

# BedSAT: Antarctica

Exploring what lies beneath using big data and modelling

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## **Impact Statement**

Antarctica's bed topography data currently has local uncertainties of hundreds of meters in elevation due to sparse and unevenly distributed radar surveys, significantly limiting our ability to predict ice sheet behaviour and sea level rise contributions. Through the BedSAT project, I am developing a novel modelling approach that integrates remote sensing data and airborne-derived estimates, with mathematical and numerical ice flow models to substantially improve bed topography resolution and accuracy. I aim to derive a continent-wide bed topography dataset and conduct sensitivity analyses of dynamic ice loss to different realisations of topographic roughness through 2300CE. My work will quantify how bed topography uncertainties affect ice mass loss projections. This improved understanding can provide more reliable sea level rise predictions, and enable evidence-based policy decisions for climate adaptation strategies. This open-source approach and FAIR data principles will ensure these improvements benefit the broader scientific community and support more effective climate change mitigation planning.

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# Antarctica's Landscape

## 1.1 Climate Impacts and Global Significance

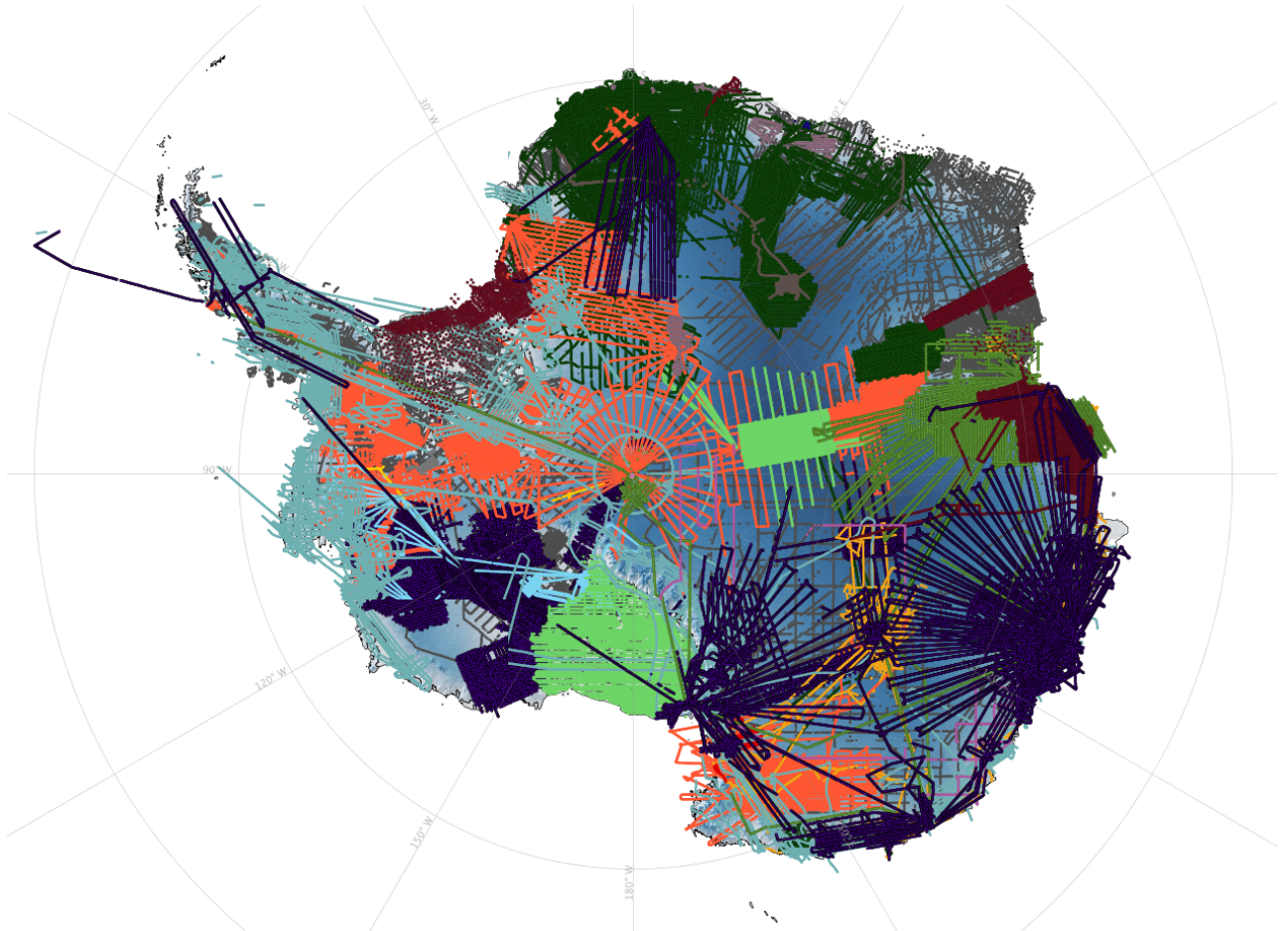
The polar regions are losing ice, and their oceans are changing rapidly [1]. The consequences of this extend to the whole planet and it is crucial for us to understand them to be able to evaluate the costs and benefits of potential mitigation.

Changes in different kinds of polar ice affect many connected systems. Of particular concern is the accelerating loss of continental ice sheets (glacial ice masses on land) in both Greenland and Antarctica, which has become a major contributor to global sea level rise [1]. Impacts extend beyond direct ice loss: as fresh water from melting ice sheets is added into the ocean, it increases ocean stratification disrupting global thermohaline circulation [2]. In addition, cold freshwater can dissolve larger amounts of  $\text{CO}_2$  than regular ocean water creating corrosive conditions for marine life [1].

While there is high confidence in current ice loss and retreat observations in many areas, there is more uncertainty about the mechanisms driving these changes and their future progression [3]. Uncertainty increases in regions with variable bed conditions, where characteristics like bed slipperiness and roughness are difficult to verify via direct observations. Other problematic areas involve the ice sheet grounding line (GL): The zone that delineates ice grounded on bedrock from ice shelves floating over the ocean. The GL retreat rate depends crucially on topographical features like pinning points [3], which lead to increased buttressing by the ice shelf on the upstream ice sheet. Although this mechanism is established, major knowledge gaps persist in mapping bed topography across Antarctic ice sheet margins, with over half of all margin areas having insufficient data within 5 km of the grounding zone [4]. Addressing this data gap through both systematic mapping and improved interpolation —utilising auxiliary data streams with more complete coverage— would significantly improve both our understanding of current ice dynamics and the accuracy of ice-sheet models projecting future changes.

