BedSAT: Antarctica

Exploring what lies beneath using big data and modelling

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Impact Statement

Antarctica's bed topography data currently has uncertainties of hundreds of meters in elevation due to sparse and unevenly distributed radar surveys, significantly limiting our ability to predict ice sheet behavior and sea level rise contributions. Through the BedSAT project, we are developing a novel modelling approach that integrates remote sensing data and airborne derived estimates with mathematical and numerical ice flow models to substantially improve bed topography resolution and accuracy. We aim to derive a continent-wide bed topography dataset and conduct sensitivity analyses of dynamic ice loss to different realisations of topographic roughness through 2300. Our work will quantify how bed topography uncertainties affect ice mass loss projections. This improved understanding can provide more reliable sea level rise predictions, and enable evidence-based policy decisions for climate adaptation strategies. Our open-source approach and FAIR data principles will ensure these improvements benefit the broader scientific community and support more effective climate change mitigation planning for vulnerable coastal communities.

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Antarctica's Landscape

Climate Impacts and Global Significance

The polar regions are losing ice, and their oceans are changing rapidly [1]. The consequences of this polar transition extend to the whole planet and it is crucial for us to understand them to be able to evaluate the costs and benefits of potential mitigation strategies.

The consequences of changes in different kinds of polar ice manifest across multiple interconnected systems. In both polar oceans, shifts in seasonal sea ice (both 'pack-ice' that moves with ocean currents and 'land-fast ice' that remains attached to the coast [2]) are altering marine ecosystems, from the production of new organic matter by photosynthetic organisms like plants and algae, to species distribution [1]. Of particular concern is the accelerating loss of continental ice sheets (permanent glacial ice masses on land) in both Greenland and Antarctica, which has become a major contributor to global sea level rise [1]. Impact extends beyond direct ice loss: As fresh water from melting ice sheets is added into the ocean, it increases ocean stratification. Cold freshwater can dissolve larger amounts of CO_2 than regular ocean water. Increased CO_2 uptake by polar oceans is creating corrosive conditions for calcifying organisms [1]. In addition, freshwater stratification threatens to disrupt global thermohaline circulation [3] by decreasing the natural mixing of the ocean layers.

Antarctica's climate response, however, differs markedly from the Arctic's uniform warming pattern. While West Antarctica has experienced warming in certain regions, East Antarctica has shown minimal overall temperature change in recent decades [1]. These observations carry low confidence due to limited data availability and high variability [1]. This asymmetric response is partly explained by the Southern Ocean's unique ability to absorb and mix heat into its depths [4].

To make matters more challenging, there is significant uncertainty in the timing and magnitude of Antarctica's ice loss, largely due to unknowns in ice sheet properties and associated flow processes [5]. Uncertainty increases in regions with variable bed conditions, where characteristics like "slipperiness" and roughness are difficult to verify via direct samples. Additionally, our assumptions about temperature and depth-dependent parameters like viscosity affect several key physical processes. How we use these model variables to constrain ice dynamics where we have data gaps is crucial for our ability to accurately predict ice sheet behavior [6].

Topography of Antarctica

Bed topography is one of the most crucial boundary conditions that influences ice flow and loss from the Antarctic Ice Sheet (AIS) [7]. Bed topography datasets are typically generated from airborne radar surveys, which are sparse and unevenly distributed across the Antarctic continent (see figure 2.1). Interpolation schemes to "gap fill" these sparse datasets yield bed topography estimates that have high uncertainties (i.e. multiple hundreds of metres in elevation uncertainty; Morlighem et al., 2020), which propagate in simulations of AIS evolution under climate change [8]. Given the logistical challenges of accessing large parts of the Antarctic continent, there is a crucial need for alternative approaches that integrate diverse and possibly more spatially complete data streams – including satellite data – to derive bed topography.

