Bedmap3 observational data - analysis

Aim

The goal to characterise Antarctic subglacial landscape features through statistical and spectral analysis of bedrock topography. Historical radio-echo sounding (RES) flight lines provide bedrock elevation measurements. Contemporary Reference Elevation Model of Antarctica (REMA) is used to calculate mean ice thickness and its spatial variability. Local ice surface gradients derived from REMA are also used to infer ice flow direction.

Method (`analyse_bedrock`)

0. Data Ingestion & Cleaning

- **Explicit Error Handling**: The script implements `try-except` blocks for each file to ensure that a single missing or corrupted dataset does not halt the entire analysis.
- **Invalid Data Handling:** Explicitly filters invalid values for bedrock altitude and trajectory IDs.
- **Null Reporting**: The console now outputs the exact count of filtered nulls for each region to provide immediate feedback on dataset quality.
- **`load_datasets()` function** Implements a **dictionary-based configuration**. The user defines a list of `target_files` with labels and optional subsetting functions, making it trivial to add or remove regions. The `load_datasets()` function returns a **list of dictionaries** (`all_dfs`), each containing both the descriptive `name` and the `data`
- The `analyse_bedrock()` function loop then iterates through this list and proceeds to process each region separately to analyze the topography.

1. Data Segmentation & Pre-processing

Gap Detection: The script uses `detect_data_gaps` with a `gap_threshold=2000` (2km) to identify discontinuities.

Data Splitting: split_into_segments breaks each trajectory into independent chunks. This prevents the spectral analysis from "smearing" data across large physical voids.

Length Filtering: Chunks shorter than `min_segment_length=50` points are discarded to ensure data validity for the Lomb-Scargle periodogram.

Thickness Threshold: The script introduces a strict **validity check for ice thickness**. Segments are now automatically discarded if less than **20%** of the data has valid thickness measurements.

2. Spatial Domain Analysis (Per Segment)

Sliding Windows (50 km overlapping frames) implement **Bedrock Detrending:**
This means that local linear trends are removed from the bed elevation to isolate the **topographic residuals** (relief) required for spectral analysis in each segment, preserving valley features and calculating relief to identify mountains.
The plots display the global trend rather than the local detrending.

REMA Surface Integration: For each segment, surface elevation is extracted from the REMA mosaic.

2.1. Flight Line Distance Calculation

The input data provides coordinates in latitude/longitude (WGS84) but lacks reliable along-track distance information. To enable spectral analysis, the script converts coordinates to Antarctic Polar Stereographic (EPSG:3031) using `pyproj` , then calculates cumulative along-track distance in metres.

```
# make the transformer function
`transformer = Transformer.from_crs("EPSG:4326", "EPSG:3031", always_xy=True)`

# --- COORDINATE TRANSFORMATION ---
longs = line['longitude (degree_east)'].values
lats = line['latitude (degree_north)'].values

# Convert to Meters (EPSG:3031)
# Note: pyproj expects (Long, Lat) order for x, y output
x, y = transformer.transform(longs, lats)

# Vectorized Euclidean Distance Calculation (in projected space)
dx_seg = np.diff(x) # Δx array
dy_seg = np.diff(y) # Δy array
segment_distances = np.sqrt(dx_seg**2 + dy_seg**2)

# Cumulative distance starting at 0
dist = np.concatenate([[0], np.cumsum(segment_distances)])
```

2.2. Bedrock Spectral Signal Processing

refs: [Jordan_2017, VanderPlas_2018]
Frequency domain analysis for non-periodic and unevenly spaced real-world data

Frequency Range Definition: To ensure consistency across flight lines and regions, the script enforces a fixed spectral analysis window of 250m – 50km. The minimum frequency (longest detectable wavelength) is set by the sliding window size, marking the transition to sub-kilometre roughness; overlapping measurements are excluded. The maximum frequency (shortest wavelength) is determined by the median sampling interval. Frequencies are logarithmically distributed using `numpy.geomspace`.

Welch-like averaging apprroach: A rectangular window is applied to each sliding segment, preserving signal energy and providing the finest frequency resolution. Although rectangular windows exhibit greater spectral leakage than tapered alternatives, this trade-off is acceptable given the power law fitting approach.
Window Size: 50 km. **Step Size:** 10 km.
Overlap: $\frac{50k-10k}{50k} = 0.8 =$ 80% between consecutive windows.

Lomb-Scargle Periodogram: This algorithm performs least-squares spectral analysis without requiring resampling or uniform spacing. Instead, it fits sinusoids directly to the data at their original along-track positions—well suited to radar data, which is rarely evenly spaced due to variations in aircraft ground speed.
The script calculates a Lomb-Scargle periodogram for each overlapping window, then averages these to produce a robust power spectral density (PSD). A two-pass power law fit is applied to this averaged PSD.

1. **First Pass:** The script fits an initial power law to the PSD and identifies significant peaks in the residuals.
2. **Second Pass:** These peaks are masked to isolate the background roughness spectrum, and the power law is refitted to the *cleaned* data.

Finally, the script calculates the **whitened spectrum** by dividing the full PSD by the cleaned power-law fit. This flattens the background roughness trend, making spectral peaks associated with larger bedforms more prominent.

The spectral slope (β) is saved and displayed in the final results summary. This parameter characterises the scale-dependent roughness of the terrain:

$$\mathrm{PSD}(f) \propto f^{-\beta} \Leftrightarrow \mathrm{PSD}(f) \propto \lambda^{\beta}$$

Hurst exponent H is a measure of surface self-affinity, is derived from the spectral slope as:

$$H = \frac{\beta - 1}{2}$$

this metric is reported in the final results to characterize surface self-affinity.

Beta (β)	Landscape Type	Interpretation
~2.0	**"Brownian" / Fractal**	**Hard Bed.** Typical of crystalline shields (granite/gneiss). The ice has to "stick and slip" over these jagged teeth. Friction is high. The curve is flatter. This means there is a lot more "power/energy/amplitude" at high frequencies (small wavelengths).
> 2.5	**Smoothed / Draped**	**Soft Bed.** Likely a sedimentary basin (mud/till). The ice can slide rapidly over this deformable sludge. This is often where you find fast-flowing **Ice Streams**. The curve is steep. This means the "power" drops off very fast at high frequencies.
< 1.5	**White Noise**	**Chaotic.** Can indicate data noise, OR **shattered crystalline bedrock** (e.g., deep glacial troughs)

Peak Finding: The script uses `scipy.signal.find_peaks` to identify peaks in the whitened spectrum that exceed the background trend by a factor of two or more. These peaks correspond to dominant topographic wavelengths within each flight line. Detected wavelengths are aggregated across all flight lines to assess whether a *characteristic* periodicity exists at the basin scale.

A minimum sampling floor is enforced when calculating the Nyquist limit: `limit_spacing = max(dx_median, 15.0)`. This ensures a minimum spacing of 15m per window, preventing aliasing artifacts caused by GPS jitter or overlapping points at shorter wavelengths.

The script compiles summary statistics, calculating the **mean** and **range (min, max)** for all metrics across flight lines.

Only *confirmed* wavelengths are included in the final statistics. A wavelength is confirmed if $\lambda < L/2$, where L is the segment length. Wavelengths exceeding this threshold are classified as *candidates* and excluded, since fewer than two complete cycles fall within the observation window (making them indistinguishable from linear trends or red noise.)
The detectable range is explicitly constrained to 250m – 50km. Wavelengths longer than the observation window cannot be reliably resolved and introduce spectral leakage, causing trends to mimic periodic signals.

Note: Wavelengths > L/2 are excluded because they cannot be distinguished from **linear trends or red noise** within the observation window

2.3. Plotting(`plot_spectra`)

This function generates a three-panel figure:

- **`ax1` :** Bedrock elevation profile (black line).
- **`ax2` :** Log-log plot of wavelength versus PSD, with the power-law fit overlaid.
- **`ax3` :** Whitened spectrum (residuals), used to assess whether detected wavelengths represent physical bedforms or noise.

Peak wavelengths are marked with coloured points: **blue** for the minimum detected wavelength and **red** for the maximum.

2.4. Robust RMS Roughness (Butterfly Filter)

Standard RMS roughness is typically calculated as the standard deviation of the detrended bed elevation. However, this method captures *all* spectral energy, including high-frequency instrumentation noise (<250m) and potential aliasing artifacts. This creates a disconnect with the spectral analysis (Section 2.2), which explicitly restricts the Power Law fit (β) to the geological band of 250m−50km. To ensure statistical consistency between the reported roughness amplitude (RMS) and roughness texture (β), the spatial data must be band-limited to the same range.

Surface Smoothing: A low-pass stencil (a costumised 4th order bandpass Butterworth filter (from `scipy.signal`)) is applied to REMA to suppress local surface noise (dunes) and isolate the **regional driving stress** direction.

- The filter defines cutoffs (normalised by the Nyquist frequency)
  - **High cutoff**: 50 km (matches the window width)
  - **Low cutoff**: 250 m
- **Design**
  - Butterworth flat passband
  - Second-order sections (`output='sos'`)
  - **Forward-Backward filtering** (`sosfiltfilt`) eliminates phase distortion
    - Forward: (introduces phase lag when filtering elevation profile),
    - Backward: (reverses phase lag by introducing an identical backward lag when filtering the elevation profile) **Result:** The two lags cancel out perfectly. This ensures that roughness peaks in the filtered data align exactly with the geographic location of the bedrock features.

3. Other Statistical analysis

3.1 Ice Thickness

The workflow calculates ice thickness by integrating local radar-derived bedrock data with the **Reference Elevation Model of Antarctica (REMA)** surface mosaic.

Surface Elevation Extraction

Surface elevations are extracted from the REMA DEM at specific flight track coordinates using **Bilinear Interpolation**.

```
from scipy.ndimage import map_coordinates
# Interpolate:
# order=1 is Bilinear (linear in x, linear in y)
# mode='nearest' handles edges gracefully
elevations = map_coordinates(data, [rows, cols], order=1, mode='nearest')
```

- **Coordinate Mapping:** Projected world coordinates (EPSG:3031) are mapped to image pixel indices using an inverse affine transform.
- **Interpolation:** A first-order spline interpolation (`order=1`) calculates the surface height at the exact point of the radar measurement, ensuring high spatial accuracy that matches JR's Fortran-based method.

