

# Simulating phase contrast imaging for materials with variable density

Ana Fabela Hinojosa <sup>1</sup>

Supervisor:  
Assoc. Prof. Marcus Kitchen  
SCHOOL OF PHYSICS & ASTRONOMY



MONASH University

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<sup>1</sup>acfab1@student.monash.edu.au

“The questions are important. I have thought harder about them than I have about the answers I already have. That is the first thing I know for sure: If the questions don’t make sense, neither will the answers.”

Kurt Vonnegut.

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### **Abstract**

This work studies the theoretical perspective of coherent X-ray imaging. The aim is to determine whether a density difference alone can give phase contrast and, if so, whether the phase contrast is likely to be detected in real experiments. Using simulations of cylinder-shaped sample materials subjected to X-ray wave-fields of different energies, the importance of material density in phase contrast imaging is demonstrated. In addition, this work investigates possible improvements to the numerical techniques used in the field of X-ray imaging and presents an improved accuracy method for phase contrast based on the Transport-of-intensity equation.

# Chapter 1

## Introduction

This chapter describes the theoretical background of the field of X-ray optics. Specifically, the mathematical formalism used in phase contrast imaging and in phase retrieval. The origin of X-ray optics lies with Maxwell's classical electrodynamics[1]. Since Maxwell's equations describe electromagnetic field disturbances as waves, the spatial and temporal evolution of electromagnetic fields in space can be described by wave optics. Under the wave formalism, there has been substantial progress in the theoretical understanding of the interaction of X-rays with matter, and in the knowledge of how to exploit these interactions experimentally.

### 1.1 X-ray phase contrast imaging

#### Helmholtz equation and the Angular spectrum formulation

The Helmholtz equation represents a time-independent form of the wave equation and it is a central equation of diffraction theory[2][3]. In general, X-ray imaging is done with polychromatic X-rays with non-trivial spectra[2]. It is possible to use a method known as the angular spectrum formulation (ASF) on a wave function in order to greatly simplify radiological imaging. The ASF comprises spectral decomposition of a wave-function into monochromatic components. The separation of each component with an individual fixed angular frequency (i.e. a trivial dependence in time), transforms the wave equation into a time-independent partial differential equation (PDE) that is applied individually to each monochromatic component. After the individual analysis of each component is done, a recombination of the monochromatic components results in the complete description of the polychromatic process[2][3].

The complex scalar wave-field function  $\Psi(\mathbf{r}, t) = \sqrt{I(\mathbf{r}, t)}\exp(i\phi(\mathbf{r}, t))$  obeys the wave equation in a vacuum

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2\right) \Psi(\mathbf{r}, t) = 0, \quad (1.1)$$

where  $c$  is the speed of light and  $\nabla$  is the Laplacian operator. The spectral decomposition of  $\Psi(\mathbf{r}, t)$  is done via the Fourier transform

$$\Psi(\mathbf{r}, t) = \frac{1}{\sqrt{2\pi}} \int_0^\infty \psi_\omega(\mathbf{r}) e^{-i\omega t} d\omega, \quad (1.2)$$

notice that the integral only considers a positive integration range (due to analyticity concerns[1]). In this equation the spatial wave-function subscript  $\omega$  indicates functional dependence on this quantity[3]. Evaluating equation (1.1) using equation (1.2) yields the Helmholtz equation for the vacuum

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2\right) \Psi(\mathbf{r}, t) = 0, \quad (1.3)$$

$$\therefore \int_0^\infty \frac{1}{\sqrt{2\pi}} \left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2\right) \psi_\omega(\mathbf{r}) e^{-i\omega t} d\omega = 0, \quad (1.4)$$

$$\therefore \int_0^\infty \frac{1}{\sqrt{2\pi}} \left(\frac{-\omega^2}{c^2} \psi_\omega(\mathbf{r}) e^{-i\omega t} - \nabla^2 \psi_\omega(\mathbf{r}) e^{-i\omega t}\right) d\omega = 0, \quad (1.5)$$

$$(k^2 + \nabla^2) \psi_\omega(\mathbf{r}) = 0, \quad (1.6)$$

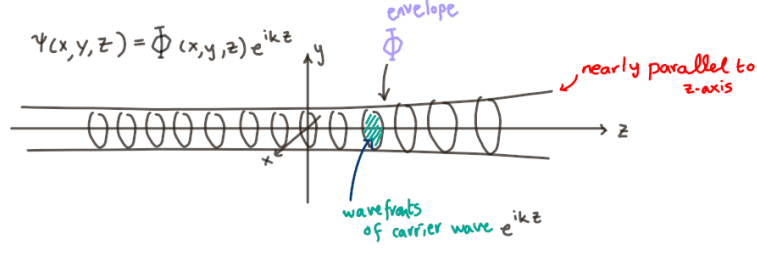
where the wave vector  $k = \omega/c$ .

#### Paraxial fields

In most situations involving phase-contrast imaging, X-ray fields behave as paraxial fields. Electromagnetic (EM) energy in this context is concentrated within a small region about the beam axis. A beam is an EM wave propagating in free space with small divergence (i.e. spread) away from the propagation axis, hence the origin of the name *paraxial*.

In the previous section  $\Psi(\mathbf{r}, t)$  described a complex scalar wave-field, in this section the paraxiality condition is added. Under this approximation the complex disturbance  $\psi_\omega(\mathbf{r})$  present in the Helmholtz equation is expressed as a product of a z-directed plane wave and a perturbing envelope[1][2]

$$\psi_\omega(\mathbf{r}) = \Phi(\mathbf{r})\exp(ikz), \quad (1.7)$$



**Figure 1.1:** A cartoon of a paraxial wave-field  $\psi_\omega(\mathbf{r})$  displaying beam propagation properties along the z-axis. The slowly varying complex envelope is modulated by a carrying plane wave.

Note that the longitudinal variation of the complex envelope  $\Delta z = \lambda = 2\pi/k$  is required to be smaller than the complex envelope itself

$$\frac{\Delta\Phi(\mathbf{r})}{\Phi(\mathbf{r})} \leq 1 \quad (1.8)$$

using equation (1.8) and the longitudinal variation  $\Delta z$  in the Helmholtz equation yields the paraxial Helmholtz equation in the vacuum

$$\left(\nabla_\perp^2 + 2ik\frac{\partial}{\partial z}\right)\Phi(\mathbf{r}_\perp, z) = 0 \quad (1.9)$$

### Refractive index and the projection approximation

In the presence of static, non-magnetic, scattering material media the complex scalar X-ray wave-field can still be studied by using the inhomogeneous Helmholtz equation