# Topography of Antarctica

Bed topography is one of the most crucial boundary conditions that influences ice flow and loss from the Antarctic Ice Sheet (AIS) [5]. Bed topography datasets are typically generated from airborne radar surveys, which are sparse and unevenly distributed across the Antarctic continent (see figure 2.1). Interpolation schemes to “gap fill” these sparse datasets yield bed topography estimates that have high uncertainties (i.e. multiple hundreds of metres in elevation uncertainty; Morlighem et al., 2020) which propagate through simulations of AIS evolution under climate change [6]. Given the logistical challenges of accessing large parts of the Antarctic continent, there is a crucial need for alternative approaches that integrate diverse and possibly more spatially complete data streams – including satellite data.



**Figure 2.1:** Distribution of BedMAP{1,2,3} data tracks (Source: bedmap.scar.org).

## 2.1 Approaches to Bed Topography Reconstruction

An objective of my research is to understand the bed topography itself and how it influences ice dynamics. There are two ways to infer information about this relationship: Through forward modelling, with assumptions of the bed conditions; and through inverse modelling that relies on surface observations.

- **Forward models**

The aim of forward models is to see how bed properties impact ice dynamics. A key example is using a large ensemble of bed topographies to investigate how bed uncertainties impact simulated ice mass loss. Geostatistical methods can be used to generate bed topographies that either preserve elevation or texture:

- **Geostatistics** is comprised of statistical methods specialized for analyzing spatially correlated data. In glaciology, this approach is used to interpolate between sparse measurements and characterise spatial patterns in bed properties, often employing techniques like kriging [7].

- **Inversion models**

The aim of these models is to understand bed properties through knowledge of surface or other variables. A key example is the retrieval of bed topography or basal slipperiness from surface elevation and velocities.

- **Control method inversion:** A variational approach that minimizes mismatches between observed and simulated fields through a cost function approach. Remote sensing data and theoretical ice flow models are used to obtain basal conditions [8]. Often needs regularization terms to prevent non-physical features or over-fitting [9].
- **4dvar:** Four-dimensional variational data assimilation - Similar to the control method inversion algorithm, but adds a time dimension. Used to optimize model parameters and initial conditions [9]. Can handle time-varying data and evolving glacier states, making it more suitable for dynamic systems unlike control methods, this makes them more computationally demanding [9].
- **Mass conservation:** Used to constrain inversion models and fill data gaps by employing physical conservation laws, particularly effective for reconstructing bed topography where direct measurements are sparse [5,10]. Requires (contemporary) measurements of ice thickness at the inflow boundary to properly constrain the system [9].
- **Markov Chain Monte Carlo (MCMC):** A probabilistic method that generates sample distributions to quantify uncertainties in ice sheet parameters and models [9]. While powerful for uncertainty quantification, these methods remain computationally intensive for continental-scale ice sheet models [9].
- **EnKF** Ensemble Kalman Filter. A sequential data assimilation method that uses an ensemble of model states to estimate uncertainty and update model parameters based on observations [9].

Despite revolutionary advances in satellite technology that provide unprecedented surface detail, a key challenge in glaciology remains: how to fully utilize this wealth of information in regions where our understanding of subglacial conditions is limited. My

research aims to develop an integrated method combining forward and inverse modeling to improve bed topography estimates by leveraging high-resolution satellite surface data in regions where radar data is sparse.

## 2.2 Theoretical Frameworks

Understanding how bed features manifest in surface observations requires a theoretical framework that connects these two domains. The modelling approach used in this project relies on two different theoretical frameworks that relate bed topography and surface features. Using synthetic data, observations and these modelling frameworks, my goal is understanding the limitations of each approach and how they can be improved.

### 2.2.1 Ice Flow Over Bedrock Perturbations - Budd 1970

The first framework was originally developed by Budd [11]. This model relates ice flow over bedrock perturbations to surface expressions using a two-dimensional biharmonic stress equation. The modelling carried out by Budd determined ice-sliding velocities for wide ranges of roughness, normal stress, and shear stress relevant to real glaciers [11]. Budd's framework fits in my project as a means of verifying the validity of the physics in my ice-sheet model, since it describes important effects of bedrock disturbances on the transient evolution of the transferred basal disturbances onto the surface. The theory makes several key predictions that have been confirmed through spectral analysis of real ice cap profiles:

1. A wavelength of minimum damping occurs at approximately 3.3 times the ice thickness,
2. surface undulations exhibit a  $\pi/2$  phase lag relative to bedrock features with steepest surface slopes occurring over the highest bedrock points, and
3. the amplitude reduction depends systematically on ice speed, viscosity, thickness, and wavelength.

Importantly, Budd's theory demonstrates that energy dissipation and basal stress patterns are maximized for bedrock irregularities with wavelengths several times the ice thickness, while smaller-scale bedrock variations decay exponentially with distance into the ice and have minimal impact on overall ice motion. This selective filtering of bedrock signals provides crucial insights for understanding which scales of bed topography most significantly influence ice dynamics.

A critical aspect of Budd's theoretical framework is understanding how ice rheology affects the bed-to-surface transfer relationships. Glen's flow law typically employs a stress exponent  $n \approx 3$  for ice under most natural conditions, reflecting the strongly nonlinear relationship between stress and strain rate. Budd's analysis revealed that under certain low-stress conditions, ice deformation can behave more linearly ( $n \approx 1$ ) than conventional wisdom suggests. This rheological distinction has profound implications for bed-to-surface transfer functions: because linear rheology ( $n = 1$ ) may produce different amplitude dampening and phase relationships compared to nonlinear rheology ( $n = 3$ ), particularly for wavelengths around the critical 3.3 times ice thickness scale. My current modelling work systematically explores this by generating forward models for multiple synthetic bedrock profiles across four scenarios combining rheological assumptions ( $n = 1$  vs  $n = 3$ ) with basal boundary conditions (no-slip vs sliding), enabling direct comparison of how these physical assumptions affect the detectability and reconstruction of bed features from surface observations. Understanding these differences is essential for developing robust inversion methods, as the choice of rheological model fundamentally determines



the mathematical relationship between observable surface expressions and the underlying bed topography I seek to reconstruct.

Crucially, Budd’s work established the concept of frequency-dependent transfer functions that act as "filters" between bed and surface topography. This transfer function approach, expressed as  $\psi(\omega) = \frac{\text{surface amplitude}}{\text{bed amplitude}}$  for wavelength  $\lambda = 2\pi/\omega$ , provides a direct mathematical framework for inversion. By inverting these transfer functions, one can theoretically reconstruct bed topography from surface observations, particularly for wavelengths where the damping factor is minimal and the signal-to-noise ratio is optimal.

