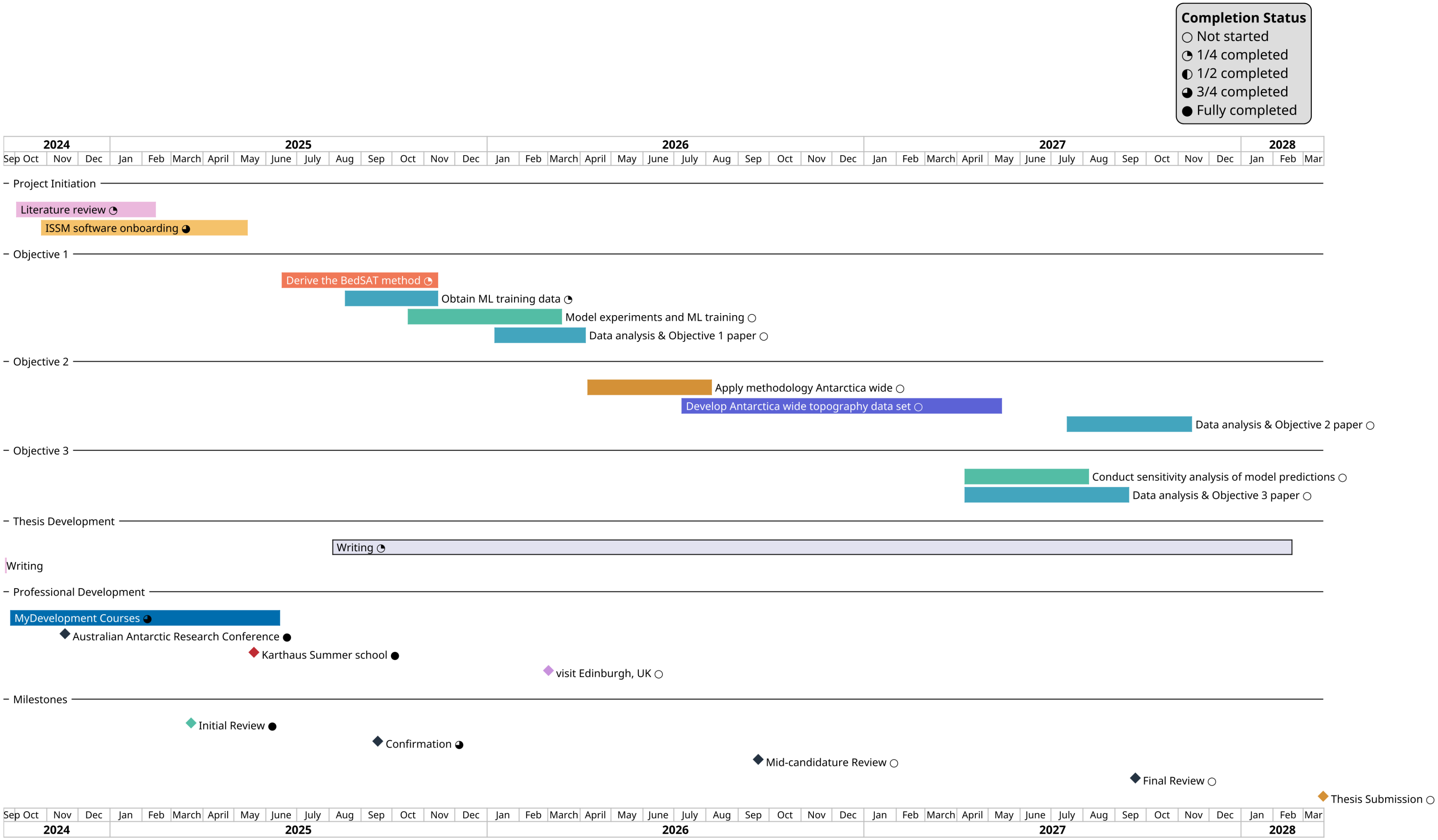
Project timeline



Glossary of Key Terms

1. **Adaptive Mesh Refinement**: A technique used to refine the mesh in regions of high variability or complexity, enhancing model accuracy and efficiency.
2. **Asperity**: A protrusion or bump on a surface. Roughness The unevenness of a surface, characterized by the size and distribution of asperities.
3. **Basal Drag Coefficient**: A parameter representing the frictional resistance between the ice sheet base and the underlying bedrock.
4. **Basal Melt Rates**: The rate at which the underside of a glacier or ice sheet melts due to contact with warmer ocean water.
5. **Basal sliding**: Sliding occurring at the base of a glacier.
6. **Bed Topography**: The shape and elevation of the bedrock underlying a glacier or ice sheet.
7. **Bedrock Topography**: The shape and elevation of the solid rock surface beneath a glacier or ice sheet.
8. **Blatter-Pattyn Approximation (BP)**: A higher-order ice flow model that incorporates longitudinal stresses, making it suitable for simulating fast-flowing ice streams and regions with significant vertical shear.
9. **Calving**: The process by which icebergs break off from the edge of a glacier or ice sheet.
10. **Coefficient of sliding friction (μ)**: The ratio of the shear stress to the normal stress during steady-state sliding.
11. **Coefficient of static friction (μ_s)**: The ratio of the limiting static shear stress to the normal stress, indicating the resistance to sliding from rest.
12. **DOI (Digital Object Identifier)**: A persistent digital identifier assigned to research outputs and scientific data that provides a permanent link to its location on the internet, ensuring long-term accessibility and citability.
13. **Data Assimilation**: The process of incorporating observational data into a model to improve its accuracy and predictive capabilities.
14. **Discrete wavelet transform (DWT)**: A mathematical technique that decomposes a signal into different frequency components while preserving spatial information. In glaciology, it's used to analyze bed topography at multiple spatial scales.

15. **FAIR principles**: A set of guiding principles for scientific data management and stewardship: Findable, Accessible, Interoperable, and Reusable. These principles are designed to optimize data reuse by both humans and machines.

16. **Finite Element Method (FEM)**: A numerical method for solving partial differential equations by dividing the domain into smaller elements and approximating the solution within each element.

17. **Fourier Transform**: A mathematical tool used to decompose a signal, such as surface elevation data, into its constituent frequencies. This allows for analysis of specific spatial scales and features.

