✡ · why — compress names in unsrt_mod?

· edit Refs

2. Want to talk about proj structure for lit Rev.

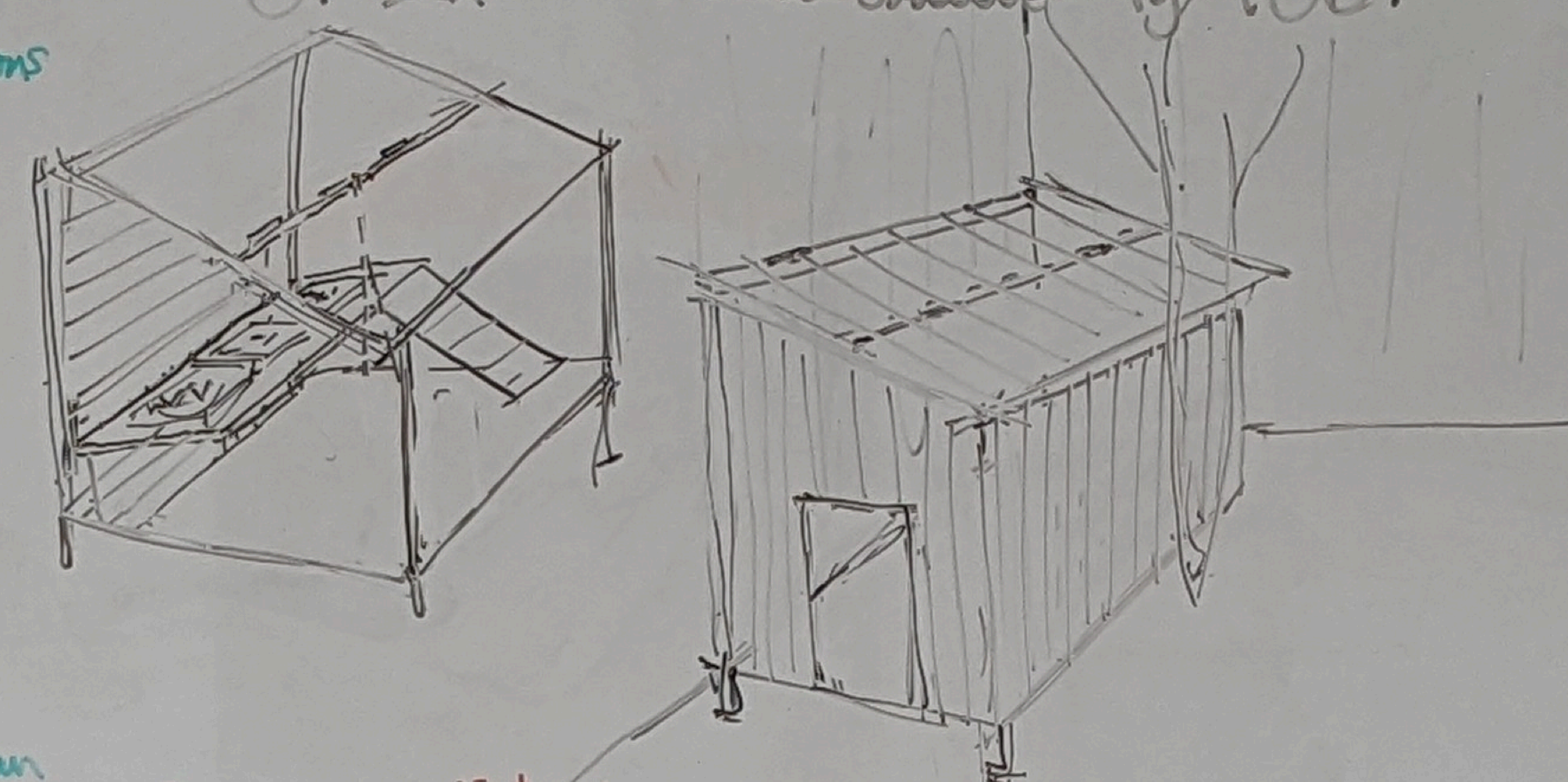
1. Open systems: Ellena Ostrovskaya

0. 2x2 Hamiltonian Pg 986.

OTHER Queries

· Dual Hamiltonians ?

· Parity anomaly deets



(1) Want to verify whether RK4 & $\hat{U}$ time evolutions being distinct for the HH makes sense

I don't think evolutions for HH should differ between them.

a.- Constructing $\hat{U}$:

we know from Bender's review that:

the unitary: $\hat{U} = e^{-i\hat{H}t}$ applies to ub. PT-symmetric $\hat{H}$.

pseudo-Hermicity

∃ a hermitian η: $\hat{H}^\dagger = \eta^\dagger \hat{H} \eta$ ⇒ $\hat{H}$ is pseudo Hermitian

intertwining operator

Mostafazadeh: $\eta = P$

Then can we write: $\hat{U}' = \eta^\dagger \hat{U} \eta$?

$\therefore \hat{U}'|\psi(0)\rangle = \eta^\dagger \hat{U} \eta \eta^\dagger |\psi(0)\rangle$

$\hat{U}(t) = \sum_n |n\rangle e^{-iE_n t} \langle n|$

what basis

How am I writing $\hat{U}$?

$M_{ij} = \langle \psi_i | HH | \psi_j \rangle$

$U = \text{expm}(-iMt)$

2. Want to talk about proj structure for lit Rev.

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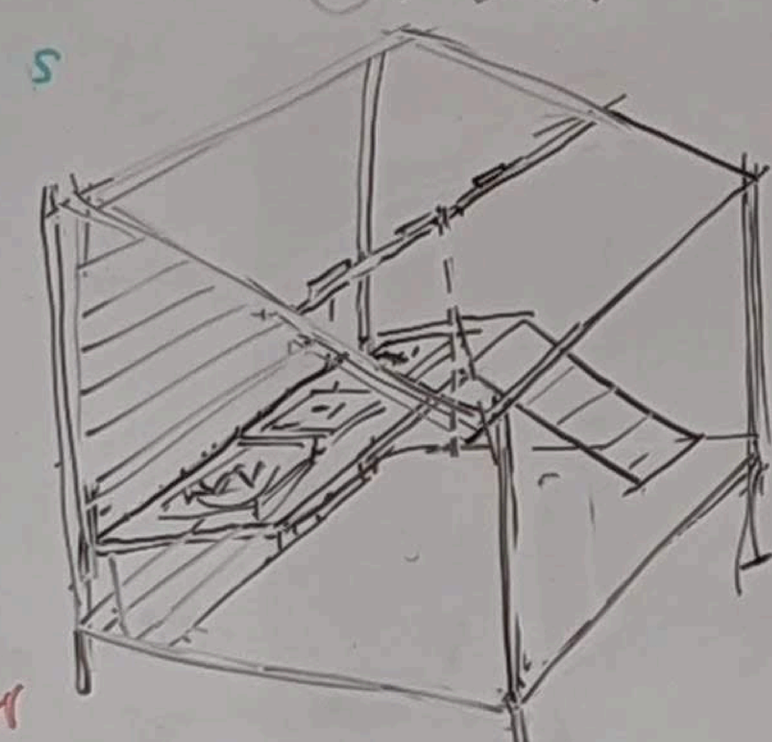
The CPT inner-product.

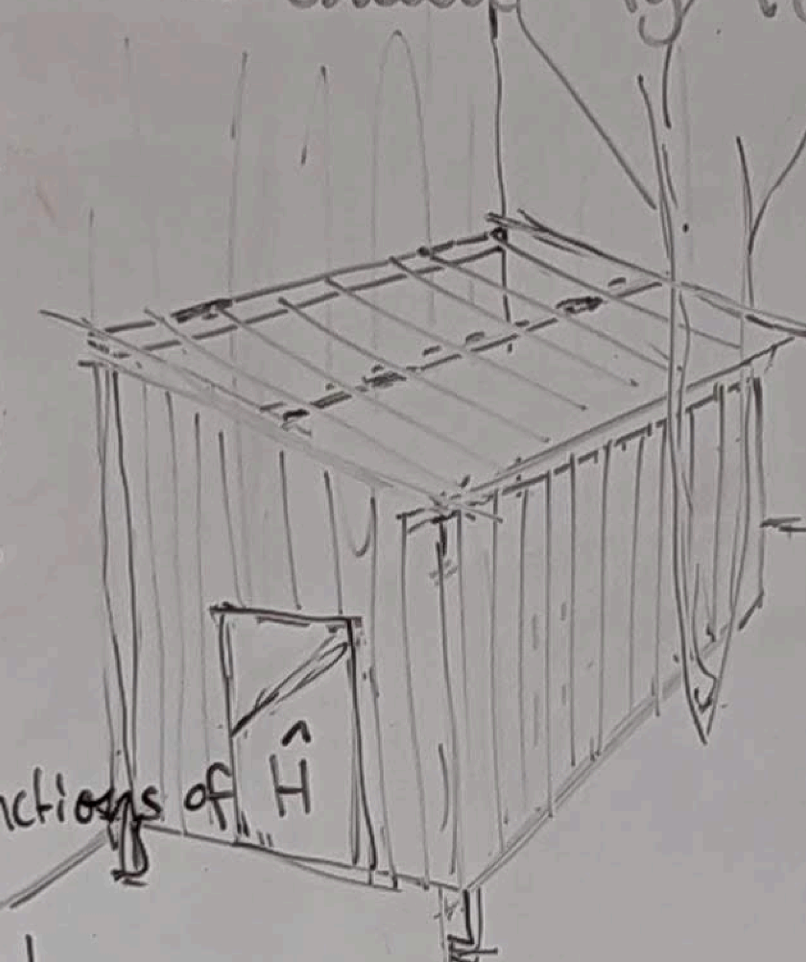
$[C, H] = 0$ — Commutation represents a symmetry

$[C, PT] = 0$

C acts similarly to the charge conjugation operator

$$\langle \psi | \chi \rangle^{CPT} = \int dx \, \underbrace{\psi^{CPT}(x)}_{\psi^{CPT}(x) = \int dy \, C(x,y) \psi^*(-y)} \chi(x)$$





from constructing C

→ CPT inner product is:

phase independent

positive definite norm

$$C = \sum_n \underbrace{|n\rangle \langle n|}_{\text{eigenfunctions of } \hat{H}}$$

$\hat{H}$'s eigenfunctions

$C = \sum_n |\psi_n\rangle \langle\psi_n|$

about proj structure for it

1. Open systems: Ellena Ostrovskaya

0. 2x2 Hamiltonian - Pg 986.

OTHER Queries

- Dual Hamiltonians
- parity anomaly deals

Normalization of $|\psi_n\rangle$

iff unbroken PT-symmetry then $[\hat{H}, PT] = 0$

$PT : |n\rangle$ has eigenvalues $\lambda = e^{i\alpha}$ α, λ depend on n

our normalised eigenfunctions are $\phi_n(x) \equiv e^{-i\alpha/2}\psi_n(x)$

$\Rightarrow \phi_n(x)$ is simultaneously an eigenfunction of $\hat{H}$ & PT (with e'value 1)

The claim is that: $\mathrm{sgn}(|\phi_n(x)|^2) = \underbrace{(-1)^n}_{\text{This is why we want to use the C operator \& the CPT inner product}}$ $\forall n, \forall \epsilon > 0$

about proj structure for lit Rev.

OTHER Queries

- Dual Hamiltonians ?
- parity anomaly deals

$\langle \psi, \varphi \rangle$

1. Open systems: Ellena Ostrovskaya

0. 2x2 Hamiltonian → Pg 986

∞. write to CMBender.

see Pg 979 (MSNHH)

Verify mathematically PT inner product is $(-1)^n$

Verify C operator solves this.

$\boxed{\lambda = e^{i\alpha}}$ n

eigenstates $\hat{H}, PT$

? PT normalized to make $(-1)^n$

$\phi_n(x) = e^{-i\alpha/2} \psi_n(x)$

- PT norm $\neq$ Probability density

$$(\psi_n, \psi_m) \equiv \int_C dx\, [\psi_n(x)]^{PT} \psi_m(x) = \int_C dx\, (\psi_n(-x))^* \psi_m(x)$$

$$C = \sum_n |\phi_n\rangle\langle\phi_n| \,?$$

eigenfuncs: $\psi(x)$

$H = H^{PT}$

$H = \frac{p^2}{2m} + x^2(ix)^\epsilon$

$|\psi\rangle$

CPT inner prod.

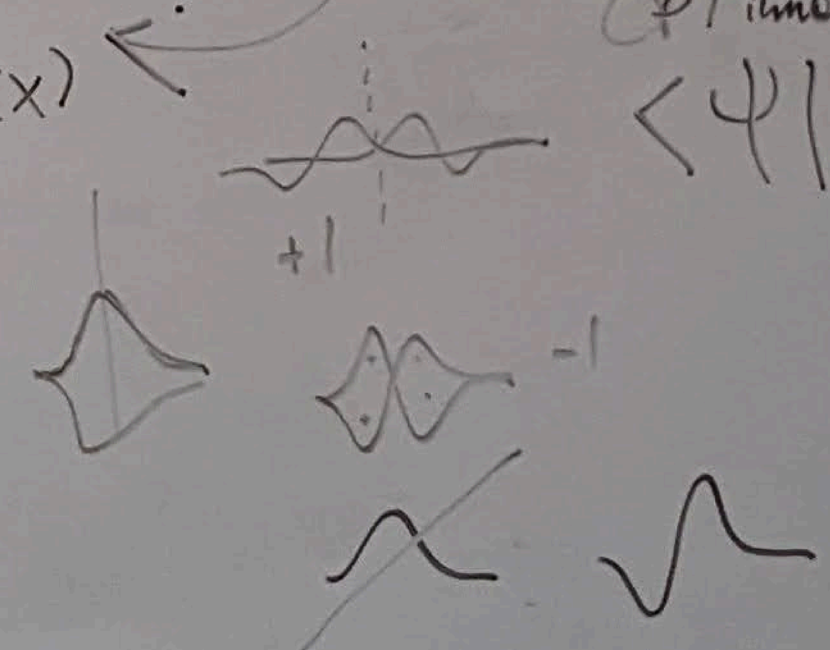
$$\langle \psi | \chi \rangle^{CPT} = \int dx\, \underbrace{\psi^{CPT}(x)}_{\int dy\, C(x,y)\, \psi^*(-y)} \chi(x)$$

$$\text{Sign}\left(|?_n(x)|^2 \right) = (-1)^n = \int_C dx\, (\phi(-x))^* \phi(x)$$

?

is this ψ or ϕ

$\langle \phi_n | \hat{x} | \phi_m \rangle$



* phase related?

Hypoth: This occurs for ψ_n

about proj structure for lit Rev.

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$\langle \psi, \varphi \rangle$

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$$C = \sum_n |\phi_n\rangle\langle\phi_n| \stackrel{?}{=}$$

eigenfuncs: $\psi(x)$

$$H = H^{PT}$$

$$H = \frac{p^2}{2m} + x^2(ix)^\epsilon$$

$|\psi\rangle$

CPT inner prod:

$$\langle\psi|\chi\rangle^{CPT} = \int dx\,\psi^{CPT}(x)\chi(x)$$

$$\int dy\, C(x,y)\,\psi^*(-y)$$

$$\text{Sign}\left(|?_n(x)|^2\right) = (-1)^n = \int_C dx\,(\phi(-x))^*\phi(x) \; ?$$

is this ψ or ϕ

$\langle\phi_n|\hat{x}|\phi_m\rangle$

+1 -1

* phase related ?

Hypoth: This occurs for ψ_n

New refs:

[2] Bender 2005 Contemp. Phys 46 277
[3] Bender, Boettcher 1998 Phys. Rev. Letter 80 5243: Runga-Kutta 4?
[10] Scholtz et al 1992 Ann. Phys. 213 74 : Quasi Hermiticity
[44] Ahmed et al 2005 J. Phys. A : Math. Gen 38 L627: Analysis $(-x^4)$
[64] Mostafazadeh, Batal 2004 J. Phys. A : Math. Gen 37 11645 : $\eta = P$

* Check Pablo's links

$$PT\hat{H} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} T \begin{pmatrix} re^{i\theta} & s \\ s & re^{-i\theta} \end{pmatrix} \qquad r, \theta, s \in \mathbb{R}$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} re^{-i\theta} & s \\ s & re^{i\theta} \end{pmatrix}$$

$$= \begin{pmatrix} s & re^{i\theta} \\ re^{-i\theta} & s \end{pmatrix}$$

$$PT\hat{H}PT = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} T \begin{pmatrix} re^{i\theta} & s \\ s & re^{-i\theta} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} T$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} re^{-i\theta} & s \\ s & re^{i\theta} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} T$$

$$= \begin{pmatrix} s & re^{i\theta} \\ re^{-i\theta} & s \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} T$$

$$= \begin{pmatrix} re^{i\theta} & s \\ s & re^{-i\theta} \end{pmatrix} T$$

M. Swanson

WAT $\begin{cases} \text{dual space} \\ \text{Complexification (Bender)} \leftarrow \text{This is} \\ \text{Bessel functions} \\ \text{Bogoliubov transformation} \end{cases}$

$$H = \alpha a^2 + \beta a^{\dagger 2} + \omega a^\dagger a + \frac{1}{2}\omega$$

$\alpha, \beta, \omega \in \mathbb{R}$ (inverse time parameters?)