Thickness Derivation

Ice thickness (H) is determined by the difference between the interpolated surface elevation (z_s) and the radar-measured bedrock altitude (z_b):

$$H = z_s - z_b$$

Data Validation

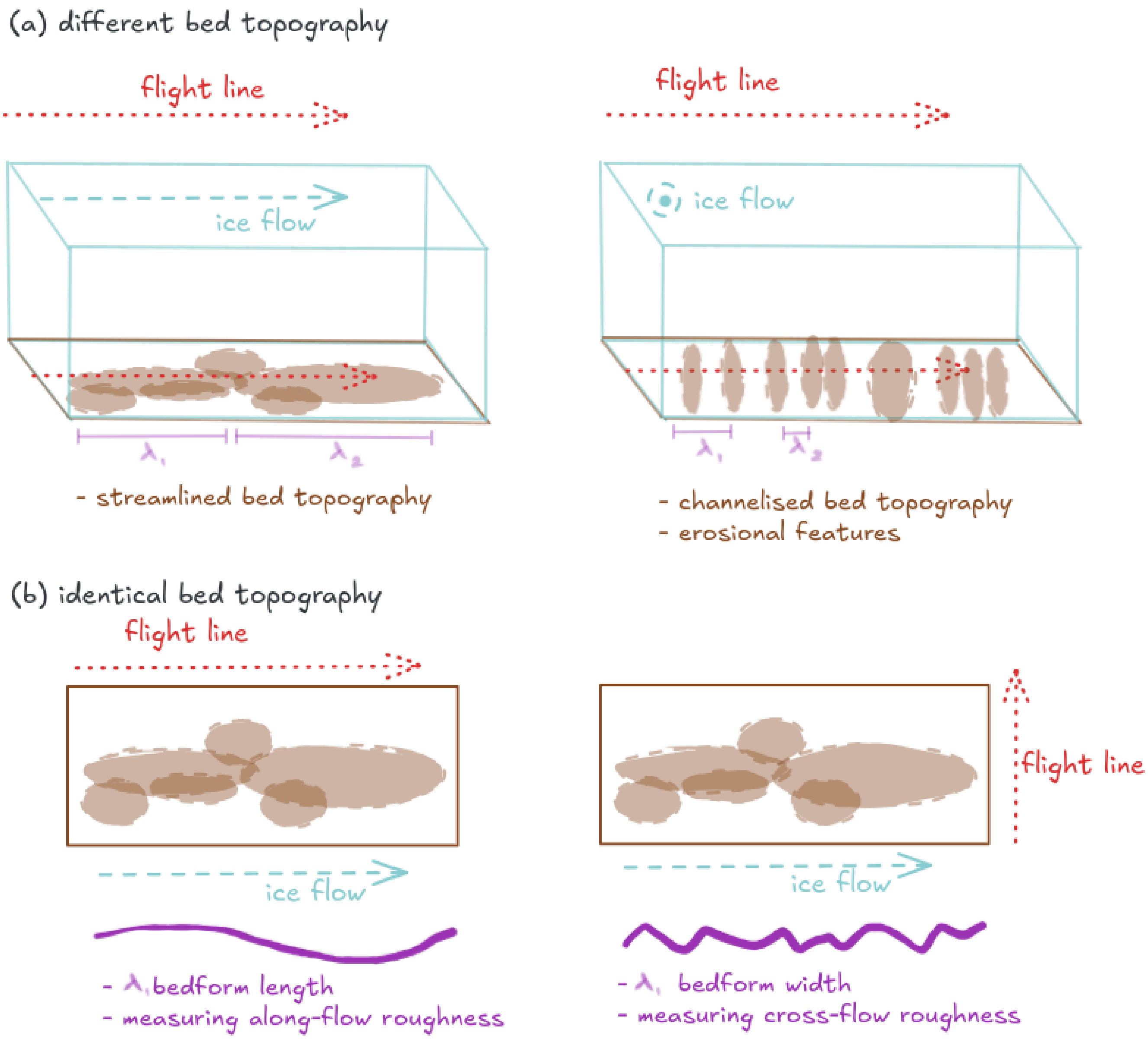
To ensure physical consistency, the script applies two primary filters:

- **NoData Handling:** REMA "nodata" values are converted to NaNs to prevent interpolation artifacts.
- **Physical Constraint:** Any calculated thickness values <0 (where the measured bed appears higher than the surface) are set to NaN to maintain dataset integrity.

3.2 Determining "True" Flow Direction For Each Flight Line

ref [McCormack_2019]

The physical interpretation of detected wavelengths depends on local ice flow direction, as bedrock roughness is anisotropic—varying with measurement orientation.



figure

Schematic illustrating the anisotropic interpretation of detected wavelengths.

(a) Different beds, same flight line orientation: detected wavelengths (λ) differ due to genuine geological variation.

(b) Same bed, different flight line orientation: detected wavelengths differ because orthogonal transects sample different dimensions of the same features.

A detected wavelength may therefore represent either:

Bedform length — when the flight line is parallel to ice flow.

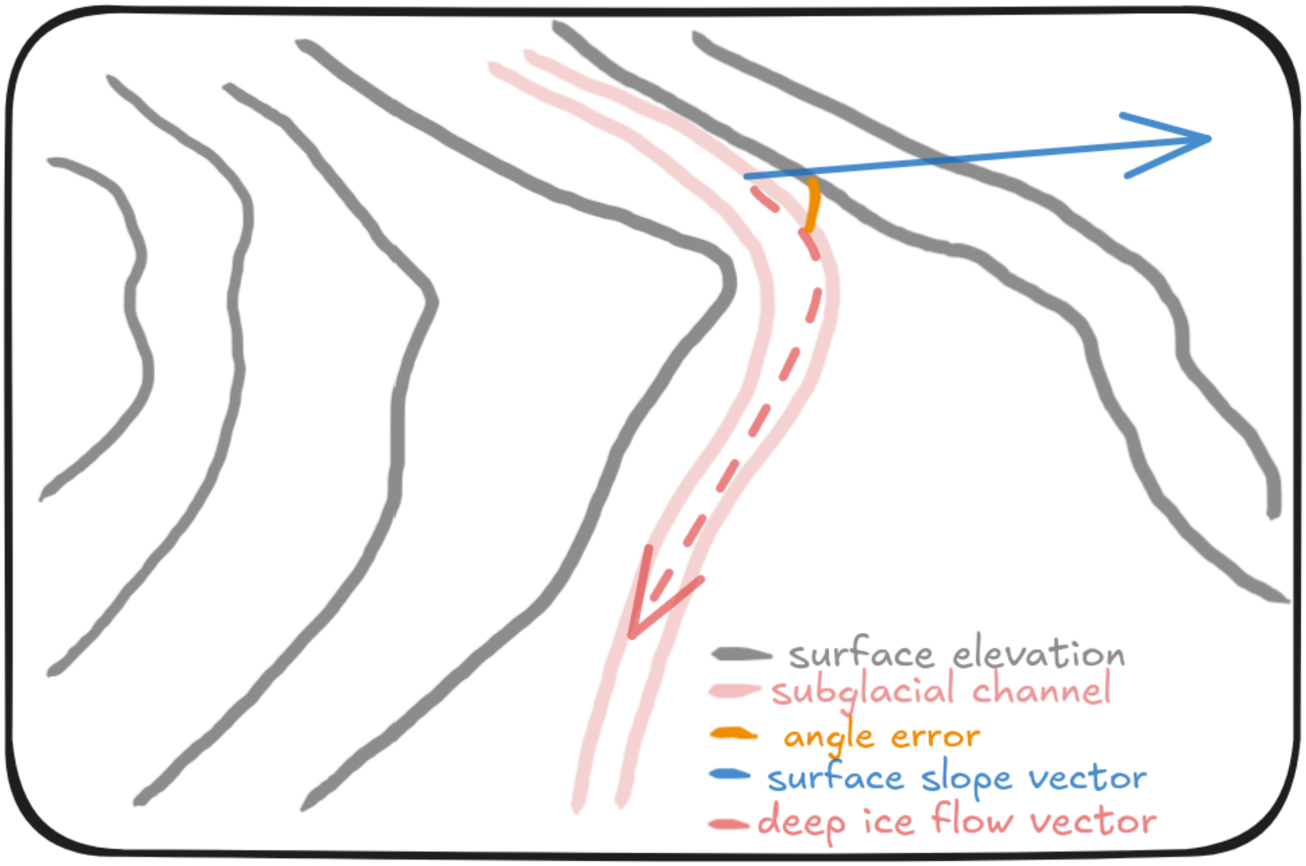
Bedform spacing — when the flight line is perpendicular to ice flow.

As described in Section 2.2, the spectral slope β characterises the scale-dependent roughness of the terrain. Its value varies systematically with flight line orientation relative to ice flow:

- **Parallel to ice flow:** Flight lines sample along-flow roughness. Higher β values indicate reduced high-frequency power, consistent with streamlined, glacially smoothed terrain.
- **Perpendicular to ice flow:** Flight lines sample cross-flow roughness. Lower β values indicate greater high-frequency power, reflecting rougher, less modified terrain.

Local Surface Gradient

In deep, fast-flowing channels, the direction the **surface is tilting** (Slope) is often **NOT** the same direction the **bottom ice is moving** (Flow).



McCormack et al. (2019) addressed this mismatch by smoothing the surface DEM with a spatially varying triangular filter width = 8–10 $\times$ ice thickness (~10×H). The stencil is used **only** to derive flow orientation and does not modify the bedrock elevations used in the spectral power-law fit. This filtering removes local surface undulations that are decoupled from the bed, leaving only the broad regional slope that actually drives deep ice flow. The smoothed slopes showed strong agreement with observed satellite velocity fields (MEaSUREs v2).

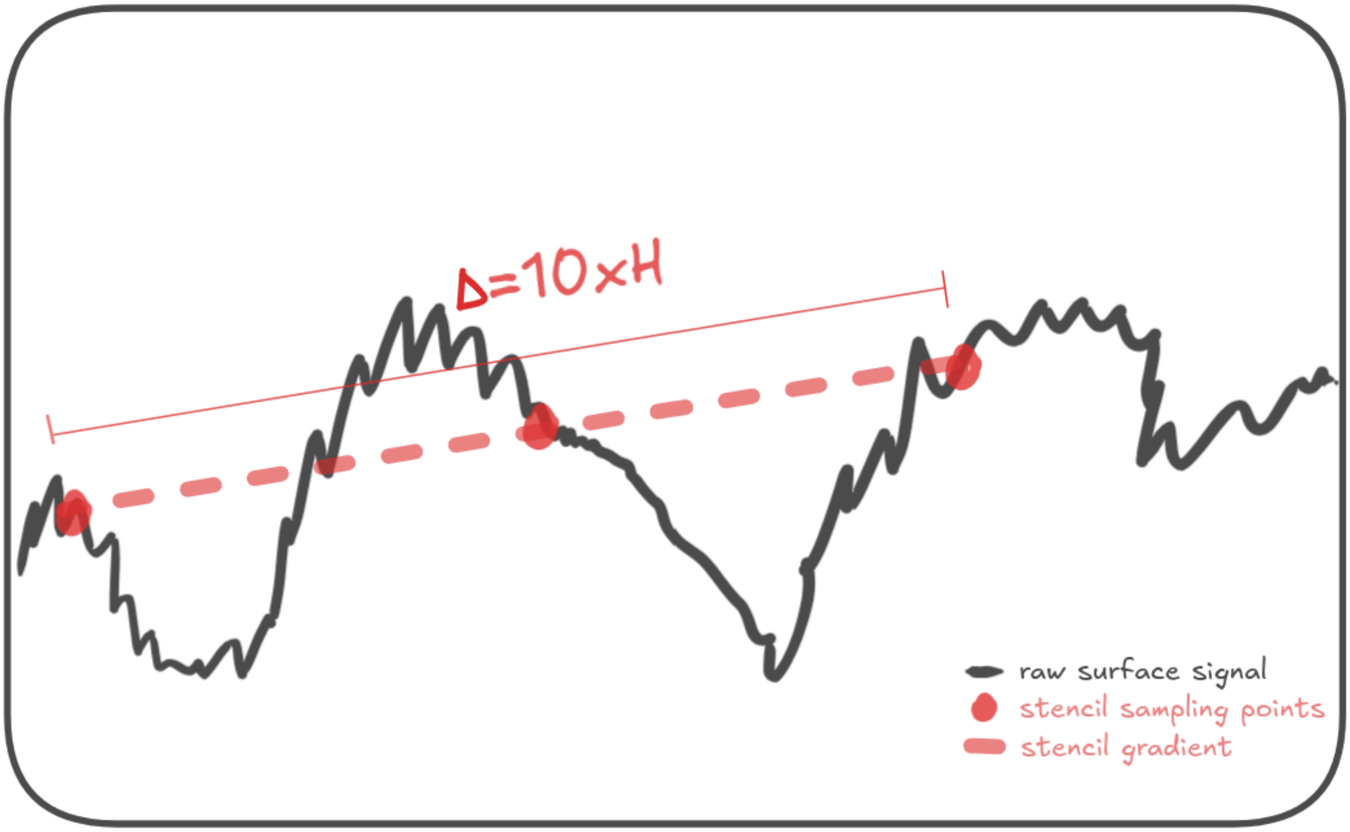
The regional **ice flow vector** is estimated from the REMA mosaic (100 m resolution) using the surface elevation gradient:

$$v_{flow} \propto -\left(\frac{dS}{dx}, \frac{dS}{dy}\right)$$

where S is ice surface elevation. Gradients are sampled at flight line coordinates, allowing the **incidence angle** between each flight path and the local ice flow direction to be calculated.
The script utilises a **central finite difference stencil** to capture this large-scale flow direction by sampling the elevation around four neighbouring points:

$$\frac{\partial S}{\partial x} \approx \frac{z_{i+1,j} - z_{i-1,j}}{2\Delta x},$$
$$\frac{\partial S}{\partial y} \approx \frac{z_{i,j+1} - z_{i,j-1}}{2\Delta y},$$

Where $\Delta x = \Delta y = 5 \times H$.