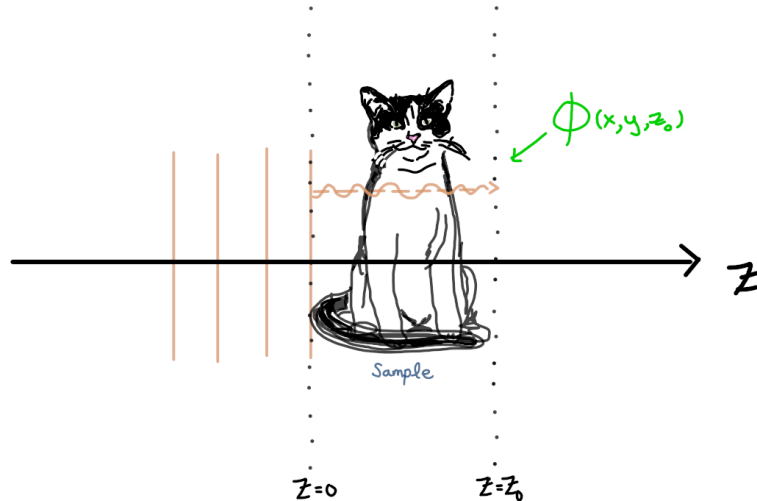
$$(k^2 n^2(\mathbf{r}) + \nabla^2)\Psi(\mathbf{r}, t) = 0, \quad (1.10)$$

where  $n(\mathbf{r})$  is the position dependent and complex form for the refractive index, the real part of which corresponds to the refractive index and the imaginary part of this complexified refractive index can be related to the absorptive properties of a sample[1].

$$n(\mathbf{r}) = 1 - \delta(\mathbf{r}) + i\beta(\mathbf{r}), \quad (1.11)$$

where  $|\delta|, |\beta| \ll 1$ . Using the complex refractive index in the inhomogeneous paraxial Helmholtz equation yields the *projection approximation*, which is an expression that describes the phase shift and attenuation undergone by an X-ray wave moving across a sample

$$\Phi(\mathbf{r}_\perp, z_0) = \exp\left(-ik \int_0^{z_0} (\delta - i\beta) dz\right) \Phi(\mathbf{r}_\perp, 0) \quad (1.12)$$



**Figure 1.2:** Schematic diagram of the projection approximation adapted from references [2] and [3]. The sample is contained within the space between  $z = 0$  and  $z = z_0$ . The sample is described by its refractive index  $n(\mathbf{r})$  which differs from the the refractive index of the air volume that surrounds the sample. In this figure one can see that the path of X-rays passing through an object can be described by defining a surface (also known as the contact plane) immediately downstream from the irradiated object (i.e.  $z = z_0$ ). at which the transferred transverse intensity and phase changes of the incident X-rays are imprinted[2].



The projection approximation assumes that X-ray flow may be well approximated by straight lines parallel to  $z$ [1] and that most X-rays passing through an object do not actually interact with the sample material (i.e. minimal to no scattering occurs within the sample, a fair assumption due to small magnitude of the complex *refractive index* of X-rays). Neglecting scattering effects is mathematically equivalent to discarding the transverse Laplacian term in equation (1.9)[2], in addition, if X-ray-matter interaction must be taken into account in equation (1.9) then we require the inhomogeneous paraxial Helmholtz equation

$$\left(2i\frac{\partial}{\partial z} + k(n^2(\mathbf{r}) - 1)\right)\Phi(\mathbf{r}) = 0. \quad (1.13)$$

### Intensity and phase-shift

Refraction is a property that may be augmented by the intensity attenuation due to the imaged object. This latter quantity may be obtained by taking the squared modulus of equation (1.12), to give the *Beer-Lambert law*[1]

$$I(\mathbf{r}_\perp, z = z_0) = I^{\text{in}} e^{-\mu T(\mathbf{r}_\perp)}, \quad (1.14)$$

where  $T(\mathbf{r}_\perp)$  is the projected thickness of the object in the direction of the beam propagation, and,  $I^{\text{in}}$  the incident light intensity to the object. This expression is also known as the “contact image”. Equation (1.14) relates the imaginary part of the refractive index  $\beta$  to the *linear attenuation coefficient*  $\mu = 2k\beta$ . Since we are working with a wave picture, refraction is associated with wave-front deformation rather than ray deflection, therefore the projection approximation term involving the refractive index  $\delta$  (refractive properties of the sample), quantifies the deformation of the X-ray wave-fronts due to passage through the sample. Physically, for each fixed transverse coordinate  $(x, y)$ , phase-shifts (and the associated wave-front deformations) are continuously accumulated along energy-flow streamlines[1].

$$\Delta\phi(\mathbf{r}_\perp) = -k\delta T(\mathbf{r}_\perp), \quad (1.15)$$

where  $k$  is the X-ray wave-number, and  $\delta$  the refractive index decrement.

### Transport-of-intensity equation

The transport-of-intensity equation (TIE) expresses the intensity and phase evolution of a paraxial monochromatic scalar electromagnetic or matter wave on propagation[4]. The TIE is also used to quantify the contrast (i.e. refractive (phase) effects) present in propagation-based X-ray phase contrast images[1]. More concisely, the TIE physically describes the change in the light intensity (eq.1.14) and phase (eq.1.15) as the divergence of the transverse energy-flow vector known as the Poynting vector ( $\mathbf{S}$ ). If the divergence of this vector is positive, (i.e. the wave-field is locally expanding), optical energy moves away from the local optic axis and so the longitudinal derivative of intensity will be negative. The converse occurs if the divergence of the Poynting vector is negative, and so the longitudinal derivative of intensity will be positive[1].

$$-\nabla_\perp[I(\mathbf{r})\nabla_\perp\phi(\mathbf{r})] = k\frac{\partial I(\mathbf{r})}{\partial z}. \quad (1.16)$$

where  $k$  is the wave-number,  $I(\mathbf{r})$  is the intensity and  $\phi(\mathbf{r})$  is the phase of the X-ray beam, the Poynting vector  $\mathbf{S} \propto I(\mathbf{r})\nabla_\perp\phi(\mathbf{r})$ .

## 1.2 X-ray phase retrieval

Decoding of X-ray phase contrast images is essentially the reverse problem to phase contrast imaging and it is known as *phase retrieval*. In general inverse problems are more complex than forward problems. Often inverse problems are unsolvable, and, even if there exist possible solutions to these problems, these may not be unique or stable with regards to realistic experimental conditions. Phase retrieval seeks to reconstruct both the intensity and phase of the input field, given only the intensity of the output field of the imaging system[1][4]. Under the projection approximation (described in section 1.1), it is clear that both the intensity (1.14) and the phase (1.15) on the contact plane (Figure 1.1) are both described by the projected thickness ( $T(\mathbf{r}_\perp)$ ) of the sample object. Using this approximation, and assuming that the projected thickness of a homogeneous material sample is obtained from a single propagation-based phase contrast image. Paganin et al.(2002) developed a fast and deterministic method for quantitative phase extraction[4] using the TIE to propagate a wave-field’s intensity and phase by a sufficiently small distance away from the sample. Substituting the contact plane intensity and phase into equation (1.16) returns a non linear equation in  $T(\mathbf{r}_\perp)$

$$-\frac{\delta}{\mu}I^{\text{in}}\nabla_\perp^2 e^{-\mu T(\mathbf{r}_\perp)} = \frac{\partial}{\partial z}I(\mathbf{r}_\perp, z = 0). \quad (1.17)$$

