$\neg(\forall\epsilon, \exists\delta) \Leftrightarrow (\exists\epsilon, \forall\delta)$...
contrdctn: neg definitions, flip ineq in conclusion.
$\frac{d}{dx}a^x = a^x \log a$
$*\sqrt{a} - \sqrt{b} = \frac{a-b}{\sqrt{a}+\sqrt{b}}$
$*\frac{a}{b+\sqrt{c}} \times 1$ (denom.opp.sign)
* contrapos: if A ⇒ B, then ¬ B ⇒ ¬ A.
$\neg(A \text{ and } B) \Leftrightarrow \neg A \text{ or } \neg B$

ineq algebra
(i) If $x \le y$ and $z \in \mathbb{R}$ then $x + y \le y + z$
(ii) If $x \le y$ and $c \ge 0$ then $cx \le cy$.
(iii)If $x \le y$ and $c \le 0$ then $cx \ge cy$.
(iv) If $\frac{c}{x} \le y$ and $c \ge 0$ then $x \ge \frac{c}{y}$.

Abs.val and inequalities
(i) $|a| = a$, if $a \ge 0, |a| = -a$ if $a \le 0$.
(ii) $|a| \le m \Leftrightarrow -m \le a \le m$. In particular, $-|a| \le a \le |a|$.
(iii) Triangle ineq $|a \pm b| \le |a| + |b|$.

Indtn
Prove that a_n is hippo $\forall n \in \mathbb{N}$
1. Base case: do the first term(s) satisfy hippo?
2. Inductive step: prove that by assuming a_n hippo $\Rightarrow a_{n+1}$ is hippo.

characterisations of s = sup(A)
s is a supremum of A ⇔ (i) s is an UB of A and (ii) $\forall\epsilon > 0, \exists a \in A : s - \epsilon \le a \le s$. (or) (i) and (iii) if z is an UB of A, then $z \ge s$.

Archimedean rty
$\forall x > 0, \forall y \in \mathbb{R}, \exists n \in \mathbb{N} : nx > y$.

AoC Every non empty set of the reals that is bd above has a supremum.

NIP For each $n \in \mathbb{N}$, assume we have a clsd int $I_n = [a_n, b_n] = \{x \in \mathbb{R} : a_n \le x \le b_n\}$. Assume each $I_n \supseteq I_{n+1}$ then the resulting sequence of nested ints has $\bigcap_n I_n \ne \emptyset$.
let $A = \{a_n : n \in \mathbb{N}\}$, by AoC, A has a sup s (check). Then $s \in \bigcap_n I_n$ if $s \in I_n \forall n \in \mathbb{N}$. $s \ge a_n$ by def of A, if b_n is an UB of A (...), then $s \le b_n$.(...) $\therefore a_n \le s \le b_n \Rightarrow s \in I_n$

2.27 An int I is cmpct ⇔ I is bd and it contains its endpoints, i.e. I is an int of the form [a, b]: $a, b \in \mathbb{R}$

PCSif $f : A \to \mathbb{R}$ is conton A and $K \subset A$ is cmpct. Then $f(K)$ is cmpct.
$f(K)$ is cmpct.
Let $(y_n) \in f(K)$: $y_n = f(x_n) \Leftarrow (x_n) \in K$.
Translate the fact that K is cmpct, and use the cnty of f along the SCC.

Density of $\mathbb{Q} \in \mathbb{R}$ $\forall a, b$ with $a < b, \exists r : a < r < b$.
(or) Given any $y \in \mathbb{R}, \exists(q_n) \in \mathbb{Q}$ converging to y.
¬op ⇏ clsd

HB a set $A \in \mathbb{R}$ is cmpct ⇔ A clsd and bd.

clsdness A set $F \in \mathbb{R}$ is clsd ⇔ every cauchy sequence in F has a lim in F.

seq cv
(1) $|u_n - l| \le \epsilon$
(2) find N_ϵ and impose that n must be such that this upper bound is less that ϵ.
(3) Take any M that is greater than N_ϵ, so whenever $n \ge M \ge N_\epsilon$, $\Rightarrow |u_n - l| \le \epsilon$.

Indeterminate lim
(1) Try and factorise the dominant term, to simplify the expression and find the lim or prove it DNE.
(2) Use these to find the dominant term:$(\forall k \in \mathbb{N})$
(i) $lim_{n\to\infty} n^{-k}e^n = \infty$
(ii) $lim_{n\to\infty} n^{-k}e^n = 0$,
(iii) $lim_{n\to\infty} \frac{\log(n)}{n} = 0$.