18. **Full-Stokes (FS)**: The most comprehensive and computationally expensive ice flow model, accounting for all stress components. Essential for accurately simulating ice flow near grounding lines.

19. **Global Mean Sea-Level Rise (SLR)**: The average increase in sea level across the globe due to various factors, including melting of glaciers and ice sheets, and thermal expansion of ocean water.

20. **Grounding Line**: The boundary where the ice sheet transitions from grounded ice to floating ice (ice shelf).

21. **Grounding line retreat**: The inland migration of the grounding line (where ice transitions from grounded to floating) due to factors such as increased basal melting or dynamic thinning. A key indicator of ice sheet instability and potential mass loss.

22. **Ice-Penetrating Radar**: A remote sensing technique used to map the bedrock topography beneath glaciers and ice sheets by transmitting radar waves through the ice.

23. **Ice shelf**: A floating extension of a land-based ice sheet, typically hundreds of meters thick. Ice shelves are important for buttressing ice flow from the interior and their interaction with ocean water can significantly influence ice sheet stability.

24. **Ice Sheet System Model (ISSM)**: A finite element, thermomechanical ice flow model that incorporates SIA, SSA, BP, and FS formulations to simulate ice sheet behaviour at various complexities and spatial resolutions.

25. **InSAR**: Interferometric Synthetic Aperture Radar, a remote sensing technique used to measure ice surface velocity.

26. **Interferometry**: A remote sensing technique that combines multiple radar images to measure surface deformation or elevation changes with high precision. In glaciology, it's particularly useful for measuring ice flow velocities and surface elevation changes.

27. **Inversion**: A mathematical technique used to infer unknown parameters, such as bed topography, from observations of other variables, such as surface elevation and velocity.

28. **Limiting dynamic shear stress (T_m)**: The shear stress at which a glacier or ice block transitions from steady-state sliding to accelerated sliding.

29. **Kriging**: A geostatistical interpolation method that estimates unknown values at specific points by calculating weighted averages of known values from surrounding points, while accounting for spatial correlation and providing uncertainty estimates.

30. **Land-fast ice**: Sea ice that remains attached to the coast, seafloor, or grounded icebergs. This stationary ice forms a stable platform along coastal areas and plays a crucial role in local ecosystems and climate processes

31. **Limiting static shear stress (T_S)**: The minimum shear stress required to initiate sliding from a resting position.

32. **Linear Perturbation Analysis**: A technique that examines the response of a system to small perturbations in its parameters, assuming a linear relationship between the perturbation and the response.