### 2.2.2 Ice flow perturbation analysis - Ockenden 2023

The second framework in my analysis builds upon these foundational concepts through some of the recent work by Ockenden et al. in [12], which uses observed surface perturbations (in velocity and elevation) to invert for unknown basal perturbations. Ockenden et al. improve from their previous work in [13] by using full-Stokes transfer functions, which greatly improves their method when dealing with steep topography where the shallow-ice-stream approximation breaks down. Ockenden et al. find this is crucial for better resolving the topographic features they are interested in. The core principle of the method by Ockenden et al. (2023) relies on the fact that variations in basal topography, slipperiness, and roughness cause measurable disturbances to the surface flow of the ice. Through linear perturbation analysis, they establish a systematic relationship between surface observations and bed conditions. This relationship can be expressed as  $y = f(x)$ , where  $y$  represents surface measurements (velocity and topography),  $x$  represents bed properties (topography and slipperiness), and  $f$  is the forward model transfer function refined by Gudmundsson and Raymond in 2008 [14].

In their work Ockenden et al apply this framework in reverse  $x = f^{-1}(y)$ , to infer the bedrock from modern, high-resolution satellite data estimates. A restrictive assumption in the modeling design by Ockenden et al might be their assumption of “constant viscosity”, in glaciology this is equivalent to assuming a linear rheology, where the stress exponent,  $n$ , is equal to 1. This means that the strain rate is directly proportional to the stress. This is in contrast to the more commonly used non-linear Glen’s Flow Law, where  $n$  is typically around 3 or even 4. This earliest phase in my PhD project has as a goal to determine whether treating the rheology of ice as linear is adequate. Ockenden et al account for a non-linear sliding law at the base of the ice, mentioning the “sliding law parameter  $m$ ”. However, the transfer of stress through the body of the ice—the core of the perturbation analysis—relies on the constant viscosity assumption from the foundational work of Gudmundsson and Raymond 2008.

### 2.2.3 Bridging Classical and Modern Approaches

While both frameworks address the fundamental bed-to-surface relationship, they operate at different levels of complexity and make different assumptions. Budd’s approach provides the fundamental physical understanding of how specific wavelengths propagate through ice, establishing theoretical limits on what bed features can be detected from surface observations. Ockenden’s method extends this to practical applications using real satellite data but relies on linearised assumptions that may break down under certain conditions. My research aims to bridge these approaches by combining Budd’s rigorous transfer function analysis with comprehensive forward modeling that relaxes some of the

restrictive assumptions inherent in linear perturbation methods. By systematically exploring how different rheological models ( $n = 1$  vs  $n = 3$ ) and basal conditions affect the bed-to-surface transfer functions, my work aims to develop a robust inversion method that can better handle the nonlinear physics of ice flow while maintaining the theoretical rigor established by Budd’s foundational analysis.

## 2.3 Current Opportunities

Current Antarctic bed topography reconstruction methods fail to utilize the wealth of presently available satellite-derived surface data. While mathematical models linking bed to surface through ice dynamics (such as those by Ockenden and Budd) provide a foundation for inferring bed topography from satellite data, they have significant limitations. My approach with BedSAT builds upon theoretical foundations and recent inversion methods to better understand how bed conditions—including slipperiness, roughness, and pinning points—affect both grounding line retreat rates and their surface expressions. BedSAT will connect surface observations with bed topography using more realistic rheological and geometric assumptions through an iterative process: initially inverted bed topography will feed into ice dynamics models with these improved assumptions, allowing comparison between model predictions and established datasets like NASA’s ITS\_LIVE. I expect to utilise Machine learning methods to systematize this process, enhancing the analytical capabilities for the project’s final phase.

# Methods

## 3.1 Aims

My research plan is structured around these three broad research questions:

1. How does the bed topography manifest on the ice surface?
2. To what extent do interpolation uncertainties in bed topography datasets affect the accuracy of Antarctic Ice Sheet evolution simulations under different climate change scenarios?
3. What is the impact of variable bed conditions and topography on the rate of grounding line (GL) retreat in continental ice sheets?

Underpinning these research questions are the following objectives (O):

- O1: Develop an ice sheet modelling approach to assimilate satellite remote sensing datasets to improve knowledge of the bed (BedSAT) informed by mathematical models of ice flow over topography;
- O2: Derive a new bed topography for Antarctica using BedSAT;
- O3: Evaluate the impact of the improved bed topography on projections of ice mass loss from Antarctica under climate warming through sensitivity analyses.

## 3.2 Research plan methodology

In order to achieve these objectives, each will be addressed in sequential phases. My primary focus is currently on O1: Deriving the BedSAT method. As the initial phase of O1, I am working on an investigation on the influence of different combinations of rheological and sliding law assumptions in ice sheet modeling. The goal of this investigation is to systematically understand the forward problem —how the bed affects the surface under different physical rules— to then use that knowledge to build a better inverse model (BedSAT).

### 3.2.1 Foundational Analysis of Bed-to-Surface Signal Transfer

The first critical step is to systematically quantify how fundamental physical assumptions influence the expression of subglacial topography at the ice surface. This directly addresses my first research question: "How does the bed topography manifest on the ice surface?".

This work will leverage the Ice-sheet and Sea-level System Model (ISSM) with a custom-built computational framework based on a synthetic bed topography database. This systematic study will verify and validate the necessary set of constraints on bed-to-surface transfer functions that account for realistic ice dynamics. This comprehensive analysis will form the basis of the the first peer-reviewed manuscript of this PhD.

### 3.2.2 Development of the BedSAT Inversion Framework

By understanding how rheology and sliding conditions alter the surface expression of the bed, I can develop more physically robust transfer functions for the inversion process. The inversion model will be developed and tested using a regional catchment in Antarctica with extensive radar data, such as the Aurora Subglacial Basin (this data can be found in works such as [15]). The model will be constrained by available observations of surface velocity, thermal distribution, and ice thickness, this will allow for direct validation of the inversion results against known bed configurations. Furthermore, the robustness of the model will be ensured through grid independence testing and a sensitivity analysis of model assumptions. See Chapter 4 for detailed information on the progress of this work.

### 3.2.3 Derive a new bed topography for Antarctica using BedSAT

I will apply the validated BedSAT methodology from O1 to the entire Antarctic continent, deriving a new continent-wide bed topography dataset. Using covariance properties from existing radar surveys, I will generate multiple realisations of the bed, each with unique and statistically-consistent topographic roughness.

### 3.2.4 Evaluate the impact of the improved bed topography

The new bed topography datasets will be used to conduct a sensitivity analysis of ice sheet model projections to 2300 CE. This will investigate the impact of the improved topography and different roughness realisations on ice dynamics, subglacial hydrology, and overall ice mass loss from Antarctica, directly addressing the project's main research questions.

*Note: Detailed methodological outlines for O2 and O3 will be developed following the completion and refinement of the BedSAT method in O1.*

# Progress

## 4.1 Writing Contributions

### 4.1.1 Synthetic bed topographies for Antarctica and their utility in ice sheet modelling

I contributed to the investigation and writing of the manuscript titled "Synthetic bed topographies for Antarctica and their utility in ice sheet modelling," which has been submitted to the journal "Proceedings of the Royal Society A". This comprehensive review and case study examines the various methods used to generate synthetic bed topographies for Antarctica, assessing their underlying objectives, associated uncertainties, and their impact on ice sheet model projections of sea level rise.