$a, a^\dagger$ bosonic creation & anihilation operators (HO)

$[a, a^\dagger] = 1$, $a|0\rangle = 0 \Rightarrow \langle 0| a^\dagger = 0$, $|n\rangle = \frac{a^{\dagger n}}{\sqrt{n!}}|0\rangle$

Eigenvalues : $\omega^2 \geq 4\alpha\beta$

& States are orthonormal

< complete

indefinite metric $\rightsquigarrow$ modifiying Inner Product through the action of an operator

Hamiltonian

Similar to charge operator in particle phys. $\rightsquigarrow$ CPT :

$$\langle f | g \rangle = \int_C dx \, [CPT f(x)] \, g(x)$$

dynamically determined from

C is contour in $\mathbb{C}$

How?

$$PT\hat{H} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} T \begin{pmatrix} re^{i\theta} & s \\ s & re^{-i\theta} \end{pmatrix} \qquad r, \theta, s \in \mathbb{R}$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} re^{-i\theta} & s \\ s & re^{i\theta} \end{pmatrix}$$

$$= \begin{pmatrix} s & re^{i\theta} \\ re^{-i\theta} & s \end{pmatrix}$$

$$PT\hat{H}PT = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} T \begin{pmatrix} re^{i\theta} & s \\ s & re^{-i\theta} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} T$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} re^{-i\theta} & s \\ s & re^{i\theta} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} T$$

$$= \begin{pmatrix} s & re^{i\theta} \\ re^{-i\theta} & s \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} T$$

$$= \begin{pmatrix} re^{i\theta} & s \\ s & re^{-i\theta} \end{pmatrix} T$$

Pseudo Hermiticity

$$H^\dagger = PHP$$

If I redefine $\hat{C}_1, \hat{C}_2$ to

$$C_1 = \frac{1}{\sqrt{2\cos\alpha}} \begin{pmatrix} e^{i\alpha/2} \\ e^{-i\alpha/2} \end{pmatrix} \quad \& \quad C_2 = \frac{1}{\sqrt{2\cos\alpha}} \begin{pmatrix} e^{-i\alpha/2} \\ -e^{i\alpha/2} \end{pmatrix}$$

$$(C_1, C_1) = \int dx \, \left[C_1(x) \right]^{PT} C_1(x) \quad , x \in \mathbb{R}$$

$$\left(\langle x | C_1 \rangle \right)^{PT} \langle x | C_1 \rangle$$

$$\langle C_1 | -x \rangle \langle x | C_1 \rangle = \langle x | \underbrace{C_1 \rangle \langle C_1}_{} | -x \rangle$$

$$\begin{pmatrix} e^{i\alpha/2} \\ e^{-i\alpha/2} \end{pmatrix} \left(e^{i\alpha/2}, e^{-i\alpha/2} \right) = \begin{pmatrix} e^{i\alpha} & 1 \\ 1 & e^{-i\alpha} \end{pmatrix}$$

$$(x_1, x_2) \begin{pmatrix} e^{i\alpha} & 1 \\ 1 & e^{-i\alpha} \end{pmatrix} \begin{pmatrix} -x_1 \\ -x_2 \end{pmatrix} \overset{\text{Naive}}{=} \left(x_1 e^{i\alpha} + x_2 \, , \, x_1 + x_2 e^{-i\alpha} \right) \begin{pmatrix} -x_1 \\ -x_2 \end{pmatrix} = -x_1^2 e^{i\alpha} - 2x_2 x_1 - x_2^2 e^{-i\alpha}$$

If I redefine $\hat{C}_1, \hat{C}_2$ to

$$C_1 = \frac{1}{\sqrt{2\cos\alpha}}\begin{pmatrix} e^{i\alpha/2} \\ e^{-i\alpha/2} \end{pmatrix} \; \& \; C_2 = \frac{1}{\sqrt{2\cos\alpha}}\begin{pmatrix} e^{-i\alpha/2} \\ -e^{i\alpha/2} \end{pmatrix}$$

$$|\uparrow_z\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |\downarrow_z\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

projection onto $\{|\uparrow_z\rangle, |\downarrow_z\rangle\}$ since this basis spans $\mathbb{C}P^1$

$$\langle \uparrow_z | C_1 \rangle = \frac{1}{\sqrt{2\cos\alpha}}(1, 0)\begin{pmatrix} e^{i\alpha/2} \\ e^{-i\alpha/2} \end{pmatrix} = \frac{1}{\sqrt{2\cos\alpha}} e^{i\alpha/2}$$

$$\langle \downarrow_z | C_1 \rangle = \frac{1}{\sqrt{2\cos\alpha}} e^{-i\alpha/2}$$

$$\langle \uparrow_z | C_2 \rangle = \frac{1}{\sqrt{2\cos\alpha}} e^{-i\alpha/2}$$

$$\langle \downarrow_z | C_2 \rangle = \frac{-1}{\sqrt{2\cos\alpha}} e^{i\alpha/2}$$

$$\therefore E_+ = \begin{pmatrix} \langle \uparrow | C_1 \rangle \\ \langle \downarrow | C_1 \rangle \end{pmatrix} = \frac{1}{\sqrt{2\cos\alpha}}\begin{pmatrix} e^{i\alpha/2} \\ e^{-i\alpha/2} \end{pmatrix}$$

$$\therefore E_- = \begin{bmatrix} \langle \uparrow | C_2 \rangle \\ \langle \downarrow | C_2 \rangle \end{bmatrix} = \frac{1}{\sqrt{2\cos\alpha}}\begin{pmatrix} e^{-i\alpha/2} \\ -e^{i\alpha/2} \end{pmatrix}$$

PT-INNer Product

$$[|E_+\rangle]^{PT}(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|)|E_-\rangle = (\langle E_+|)^{P}|\uparrow\rangle\langle\uparrow|E_-\rangle + (\langle E_+|)^{P}|\downarrow\rangle\langle\downarrow|E_-\rangle$$

P How does the parity operator act on spin?

$$\vec{E}_+ = \frac{1}{\sqrt{2\cos\alpha}}\begin{pmatrix} e^{i\alpha/2} \\ e^{-i\alpha/2}\end{pmatrix} \quad \& \quad \vec{E}_- = \frac{1}{\sqrt{2\cos\alpha}}\begin{pmatrix} e^{-i\alpha/2} \\ -e^{i\alpha/2}\end{pmatrix}$$

$$|\uparrow_z\rangle = \begin{pmatrix}1\\0\end{pmatrix}, \quad |\downarrow_z\rangle = \begin{pmatrix}0\\1\end{pmatrix} \quad \rightsquigarrow \quad P = \begin{pmatrix}0&1\\1&0\end{pmatrix}$$

$$(\vec{E}_+,\vec{E}_+) = \frac{1}{2\cos\alpha}(PTE_+)^T E_+ = \frac{1}{2\cos\alpha}\left(\begin{pmatrix}0&1\\1&0\end{pmatrix} T \begin{pmatrix}e^{i\alpha/2}\\e^{-i\alpha/2}\end{pmatrix}\right)^T \begin{pmatrix}e^{i\alpha/2}\\e^{-i\alpha/2}\end{pmatrix} = \frac{1}{2\cos\alpha}\left(e^{i\alpha/2}, e^{-i\alpha/2}\right)\begin{pmatrix}e^{i\alpha/2}\\e^{-i\alpha/2}\end{pmatrix} = +1$$

PT orthogonal

$$(\vec{E}_+,\vec{E}_-) = \frac{1}{2\cos\alpha}(PTE_+)^T E_- = \frac{1}{2\cos\alpha}\left(\begin{pmatrix}0&1\\1&0\end{pmatrix} T \begin{pmatrix}e^{i\alpha/2}\\e^{-i\alpha/2}\end{pmatrix}\right)^T \begin{pmatrix}e^{-i\alpha/2}\\-e^{i\alpha/2}\end{pmatrix} = \frac{1}{2\cos\alpha}\left(e^{i\alpha/2}, e^{-i\alpha/2}\right)\begin{pmatrix}e^{-i\alpha/2}\\-e^{i\alpha/2}\end{pmatrix} = \left(\frac{1-1}{2\cos\alpha}\right) = 0$$

$$(\vec{E}_-,\vec{E}_+) = \frac{1}{2\cos\alpha}(PTE_-)^T E_+ = \frac{1}{2\cos\alpha}\left(\begin{pmatrix}0&1\\1&0\end{pmatrix} T \begin{pmatrix}e^{-i\alpha/2}\\-e^{i\alpha/2}\end{pmatrix}\right)^T \begin{pmatrix}e^{i\alpha/2}\\e^{-i\alpha/2}\end{pmatrix} = \frac{1}{2\cos\alpha}\left(-e^{-i\alpha/2}, e^{i\alpha/2}\right)\begin{pmatrix}e^{i\alpha/2}\\e^{-i\alpha/2}\end{pmatrix} = \frac{(-1+1)}{2\cos\alpha} = 0$$

$$(\vec{E}_-,\vec{E}_-) = \frac{1}{2\cos\alpha}(PTE_-)^T E_- = \frac{1}{2\cos\alpha}\left(\begin{pmatrix}0&1\\1&0\end{pmatrix} T \begin{pmatrix}e^{-i\alpha/2}\\-e^{i\alpha/2}\end{pmatrix}\right)^T \begin{pmatrix}e^{-i\alpha/2}\\-e^{i\alpha/2}\end{pmatrix} = \frac{1}{2\cos\alpha}\left(-e^{-i\alpha/2}, e^{i\alpha/2}\right)\begin{pmatrix}e^{-i\alpha/2}\\-e^{i\alpha/2}\end{pmatrix} = -1$$

metric signature: $(+,-)$

$$\hat{H} = \begin{pmatrix} re^{i\theta} & s \\ s & re^{-i\theta} \end{pmatrix}$$

eigenvalues

$$\det(\hat{H} - \mathbb{1}\lambda) = \begin{vmatrix} re^{i\theta} - \lambda & s \\ s & re^{-i\theta} - \lambda \end{vmatrix} = (re^{i\theta} - \lambda)(re^{-i\theta} - \lambda) - s^2 = r^2 e^{i\theta} e^{-i\theta} - \lambda r e^{i\theta} - \lambda r e^{-i\theta} + \lambda^2 - s^2$$

$$= \underbrace{\lambda^2}_{A} + \underbrace{r^2 - s^2}_{C} - \lambda \underbrace{r(e^{i\theta} + e^{-i\theta})}_{B}$$

$$\therefore \lambda = \frac{2r\cos\theta}{2} \pm \frac{\sqrt{(2r\cos\theta)^2 - 4(r^2 - s^2)}}{2}$$

$$\therefore \lambda = r\cos\theta \pm \sqrt{r^2(\cos^2\theta - 1) + s^2}$$

$$\therefore \lambda_{1,2} = r\cos\theta \pm \sqrt{s^2 - r^2\sin^2\theta}$$

eigenvectors

for $\lambda_1 = r\cos\theta + (s^2 - r^2\sin^2\theta)^{\frac{1}{2}}$

$$\hat{H}\vec{E}_+ = \lambda_1 \vec{E}_+ \rightarrow \begin{pmatrix} re^{i\theta} & s \\ s & re^{-i\theta} \end{pmatrix} \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} = r\cos\theta + (s^2 - r^2\sin^2\theta)^{\frac{1}{2}} \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}$$

$$\begin{pmatrix} re^{i\theta}\epsilon_1 + s\epsilon_2 \\ s\epsilon_1 + re^{-i\theta}\epsilon_2 \end{pmatrix} = \begin{pmatrix} r\cos\theta\,\epsilon_1 + (s^2 - r^2\sin^2\theta)^{\frac{1}{2}}\epsilon_1 \\ r\cos\theta\,\epsilon_2 + (s^2 - r^2\sin^2\theta)^{\frac{1}{2}}\epsilon_2 \end{pmatrix} \rightarrow \begin{cases} (1)\ re^{i\theta}\epsilon_1 + s\epsilon_2 = r\cos\theta\,\epsilon_1 + (s^2 - r^2\sin^2\theta)^{\frac{1}{2}}\epsilon_1 \\ (2)\ s\epsilon_1 + re^{-i\theta}\epsilon_2 = r\cos\theta\,\epsilon_2 + (s^2 - r^2\sin^2\theta)^{\frac{1}{2}}\epsilon_2 \end{cases}$$

(1) : $s\epsilon_2 = r\cos\epsilon_1 + s(1 - \frac{r^2}{s}\sin^2\theta)^{\frac{1}{2}}\epsilon_1 - re^{i\theta}\epsilon_1$

$$= \frac{r}{s}\epsilon_1\left(\frac{e^{i\theta} + e^{-i\theta}}{2}\right) - \frac{r}{s}e^{i\theta}\epsilon_1 + \left(1 - \frac{r^2}{s^2}\sin^2\theta\right)^{\frac{1}{2}}\epsilon_1$$

let $\frac{r}{s}\sin\theta = \sin\alpha$

$$\therefore \epsilon_2 = \epsilon_1\left(-\left(\frac{e^{i\alpha} - e^{-i\alpha}}{2}\right) + (1 - \sin^2\alpha)^{\frac{1}{2}}\right)$$

$$\epsilon_2 = \epsilon_1(-i\sin\alpha + |\cos\alpha|)$$

(2) $\epsilon_1 = \frac{r}{s}\cos\theta\,\epsilon_2 + (1 - \frac{r^2}{s^2}\sin^2\theta)^{\frac{1}{2}}\epsilon_2 - \frac{r}{s}e^{-i\theta}\epsilon_2$

$$\therefore \epsilon_1 = \epsilon_2\left(\frac{r}{s}\left(\frac{e^{i\theta} + e^{-i\theta}}{2} - e^{-i\theta}\right) + \left(1 - \frac{r^2}{s^2}\sin^2\theta\right)^{\frac{1}{2}}\right)$$

$$\therefore \epsilon_1 = \epsilon_2(i\sin\alpha + |\cos\alpha|)$$

I'm Trying to make the C operator for Swanson's $\hat{H}$ as presented in Bender's review pt 5.

$$C = \begin{pmatrix} (PT|E_+\rangle)^T\begin{bmatrix}1&0\\0&0\end{bmatrix}|E_+\rangle + (PT|E_+\rangle)^T\begin{bmatrix}0&0\\0&1\end{bmatrix}|E_+\rangle & (PT|E_+\rangle)^T\begin{bmatrix}1&0\\0&0\end{bmatrix}|E_-\rangle + (PT|E_+\rangle)^T\begin{bmatrix}0&0\\0&1\end{bmatrix}|E_-\rangle \\ (PT||E_-\rangle)^T\begin{bmatrix}1&0\\0&0\end{bmatrix}|E_+\rangle + (PT|E_+\rangle)^T\begin{pmatrix}0&0\\0&1\end{pmatrix}|E_-\rangle & (PT|E_-\rangle)^T\begin{pmatrix}1&0\\0&0\end{pmatrix}|E_-\rangle + (PT|E_-\rangle)^T\begin{bmatrix}0&0\\0&1\end{bmatrix}|E_-\rangle \end{pmatrix}$$

$$= \frac{1}{2\cos\alpha}\begin{pmatrix} \left(P\begin{bmatrix}e^{i\alpha/2}\\e^{-i\alpha/2}\end{bmatrix}\right)^T\begin{bmatrix}1&0\\0&0\end{bmatrix}\begin{bmatrix}e^{i\alpha/2}\\e^{-i\alpha/2}\end{bmatrix} + \left(P\begin{bmatrix}e^{i\alpha/2}\\e^{-i\alpha/2}\end{bmatrix}\right)^T\begin{bmatrix}0&0\\0&1\end{bmatrix}\begin{bmatrix}e^{i\alpha/2}\\e^{-i\alpha/2}\end{bmatrix} & g \\ g & (i)^2\left(\left(P\begin{bmatrix}e^{-i\alpha/2}\\-e^{i\alpha/2}\end{bmatrix}\right)^T\begin{bmatrix}1&0\\0&0\end{bmatrix}\begin{bmatrix}e^{-i\alpha/2}\\-e^{i\alpha/2}\end{bmatrix} + \left(P\begin{bmatrix}e^{-i\alpha/2}\\-e^{i\alpha/2}\end{bmatrix}\right)^T\begin{bmatrix}0&0\\0&1\end{bmatrix}\begin{bmatrix}e^{-i\alpha/2}\\-e^{i\alpha/2}\end{bmatrix}\right) \end{pmatrix}$$

where $\frac{r}{s}\sin\theta = \sin\alpha$, g is interaction $|E_\pm\rangle$

for example: HOW DOES P act on $e^{i\alpha}$?

$$P\begin{bmatrix}e^{-i\alpha/2}\\-e^{i\alpha/2}\end{bmatrix} = ?$$

$$C = \begin{pmatrix} (PT|E_+\rangle)^T|\uparrow\rangle\langle\uparrow|E_+\rangle + (PT|E_+\rangle)^T|\downarrow\rangle\langle\downarrow|E_+\rangle & (PT|E_+\rangle)^T|\uparrow\rangle\langle\uparrow|E_-\rangle + (PT|E_+\rangle)^T|\downarrow\rangle\langle\downarrow|E_-\rangle \\ (PT|E_-\rangle)^T|\downarrow\rangle\langle\downarrow|E_+\rangle + (PT|E_+\rangle)^T|\downarrow\rangle\langle\downarrow|E_-\rangle & (PT|E_-\rangle)^T|\uparrow\rangle\langle\uparrow|E_-\rangle + (PT|E_-\rangle)^T|\downarrow\rangle\langle\downarrow|E_-\rangle \end{pmatrix}$$

$$= \frac{1}{2\cos\alpha}\begin{bmatrix} \left(\begin{bmatrix}0&1\\1&0\end{bmatrix}\begin{bmatrix}e^{-i\alpha/2}\\e^{i\alpha/2}\end{bmatrix}\right)^T\begin{pmatrix}1\\0\end{pmatrix}(1,0)\begin{bmatrix}e^{i\alpha/2}\\e^{-i\alpha/2}\end{bmatrix} + \left(\begin{bmatrix}0&1\\1&0\end{bmatrix}\begin{bmatrix}e^{-i\alpha/2}\\e^{i\alpha/2}\end{bmatrix}\right)^T\begin{pmatrix}0\\1\end{pmatrix}(0,1)\begin{pmatrix}e^{i\alpha/2}\\e^{-i\alpha/2}\end{pmatrix} & g \\ g & (i)^2\left(\begin{bmatrix}0&1\\1&0\end{bmatrix}\begin{bmatrix}e^{i\alpha/2}\\-e^{-i\alpha/2}\end{bmatrix}\right)^T\begin{pmatrix}0\\1\end{pmatrix}(0,1)\begin{bmatrix}e^{-i\alpha/2}\\-e^{i\alpha/2}\end{bmatrix} + \left(\begin{bmatrix}0&1\\1&0\end{bmatrix}\begin{bmatrix}e^{i\alpha/2}\\-e^{-i\alpha/2}\end{bmatrix}\right)^T\begin{bmatrix}0\\1\end{bmatrix}[0,1]\begin{bmatrix}e^{-i\alpha/2}\\-e^{i\alpha/2}\end{bmatrix} \end{bmatrix}$$

Eg. $C_{11} = (e^{i\alpha/2}, e^{-i\alpha/2})\begin{pmatrix}1\\0\end{pmatrix}(1\ 0)\begin{bmatrix}e^{i\alpha/2}\\e^{-i\alpha/2}\end{bmatrix} + (e^{i\alpha/2}, e^{-i\alpha/2})\begin{pmatrix}0\\1\end{pmatrix}(0,1)\begin{pmatrix}e^{i\alpha/2}\\e^{-i\alpha/2}\end{pmatrix}$

$$= (e^{i\alpha/2})e^{i\alpha/2} + (e^{-i\alpha/2})e^{-i\alpha/2}$$

$$\therefore C_{11} = e^{i\alpha} + e^{-i\alpha} = 2i\sin\alpha$$

← 2 LOOP RGE

$g\,T$

$\frac{g^{3/2}T}{\sqrt{\pi}}$ $\mathcal{L}_{NNC}$ $\xleftarrow{\ \ }$ t_0 $\left\{\begin{array}{l}\text{COULD +} \\ \text{TENKATEN}\end{array}\right.$

$g^2 T$ $\bar{\mathcal{L}}_3$ $\vec{A},\ \chi,\ C,\ H$ $S_3 = \int d^3x \left[V_{eff} \right.$

g^2

$g^4 \leftarrow$ TWO LOOP

$\frac{g}{i} = C_{21} = \left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} e^{i\alpha/2} \\ -e^{-i\alpha/2} \end{bmatrix} \right)^T \begin{bmatrix} 1 \\ 0 \end{bmatrix} [1, 0] \begin{bmatrix} e^{i\alpha/2} \\ e^{-i\alpha/2} \end{bmatrix}$

$+ \left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{bmatrix} e^{i\alpha/2} \\ -e^{-i\alpha/2} \end{bmatrix} \right)^T \begin{bmatrix} 0 \\ 1 \end{bmatrix} [0, 1] \begin{bmatrix} e^{i\alpha/2} \\ e^{-i\alpha/2} \end{bmatrix}$ $+ Z(\phi)(\partial_\mu C)^2 +$

Eg.