Charact.of bd seq $(u_n)_{n\in\mathbb{N}}$ is bd ⇔ $(|u_n|)$ is bd above, that is
$\exists M \in \mathbb{R} : |u_n| \le M, \forall n \in \mathbb{N}$.
(similarly for below)

MCT If a sequence is monotone and bd ⇒ cv.
*NIP, and using charact of sup(A) and monotonicity...

BW Every bd sequence contains a cv sbq. Compactness!

SLT sbqs of cv seq cv to the lim of the original sequence.shows how some seqs, don't cv.

ℝ is complete a sequence cv in ℝ if it is cauchy.

2.12 (cv seq are bd)
cv ⇒ bd
not bd⇒ not cv
$\epsilon = 1$ and find N_ϵ from $u_n \to \ell$ as $n \to \infty$. by Triang. ineq $|u_n| = |u_n - \ell + \ell| \le |u_n - \ell| + |\ell| \le 1 + |\ell|$. $M = max(|u_1|, \dots, |u_{N_\epsilon - 1}|, 1 - \ell)$ is $UB(|u_n|)$, Char.Bd.Seq. $\Rightarrow (u_n)$ is Bd.

2.31
1.cauchy ⇔ cv.
2.cauchy seq are bd.
*pr.2.12 (change ℓ for u_p). use $\epsilon/2$ instead and if $n, p \ge N/2$, by triang.ineq $|u_n - u_p| = |u_n - \ell + \ell - u_p|$...

2.14 (Uniqueness of the lim)
If $(a_n)_{n\in\mathbb{N}}$ has a lim then this lim is unique and it is denoted by $lim_{n\to\infty} a_n$
cntradtn. Assume $\ell < \ell'$, take $\epsilon = (\ell' - \ell)/3$. Translate the defs, find the ranks, and by triang. Ineq. $|\ell' - \ell| \le |\ell - u_n| + |u_n - \ell'| = 2\epsilon = 2/3(\ell' - \ell) \Rightarrow \ell' = \ell$.

ALT (seq) Let $(a_n) and (b_n)$ be two cv seq:
$lim_{n\to\infty} a_n = a$
and $lim_{n\to\infty} b_n = b$.
(i) $\forall c \in \mathbb{R}$, $Lim\ c\, a_n = ca$.
(ii) lim of sum is sum of lims.
(iii) lim of prod is prod of lims.
(iv) lim of quot is quot of lims (if lims exist).
(i)$|ca_n - ca| = |c||a_n - a|$ where $c \ne 0$ translate "$a_n \to a$" with $\epsilon/|c|$ find the rank, etc...
(ii) $|a_n + b_n - (a+b)| \le |a_n - a| + |b_n - b|$. $\epsilon > 0$, find each rank N_ϵ, M_ϵ and use the max as the rank of the sum.
(iii) Triangle ineq:$|a_n b_n - ab|$ ...by 2.12 since $a_n b_n$ it is bd by M it is ≤ Msomething.Take $\epsilon > 0$ and translate the lims of (a_n) and (b_n) respectively, with $\epsilon/(2M)$ and $\epsilon/(2|b|)$, take the max of the ranks to conclude.
(iv) using (iii) allow $a_n = 1, \forall n$

OLT Let $lim_{n\to\infty} a_n = a$ and $lim_{n\to\infty} b_n = b$:
(i) if $a_n \le b_n$ then $a \le b$.
(ii) if $a_n \ge 0, \forall n \in \mathbb{N} \Rightarrow a \ge 0$.
(iii) if $\exists z \in \mathbb{R} : z \le b_n \forall n \in \mathbb{N} \Rightarrow z \le b$.
cntradtn let $b < a$ and take $\epsilon = (a - b)/3$. Take N_ϵ for $a_n \to a$ and M_ϵ for $b_n \to b$. Consider $n = max(N_\epsilon, M_\epsilon)$ so $|a_n - a| \le \epsilon$ and $|b_n - b| \le \epsilon$ by assumption in OLT $a_n \le b_n$ so $a - \epsilon \le a \le b \le b + \epsilon$ write ϵ to show cntradtn with $b < a$.

Squeeze (seq)
Let $(u_n), (v_n)$ and (w_n) be three seq. If (u_n) and (w_n) cv to ℓ and if $u_n \le v_n \le w_n$, $\forall n \in \mathbb{N}$, then $(v_n) \to \ell$.

important series
(1) Geometric:by ALT $\sum ar^n = \frac{r}{1-r}$ iff $|r| < 1$.
(2) p-series: $\sum \frac{1}{n^p}$ cv iff $p > 1$.
(3) alternating harmonic: $\sum \frac{x^n}{n} \to \log 2$.
prod of series
$\sum_i \sum_j a_{ij} \ne \sum_j \sum_i a_{ij}$

Ratio test Let (a_n) be non zero reals and assume that $lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = r$. Then if:
(i) $r < 1, \sum a_n$ is absolutely cv.
(ii) $r > 1, \sum a_n$ dv.

|cv‖ If $\sum|a_n|$ cv $\Rightarrow \sum a_n$ cv abs. if $\sum a_n$ cv abs, then any rearrangement of the series cv to the same lim.