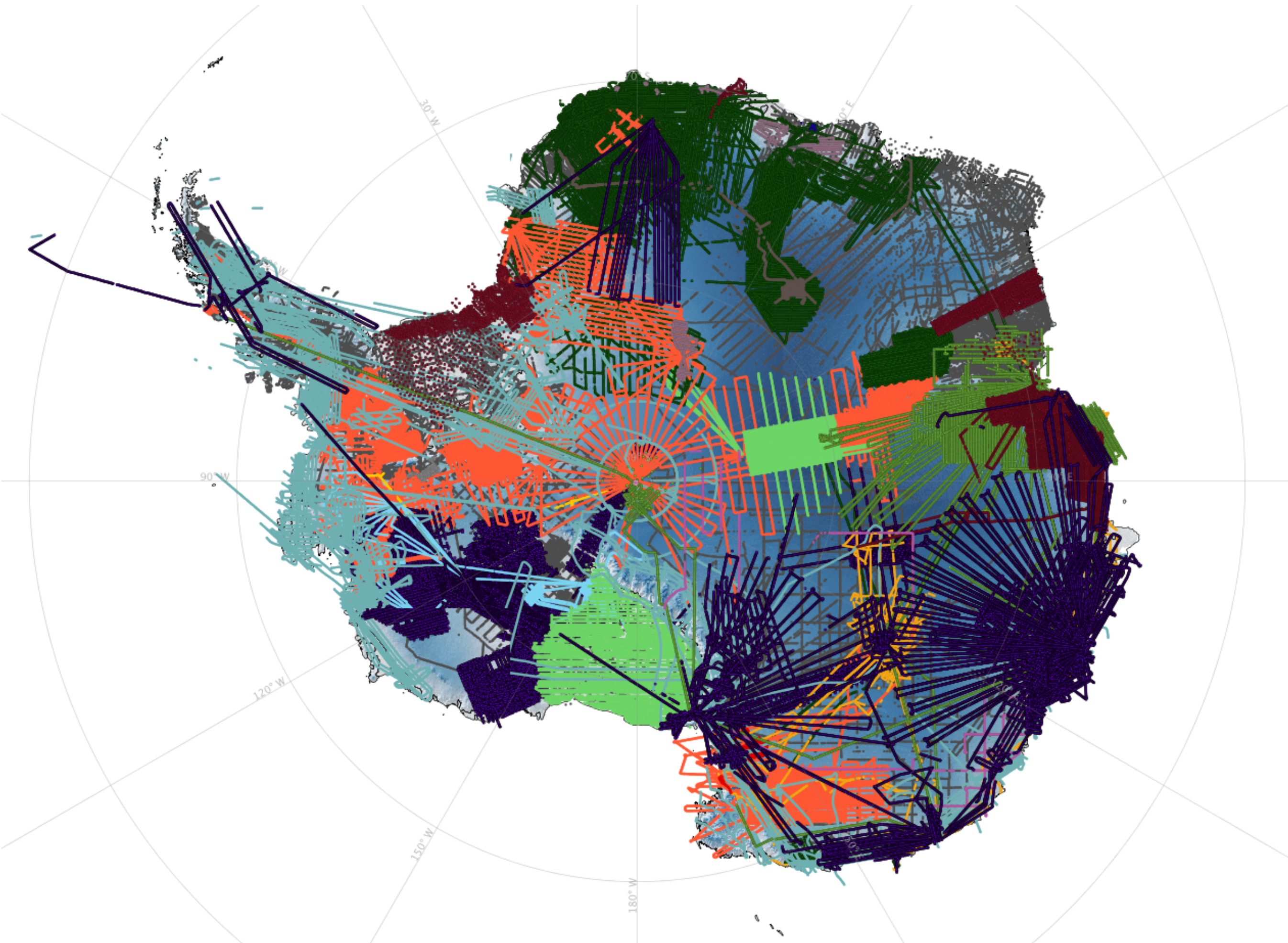

Figure 2.1: Distribution of BedMAP{1,2,3} data tracks (Source: bedmap.scar.org).

Obtaining information about the conditions at the base of the ice-sheet often relies on indirect modelling methods like

- **Inversions**
  - **Mass conservation**: Used to constrain inversion models and to fill data gaps by taking advantage of the physical laws of conservation of mass and momentum [7, 9]. (Monte Carlo Mass Conservation: Random sampling solves mass conservation equations [10]).
  - **Control method inversion**: Basal conditions distribution information is obtained from remote sensing data and theoretical ice flow models [11].
  - **Statistical inversion**: Study the simultaneous retrieval (or update) of bed topography and basal slipperiness from surface topography and velocity measurements [11]. See section 2.2 for an outlined example using **linear perturbation analysis**.
- **4dvar**: Four-dimensional variational data assimilation - Minimizes the difference between model predictions and observations across a time window. Mainly used to optimize model parameters and initial conditions [12].
- **Geostatistics** Statistical methods specialized for analyzing spatially correlated data. In glaciology, it's used to interpolate between sparse measurements and characterize spatial patterns in bed properties, often employing techniques like kriging [13].
- **EnKF** Ensemble Kalman Filter A sequential data assimilation method that uses an ensemble of model states to estimate uncertainty and update model parameters based on observations [12].

2.1 A case study

Critical Factors Influencing Thwaites Glacier's Future Evolution

Thwaites Glacier in West Antarctica represents one of the most impactful potential contributors to the mean global sea-level rise (SLR), with an estimated contribution of 0.59 meters [14]. The glacier's future evolution is of particular concern because it could trigger a broader collapse of the West Antarctic Ice Sheet (WAIS) [8, 14]. Understanding the factors that control its susceptibility to instabilities, including the marine ice sheet instability is therefore crucial for accurate sea-level rise projections [8].

Two primary factors control Thwaites Glacier's evolution: Ocean-driven basal melt rates and bedrock topography [8]. The Castleman et al. [8] study reveals that the glacier is highly sensitive to changes in bedrock topography within current measurement error bounds, highlighting the critical need for more accurate topographical data.