This stencil acts as a **low-pass filter**, ignoring roughness features smaller than the stencil width. This achieves a similar goal to McCormack's smoothing (suppressing local noise to find regional flow) but is faster to calculate.
Note: flat regions are handled by calculating the flow magnitude and imposing a value of 1 when encountering zero-valued gradient magnitudes.

The script categorises flight lines by orientation relative to flow using the dot product to calculate angles between the flight line and flow direction, ignoring upstream/downstream direction. Orientation thresholds are: Parallel < 30°, Oblique 30–60°, and Perpendicular > 60°.

Current input & Results

Note: data is organized into three main datasets:

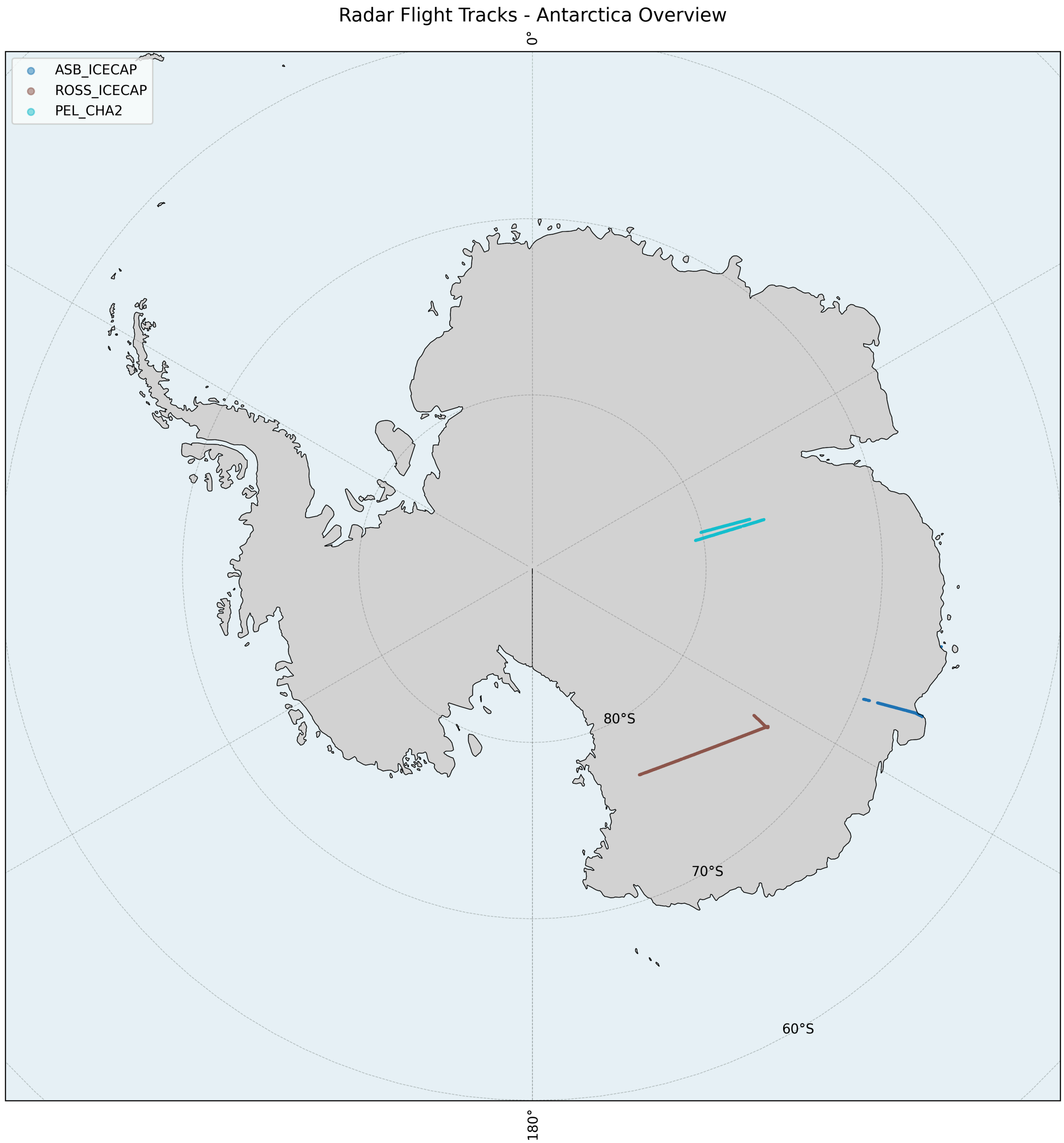
- bedmap3/: Recent data (2013-2019)
  CSV File Format
- **Header**: Metadata about the survey (lines starting with #)
- **Column headers**:

1. trajectory_id,
2. trace_number,
3. **longitude**,
4. **latitude,**
5. date, time_UTC,
6. surface_altitude,
7. land_ice_thickness,
8. **bedrock_altitude,**
9. two_way_travel_time,
10. aircraft_altitude,
11. along_track_distance

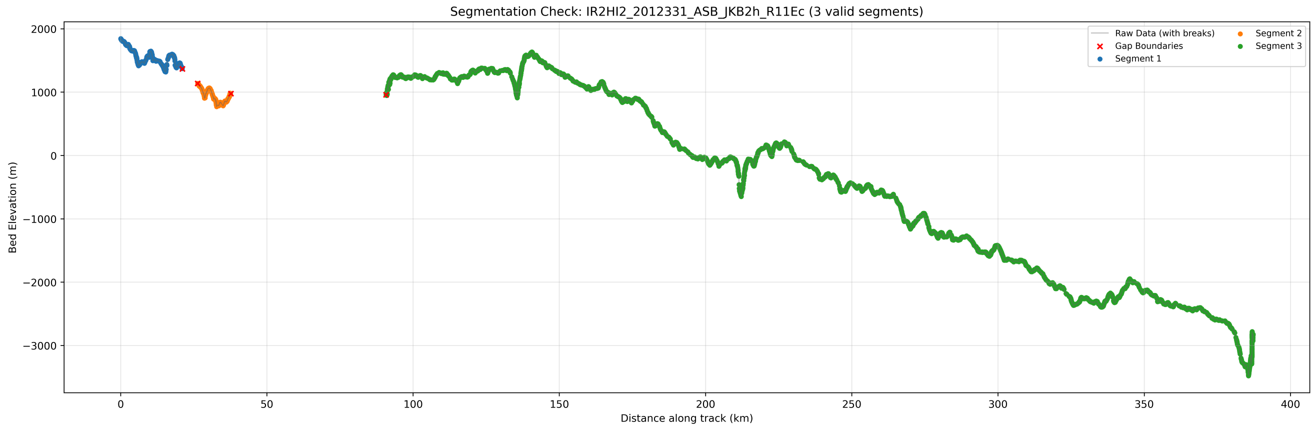
Data tracks: Target files

Jason's 500m_grid.zsh script shows exactly what locations were culled

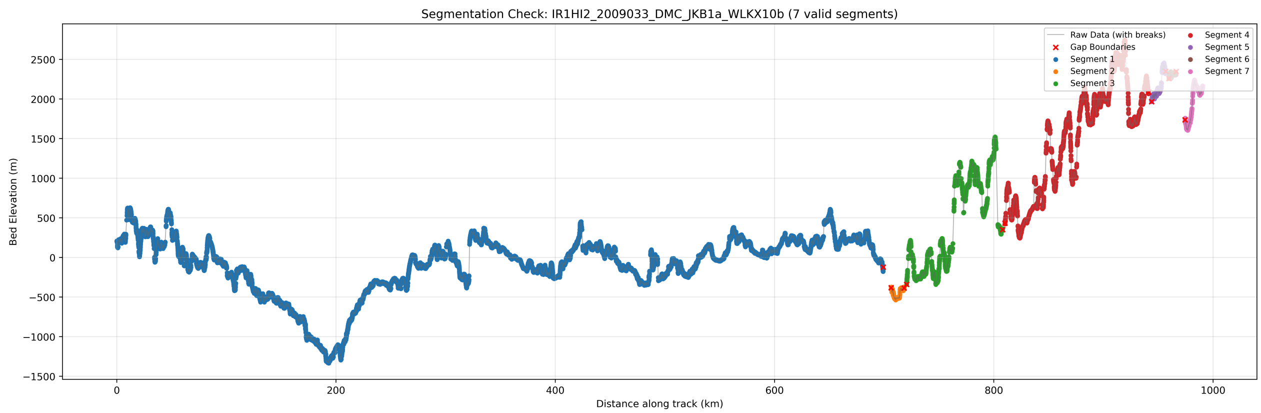
ASB: UTIG flight segment 2012331_ASB_JKB2h_R11E
ROSS : UTIG flight segment IR1HI2_2009033_DMC_JKB1a_WLKX10b
PEL: PRIC_2016_CHA2_AIR_BM3, (last 3 segments)



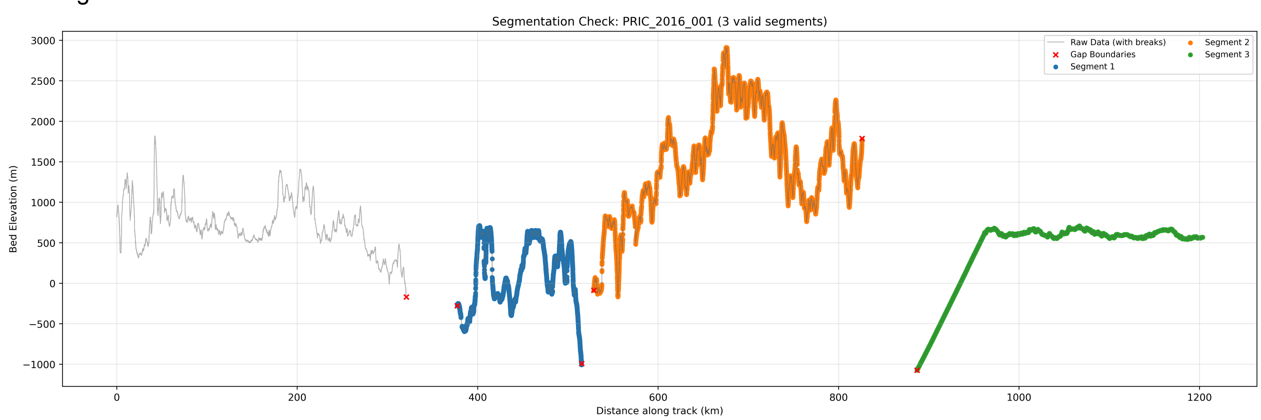
- **ASB** region: UTIG flight segment 2012331_ASB_JKB2h_R11Ec



- **ROSS** region: UTIG flight segment IR1HI2_2009033_DMC_JKB1a_WLKX10b



- **PEL** region:



Current output (for a noise floor of 15 m)

