5. Transfer Function Analysis

determines how to measure the displacement of the ice surface profile relative to the amplitude and phase of the bedrock. In the 3. flowline_synthetic.py simulations it can observed that bedrock perturbations of a given wavelength can effectively transfer their profile onto the steady-state surface shape.

The **damping factor** (ψ) from Budd (1970) is the ratio of basal to surface amplitudes.

$$\psi = A_b / A_s$$

Budd also presents an equation for the damping factor which includes a viscosity parameter. The damping factor increases with the square of the ice thickness and is inversely proportional to velocity and the viscosity parameter. It also notes that the damping factor has a frequency dependence with a minimum for wavelengths between 3 and 4 times the ice thickness.
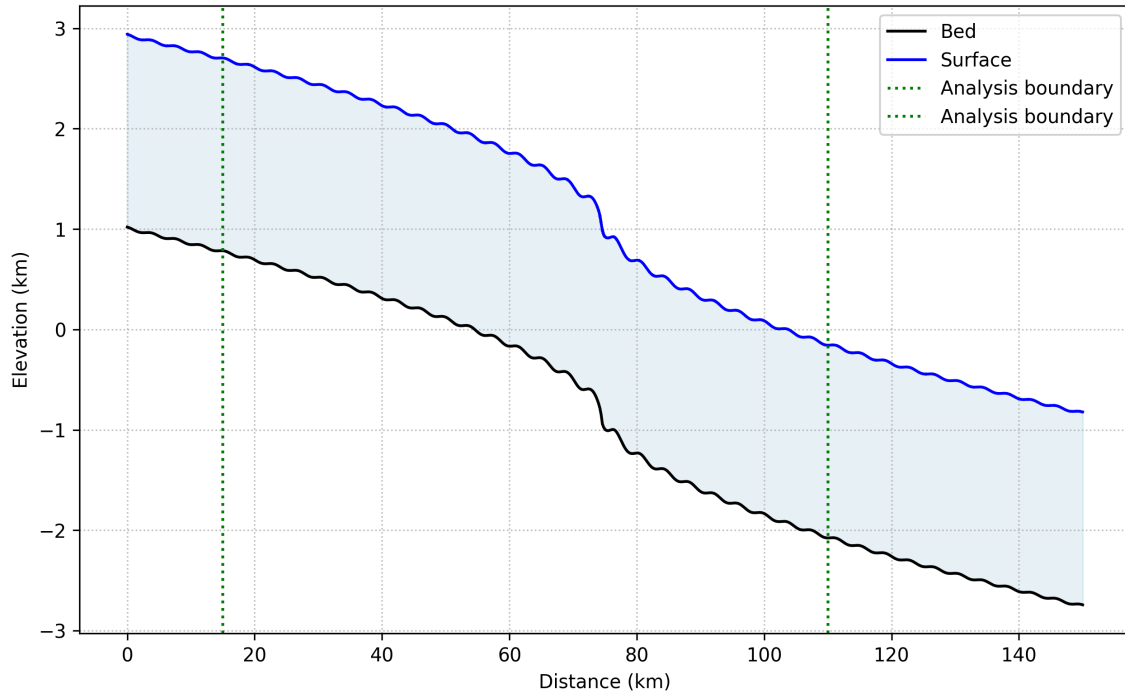
Page 46 in Budd (1970) mentions that Equation (4.34) for ψ may be considered a "transfer function" or "filter" for calculating the surface profile from the bedrock profile.
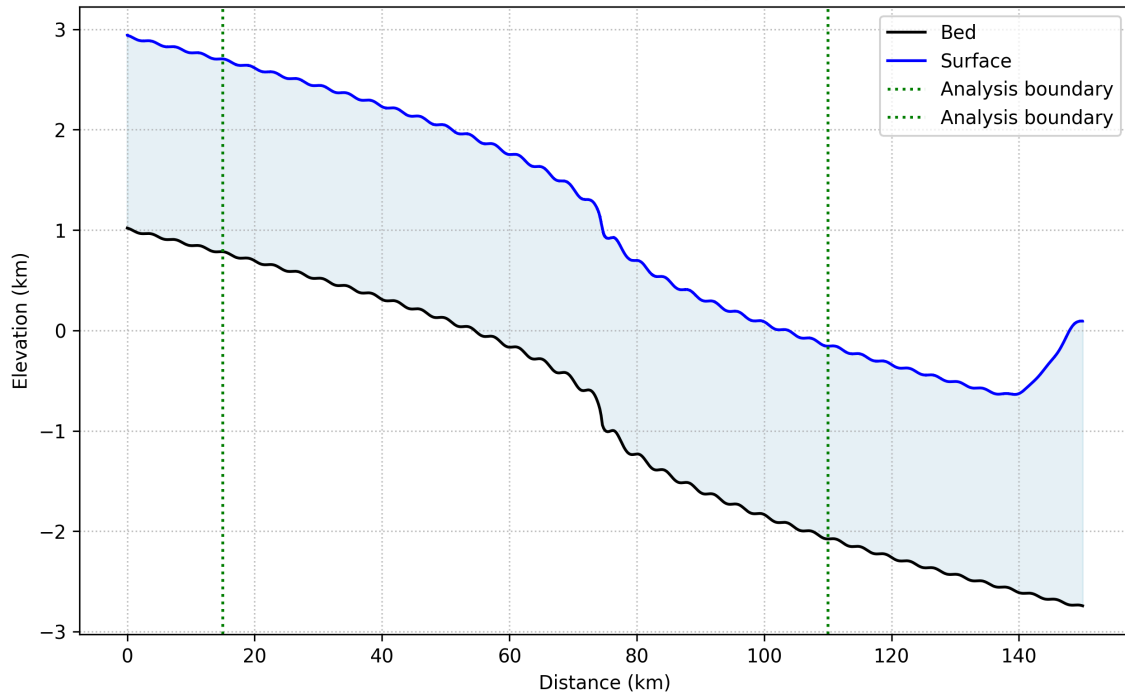
Spectral Analysis Approach