The glacier's vulnerability is further complicated by Marine Ice Sheet Instability (MISI) [8], a feedback mechanism where warmer ocean temperatures accelerate ice shelf melting and calving. Buttressing ice shelves have a stabilizing effect on the ice sheet and can potentially suppress or delay MISI [15]. For ice sheets on retrograde topographies (such as Thwaites glacier), this stabilizing effect fails. In these cases, the Grounding

Line (GL)—the transition from grounded to floating ice—undergoes retreat. Once initiated on such topography, grounding line retreat can accelerate [8]. Ocean-driven basal melt rates present another significant source of uncertainty in ice-sheet model simulations. This uncertainty stems from multiple factors [8]: the stochastic nature of ocean circulation patterns, temporal variability in ocean forcing, limitations in current ocean models, and limited direct ocean observations underneath ice shelves. These factors make it particularly difficult to predict how the ice sheet will respond to future ocean warming scenarios [8].

Current methods to generate topography datasets rely primarily on ice-penetrating radar, which presents significant challenges. While radar can provide direct measurements along specific tracks, interpolation is used to fill gaps between these tracks. This interpolation introduces uncertainties, particularly in areas where measurements are sparse. These uncertainties significantly affect our ability to model the glacier's future behavior accurately.

To quantify these uncertainties, Castleman et al. [8] employed two-dimensional discrete wavelet transform (DWT) to systematically analyze and modify bedrock topography data. Their method decomposes a bedrock elevation map $B(x, y)$ into four distinct subarrays: $\mathbf{A}_n$ (low-frequency approximation), and three high-frequency components - $\mathbf{H}_n$ (horizontal), $\mathbf{V}_n$ (vertical), and $\mathbf{D}_n$ (diagonal), where n indicates the decomposition level.

The approach selectively amplified the high-frequency components using a multiplier $\alpha > 1$, creating modified arrays $\mathbf{H}'_n = \alpha\mathbf{H}_n$, $\mathbf{V}'_n = \alpha\mathbf{V}_n$, and $\mathbf{D}'_n = \alpha\mathbf{D}_n$. This approach allows to introduce realistic perturbations into bedrock topography models and assess how varying spatial and vertical resolutions affect SLR projections.

One of the study's most significant findings relates to the importance of "pinning points" - bedrock features that can temporarily halt or slow grounding line retreat. While their wavelet-based method could potentially bias results toward more effective pinning points, the study revealed that the location of bedrock features relative to the grounding line and their deviation from mean bed elevation were more significant than feature amplification [8]. Through this analysis, they established crucial requirements for future bedrock measurements: 2 km spatial resolution and ± 8 meters vertical accuracy, particularly near the grounding line, to keep SLR uncertainty within ± 2 centimeters.

This study underscores the critical importance of accurate bedrock topography measurements for reliable SLR projections. The findings provide clear guidelines for future mapping efforts and highlight the need for focused attention on grounding line regions where bedrock features have the most significant impact on glacier stability.

2.2 Ice Sheet Bed Reconstruction via Surface Data Inversion

An important observation is that features in the ice bed often show up as subtle patterns in the surface topography above them [6]. These bed conditions - both their shape and mechanical properties - significantly influence how ice flows, with even small changes at the bed potentially leading to large differences in predicted ice loss rates. To address this challenge, is common to use inversion methods to understand the geophysical conditions at the ice sheet bed. The relationship between bed and surface characteristics is core to inversion methods that attempt to reconstruct bed properties from surface observations. Unfortunately, these methods cannot paint a complete picture of the ice sheet model.

Inversion methods require careful tuning of parameters which are not directly observable (e.g. the basal friction coefficient), so that modelled surface velocity matches observations. The mathematical framework for these inversions can be achieved via steady-state **linear perturbation analysis** of shallow stream approximation (SSA) [16].

Theoretical Framework

Inversion is based on the principle that variations in basal topography, slipperiness, and roughness cause disturbances to the surface flow of the ice. By measuring these disturbances in surface velocity and topography, and using equations like the shelfy-stream approximation (SSA) to relate those disturbances back to their source, we can estimate the basal conditions. The indirect measurements of basal properties x and y are related through $y = f(x)$, where f is referred to as the forward model, this makes $x = f^{-1}(y)$ the inversion. The indirect measurements y can be measurements of velocity and topography along the upper surface of a glacier, while the quantity x to be estimated represents basal topography and basal slipperiness [16]. The inversion method developed by Ockenden et al. introduces perturbations to study how small changes in ice thickness (h), surface elevation (s), basal topography (b), and ice velocity (u) affect ice flow. This means that the method is most accurate when applied to small perturbations, with the assumption that the perturbations are small relative to the mean of each studied property based on a reference state. The method assumes:

1. A linear viscous medium ($n = 1$)
2. Non-linear sliding law ($m > 0$)
3. Steady-state conditions
4. Spatially constant zero-order solutions

The SSA system described in Ockenden et al. [6] is as follows:

$$\partial_x(4h\eta\partial_x u + 2h\eta\partial_y v) + \partial_y(h\eta(\partial_x v + \partial_y u)) - (u/c^{1/m}) = \rho g h(\partial_x(s)\cos(\alpha) - \sin(\alpha)) \tag{2.1}$$

$$\partial_y(4h\eta\partial_y v + 2h\eta\partial_x u) + \partial_x(h\eta(\partial_y u + \partial_x v)) - (v/c^{1/m}) = \rho g h(\partial_y(s)\cos(\alpha) \tag{2.2}$$