$g = (PT|E_-\rangle)^T |\uparrow\rangle\langle\uparrow| E_+\rangle$

$+ (PT|E_-\rangle)^T |\downarrow\rangle\langle\downarrow| E_+\rangle$ $\Lambda \sim M_{eff}$

i factor

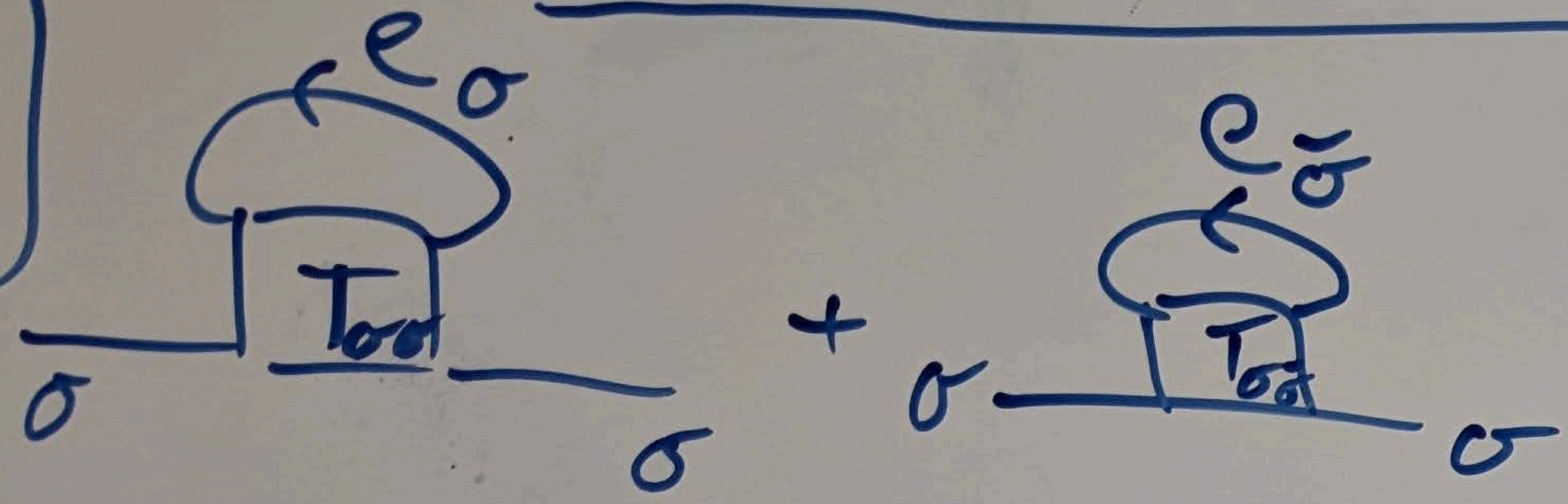
$-ig = C_{21} = (-e^{-i\alpha/2})\, e^{i\alpha/2} + (e^{i\alpha/2})\, e^{-i\alpha/2}$

$-ig = (-1 + 1)$ $\partial_\mu C$

γ^2

$$\sum_k k\cdot\lambda\left(X_{k\sigma} X_{k\bar{\sigma}} + h.c.\right)$$

$$N = \sum_k \int d\omega\, e \ldots (A$$



$$PE_-^* = \frac{-i}{\sqrt{2\cos\alpha}}\begin{pmatrix} -e^{-i\alpha/2} \\ e^{i\alpha/2} \end{pmatrix} \quad E_+ = \frac{1}{\sqrt{2\cos\alpha}}\left(\ldots \right.$$

$$E_+ \cdot E_- = \frac{i}{2\cos\alpha}(1-1) = 0$$

$$(PT E_+)\cdot E_+ = \frac{1}{2\cos\alpha}\left(e \ldots \right.$$

$$C_{12} = \frac{1}{2\cos\alpha}\left(1 - 1(-1)\right) = \frac{1}{\cos\alpha}$$

$$C(x,y) = \sum_n \phi_n(x)\phi_n(y)$$

$$(PT E_-)\cdot E_- = \frac{1}{2\cos\alpha}\left(-e^{-i\alpha/2} \ldots \right.$$

$$C_{ij} = E_+(i)E_+(j) + E_-(i)E_-(j)$$

$$C_{11} = \frac{1}{2\cos\alpha}\left(e^{i\alpha} - e^{-i\alpha}\right) = \frac{+i\sin\alpha}{\cos\alpha}$$

$$= \frac{1}{2\cos\alpha}\left(-e^{-i\alpha} + e \ldots \right.$$

$$\vec{E}_+ = \frac{1}{\sqrt{2\cos\alpha}} \begin{pmatrix} e^{\cdots} \\ e^{-i\alpha/2} \end{pmatrix}$$

$$H = \sum_{k\sigma} \epsilon^x_k X^\dagger_{k\sigma} X_{k\sigma} + \sum_{k,\sigma} \epsilon^e_k e^\dagger_{k\sigma} e_{k\sigma}$$

$$= (E_+|\uparrow X \uparrow|E_+\rangle + (E_+|\downarrow X \downarrow|E_+)$$

$$+ \sum_{\substack{k,k'\\ \sigma,\sigma'}} g_{\sigma\sigma'} X^\dagger_\sigma X_\sigma e^\dagger_{\sigma'} e_{\sigma'}$$

$$\langle E| \quad (PT|E_+\rangle)^T$$

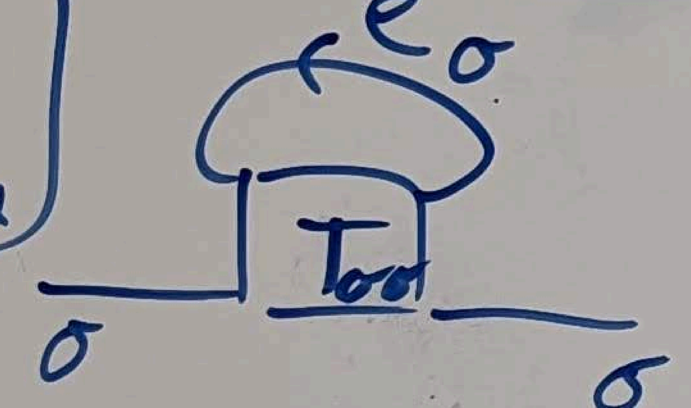
Bender's:
$$\frac{1}{\cos\alpha}\begin{bmatrix} i\sin\alpha & 1 \\ 1 & -i\sin\alpha \end{bmatrix}$$

$$\overset{?}{+} \sum_k k\cdot\lambda (X^\dagger_{k\sigma} X_{k\bar\sigma} + h.c.)$$

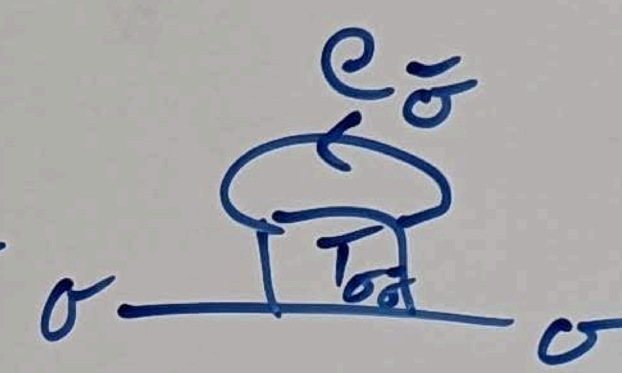
$$(u,v) = \underbrace{(PT\,u)^T \cdot v}$$

Mine's:
$$\frac{1}{\cos\alpha}\begin{bmatrix} i\sin\alpha & 0 \\ 0 & -i\sin\alpha \end{bmatrix}$$

$(\cdots u)$



$+$



$$\frac{e^{i\alpha} + e^{-i\alpha}}{2\cos\alpha} = 1$$

assu

$$N = \sum_k$$

$$PE^*_- = \frac{-i}{\sqrt{2\cos\alpha}} \begin{pmatrix} e^{-i\alpha/2} \\ e^{i\alpha/2} \end{pmatrix} E_-$$

$$U(t) = e^{-iHt}$$
$$= \sum_n |n\rangle\langle n| e^{-iE_n t}$$

$$(PT\,E_+$$

$$U_{11} = \left(PT\begin{pmatrix}1\\0\end{pmatrix}\right)\cdot E_+\left)\left(PT(E_+)\right)\cdot\begin{pmatrix}1\\0\end{pmatrix} e^{-iE_+ t}$$

$$+ \dots \quad E_-$$

$$E_+ \cdot E_- = \frac{i}{2\cos\alpha}(1-1) = 0$$

$$E_- \quad C_{12} = \frac{1}{2\cos\alpha}(1 - 1(-1))$$
$$= \frac{1}{\cos\alpha}$$

$$C(x,y) = \sum_n \phi_n(x)\phi_n(y)$$

$$(PT\,E)\cdot$$

$$C_{ij} = E_+(i)E_+(j) + E_-(i)E_-(j)$$

$$C_{11} = \frac{1}{2\cos\alpha}(e^{i\alpha}\cdots = \frac{i\sin\alpha}{\cos\alpha}$$

$$\vec{E}_+ = \frac{1}{\sqrt{2\cos\alpha}}\begin{pmatrix} e^{i\alpha/2} \\ e^{-i\alpha/2} \end{pmatrix} \qquad \vec{E}_- = \frac{i}{\sqrt{2\cos\alpha}}\begin{pmatrix} e^{-i\alpha/2} \\ -e^{i\alpha/2} \end{pmatrix}$$

$$C_{12} = E_+E_- + E_+E_-$$

$$C_{21} = E_-E_+ + E_+E_-$$

$$C_{22} = E_-E_+ + E_-E_- \text{ ? MAYBE}$$

$$C_{ij} = \vec{E}_+(i)\cdot\vec{E}_+(j) + \cancel{\vec{E}_-(i)\cdot\vec{E}_-(j)} \rightarrow 0 \text{ for } ii \;\&\; 0 \text{ for } jj$$

$$C_{11} = E_+E_+ + E_+E_-$$

$$C_{11} = \frac{1}{2\cos\alpha}\left(e^{i\alpha/2}, e^{-i\alpha/2}\right)\begin{pmatrix} e^{i\alpha/2} \\ e^{-i\alpha/2} \end{pmatrix} + \frac{i}{2\cos\alpha}\left(-e^{i\alpha/2}, e^{-i\alpha/2}\right)\begin{pmatrix} e^{-i\alpha/2} \\ -e^{i\alpha/2} \end{pmatrix}$$

$$= \frac{1}{2\cos\alpha}\left(\left(e^{i\alpha/2}\right)e^{i\alpha/2} + \left(e^{-i\alpha/2}\right)e^{-i\alpha/2} + i\left(\left(e^{i\alpha/2}\right)e^{-i\alpha/2} - \left(e^{i\alpha/2}\right)e^{-i\alpha/2}\right)\right)$$

$$= \frac{1}{2\cos\alpha}\left(e^{i\alpha} + e^{-i\alpha} + 1 - 1\right) = \frac{1}{\cos\alpha}\left(i\sin\alpha\right)$$

$$C\begin{bmatrix} c_{ii} & c_{ij} \\ c_{ji} & c_{jj} \end{bmatrix}$$

$$c_{ii} = C_{11}$$

$$\vec{E}_+ = \frac{1}{\sqrt{2\cos\alpha}} \begin{bmatrix} e^{i\alpha/2} \\ e^{-i\alpha/2} \end{bmatrix} \quad \& \quad \vec{E}_- = \frac{i}{\sqrt{2\cos\alpha}} \begin{bmatrix} e^{-i\alpha/2} \\ -e^{i\alpha/2} \end{bmatrix}$$

HOW DOES THE INDEXING WORK?

$$C_{ij} = \vec{E}_+(i) \cdot \vec{E}_+(j) + \vec{E}_-(i) \cdot \vec{E}_-(j)$$

$C_{++} =$

C_{+-}

C_{-+}

C_{--}

$$\begin{array}{c|cc} & + & - \\ \hline + & C_{ii} & C_{ij} \\ - & C_{ji} & C_{jj} \end{array}$$

$$\begin{bmatrix} \vec{E}_+ \cdot \vec{E}_+ + \vec{E}_- \vec{E}_- ? & ? \\ ? & \vec{E}_+ \vec{E}_+ + \vec{E}_- \cdot \vec{E}_- ? \end{bmatrix}$$

I don't understand

Bender + Jesper say that

$$C_{12} = \frac{1}{2\cos\alpha}(1 - 1(-1))$$

$$= \frac{1}{\cos\alpha}(? - ?\,(i^2))$$

$$\hat{H} = \begin{pmatrix} s & re^{-i\theta} \\ re^{i\theta} & u \end{pmatrix} \qquad r, s, u, \theta \in \mathbb{R} \qquad \text{evals } \omega^2 = (s-u)^2 + 4r^2$$

$$\text{or } \hat{H} = \frac{1}{2}(s+u)\hat{\mathbb{1}} + \frac{\omega}{2}(\sigma \cdot \hat{n})$$

where: $\hat{n} = \frac{1}{\omega}(2r\cos\theta, 2r\sin\theta, s-u)$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

What is: $\sigma \cdot \hat{n} = \frac{1}{\omega}\left[\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} 2r\cos\theta + \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} 2r\sin\theta + \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}(s-u)\right]$

$$= \frac{2}{\omega}\left[\begin{bmatrix} 0 & r\cos\theta \\ r\cos\theta & 0 \end{bmatrix} + \begin{bmatrix} 0 & -ir\sin\theta \\ ir\sin\theta & 0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} (s-u) & 0 \\ 0 & -(s-u) \end{bmatrix}\right]$$

$$\sigma \cdot \hat{n} = \frac{2}{\omega}\begin{bmatrix} \frac{s-u}{2} & r(\cos\theta - i\sin\theta) \\ r(\cos\theta + i\sin\theta) & -\frac{(s-u)}{2} \end{bmatrix} = \frac{2}{\omega}\begin{bmatrix} \frac{s-u}{2} & re^{-i\theta} \\ re^{i\theta} & -\frac{(s-u)}{2} \end{bmatrix}$$

for the unitary: $\hat{U} = e^{-i\hat{H}t/\hbar}$. Let $\vec{\psi}_i = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\vec{\psi}_f = \begin{bmatrix} a \\ b \end{bmatrix}$

we assume: $|a|^2 + |b|^2 = 1$

$$\therefore |\psi_f\rangle = e^{-i\hat{H}t/\hbar}|\psi_i\rangle$$

$$= e^{-\frac{it}{\hbar}\left[\frac{1}{2}(s+u)\mathbb{1} + \frac{1}{2}\omega\,\sigma\cdot\hat{n}\right]}|\psi_i\rangle$$

$$= \exp\left[\frac{-it}{2\hbar}\left(\begin{bmatrix} s+u & 0 \\ 0 & s+u \end{bmatrix} + \cancel{\omega}\left(\frac{2}{\cancel{\omega}}\begin{bmatrix} \frac{s-u}{2} & re^{-i\theta} \\ re^{i\theta} & \frac{u-s}{2} \end{bmatrix}\right)\right)\right]|\psi_i\rangle$$

$$= \exp\left[\frac{-it}{2\hbar}\left(\begin{bmatrix} s+u & 0 \\ 0 & s+u \end{bmatrix} + \begin{bmatrix} s-u & 2re^{-i\theta} \\ 2re^{i\theta} & u-s \end{bmatrix}\right)\right]|\psi_i\rangle$$

$$= \exp\left[\frac{-it}{2\hbar}\left(\begin{bmatrix} 2s & 2re^{-i\theta} \\ 2re^{i\theta} & 2u \end{bmatrix}\right)\right]|\psi_i\rangle \qquad \text{let } \phi = \frac{-\omega t}{2\hbar}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \underbrace{\exp\left[\frac{-it}{\hbar}\begin{bmatrix} s & re^{-i\theta} \\ re^{i\theta} & u \end{bmatrix}\right]\cdot\begin{bmatrix} 1 \\ 0 \end{bmatrix}} = \exp\left[\frac{-it}{2\hbar}(s+u)\hat{\mathbb{1}}\right](\cos\phi\,\hat{\mathbb{1}} + i\sin\phi(\sigma\cdot\hat{n}))\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\hat{H} = \begin{pmatrix} s & re^{-i\theta} \\ re^{i\theta} & u \end{pmatrix} \quad r, s, u, \theta \in \mathbb{R} \qquad \text{evals } \omega^2 = (s-u)^2 + 4r^2$$

$$\text{or } \hat{H} = \frac{1}{2}(s+u)\hat{\mathbb{1}} + \frac{\omega}{2}(\sigma \cdot \hat{n})$$

where: $\hat{n} = \frac{1}{\omega}(2r\cos\theta, 2r\sin\theta, s-u)$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

what is: $\sigma \cdot \hat{n} = \frac{1}{\omega}\left[\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} 2r\cos\theta + \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} 2r\sin\theta + \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}(s-u)\right]$

$$= \frac{2}{\omega}\left[\begin{bmatrix} 0 & r\cos\theta \\ r\cos\theta & 0 \end{bmatrix} + \begin{bmatrix} 0 & -ir\sin\theta \\ ir\sin\theta & 0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} (s-u) & 0 \\ 0 & -(s-u) \end{bmatrix}\right]$$

$$\sigma \cdot \hat{n} = \frac{2}{\omega}\begin{bmatrix} \frac{s-u}{2} & r(\cos\theta - i\sin\theta) \\ r(\cos\theta + i\sin\theta) & -\frac{(s-u)}{2} \end{bmatrix} = \frac{2}{\omega}\begin{bmatrix} \frac{s-u}{2} & re^{-i\theta} \\ re^{i\theta} & -\frac{(s-u)}{2} \end{bmatrix}$$

for the unitary: $\hat{U} = e^{-i\hat{H}t/\hbar}$. Let $\vec{\psi}_i = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\vec{\psi}_f = \begin{bmatrix} a \\ b \end{bmatrix}$

we assume: $|a|^2 + |b|^2 = 1$

$$\therefore |\psi_f\rangle = e^{-i\hat{H}t/\hbar}|\psi_i\rangle$$

$$= e^{-\frac{it}{\hbar}\left[\frac{1}{2}(s+u)\hat{\mathbb{1}} + \frac{1}{2}\omega\, \sigma \cdot \hat{n}\right]}|\psi_i\rangle \qquad \text{let } \phi = -\frac{\omega t}{2\hbar}$$

$$= \exp\left[-\frac{it}{2\hbar}(s+u)\hat{\mathbb{1}}\right] \underbrace{\exp\left[i\phi\, \sigma \cdot \hat{n}\right]}_{=} |\psi_i\rangle$$

$$= \exp\left[\frac{i\phi}{\omega}(s+u)\hat{\mathbb{1}}\right]\left(\cos\phi\, \underset{\text{Matrix}}{\hat{\mathbb{1}}} + i\sin\phi\, \underset{\text{Matrix}}{(\sigma \cdot \hat{n})}\right)\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$H = \begin{pmatrix} s & re^{-i\theta} \\ re^{i\theta} & u \end{pmatrix} = \underbrace{\frac{1}{2}(s+u)\hat{\mathbb{1}} + \frac{1}{2}\omega\sigma\cdot\hat{n}}$$

$$e^{A+B} \neq e^{A}e^{B}$$

$$= \underbrace{e^{At}e^{Bt}}e^{-[At,Bt]} + O(t^2)$$

$$C = \frac{rP}{1-(1+r)^{N}}$$

$$|\psi_f\rangle = e^{\frac{-it}{2\hbar}\left[(s+u)\hat{\mathbb{1}} + \omega(\sigma\cdot\hat{n})\right]}|\psi_i\rangle$$

$$e^{i\phi\sigma\cdot\hat{n}} = \cos\phi\,\hat{\mathbb{1}} + i\sin\phi(\sigma\cdot\hat{n})$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \exp\left(\frac{-it}{2\hbar}\begin{pmatrix} s+u & 0 \\ 0 & s+u \end{pmatrix}\right)\underbrace{\left(\cos\phi\,\hat{\mathbb{1}} + i\sin\phi\,\sigma\cdot\hat{n}\right)}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\frac{i\sin\phi}{\omega}\begin{pmatrix} s-u & 2re^{-i\theta} \\ 2re^{i\theta} & u-s \end{pmatrix}$$

$$\hat{U} = e^{iHt/\hbar}$$

$$\underbrace{\exp\left[\frac{-it}{2\hbar}\begin{bmatrix} s+u & 0 \\ 0 & s+u \end{bmatrix}\right]\left(\begin{bmatrix} \cos\phi & 0 \\ 0 & \cos\phi \end{bmatrix} + \frac{i\sin\phi}{\omega}\begin{bmatrix} s-u & 2re^{-i\theta} \\ 2re^{i\theta} & u-s \end{bmatrix}\right)}_{A[\substack{1\\0}]}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \ddot{\smile} & \ddot{\frown} \\ \ddot{=} & \ddot{\smile} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$A\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