My contribution was focused on the literature review of key methodologies used to generate these topographies. I was responsible for authoring the descriptions for several prominent techniques, including:

- **Mass Conservation:** Detailing how this physics-informed approach, as implemented in widely-used datasets like BedMachine [5] and the TELVIS algorithm [16], is used to reconstruct bed topography by ensuring the continuity of ice volume across the glacial system.
- **Ensemble Kalman Filter (EnKF):** Summarising this data assimilation technique which uses an ensemble of model states to estimate and update system parameters based on new observations, thereby tracking the transient evolution of an ice sheet. some relevant works include [17, 18].
- **Linear Perturbation Theory:** Referenced works using this method are included and discussed in section 2.2 of this milestone report.

By describing these distinct approaches, their physical assumptions, and their limitations, my work helped to establish the theoretical context for the paper's case study on the Aurora Subglacial Basin. This review framed the comparison between different types of synthetic beds ("elevation-preserving" vs. "texture-preserving") and underscored how methodological choices in bed generation can significantly influence projections of future sea level contributions

### 4.1.2 SMUG

I have also contributed to the investigation and writing of a manuscript detailing a new interpolation method for sparse and unevenly sampled data. The manuscript is titled: “Antarctic bed topography estimation using a Stochastic Meshless Uncertainty Gridding (SMUG) method”.

My contribution involved conducting a review of existing interpolation techniques helping to author the introductory section of the manuscript. My writing establishes the scientific context and rationale for the development of SMUG. I analysed several established methods used in previous Antarctic bed topography datasets, including:

- Inverse Distance Weighting (IDW): Used in Bedmap1 [19], a straightforward method that can produce overly smooth surfaces and struggle with highly variable data.
- Kriging: A geostatistical method that provides uncertainty estimates but often requires subjective, expert-driven parameter selection, which can introduce bias. This method is evaluated in Bedmap2 [20] and Bedmap3 [21] and found to produce less accurate results than other methods such as spline interpolation.
- Spline Interpolation (e.g., Topogrid): The key technique in Bedmap2 [20], this method demonstrated good performance but faced challenges in optimising smoothing parameters and honouring all data points.
- Mass Conservation Methods: Implemented in BedMachine [5], this physics-informed approach improves accuracy in data-sparse regions but requires additional datasets (like ice velocity) that are not always available.

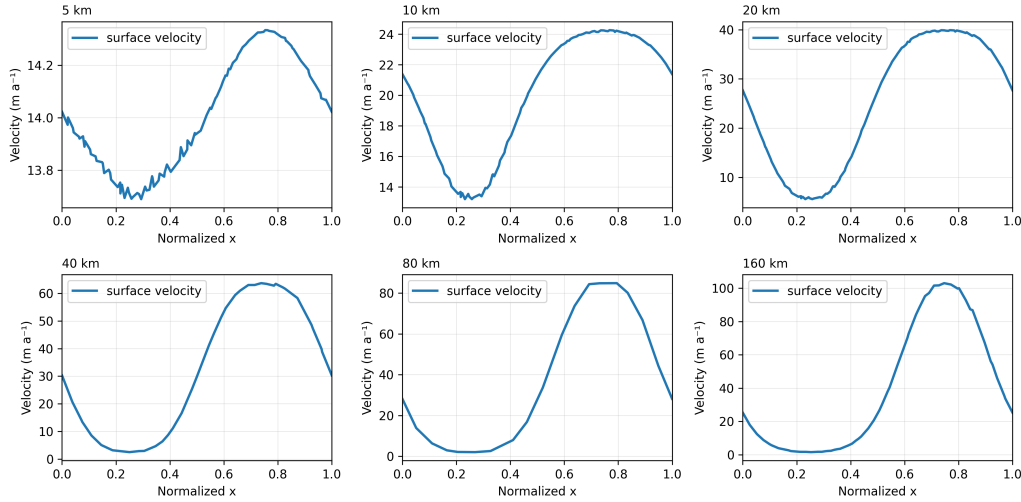
Through my investigation, I identified and articulated key limitations and research gaps inherent in these widely-used techniques. Specifically, my writing highlighted the common difficulties in providing robust uncertainty estimates, avoiding systematic biases, and capturing the spatial correlation of errors realistically. This analysis sets the stage for the manuscript to introduce SMUG as a method designed to overcome these specific shortcomings, setting the foundation upon which the novelty and significance of the SMUG method were demonstrated in our manuscript.

## 4.2 Recreating ISMIP-HOM

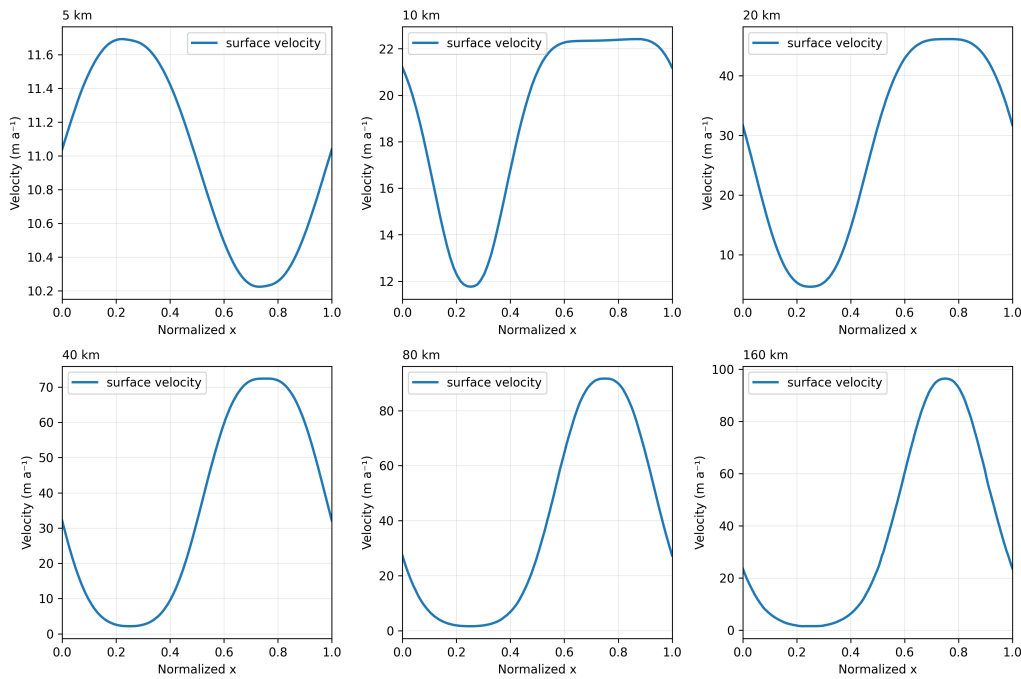
As a first step in validating the computational framework for this project and to build a understanding foundation of the capabilities and functionality of ISSM, I replicated a series of benchmark experiments from the Ice Sheet Model Intercomparison Project for Higher-Order Models (ISMIP-HOM) [22]. Successfully replicating these benchmarks demonstrates that the simulation setup is configured accurately capturing the fundamental physics of ice flow.

