

BedSAT: Antarctica

Exploring what lies beneath using big data and modelling

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Impact Statement

Antarctica's bed topography data currently has uncertainties of hundreds of meters in elevation due to sparse and unevenly distributed radar surveys, significantly limiting our ability to predict ice sheet behavior and sea level rise contributions. Through the BedSAT project, we are developing a novel modeling approach that integrates remote sensing data with mathematical ice flow models to substantially improve bed topography resolution and accuracy. By creating a new continent-wide bed topography dataset and conducting sensitivity analyses through to 2300, we will quantify how bed topography uncertainties affect ice mass loss projections. This improved understanding will directly enhance global climate models, provide more reliable sea level rise predictions, and enable evidence-based policy decisions for climate adaptation strategies. Our open-source approach and FAIR data principles will ensure these improvements benefit the broader scientific community and support more effective climate change mitigation planning for vulnerable coastal communities.

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Antarctica's Landscape

Climate Impacts and Global Significance

The polar regions are losing ice, and their oceans are changing rapidly [1]. The consequences of this polar transition extend to the whole planet and it is crucial for us to understand them to be able to evaluate the costs and benefits of potential mitigation strategies.

The consequences of changes in different kinds of polar ice manifest across multiple interconnected systems. In both polar oceans, shifts in seasonal sea ice (both ‘pack-ice’ that moves with ocean currents and ‘land-fast ice’ that remains attached to the coast [2]) are altering marine ecosystems, from the production of new organic matter by photosynthetic organisms like plants and algae, to species distribution [1]. Of particular concern is the accelerating loss of continental ice sheets (permanent glacial ice masses on land) in both Greenland and Antarctica, which has become a major contributor to global sea level rise [1]. The impact extends beyond direct ice loss: as fresh water from melting ice sheets is added into the ocean, it increases ocean stratification. Cold freshwater can dissolve larger amounts of CO₂ than regular ocean water. This increased CO₂ uptake by polar oceans is creating corrosive conditions for calcifying organisms [1]. In addition, freshwater stratification threatens to disrupt global thermohaline circulation [3] by decreasing the natural mixing of the ocean layers.

Antarctica's climate response, however, differs markedly from the Arctic's uniform warming pattern. While West Antarctica has experienced warming in certain regions, East Antarctica has shown minimal overall temperature change in recent decades [1]. These observations carry low confidence due to limited data availability and high variability [1]. This asymmetric response is partly explained by the Southern Ocean's unique ability to absorb and mix heat into its depths [4].

To make matters more challenging, there is significant uncertainty in the timing and magnitude of Antarctica's ice loss, largely due to unknowns in ice sheet properties and associated flow processes [5]. Several key physical processes remain poorly constrained in current models. These include bedrock characteristics like ground “slipperiness” and roughness, as well as our assumptions of temperature and depth dependent parameters like viscosity, which is often assumed to be constant. Models might also suffer undesirable three-dimensional effects, where radar profiles may not perfectly align with surface velocity fields, effectively neglecting real transverse ice dynamical effects. Common modeling approximations often break down in regions with variable bed conditions - all of which impact our ability to accurately predict ice sheet behavior [6].

Topography of Antarctica

Bed topography is one of the most crucial boundary conditions that influences ice flow and loss from the Antarctic Ice Sheet (AIS) [7]. Bed topography datasets are typically generated from airborne radar surveys, which are sparse and unevenly distributed across the Antarctic continent (see figure 2.1). Interpolation schemes to "gap fill" these sparse datasets yield bed topography estimates that have high uncertainties (i.e. multiple hundreds of metres in elevation uncertainty; Morlighem et al., 2020), which propagate in simulations of AIS evolution under climate change [8]. Given the logistical challenges of accessing large parts of the Antarctic continent, there is a crucial need for alternative approaches that integrate diverse and possibly more spatially complete data streams – including satellite data – to derive bed topography.