This phase retrieval method[4] approximates the right hand side of equation 2.1 by the first order finite difference formula

$$\frac{\partial}{\partial z}I(\mathbf{r}_\perp, z = 0) \approx \frac{I(\mathbf{r}_\perp, z = z_0) - I^{\text{in}}e^{-\mu T(\mathbf{r}_\perp)}}{z_0}, \quad (1.18)$$

replacing equation (1.18) into equation (2.1) and rearranging to find the ratio of the intensity at the detector plane (i.e. the phase contrast image  $I(\mathbf{r}_\perp, z = z_0)$ ) to the intensity initially incident to the object

$$\left( \frac{-z_0 \delta}{\mu} \nabla_\perp^2 + 1 \right) e^{-\mu T(\mathbf{r}_\perp)} = \frac{I(\mathbf{r}_\perp, z = z_0)}{I^{\text{in}}}. \quad (1.19)$$

The construction of the phase retrieval algorithm requires that the contact image and the phase contrast image are represented as Fourier integrals[4]

$$I^{\text{in}} e^{-\mu T(\mathbf{r}_\perp)} = \frac{I^{\text{in}}}{2\pi} \int \int \mathcal{F} \left\{ e^{-\mu T(\mathbf{r}_\perp)} \right\} e^{i\mathbf{k}_\perp \cdot \mathbf{r}_\perp} d\mathbf{k}_\perp \quad (1.20)$$

$$I(\mathbf{r}_\perp, z = z_0) = \frac{1}{2\pi} \int \int \mathcal{F} \{ I(\mathbf{r}_\perp, z = z_0) \} e^{i\mathbf{k}_\perp \cdot \mathbf{r}_\perp} d\mathbf{k}_\perp \quad (1.21)$$

where  $\mathcal{F} \{ \}$  represents Fourier transformation. Substituting these equations in equation (1.19)

$$\mathcal{F} \left\{ e^{-\mu T(\mathbf{r}_\perp)} \right\} = \mu \frac{\mathcal{F} \{ I(\mathbf{r}_\perp, z = z_0) \} / I^{\text{in}}}{z_0 \delta |\mathbf{k}_\perp|^2 + \mu}. \quad (1.22)$$

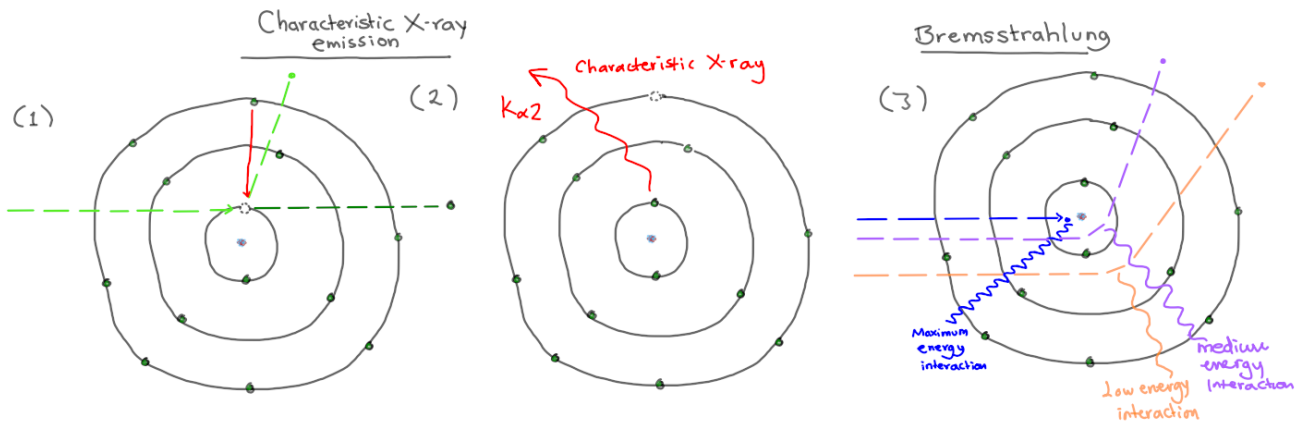
Taking the inverse Fourier transform of equation (1.22) and solving for  $T(\mathbf{r}_\perp)$

$$T(\mathbf{r}_\perp) = -\frac{1}{\mu} \log_e \left( \mathcal{F}^{-1} \left\{ \mu \frac{\mathcal{F} \{ I(\mathbf{r}_\perp, z = z_0) \} / I^{\text{in}}}{z_0 \delta |\mathbf{k}_\perp|^2 + \mu} \right\} \right). \quad (1.23)$$

Since the projected thickness of the homogeneous sample has been recovered from the TIE, the intensity and phase of the radiation at the contact plane can be calculated.

### 1.3 Characteristic X-rays

X-rays are commonly produced by a phenomenon known as X-ray fluorescence (XRF). A heated filament emits electrons by thermionic emission, these electrons are accelerated by a high voltage and collide with a metal target. In simple terms, the accelerated electrons interact with the atoms comprising the metal target by (i) impacting a shell electron in the target and dislodging it causing photoelectron emission, (ii) having their path deviated without having any actual contact with the target atoms, (iii) directly impacting the nucleus. Characteristic X-rays are created by the first interaction described above. As the incident electron releases an inner shell electron, this leaves a vacancy in the atom's inner shell. The vacancy is quickly filled by an outer shell electron which causes photon emission with an energy equal to the energy difference between the inner and outer shells. The term "characteristic" refers to the fact that each element has a distinct pattern of X-ray photoelectron emission. The second interaction described above is known as Bremsstrahlung (Background/ Breaking) radiation. A source of Bremsstrahlung radiation occurs due to incident electrons interacting with the nuclear electric field. The electron paths may be deflected or decelerated and recaptured by the atom into an energy electron shell.



**Figure 1.3:** Cartoons of accelerated electrons interacting with a target atoms. On the left two cartoons, characteristic X-ray emission is represented by (1) an accelerated electron hitting an inner shell electron expelling it from the atom, while an outer shell electron promptly moves inward to fill the inner shell gap (2) releasing the characteristic X-ray. (3) The types of possible Bremsstrahlung radiation, ranging from low energy/distant to high energy interactions between the incident accelerated electrons and the atomic nucleus.

# Chapter 2

## Simulations

To determine whether density differences contribute to phase contrast and if so, whether this contribution is actually detectable in a real experiment, I created several simulations of cylinder-shaped sample materials and subjected these samples to X-ray wave-fields of different energies. In some of my simulations, I use a single cylinder made of a homogeneous material. In others I use two embedded concentric cylinder-shaped samples of materials with either an equal chemical composition and different densities or, different chemical compositions, and densities. With these simulations I demonstrate the importance of material density in phase contrast imaging, by 1) showing how distinctly, phase contrast arises between material boundaries and 2) whether the observed phase contrast is density-dependent.