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1. Fourier transform both bed and surface profiles
2. Calculate the transfer function as the ratio of bed to surface spectral
amplitudes
3. Extract phase information to identify any shifts
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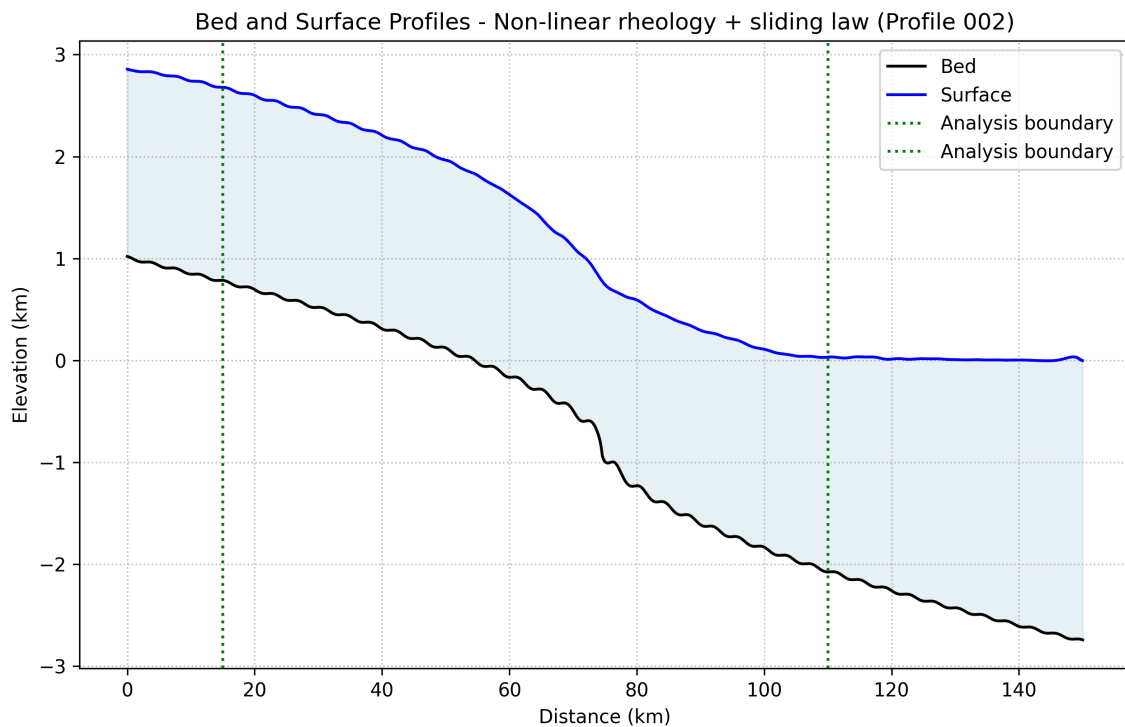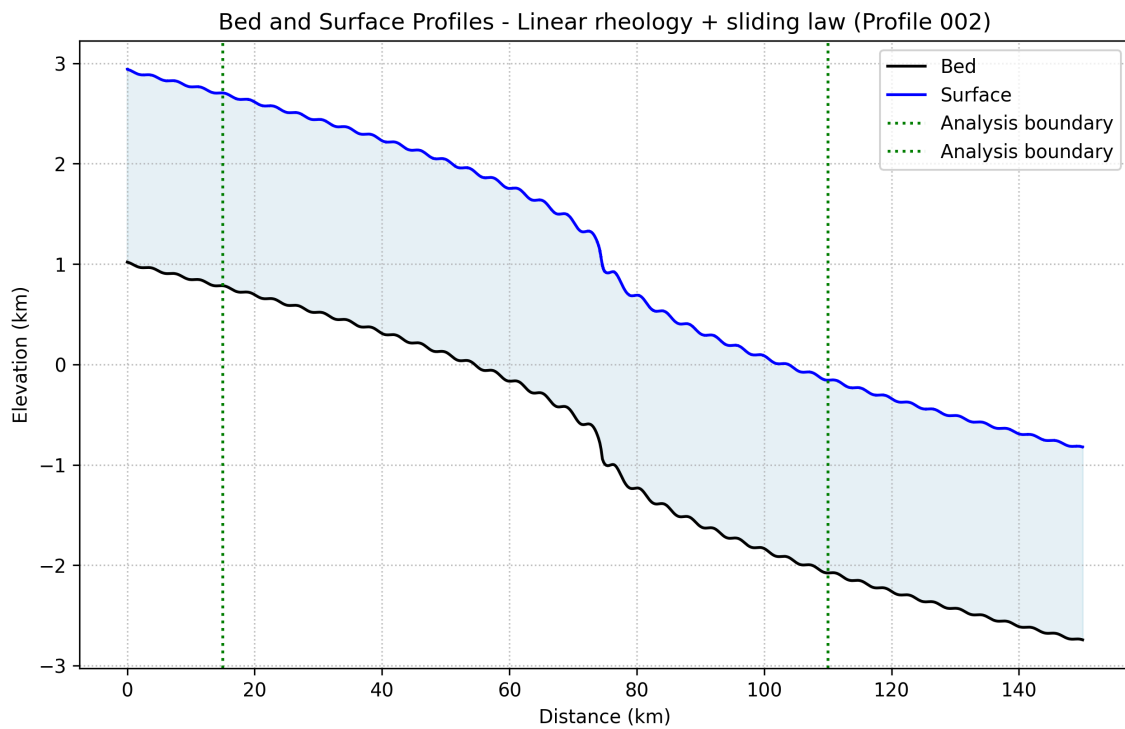
Regions used for analysis