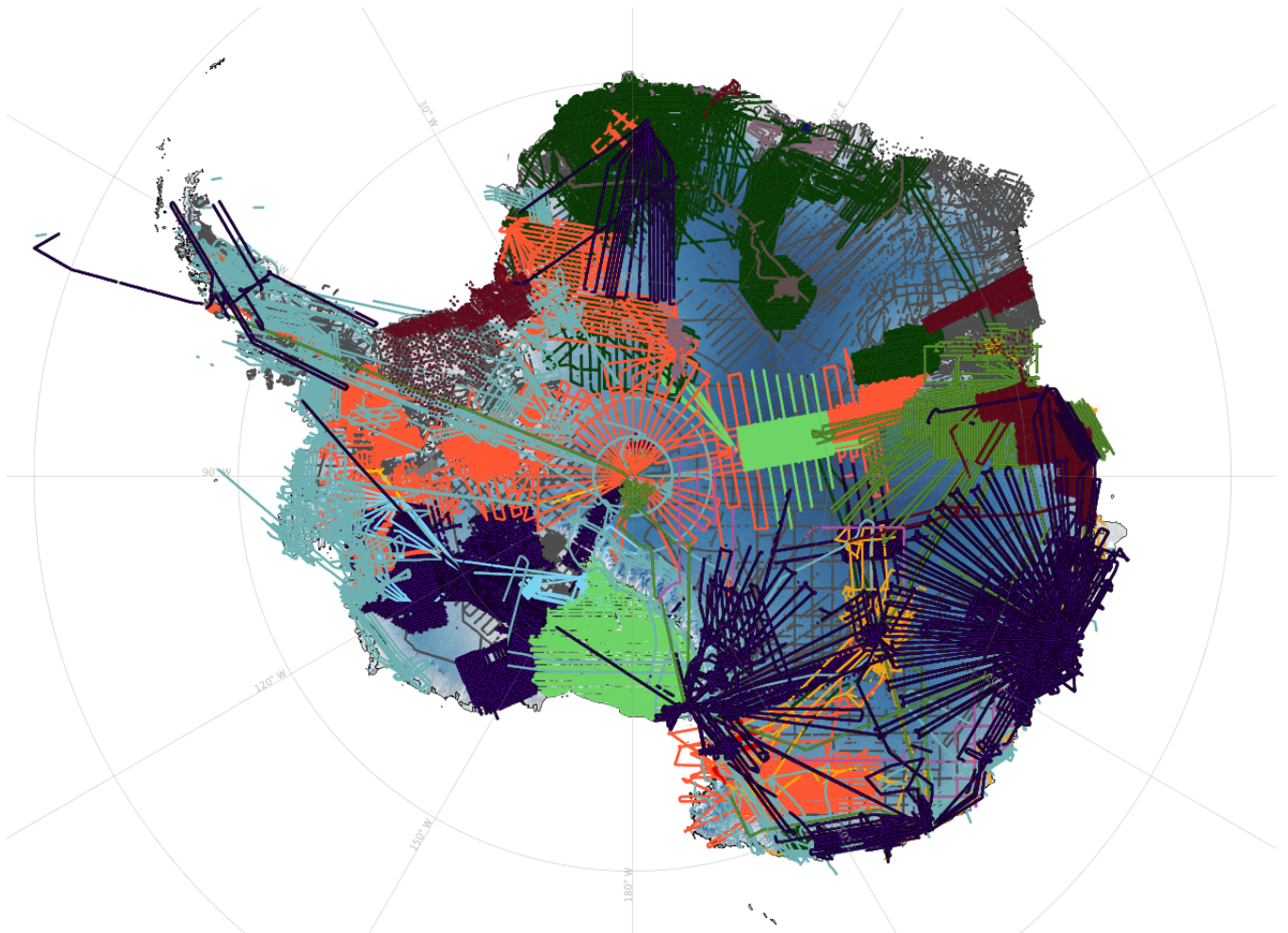


Figure 2.1: Distribution of BedMAP{1,2,3} data tracks (Source: bedmap.scar.org).