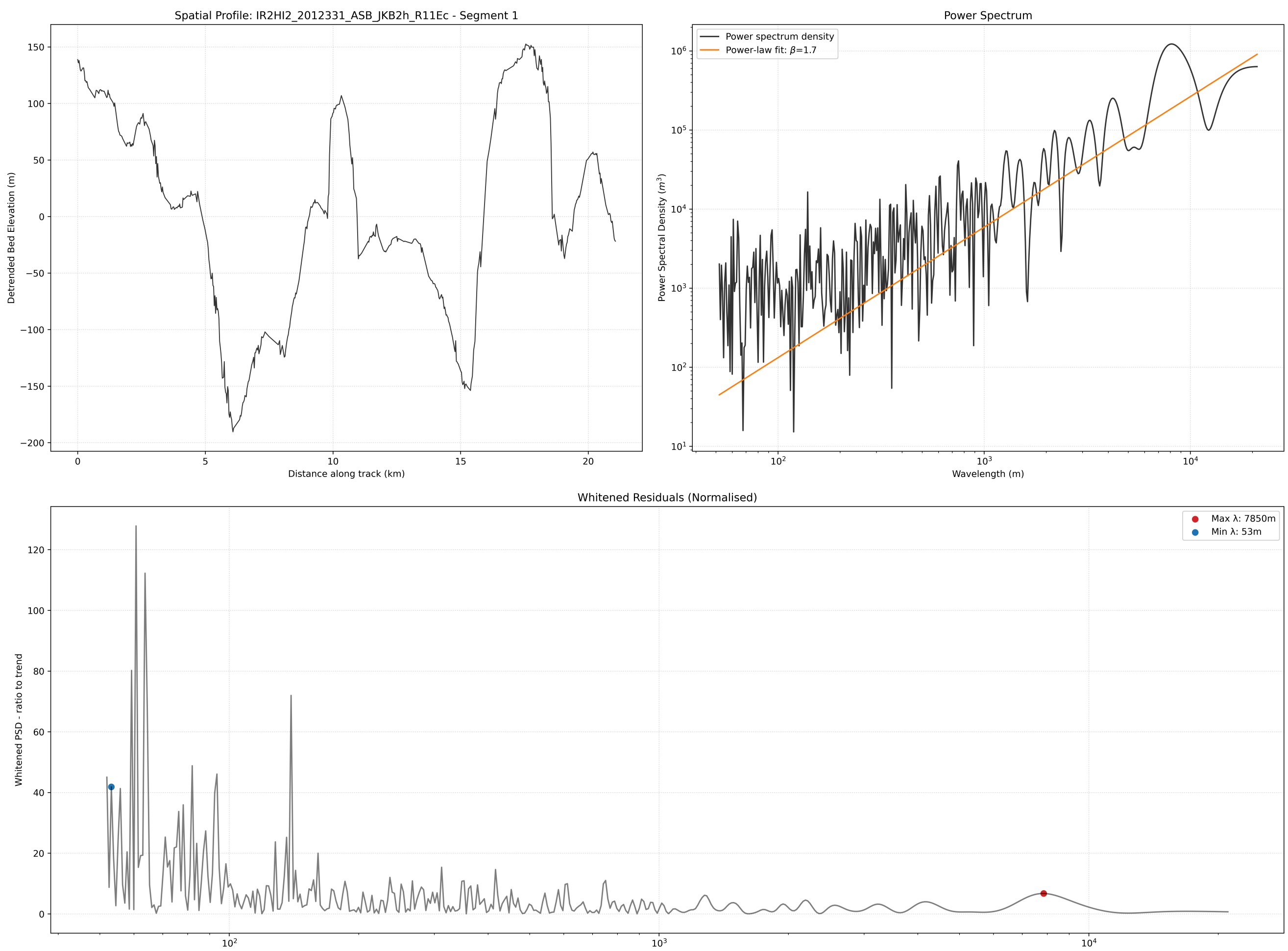
Observations

1. **Beta (β) Differences**:
   ASB (avg β≈1.7): The ASB region is quite heterogeneous, transitioning between a hard bed (segment 3: 1.3) from a soft sedimentary bed (segment 2: 2.0).
   ROSS (avg β≈2.2): Smooth sedimentary terrain (even in cross-section; likely a basin).
   PEL (avg β≈2.0): Massive vertical relief (>2km); mountainous terrain of mixed roughness.
   Moller IS
   Thwaites G
   Thwaites S
   DML

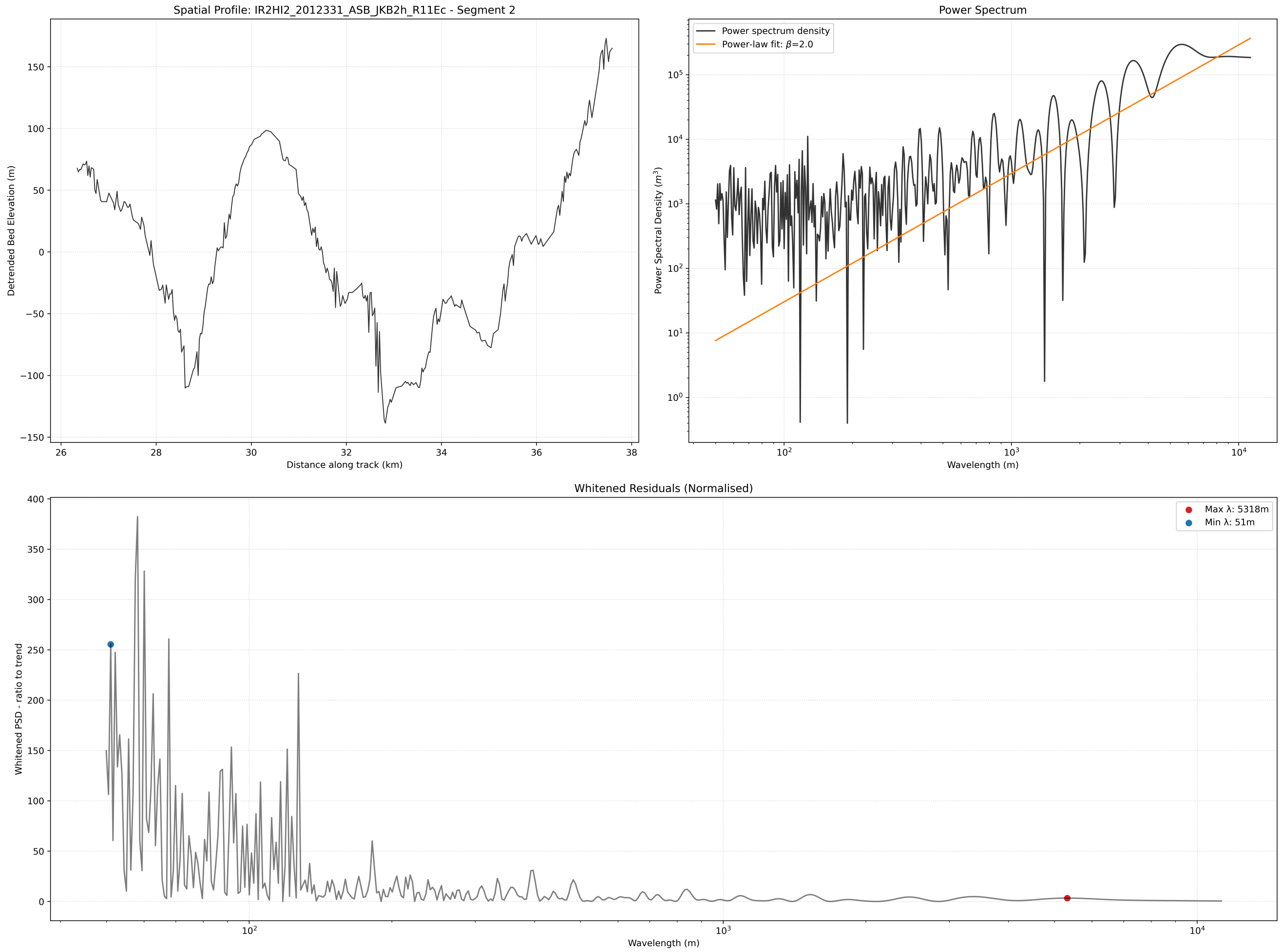
   Interpretation:

2. **Ice Thickness Validation**:
   ASB: 100% valid ice thickness coverage.
   ROSS: 100% valid ice thickness coverage.
   PEL: One segment has partial coverage (872 / 14469), likely because the flight line extends beyond the REMA mask or the coastline. THIS SEGMENT IS REMOVED
3. **Flow Orientation**:
   ASB: **Parallel** for two and **Oblique** for one segment (3 total)
   ROSS: Three **perpendicular** and four oblique **segments** (7 total)
   PEL: ALL **oblique** segments (4 total)
4. **Nyquist Limit:** The median sampling spacing is ~(20-25)m. Analysis cannot observe anything at wavelengths smaller than ~(40-50)m. The 15m minimum floor Is chosen to prevent aliasing artifacts. **Wavelengths <100 m** approach the noise floor which make them highly uncertain.

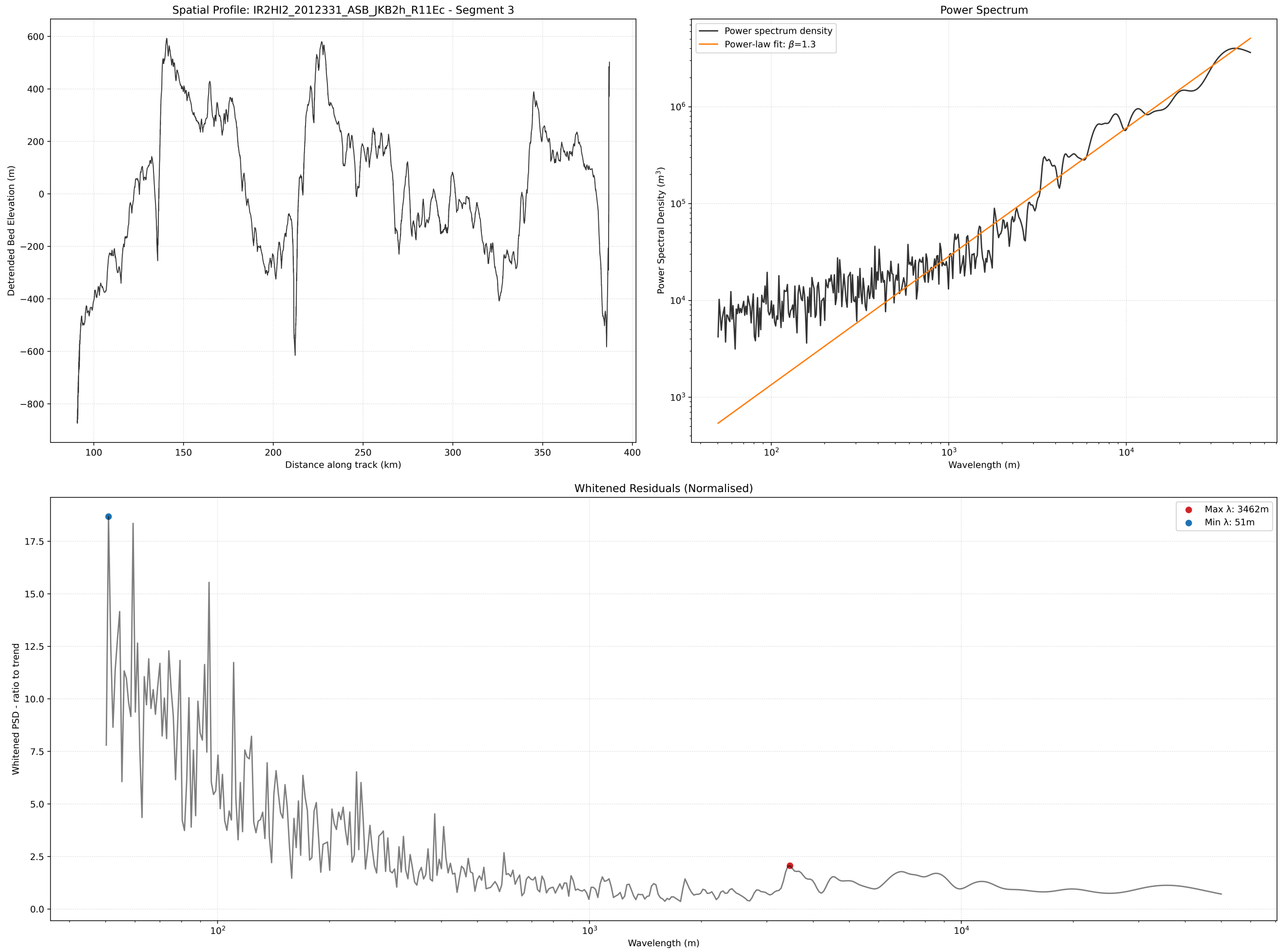
ASB Region:



1.