### 2.1 Methods

For the single cylinder simulations I use the projection approximation to solve the TIE. The TIE effects observable changes onto the intensity (1.14) and phase (1.15) when these are used as initial conditions in the propagation problem. For the two-cylinder simulations, I also use the projection approximation, but this time I model the propagation of the monochromatic X-ray wave-field using the angular spectrum formulation (ASF) (section 1.1). The ASF uses Fourier decomposition of wave-fields at a single plane into distinct component plane waves. Each plane wave component is propagated through the Fourier domain to a destination plane, and then the propagated wave-field components are reconstructed via an inverse spatial Fourier transform[5]. To implement the ASF, I use the X-ray imaging XRI library, since the existing algorithm has already implemented several fixes to undesirable instabilities that can appear in the simulations. The XRI library was built by the Monash X-ray imaging group.

#### 2.1.1 Simulating a single homogeneous cylinder

I solve the TIE in two-dimensions using the projection approximation to establish my initial conditions. Originally, my code discretises a two-dimensional space as an array with dimensions  $x, y = 1024, 512$ . I first attempt to solve the TIE in Fourier space. To calculate the wave-field's derivative terms in equation (1.16), I use the Fourier derivative theorem to write the derivatives. I define functions describing the geometry and, optical properties (i.e. refractive index:  $\delta(x, y, z)$  and  $\mu(x, y, z)$ ) of the simulated cylinder. Both  $\delta$  and  $\mu$  functions use a sigmoid function with adjustable gradient to slightly blur the edge of the cylinder and, avoid instability while calculating (1.14) and (1.15). This method is slow due to the Fourier derivatives, and the output intensity profile after propagation does not appear very stable, as several oscillations can be seen in figure 2.2.1, to fix this issue, I adjusted the ratio of the spacings in my discretised space several times to make sure that the Nyquist mode of the TIE was resolved adequately. Nevertheless I was unable to obtain a completely smooth, and realistic looking intensity profile in figure 2.2.1.

To complement this investigation, I solved the TIE in position space using finite differences (see figure 2.2.1). In this method I use `numpy.gradient` to evaluate each derivative and `scipy.ndimage.laplace` to evaluate the phase Laplacian term in the TIE. My code discretised the two-dimensional space as a 2D array  $x, y = 1024, 1024$ .

#### 2.1.2 Improving the TIE numerics

Here I want to expand on how my evaluation of the TIE produces a better solution than the regular usage of the TIE in the propagation problem. The improvement to the numerics consists on increasing the order of the derivatives used in the TIE. My method applies the fourth order Runge-Kutta (RK) evolution algorithm to propagate the intensity of the monochromatic scalar electromagnetic wave-field over a total distance in meters using the TIE. I employ the RK

evolution algorithm as

$$\begin{aligned}
k_1 &= f(z, I_n, \phi), \\
k_2 &= f\left(z + \frac{\Delta z}{2}, I_n + \frac{1}{2}k_1, \phi\right), \\
k_3 &= f\left(z + \frac{\Delta z}{2}, I_n + \frac{1}{2}k_2, \phi\right), \\
k_4 &= f(z + \Delta z, I_n + k_3, \phi), \\
I_{n+1} &= I_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) + O(\Delta z^5),
\end{aligned} \tag{2.1}$$

where the spatial evolution step is set as  $\Delta z = 1$  mm. The propagation via fourth order RK over a spatial interval is done by solving a differential equation  $f$  using four Euler-style steps (i.e. the  $k_i$  coefficients) where each Euler-style step involves one evaluation of the differential equation with slightly different input parameters. This method combines the information obtained for each solution to match a Taylor series expansion up to fourth order. The combined evaluations of the differential equation eliminate the error terms order by order, resulting in the remaining error being very small[6].

### 2.1.3 Simulating two concentric homogeneous cylinders

I simulate two concentric cylinders made of two different homogeneous materials using the angular spectrum formulation (ASF) propagation algorithm in the xri library. To find the optical parameters of the sample materials (i.e.  $\delta$  and  $\mu$ ) under certain energy X-rays, I use the X-ray imaging group “X-ray attenuation calculator” which employs the values recorded in NIST in database 66: X-Ray Form Factor, Attenuation, and Scattering Tables[7], making these values easily accessible via a GUI. I define a function for the projected thickness of the sample cylinders. The thickness function uses an adjustable width Gaussian filter to slightly blur the edge of each cylinder and, avoid instability while calculating the parameters required by the ASF propagation algorithm. The parameters calculated are  $\delta T(\mathbf{r}_\perp)$  and  $\beta T(\mathbf{r}_\perp)$ , corresponding to the refraction and absorption parameters respectively. I calculate these quantities for each of the two cylinders in each simulation. Then I combine these quantities because of the concentric feature of the cylinders.

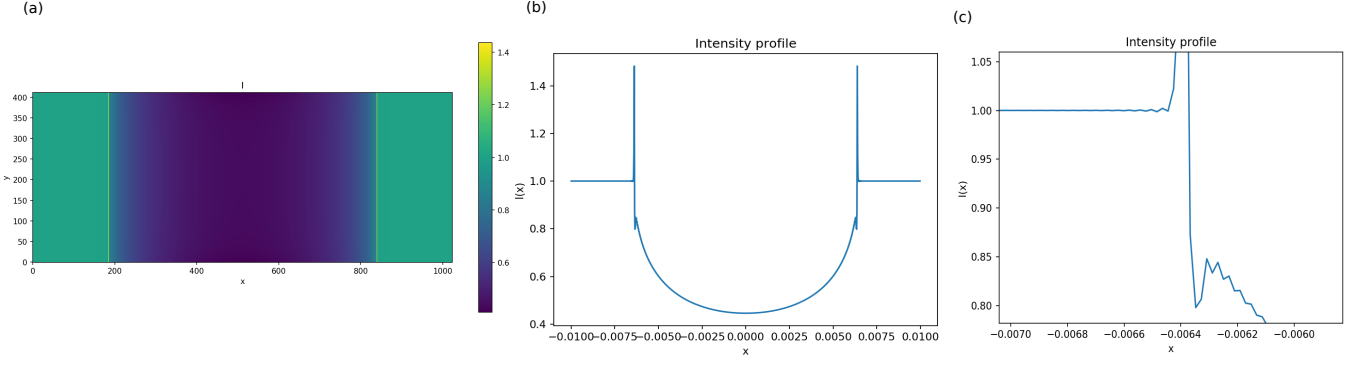
### 2.1.4 Imaging brain structures

The optical properties of grey and white matter are extremely similar. However, it is known that there is a small difference in the values of the attenuation coefficients of these materials. CT brain imaging has been reported to yield clearly demarcated tissue borders at the grey/white matter boundaries[8]. To determine if density differences alone produce phase contrast when imaging extremely similar brain materials, I simulated grey and white matter. I use the attenuation coefficients for both materials  $\mu_{GM}$  for grey matter, and  $\mu_{WM}$  for white matter, obtained from CT imaging of rabbit and kitten brain samples. This data was acquired at Japan’s SPring-8 synchrotron radiation facility using an X-ray energy of  $E = 24$  keV and a sample-to-detector distance of 5 m[9]. In my calculations, I also use experimental grey and white matter densities reported in the “IT’IS Database for thermal and electromagnetic parameters of biological tissues” (see reference [10]). To find the complex refractive index components of the whole brain (i.e.  $\delta_B$  and  $\mu_B$ ), I use the X-ray imaging group “X-ray attenuation calculator” X-Ray Form Factor, Attenuation, and Scattering Tables[7]. Using all these tools I calculated an approximate estimate for the grey and white matter refractive index decrements ( $\delta_{GM}$  and  $\delta_{WM}$ ) The calculations were simple. In one, I modelled the refractive index decrements as proportional to the ratio of the attenuation coefficient value for the full brain from reference[7] and the density of the grey or white matter values reported in reference [10] (for example for grey matter:  $\delta_{GM} = \frac{\mu_B}{\rho_{GM}} \delta_B$ ). In contrast, my second model considered the refractive index decrement to be solely proportional to the ratio between the attenuation coefficient value for the full brain and the value corresponding to either the grey or white matter attenuation coefficients (for example for grey matter:  $\delta_{GM} = \frac{\mu_B}{\mu_{GM}} \delta_B$ ).