Equations 2.2 and 2.2 are a linearised system around a reference model $(\bar{h}, \bar{s}, \bar{b}, \bar{u}, \bar{v}, \bar{c})$, leading to first-order momentum balance equations:

$$4\eta\bar{h}\partial_{xx}\Delta u + 3\eta\bar{h}\partial^2_{xy}\Delta v + \eta\bar{h}\partial^2_{yy}\Delta u - \gamma\Delta u = \rho g\bar{h}\cos(\alpha)\partial_x\Delta s - \rho g\sin(\alpha)\Delta h \tag{2.3}$$

$$4\eta\bar{h}\partial_{yy}\Delta v + 3\eta\bar{h}\partial^2_{xy}\Delta u + \eta\bar{h}\partial^2_{xx}\Delta v - \gamma\Delta v = \rho g\bar{h}\cos(\alpha)\partial_y\Delta s \tag{2.4}$$

where h represents ice thickness, s surface elevation, (u, v) horizontal components of surface velocity, c basal slipperiness, η effective ice viscosity, m sliding law parameter, ρ ice density, α mean surface slope in x-direction, and g acceleration due to gravity.

A disadvantage of the assumptions above is that they can limit how well we can model the behavior of real ice which exhibits nonlinear rheology.

Transfer Functions and Implementation

The methodology in [6] employs transfer functions. Transfer functions are mathematical expressions that describe how perturbations in basal topography and slipperiness affect surface topography and velocity. They are derived from the SSA equations

$$\begin{bmatrix} \hat{s} \\ \hat{u} \\ \hat{v} \end{bmatrix} = \begin{bmatrix} T_{sb} & T_{sc} \\ T_{ub} & T_{uc} \\ T_{vb} & T_{vc} \end{bmatrix} \begin{bmatrix} \hat{b} \\ \hat{c} \end{bmatrix}$$

The system is solved using a weighted least-squares approach, minimizing:

$$\Sigma s(s_{\text{obs}} - s_{\text{pred}})^2 + \Sigma u(u_{\text{obs}} - u_{\text{pred}})^2 + \Sigma v(v_{\text{obs}} - v_{\text{pred}})^2 \tag{2.5}$$

Inversion combines the surface data with the transfer functions to estimate the bed properties that would cause the surface changes.

Application and Limitations

Ockenden et al. report to have successfully implemented their inversion method using REMA surface elevation data (8m resolution) and NASA ITS_LIVE velocity data (120m resolution). The method performs particularly well in:

- Areas with moderate topographic gradients in the central trunk of glaciers
- Features not aligned with ice flow direction
- Medium-wavelength (5-50km) bedrock features

However, the method faces limitations in cases of:

- Steep topography where shallow-ice-stream approximation breaks down
- Features aligned with ice flow direction
- Variable slipperiness parameters
- Lack of validation data for slipperiness predictions

This work represents a significant advance in our ability to reconstruct bed conditions using surface data. Modern satellite technology provides unprecedented detail of ice sheet surface features through high-resolution elevation models and velocity measurements. However, we have yet to fully harness this wealth of surface information to improve bed topography estimates. The model in [17] relates ice flow over bedrock perturbations to surface expressions using a two-dimensional biharmonic stress equation. Budd's model simplifies the stress balance within the ice by assuming that most of the shear deformation happens at the base. It also explicitely considers longitudinal stresses and strain-rates. All these assumptions are similar to those of the SSA approach in Ockenden et al. While not directly using SSA equations or explicitly describing inverse methods, the concepts and relationships described by Budd in [17] emphasize how the stress and strain components are influenced by bed topography which in turn has a strong effect on surface features and ice flow. Similarly to Ockenden et al., Budd's model goes beyond a simple relationship between surface-slope variation and perturbation size, which is a limitation of simplified SSA models [17]. Interestingly, Budd's mathematical framework, despite being around for over five decades, is yet to be put to use in modern ice sheet modeling. This is especially surprising now that we have the computational power and high-resolution satellite data that could make it work. In this project, I aim to integrate this framework into ice sheet models using the Ice-sheet and Sea-level System Model (ISSM), potentially creating a systematic way to link what we can see on the surface to what's happening at the bed - especially in areas where we don't have good bed measurements.

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Objectives and Methodology

The overall aim of this project is to derive a new Antarctic bed topography using remote sensing data, airborne derived-estimates of the bed and ice sheet modelling. Using the new bed topography to improve understanding of the impact of fine-scale topographic roughness on ice and subglacial hydrological flow, and projections of ice mass loss under climate warming.

The specific objectives are:

1. Develop an ice sheet modelling approach to assimilate satellite remote sensing datasets to improve knowledge of the bed (BedSAT) informed by mathematical models of ice flow over topography;

2. Derive a new bed topography for Antarctica using BedSAT;

3. Conduct sensitivity analyses to understand the impact of the improved bed topography on projections of ice mass loss from Antarctica under climate warming.

