BEDSAT:Antarctica

random writings

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Abstract

Antarctica has been losing ice mass over recent decades, contributing to rising sea levels (SLR). There is significant uncertainty regarding the extent and timing of this ice loss. One key factor influencing ice flow and loss is bed topography, typically derived from sparse airborne radar surveys. The uncertainty in the data can impact simulations of the Antarctic Ice Sheet (AIS) evolution under climate change. We need alternative approaches to surveying Antarctica and given the logistical challenges we plan on modelling the bed topography. Our model's accuracy (surely?) depends on the spacing of ice thickness measurements and uncertainties in ice velocity (SOMETHING I AM WORKING ON understanding rn) and surface mass balance. BedSAT, aims to leverage the mathematical relationship between ice surface elevation (data we have?) and bed topography to estimate the actual bed topography of a surrounding spatial region (in 2D?).

Ana's plan

I've asked ChatGPT to build me a syllabus:

- 1. Fluid Mechanics: Conservation of mass, momentum (Navier-Stokes equations), incompressible flows.
- 2. Rheology and Non-Newtonian Fluids

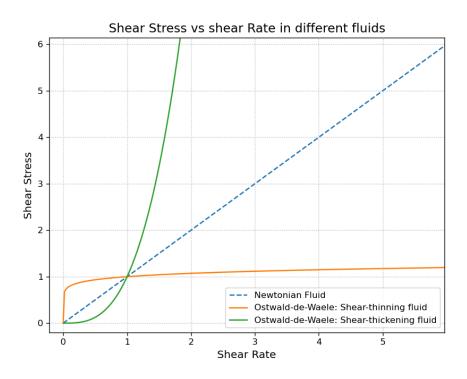


Figure 1.1: Newtonian vs. non-Newtonian behaviour, Shear Stress and shear Rate relationships.

- 3. Stokes' equations. Methods for solving PDEs, boundary conditions, numerical methods.
- 4. Glaciology Key Topics: Ice deformation, creep processes, thermal effects.
- 5. Numerical Methods for solving the Stokes equations can help since these equations often don't have analytical solutions.

Why Understanding Antarctica's Landscape Matters: Climate Impacts and Global Significance

The polar regions are losing ice, and their oceans are changing rapidly [1]. The consequences of this polar transition extend to the whole planet and it is crucial for us to understand them and plan for changes.

- Climate-induced changes in seasonal ice extent and thickness are affecting sea ice and ocean layers, which impacts marine plant growth (highly likely). This alters ecosystems (moderately likely). The timing and amount of plant growth have changed in both polar oceans, varying by location (highly likely). In Antarctica, these changes relate to retreating glaciers and sea ice change (moderately likely). In the Arctic, they've affected the types, locations, and numbers of marine species, changing ecosystem structure (moderately likely) [1].
- The rapid ice loss from the Greenland and Antarctic ice sheets during the early 21st century has increased into the near present day, adding to the ice sheet contribution to global sea level rise (SLR)(extremely likely) [1].
- Both polar oceans will be increasingly affected by CO₂ uptake, causing conditions corrosive for calcium carbonate shell-producing organisms (high confidence), with associated impacts on marine organisms and ecosystems (medium confidence) [1].
- Thermohaline circulation changes: a large-scale system of ocean currents driven by differences in water temperature (thermo) and salinity (haline), these factors affect the density of seawater. Thermohaline circulation plays a critical role in regulating Earth's climate and distributing heat and nutrients across the globe. Warm surface waters flow from the tropics toward the poles, where they cool and sink, forming deep-water currents. These deep waters then travel along the ocean floor toward the equator, eventually rising to the surface through a process called upwelling, bringing cold, nutrient-rich waters to the surface [2]. Altering the salt concentration in the Antarctic ocean due to the melting of the ice sheet could result in disruptions to this circulation and could have significant consequences for global weather systems.

Unlike the Arctic, which has seen uniform warming, Antarctica's temperature changes have been less consistent. West Antarctica has warmed in some parts. East Antarctica hasn't shown significant overall change. in the past 3-5 decades. There's low confidence in these observations due to limited data and high variability [1].

Atmospheric factors influencing Antarctic climate:

• Southern Annular Mode (SAM)

Recent changes in the Southern Annular Mode (SAM): The SAM has been mostly positive in recent decades during summer. This means stronger westerly winds around Antarctica. This positive phase is unprecedented in at least 600 years. It's associated with cooler conditions over Antarctica [1].

Causes of SAM changes: Ozone depletion was likely the main driver of SAM changes from the late 1970s to late 1990s. Since 2000, tropical sea surface temperatures have played a stronger role in influencing SAM [1].