2x2

$$\begin{bmatrix} \ddot{\smile}+0 \\ \ddot{-}+0 \end{bmatrix} = \begin{bmatrix} \ddot{\smile} \\ \ddot{-} \end{bmatrix}$$

$$3. \quad \begin{bmatrix} a \\ b \end{bmatrix} = \exp\Bigg(\overbrace{\frac{-it}{2\hbar}\underbrace{\begin{bmatrix} s+u & 0 \\ 0 & s+u \end{bmatrix}}_{C}\Bigg(\underbrace{\begin{bmatrix} \cos\phi & 0 \\ 0 & \cos\phi \end{bmatrix}}_{A} + \frac{i\sin\phi}{\omega}\underbrace{\begin{bmatrix} s-u & 2re^{-i\theta} \\ 2re^{i\theta} & u-s \end{bmatrix}}_{B}\Bigg)}^{M}\Bigg)\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

let $\alpha = \dfrac{i\sin\phi}{\omega}$

$$A+\alpha B = \begin{bmatrix} \cos\phi+\alpha(s-u) & \alpha 2re^{-i\theta} \\ \alpha 2re^{i\theta} & \cos\phi+\alpha(u-s) \end{bmatrix}$$

$$C(A+\alpha B) = \begin{bmatrix} s+u & 0 \\ 0 & s+u \end{bmatrix}\begin{bmatrix} \cos\phi+\alpha(s-u) & \alpha 2re^{-i\theta} \\ \alpha 2re^{i\theta} & \cos\phi+\alpha(u-s) \end{bmatrix}$$

$$= \begin{bmatrix} (s+u)(\cos\phi+\alpha(s-u)) & (s+u)\alpha 2re^{-i\theta} \\ (s+u)\alpha 2re^{i\theta} & (s+u)(\cos\phi+\alpha(u-s)) \end{bmatrix}$$

$$= (s+u)(A+\alpha B)$$

$$M = \frac{-it}{2\hbar\omega}(s+u)\begin{bmatrix} \omega\cos\phi+i\sin\phi(s-u) & 2re^{-i\theta}i\sin\phi \\ 2re^{i\theta}i\sin\phi & \omega\cos\phi+i\sin\phi(u-s) \end{bmatrix}$$

$$M_{11} = (\omega\cos\phi + s\cdot i\sin\phi - ui\sin\phi)\left(\frac{-it}{2\hbar\omega}(s+u)\right)$$

$$M_{12} = 2re^{-i\theta}i\sin\phi\,(\cdots)$$

$$M_{21} = 2re^{i\theta}i\sin\phi\,(\cdots)$$

$$M_{22} = (\omega\cos\phi + ui\sin\phi - s\cdot i\sin\phi)(\cdots)$$

$$\hat{H} = \begin{bmatrix} s & re^{-i\theta} \\ re^{i\theta} & u \end{bmatrix} \overset{?}{=} \frac{1}{2}(s+u)\hat{\mathbb{1}} + \frac{1}{2}\omega\hat{\sigma}\cdot\hat{n}$$

0. in General : $e^{AB} \neq e^{A}e^{B}$ But yes if $[A,B]=0$

1. Why is $e^{i\phi\hat{\sigma}\cdot\hat{n}} = \cos\phi\,\hat{\mathbb{1}} + i\sin\phi(\hat{\sigma}\cdot\hat{n})$?

   * (Slack problem) *

2. $\hat{\sigma}\cdot\hat{n} = \dfrac{i\sin\phi}{\omega}\begin{pmatrix} s-u & 2re^{-i\theta} \\ 2re^{i\theta} & u-s \end{pmatrix}$

3. time evolution $\left(H = \begin{bmatrix} s & re^{-i\theta} \\ re^{i\theta} & u \end{bmatrix}\right)$ H.H.

$$e^{\frac{-it}{2\hbar}\left((s+u)\hat{\mathbb{1}} + \omega(\hat{\sigma}\cdot\hat{n})\right)}|\psi_i\rangle = |\psi_f\rangle$$

where $|\psi_i\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $|\psi_f\rangle = \begin{bmatrix} a \\ b \end{bmatrix}$, $|a|^2+|b|^2=1$

EXPONENTIATING A MATRIX:

$$\therefore \exp[M]\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \sum_{n=0}^{\infty}\frac{M^n}{n!}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

for $n=0$:

$= \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

for $n=1$:

$$= M\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{-it}{2\hbar\omega}(s+u)\begin{bmatrix} \omega\cos\phi + s\cdot i\sin\phi - ui\sin\phi & 2re^{-i\theta}i\sin\phi \\ 2re^{i\theta}i\sin\phi & \omega\cos\phi + ui\sin\phi - s\cdot i\sin\phi \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$3. \quad \begin{bmatrix} a \\ b \end{bmatrix} = \exp\Bigg(\overbrace{\frac{-it}{2\hbar} \underset{C}{\begin{bmatrix} s+u & 0 \\ 0 & s+u \end{bmatrix}} \Bigg(\underset{A}{\begin{bmatrix} \cos\phi & 0 \\ 0 & \cos\phi \end{bmatrix}} + \frac{i\sin\phi}{\omega} \underset{B}{\begin{bmatrix} s-u & 2re^{-i\theta} \\ 2re^{i\theta} & u-s \end{bmatrix}} \Bigg)}^{M} \Bigg) \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$M \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{-it}{2\hbar\omega}(s+u) \begin{bmatrix} \omega\cos\phi + s\cdot i\sin\phi - ui\sin\phi & 2re^{-i\theta} i\sin\phi \\ 2r\cdot e^{i\theta} i\cdot\sin\phi & \omega\cos\phi + ui\sin\phi - s\cdot i\sin\phi \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \frac{-it}{2\hbar\omega}(s+u) \begin{bmatrix} \omega\cos\phi + s\cdot i\sin\phi - ui\sin\phi \\ 2r\cdot e^{i\theta} i\cdot\sin\phi \end{bmatrix}$$

$$\therefore |\psi_f\rangle = \exp(M)|\psi_i\rangle \Rightarrow \begin{bmatrix} a \\ b \end{bmatrix} \approx \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \frac{it}{2\hbar\omega}(s+u) \begin{bmatrix} \omega\cos\phi + s\cdot i\sin\phi - ui\sin\phi \\ 2re^{i\theta} i\sin\phi \end{bmatrix}$$

$$\hat{H} = \begin{bmatrix} s & re^{-i\theta} \\ re^{i\theta} & u \end{bmatrix} \stackrel{?}{=} \frac{1}{2}(s+u)\hat{\mathbb{1}} + \frac{1}{2}\omega\hat{\sigma}\cdot\hat{n}$$

0. in General: $e^{AB} \neq e^{A}e^{B}$ But yes if $[A,B]=0$

1. Why is $e^{i\phi\hat{\sigma}\cdot\hat{n}} = \cos\phi\,\hat{\mathbb{1}} + i\sin\phi\,(\hat{\sigma}\cdot\hat{n})$?

* (Slack problem) *

2. $\hat{\sigma}\cdot\hat{n} = \dfrac{i\sin\phi}{\omega}\begin{pmatrix} s-u & 2re^{-i\theta} \\ 2re^{i\theta} & u-s \end{pmatrix}$

3. time evolution $\left(H = \begin{bmatrix} s & re^{-i\theta} \\ re^{i\theta} & u \end{bmatrix}\right)$ H.H.

$$e^{\frac{-it}{2\hbar}\left((s+u)\hat{\mathbb{1}} + \omega(\hat{\sigma}\cdot\hat{n})\right)}|\psi_i\rangle = |\psi_f\rangle$$

where $|\psi_i\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $|\psi_f\rangle = \begin{bmatrix} a \\ b \end{bmatrix}$, $|a|^2 + |b|^2 = 1$

EXPONENTIATING A MATRIX:

$$\therefore \exp[M]\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \sum_{n=0}^{\infty} \frac{M^n}{n!} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

for $n=0$:

$= \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

for $n=1$:

$$= M\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{-it}{2\hbar\omega}(s+u) \begin{bmatrix} \omega\cos\phi + s\cdot i\sin\phi - ui\sin\phi & 2re^{-i\theta} i\sin\phi \\ 2re^{i\theta} i\sin\phi & \omega\cos\phi + ui\sin\phi - s\cdot i\sin\phi \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$3. \quad \begin{bmatrix} a \\ b \end{bmatrix} = \exp\Bigg(\overbrace{\underbrace{\frac{-it}{2\hbar}\begin{bmatrix} s+u & 0 \\ 0 & s+u \end{bmatrix}}_{C}\Bigg(\underbrace{\begin{bmatrix} \cos\phi & 0 \\ 0 & \cos\phi \end{bmatrix}}_{A} + \frac{i\sin\phi}{w}\underbrace{\begin{bmatrix} s-u & 2re^{-i\theta} \\ 2re^{i\theta} & u-s \end{bmatrix}}_{B}\Bigg)}^{M}\Bigg)\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$M\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{-it}{2\hbar w}(s+u)\begin{bmatrix} w\cos\phi + s\cdot i\sin\phi - ui\sin\phi & 2re^{-i\theta}i\sin\phi \\ 2re^{i\theta}i\cdot\sin\phi & w\cos\phi + ui\sin\phi - s\cdot i\sin\phi \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \frac{-it}{2\hbar w}(s+u)\begin{bmatrix} w\cos\phi + s\cdot i\sin\phi - ui\sin\phi \\ 2re^{i\theta}i\cdot\sin\phi \end{bmatrix}$$

$$\therefore |\psi_f\rangle = \exp(M)|\psi_i\rangle \Rightarrow \begin{bmatrix} a \\ b \end{bmatrix} \approx \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \frac{it}{2\hbar w}(s+u)\begin{bmatrix} w\cos\phi + s\,i\sin\phi - ui\sin\phi \\ 2re^{i\theta}i\sin\phi \end{bmatrix}$$

$$\hat{H} = \begin{bmatrix} s & re^{-i\theta} \\ re^{i\theta} & u \end{bmatrix} \stackrel{?}{=} \frac{1}{2}(s+u)\hat{\mathbb{1}} + \frac{1}{2}w\hat{\sigma}\cdot\hat{n}$$

0. in General: $e^{\hat{A}\hat{B}} \neq e^{\hat{A}}e^{\hat{B}}$ But yes if $[A,B]=0$

1. Why is $e^{i\phi\hat{\sigma}\cdot\hat{n}} = \cos\phi\,\hat{\mathbb{1}} + i\sin\phi\,(\hat{\sigma}\cdot\hat{n})$?
   * (Slack problem) *

2. $\hat{\sigma}\cdot\hat{n} = \frac{i\sin\phi}{w}\begin{pmatrix} s-u & 2re^{-i\theta} \\ 2re^{i\theta} & u-s \end{pmatrix}$

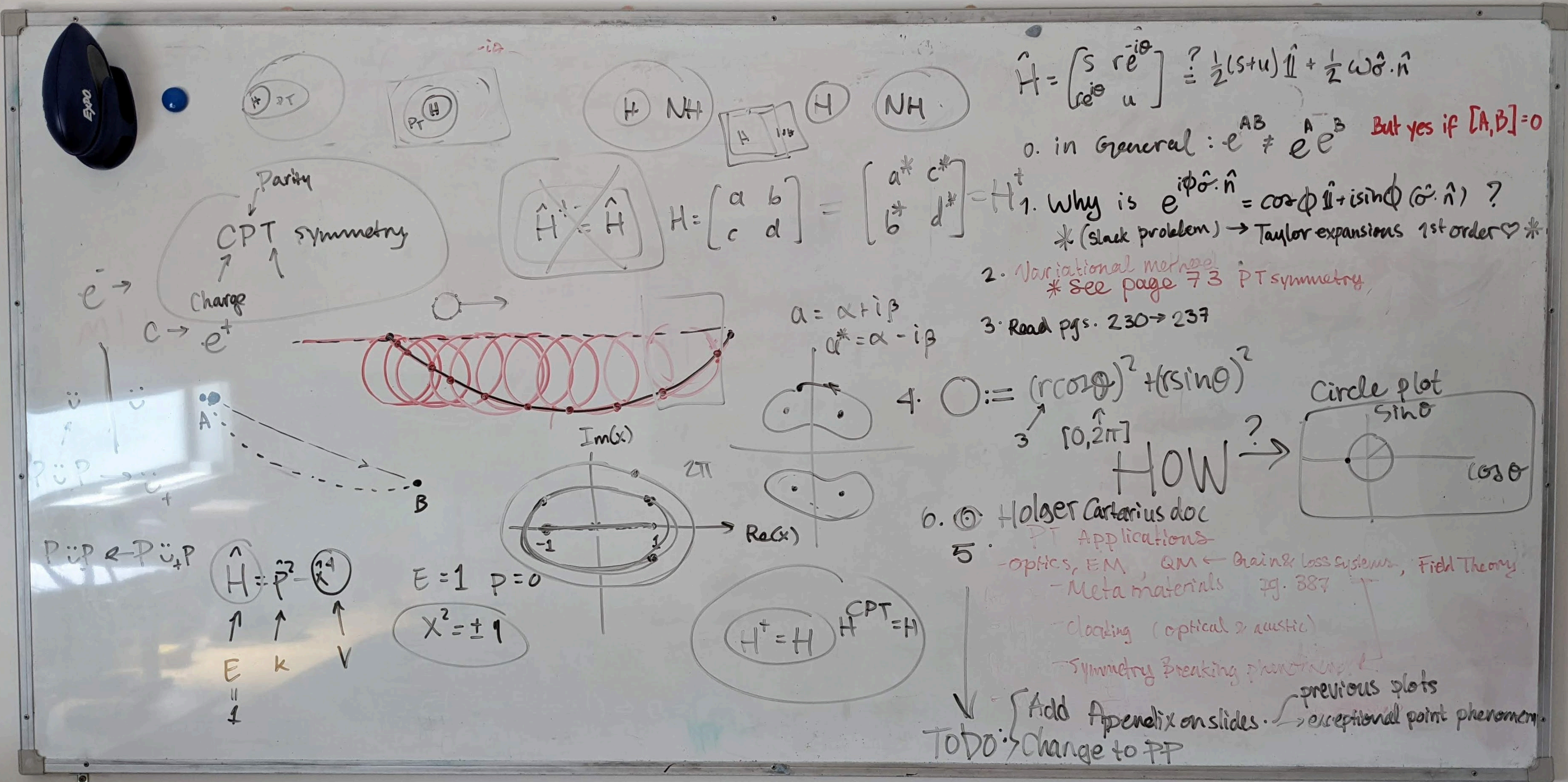
3. time evolution $\left(H = \begin{bmatrix} s & re^{-i\theta} \\ re^{i\theta} & u \end{bmatrix}\right)$ H.H.
   $e^{\frac{-it}{2\hbar}((s+u)\hat{\mathbb{1}} + w(\hat{\sigma}\cdot\hat{n}))}|\psi_i\rangle = |\psi_f\rangle$
   where $|\psi_i\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $|\psi_f\rangle = \begin{bmatrix} a \\ b \end{bmatrix}$, $|a|^2 + |b|^2 = 1$

EXPONENTIATING A MATRIX:

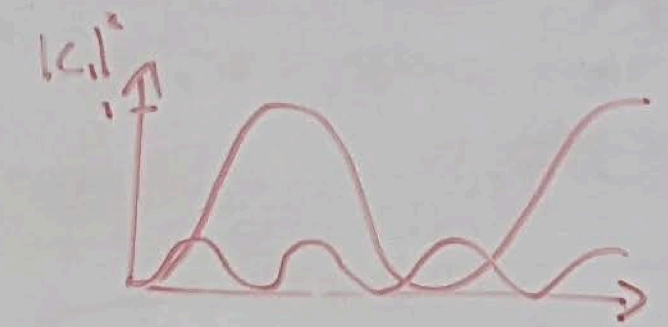
$$\therefore \exp[M]\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \sum_{n=0}^{\infty}\frac{M^n}{n!}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

for $n=0$:
$= \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

for $n=1$:
$$= M\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{-it}{2\hbar w}(s+u)\begin{bmatrix} w\cos\phi + s\cdot i\sin\phi - ui\sin\phi & 2re^{-i\theta}i\sin\phi \\ 2re^{i\theta}i\sin\phi & w\cos\phi + ui\sin\phi - s\cdot i\sin\phi \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

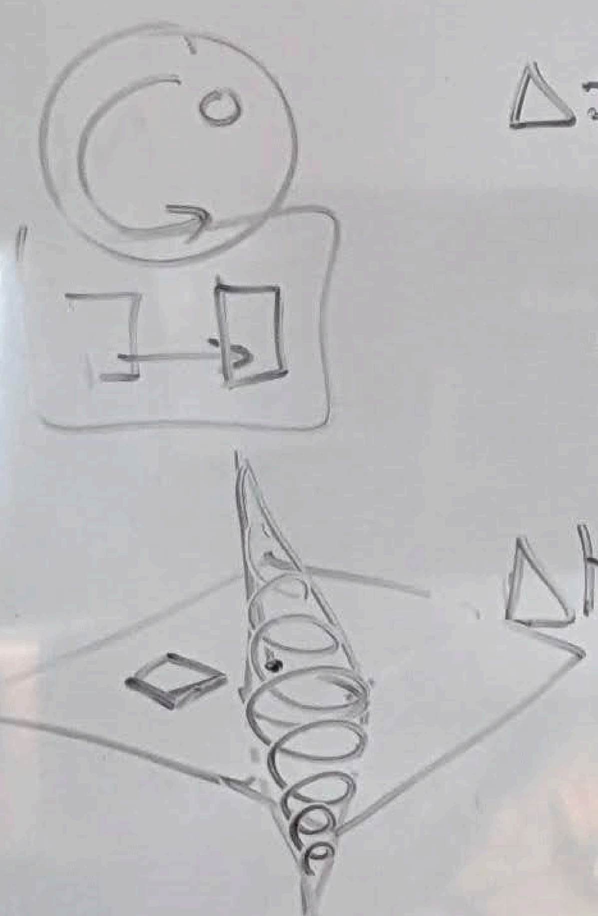


PT
H NH
H NH
Parity
CPT symmetry
Charge
$e^- \rightarrow$
$C \rightarrow e^+$
$\hat{H}^\dagger = \hat{H}$
$H = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^* & c^* \\ b^* & d^* \end{bmatrix} = H^\dagger$
$a = \alpha + i\beta$
$a^* = \alpha - i\beta$
A
B
Im(x)
Re(x)
2π
-1
1
$\hat{H} = \hat{p}^2 - \hat{x}^4$
E k V
E = 1
E = 1 p = 0
$x^2 = \pm 1$
$H^\dagger = H$
$H^{CPT} = H$
$\hat{H} = \begin{bmatrix} s & re^{-i\theta} \\ re^{i\theta} & u \end{bmatrix} \stackrel{?}{=} \frac{1}{2}(s+u)\mathbb{1} + \frac{1}{2}\omega\hat{\sigma}\cdot\hat{n}$
0. in General : $e^{AB} \neq e^A e^B$ But yes if $[A,B]=0$
1. Why is $e^{i\phi\hat{\sigma}\cdot\hat{n}} = \cos\phi\,\mathbb{1} + i\sin\phi\,(\hat{\sigma}\cdot\hat{n})$?
* (slack problem) → Taylor expansions 1st order ♡ *
2. Variational method
* see page 73 PT symmetry
3. Read pgs. 230 → 237
4. ◯ := $(r\cos\theta)^2 + (r\sin\theta)^2$
3 [0, 2π]
HOW ? →
Circle plot
sinθ
cosθ
6. Holger Cartarius doc
5. PT Applications
- optics, EM, QM ← Gain & Loss systems, Field Theory
- Meta materials pg. 387
- Cloaking (optical & acustic)
- Symmetry Breaking phenomena
TODO: Add Apendix on slides. → previous plots → exceptional point phenomena
Change to PP