Part of my recreation focused on the first four diagnostic experiments (A, B, C, and D), which test a model’s ability to simulate ice flow under a range of conditions. Experiments A and B involve flow over a sinusoidally varying bed topography (a "bumpy" 3D bed and a "rippled" 2D bed, respectively) with no basal sliding. These experiments are designed to evaluate the model’s handling of longitudinal and vertical stress gradients induced by basal topography. Conversely, Experiments C and D feature a flat bed but introduce

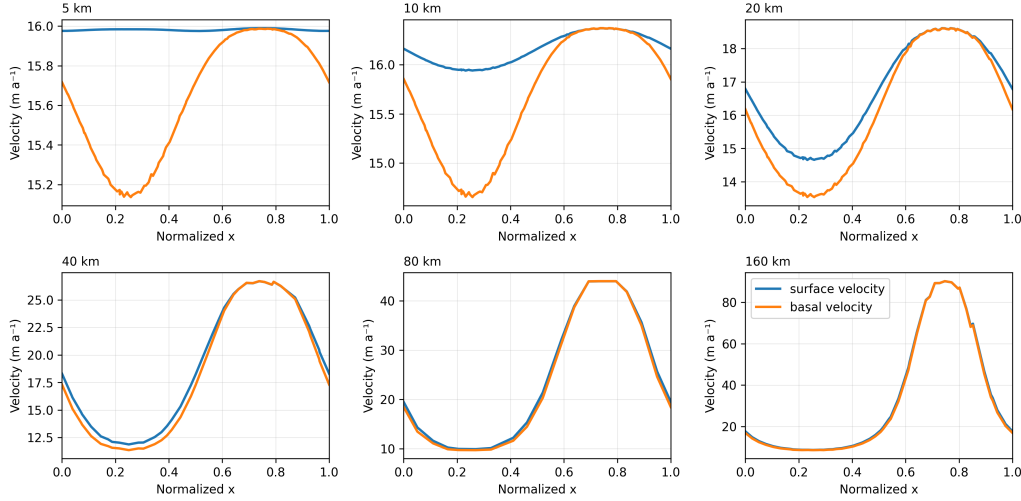
spatially variable basal friction, simulating the dynamics of an ice stream with slippery and sticky patches. All my models utilised full-Stokes equations (FS).



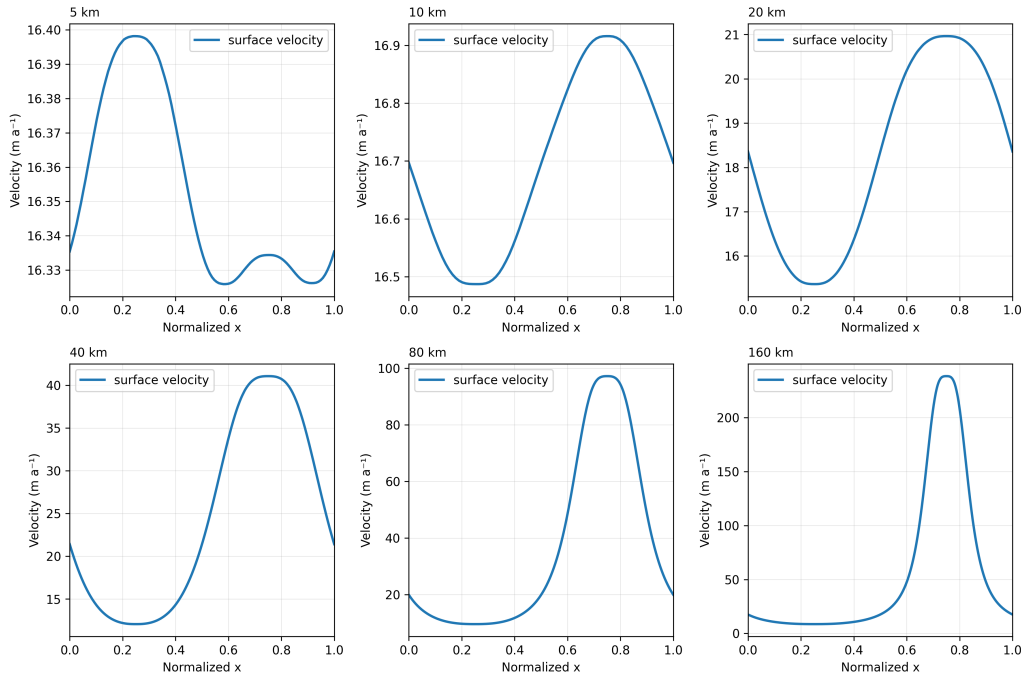
**Figure 4.1:** ISSM recreation of ISMIP-HOM Experiment A: Ice flow over a bumpy bed. The panels show the the surface velocity for a 3D ice flow simulation over a sinusoidal bed with no basal sliding ( $v_b = 0$ ). Each panel corresponds to a different domain length ( $L$ ), from 5 km to 160 km.



**Figure 4.2:** ISSM recreation of ISMIP-HOM Experiment B: Ice flow over a rippled bed. The panels show the surface velocity for a 2D flowline simulation. The setup is identical to Experiment A, but the basal topography does not vary in the y-direction, isolating longitudinal stress effects.



**Figure 4.3:** ISSM recreation of ISMIP-HOM Experiment C: Ice stream flow I. The panels show both surface (blue) and basal (orange) velocity for a 3D simulation over a flat bed where basal motion is governed by a spatially variable friction coefficient,  $\beta^2(x, y)$ .



**Figure 4.4:** ISSM recreation of ISMIP-HOM Experiment D: Ice stream flow II. The panels show the surface velocity for a 2D flowline over a flat bed with variable basal friction. The setup is identical to Experiment C, but the friction coefficient varies only in the x-direction,  $\beta^2(x, y)$ .