33. **Marine Ice Sheet Instability (MISI)**: A process where the grounding line of an ice sheet retreats into deeper water, leading to accelerated ice discharge and potentially unstoppable collapse.

34. **Mass conservation**: A fundamental physical principle stating that mass cannot be created or destroyed in a closed system. In glaciology, it's used to constrain ice flow models and ensure physically realistic solutions when reconstructing bed topography.

35. **Momentum Balance**: The fundamental physical principle describing how forces control ice motion, expressed through the Navier-Stokes equations. In ice sheet modeling, it accounts for the balance between internal stresses, gravitational driving forces, and resistive forces (including drag at the bed and lateral margins).

36. **Normal stress (N)**: The force acting perpendicular to a surface, per unit area. In the context of glaciers, it is primarily the weight of the overlying ice.

37. **Null Space**: The set of all possible solutions to an inverse problem that do not contribute to the observed data. In the case of the work of **??**, features aligned with ice flow fall within the null space and cannot be resolved by the inversion.

38. **Ocean forcing**: The influence of oceanic conditions (temperature, salinity, currents) on ice sheet behavior, particularly through melting at the base of ice shelves and at the grounding line. A key factor in ice sheet modeling and future projections.

39. **Orthorectification**: The process of removing geometric distortions from satellite or aerial imagery to create a planimetrically correct image where all pixels are viewed from directly above, essential for accurate surface elevation measurements.

40. **Pack-ice**: Sea ice that moves with ocean currents and winds, typically found in polar regions. Unlike land-fast ice, pack-ice is mobile and can form dense fields of ice floes that vary in size and thickness.

41. **Pinning Point**: A topographic feature, such as a ridge or mountain, that can slow or temporarily halt the retreat of a glacier's grounding line.

42. **Regelation**: The process of melting under pressure and refreezing at lower pressure, potentially contributing to ice sliding.

43. **Retrograde Bedrock Slope**: A bedrock slope that deepens inland, making the ice sheet more susceptible to marine ice sheet instability.

44. **Rheology**: The study of how materials deform and flow under stress. In glaciology, it refers to the flow properties of ice.

45. **Shallow Ice Approximation (SIA)**: A simplified ice flow model that considers only vertical shear stresses and neglects horizontal stress gradients. Suitable for slow-moving ice in the interior of ice sheets.

46. **Shallow Shelf Approximation (SSA)**: A simplified ice flow model that neglects vertical shear stresses and assumes depth-independent horizontal velocity. Appropriate for modelling floating ice shelves and fast-flowing ice streams.

47. **Shallow-Ice-Stream Approximation**: A simplification of the ice flow equations that assumes the ice thickness is much smaller than the horizontal extent of the glacier, allowing for analytical solutions.

48. **Shear stress (T)**: The force acting parallel to a surface, per unit area. In the context of glaciers, it is the force driving glacier motion.

49. **Sliding**: The movement of a glacier over its bed by sliding rather than internal deformation.

50. **Slipperiness**: A measure of the ease with which ice can slide over its bed. It encompasses the influence of basal conditions like geology, hydrology, and sediment characteristics.

51. **Steady-state**: A condition where the glacier's flow and properties are constant over time, assuming a balance between ice accumulation and loss.

52. **Steady-state velocity (V_b)**: The constant velocity reached by a glacier or ice block when the driving shear stress is balanced by resisting forces.

53. **Stress Balance**: The equilibrium between the forces acting on an ice sheet, including gravity, basal friction, and internal ice stresses.

54. **Surface mass balance**: The net difference between accumulation (snowfall, rain) and ablation (melting, sublimation, wind erosion) at the ice sheet surface. A fundamental parameter in ice sheet mass budget calculations.

55. **Thermohaline circulation**: The global ocean circulation system driven by differences in temperature (thermo) and salinity (haline). This system, often called the "global conveyor belt," plays a crucial role in global heat distribution and climate regulation

56. **Temperate ice**: Ice at or near its pressure-melting point.

57. **Transfer Functions**: Mathematical equations that describe the relationship between perturbations in bed properties and the resulting changes in surface variables.

58. **Volume Above Floatation (VAF)**: The volume of an ice sheet that is grounded on bedrock and contributes to sea-level rise if it melts or slides into the ocean.

59. **Wavelet Decomposition**: A mathematical technique that analyzes a signal by decomposing it into different frequency components at various spatial scales.

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