## 2.2 Results

### 2.2.1 Simulating a single homogeneous cylinder

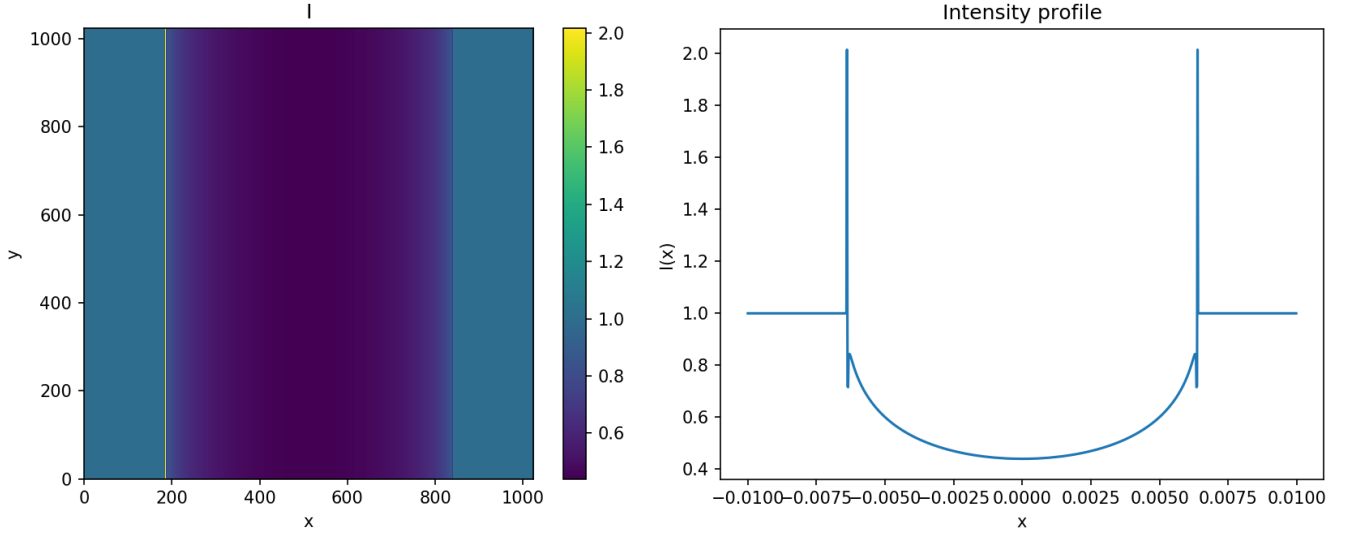
The phase contrast images visible in figure 2.2.1 were obtained using a Fourier TIE method. This method was not able to simultaneously match the expected phase contrast peak height, at this energy while at the same time remaining fully stable. The parameters used to obtain this image were chosen to match a silver source, specifically the k-alpha 1 transition. The simulated material for this cylinder is water. As explained in section 2.1.4 the optical parameters for these simulations were obtained from the “X-ray attenuation calculator”.



**Figure 2.1:** Phase contrast image (a), cross section (b), and enhanced view of (b) showing small ringing artefacts (instability)(c) obtained using the Fourier TIE method. The parameters used to obtain this image were an X-ray energy of 22.1629 keV (corresponding to a Ag k-alpha 1 source), a refractive index decrement for the water cylinder of  $\delta = 4.68141 \times 10^{-7}$  and an attenuation coefficient of  $\mu = 64.38436 \text{ m}^{-1}$ . The cylinder radius was  $R = 6.375 \text{ mm}$ . The sigmoid function blurred 0.5 pixels over the edge of the imaged cylinder. The propagation distance used was  $z = 1 \text{ m}$ .

From observing the results in figure 2.2.1, I hypothesize that the underlying reason why my Fourier TIE method didn't work as expected was due to an effect known as Gibbs phenomenon, this effect occurs when the  $n$ th partial sum of a Fourier series undergoes large oscillations near regions with jump discontinuities[11] (i.e. like the phase contrast fringes). Discovering that higher degree polynomial interpolation does not always improve accuracy was interesting and certainly unexpected.

The phase contrast images visible in figure 2.2.1 were obtained using a finite differences TIE and RK method. As can be seen here, the phase contrast peaks are much higher than the profile found using the Fourier method in figure 2.2.1. The parameters used to obtain figure 2.2.1 were chosen to be identical to those used to obtain figure 2.2.1, with the exception of the sigmoid blurring parameter, which was softer under the finite differences method.



**Figure 2.2:** Phase contrast image and cross section. The parameters used to obtain this image were an X-ray energy of 22.1629 keV (corresponding to a Ag k-alpha 1 source), a refractive index decrement for the water cylinder of  $\delta = 4.68141 \times 10^{-7}$  and an attenuation coefficient of  $\mu = 64.38436 \text{ m}^{-1}$ . The cylinder radius was  $R = 6.375 \text{ mm}$ . The sigmoid function blurred 0.14 pixels over the edge of the imaged cylinder. The propagation distance used was  $z = 1 \text{ m}$ . The apparent asymmetry of the two-dimensional phase contrast image is due to aliasing.

This result presented an apparent improvement in the height of the expected phase contrast fringes when compared to the TIE method implemented by the xri library.

## 2.3 Comparing to xri's methods

### 2.3.1 Shifting fringes

It is important to mention that the angular spectrum formulation (ASF) is the 'gold standard' intensity propagation algorithm to compare against. The ASF demonstrates that the phase contrast fringe position will shift with increasing radiation energy. Another noteworthy fact is that under these same conditions the standard TIE algorithm phase contrast fringes never budge as can be seen in figure 2.3.2. I figured that if the addition of the fourth order Runge-Kutta algorithm to the TIE (TIE+RK) allowed shifts of phase contrast fringes with changing radiation energy, then this will be proof that the TIE+RK method is a better approach than the standard finite difference TIE approach.