$\hat{f}(k) \quad f(x) \quad \int f(x)\,dx = 1$

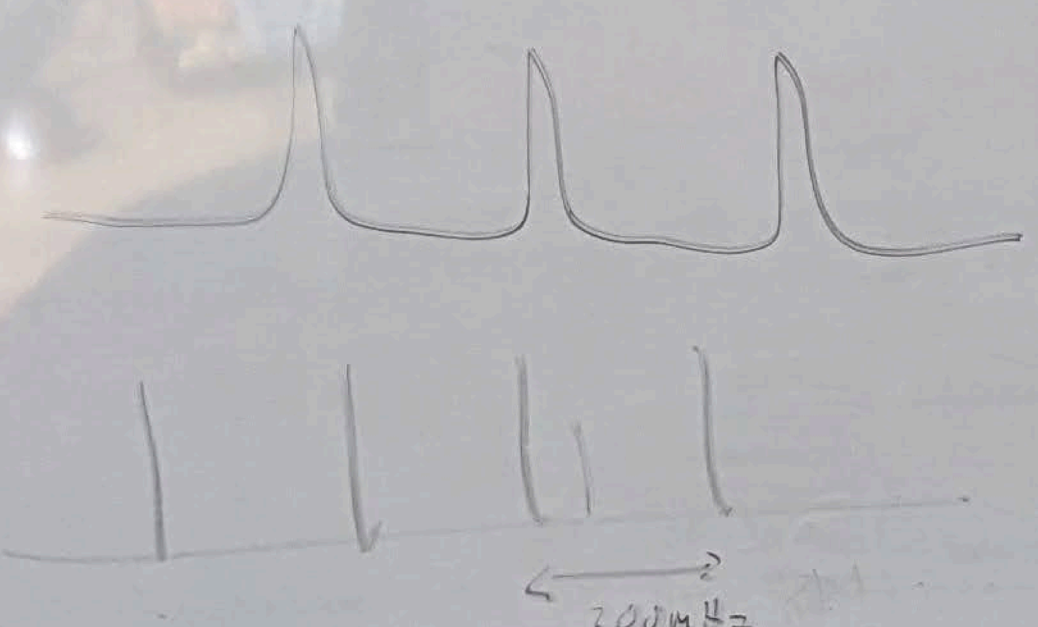
$\Delta\omega\Delta t \geq \frac{1}{2}$

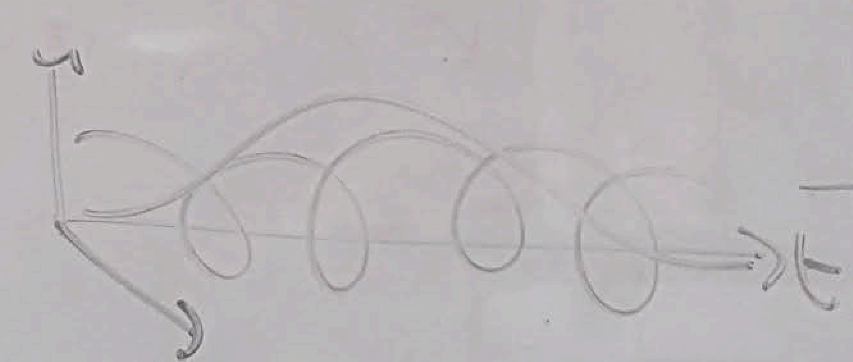


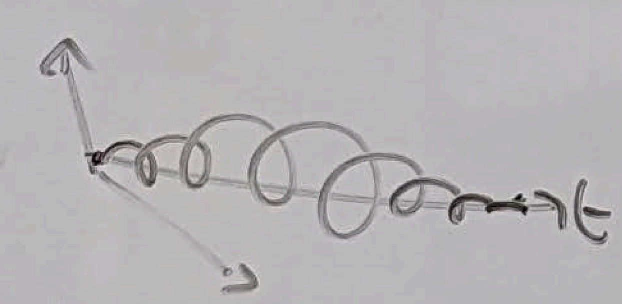
$\Delta x = \sqrt{\int (x-\mu)^2 f\,dx}$

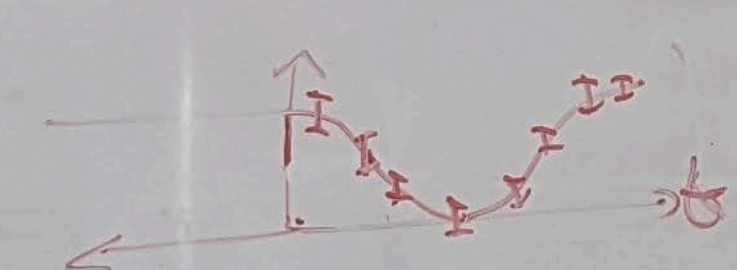
$\mu = \int x f(x)\,dx$

$\Delta k \Delta x \geq \frac{1}{2}$





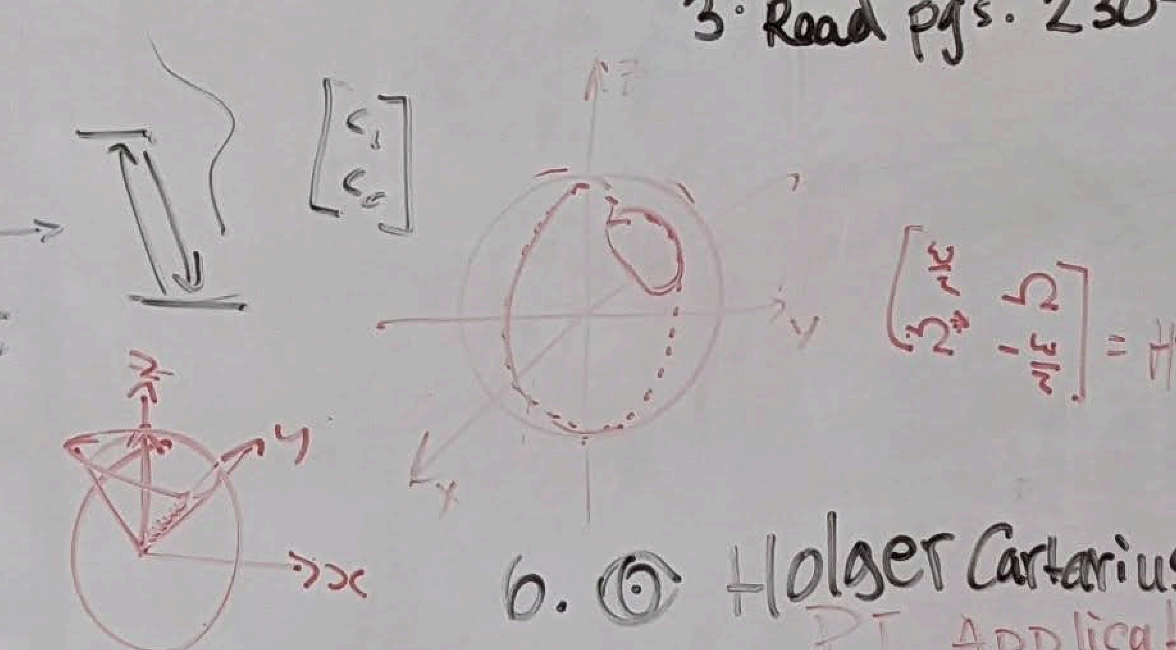




$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi$

$E\psi = (K + V)\psi$

$e^{i(kx - \omega t)}$



$\hat{H} = \begin{bmatrix} s & re^{-i\theta} \\ re^{i\theta} & u \end{bmatrix} \stackrel{?}{=} \frac{1}{2}(s+u)\hat{\mathbb{1}} + \frac{1}{2}\omega\hat{\sigma}\cdot\hat{n}$

0. in General : $e^{AB} \neq e^{A}e^{B}$ But yes if $[A,B]=0$

2. * see page 73 PT symmetry

3. Read pgs. 230 → 237

6. Holger Cartarius doc

5. PT Applications

- optics, EM, QM ← Gain & Loss systems, Field Theory
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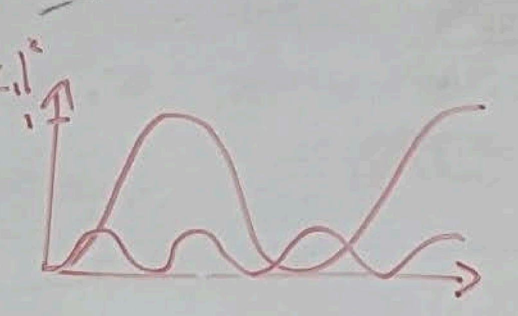


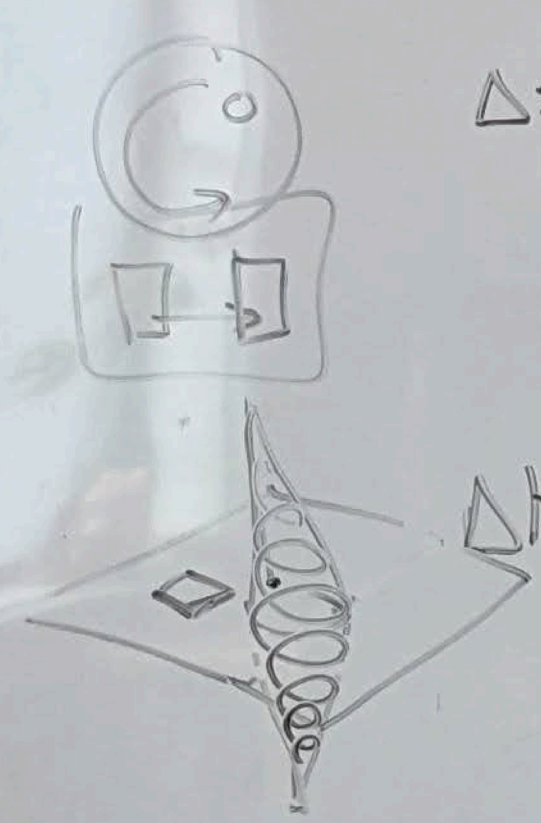
$\hat{f}(k)$ $\quad f(x)$ $\quad \int f(x)\,dx = 1$

$\Delta\omega\,\Delta t \geq \frac{1}{2}$

$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + U\right)\psi$

$E\psi = (K + U)\psi$

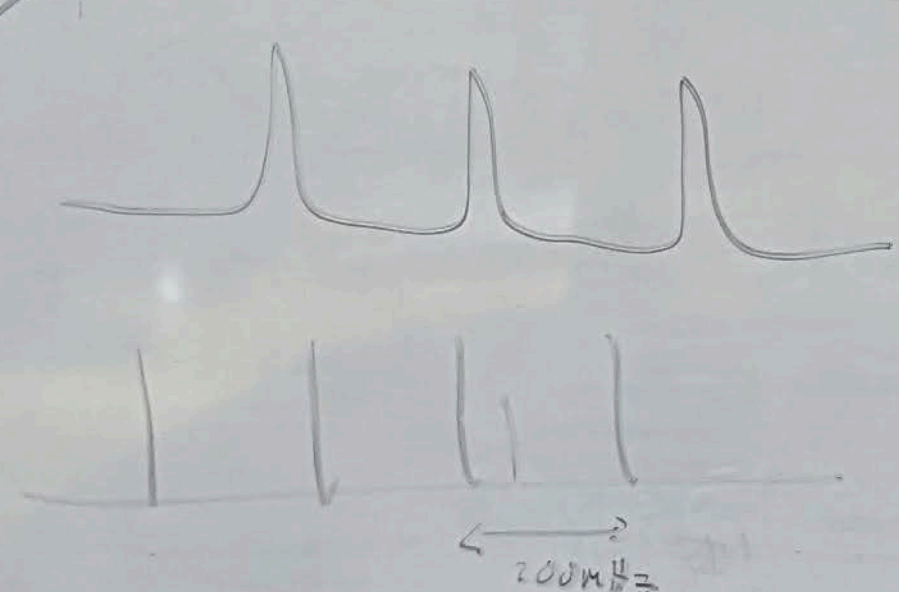
$e^{i(kx - \omega t)}$

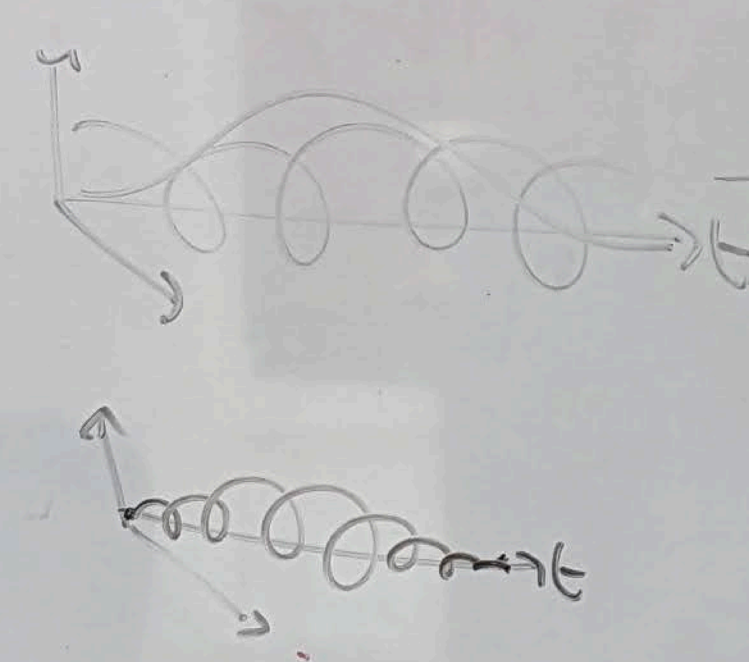


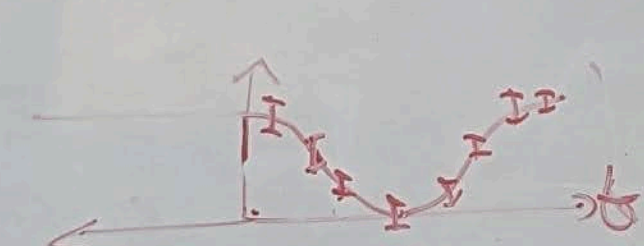
$\Delta x = \sqrt{\int (x-\mu)^2 f\,dx}$

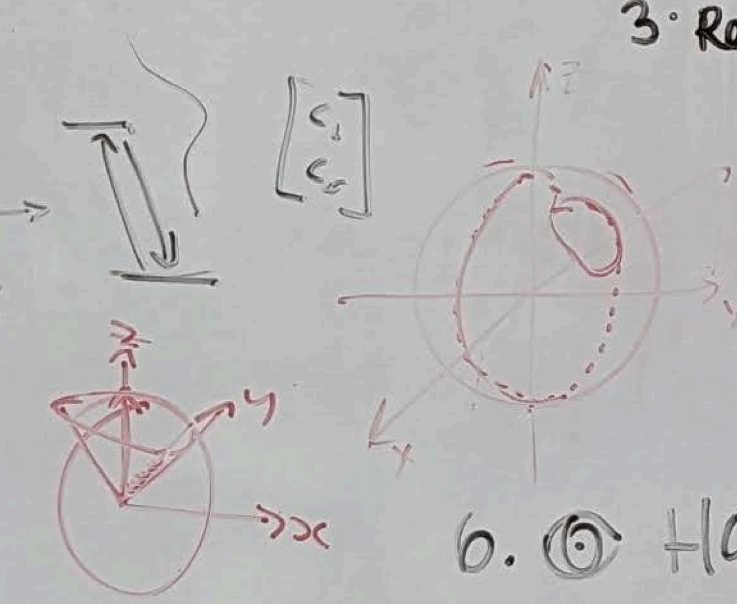
$\mu = \int x f(x)\,dx$

$\Delta k\,\Delta x \geq \frac{1}{2}$









$\begin{bmatrix} \frac{\omega}{2} & \Omega \\ \Omega^* & -\frac{\omega}{2} \end{bmatrix} = H$

$\hat{H} = \begin{bmatrix} s & re^{-i\theta} \\ re^{i\theta} & u \end{bmatrix} \stackrel{?}{=} \frac{1}{2}(s+u)\hat{\mathbb{1}} + \frac{1}{2}\omega\hat{\sigma}\cdot\hat{n}$

0. in General : $e^{AB} \neq e^{A}e^{B}$ But yes if $[A,B]=0$

2. * see page 73 PT symmetry

3. Read pgs. 230 → 237

6. Holger Cartarius doc

5. PT Applications

- Optics, EM, QM ← Gain & Loss systems, Field Theory
  - Meta materials pg. 387
  - Cloaking (optical & acustic)
  - Symmetry Breaking phenomena

TODO: { Add Apendix on slides. → previous plots, → exceptional point phenomena.

2019

This board will be interactive in [10] years

[Y] Jesper

[N] Meer

Coffee

- Input-output problem (?)
- Modelling non-linearities "properly" in microcavity
- Droplets of light
- "Saturation"
  - Re Σ, Im Σ
  - Residue
  - Composite nature

$$|\lambda\rangle \longrightarrow \begin{pmatrix} \langle 1|\lambda\rangle \\ \langle 2|\lambda\rangle \\ \langle 3|\lambda\rangle \end{pmatrix}$$

PT normalised: $|PT(v_n)\cdot v_n| = 1 \Rightarrow v_n \rightarrow \frac{v_n}{\sqrt{|PT(v_n)\cdot v_n|}}$

Hermitian

$\hat{H}$, $|n\rangle$, $\langle n|$, $E_n \Rightarrow \hat{H}|n\rangle = E_n|n\rangle$

$\langle n|\hat{H} = E_n\langle n|$

$\langle n|m\rangle = \delta_{nm}$

$\hat{\mathbb{1}} = \sum_n |n\rangle\langle n|$

$\langle n|\hat{A}\hat{B}|m\rangle = (\mathbb{A}\mathbb{B})_{nm}$

$\langle n|\hat{A}|m\rangle = A_{nm}$

$\hat{A} = \sum_{n,m} |n\rangle A_{nm}\langle m|$

$f(\hat{H}) = \sum_n f(E_n)|n\rangle\langle n|$

Non-Hermitian

$\mathbb{H}$, $|n\rangle$, v_n, $\langle n|$, $CPT(v_n)$, E_n

$\hat{H}|n\rangle = E_n|n\rangle$

$\mathbb{H}v_n = E_n v_n$

$H = \sum_n E_n|n\rangle\langle n|$

$\mathbb{H} = \sum_n E_n v_n [CPT(v_n)]^T$

$\Rightarrow (v_n^{CPT})^T \mathbb{H} = E_n (v_n^{CPT})^T$

$(v_n^{CPT})^T v_m \equiv (CP\, v_n^*) \cdot v_m = \delta_{nm}$

$C_{ij} = \sum_n v_n(i) v_n(j)$

$|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$

$= \sum_n e^{-iE_n t}|n\rangle\underbrace{\langle n|\psi(0)\rangle}$

$\psi(x,t) = \langle x|\psi(t)\rangle$

$= \sum_n e^{-iE_n t}\overbrace{\langle x|n\rangle}^{\phi_n(x)}\langle n|\psi(0)\rangle$

$\langle n|\psi(0)\rangle = \int_C dx\, \underbrace{\langle n|x\rangle}\overbrace{\langle x|\psi(0)\rangle}^{\propto e^{-i\delta x^2}}$