The first phase of the project (objective 1) is to derive the BedSAT method. This will involve the integration of the Budd [17] mathematical model relating ice surface elevation and bed topography into ISSM, and the development of a methodology for the data assimilation into ISSM. I will use a regional catchment in Antarctica for which relatively more radar data are available and has an indicative range of topography features, e.g. the Aurora Subglacial Basin, East Antarctica, extensively surveyed by the ICECAP project for airborne geophysics [18]. The second phase of the project (objective 2) will apply the methodology developed in objective 1 to the whole Antarctic continent, deriving a continent-wide bed topography dataset. Using covariance properties from existing radar surveys, I will generate a number of realisations of bed topography with unique high-resolution, and statistically-consistent topographic roughness. The third phase of the project will use the new bed topography datasets to conduct a sensitivity analysis of ice sheet model projections to 2300 CE, investigating the impact of the new topography and different realisations of roughness on ice and subglacial hydrological flow and ice mass loss from Antarctica.

Plan:

Objective 1

1. Develop a method to interpolate topography that ensures consistent surface expressions with observations

2. Reduce RMS error between observations and model predictions

Key Investigation Areas

1. **Model Development Strategy**
   - Focus on invariant bed traction over the time-frame
   - Consider thermal distribution, velocity field, and ice thickness
   - Perform analysis to identify which topographical feartures affecting surface expression amplitude and positions
   - Note: Ice behavior varies with thickness. Simulate the following different scenarios: (thick ice → slippery base, thin ice → sticky base)
2. **Transfer Functions**
   - Develop rapid transfer function construction methods
   - Use for various conditions and domains
   - Validate against ICECAP cross-section radargrams
   - Analyze spatial covariance of existing data
   - Consider friction roughness and high amplitude variations
3. **Model Validation**
   - Quantify error reduction
   - Identify model breaking points
   - Use SVD (Singular Value Decomposition)
   - Test grid independence
   - Evaluate sensitivity to assumptions

Confounding Factors to Consider

Basal friction coefficient effects on:

- Sliding behavior
- Rheological properties
- Thermomechanical responses (stiff vs soft ice)
- Slippery spots

Workflow Plan

1. **Initial Modeling Phase**
    - Exclude surface elevation initially (reserve for control model)
    - Use 2D Gaussian with 3× ice thickness
    - Analyze REMA spectral components at various frequencies
    - Determine reasonable Signal-to-Noise ratio levels
2. **Inversion Development** Parameters to consider:
    - Basal traction
    - Internal temperature distribution
    - Heat flux
    - Additional rheological parameters
3. **Model Testing**
    - Test with ensemble of topographical conditions
    - Experiment with Gaussian features of different sizes
    - Focus on surface expressions with meaningful results
    - Consider Gausberg domain
    - Use Jameson cross-flow features (ranging from shallow to deep ice, sticky to sliding bed)
    - Incorporate ICECAP data

Resources

The project will require high performance computing resources (including compute and storage) from the National Computing Infrastructure (NCI). We anticipate requiring ∼250 k Service Units (SU) each quarter, and up to 500 TB of storage. These resources are already available via a Flagship between NCI and the Monash-led Australian Research Council project Securing Antarctica's Environmental Future (SAEF).

Currently available data and Framework

The project will make use of a number of new remote sensing datasets and software tools.

1. **Reference Elevation Model of Antarctica (REMA)**
   REMA provides a high-resolution (2-metre) terrain map of nearly the entire continent, allowing for precise measurements of elevation changes over time. REMA supports various remote sensing activities, such as image orthorectification and interferometry, and aids in geodynamic and ice flow modeling, grounding line mapping, and surface process studies. Constructed from hundreds of thousands of Digital Elevation Models (DEMs) derived from high-resolution Maxar satellite imagery (including WorldView and GeoEye data), REMA is calibrated with Cryosat-2 and ICESat altimetry, ensuring high elevation accuracy with uncertainties of less than 1 meter over most areas [19].

2. **ITS_LIVE Antarctic surface velocities and elevation**
   The NASA-administered ITS_LIVE website provides automated, high-resolution datasets of Antarctic surface velocities and ice surface elevation change, derived from satellite observations. The datasets are available on annual timesteps from 1985 to present. ITS_LIVE employs various statistical and computational methods to process data from satellites including Landsat and Sentinel, ensuring precise and timely updates for scientific research [20].

3. **BedMachine Antarctica**
   A high-resolution map of Antarctic subglacial bed topography that provides unprecedented detail of basal features. The dataset combines multiple ice thickness measurements with mass conservation principles, satellite-derived ice flow velocities, and surface mass balance from regional atmospheric models. This methodology has led to significant corrections in known glacier depths (e.g., 200m deeper for Pine Island Glacier) and revealed previously unknown features, with bed slopes found to be steeper in 62% of the mapped area compared to previous datasets [7].

4. **BedMAP**
   A suite of gridded products describing surface elevation, ice-thickness and the seafloor and subglacial bed elevation of Antarctica, based on a compilation of data collected by a large number of researchers using a variety of techniques, with the aim of representing a snap-shot of understanding of the Antarctic region [21].