Bed and Surface Profiles - Linear rheology + frozen bed (Profile 002)

Bed and Surface Profiles - Non-linear rheology + frozen bed (Profile 002)

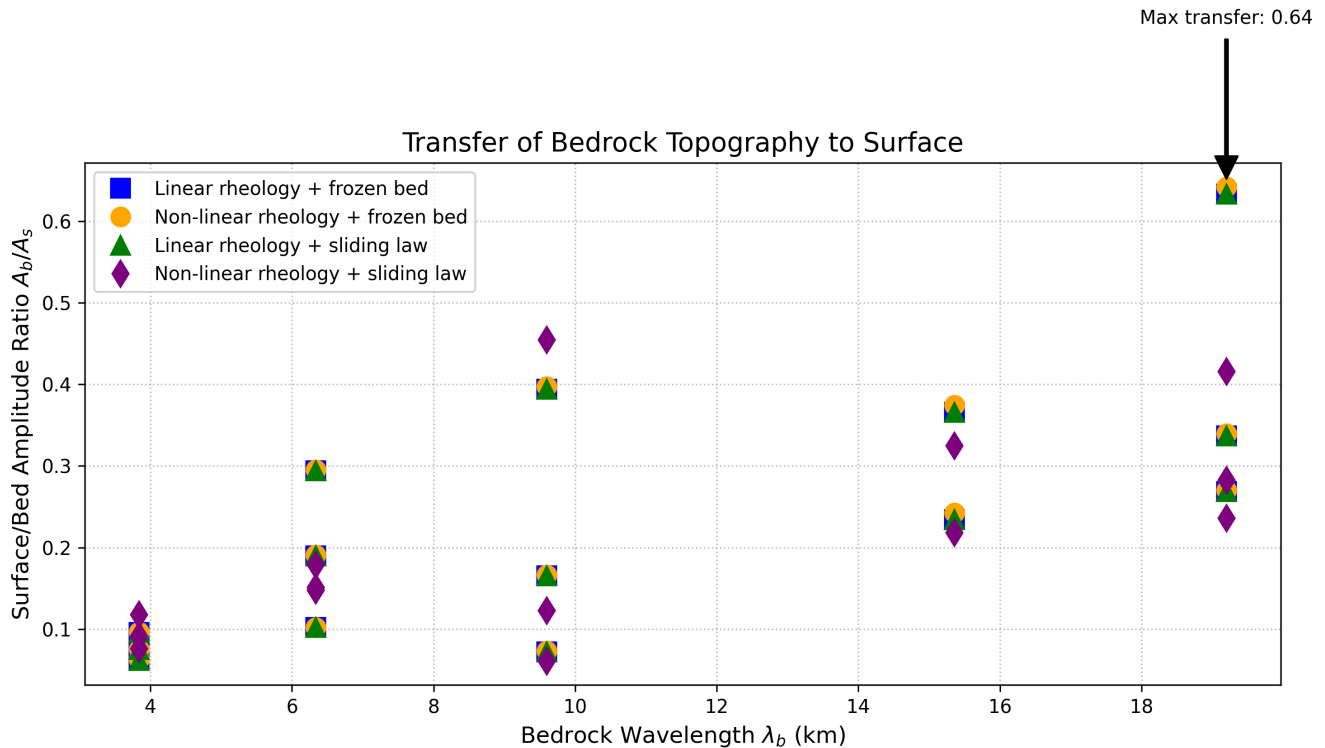**Bed and Surface Profiles - Linear rheology + sliding law (Profile 002)**

Bed and Surface Profiles - Non-linear rheology + sliding law (Profile 002)

Non-linear rheology results in a highly deformed terminus.