Obtaining information about the conditions at the base of the ice-sheet often relies on indirect modelling methods like

- **Inversions**
 - **Mass conservation:** Used to constrain inversion models and to fill data gaps by taking advantage of the physical laws of conservation of mass and momentum [7, 9].
 - **Control method inversion:** Basal conditions distribution information is obtained from remote sensing data and theoretical ice flow models [10].
 - **Statistical inversion:** Study the simultaneous retrieval (or update) of bed topography and basal slipperiness from surface topography and velocity measurements [10].
- **4dvar:** Four-dimensional variational data assimilation - Minimizes the difference between model predictions and observations across a time window. Mainly used to optimize model parameters and initial conditions [11].
- **Monte Carlo Mass Conservation:** Random sampling solves mass conservation equations [12].
- **Geostatistics** Statistical methods specialized for analyzing spatially correlated data. In glaciology, it's used to interpolate between sparse measurements and characterize spatial patterns in bed properties, often employing techniques like kriging [13].
- **EnKF** Ensemble Kalman Filter A sequential data assimilation method that uses an ensemble of model states to estimate uncertainty and update model parameters based on observations [11].
- **linear perturbation analysis** for an outlined example see section 2.2.

2.1 A case study

Critical Factors Influencing Thwaites Glacier's Future Evolution

Thwaites Glacier in West Antarctica represents one of the most impactful potential contributors to global sea-level rise (SLR), with an estimated contribution of 0.59 meters. The glacier's future evolution is of particular concern because it could trigger a broader collapse of the West Antarctic Ice Sheet. Understanding the factors that control its stability is therefore crucial for accurate sea-level rise projections.

Two primary factors control Thwaites Glacier's evolution [8]: Ocean-driven basal melt rates and bedrock topography. Their study reveals that the glacier is highly sensitive to even small changes in bedrock topography within current measurement error bounds, highlighting the critical need for more accurate topographical data.

Current measurement methods rely primarily on ice-penetrating radar, which presents significant challenges. While radar can provide direct measurements along specific tracks, scientists must use interpolation to fill gaps between these tracks. This interpolation introduces statistical uncertainties, particularly in areas where measurements are sparse

or rapidly changing. These uncertainties significantly affect our ability to model the glacier's future behavior accurately.

To quantify these uncertainties, Castleman et al. [8] employed two-dimensional discrete wavelet transform (DWT) to systematically analyze and modify bedrock topography data. Their method decomposes a bedrock elevation map $B(x, y)$ into four distinct sub-arrays: \mathbf{A}_n (low-frequency approximation), and three high-frequency components - \mathbf{H}_n (horizontal), \mathbf{V}_n (vertical), and \mathbf{D}_n (diagonal), where n indicates the decomposition level.

The researchers selectively amplified the high-frequency components using a multiplier $\alpha > 1$, creating modified arrays $\mathbf{H}'_n = \alpha\mathbf{H}_n$, $\mathbf{V}'_n = \alpha\mathbf{V}_n$, and $\mathbf{D}'_n = \alpha\mathbf{D}_n$. This mathematical approach allowed them to introduce realistic perturbations into bedrock topography models and assess how varying spatial and vertical resolutions affect SLR projections.

One of the study's most significant findings relates to the importance of "pinning points" - bedrock features that can temporarily halt or slow grounding line retreat. While their wavelet-based method could potentially bias results toward more effective pinning points, the study revealed that the location of bedrock features relative to the grounding line and their deviation from mean bed elevation were more significant than feature amplification [8]. Through this analysis, they established crucial requirements for future bedrock measurements: 2 km spatial resolution and ± 8 meters vertical accuracy, particularly near the grounding line, to keep SLR uncertainty within ± 2 centimeters.

The glacier's vulnerability is further complicated by Marine Ice Sheet Instability (MISI) [8], a feedback mechanism where warmer ocean temperatures accelerate ice shelf melting and calving. This process is particularly concerning for Thwaites Glacier due to its retrograde bedrock slope [8], which can accelerate grounding line retreat once initiated.

Ocean-driven basal melt rates present another significant source of uncertainty in ice-sheet model simulations. The challenge stems from the stochastic nature of ocean circulation patterns, temporal variability in ocean forcing, and limitations in current ocean models and limited direct ocean observations underneath ice shelves. [8]. These factors make it particularly difficult to predict how the ice sheet will respond to future ocean warming scenarios [8].

This study underscores the critical importance of accurate bedrock topography measurements for reliable SLR projections. The findings provide clear guidelines for future mapping efforts and highlight the need for focused attention on grounding line regions where bedrock features have the most significant impact on glacier stability.