Spatial Profile: IR2HI2_2012331_ASB_JKB2h_R11Ec - Segment 2
Detrended Bed Elevation (m)
Distance along track (km)
Power Spectrum
Power spectrum density
Power-law fit: β=2.0
Power Spectral Density (m^3)
Wavelength (m)
Whitened Residuals (Normalised)
Max λ: 5318m
Min λ: 51m
Whitened PSD - ratio to trend
Wavelength (m)

2.



3.

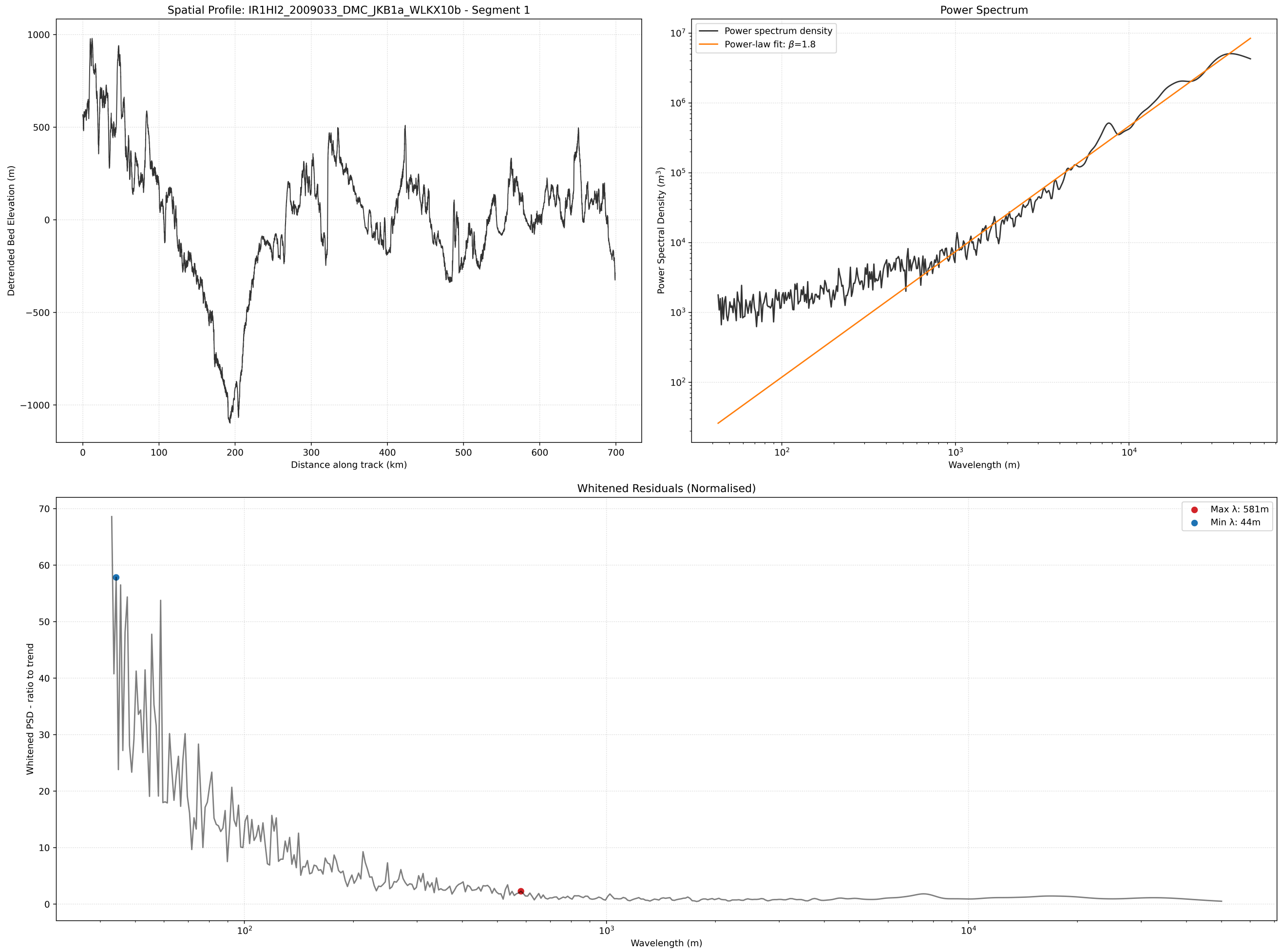
Terminal

```
✓ ASB_ICECAP loaded: 5756 rows (Filtered 11772 nulls)

Starting analysis of ASB_ICECAP...
  Valid data points: 5756
  Unique trajectories: 2
  IR2HI2_2012331_ASB_JKB2h_R11Ec: Found 2 gaps (>=2000m) in data
    > Segment 1: Rows 0 to 498 (498 points), Length: 21.06 km
    > Segment 2: Rows 499 to 809 (310 points), Length: 11.25 km
    > Segment 3: Rows 810 to 5744 (4934 points), Length: 296.46 km
3 data segments found
 > Segment 1: Valid ice thickness for count: 498 / 498
 >>>>>>>>>: ASB_ICECAP | IR2HI2_2012331_ASB_JKB2h_R11Ec | Segment 1: Parallel (incidence: 15.4)
 > Segment 2: Valid ice thickness for count: 310 / 310
 >>>>>>>>>: ASB_ICECAP | IR2HI2_2012331_ASB_JKB2h_R11Ec | Segment 2: Parallel (incidence: 12.1)
 > Segment 3: Valid ice thickness for count: 4934 / 4934
 >>>>>>>>>: ASB_ICECAP | IR2HI2_2012331_ASB_JKB2h_R11Ec | Segment 3: Oblique (incidence: 32.2)
  Trajectory IR2HI2_2012331_ASB_JKB2h_R11Ec: 3 segments, combined median spacing = 25.3m, Nyquist =
50.7m
ASB_ICECAP is finished processing

---
Analysed 1 regions

=== ASB_ICECAP SUMMARY ===
------------------------------------------------------------
  RESULTS SUMMARY (1 trajectories aggregated)
------------------------------------------------------------
```

```
VERTICAL RELIEF (Max-Min):
  -> 2002.5m (Single Value)
AVG SEGMENT LENGTH:
  -> 109589.7m (Single Value)
POWER LAW EXPONENT (Beta):
  -> Mean: 1.7 | Range: [1.3, 2.0]
HURST EXPONENT:
  -> Mean: 0.3 | Range: [0.2, 0.5]
MEAN ICE THICKNESS:
  -> 1974.1m (Single Value)
............................................................
FLOW ORIENTATION (All Segments):
  -> Oblique: 1 segments (33.3%)
  -> Parallel: 2 segments (66.7%)

  STRATIFIED BETA (Roughness) BY ORIENTATION:
  -> Parallel       : Mean Beta = 1.82  (n=2)
  -> Oblique        : Mean Beta = 1.33  (n=1)
  -> Perpendicular  : No segments found
............................................................
CONFIRMED WAVELENGTHS (Physically valid < L/2):
  -> Count: 180
  -> Mean: 397.8m | Range: [50.9, 7850.1]m
............................................................
LARGEST LOCAL STRUCTURES (50km Window):
  -> Relief: 1555.5m at km 170.7
............................................................
============================================================

real	13m52.082s
user	12m31.830s
sys	1m24.848s
```

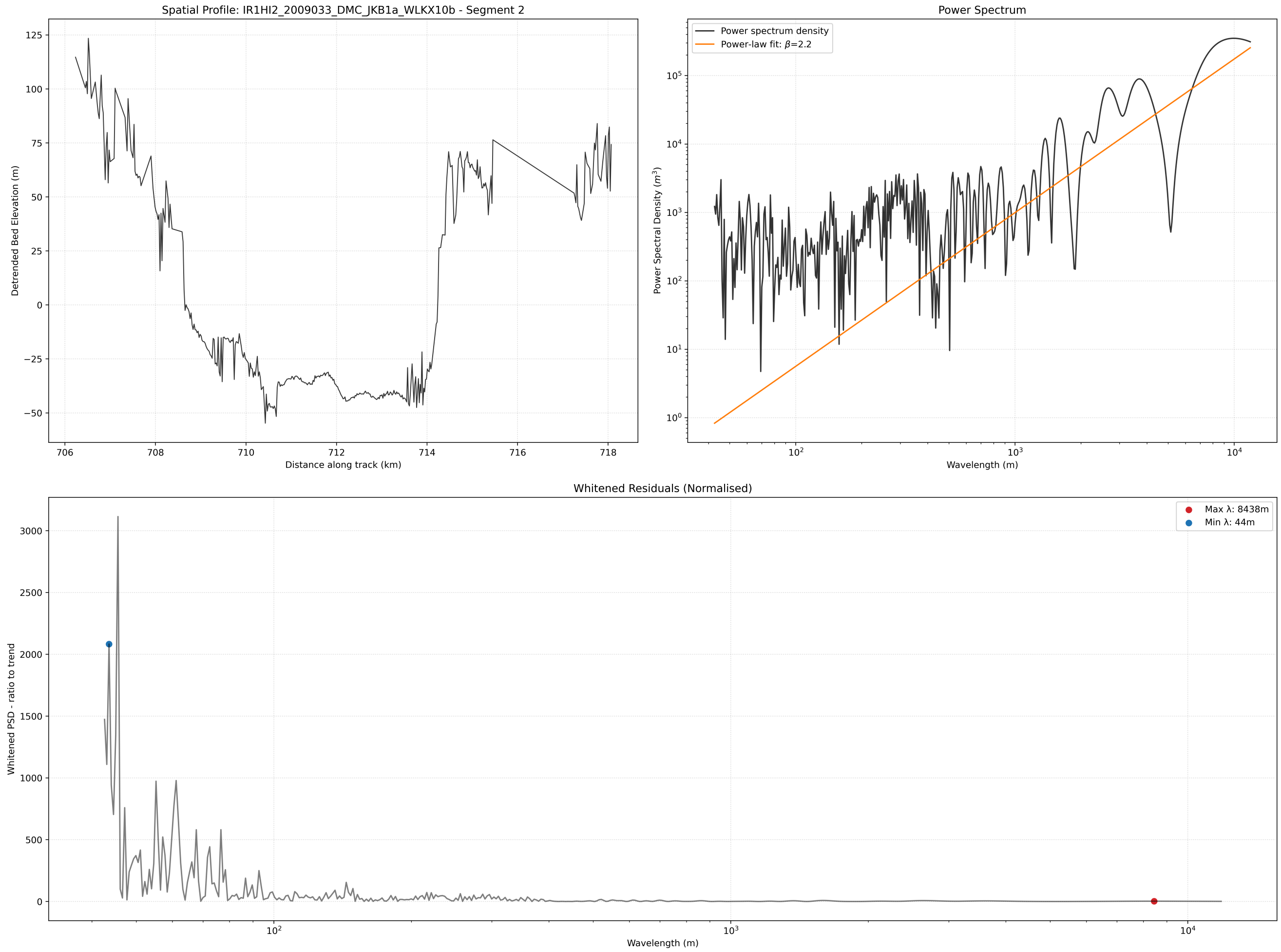
On average $\beta < 2$ even when sampled along-flow suggests the bed is **highly resistant to erosion** or is a transition zone. This is interesting and unexpected since its a basin below sea level (we would expect high sediment)