$\langle n|x\rangle = \phi_n^{CPT}(x)$

$= \phi_m(x)\int_C dy \sum_m \phi_m(y)\, \phi_n^*(-y)$

2 ↓↓ / 23.5

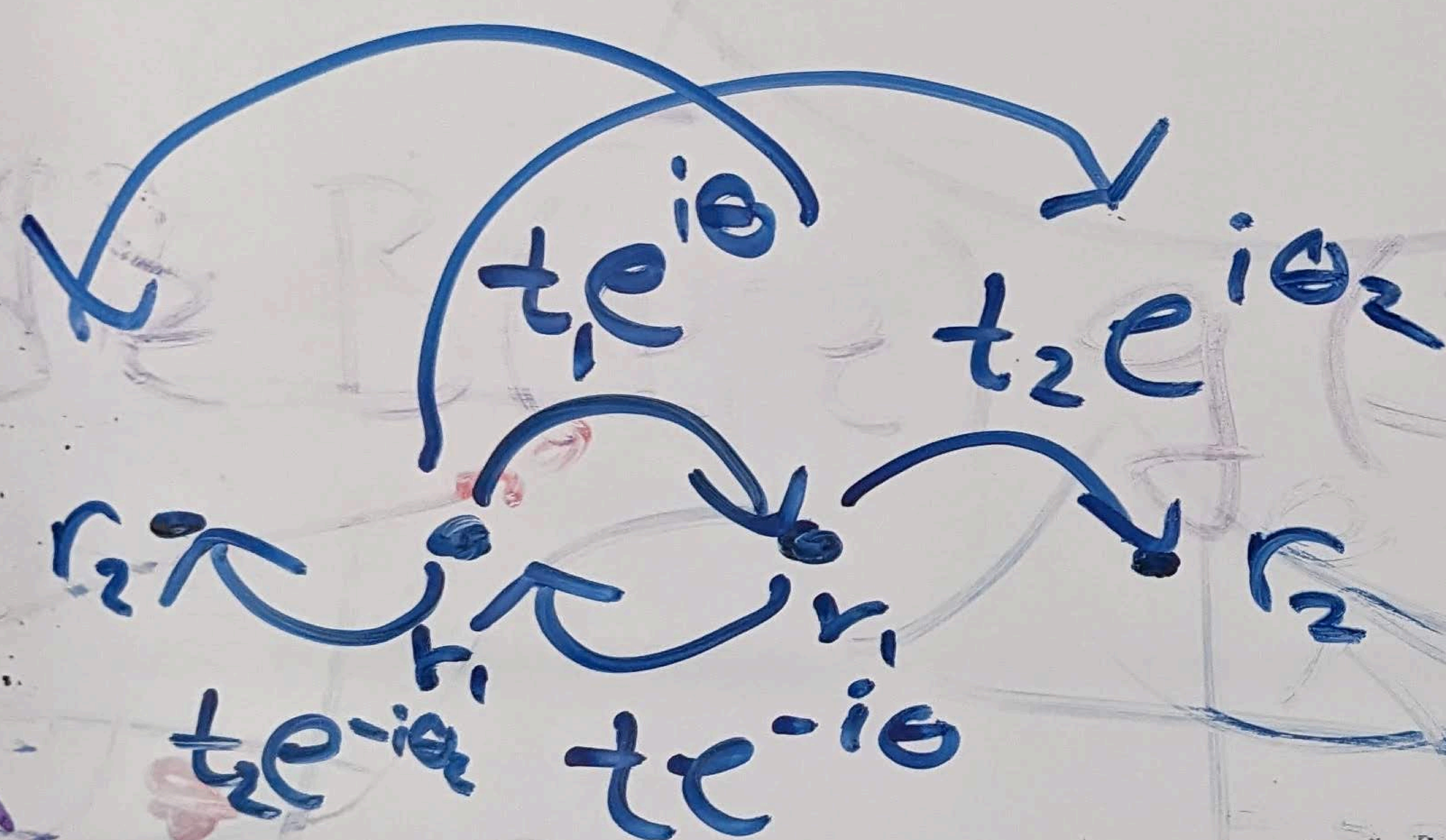
T

0↓

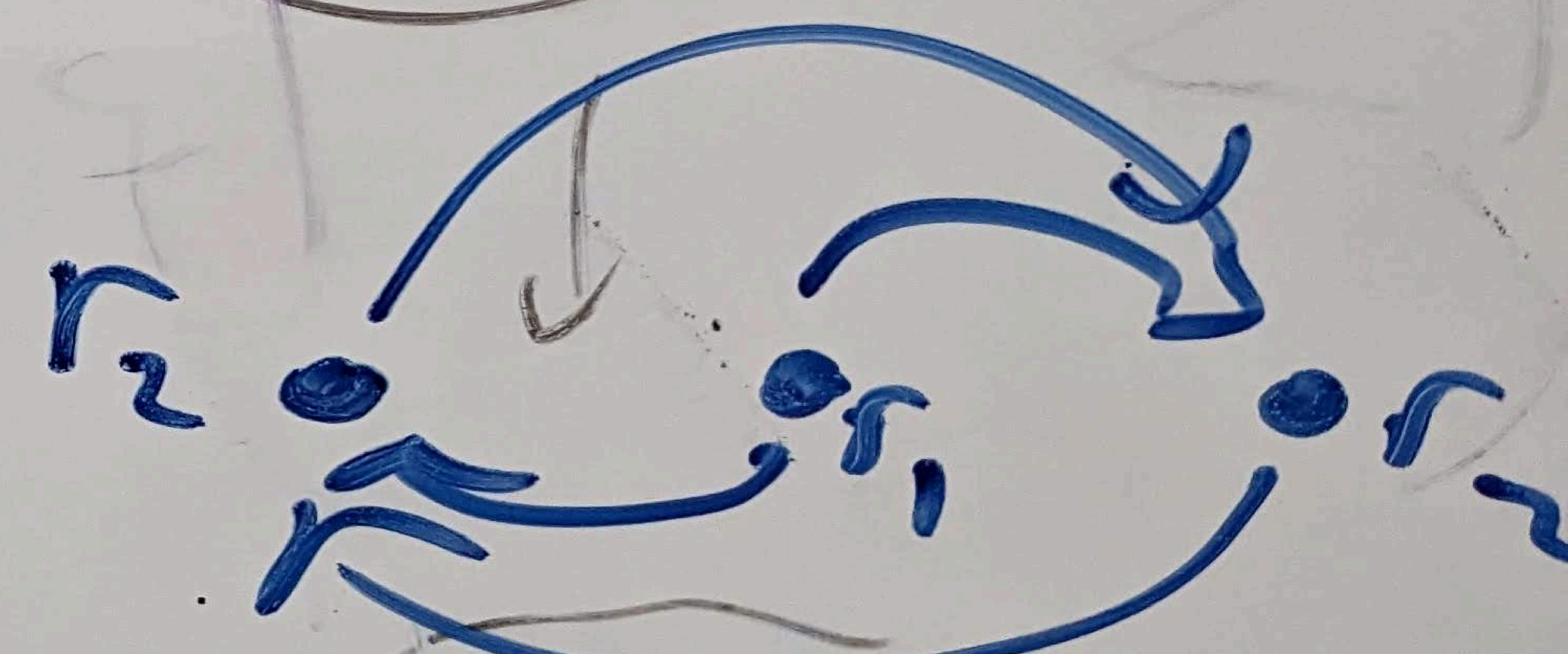
$$\begin{pmatrix} r & te^{i\theta} \\ te^{-i\theta} & r \end{pmatrix}$$

$F = E -$

9↓



0↑



$$(E_\pm, E_\pm) = (PT E_\pm)^T E_\pm$$

(E_+, E_+)

$$\hookrightarrow = \left(\frac{1}{\sqrt{2\cos\alpha}}\right)^2 \left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tau \begin{pmatrix} e^{i\alpha/2} \\ e^{-i\alpha/2} \end{pmatrix} \right)^T \begin{pmatrix} e^{i\alpha/2} \\ e^{-i\alpha/2} \end{pmatrix}$$

$$= \frac{1}{2\cos\alpha} \left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} e^{-i\alpha/2} \\ e^{i\alpha/2} \end{pmatrix} \right)^T \begin{pmatrix} e^{i\alpha/2} \\ e^{-i\alpha/2} \end{pmatrix}$$

$$= \frac{1}{2\cos\alpha} \left(e^{i\alpha/2}, e^{-i\alpha/2} \right) \begin{pmatrix} e^{i\alpha/2} \\ e^{-i\alpha/2} \end{pmatrix}$$

$$= \frac{1}{2\cos\alpha} \left(e^{i\alpha} + e^{-i\alpha} \right) = 1$$

$$(E_+, E_-) = \frac{1}{2\cos\alpha} \left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tau \begin{pmatrix} e^{i\alpha/2} \\ e^{-i\alpha/2} \end{pmatrix} \right)^T \begin{pmatrix} e^{-i\alpha/2} \\ -e^{i\alpha/2} \end{pmatrix}$$

$$= \frac{1}{2\cos\alpha} \left(e^{i\alpha/2}, e^{-\alpha/2} \right) \begin{pmatrix} e^{-i\alpha/2} \\ -e^{i\alpha/2} \end{pmatrix}$$

$$= \frac{1}{2\cos\alpha} (1 - 1) = 0$$

$$(E_-, E_+) = \frac{1}{2\cos\alpha} \left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tau \begin{pmatrix} e^{-i\alpha/2} \\ -e^{i\alpha/2} \end{pmatrix} \right)^T \begin{pmatrix} e^{i\alpha/2} \\ e^{-i\alpha/2} \end{pmatrix}$$

$$= \frac{1}{2\cos\alpha} \left(-e^{-i\alpha/2}, e^{i\alpha/2} \right) \begin{pmatrix} e^{i\alpha/2} \\ e^{-i\alpha/2} \end{pmatrix}$$

$$= \frac{1}{2\cos\alpha} (-1 + 1) = 0$$

$$(E_-, E_-) = \frac{1}{2\cos\alpha} \left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tau \begin{pmatrix} e^{-i\alpha/2} \\ -e^{i\alpha/2} \end{pmatrix} \right)^T \begin{pmatrix} e^{-i\alpha/2} \\ -e^{i\alpha/2} \end{pmatrix}$$

$$= \frac{1}{2\cos\alpha} \left(-e^{i\alpha/2}, e^{i\alpha/2} \right) \begin{pmatrix} e^{-i\alpha/2} \\ -e^{i\alpha/2} \end{pmatrix}$$

$$= \frac{-1}{2\cos\alpha} \left(e^{-i\alpha} + e^{i\alpha} \right) = -1$$

$$(u, v) = (PTu) \cdot v$$

if $s^2 > r^2 \sin^2\theta$ $\rightsquigarrow$ Broken PT symmetry!

$\therefore$ α becomes imaginary

$\Rightarrow$ PT norm vanishes.

$$\vec{E}_+ = \frac{1}{\sqrt{2\cos\alpha}}\begin{pmatrix} e^{i\alpha/2} \\ e^{-i\alpha/2}\end{pmatrix} \quad \& \quad \vec{E}_- = \frac{i}{\sqrt{2\cos\alpha}}\begin{pmatrix} e^{-i\alpha/2} \\ -e^{i\alpha/2}\end{pmatrix}$$

$$C_{ii} = E_+(i)E_+(i) + E_-(i)E_-(i)$$

$$= \left(\frac{1}{\sqrt{2\cos\alpha}}\right)^2\left(e^{i\alpha/2}\cdot e^{i\alpha/2}\right) + \left(\frac{i}{\sqrt{2\cos\alpha}}\right)^2\left(e^{-i\alpha/2}\cdot e^{-i\alpha/2}\right)$$

$$= \frac{1}{2\cos\alpha}\left(e^{i\alpha} + (-1)e^{-i\alpha}\right)$$

$$= \frac{1}{2\cos\alpha}\left(e^{i\alpha} - e^{-i\alpha}\right) = \frac{1}{\cos\alpha}(i\sin\alpha)$$

$$C_{ij} = E_+(i)E_+(j) + E_-(i)E_-(j)$$

$$= \frac{1}{2\cos\alpha}\left(\left(e^{i\alpha/2}\cdot e^{-i\alpha/2}\right) + (i)^2\left(e^{-i\alpha/2}\cdot\left(-e^{i\alpha/2}\right)\right)\right)$$

$$= \frac{1}{2\cos\alpha}\left(1 + (-1)(-1)\right)$$

$$= \frac{1}{\cos\alpha}$$

$$C_{ji} = E_+(j)E_+(i) + E_-(j)E_-(i)$$

$$= \left(\frac{1}{\sqrt{2\cos\alpha}}\right)^2\left(e^{-i\alpha/2}\left(e^{i\alpha/2}\right) + (i)^2\left(-e^{i\alpha/2}\right)e^{-i\alpha/2}\right)$$

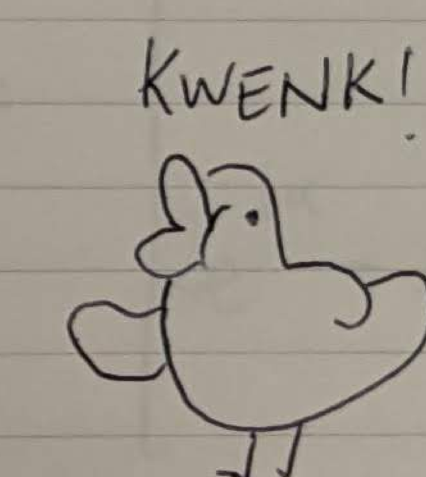
$$= \frac{1}{2\cos\alpha}\left(1 - 1(-1)\right) = \frac{1}{\cos\alpha}$$

$$C_{jj} = E_+(j)E_+(j) + E_-(j)E_-(j)$$

$$C_{jj} = \frac{1}{2\cos\alpha}\left(e^{-i\alpha/2}\cdot e^{-i\alpha/2} + (i)^2\left(-e^{i\alpha/2}\right)\left(-e^{i\alpha/2}\right)\right)$$

$$C_{jj} = \frac{1}{2\cos\alpha}\left(e^{-i\alpha} \cancel{+} - e^{i\alpha}\right) = \frac{-i\sin\alpha}{\cos\alpha}$$

$$\therefore\ C = \frac{1}{\cos\alpha}\begin{bmatrix} i\sin\alpha & 1 \\ 1 & -i\sin\alpha \end{bmatrix}$$



06/09/2022

Creating C - operator

we have $\hat{H} = \begin{pmatrix} re^{i\theta} & s \\ s & re^{-i\theta} \end{pmatrix}$

with eigenvectors

$$\vec{E}_+ = \frac{1}{\sqrt{2\cos\alpha}} \begin{pmatrix} e^{i\alpha/2} \\ e^{-i\alpha/2} \end{pmatrix} \quad \& \quad \vec{E}_- = \frac{\overset{\text{arbitrary scaling factor}}{i}}{\sqrt{2\cos\alpha}} \begin{pmatrix} e^{-i\alpha/2} \\ -e^{i\alpha/2} \end{pmatrix}$$

with corresponding eigenvalues:

where $\frac{r}{s}\sin\theta = \sin\alpha$

$$E_\pm = r\cos\theta \pm \sqrt{s^2 - r^2\sin^2\theta}$$

$(PTE_\pm)$

$$\text{Here } C = \begin{bmatrix} \langle \cancel{E_+} | \uparrow \rangle\langle \uparrow | E_+ \rangle + \langle \cancel{E_+} | \downarrow \rangle\langle \downarrow | E_+ \rangle & \langle \cancel{E_+} | \uparrow \rangle\langle \uparrow | E_- \rangle + \langle \cancel{E_+} | \downarrow \rangle\langle \downarrow | E_- \rangle \\ \vdots & \\ \langle \cancel{E_-} | \uparrow \rangle\langle \uparrow | E_+ \rangle + \langle \cancel{E_-} | \downarrow \rangle\langle \downarrow | E_+ \rangle & \langle \cancel{E_-} | \uparrow \rangle\langle \uparrow | E_- \rangle + \langle \cancel{E_-} | \downarrow \rangle\langle \downarrow | E_- \rangle \end{bmatrix}$$

is a 2x2 matrix where $|\uparrow\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ & $|\downarrow\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Note that: $C(x,y) = \sum_{n=0}^{\infty} \phi_n(x)\phi_n(y)$

in the continuous representation

but since we have a 2-Level System this definition is not useful as it is written

But we must notice the product: wavefunction is $\overset{\text{twice}}{\text{evaluated}}$ at different points of its domain & then multiplied

in 2x2 system:

$$C_{11} = \langle E_+ | \uparrow \rangle\langle \uparrow | E_+ \rangle + \langle E_+ | \downarrow \rangle\langle \downarrow | E_+ \rangle$$

$$= \frac{1}{2\cos\alpha}\left((e^{i\alpha/2}, e^{-i\alpha/2}) \begin{pmatrix} 1 \\ 0 \end{pmatrix} (1,0) \begin{pmatrix} e^{i\alpha/2} \\ e^{-i\alpha/2} \end{pmatrix} + (e^{i\alpha/2}, e^{-i\alpha/2}) \begin{pmatrix} 0 \\ 1 \end{pmatrix} (0,1) \begin{pmatrix} e^{i\alpha/2} \\ e^{-i\alpha/2} \end{pmatrix} \right)$$

$$= \frac{1}{2\cos\alpha}\left((e^{i\alpha/2}) e^{i\alpha/2} + (e^{-i\alpha/2}) e^{-i\alpha/2} \right)$$

$$C_{11} = \frac{1}{\cos\alpha} i\sin\alpha$$

$$C_{22} = \langle E_+ | \uparrow \rangle\langle \uparrow | E_- \rangle + \langle E_+ | \downarrow \rangle\langle \downarrow | E_- \rangle$$

$$C_{12} = \frac{i}{2\cos\alpha}\left((e^{i\alpha/2}, e^{-i\alpha/2}) \begin{pmatrix} 1 \\ 0 \end{pmatrix} (1,0) \begin{pmatrix} e^{-i\alpha/2} \\ -e^{i\alpha/2} \end{pmatrix} + (e^{i\alpha/2}, e^{-i\alpha/2}) \begin{pmatrix} 0 \\ 1 \end{pmatrix} (0\,1) \begin{pmatrix} e^{-i\alpha/2} \\ -e^{i\alpha/2} \end{pmatrix} \right)$$

$$C_{12} = \frac{i}{2\cos\alpha}\left((e^{i\alpha/2}) e^{-i\alpha/2} + (e^{-i\alpha/2})(-e^{i\alpha/2}) \right)$$

$$\therefore C_{12} = \frac{i}{2\cos\alpha}\left(1 + (-1) \right)$$

$$C_{12} = 0$$

but I want it to be $= 1$

because Bender says that

$$C = \frac{1}{\cos\alpha}\begin{bmatrix} i\sin\alpha & 1 \\ 1 & -i\sin\alpha \end{bmatrix}$$

No. 31/08/2022

DATE

$$\hat{H} = \begin{pmatrix} re^{i\theta} & s \\ s & re^{-i\theta} \end{pmatrix} \qquad r, s, \theta \in \mathbb{R}$$

is PT symmetric: * recall that $[P, T] = 0$

$$PT\hat{H}PT = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} T \begin{pmatrix} re^{i\theta} & s \\ s & re^{-i\theta} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} T$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} re^{-i\theta} & s \\ s & re^{i\theta} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} s & re^{i\theta} \\ re^{-i\theta} & s \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} re^{i\theta} & s \\ s & re^{-i\theta} \end{pmatrix} = \hat{H}$$

its eigenvalues are

$$\det(\hat{H} - \mathbb{I}\lambda) = \begin{vmatrix} re^{i\theta} - \lambda & s \\ s & re^{-i\theta} - \lambda \end{vmatrix} = r^2(e^{i\theta} e^{-i\theta}) - re^{i\theta}\lambda - re^{-i\theta}\lambda + \lambda^2 - s^2$$

$$\text{let: } r^2(1) - r\lambda(e^{i\theta} + e^{-i\theta}) + \lambda^2 - s^2 = 0$$

$$\therefore r^2 - 2r\cos\theta\lambda + \lambda^2 - s^2 = 0$$

$$\Rightarrow r^2 - s^2 - 2r\cos\theta\lambda + \lambda^2 = 0$$

$$\Rightarrow \underbrace{(r+s)(r-s)} - 2r\cos\theta\lambda + \lambda^2 = 0$$

$$= a - 2r\cos\theta\lambda + \lambda^2 = 0$$

No.

