A summary of: Simulating phase contrast imaging for materials with variable density

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August 17, 2021,

Report 1 due date: Friday 26$^{\text{th}}$ August, 2021

1 Current objective

In this project I study the theoretical perspective of coherent X-ray imaging. My current focus is to investigate via a simulation how the phase of incident X-rays changes as the density of a sample material in an arbitrary imaging system changes. My investigation will be extended by simulating two distinct materials under the same wave-field and see if and how distinctly phase contrast occurs. The eventual goal of this simulation is to verify if successful phase retrieval can be done of the imaged objects given their variable densities.

2 The fundamentals of X-ray imaging theory

2.0.1 Helmholtz equation and the Angular spectrum formulation

2.0.2 Paraxial fields

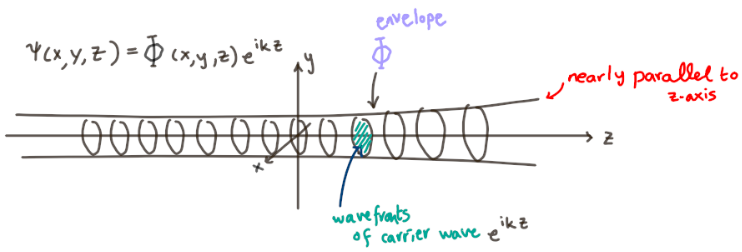

Figure 1: cartoon of a paraxial wave-field $\psi_\omega(\vec{r})$ displaying beam propagation properties along the z-axis. The slowly varying complex envelope is modulated by a carrying plane wave.

2.0.3 Refractive index and the projection approximation

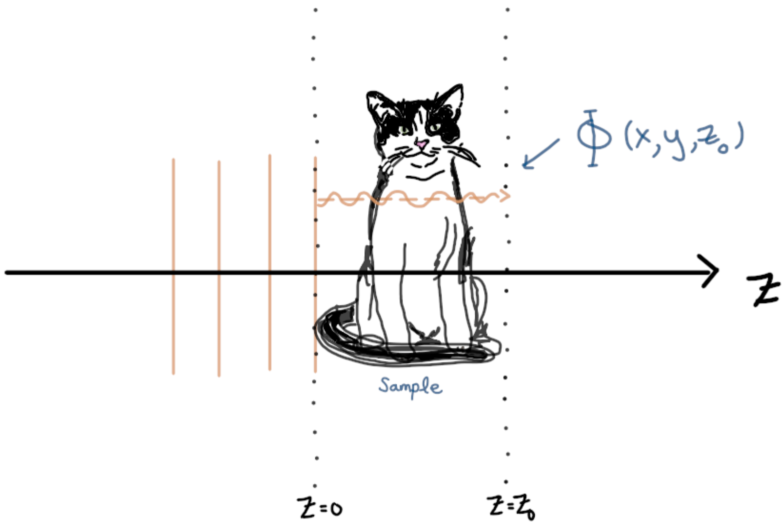

Figure 2: Schematic diagram of the projection approximation adapted from references [1] and [2]. The sample and is contained within the space between $z = 0$ and $z = z_0$. The sample is described by its refractive index $n(\vec{r})$ which differs from the the refractive index of the air volume that surrounds the sample.In this figure one can see that the path of X-rays passing through an object can be described by defining a surface immediately downstream from the irradiated object (i.e. $z = z_0$) at which the transferred transverse intensity and phase changes of the incident X-rays are imprinted[1].

2.0.4 Phase-shift

2.0.5 Transport-of-intensity equation

3 Investigations

The simulation that I am presently working on involves X-ray imaging a cylinder made of a single material (i.e. monomorphic) and constant density. My supervisor and I are curious to see if we can verify a claim in one of the texts I am using in my research. The claim states that the stability of an algorithm first developed by Paganin et al. (2002)[3][1] to be used in propagation based phase contrast imaging (PBI). This algorithm depends among other factors on the ratio between δ and μ, both of which are proportional to the density of the medium, The conclusion from this claim is essentially that changes in material density throughout the material would not affect the imaging process. My supervisor suspects that this statement might not be entirely correct and we want to verify if the stability of the Paganin et al. (2002) algorithm is dependent instead on the ratio of the differences in the refraction and attenuation coefficients $\Delta\delta$ and $\Delta\mu$.

3.0.1 Current progress

I am inspiring some of my work in a paper by Beltran et al. (2010), their approach requires just a single phase-contrast projection, and enables the user to focus on a material of interest. In the case of my simulation, I set the complex refractive index, and thickness of the imaged cylinder based on the parameters given in Beltran et al.(2010)[1]. For simplicity, my supervisor and I decided that I should break my simulation into simpler tasks. Therefore my first simulation will be that of a single monomorphic cylinder, and later on I will work on 2 cylinders, one embedded on another, both these cylinders will have different densities, since I want to understand how much density difference is needed before I see phase contrast in my simulation. To solve the transport-of-intensity equation (TIE) and go beyond the projection approximation I have designed a differential equation solver using the fourth order Runge-Kutta algorithm. Since there is an intrinsic link between the phase and the TIE, I decided to solve re-write equation (**??**) in differential form and solve it simultaneously to the TIE in my code. So far I have been able to recover the phase gained by the incident X-ray wave-fronts as they cross the cylindrical material I simulated.