ROSS Region: IR1HI2_2009033_DMC_JKB1a_WLKX10b

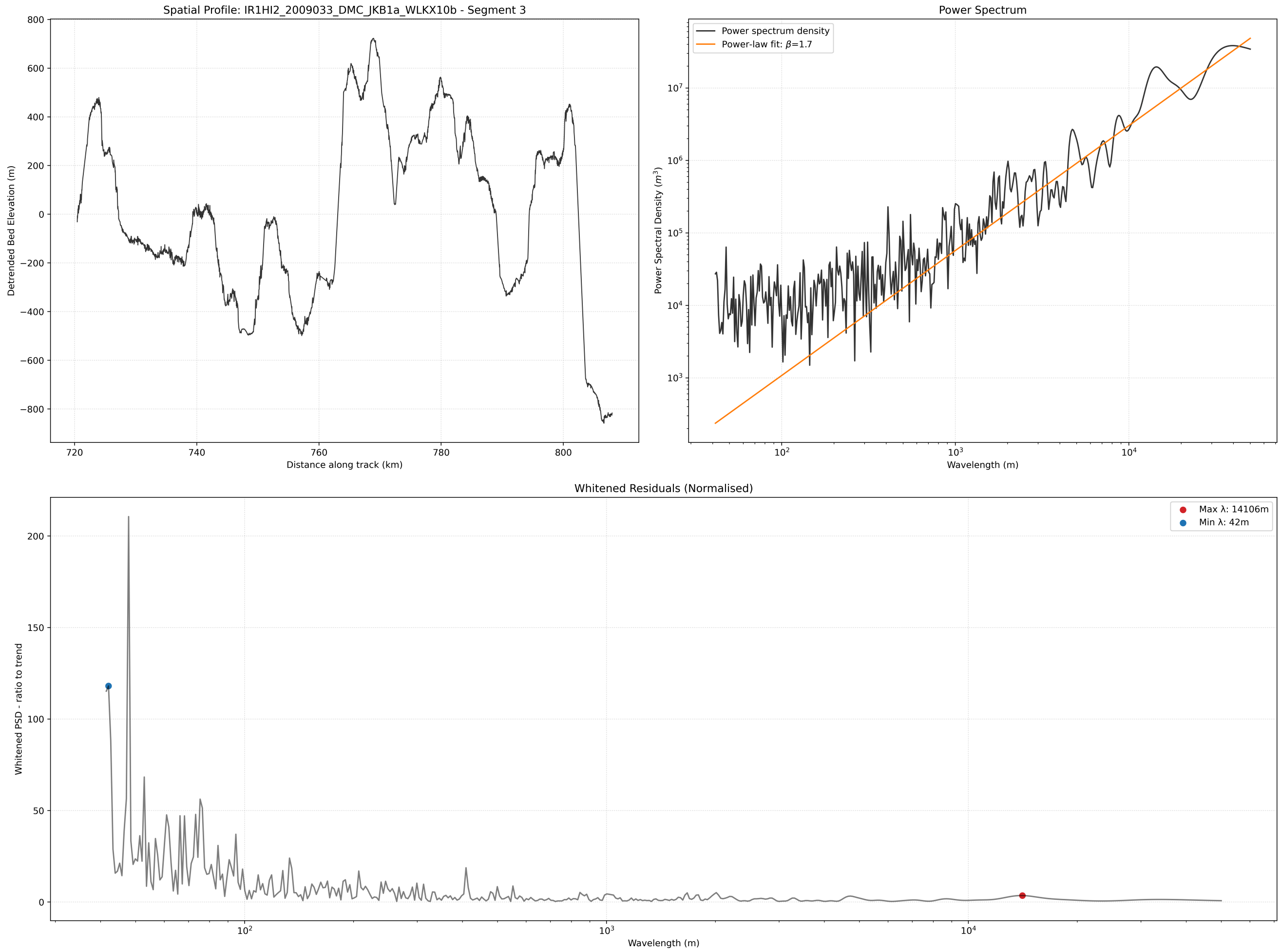


Spatial Profile: IR1HI2_2009033_DMC_JKB1a_WLKX10b - Segment 1
Detrended Bed Elevation (m)
Distance along track (km)
Power Spectrum
Power spectrum density
Power-law fit: β=1.8
Power Spectral Density (m^3)
Wavelength (m)
Whitened Residuals (Normalised)
Max λ: 581m
Min λ: 44m
Whitened PSD - ratio to trend
Wavelength (m)

1.

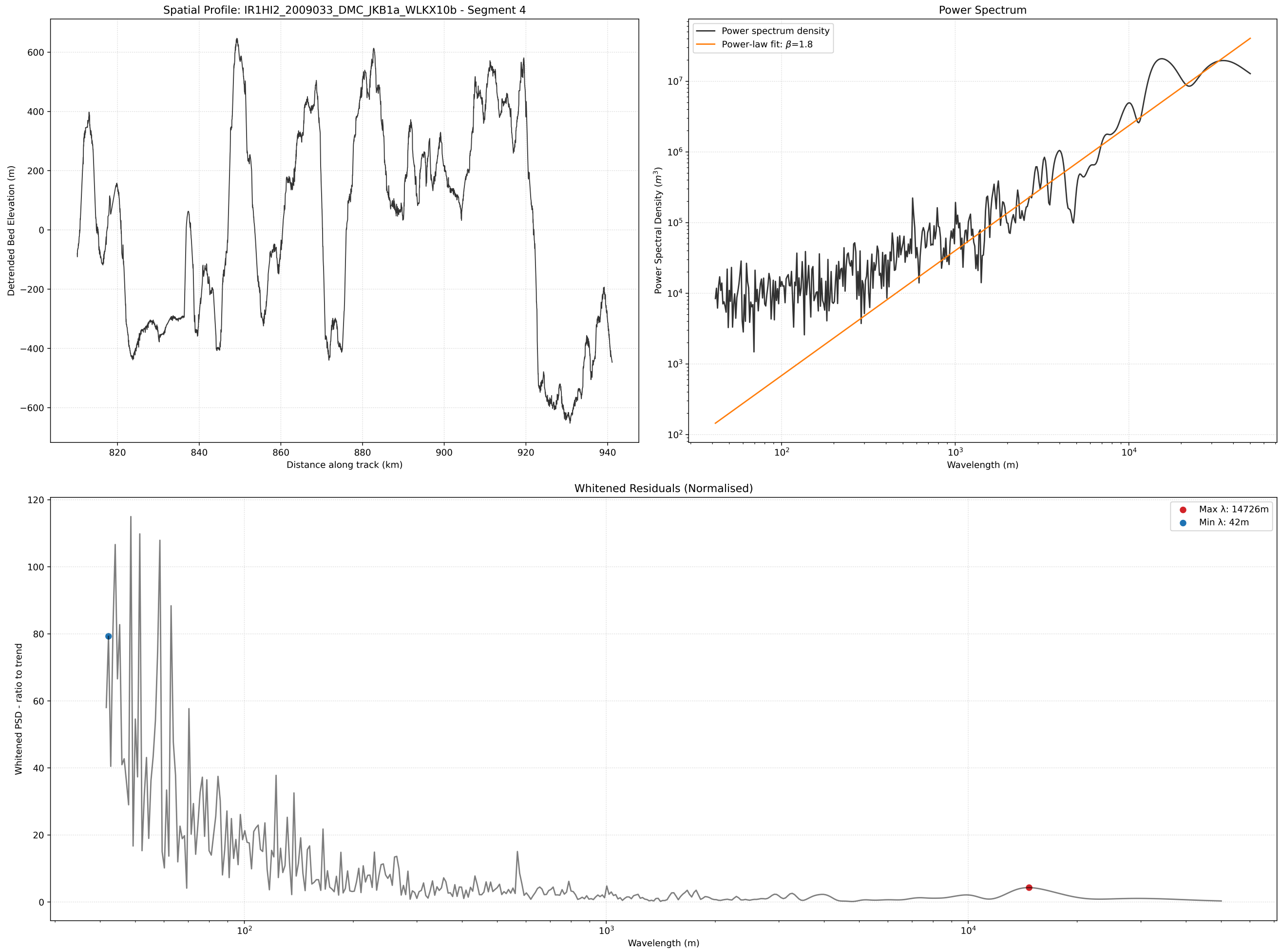

Spatial Profile: IR1HI2_2009033_DMC_JKB1a_WLKX10b - Segment 2
Detrended Bed Elevation (m)
Distance along track (km)
Power Spectrum
Power spectrum density
Power-law fit: β=2.2
Power Spectral Density (m³)
Wavelength (m)
Whitened Residuals (Normalised)
Max λ: 8438m
Min λ: 44m
Whitened PSD - ratio to trend
Wavelength (m)

2.



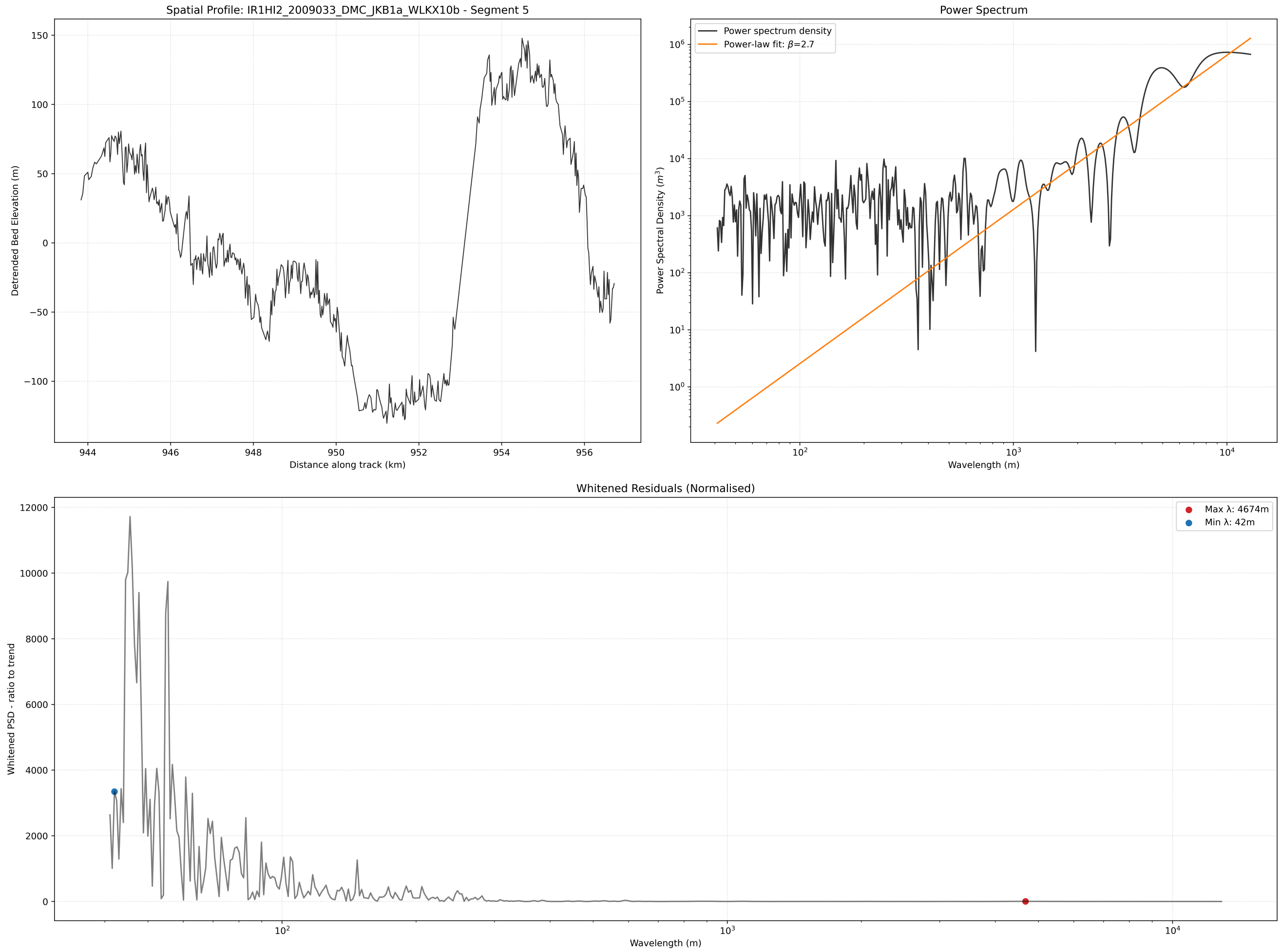
Spatial Profile: IR1HI2_2009033_DMC_JKB1a_WLKX10b - Segment 3
Detrended Bed Elevation (m)
Distance along track (km)
Power Spectrum
Power spectrum density
Power-law fit: β=1.7
Power Spectral Density (m³)
Wavelength (m)
Whitened Residuals (Normalised)
Max λ: 14106m
Min λ: 42m
Whitened PSD - ratio to trend
Wavelength (m)

3.