Other influences on Antarctic climate: Tropical sea surface temperatures can affect Antarctic temperatures and Southern Hemisphere mid-latitude circulation [2].

- Pacific South American mode
- Zonal-wave 3

Other factors influencing Antarctic climate:

• Antarctica isn't warming as much as the Arctic because the Southern Ocean surrounding Antarctica absorbs and mixes heat deep into the ocean [3].

Topographic Models of Antarctica: A Review

Numerical modelling and sedimentary sequence interpretation suggest cyclical periods of ice-sheet expansion and retreat [?]. Using ice-penetrating radar data to generate a new basal bed topography of the Aurora Subglacial Basin (ASB) in east Antarctica is characterised by a fjord landscape (this land is under \sim under 2-4.5 km of ice). The ASB has a potentially significant influence on the east Antarctic ice-sheet (EAIS), however there is high uncertainty in estimates of past and present global sea level changes due to the scarcity of bed data [?]. This uncertainty also limits the accuracy of models used to predict future ice sheet growth or decay.

Methods in [?]

- 1. A ski-equipped airplane (DC-3T) carried a radar system (HiCARS), which can see through ice. HiCARS sends signals that bounce back to show the thickness of the ice and the shape of the land beneath it.
- 2. The plane flew back and forth over a large area, covering distances of around 1,000 km. The flights took place over two different periods in 2008–2009 and 2009–2010.
- 3. The radar data was cleaned up (processed) to improve accuracy, and they used a special radar system that helps reduce distortions (errors) in the measurements. [HOW?]
- 4. Thickness of the ice was measured using the time it took for the radar signals to travel through the ice and back, assuming the radar signals move through the ice at a specific speed (169 meters per microsecond).
- 5. The height of the land below the ice was calculated by looking at the radar-determined surface of the ice. [WHAT?]
- 6. The radar data was combined with other existing datasets (BEDMAP) to improve the overall picture. They used a computer algorithm to fill in gaps where they didn't have direct measurements. [WHICH?]
- 7. Determining how rough or uneven the land under the ice was, by using a statistical measure called the "root mean squared (rms) deviation."

In short, Young et al. used advanced radar technology on an airplane to map the ice thickness and the landscape beneath it in a region of Antarctica, combining this data with previous maps for a better overall picture.

Ice is weird

Ice is described as non-Newtonian because its viscosity (ν) is not constant. A Newtonian fluid (like water) has a constant viscosity regardless of the flow conditions, while a non-Newtonian fluid's viscosity changes based on factors like strain rate. The viscosity of ice depends on how much it's deforming (shear rate). Ice is a slow, shear-thinning fluid. "shear-thinning" means that the viscosity of ice decreases with increasing strain rate. This means that under more strain forces ice becomes "softer" and flows more easily. "Slow" means that the flow of ice occurs at very low velocities, that means that the change in flow velocity (time dependent and convective) are approximately zero

$$\rho(\vec{u_t} + \vec{u}\dot{\nabla}\vec{u}) \approx 0,\tag{4.1}$$

This assumption greatly simplifies the Navier-Stokes equations for ice flow. The equations we use to are the incompressibility condition (the divergence of the velocity field is zero)

$$\nabla \vec{u}) = 0, \tag{4.2}$$

the force balance equation, i.e. the pressure gradient, the divergence of the stress tensor (viscous forces within the ice) and the gravitational body force acting on the ice all cancel out

$$-\nabla p + \nabla \dot{\tau}_{ij} + \rho g, \tag{4.3}$$

And finally Glen flow law

$$D_{ij} = A\tau^n \tau_{ij}, \tag{4.4}$$

where the strain rate tensor D_{ij} , which describes how fast the ice is deforming is proportional to the deviatoric stress tensor τ_{ij} and it's magnitude τ , which accounts for the stress caused by deformation (as opposed to isotropic stress like pressure). is the flow law exponent, which determines how strongly the flow rate depends on stress. For ice, Glen's law uses n=3, which implies a nonlinear relationship between stress and strain rate, meaning the flow rate accelerates rapidly with increased stress [4].

This model does not have time derivatives!

4.1 Ice flow over bedrock perturbations

This 1970's rheology paper by Budd explains how bedrock irregularities beneath a glacier affect the surface shape of the ice mass. Budd develops a mathematical model to describe the flow of ice over these undulations, considering the ice as a viscous fluid that deforms under stress. The model predicts that the surface shape of the glacier will mirror the bedrock undulations, but shifted out of phase by approximately $\frac{\pi}{2}$ radians. The paper

analyzes the damping of different wavelengths of bedrock undulations, finding that waves with a length roughly three times the ice thickness are minimally damped, while shorter or longer waves are significantly damped out. Finally, the implications of this theory proposes the potential for ice to flow uphill, concluding that bedrock undulations with wavelengths several times the ice thickness are most important in controlling ice motion.