Results

Wavelength transfer

Transfer of Bedrock Topography to Surface

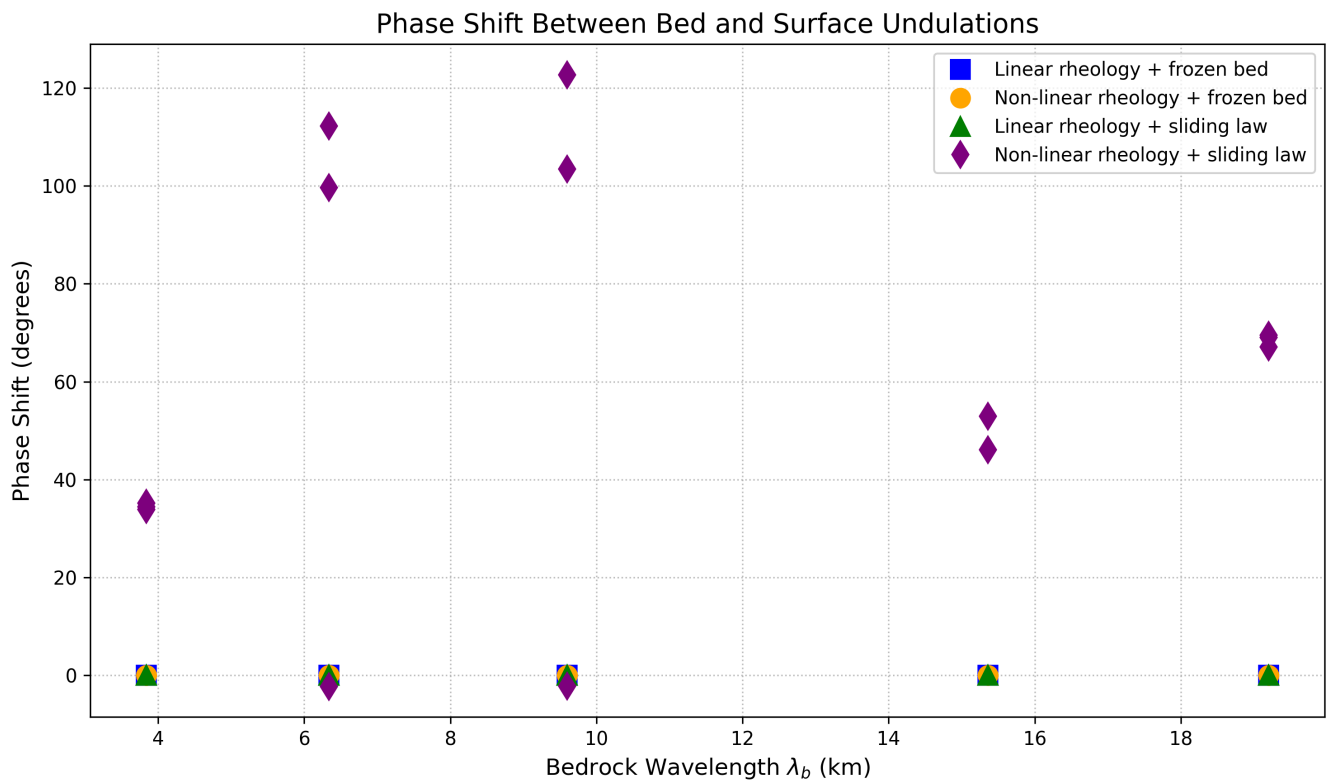Fraction of the bedrock's elevation profile that is visible at the ice surface.
The **linear rheology models** show a maximum ratio with at best only 57% of the bedrock amplitude appearing at the surface.

NOTE THAT Vertical Spread in Data is large

The linear rheology cases show identical behavior regardless of basal conditions (frozen vs sliding) means that there is nuance to Budd's theory.