5. **Ice-sheet and Sea-level System Model (ISSM)**
   ISSM is a finite-element numerical ice sheet model. It has been used to simulate the Antarctic Ice Sheet's response to various climate scenarios and assess future mass loss contributions to sea level rise [9, 10]. The mesh can be refined to better capture variations in ice flow and driving stresses, enhancing the simulation's accuracy of surface elevation changes and ice dynamics. This project will involve numerical modeling using advanced mathematical approaches, including the Blatter-Pattyn approximation to the full Stokes equations for ice flow (i.e. conservation of momentum equations). The Blatter-Pattyn approximation strikes a balance between the computationally intensive full Stokes equations and the simpler shallow ice approximation (SIA), retaining vertical shearing and longitudinal stress gradients. This makes it ideal for modeling the dynamics of fast-flowing ice streams and ice shelves at the continental scale, enhancing simulation accuracy while being computationally feasible. Additionally, data assimilation, machine learning, and geostatistics will be employed, with the full Stokes equations used if necessary [22]

Data management and archiving

Data will be published adhering to FAIR principles (Findable, Accessible, Interoperable, Reusable), ensuring transparency and accessibility. The final bed topography datasets will be published at the Australian Antarctic Data Centre (AADC) under an open source licence. All production model outputs will be published with unique DOIs at repositories aligned with the corresponding journal articles. Model outputs – including production and other outputs – will be archived to tape at NCI using existing SAEF resources, as well as backed up to storage available through Monash MASSIVE M3 account aligned with project supervisor Dr McCormack. All journal articles published through this project will be open source, and tier 1 journals will be targeted.

Risk

The project is highly feasible and low risk, given that it is a desk-based modelling and data assimilation project. All the data to be used in this project are freely available for download, and project supervisors are experts in ice sheet modelling using ISSM.

Fieldwork

Fieldwork is not necessary to achieve the objectives of the project; however, there may be the opportunity to participate in fieldwork through the ICECAP airborne geophysics project (led CI of ICECAP is project supervisor Dr Jason Roberts, Australian Antarctic Division), which will be instrumental in training of geophysical instruments and in developing broader expertise in the field.

Career Development

Conferences
At least one conference will be attended each year. An international conference relevant to the discipline, e.g. the European Geophysical Union General Assembly, will be attended in the final year of the project.

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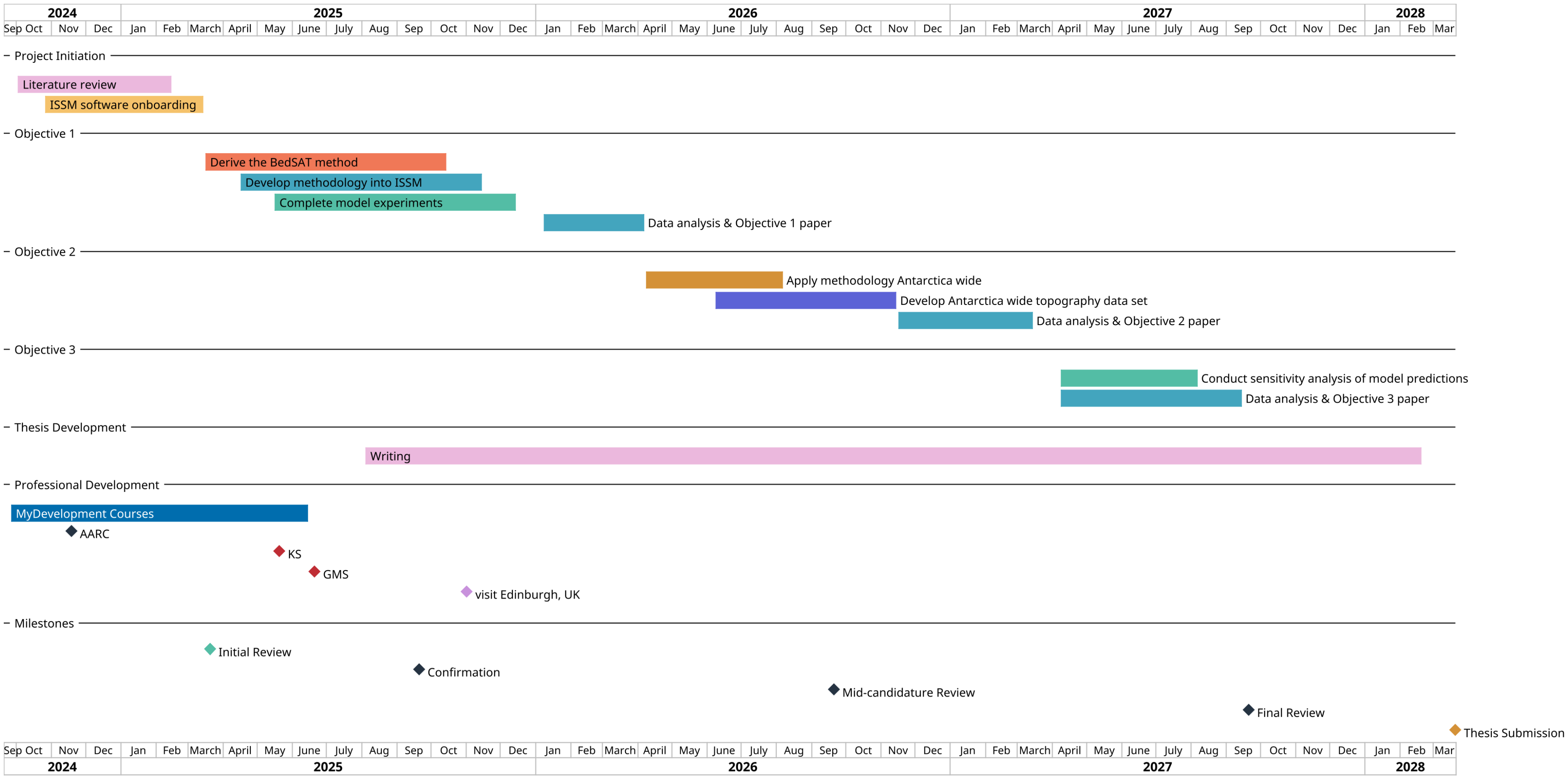
Project timeline