DATE

$$\therefore \lambda = \frac{2r\cos\theta \pm \sqrt{(-2r\cos\theta)^2 - 4a}}{2}$$

where Δ is

$$\Rightarrow \lambda = \frac{\cancel{2}r\cos\theta}{\cancel{2}} \pm \frac{\cancel{2}\sqrt{r^2\cos^2\theta - a}}{\cancel{2}} \quad : \; r^2\cos^2\theta - (r^2 - s^2)$$

2 real values of λ iff $r^2\cos^2\theta - a \geq 0$ (unbroken)
otherwise λ is complex conjugate pairs (broken)
(There are 2 regions to consider)

$$r^2\cos^2\theta - r^2 + s^2$$

$$= r^2(\cos^2\theta - 1) + s^2$$

$$= -r^2\sin^2\theta + s^2$$

Then the eigenvalues are:

$$\lambda = r\cos\theta \pm \sqrt{s^2 - r^2\sin^2\theta}$$

in the region of unbroken PT symmetry (unitary!)

$$0 = (\hat{H} - \lambda\mathbb{I})\tilde{c} = \lambda\tilde{c} \Leftrightarrow \begin{pmatrix} re^{i\theta} - \lambda_i & s \\ s & re^{-i\theta} - \lambda_i \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \lambda_i \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = 0$$

for $\lambda_1 = r\cos\theta + \sqrt{s^2 - r^2\sin^2\theta}$

$$\begin{pmatrix} re^{i\theta} & s \\ s & re^{-i\theta} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = r\cos\theta + (s^2 - r^2\sin^2\theta)^{\frac{1}{2}} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} c_1 re^{i\theta} + sc_2 \\ sc_1 + re^{-i\theta}c_2 \end{pmatrix} = \begin{pmatrix} r\cos\theta\, c_1 + (s^2 - r^2\sin^2\theta)^{\frac{1}{2}} c_1 \\ r\cos\theta\, c_2 + (s^2 - r^2\sin^2\theta)^{\frac{1}{2}} c_2 \end{pmatrix}$$

31/08/22

$$\hat{H} = \begin{pmatrix} re^{i\theta} & s \\ s & re^{-i\theta} \end{pmatrix} \qquad r, s, \theta \in \mathbb{R}$$

$\hat{H}$ is PT symmetric

recall that $[P,T]=0$

$$PT\hat{H}PT = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} T \begin{pmatrix} re^{i\theta} & s \\ s & re^{-i\theta} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} T$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} re^{-i\theta} & s \\ s & re^{i\theta} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} s & re^{i\theta} \\ re^{-i\theta} & s \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} re^{i\theta} & s \\ s & re^{-i\theta} \end{pmatrix} = \hat{H}$$

it's eigenvalues are

$$\det(\hat{H} - \mathbb{I}\lambda) = 0 \Rightarrow \lambda = r\cos\theta \pm \sqrt{s^2 - r^2\sin^2\theta}$$

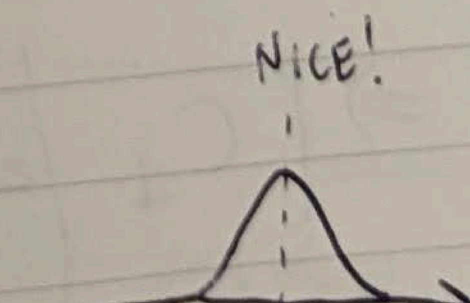
the case where PT-symmetry is unbroken:

$$\Delta = s^2 - r^2 \sin\theta \geq 0$$

for $\lambda_1 = r\cos\theta \pm \sqrt{s^2 - r^2\sin^2\theta}$

$$\begin{pmatrix} re^{i\theta} & s \\ s & re^{-i\theta} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = r\cos\theta + (s^2 - r^2\sin^2\theta)^{\frac{1}{2}} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} c_1 re^{i\theta} + sc_2 \\ sc_1 + re^{-i\theta} c_2 \end{pmatrix} = \begin{pmatrix} r\cos\theta c_1 + (s^2 - r^2\sin^2\theta)^{\frac{1}{2}} c_1 \\ r\cos\theta c_2 + (s^2 - r^2\sin^2\theta)^{\frac{1}{2}} c_2 \end{pmatrix}$$



See Time evolution : Lindbladian equation

(1) $C_1 re^{i\theta} + C_2 S = r\cos\theta C_1 + (s^2 - r^2\sin^2\theta)^{\frac{1}{2}} C_1$

$\Rightarrow C_2 S = r\cos\theta C_1 + (s^2 - r^2\sin^2\theta)^{\frac{1}{2}} C_1 - C_1 re^{i\theta}$

$\Rightarrow C_2 S = (r\cos\theta + s(1 - \frac{r^2}{s^2}\sin^2\theta)^{\frac{1}{2}} - re^{i\theta}) C_1$

$\Rightarrow C_2 S = r(-\frac{e^{i\theta} + e^{-i\theta}}{2} + \frac{s}{r}(1 - \frac{r^2}{s^2}\sin^2\theta)^{\frac{1}{2}}) C_1$

let $\frac{r}{s}\sin\theta = \sin\alpha$

$\Rightarrow C_2 S = r(-(\frac{e^{i\theta} - e^{-i\theta}}{2}) + \frac{s}{r}(1 - \sin^2\alpha)^{\frac{1}{2}}) C_1$

$\Rightarrow C_2 S = r(-i\sin\theta + \frac{s}{r}|\cos\alpha|) C_1$

$\Rightarrow C_2 = (-\frac{r}{s} i\sin\theta + \cos\alpha) C_1$

$\Rightarrow \boxed{C_2} = (\cos\alpha - i\sin\alpha) C_1 = \boxed{e^{-i\alpha} C_1}$

(2) $SC_1 = r\cos\theta C_2 + (s^2 - r^2\sin^2\theta)^{\frac{1}{2}} C_2 - re^{-i\theta} C_2$

$\Rightarrow SC_1 = (r\cos\theta + s(1 - \frac{r^2}{s^2}\sin^2\theta)^{\frac{1}{2}} - re^{-i\theta}) C_2$

$\Rightarrow SC_1 = (r(\frac{e^{i\theta} - e^{-i\theta}}{2}) + s(1 - \sin^2\alpha)^{\frac{1}{2}}) C_2$

$\Rightarrow SC_1 = (r(i\sin\theta) + s|\cos\alpha|) C_2$

$\Rightarrow \boxed{C_1} = (\cos\alpha + i\sin\alpha) C_2 = \boxed{e^{i\alpha} C_2}$

for $X_2 = r\cos\theta - \sqrt{s^2 - r^2\sin^2\theta}$

$$\begin{pmatrix} re^{i\theta} & s \\ s & re^{-i\theta} \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = r\cos\theta - (s^2 - r^2\sin^2\theta)^{\frac{1}{2}} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

$$\begin{pmatrix} re^{i\theta} C_1 + SC_2 \\ SC_1 + re^{-i\theta} C_2 \end{pmatrix} = \begin{pmatrix} r\cos\theta C_1 - (s^2 - r^2\sin^2\theta)^{\frac{1}{2}} C_1 \\ r\cos\theta C_2 - (s^2 - r^2\sin^2\theta)^{\frac{1}{2}} C_2 \end{pmatrix}$$

(1) : $re^{i\theta} C_1 + SC_2 = (r\cos\theta - (s^2 - r^2\sin^2\theta)^{\frac{1}{2}}) C_1$

Here, I made an error

$\Rightarrow SC_2 = (r\cos\theta - s(1 - \frac{r^2}{s^2}\sin^2\theta)^{\frac{1}{2}} - re^{i\theta}) C_1$

$\Rightarrow SC_2 = (r(-\frac{e^{i\theta} + e^{-i\theta}}{2}) - s(1 - \sin^2\alpha)^{\frac{1}{2}}) C_1$

$\Rightarrow SC_2 = (-r(\frac{e^{i\theta} - e^{-i\theta}}{2}) - s\cos\alpha) C_1$

$\Rightarrow C_2 = (-\frac{ir}{s}\sin\theta - \cancel{s}\cos\alpha) C_1$

$\Rightarrow C_2 = (-\cos\alpha - i\sin\alpha) C_1$

$\Rightarrow \boxed{C_2 = -e^{-i\alpha} C_1}$ should be $e^{-i\alpha} C_1$

(2) : $SC_1 + re^{-i\theta} C_2 = r\cos\theta C_2 - (s^2 - r^2\sin^2\theta)^{\frac{1}{2}} C_2$

$\Rightarrow SC_1 = (r(\frac{e^{i\theta} - e^{-i\theta}}{2}) - s(1 - \frac{r^2}{s^2}\sin\theta)^{\frac{1}{2}}) C_2$

$\Rightarrow C_1 = (i\frac{r}{s}\sin\theta - \cancel{s}|\cos\alpha|) C_2$

$\Rightarrow \boxed{C_1} = (-\cos\alpha + i\sin\alpha) C_2 = \boxed{-e^{-i\alpha} C_2}$

Our eigenvectors are

$$\tilde{C}_1 = \begin{pmatrix} e^{i\alpha} \\ e^{-i\alpha} \end{pmatrix} \quad \& \quad \tilde{C}_2 = \begin{pmatrix} e^{-i\alpha} \\ -e^{i\alpha} \end{pmatrix}$$

with eigenvalues $\lambda_1 = r\cos\theta + \sqrt{s^2 - r^2\sin^2\theta}$

$\lambda_2 = r\cos\theta - \sqrt{s^2 - r^2\sin^2\theta}$

are our vectors normalised?

$$\hat{C}_1 = \frac{\tilde{C}_1}{|\tilde{C}_1|} = \frac{1}{\sqrt{e^{2i\alpha} + e^{-2i\alpha}}} \begin{pmatrix} e^{i\alpha} \\ e^{-i\alpha} \end{pmatrix} = \frac{1}{\sqrt{2\cos 2\alpha}} \begin{pmatrix} e^{i\alpha} \\ e^{-i\alpha} \end{pmatrix}$$

now they are

$$\hat{C}_2 = \frac{\tilde{C}_2}{|\tilde{C}_2|} = \frac{1}{\sqrt{(e^{-i\alpha})^2 + (-e^{i\alpha})^2}} \begin{pmatrix} e^{-i\alpha} \\ -e^{i\alpha} \end{pmatrix} = \frac{1}{\sqrt{2\cos 2\alpha}} \begin{pmatrix} e^{-i\alpha} \\ -e^{i\alpha} \end{pmatrix}$$

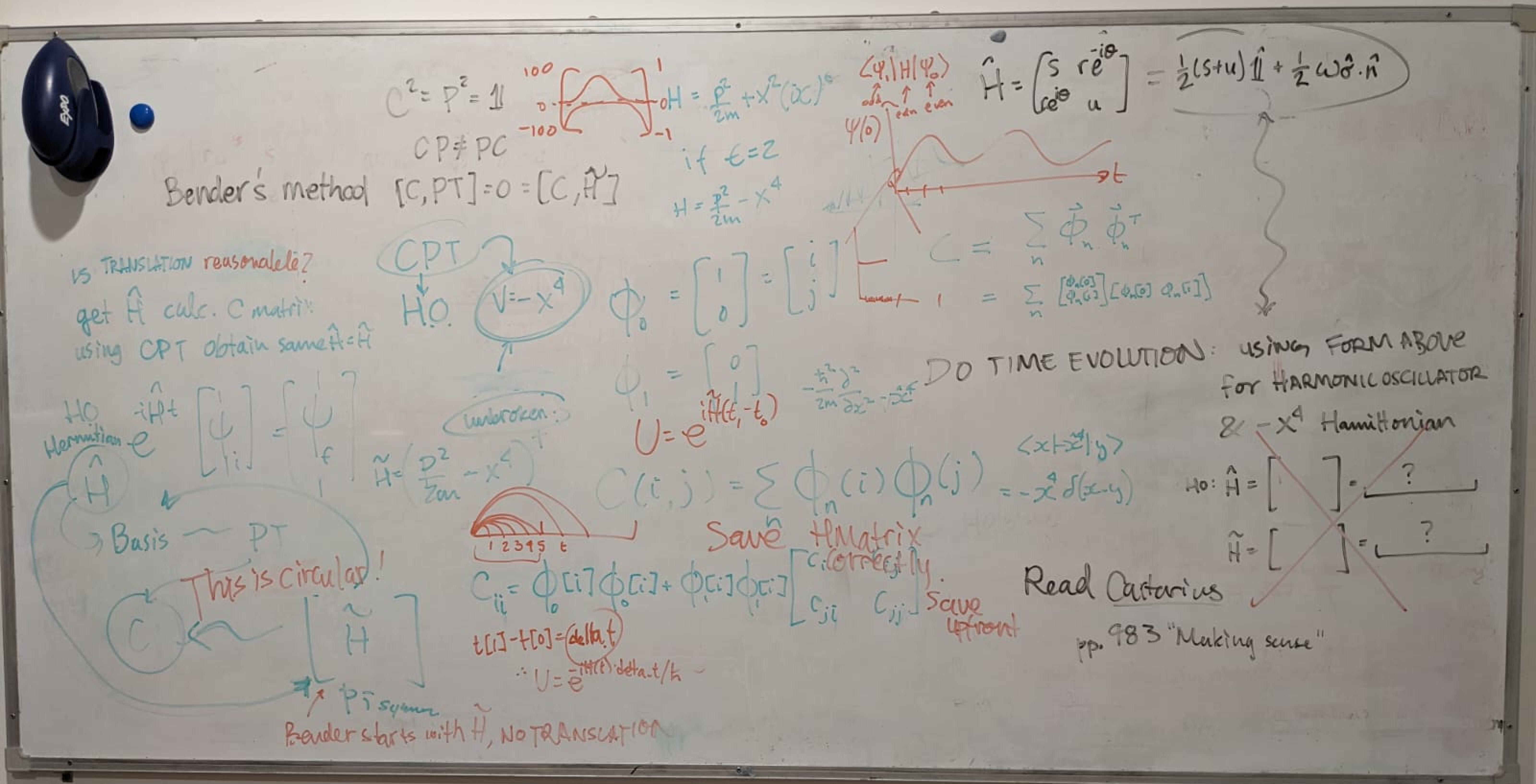
are they orthogonal?

$$\langle \tilde{C}_1 | \tilde{C}_2 \rangle = \left(\frac{1}{\sqrt{2\cos 2\alpha}}\right)\left(\frac{1}{\sqrt{2\cos 2\alpha}}\right) \left(e^{i\alpha}, e^{-i\alpha}\right) \begin{pmatrix} e^{-i\alpha} \\ -e^{i\alpha} \end{pmatrix}$$

$$= \frac{1}{2\cos 2\alpha}\left(e^{i\alpha} e^{-i\alpha} + e^{-i\alpha}(-e^{i\alpha})\right)$$

$$= \frac{1}{2\cos 2\alpha}\left(1 + (-1)\right) = 0$$

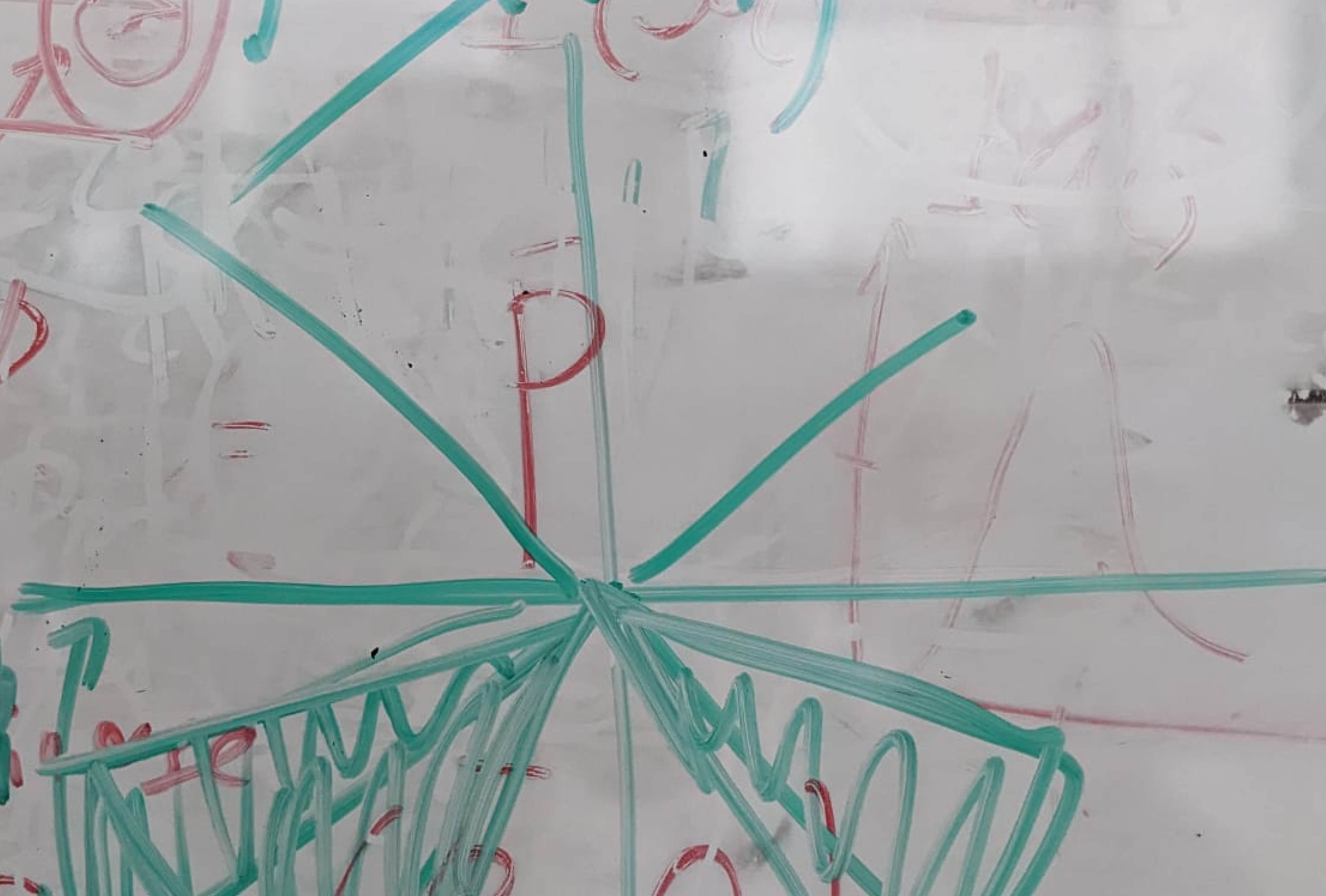
Yup!


C² = P² = 1
CP ≠ PC
Bender's method [C,PT] = 0 = [C,H]
H = p²/2m + x²(ix)^ε
if ε = 2
H = p²/2m − x⁴
⟨ψ₁|H|ψ₀⟩
odd even even
ψ(0)
t
Ĥ = [s re^{-iθ}; re^{iθ} u] = ½(s+u)𝟙 + ½ωσ̂·n̂
C = Σₙ φₙ φₙᵀ
1 = Σₙ [φₙ[0] φₙ[1]][φₙ[0] φₙ[1]]
is TRANSLATION reasonable?
get Ĥ calc. C matrix
using CPT obtain same H̃ = Ĥ
CPT
H.O.
V = −x⁴
φ₀ = [1 0] = [i j]
φ₁ = [0 1]
U = e^{iH̃(t₁−t₀)}
−ħ²/2m ∂²/∂x² − x⁴
DO TIME EVOLUTION: USING FORM ABOVE for HARMONIC OSCILLATOR & −x⁴ Hamiltonian
HO
Hermitian
e^{−iĤt}
[ψ_i] = [ψ_f]
unbroken
H̃ = (p²/2m − x⁴)
C(i,j) = Σₙ φₙ(i) φₙ(j)
⟨x|−x⁴|y⟩ = −x⁴ δ(x−y)
HO: Ĥ = [ ] = ?
H̃ = [ ] = ?
Basis ~ PT
This is circular!
Save H Matrix correctly
1 2 3 4 5 t
C_{ij} = φ₀[i]φ₀[j] + φ₁[i]φ₁[j]
[C_{ii} C_{ij}; C_{ji} C_{jj}]
Save upfront
Read Cartarius
pp. 983 "Making sense"
t[1] − t[0] = delta.t
∴ U = e^{−iH̃(t)·delta.t/ħ}
PT symm
Bender starts with H̃, NO TRANSLATION

$$\int 1 \times e^{i\phi}$$

phase

$$\cancel{\int d(x+iy)}$$



board will be interactive in [10] years

[Y] Jesper

[N] Meer

Coffee

- Input-output problem (?)
- Modelling non-linearities "properly" in microcavity
- Droplets of light
- "Saturation"
  - Re Σ, Im Σ
  - Residue
  - Composite nature

$$\hat{H} \rightarrow |n\rangle, E_n \rightarrow \hat{C}$$

$$\underbrace{\langle n|m\rangle}_{CPT} = \delta_{nm}$$

$$\hat{\mathbb{1}} = \sum_n |n\rangle\langle n|$$

$$|\psi\rangle = \sum_m C_m |m\rangle$$

$$\begin{pmatrix} E_1 & & 0 \\ & E_2 & \\ 0 & & E_3 \; \ddots \end{pmatrix} = H_{nm}$$

$$\hat{H} = \hat{\mathbb{1}}\,\hat{H}\,\hat{\mathbb{1}} = \sum_{n,m} |n\rangle \overbrace{\langle n|\hat{H}|m\rangle}^{H_{nm}} \langle m| = \sum_n E_n |n\rangle\langle n|$$

$$e^{-i\hat{H}t} = \sum_n e^{-iE_n t} |n\rangle\langle n|$$

$$\langle x|\psi\rangle = \psi(x) \,?$$

$$\hat{\mathbb{1}} = \int dx\, |x\rangle\langle x| \,?$$

$$\psi(x, t=0) = e^{-x^2}$$

CPT $|x\rangle = |x\rangle$

$$|\psi(t)\rangle = e^{-i\hat{H}t}|\psi\rangle$$

Stoke's wedge

$$\langle x|\psi(t)\rangle = \sum_n e^{-iE_n t} \underbrace{\langle x|n\rangle}_{\text{WKB}} \int dy \underbrace{\langle n|y\rangle}_{\phi_{n,CPT}(y)} \langle y|\psi\rangle$$

WKB $\leftarrow$ $\phi_{n,CPT}(y)$

$$\langle \phi|\psi\rangle = \int dx\, \phi_{CPT}(x)\, \psi(x)$$

How am I constructing the operator rn?