The results from these simulations demonstrate a strong agreement with the published findings in [22]. For all experiments and across the different prescribed length scales ( $L = 5$  km to 160 km), the calculated surface velocities closely matched the behaviour of the full-Stokes (FS) models from the original ISMIP-HOM. This successful validation confirms that my computational framework is robust and reliably simulates complex ice dynamics. This verification establishes a solid foundation for the application of my framework to more complex simulation settings and its subsequent research questions. In subsection 4.2.1 I work on extending the findings in Pattyn et al, 2008. To investigate the transient



(Prognostic) experiment F where the free surface is allowed to relax until a steady state is reached for zero surface mass balance [22]

### 4.2.1 Transient evolution ISMIP-HOM

Building upon the diagnostic ISMIP-HOM experiments, this work extends the prognostic experiment F to systematically investigate the combined effects of rheology and basal sliding within a benchmark ice sheet model. The original experiment F included two scenarios: one with a frozen bed (no-slip) and another with linear sliding. My study expands upon these conditions by also incorporating non-linear rheology. This addition generates four distinct scenarios for comparison:

- S1 No-slip (frozen) bed + Linear rheology ( $n = 1$ ).
- S2 No-slip (frozen) bed + Non-linear rheology ( $n = 3$ ).
- S3 Linear sliding + Linear rheology ( $n = 1$ ).
- S4 Linear sliding + Non-linear rheology ( $n = 3$ ).

While the original study by Pattyn et al. (2008) covered scenarios S1 and S3, understanding the impact of rheological assumptions is crucial for modern ice sheet modelling. During periods of rapid grounding line retreat, uncertainty in the Glen flow law exponent  $n$  has been found to cause a larger spread in ice-loss projections than uncertainty in climate forcing [23]. Therefore, explicitly testing these different physical conditions is a critical step. This foundational analysis, validated against a well-established benchmark, will provide the necessary confidence in the modelling framework before extending the work to a suite of more complex synthetic bed topographies.

In section 4.3, I extend these principles to investigate the transfer of more complex synthetic bed topography signals to the ice surface.

## 4.3 Rheology and Sliding Study

## 4.4 The Computational Framework of this Study

### 4.4.1 Synthetic Bedrock Generation

### 4.4.2 Ice Flow Simulation

The experimental design is built around four benchmark experiments mentioned in subsection 4.2.1 to test different physical conditions.

### 4.4.3 Data Processing and Visualization Tools

I have developed a set of robust, high-performance scripts to handle the large volume of data produced by the ice flow simulations. (Note that: All these scripts have individual file processing capabilities)

1. Binary to NetCDF Conversion A batch-capable tool (`batch_convert.py`) converts ISSM `.outbin` files into the standard, portable NetCDF format. This script supports parallel processing for high throughput.
2. Result Extraction and Visualization A batch script (`batch_extract_results.py`) that automatically finds and processes NetCDF files to generate visualisations of key fields like velocity and pressure.

#### 4.4.4 Scientific Analysis Tools

##### Key Findings: The Grid Independence Study

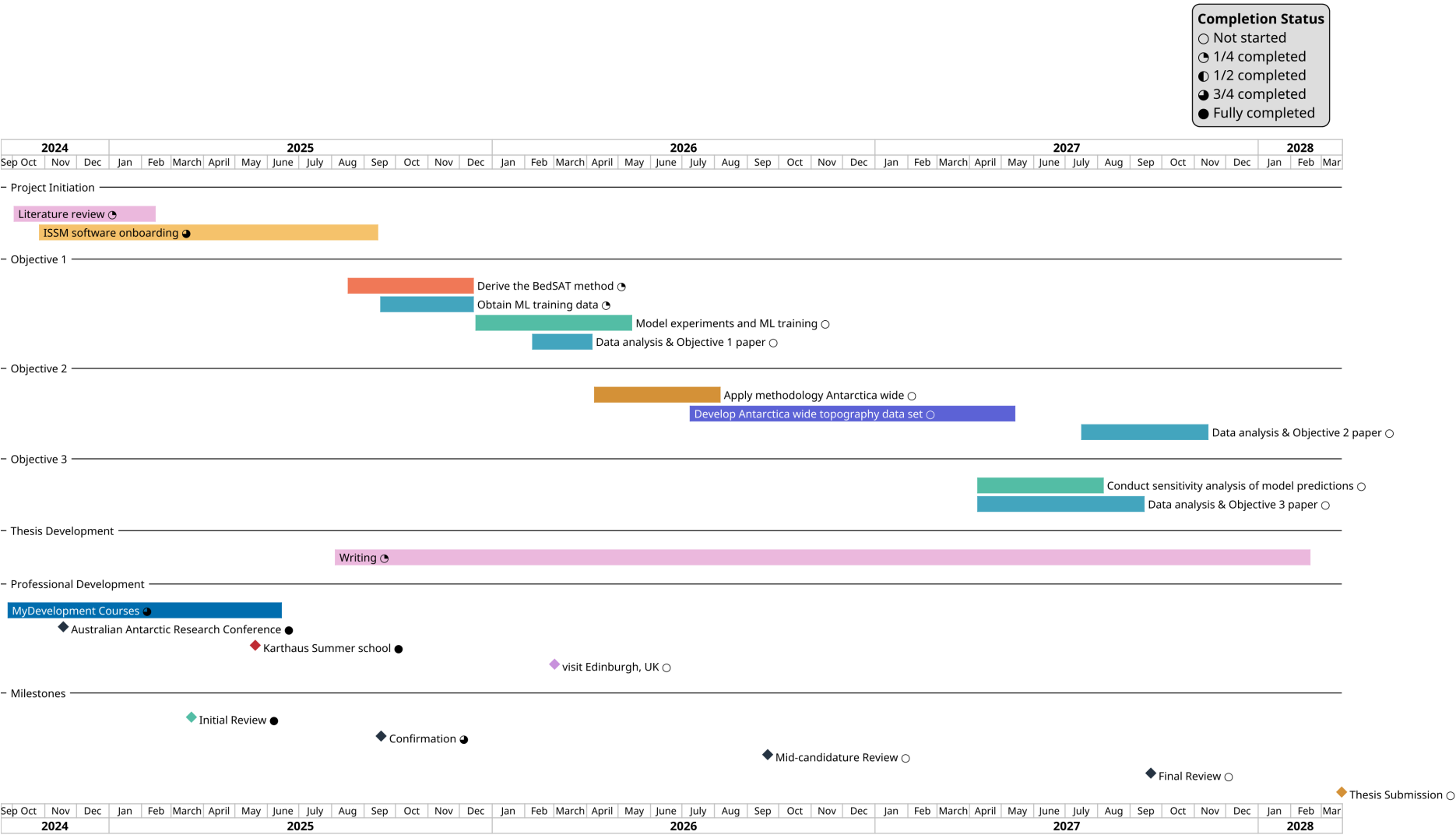
In order to perform quantitative analysis on the simulation results. I developed a convergence analysis script: `convergence_analyser.py` Grid convergence analysis is a fundamental verification technique in computational modeling that ensures numerical solutions are approaching the true solution as mesh resolution increases. The key principle is that as the grid is refined (smaller elements, more nodes), the numerical error should decrease systematically.