Budd predicted minimum damping (maximum transfer) at wavelengths of approximately 3.3× ice thickness. But these results suggest more complex patterns, with maximum transfer occurring at longer wavelengths, especially for linear rheology cases.
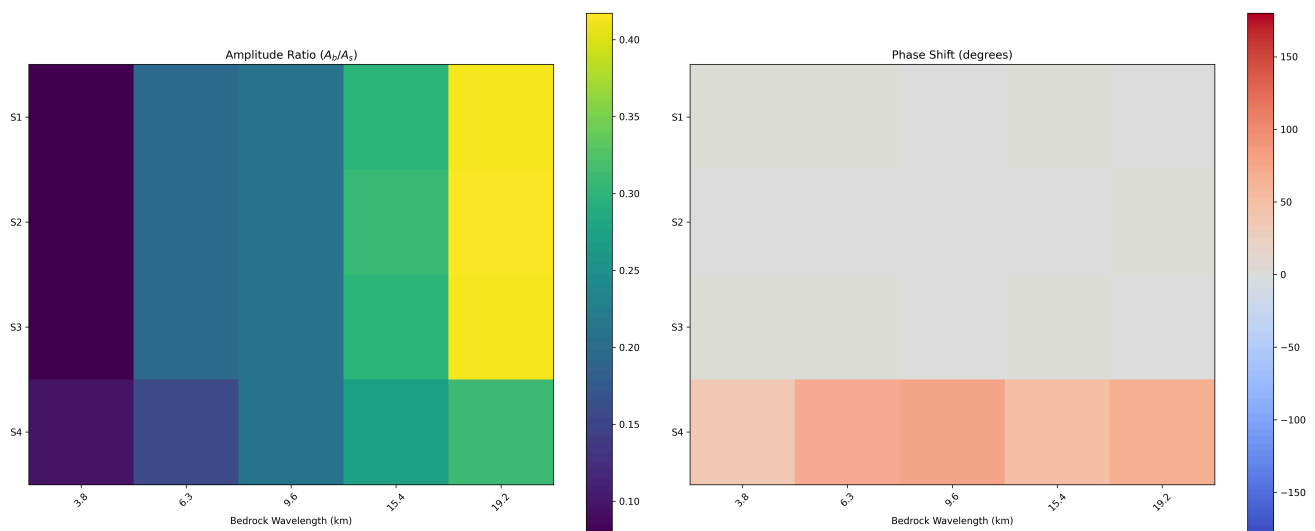
Phase shift

Phase Shift Between Bed and Surface Undulations

Non-linear rheology + SLIDING creates a more complex stress transmission pattern where bed features influence the surface at a downstream location.

Some non-linear rheology cases show negative phase shifts (around -55° to -75°), These correspond to profiles 50 and 53 at wavelengths of 6.3km and 9.6km correspondingly. 🪐 WHAT IS HAPPENING this isn't directly addressed in Budd's theory.

Matrices

Amplitude ratio

Damping occurs in all models

The S1, S2, S3 models demonstrate the maximum amplitude ratio.

Phase shift

ONLY S4 shows phase shift. However Budd predicted that "the phase shift between surface and bed slopes should be approximately π/2 (90 degrees)." and he didn't specify that this was exclusive to a non-linear case. In these results linear rheology cases show almost no phase shift. while only your non-linear rheology + sliding cases show significant phase shifts (30°-120°).

> ✏️ **WHAT WE EXPECT from Budd's Theory**
>
> ☐ The phase shift between surface and bed slopes should be approximately $\frac{\pi}{2}$ (90 degrees).
>
> ☑ ~~The bedrock slope amplitude (A_b) should be larger than the surface slope amplitude (A_s)~~
>
> ☐ The ratio between them should be minimum when the wavelength is ≈ 3.3 times the ice thickness.
>
> ☐ Very long and very short waves are expected to be heavily damped.

> ✏️ **The questions:**
>
> 1. Is the common assumption of linear flow (n=1) sufficient for bedrock topography reconstruction?
> 2. How do different rheological assumptions affect the relationship between bed features and surface expressions?
> 3. Should I use another kind of basal boundary condition?
> 4. How does rheology impact the need for iteration in inverse modeling to reduce residuals?