2.2 Ice Sheet Bed Reconstruction via Surface Data Inversion

Direct observations of ice sheet bed conditions through methods like airborne and ground-based ice penetrating radar or seismic surveying remain sparse, making interpolation and inversion methods necessary for understanding the geophysics. The shape and mechanical properties of the bed significantly influence ice flow behavior, with even small changes in bed conditions potentially causing large variations in predicted ice loss rates. An important observation is that bed features often manifest as subtle signatures in the surface topography above them [6]. The relationship between bed and surface characteristics is core to inversion methods that attempt to reconstruct bed properties from surface observations. Unfortunately, these methods cannot paint a complete picture of the ice sheet model. Inversion methods require careful tuning of parameters which are not di-

rectly observable (e.g. the basal friction coefficient), so that modelled surface velocity matches observations. The mathematical framework for these inversions can be achieved via steady-state **linear perturbation analysis** of shallow-ice-stream equations [14].

Theoretical Framework

The foundation of modern bed reconstruction methods lies in the shallow-ice-stream equations (SISEs) of motion (MacAyeal, 1989). These equations describe the relationship between ice flow and bed conditions:

$$\partial_x(4h\eta\partial_x u + 2h\eta\partial_y v) + \partial_y(h\eta(\partial_x v + \partial_y u)) - (u/c^{1/m}) = \rho gh(\partial_x(s)\cos(\alpha) - \sin(\alpha)) \quad (2.1)$$

$$\partial_y(4h\eta\partial_y v + 2h\eta\partial_x u) + \partial_x(h\eta(\partial_y u + \partial_x v)) - (v/c^{1/m}) = \rho gh(\partial_y(s)\cos(\alpha)) \quad (2.2)$$

where h represents ice thickness, s surface elevation, (u, v) horizontal components of surface velocity, c basal slipperiness, η effective ice viscosity, m sliding law parameter, ρ ice density, α mean surface slope in x -direction, and g acceleration due to gravity.

Inversion Methodology

The inversion method developed by Ockenden et al. introduces perturbations to study how small changes in ice thickness (h), surface elevation (s), basal topography (b), and ice velocity (u) affect ice flow. The method assumes:

1. A linear viscous medium ($n = 1$)
2. Non-linear sliding law ($m > 0$)
3. Steady-state conditions
4. Spatially constant zero-order solutions

The system is linearised around a reference model $(\bar{h}, \bar{s}, \bar{b}, \bar{u}, \bar{v}, \bar{c})$, leading to first-order momentum balance equations:

$$4\eta\bar{h}\partial_{xx}\Delta u + 3\eta\bar{h}\partial_{xy}^2\Delta v + \eta\bar{h}\partial_{yy}^2\Delta u - \gamma\Delta u = \rho g\bar{h}\cos(\alpha)\partial_x\Delta s - \rho g\sin(\alpha)\Delta h \quad (2.3)$$

$$4\eta\bar{h}\partial_{yy}\Delta v + 3\eta\bar{h}\partial_{xy}^2\Delta u + \eta\bar{h}\partial_{xx}^2\Delta v - \gamma\Delta v = \rho g\bar{h}\cos(\alpha)\partial_y\Delta s \quad (2.4)$$

Transfer Functions and Implementation

The methodology employs transfer functions to describe relationships between bed properties and surface characteristics:

$$\begin{bmatrix} \hat{s} \\ \hat{u} \\ \hat{v} \end{bmatrix} = \begin{bmatrix} T_{sb} & T_{sc} \\ T_{ub} & T_{uc} \\ T_{vb} & T_{vc} \end{bmatrix} \begin{bmatrix} \hat{b} \\ \hat{c} \end{bmatrix}$$

The system is solved using a weighted least-squares approach, minimizing:

$$\Sigma s(s_{\text{obs}} - s_{\text{pred}})^2 + \Sigma u(u_{\text{obs}} - u_{\text{pred}})^2 + \Sigma v(v_{\text{obs}} - v_{\text{pred}})^2 \quad (2.5)$$

Application and Limitations

The method has been successfully implemented using REMA surface elevation data (8m resolution) and NASA ITS_LIVE velocity data (120m resolution). It performs particularly well in:

- Areas with moderate topographic gradients in the central trunk of glaciers
- Features not aligned with ice flow direction
- Medium-wavelength (5-50km) bedrock features

However, the method faces limitations in cases of:

- Steep topography where shallow-ice-stream approximation breaks down
- Features aligned with ice flow direction
- Variable slipperiness parameters
- Lack of validation data for slipperiness predictions

This methodology represents a significant advance in our ability to reconstruct bed conditions using surface data, particularly in regions where direct measurements are sparse or unavailable.