Numerics

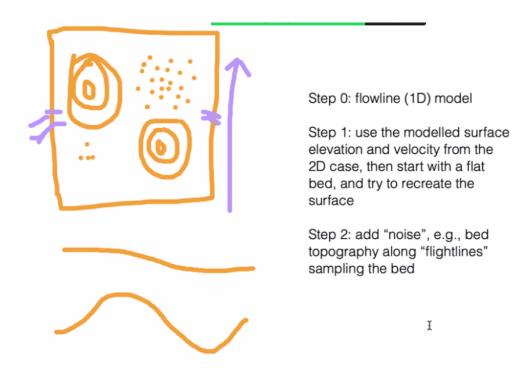


Figure 5.1: TODO

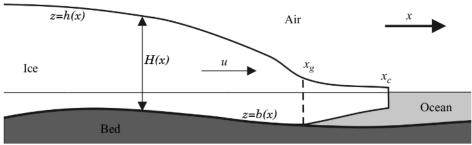


figure modified from Schoof (2007)

- coordinates t, x, y, z (with z vertical, positive upward)
- subscripts for partial derivatives $u_x = \partial u/\partial x$
- H = ice thickness
- h = ice surface elevation
- b = bedrock surface elevation
- ightharpoonup T = temperature
- $\mathbf{u} = (u, v, w) = \text{ice velocity}$
- ρ = density of ice
- $ho_{w} =$ density of ocean water
- ▶ g = acceleration of gravity
- ightharpoonup n = 3 Glen flow law exponent = 3
- A = A(T) = ice softness in Glen law ($\mathbf{D}_{ij} = A(T)\tau^{n-1}\tau_{ij}$)

Figure 5.2: figure taken from [4]

5.1 ISSM: Continental-Scale Ice Sheet Modelling

The Ice Sheet System Model (ISSM) was developed and is used for simulating ice sheet flow at continental scales. ISSM, a finite element, thermomechanical model, incorporates high-order stresses and high spatial resolution capabilities [5]. The Larour et al. (2012) ISSM paper discusses the different ice flow models within the software, including the full-Stokes, Blatter-Pattyn, Shallow-Shelf, and Shallow Ice Approximations, highlighting their individual strengths and limitations. It also explores numerical methods employed, such as static adaptive mesh refinement and inverse methods for parameter estimation. Finally, the study showcases the application of ISSM to the Greenland Ice Sheet, demonstrating its capacity to model the ice flow velocity with high accuracy, using data assimilation techniques to infer the basal drag coefficient.

Glossary of Key Terms

- 1. Temperate ice: Ice at or near its pressure-melting point.
- 2. Sliding: The movement of a glacier over its bed by sliding rather than internal deformation.
- 3. Basal sliding: Sliding occurring at the base of a glacier.
- 4. Normal stress (N): The force acting perpendicular to a surface, per unit area. In the context of glaciers, it is primarily the weight of the overlying ice.
- 5. Shear stress (T): The force acting parallel to a surface, per unit area. In the context of glaciers, it is the force driving glacier motion.
- 6. Limiting static shear stress (TS): The minimum shear stress required to initiate sliding from a resting position.
- 7. Coefficient of static friction (µs): The ratio of the limiting static shear stress to the normal stress, indicating the resistance to sliding from rest.
- 8. Steady-state velocity (Vb): The constant velocity reached by a glacier or ice block when the driving shear stress is balanced by resisting forces.
- 9. Limiting dynamic shear stress (Tm): The shear stress at which a glacier or ice block transitions from steady-state sliding to accelerated sliding.
- 10. Coefficient of sliding friction (μ): The ratio of the shear stress to the normal stress during steady-state sliding.
- 11. Regelation: The process of melting under pressure and refreezing at lower pressure, potentially contributing to ice sliding.
- 12. Asperity: A protrusion or bump on a surface. Roughness The unevenness of a surface, characterized by the size and distribution of asperities.

13.

WHAT ARE THESE

- Channel incision
- Alpine style glaciation

${\rm TOOLS}$

- $\bullet\,$ ICECAP aero geophysical programme
- BEDMAP

MATHS

- Lagrangian interpolation
- $\bullet\,$ natural-neighbour interpolation

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