C-operator (normalised states):

products = [ ]
for ψ in normalised states:

products.append($\psi \cdot \psi$)

$$\psi_j = \sum_i \psi_{ji} \psi_{ij}$$

single numbers!

$\therefore$ # products = $[\psi_0, \psi_1, \psi_2 \dots \psi_j]$

$$\text{C-op} = \sum_n \boxed{(-1)^n (PT(\psi_{j}) \cdot \psi_{ij})_n} = \sum_n C_n$$

my current operator is too large

$\Leftrightarrow C^2 \approx 100$ not $\mathbb{1}$

Why is it so large?

$n \in (0, 9)$

$i, j \in (0, 4096)$

Tests:

$P^2 = \mathbb{1}$ Struggling with P_states ie. $P\psi(x) \stackrel{?}{=} \psi(-x)$

$C^2 = \mathbb{1}$

$[C, H] = 0$

Creating C according to Jesper:

$$C_{ij} = \sum_n (-1)^n ((PT(V_{ji})) \cdot V_{ij}))_n$$

- Input-output problem(?)
- Modelling non-linearities "properly" in microcavity
- Droplets of light
- "Saturation"
  - Re Σ, Im Σ
  - Residue
  - Composite nature

Coffee -1!

$$Z_{int} = \sum_{p} e^{-\beta \epsilon_p}$$

$\{1, 3, 5, \ldots\}$

$$Q(x) = E_{WKB} - V(x)$$

$$x_0 = \sqrt{E_{WKB}}$$

$$\Psi_{WKB}(x, E_{WKB}) \approx \frac{c_1}{Q(x)^{\frac{1}{4}}} e^{+i\int_{-x_0}^{x} \sqrt{Q(x)}\, dx} + \frac{c_2}{Q^{\frac{1}{4}}} e^{-i\ldots\sqrt{E}\int_{1}^{x/\sqrt{E}} \sqrt{1-y^2}\, dy}$$

Δx

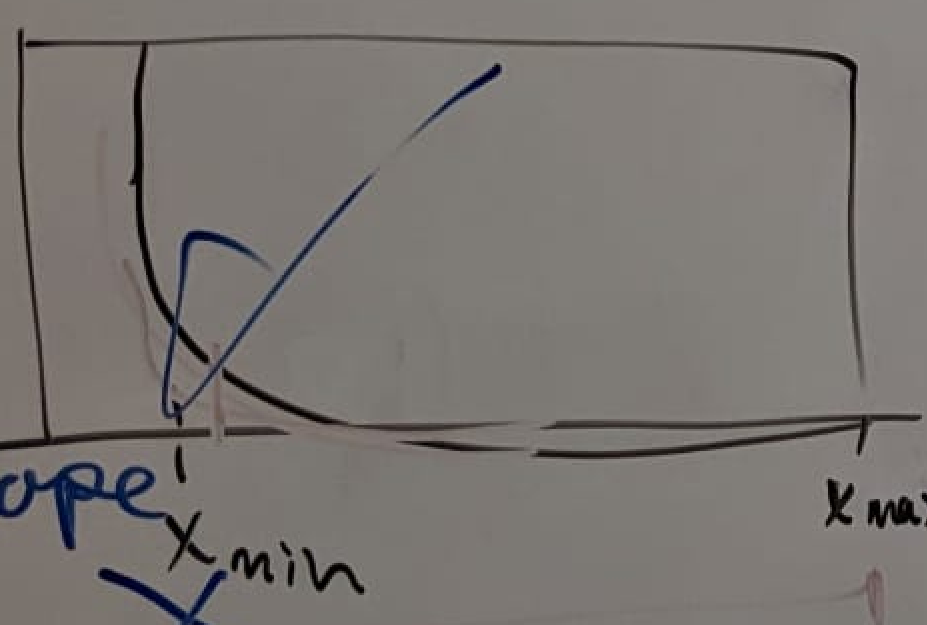
$$\int_{\sqrt{E}}^{x} \sqrt{E - x^2}\, dx$$

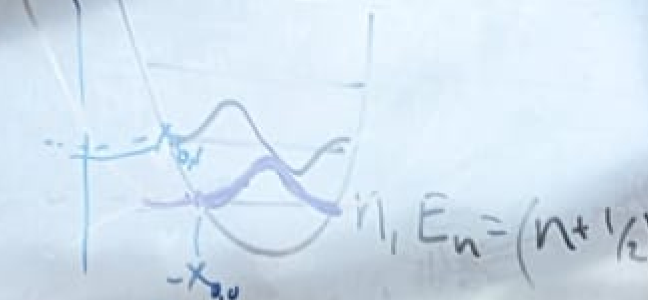
$$= x[1] - x[0] = \sqrt{E}\, \frac{1}{2}\left(y\sqrt{1-y^2} + \sin^{-1}(y)\right)\Big|_{1}^{x/\sqrt{E}}$$

$$= \sqrt{E}\left(\frac{1}{2} y \sqrt{1-y^2} + \cos^{-1}(y)\right)\Big|^{y = x/\sqrt{E}}$$

$x \to -x$

$i \to -i$



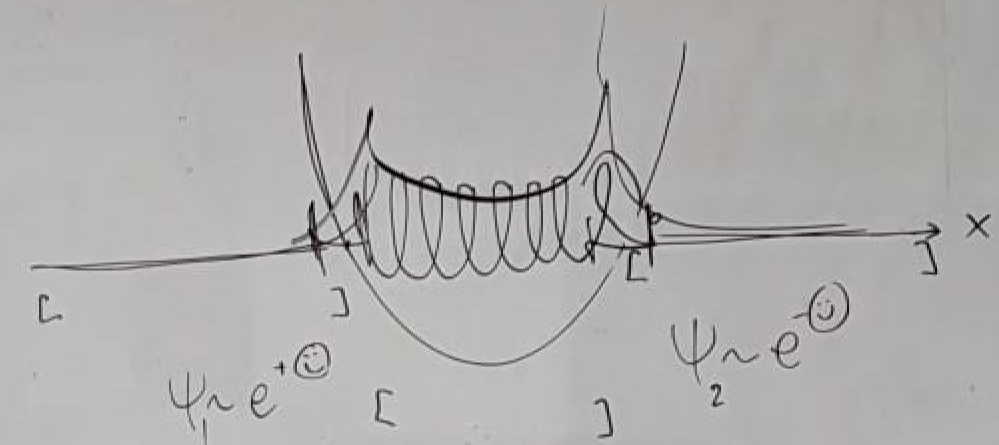


Tests:

$P^2 = \mathbb{1}$

$C^2 = \mathbb{1}$

$[C, H] = 0$



$\psi_1 \sim e^{+☺}$ $\psi_2 \sim e^{-☺}$

$\psi_1 \to 0 \quad x \to -\infty$

$\psi(x) \quad e^{\frac{i}{\hbar} \text{Integral}}$

$$\int_C \sqrt{E - V(x)}\, dx = \left(n + \tfrac{1}{2}\right)\pi$$

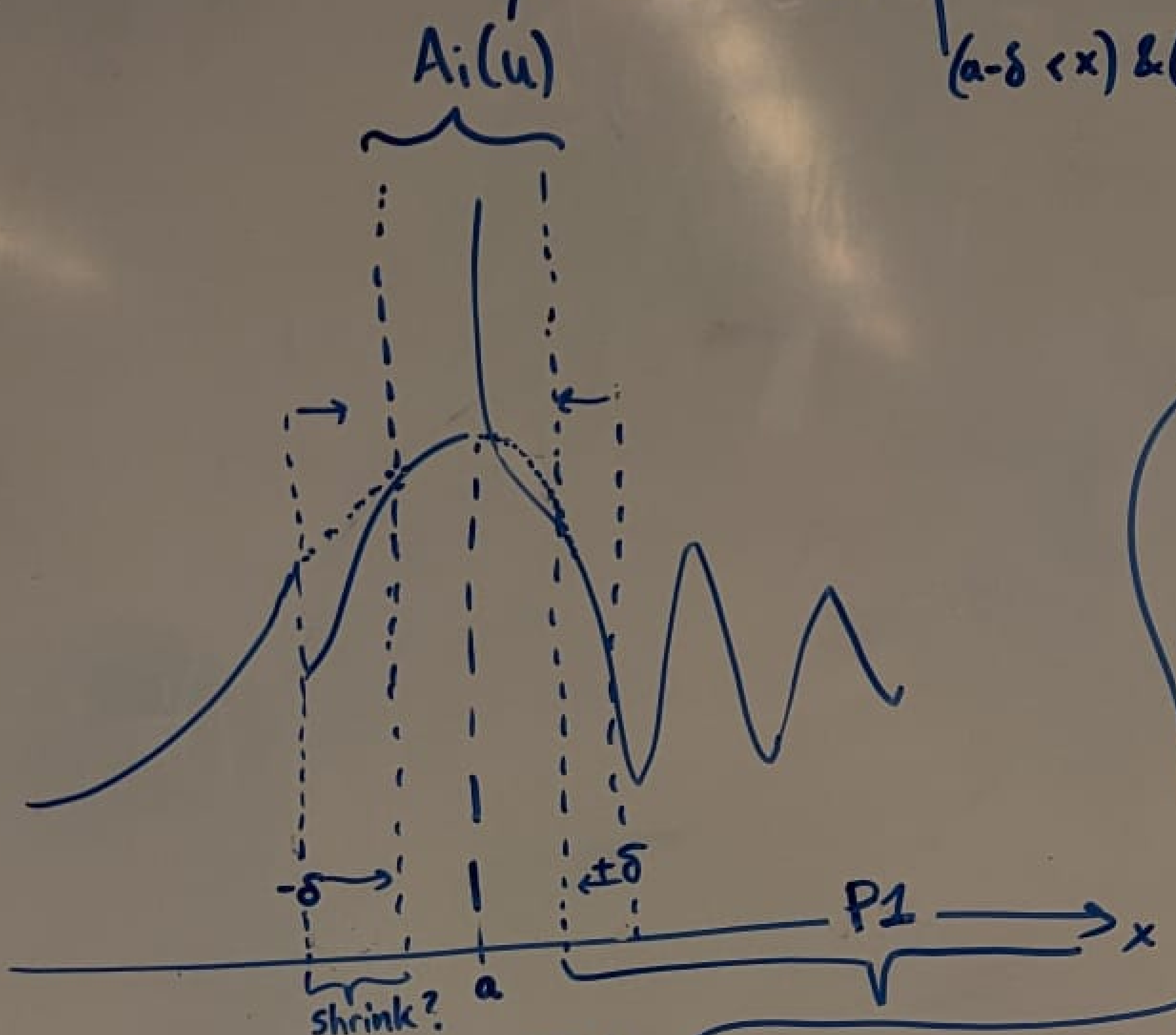
WKB Library?

make separate file

```
ψ = np.zeros(nx, dtype=complex)
ψ[x<a] =
ψ[(a<x) & (x<b)] = ∫^x P(x[(a<x) & (x<l)])
ψ[(a-ε<x) & (x<a+ε)] = ~ Ai(x(~~))
```

$F_0 = -V'(a)$

$u = F_0^{1/3}(a-x) \Big|_{(a-\delta < x)\,\&\,(x < a+\delta)}$



$\ast$ in $\delta = 0.2$ could be smaller

P1 I'm rewriting WKB from Ai(u) region

(15.27): $\psi_1 \approx \frac{1}{\sqrt{p(x)}} \cos\left(\int_a^x p(y)\,dy - \frac{\pi}{4}\right)$

or

(15.28): $\psi_2 \approx \frac{1}{\sqrt{p(x)}} \cos\left(-\int_x^b p(y)\,dy + \frac{\pi}{4}\right)$

what do I want in $a<x<b$?

$\psi_1 \underset{?}{=} \psi_2$

the ψ_1 & ψ_2 formulas agree up to multiplication by a constant in $(a+\delta, b-\delta)$

iff $\int_a^b p(x)\,dx = (n+\frac{1}{2})\pi$ (😮)

we get this by subtracting the phases of ψ_1 & ψ_2

$$\int_a^x p(y)\,dy - \frac{\pi}{4} - \left(-\int_x^b p(y)\,dy + \frac{\pi}{4}\right) = \int_a^x + \int_x^b - \frac{\pi}{2} = \int_a^b - \frac{\pi}{2}$$

$$\int_a^b - \frac{\pi}{2} = \pi n \Leftrightarrow \text{(😮)}$$

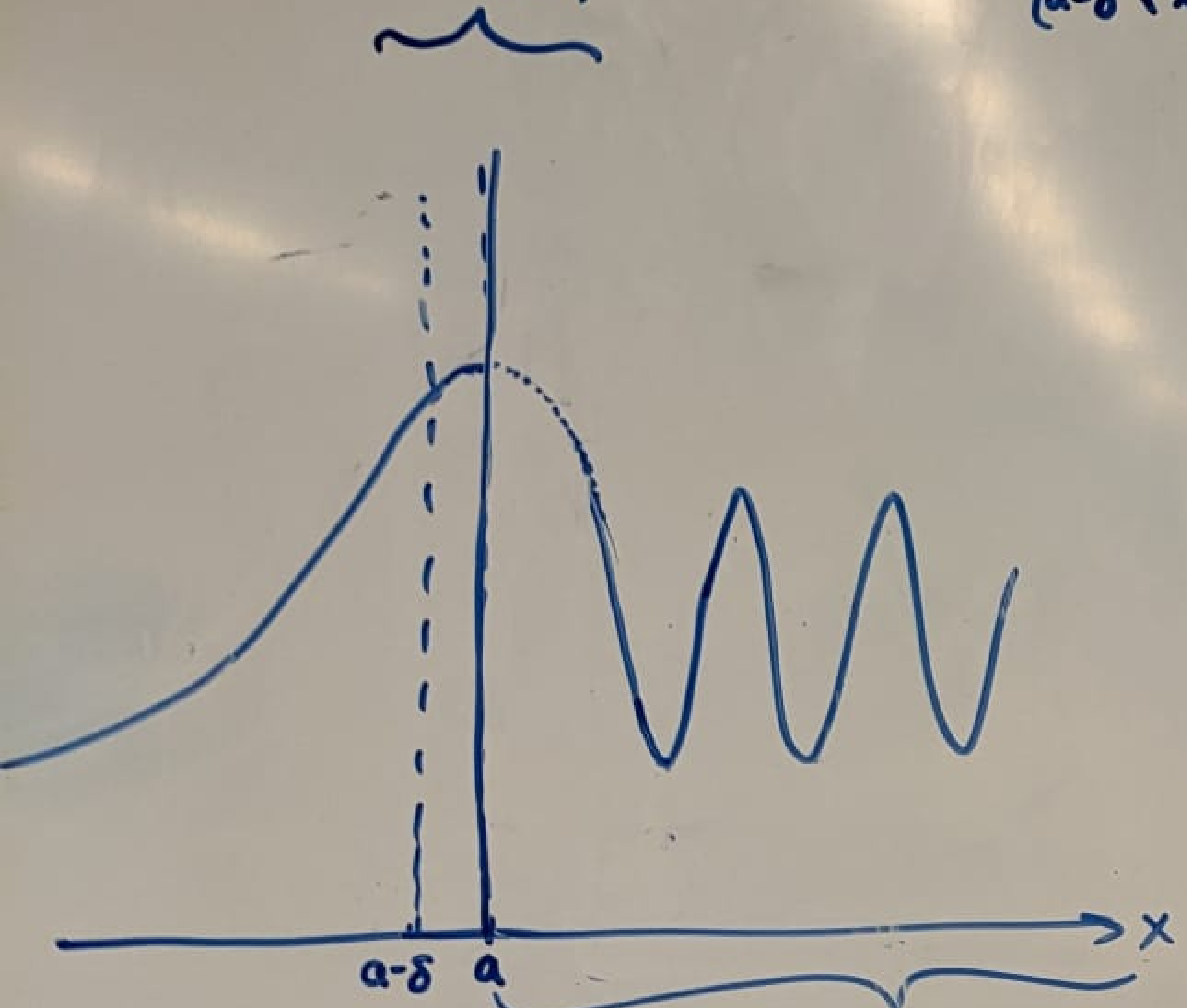
□

$F_0 = -V'(a)$

$u = F_0^{\frac{1}{3}}(a-x)$

$Ai(u)$

$\Big|_{(a-\delta < x) \,\&\, (x < a+\delta)}$



(15.27): $\psi_1 \approx \frac{1}{\sqrt{p(x)}} \cos\Big(\underbrace{\int_a^x p(y)dy}_{\theta} - \frac{\pi}{4}\Big)$

or

(15.28): $\psi_2 \approx \frac{1}{\sqrt{p(x)}} \cos\Big(\underbrace{-\int_x^b p(y)dy}_{\xi} + \frac{\pi}{4}\Big)$

what do I want in $a<x<b$?

$\psi_1 = c\psi_2$ let $\theta = \int_a^x p(y)dy$ & $\xi = -\int_x^b p(y)dy$

ie. the ψ_1 & ψ_2 formulas agree up to multiplication by a constant in (a,b)

$$\cos\left(\theta - \frac{\pi}{4}\right) = C\cos\left(\xi + \frac{\pi}{4}\right)$$

$$\cos\theta\cos\frac{\pi}{4} + \sin\theta\sin\frac{\pi}{4} = C\left(\cos\xi\cos\frac{\pi}{4} - \sin\xi\sin\frac{\pi}{4}\right)$$

$$\frac{1}{\sqrt{2}}(\cos\theta + \sin\theta) = \frac{C}{\sqrt{2}}(\cos\xi - \sin\xi)$$

$$\therefore \cos\int_a^x + \sin\int_a^x = C\left(\cos\left(-\int_x^b\right) - \sin\left(-\int_x^b\right)\right)$$

$$= C\left(\cos\int_x^b + \sin\int_x^b\right)$$

$$\therefore \cos\int_a^x - c\cos\int_x^b = c\sin\int_x^b - \sin\int_a^x$$

$$\psi_1 = c\psi_2 \iff \cos\int_a^x - c\cos\int_x^b = \sin\int_a^x - c\sin\int_x^b$$