1591 words in this section.

Objectives

The overall aim of this project is to derive a new Antarctic bed topography using remote sensing data and ice sheet modelling, and use the new bed topography in ice sheet modelling to improve understanding of the impact of fine-scale topographic roughness on ice and subglacial hydrological flow, and projections of ice mass loss under climate warming.

The specific objectives are:

1. Develop an ice sheet modelling approach to assimilate satellite remote sensing datasets to improve knowledge of the bed (BedSAT) informed by mathematical models of ice flow over topography;
2. Derive a new bed topography for Antarctica using BedSAT;
3. Conduct sensitivity analyses to understand the impact of the improved bed topography on projections of ice mass loss from Antarctica under climate warming.

121 words in this section.

Methodology

The first phase of the project (objective 1) is to derive the BedSAT method. This will involve the integration of the Budd [15] mathematical model relating ice surface elevation and bed topography into ISSM, and the development of a methodology for the data assimilation into ISSM. I will use a regional catchment in Antarctica for which relatively more radar data are available and has an indicative range of topography features, e.g. the Aurora Subglacial Basin, East Antarctica, whose margins have been extensively surveyed by the ICECAP project for airborne geophysics [16]. The second phase of the project (objective 2) will apply the methodology developed in objective 1 to the whole Antarctic continent, deriving a continent-wide bed topography dataset. Using covariance properties from existing radar surveys, I will generate a number of realisations of bed topography with unique high-resolution, and statistically-consistent topographic roughness. The third phase of the project will use the new bed topography datasets to conduct a sensitivity analysis of ice sheet model projections to 2300 CE, investigating the impact of the new topography and different realisations of roughness on ice and subglacial hydrological flow and ice mass loss from Antarctica.

Currently available data and Framework

The project will make use of a number of new remote sensing datasets, namely the Reference Elevation Model of Antarctica (REMA), ice surface velocities from NASA's ITS_LIVE, and the state-of-the-art Ice-sheet and Sea-level System Model (ISSM).

1. Reference Elevation Model of Antarctica (REMA)

REMA provides a high-resolution (2-metre) terrain map of nearly the entire continent, allowing for precise measurements of elevation changes over time. REMA supports various remote sensing activities, such as image orthorectification and interferometry, and aids in geodynamic and ice flow modeling, grounding line mapping, and surface process studies. Constructed from hundreds of thousands of Digital Elevation Models (DEMs) derived from high-resolution Maxar satellite imagery (including WorldView and GeoEye data), REMA is calibrated with Cryosat-2 and ICESat altimetry, ensuring high elevation accuracy with uncertainties of less than 1 meter over most areas [17].

2. ITS_LIVE Antarctic surface velocities and elevation

The NASA-administered ITS_LIVE website provides automated, high-resolution datasets of Antarctic surface velocities and ice surface elevation change, derived from satellite observations. The datasets are available on annual timesteps from

1985 to present. ITS_LIVE employs various statistical and computational methods to process data from satellites including Landsat and Sentinel, ensuring precise and timely updates for scientific research [18].

3. **BedMachine**

[19].

4. **Ice-sheet and Sea-level System Model (ISSM)** [20]

ISSM is a finite-element numerical ice sheet model. It has been used to simulate the Antarctic Ice Sheet's response to various climate scenarios and assess future mass loss contributions to sea level rise [9, 10]. The mesh can be refined to better capture variations in ice flow and driving stresses, enhancing the simulation's accuracy of surface elevation changes and ice dynamics. This project will involve numerical modeling using advanced mathematical approaches, including the Blatter-Pattyn approximation to the full Stokes equations for ice flow (i.e. conservation of momentum equations). The Blatter-Pattyn approximation strikes a balance between the computationally intensive full Stokes equations and the simpler shallow ice approximation (SIA), retaining vertical shearing and longitudinal stress gradients. This makes it ideal for modeling the dynamics of fast-flowing ice streams and ice shelves at the continental scale, enhancing simulation accuracy while being computationally feasible. Additionally, data assimilation, machine learning, and geostatistics will be employed, with the full Stokes equations used if necessary.