For the analysis, I compared coarser and finer meshes to a baseline reference mesh (I used resolution scaling factors of 0.5, 1.5 and 2.0). The primary metric of this script is the L2 relative error, a global, scale-dependent measure that quantifies the overall difference between two solutions. In my analysis, I chose a convergence threshold of 1% when comparing the refined mesh solutions to the baseline. This script generates a standardised  $2 \times 2$  plot ?? to provide a comprehensive view of convergence, showing visual velocity comparisons of surface and basal velocities for each mesh resolution tested, numerical L2 error bars, and the maximum velocity at every time step.

##### Phase Analysis

##### Transient Analysis

# Project timeline



# Bibliography

- [1] M. Meredith, M. Sommerkorn, S. Cassotta, C. Derksen, A. Ekaykin, A. Hollowed, G. Kofinas, A. Mackintosh, J. Melbourne-Thomas, M. M. C. Muelbert, G. Ottersen, H. Pritchard, and E. A. G. Schuur. *The Ocean and Cryosphere in a changing climate*, pages 203—320. Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA (2019). DOI: [10.1017/9781009157964.005](https://doi.org/10.1017/9781009157964.005). 4
- [2] S. S. Jacobs. *Bottom water production and its links with the thermohaline circulation*. Antarctic Science **16**, 427–437 (2004). DOI: [10.1017/S095410200400224X](https://doi.org/10.1017/S095410200400224X). 4
- [3] B. Fox-Kemper, H. T. Hewitt, C. Xiao, G. Aðalgeirsdóttir, S. S. Drijfhout, T. L. Edwards, N. R. Golledge, M. Hemer, R. E. Kopp, G. Krinner, A. Mix, D. Notz, S. Nowicki, I. S. Nurhati, L. Ruiz, J.-B. Sallée, A. B. A. Slangen, and Y. Yu. *Ocean, Cryosphere and Sea Level Change*. In V. Masson-Delmotte, P. Zhai, A. Pirani, S. L. Connors, C. Péan, S. Berger, N. Caud, Y. Chen, L. Goldfarb, M. I. Gomis, M. Huang, K. Leitzell, E. Lonnoy, J. B. R. Matthews, T. K. Maycock, T. Waterfield, O. Yelekçi, R. Yu, and B. Zhou (editors), *Climate Change 2021: The Physical Science Basis. Contribution of Working Group I to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change*, chapter 9, pages 1211–1362. Cambridge University Press, Cambridge, UK and New York, NY, USA (2021). DOI: [10.1017/9781009157896.011](https://doi.org/10.1017/9781009157896.011). 4
- [4] R. A. Group. *RINGS: Collaborative international effort to map all Antarctic ice-sheet margins*. Scientific Committee on Antarctic Research (2022). DOI: <https://doi.org/10.5281/zenodo.6638327>. 4
- [5] M. Morlighem, E. Rignot, T. Binder, D. Blankenship, R. Drews, G. Eagles, O. Eisen, F. Ferraccioli, R. Forsberg, P. Fretwell, V. Goel, J. S. Greenbaum, H. Gudmundsson, J. Guo, V. Helm, C. Hofstede, I. Howat, A. Humbert, W. Jokat, N. B. Karlsson, W. L. Lee, K. Matsuoka, R. Millan, J. Mouginot, J. Paden, F. Pattyn, J. Roberts, S. Rosier, A. Ruppel, H. Seroussi, E. C. Smith, D. Steinhage, B. Sun, M. R. van den Broeke, T. D. van Ommen, M. van Wessem, and D. A. Young. *Deep glacial troughs and stabilizing ridges unveiled beneath the margins of the Antarctic ice sheet*. Nature Geoscience **13**, 132 (2020). DOI: [10.1038/s41561-019-0510-8](https://doi.org/10.1038/s41561-019-0510-8). 5, 6, 13, 14
- [6] B. A. Castleman, N. J. Schlegel, L. Caron, E. Larour, and A. Khazendar. *Derivation of bedrock topography measurement requirements for the reduction of uncertainty in ice-sheet model projections of Thwaites Glacier*. The Cryosphere **16**, 761 (2022). DOI: [10.5194/tc-16-761-2022](https://doi.org/10.5194/tc-16-761-2022). 5
- [7] E. J. MacKie, D. M. S. J. Caers, M. R. Siegfried, and C. Scheidt. *Antarctic Topographic Realizations and Geostatistical Modeling Used to Map Sub-*

- glacial Lakes*. Journal of Geophysical Research: Earth Surface **125** (2020). DOI: <https://doi.org/10.1029/2019JF005420>. 6
- [8] J. D. Rydt, G. H. Gudmundsson, H. F. J. Corr, and P. Christoffersen. *Surface undulations of Antarctic ice streams tightly controlled by bedrock topography*. The Cryosphere **7**, 407 (2013). DOI: [10.5194/tc-7-407-2013](https://doi.org/10.5194/tc-7-407-2013). 6
- [9] M. Morlighem and D. Goldberg. *Data Assimilation in Glaciology*, pages 93–111. Cambridge University Press (2024). DOI: [10.1017/9781009180412.007](https://doi.org/10.1017/9781009180412.007). 6
- [10] M. Morlighem, C. N. Williams, E. Rignot, L. An, J. E. Arndt, J. L. Bamber, G. Catania, N. Chauché, J. A. Dowdeswell, B. Dorschel, I. Fenty, K. Hogan, I. Howat, A. Hubbard, M. Jakobsson, T. M. Jordan, K. K. Kjeldsen, R. Millan, L. Mayer, J. Mouginot, B. P. Y. Noël, C. O’Cofaigh, S. Palmer, S. Rysgaard, H. Seroussi, M. J. Siegert, P. Slabon, F. Straneo, M. R. van den Broeke, W. Weinrebe, M. Wood, and K. B. Zinglarsen. *Bedmachine v3: Complete bed topography and ocean bathymetry mapping of greenland from multibeam echo sounding combined with mass conservation*. Geophysical Research Letters **44**, 11,051 (2017). ARXIV: <https://agupubs.onlinelibrary.wiley.com/doi/pdf/10.1002/2017GL074954>, DOI: <https://doi.org/10.1002/2017GL074954>. 6
- [11] W. F. Budd. *Ice Flow Over Bedrock Perturbations*. Journal of Glaciology **9**, 29 (1970). DOI: [10.3189/S0022143000026770](https://doi.org/10.3189/S0022143000026770). 8
- [12] H. Ockenden, R. G. Bingham, A. Curtis, and D. Goldberg. *Ice-flow perturbation analysis: a method to estimate ice-sheet bed topography and conditions from surface datasets*. Journal of Glaciology **69**, 1677 (2023). DOI: [10.1017/jog.2023.50](https://doi.org/10.1017/jog.2023.50). 9
- [13] H. Ockenden, R. G. Bingham, A. Curtis, and D. Goldberg. *Inverting ice surface elevation and velocity for bed topography and slipperiness beneath Thwaites Glacier*. The Cryosphere **16**, 3867 (2022). DOI: [10.5194/tc-16-3867-2022](https://doi.org/10.5194/tc-16-3867-2022). 9
- [14] G. H. Gudmundsson and M. Raymond. *On the limit to resolution and information on basal properties obtainable from surface data on ice streams*. The Cryosphere **2**, 167 (2008). DOI: [10.5194/tc-2-167-2008](https://doi.org/10.5194/tc-2-167-2008). 9
- [15] D. A. Young, A. P. Wright, J. L. Roberts, R. C. Warner, N. W. Young, J. S. Greenbaum, D. M. Schroeder, J. W. Holt, D. E. Sugden, D. D. Blankenship, and T. D. van Ommen. *A dynamic early East Antarctic Ice Sheet suggested by ice-covered fjord landscapes*. Nature **474**, 72 (2011). DOI: <https://doi.org/10.1038/nature10114>. 12
- [16] J. L. Roberts, R. C. Warner, D. Young, A. Wright, T. D. van Ommen, D. D. Blankenship, M. Siegert, N. W. Young, I. E. Tabacco, A. Forieri, A. Passerini, A. Zirizzotti, and M. Frezzotti. *Refined broad-scale sub-glacial morphology of Aurora Subglacial Basin, East Antarctica derived by an ice-dynamics-based interpolation scheme*. The Cryosphere **5**, 551 (2011). DOI: [10.5194/tc-5-551-2011](https://doi.org/10.5194/tc-5-551-2011). 13
- [17] F. Gillet-Chaulet. *Assimilation of surface observations in a transient marine ice sheet model using an ensemble Kalman filter*. The Cryosphere **14**, 811 (2020). DOI: [10.5194/tc-14-811-2020](https://doi.org/10.5194/tc-14-811-2020). 13