Glossary of Key Terms

1. **Adaptive Mesh Refinement**: A technique used to refine the mesh in regions of high variability or complexity, enhancing model accuracy and efficiency.

2. **Asperity**: A protrusion or bump on a surface. Roughness The unevenness of a surface, characterized by the size and distribution of asperities.

3. **Basal Drag Coefficient**: A parameter representing the frictional resistance between the ice sheet base and the underlying bedrock.

4. **Basal Melt Rates**: The rate at which the underside of a glacier or ice sheet melts due to contact with warmer ocean water.

5. **Basal sliding**: Sliding occurring at the base of a glacier.

6. **Bed Topography**: The shape and elevation of the bedrock underlying a glacier or ice sheet.

7. **Bedrock Topography**: The shape and elevation of the solid rock surface beneath a glacier or ice sheet.

8. **Blatter-Pattyn Approximation (BP)**: A higher-order ice flow model that incorporates longitudinal stresses, making it suitable for simulating fast-flowing ice streams and regions with significant vertical shear.

9. **Calving**: The process by which icebergs break off from the edge of a glacier or ice sheet.

10. **Coefficient of sliding friction (μ)**: The ratio of the shear stress to the normal stress during steady-state sliding.

11. **Coefficient of static friction (μ_s)**: The ratio of the limiting static shear stress to the normal stress, indicating the resistance to sliding from rest.

12. **DOI (Digital Object Identifier)**: A persistent digital identifier assigned to research outputs and scientific data that provides a permanent link to its location on the internet, ensuring long-term accessibility and citability.

13. **Data Assimilation**: The process of incorporating observational data into a model to improve its accuracy and predictive capabilities.

14. **Discrete wavelet transform (DWT)**: A mathematical technique that decomposes a signal into different frequency components while preserving spatial information. In glaciology, it's used to analyze bed topography at multiple spatial scales.

15. **FAIR principles**: A set of guiding principles for scientific data management and stewardship: Findable, Accessible, Interoperable, and Reusable. These principles are designed to optimize data reuse by both humans and machines.

16. **Finite Element Method (FEM)**: A numerical method for solving partial differential equations by dividing the domain into smaller elements and approximating the solution within each element.

17. **Fourier Transform**: A mathematical tool used to decompose a signal, such as surface elevation data, into its constituent frequencies. This allows for analysis of specific spatial scales and features.

18. **Full-Stokes (FS)**: The most comprehensive and computationally expensive ice flow model, accounting for all stress components. Essential for accurately simulating ice flow near grounding lines.

19. **Global Mean Sea-Level Rise (SLR)**: The average increase in sea level across the globe due to various factors, including melting of glaciers and ice sheets, and thermal expansion of ocean water.

20. **Grounding Line**: The boundary where the ice sheet transitions from grounded ice to floating ice (ice shelf).

21. **Grounding line retreat**: The inland migration of the grounding line (where ice transitions from grounded to floating) due to factors such as increased basal melting or dynamic thinning. A key indicator of ice sheet instability and potential mass loss.

22. **Ice-Penetrating Radar**: A remote sensing technique used to map the bedrock topography beneath glaciers and ice sheets by transmitting radar waves through the ice.

23. **Ice shelf**: A floating extension of a land-based ice sheet, typically hundreds of meters thick. Ice shelves are important for buttressing ice flow from the interior and their interaction with ocean water can significantly influence ice sheet stability.

24. **Ice Sheet System Model (ISSM)**: A finite element, thermomechanical ice flow model that incorporates SIA, SSA, BP, and FS formulations to simulate ice sheet behaviour at various complexities and spatial resolutions.

25. **InSAR**: Interferometric Synthetic Aperture Radar, a remote sensing technique used to measure ice surface velocity.

26. **Interferometry**: A remote sensing technique that combines multiple radar images to measure surface deformation or elevation changes with high precision. In glaciology, it's particularly useful for measuring ice flow velocities and surface elevation changes.

27. **Inversion**: A mathematical technique used to infer unknown parameters, such as bed topography, from observations of other variables, such as surface elevation and velocity.