558 words in this section.

Resources

The project will require high performance computing resources (including compute and storage) from the National Computing Infrastructure (NCI). We anticipate requiring ~250 k Service Units (SU) each quarter, and up to 500 TB of storage. These resources are already available via a Flagship between NCI and the Monash-led Australian Research Council project Securing Antarctica’s Environmental Future (SAEF).

Data management and archiving

Data will be published adhering to FAIR principles (Findable, Accessible, Interoperable, Reusable), ensuring transparency and accessibility. The final bed topography datasets will be published at the Australian Antarctic Data Centre (AADC) under an open source licence. All production model outputs will be published with unique DOIs at repositories aligned with the corresponding journal articles. Model outputs – including production and other outputs – will be archived to tape at NCI using existing SAEF resources, as well as backed up to storage available through Monash MASSIVE M3 account aligned with project supervisor Dr McCormack. All journal articles published through this project will be open source, and tier 1 journals will be targeted.

Risk

The project is highly feasible and low risk, given that it is a desk-based modelling and data assimilation project. All the data to be used in this project are freely available for download, and project supervisors are experts in ice sheet modelling using ISSM.

Fieldwork

Fieldwork is not necessary to achieve the objectives of the project; however, there may be the opportunity to participate in fieldwork through the ICECAP airborne geophysics project (led CI of ICECAP is project supervisor Dr Jason Roberts, Australian Antarctic Division), which will be instrumental in training of geophysical instruments and in developing broader expertise in the field.