Spatial Profile: IR1HI2_2009033_DMC_JKB1a_WLKX10b - Segment 4
Detrended Bed Elevation (m)
Distance along track (km)
Power Spectrum
Power spectrum density
Power-law fit: β=1.8
Power Spectral Density (m^3)
Wavelength (m)
Whitened Residuals (Normalised)
Max λ: 14726m
Min λ: 42m
Whitened PSD - ratio to trend
Wavelength (m)

4.



5.

Terminal

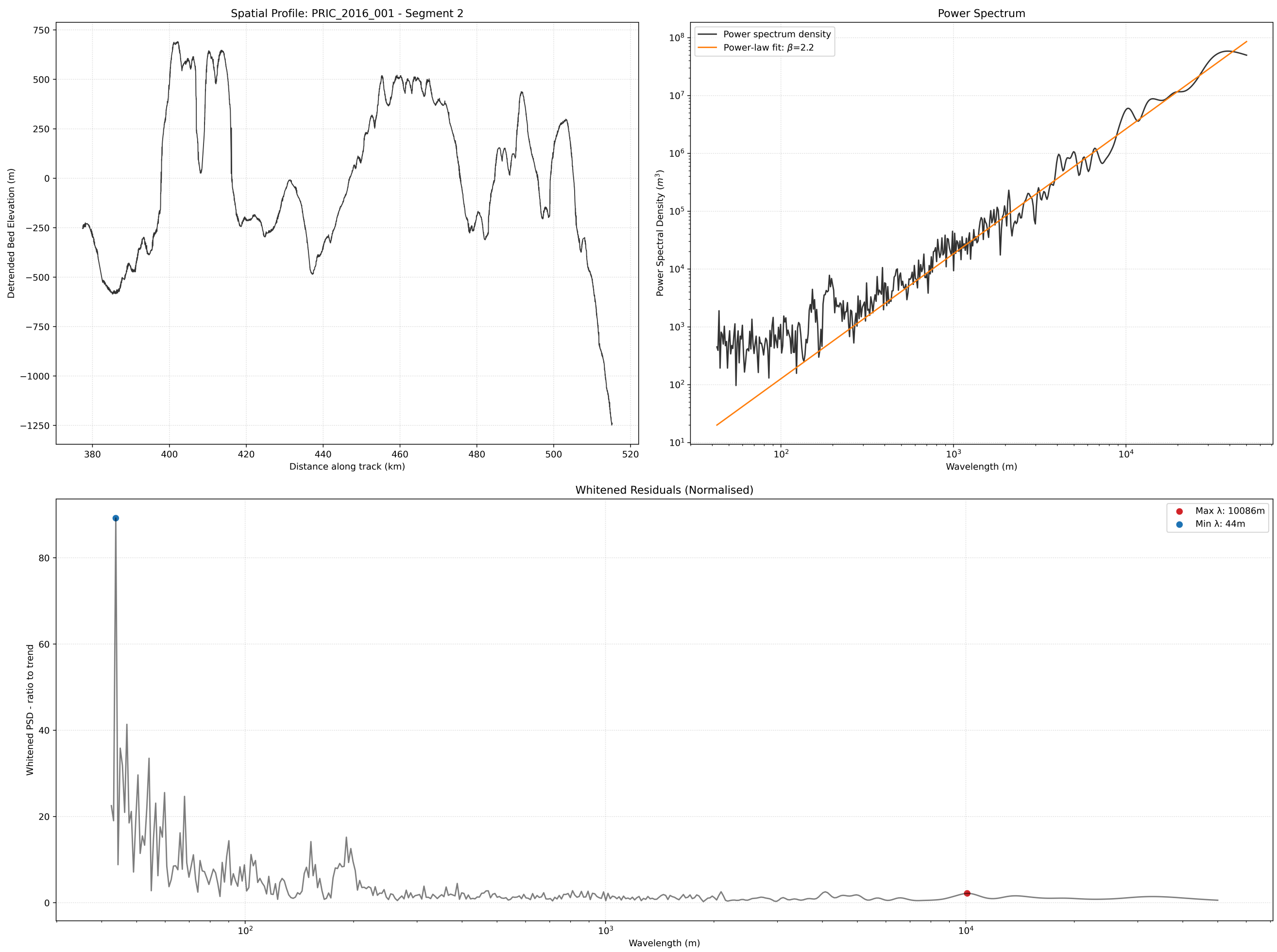
```
✓ ROSS_ICECAP loaded: 36125 rows (Filtered 11611 nulls)

Starting analysis of ROSS_ICECAP...
  Valid data points: 36125
  Unique trajectories: 1
  IR1HI2_2009033_DMC_JKB1a_WLKX10b: Found 6 gaps (>=2000m) in data
    > Segment 1: Rows 0 to 26576 (26576 points), Length: 699.01 km
    > Segment 2: Rows 26577 to 26966 (389 points), Length: 11.84 km
    > Segment 3: Rows 26967 to 30093 (3126 points), Length: 87.55 km
    > Segment 4: Rows 30094 to 34776 (4682 points), Length: 130.96 km
    > Segment 5: Rows 34777 to 35241 (464 points), Length: 12.88 km
    > Segment 6: Rows 35242 to 35491 (249 points), Length: 6.38 km
    > Segment 7: Rows 35492 to 36125 (633 points), Length: 16.13 km
7 data segments found
 > Segment 1: Valid ice thickness for count: 26576 / 26576
 >>>>>>>>>: ROSS_ICECAP | IR1HI2_2009033_DMC_JKB1a_WLKX10b | Segment 1: Oblique (incidence: 56.0)
 > Segment 2: Valid ice thickness for count: 389 / 389
 >>>>>>>>>: ROSS_ICECAP | IR1HI2_2009033_DMC_JKB1a_WLKX10b | Segment 2: Oblique (incidence: 51.9)
 > Segment 3: Valid ice thickness for count: 3126 / 3126
 >>>>>>>>>: ROSS_ICECAP | IR1HI2_2009033_DMC_JKB1a_WLKX10b | Segment 3: Perpendicular (incidence:
73.2)
 > Segment 4: Valid ice thickness for count: 4682 / 4682
 >>>>>>>>>: ROSS_ICECAP | IR1HI2_2009033_DMC_JKB1a_WLKX10b | Segment 4: Perpendicular (incidence:
69.5)
 > Segment 5: Valid ice thickness for count: 464 / 464
 >>>>>>>>>: ROSS_ICECAP | IR1HI2_2009033_DMC_JKB1a_WLKX10b | Segment 5: Perpendicular (incidence:
71.4)
```

```
 > Segment 6: Valid ice thickness for count: 249 / 249
 >>>>>>>>>: ROSS_ICECAP | IR1HI2_2009033_DMC_JKB1a_WLKX10b | Segment 6: Oblique (incidence: 44.3)
 > Segment 7: Valid ice thickness for count: 633 / 633
 >>>>>>>>>: ROSS_ICECAP | IR1HI2_2009033_DMC_JKB1a_WLKX10b | Segment 7: Oblique (incidence: 47.7)
  Trajectory IR1HI2_2009033_DMC_JKB1a_WLKX10b: 7 segments, combined median spacing = 20.9m, Nyquist =
41.8m
ROSS_ICECAP is finished processing

---
Analysed 1 regions

=== ROSS_ICECAP SUMMARY ===
------------------------------------------------------------
  RESULTS SUMMARY (1 trajectories aggregated)
------------------------------------------------------------
VERTICAL RELIEF (Max-Min):
  -> 1100.2m (Single Value)
AVG SEGMENT LENGTH:
  -> 137820.3m (Single Value)
POWER LAW EXPONENT (Beta):
  -> Mean: 2.2 | Range: [1.7, 3.0]
HURST EXPONENT:
  -> Mean: 0.6 | Range: [0.4, 1.0]
MEAN ICE THICKNESS:
  -> 2184.1m (Single Value)
............................................................
FLOW ORIENTATION (All Segments):
  -> Oblique: 4 segments (57.1%)
  -> Perpendicular: 3 segments (42.9%)

  STRATIFIED BETA (Roughness) BY ORIENTATION:
  -> Parallel       : No segments found
  -> Oblique        : Mean Beta = 2.27  (n=4)
  -> Perpendicular  : Mean Beta = 2.06  (n=3)
............................................................
CONFIRMED WAVELENGTHS (Physically valid < L/2):
  -> Count: 443
  -> Mean: 346.3m | Range: [42.0, 14725.8]m
CANDIDATE WAVELENGTHS (Statistically present > L/2):
  -> Count: 3
  -> Range: [8220m, 9882m]
............................................................
LARGEST LOCAL STRUCTURES (50km Window):
  -> Relief: 1822.9m at km 870.1
............................................................
============================================================

real    30m23.889s
user    27m35.027s
sys 2m54.729s
```

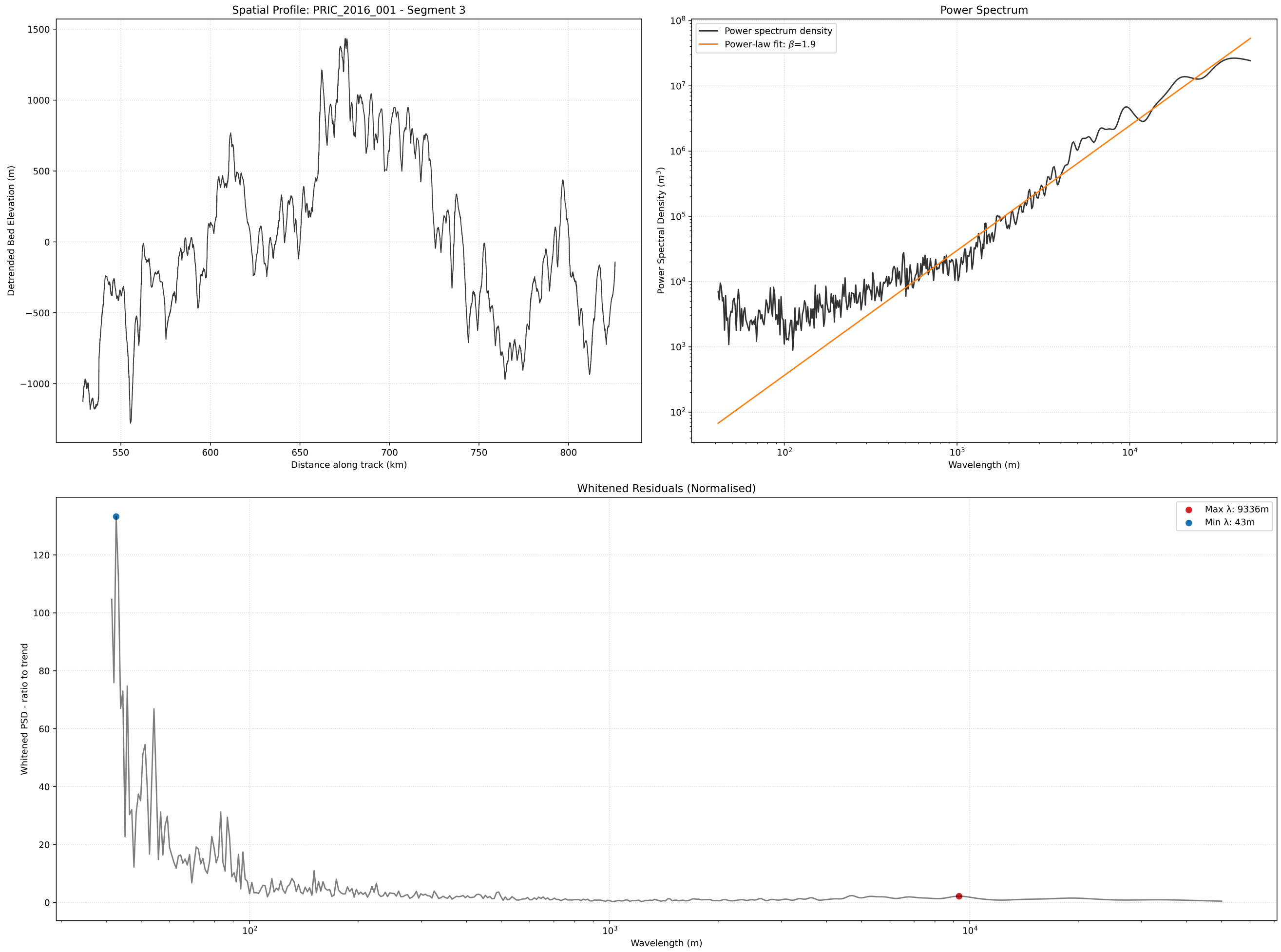
The contrast between orientation roughness confirms **bed anisotropy**:

1. Perpendicular ridging/channels
2. Oblique smoothness indicating a **soft sedimentary basin** (marine sediment deposition). The low vertical relief (1100m, the lowest of all investigated regions) further supports the idea of a flat, sediment-filled trough.