3.0.2 Results

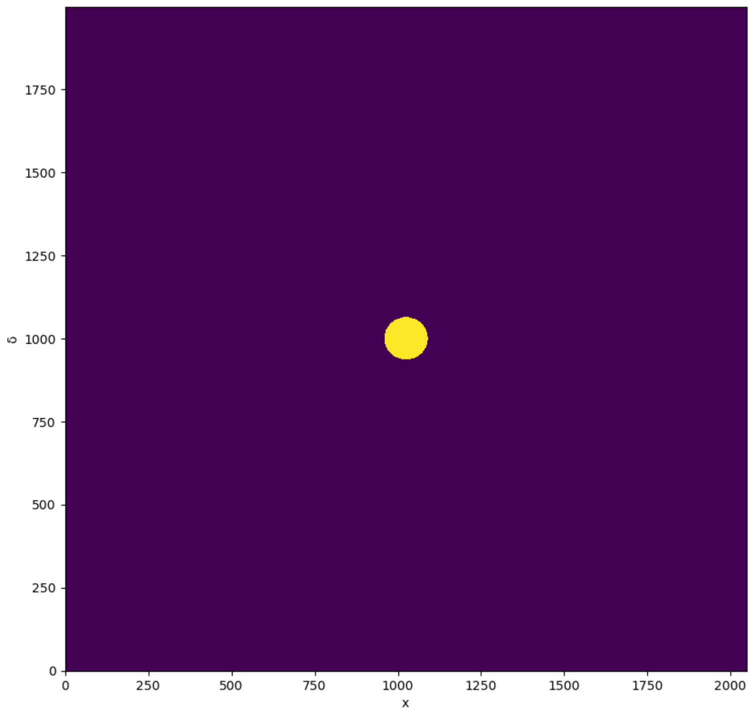

Figure 3: I made this 2D plot to verify that the cross-sectional are a of my simulated cylinder did indeed reflect the correct geometry. Here one can see a purple background representing the complex refractive index value of the vacuum $\delta = 0$ and that of the cylinder in yellow $\delta_0 = 462.8 \times 10^{-9}$ (as was reported in Beltran et al. (2010) for the PERSPEX sample).

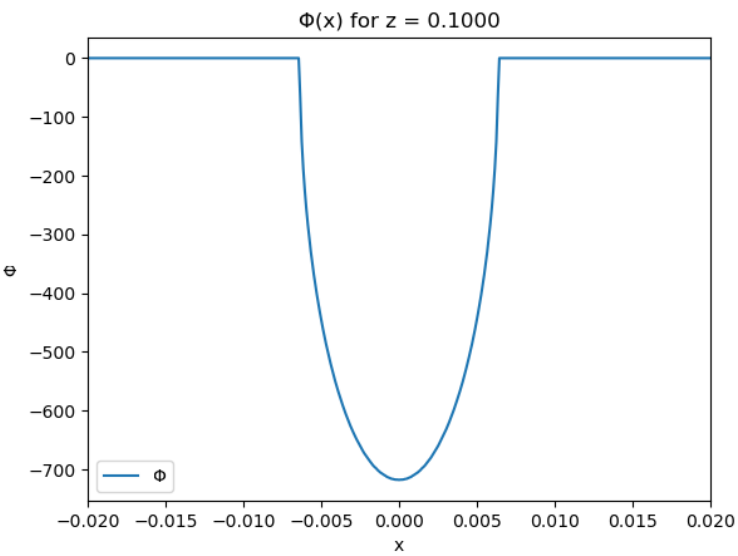

Figure 4: The position-dependent phase shift I discovered by using fourth order Runge-Kutta.

4 Future Plans

I aim to finish my simulation by creating a graphic user interface (GUI) that allows the user to modify interactively the density parameters of the imaged concentric cylinders in the simulation. The goal is to make a easily operated visual representation of the changes in phase and intensity of the X-rays as they interact with the object in-situ.

In the foreseeable future my supervisor and I plan to diversify my project by making me do some simulations about speckle analysis, the main goal there is to understand the sources and causes of speckles in images. I also plan to aid my supervisor with testing a newly obtained silver target source of X-rays in his lab apparatus. I aim to do data analysis to compare the results from the new silver target to the classic tungsten target used in the apparatus currently.

5 Conclusion

The present aim of my project is to create simulation to investigate how the phase of incident X-rays changes as the density of a sample material in an arbitrary imaging system changes. My investigation will be extended by simulating two distinct materials under the same wave-field and see if and how distinctly phase contrast occurs. The eventual goal of this simulation is to verify if successful phase retrieval can be done of the imaged objects given their variable densities. In this report I present a brief outline of the theoretical perspective of coherent X-ray imaging, a description of my current progress and future aims.

References

[1] Pelliccia D. et al. *Chapter 49 in Handbook of X-Ray Imaging: Physics and Technology*, volume 47. CRC Press Taylor & Francis Group, 1 edition, 2018.

[2] D Paganin. *Coherent X-Ray Optics*. Oxford University Press, 1 edition, 2006.

[3] Paganin D et al. Simultaneous phase and amplitude extraction from a single defocused image of a homogeneous object. *Journal of microscopy*, 206(1):33–40, 2002.