Career Development

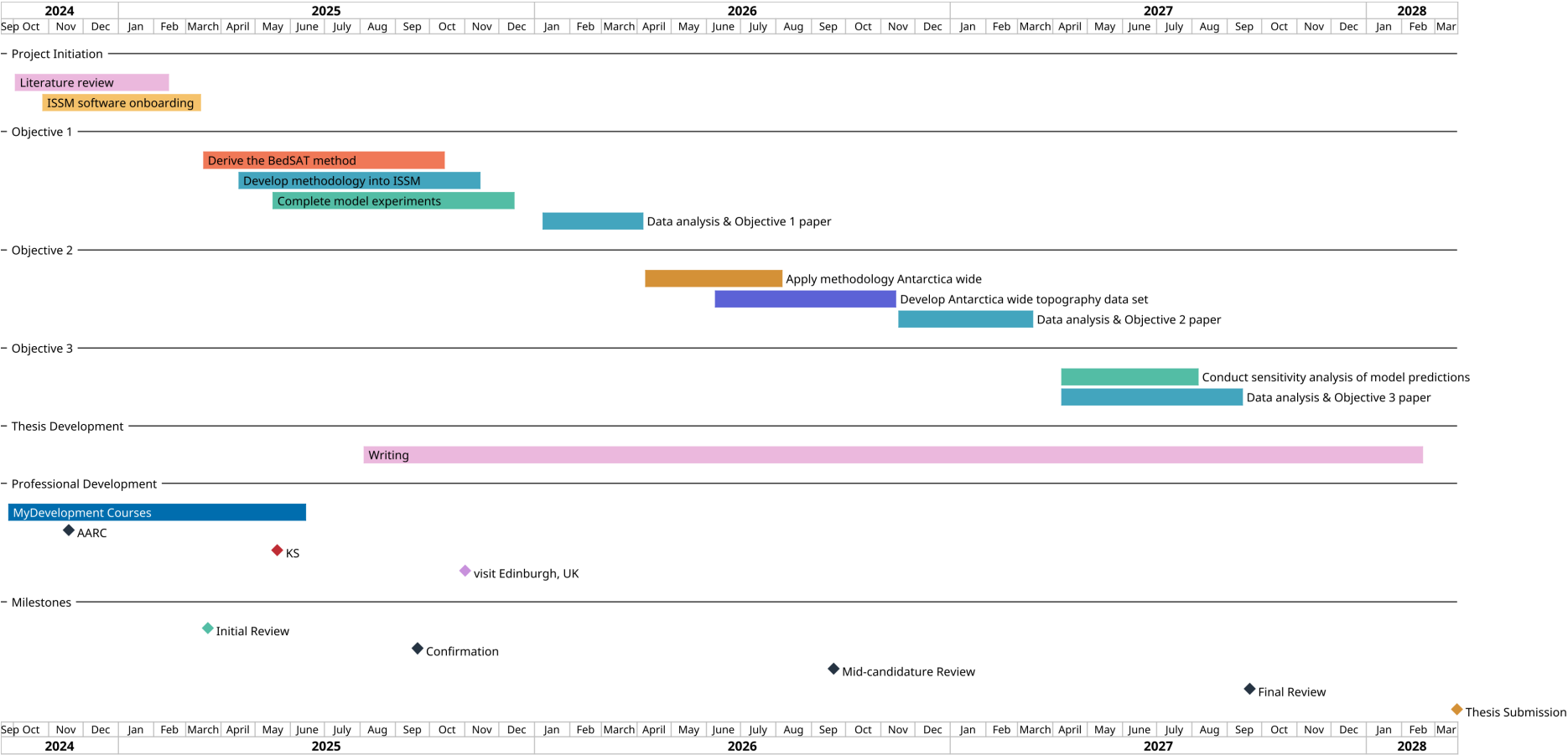
Conferences

At least one conference will be attended each year. An international conference relevant to

the discipline, e.g. the European Geophysical Union General Assembly, will be attended in the final year of the project.

303 words in this section.

Project timeline



Glossary of Key Terms

1. **Adaptive Mesh Refinement:** A technique used to refine the mesh in regions of high variability or complexity, enhancing model accuracy and efficiency.
2. **Asperity:** A protrusion or bump on a surface. Roughness The unevenness of a surface, characterized by the size and distribution of asperities.
3. **Basal Drag Coefficient:** A parameter representing the frictional resistance between the ice sheet base and the underlying bedrock.
4. **Basal Melt Rates:** The rate at which the underside of a glacier or ice sheet melts due to contact with warmer ocean water.
5. **Basal sliding:** Sliding occurring at the base of a glacier.
6. **Bed Topography:** The shape and elevation of the bedrock underlying a glacier or ice sheet.
7. **Bedrock Topography:** The shape and elevation of the solid rock surface beneath a glacier or ice sheet.
8. **Blatter-Pattyn Approximation (BP):** A higher-order ice flow model that incorporates longitudinal stresses, making it suitable for simulating fast-flowing ice streams and regions with significant vertical shear.
9. **Calving:** The process by which icebergs break off from the edge of a glacier or ice sheet.
10. **Coefficient of sliding friction (μ):** The ratio of the shear stress to the normal stress during steady-state sliding.
11. **Coefficient of static friction (μ_s):** The ratio of the limiting static shear stress to the normal stress, indicating the resistance to sliding from rest.
12. **Data Assimilation:** The process of incorporating observational data into a model to improve its accuracy and predictive capabilities.
13. **Finite Element Method (FEM):** A numerical method for solving partial differential equations by dividing the domain into smaller elements and approximating the solution within each element.
14. **Fourier Transform:** A mathematical tool used to decompose a signal, such as surface elevation data, into its constituent frequencies. This allows for analysis of specific spatial scales and features.