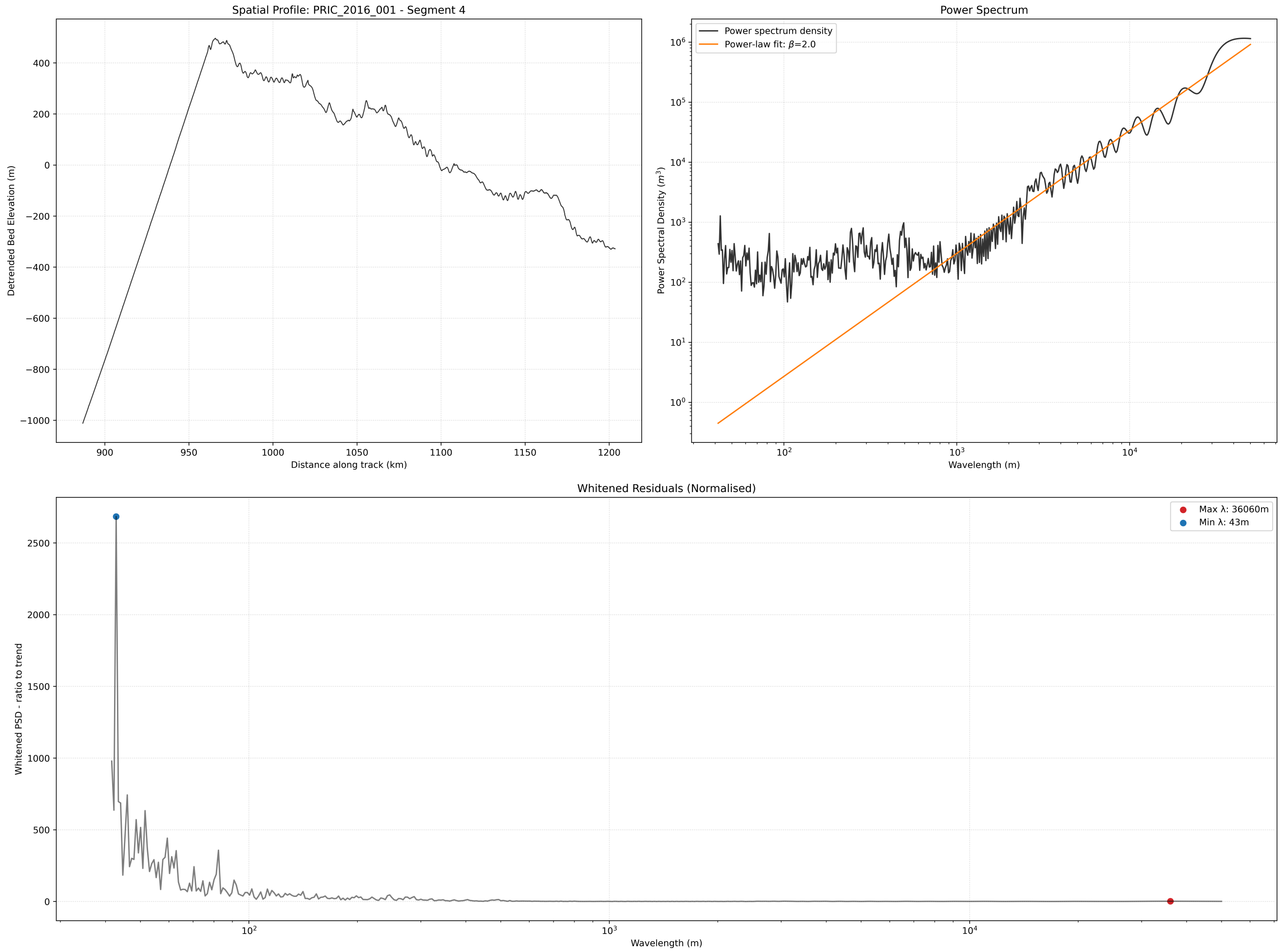
PEL region: PRIC_CHA2_2016:



2.


Spatial Profile: PRIC_2016_001 - Segment 3
Detrended Bed Elevation (m)
Distance along track (km)
Power Spectrum
Power spectrum density
Power-law fit: β=1.9
Power Spectral Density (m³)
Wavelength (m)
Whitened Residuals (Normalised)
Max λ: 9336m
Min λ: 43m
Whitened PSD - ratio to trend
Wavelength (m)

3.



4.

Terminal

```
(.venv) ana@MU00236940:~/Desktop/code/Data/Bedmap$ time python bed_analysis_15.py
✓ PEL_CHA2 loaded: 49766 rows (Filtered 4800 nulls)

Starting analysis of PEL_CHA2...
  Valid data points: 49766
  Unique trajectories: 1
  PRIC_2016_CHA2: Found 3 gaps (>=2000m) in data
    > Segment 1: Rows 0 to 14451 (14451 points), Length: 320.92 km
    > Segment 2: Rows 14452 to 20688 (6236 points), Length: 137.83 km
    > Segment 3: Rows 20689 to 34561 (13872 points), Length: 297.31 km
    > Segment 4: Rows 34562 to 49766 (15204 points), Length: 316.66 km
4 data segments found
 Skipping Segment 1: Insufficient thickness data (only 6.0% is valid)
 > Segment 2: Valid ice thickness for count: 6236 / 6236
 >>>>>>>>>: PEL_CHA2 | PRIC_2016_CHA2 | Segment 2: Oblique (incidence: 44.4)
 > Segment 3: Valid ice thickness for count: 13486 / 13872
 >>>>>>>>>: PEL_CHA2 | PRIC_2016_CHA2 | Segment 3: Oblique (incidence: 32.7)
 > Segment 4: Valid ice thickness for count: 15204 / 15204
 >>>>>>>>>: PEL_CHA2 | PRIC_2016_CHA2 | Segment 4: Oblique (incidence: 32.6)
  Trajectory PRIC_2016_CHA2: 4 segments, combined median spacing = 20.9m, Nyquist = 41.9m
PEL_CHA2 is finished processing

---
Analysed 1 regions

=== PEL_CHA2 SUMMARY ===
------------------------------------------------------------
```

```
  RESULTS SUMMARY (1 trajectories aggregated)
------------------------------------------------------------
VERTICAL RELIEF (Max-Min):
  -> 2194.1m (Single Value)
AVG SEGMENT LENGTH:
  -> 250601.8m (Single Value)
POWER LAW EXPONENT (Beta):
  -> Mean: 2.0 | Range: [1.9, 2.2]
HURST EXPONENT:
  -> Mean: 0.5 | Range: [0.5, 0.6]
MEAN ICE THICKNESS:
  -> 1696.7m (Single Value)
............................................................
FLOW ORIENTATION (All Segments):
  -> Oblique: 3 segments (100.0%)

  STRATIFIED BETA (Roughness) BY ORIENTATION:
  -> Parallel        : No segments found
  -> Oblique         : Mean Beta = 2.04  (n=3)
  -> Perpendicular   : No segments found
............................................................
CONFIRMED WAVELENGTHS (Physically valid < L/2):
  -> Count: 162
  -> Mean: 582.3m | Range: [42.6, 36575.8]m
............................................................
LARGEST LOCAL STRUCTURES (50km Window):
  -> Relief: 1817.3m at km 628.7
............................................................
============================================================

real	14m32.023s
user	13m13.627s
sys	1m25.239s
```

Highest **Vertical Relief (2143m)** and the largest detected local structure (1817m relief in a single window). The trajectory crosses a Mountain range?

Appendix

(Note that **Skewness** and **Kurtosis** are currently not being reported in final results and I am not really using them at all... these are for future usage.)

Skewness

`skewness` measures the asymmetry of a distribution.

1. Detrending: signal.detrend(elev) removes the linear trend from bedrock elevation data
2. Skewness calculation: stats.skew(detrended) computes skewness using `stats.skew`

How does `stats.skew` work?

Skewness is based on the **3rd standardized moment**

1. Calculate the Mean ($\bar{x}$):
   First, it finds the average elevation.

$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

2. Calculate the 3rd Moment (m_3):
   Next, for each data point, find the difference from the mean and cube it.
   Find average for all cubed differences

   $m_3 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^3$

   - **Why cube it?** Cubing preserves the sign. If a point is *below* the mean ($(x_i - \bar{x})$ is negative), its cubed value is still negative. This is what allows the final number to be positive or negative, telling you the *direction* of the skew.
3. Standardize It:
   The m_3 value's size depends on the units (e.g., meters). To get a universal, unit-less number, it's "standardized" by dividing it by the standard deviation cubed (σ^3).
   - First, find the standard deviation:

     $\sigma = \sqrt{\frac{1}{n}\sum(x_i - \bar{x})^2}$
   - Then, divide:

     Skewness $= \frac{m_3}{\sigma^3}$
4. The scipy.stats Part (Bias Correction):
   The formula above is a "biased" estimator. By default, stats.skew sets bias=True, which is not our preference, since we want to apply a mathematical correction factor to give a more accurate, unbiased answer for a sample of data. We therefore set it as FALSE. This correction formula is:

   $\text{Unbiased Skewness} = \left(\frac{\sqrt{n(n-1)}}{n-2}\right)\left(\frac{m_3}{s_{n-1}^3}\right)$

   (Note: It uses the $n-1$ version of standard deviation, s_{n-1}, but the core idea is the 3rd moment divided by the standard deviation cubed, all multiplied by a correction factor based on n.)

Kurtosis

`kurtosis` measures the "tailedness" or peakedness of a distribution.
`stats.kurtosis(detrended)` computes excess kurtosis using `stats.kurtosis`

How `stats.kurtosis` works?

Kurtosis is based on the **4th standardized moment**.

1. Calculate the Mean ($\bar{x}$) and Standard Deviation (s):

   Same as before.
2. Calculate the 4th Moment (m_4):

   This time, it finds the difference from the mean and raises it to the 4th power. It then finds the average.

   $m_4 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^4$

   - **Why the 4th power?** This is the key. Because it's an even power, all values become positive. But more importantly, it *massively amplifies* the effect of outliers. A point that is 2 units from the mean contributes $2^4 = 16$, while a point 3 units away contributes $3^4 = 81$. This makes the final number highly sensitive to the "fat tails" or "spikes" in the data.
3. Standardize It:
   It divides by the standard deviation to the 4th power (s^4):

   $\text{Standard Kurtosis} = \frac{m_4}{s^4}$
4. The scipy.stats Part (Excess Kurtosis):
   This is the most important concept. A perfect, normal "bell curve" has a standardized kurtosis of exactly 3.
   Statisticians find it more useful to measure kurtosis relative to this bell curve. So, they define "Excess Kurtosis" as:

$$\text{Excess Kurtosis} = \left(\frac{m_4}{s^4}\right) - 3$$

The `scipy.stats.kurtosis` function calculates this **Excess Kurtosis** by default (this is also called Fisher's definition).
- This is why a value of **0** means "normal bell curve."
- A **positive** value means "spikier than a bell curve."
- A **negative** value means "flatter than a bell curve."
Just like skewness, this function also has the option set to `bias=False` correction by default to make the number more accurate for a sample.

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