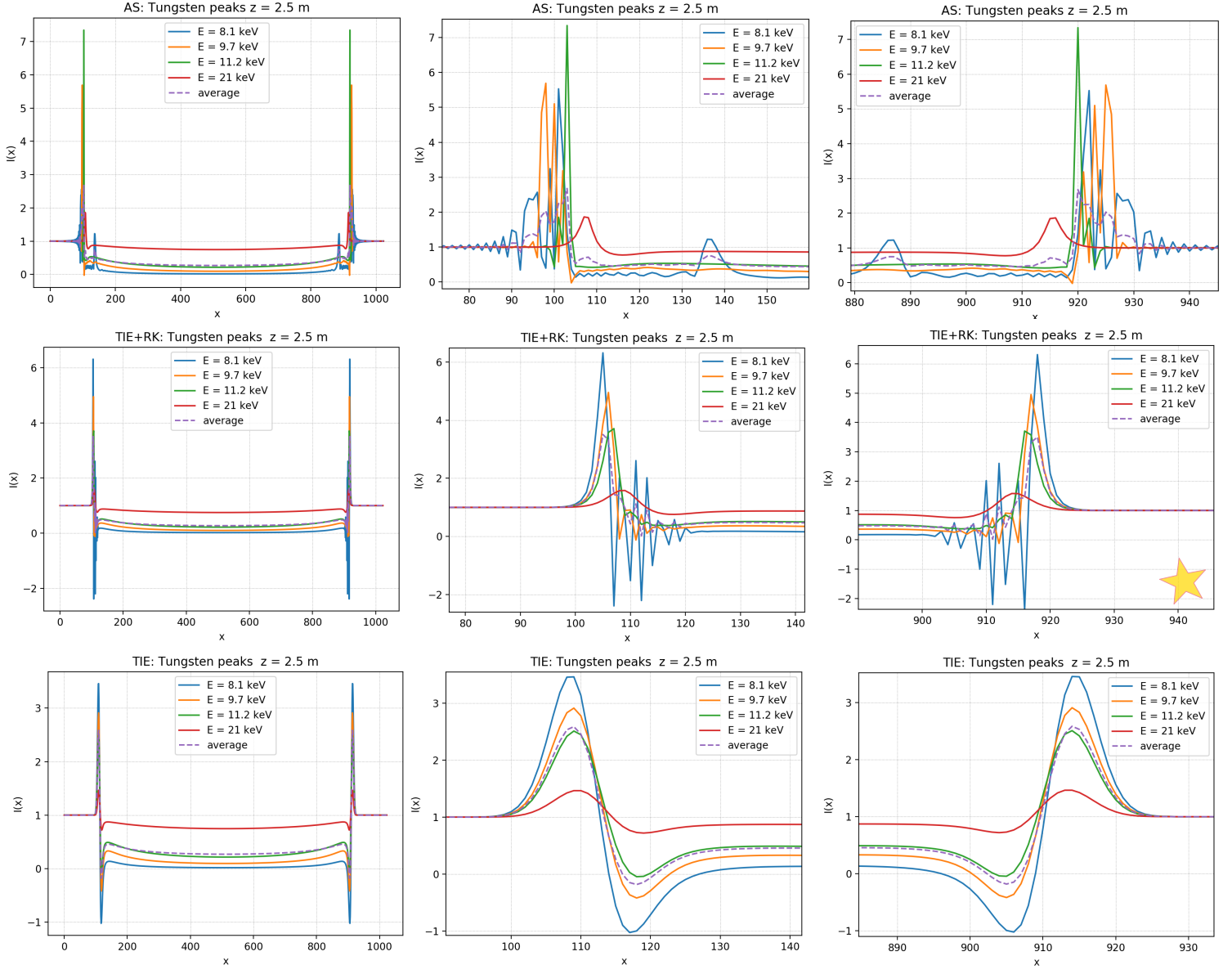


Figure 2.3



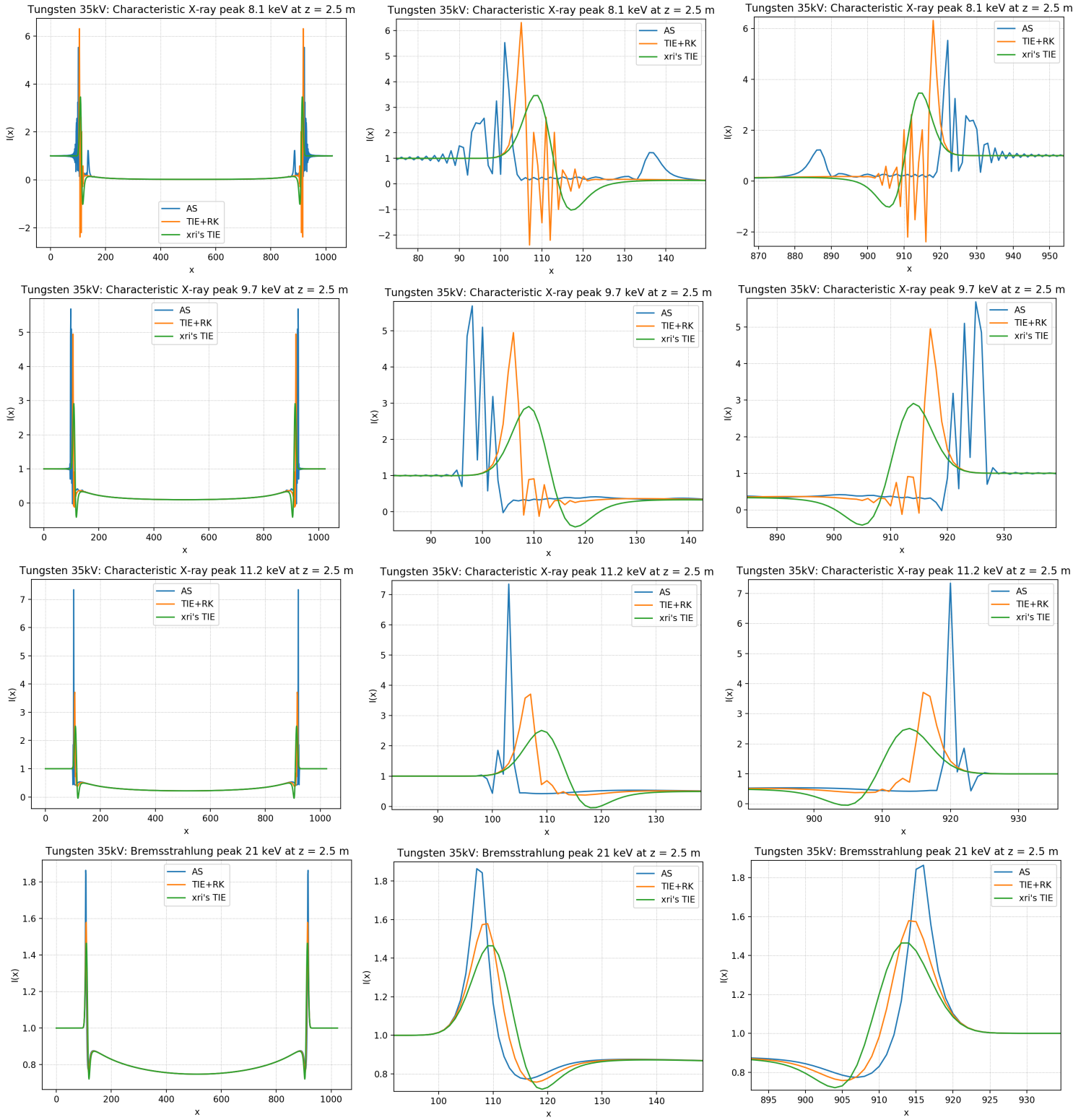
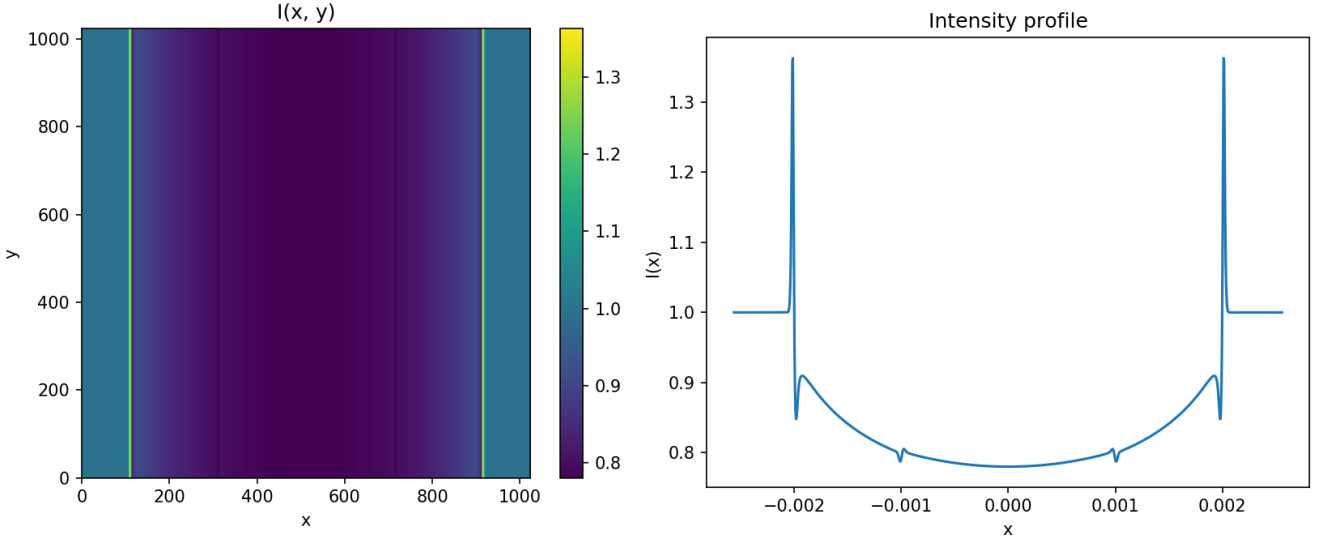