15. **Full-Stokes (FS):** The most comprehensive and computationally expensive ice flow model, accounting for all stress components. Essential for accurately simulating ice flow near grounding lines.
16. **Global Mean Sea-Level Rise (SLR):** The average increase in sea level across the globe due to various factors, including melting of glaciers and ice sheets, and thermal expansion of ocean water.
17. **Grounding Line:** The boundary where the ice sheet transitions from grounded ice to floating ice (ice shelf).
18. **Ice-Penetrating Radar:** A remote sensing technique used to map the bedrock topography beneath glaciers and ice sheets by transmitting radar waves through the ice.
19. **Ice Sheet System Model (ISSM):** A finite element, thermomechanical ice flow model that incorporates SIA, SSA, BP, and FS formulations to simulate ice sheet behaviour at various complexities and spatial resolutions.
20. **InSAR:** Interferometric Synthetic Aperture Radar, a remote sensing technique used to measure ice surface velocity.
21. **Inversion:** A mathematical technique used to infer unknown parameters, such as bed topography, from observations of other variables, such as surface elevation and velocity.
22. **Limiting dynamic shear stress (T_m):** The shear stress at which a glacier or ice block transitions from steady-state sliding to accelerated sliding.
23. **Kriging:** A geostatistical interpolation method that estimates unknown values at specific points by calculating weighted averages of known values from surrounding points, while accounting for spatial correlation and providing uncertainty estimates.
24. **Limiting static shear stress (T_S):** The minimum shear stress required to initiate sliding from a resting position.
25. **Linear Perturbation Analysis:** A technique that examines the response of a system to small perturbations in its parameters, assuming a linear relationship between the perturbation and the response.
26. **Marine Ice Sheet Instability (MISI):** A process where the grounding line of an ice sheet retreats into deeper water, leading to accelerated ice discharge and potentially unstoppable collapse.
27. **Momentum Balance:** The fundamental physical principle describing how forces control ice motion, expressed through the Navier-Stokes equations. In ice sheet modeling, it accounts for the balance between internal stresses, gravitational driving forces, and resistive forces (including drag at the bed and lateral margins).
28. **Normal stress (N):** The force acting perpendicular to a surface, per unit area. In the context of glaciers, it is primarily the weight of the overlying ice.

29. **Null Space:** The set of all possible solutions to an inverse problem that do not contribute to the observed data. In the case of the work of 2.2, features aligned with ice flow fall within the null space and cannot be resolved by the inversion.
30. **Pinning Point:** A topographic feature, such as a ridge or mountain, that can slow or temporarily halt the retreat of a glacier's grounding line.
31. **Regelation:** The process of melting under pressure and refreezing at lower pressure, potentially contributing to ice sliding.
32. **Retrograde Bedrock Slope:** A bedrock slope that deepens inland, making the ice sheet more susceptible to marine ice sheet instability.
33. **Rheology:** The study of how materials deform and flow under stress. In glaciology, it refers to the flow properties of ice.
34. **Shallow Ice Approximation (SIA):** A simplified ice flow model that considers only vertical shear stresses and neglects horizontal stress gradients. Suitable for slow-moving ice in the interior of ice sheets.
35. **Shallow Shelf Approximation (SSA):** A simplified ice flow model that neglects vertical shear stresses and assumes depth-independent horizontal velocity. Appropriate for modelling floating ice shelves and fast-flowing ice streams.
36. **Shallow-Ice-Stream Approximation:** A simplification of the ice flow equations that assumes the ice thickness is much smaller than the horizontal extent of the glacier, allowing for analytical solutions.
37. **Shear stress (T):** The force acting parallel to a surface, per unit area. In the context of glaciers, it is the force driving glacier motion.
38. **Sliding:** The movement of a glacier over its bed by sliding rather than internal deformation.
39. **Slipperiness:** A measure of the ease with which ice can slide over its bed. It encompasses the influence of basal conditions like geology, hydrology, and sediment characteristics.
40. **Steady-state:** A condition where the glacier's flow and properties are constant over time, assuming a balance between ice accumulation and loss.
41. **Steady-state velocity (V_b):** The constant velocity reached by a glacier or ice block when the driving shear stress is balanced by resisting forces.
42. **Stress Balance:** The equilibrium between the forces acting on an ice sheet, including gravity, basal friction, and internal ice stresses.
43. **Temperate ice:** Ice at or near its pressure-melting point.
44. **Transfer Functions:** Mathematical equations that describe the relationship between perturbations in bed properties and the resulting changes in surface variables.
45. **Volume Above Floatation (VAF):** The volume of an ice sheet that is grounded on bedrock and contributes to sea-level rise if it melts or slides into the ocean.

46. **Wavelet Decomposition:** A mathematical technique that analyzes a signal by decomposing it into different frequency components at various spatial scales.

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