28. **Limiting dynamic shear stress (T_m)**: The shear stress at which a glacier or ice block transitions from steady-state sliding to accelerated sliding.

29. **Kriging**: A geostatistical interpolation method that estimates unknown values at specific points by calculating weighted averages of known values from surrounding points, while accounting for spatial correlation and providing uncertainty estimates.

30. **Land-fast ice**: Sea ice that remains attached to the coast, seafloor, or grounded icebergs. This stationary ice forms a stable platform along coastal areas and plays a crucial role in local ecosystems and climate processes

31. **Limiting static shear stress (T_S)**: The minimum shear stress required to initiate sliding from a resting position.

32. **Linear Perturbation Analysis**: A technique that examines the response of a system to small perturbations in its parameters, assuming a linear relationship between the perturbation and the response.

33. **Marine Ice Sheet Instability (MISI)**: A process where the grounding line of an ice sheet retreats into deeper water, leading to accelerated ice discharge and potentially unstoppable collapse.

34. **Mass conservation**: A fundamental physical principle stating that mass cannot be created or destroyed in a closed system. In glaciology, it's used to constrain ice flow models and ensure physically realistic solutions when reconstructing bed topography.

35. **Momentum Balance**: The fundamental physical principle describing how forces control ice motion, expressed through the Navier-Stokes equations. In ice sheet modeling, it accounts for the balance between internal stresses, gravitational driving forces, and resistive forces (including drag at the bed and lateral margins).

36. **Normal stress (N)**: The force acting perpendicular to a surface, per unit area. In the context of glaciers, it is primarily the weight of the overlying ice.

37. **Null Space**: The set of all possible solutions to an inverse problem that do not contribute to the observed data. In the case of the work of 2.2, features aligned with ice flow fall within the null space and cannot be resolved by the inversion.

38. **Ocean forcing**: The influence of oceanic conditions (temperature, salinity, currents) on ice sheet behavior, particularly through melting at the base of ice shelves and at the grounding line. A key factor in ice sheet modeling and future projections.

39. **Orthorectification**: The process of removing geometric distortions from satellite or aerial imagery to create a planimetrically correct image where all pixels are viewed from directly above, essential for accurate surface elevation measurements.

40. **Pack-ice**: Sea ice that moves with ocean currents and winds, typically found in polar regions. Unlike land-fast ice, pack-ice is mobile and can form dense fields of ice floes that vary in size and thickness.

41. **Pinning Point**: A topographic feature, such as a ridge or mountain, that can slow or temporarily halt the retreat of a glacier's grounding line.

42. **Regelation**: The process of melting under pressure and refreezing at lower pressure, potentially contributing to ice sliding.

43. **Retrograde Bedrock Slope**: A bedrock slope that deepens inland, making the ice sheet more susceptible to marine ice sheet instability.

44. **Rheology**: The study of how materials deform and flow under stress. In glaciology, it refers to the flow properties of ice.

45. **Shallow Ice Approximation (SIA)**: A simplified ice flow model that considers only vertical shear stresses and neglects horizontal stress gradients. Suitable for slow-moving ice in the interior of ice sheets.

46. **Shallow Shelf Approximation (SSA)**: A simplified ice flow model that neglects vertical shear stresses and assumes depth-independent horizontal velocity. Appropriate for modelling floating ice shelves and fast-flowing ice streams.

47. **Shallow-Ice-Stream Approximation**: A simplification of the ice flow equations that assumes the ice thickness is much smaller than the horizontal extent of the glacier, allowing for analytical solutions.

48. **Shear stress (T)**: The force acting parallel to a surface, per unit area. In the context of glaciers, it is the force driving glacier motion.

49. **Sliding**: The movement of a glacier over its bed by sliding rather than internal deformation.

50. **Slipperiness**: A measure of the ease with which ice can slide over its bed. It encompasses the influence of basal conditions like geology, hydrology, and sediment characteristics.

51. **Steady-state**: A condition where the glacier's flow and properties are constant over time, assuming a balance between ice accumulation and loss.

52. **Steady-state velocity (V_b)**: The constant velocity reached by a glacier or ice block when the driving shear stress is balanced by resisting forces.

53. **Stress Balance**: The equilibrium between the forces acting on an ice sheet, including gravity, basal friction, and internal ice stresses.

54. **Surface mass balance**: The net difference between accumulation (snowfall, rain) and ablation (melting, sublimation, wind erosion) at the ice sheet surface. A fundamental parameter in ice sheet mass budget calculations.

55. **Thermohaline circulation**: The global ocean circulation system driven by differences in temperature (thermo) and salinity (haline). This system, often called the "global conveyor belt," plays a crucial role in global heat distribution and climate regulation

56. **Temperate ice**: Ice at or near its pressure-melting point.

57. **Transfer Functions**: Mathematical equations that describe the relationship between perturbations in bed properties and the resulting changes in surface variables.

58. **Volume Above Floatation (VAF)**: The volume of an ice sheet that is grounded on bedrock and contributes to sea-level rise if it melts or slides into the ocean.

59. **Wavelet Decomposition**: A mathematical technique that analyzes a signal by decomposing it into different frequency components at various spatial scales.

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