Figure 2.4

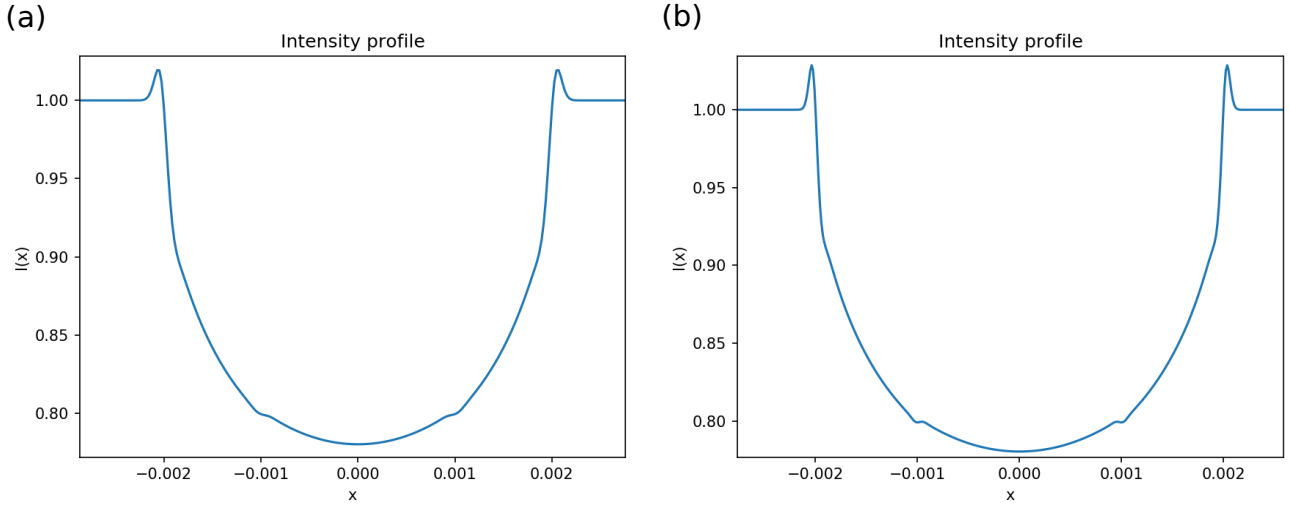
### 2.3.2 Density difference test: water and ice

As described in section 2.1.3, to test if there is a density dependence in phase contrast, I first use a sample consisting of two cylinders with the outermost one made of water and the innermost one made of ice.



**Figure 2.5:** Phase contrast image and cross section obtained with the angular spectrum formulation method developed by the X-ray imaging group. The X-ray energy used in this simulation was  $E = 22.1629$  keV. The water cylinder has a refractive index decrement  $\delta_w = 4.69337 \times 10^{-7}$ , the attenuation coefficient of water is  $\mu_w = 64.55083$  m $^{-1}$ . The ice cylinder has a refractive index decrement  $\delta_i = 4.31790 \times 10^{-7}$ , and attenuation coefficient  $\mu_i = 59.38677$  m $^{-1}$ . The density difference between these materials is  $\Delta\rho = 0.08$  gcm $^{-3}$ . The cylinder's radii were  $R_w = 2$ mm and  $R_i = 1$ mm. The propagation distance used was  $z = 2.5$  m.

With the result in figure 2.3.2. I demonstrate that any changes in material density throughout the imaged sample do affect the imaging process, and therefore the stability the propagation based imaging algorithm developed by Paganin et al. (2002) is actually dependent on the ratio of the differences in the refraction and attenuation coefficients  $\Delta\delta$  and  $\Delta\mu$  of the imaged materials. In the following simulation, I take into account the apparatus parameters and the resolution of the detector used in the lab. I made this simulation to see whether or not we would be able to see the any small fringes in a real experiment. Using magnification factors we would use in the lab (i.e. 2.5X and 4.0X) and our photon detector dimensions I resized the spatial array boundaries to match those of the detector. I also scaled the pixel size to match the pixels in the photon detector. I did this so that the pixels in the object plane are adequately sized given the expected size of the pixels in the detector plane. I also calculated the effective propagation distance by dividing the simulation propagation distance by the desired magnification factor.