- [18] Y. Choi, A. Petty, D. Felikson, and J. Poterjoy. *Estimation of the state and parameters in ice sheet model using an ensemble Kalman filter and Observing System Simulation Experiments*. *EGUsphere* **2025**, 1 (2025). DOI: [10.5194/egusphere-2025-301.13](https://doi.org/10.5194/egusphere-2025-301.13)
- [19] M. B. Lythe and D. G. Vaughan. *BEDMAP: A new ice thickness and subglacial topographic model of Antarctica*. *Journal of Geophysical Research: Solid Earth* **106**, 11335 (2001). ARXIV: <https://agupubs.onlinelibrary.wiley.com/doi/pdf/10.1029/2000JB900449>, DOI: <https://doi.org/10.1029/2000JB900449>. 14
- [20] P. Fretwell, H. D. Pritchard, D. G. Vaughan, J. L. Bamber, N. E. Barrand, R. Bell, C. Bianchi, R. G. Bingham, D. D. Blankenship, G. Casassa, G. Catania, D. Callens, H. Conway, A. J. Cook, H. F. J. Corr, D. Damaske, V. Damm, F. Ferraccioli, R. Forsberg, S. Fujita, Y. Gim, P. Gogineni, J. A. Griggs, R. C. A. Hindmarsh, P. Holmlund, J. W. Holt, R. W. Jacobel, A. Jenkins, W. Jokat, T. Jordan, E. C. King, J. Kohler, W. Krabill, M. Riger-Kusk, K. A. Langley, G. Leitchenkov, C. Leuschen, B. P. Luyendyk, K. Matsuoka, J. Mouginot, F. O. Nitsche, Y. Nogi, O. A. Nost, S. V. Popov, E. Rignot, D. M. Rippin, A. Rivera, J. Roberts, N. Ross, M. J. Siegert, A. M. Smith, D. Steinhage, M. Studinger, B. Sun, B. K. Tinto, B. C. Welch, D. Wilson, D. A. Young, C. Xiangbin, and A. Zirizzotti. *Bedmap2: improved ice bed, surface and thickness datasets for Antarctica*. *The Cryosphere* **7**, 375 (2013). DOI: [10.5194/tc-7-375-2013](https://doi.org/10.5194/tc-7-375-2013). 14
- [21] H. D. Pritchard, P. T. Fretwell, A. C. Fremand, J. A. Bodart, J. D. Kirkham, A. Aitken, J. Bamber, R. Bell, C. Bianchi, R. G. Bingham, D. D. Blankenship, G. Casassa, K. Christianson, H. Conway, H. F. J. Corr, X. Cui, D. Damaske, V. Damm, B. Dorschel, R. Drews, G. Eagles, O. Eisen, H. Eisermann, F. Ferraccioli, E. Field, R. Forsberg, S. Franke, V. Goel, S. P. Gogineni, J. Greenbaum, B. Hills, R. C. A. Hindmarsh, A. O. Hoffman, N. Holschuh, J. W. Holt, A. Humbert, R. W. Jacobel, D. Jansen, A. Jenkins, W. Jokat, L. Jong, T. A. Jordan, E. C. King, J. Kohler, W. Krabill, J. Maton, M. K. Gillespie, K. Langley, J. Lee, G. Leitchenkov, C. Leuschen, B. Luyendyk, J. A. MacGregor, E. MacKie, G. Moholdt, K. Matsuoka, M. Morlighem, J. Mouginot, F. O. Nitsche, O. A. Nost, J. Paden, F. Pattyn, S. Popov, E. Rignot, D. M. Rippin, A. Rivera, J. L. Roberts, N. Ross, A. Ruppel, D. M. Schroeder, M. J. Siegert, A. M. Smith, D. Steinhage, M. Studinger, B. Sun, I. Tabacco, K. J. Tinto, S. Urbini, D. G. Vaughan, D. S. Wilson, D. A. Young, and A. Zirizzotti. *Bedmap3 updated ice bed, surface and thickness gridded datasets for Antarctica*. *Scientific Data* **12**, 414 (2025). DOI: [10.1038/s41597-025-04672-y](https://doi.org/10.1038/s41597-025-04672-y). 14
- [22] F. Pattyn, L. Perichon, A. Aschwanden, B. Breuer, B. de Smedt, O. Gagliardini, G. H. Gudmundsson, R. C. A. Hindmarsh, A. Hubbard, J. V. Johnson, T. Kleiner, Y. Konovalov, C. Martin, A. J. Payne, D. Pollard, S. Price, M. Rückamp, F. Saito, O. Souček, S. Sugiyama, and T. Zwinger. *Benchmark experiments for higher-order and full-Stokes ice sheet models (ISMIP-HOM)*. *The Cryosphere* **2**, 95 (2008). DOI: [10.5194/tc-2-95-2008](https://doi.org/10.5194/tc-2-95-2008). 14, 16, 17
- [23] B. Getraer and M. Morlighem. *Increasing the Glen–Nye power-law exponent accelerates ice-loss projections for the Amundsen Sea Embayment, West Antarctica*. *Geo-*

physical Research Letters **52**, e2024GL112516 (2025). DOI: [10.1029/2024GL112516](https://doi.org/10.1029/2024GL112516).  
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