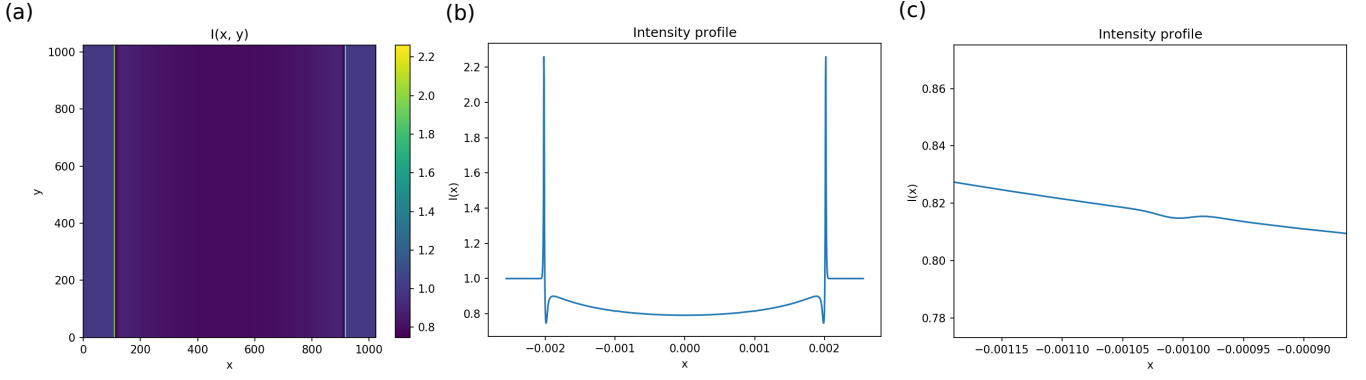
**Figure 2.6:** Phase contrast cross sections were obtained with parameters equal to those in the simulation in figure 2.3.2.

The plan was to test these results experimentally, but given that the phase contrast fringes observable in figure 2.3.2 were so small, we decided against doing the experiment, since we would not observe much in a lab setting.

### 2.3.3 Density difference test: Grey matter and white matter

To continue my tests of the importance of density in phase contrast, I assume in figure 2.3.3, that grey and white matter have the same chemical compositions but distinct densities.



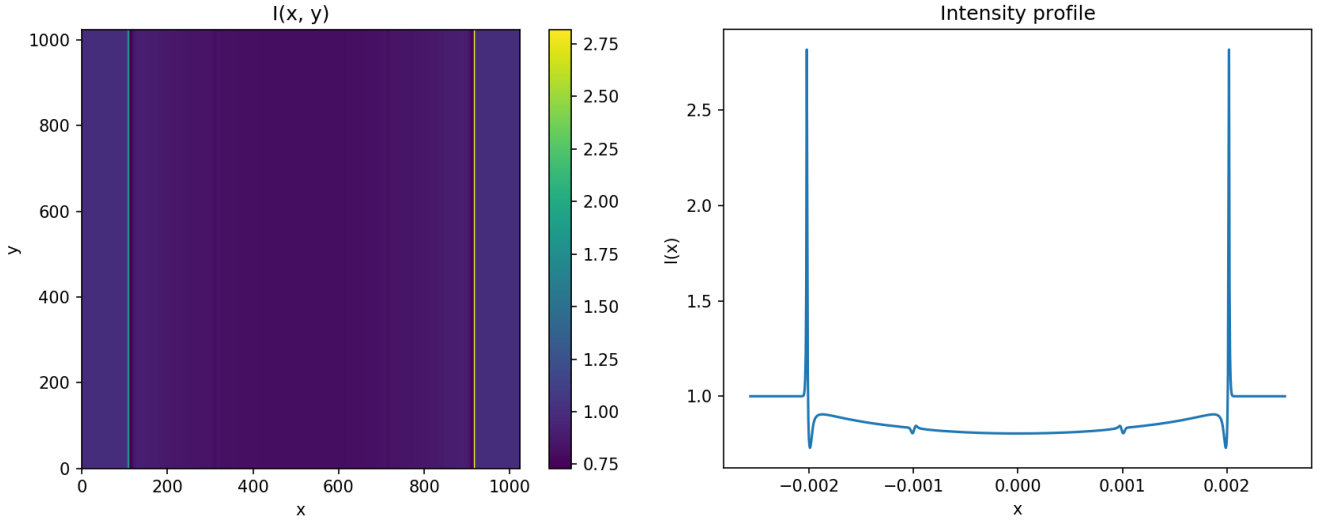


**Figure 2.7:** Phase contrast image (a), cross section (b), and enhanced view of one of the inner cylinder phase contrast fringes (c) obtained with the angular spectrum formulation method as developed by the X-ray imaging group. In (c) it can be seen that the phase contrast fringes from the inner cylinder are extremely small but can still be observed when the image in (b) is amplified enough. The X-ray energy is  $E = 24$  keV. The grey matter cylinder has a refractive index decrement  $\delta_{GM} = 4.1345 \times 10^{-7}$ , and the attenuation coefficient of grey matter is  $\mu_{GM} = 58.2978 \text{ m}^{-1}$ . The white matter cylinder has a refractive index decrement  $\delta_{WM} = 411.87 \times 10^{-7}$  and its attenuation coefficient  $\mu_{WM} = 58.0747 \text{ m}^{-1}$ . The density difference between these materials is  $\Delta\rho = 0.004 \text{ gcm}^{-3}$ [10]. The cylinders' radii were  $R_{GM} = 2 \text{ mm}$  and  $R_{WM} = 1 \text{ mm}$ . The propagation distance used was  $z = 2.5 \text{ m}$ .

My simulations demonstrate that even a small density difference like the one seen in 2.3.3 can produce phase contrast fringes. Nevertheless, my simulations only consider perfect data, therefore the fringes would be too small to see using real sample materials in a lab experiment.

### 2.3.4 Chemical difference test: Grey matter and white matter

I continue the tests of phase contrast for brain materials. As opposed to the simulation in figure 2.3.3, the simulation in figure 2.3.4 assumes that grey and white matter are different chemically. This condition returns clear fringes for both outer and inner cylinders. This simulation likely represents a more realistic output than 2.3.3 given that there are likely chemical differences between the materials in a real brain.



**Figure 2.8:** Phase contrast image and phase contrast cross section obtained with the angular spectrum formulation method developed by the X-ray imaging group. The X-ray energy used in this simulation was  $E = 24 \text{ keV}$ . The grey matter cylinder has a refractive index decrement  $\delta_{GM} = 4.591 \times 10^{-7}$ , the attenuation coefficient of grey matter is  $\mu_{GM} = 52 \text{ m}^{-1}$ [9]. The white matter cylinder has a refractive index decrement  $\delta_{WM} = 4.2631 \times 10^{-7}$  and attenuation coefficient  $\mu_{WM} = 56 \text{ m}^{-1}$ [9]. The cylinders' radii were  $R_{GM} = 2 \text{ mm}$  and  $R_{WM} = 1 \text{ mm}$ . The propagation distance used was  $z = 2.5 \text{ m}$ .

### 2.3.5 Exploring phase retrieval

## Chapter 3

## Conclusion

## Chapter 4

# Acknowledgements

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