Comparison test
Let (a_n) and (b_n) be two seq: $0 \le a_n \le b_n \forall n \in \mathbb{N}$.
If $\sum b_n$ cv⇒ $\sum a_n$ cv. elif $\sum a_n$ dv ⇒ $\sum b_n$ dv.

series with positive terms
If $\sum a_n$ is a series with positive terms that cv⇔ $\sum a_n$ is bd.
elif the series is not bd, ⇒ $\sum a_n$ dv: $lim_{n\to\infty} s_n = \infty$

Dirichlet if partial sums of $\sum x_n$ are bd(not necesarily cv) and if (y_n) satisfies $y_1 \ge y_2 \ge \dots \ge 0$ and $lim_{n\to\infty} y_n = 0$ then $\sum x_n y_n$ cv.

AST Let (a_n) be a decreasing sequence of positive reals and $lim_{n\to\infty} a_n = 0$. Then $\sum(-1)^n a_n$ cv.

n-th term test If $\sum a_n$ cv then $a_n \to 0$ as $n \to \infty$.

cauchy for series
$\sum a_n$ cv in $\mathbb{R} \Leftrightarrow \forall\epsilon > 0, \exists N \in \mathbb{N} : n \ge p \ge N, |a_p + \dots + a_n| \le \epsilon$.

Cauchy condensation test
suppose that (b_n) is decreasingand $b_n \ge 0, \forall n \in \mathbb{N}$. Then $\sum b_n$ cv iff $\sum_{n=0} 2^n b_{2^n}$ cv.

find lim of f from the def
(1)$|f(x) - \ell|$, and find a nice upper bound for this quantity in terms of x, small when $|x - c|$ is small.
(2) x must be such that this upper bound is less than ϵ.
(3)algebra on the ineq obtained to find $|x - c| \le \delta$ (δ not dependent on x).

SCL ($\exists lim?$) Let $f : A \to \mathbb{R}$ and $c \in L(A)$, then: ($lim_{x\to c} f(x) = \ell$) ⇔ (for any $(x_n) \in A$ converging to c : $x_n \ne c$, $\forall n \in \mathbb{N} \Rightarrow f(x_n) \to \ell$).

ALT (f lims)
Let $f, g \in A$ and $c \in L(A)$, assume that $lim_{x\to c} f(x) = \ell$ and $lim_{x\to c} g(x) = m$.
(i) $\forall k \in \mathbb{R}$, $lim_{x\to c} kf(x) = k\ell$
(ii) lim of the sum is sum of the lims.
(iii) lim of prod is prod of lims.
(iv) lim of quot is quot of lims (if lims exist).
SCL and ALT (seq).

4.10 cnty and lim
Let $f : A \to \mathbb{R}$ and $c \in A \cap L(A)$.
⇒ f is cont at c
⇔ $lim_{x\to c} f(x) = f(c)$.
Translating each rty. Deal with with the case x = c in the translations.

SCC Let $f : A \to R$ and $c \in A$. f is cont at c ⇔ if $\forall(x_n) \in A$ converging to c we have $f(x_n) \to f(c)$.
By 4.10 and it's hint.
Separate the case $c \in A \setminus L(A)$ (this is trivial, since any $f \in A$ is cont at any point of $A \setminus L(A) \dots$).

ACT Let $f, g \in A \to \mathbb{R}$ be cont at $c \in A$
(i)$\forall k \in \mathbb{R}$, kf is cont at c.
(ii)$f + g$ is cont at c.
(iii)fg is cont at c.
(iv) if $g \ne 0$, $\frac{f}{g}$ is cont at c.
Consequence of the SCC and the ALT for seq.

CCF (composition) Let $f : A \to \mathbb{R}$ be cont at $c \in A$ and $g : B \to \mathbb{R}$ be cont at $f(c)$. $\Rightarrow g \circ f = g(f(x))$ is cont at c.
Use twice SCC (both directions)

CMT(cntrctn) Let I be a clsd int and $f : I \to I$ be a cntrctn. Then f has a unique fixed point x*.
Moreover, if x_0 is an arbitrary point of X then the sequence (x_n) defined inductively by $x_n = f(x_{n-1})$ cv to x*.
def of cntrctn.
∃ fixed point (*indtn) (x_n) : $d(x_n, x_{n+1}) = def.of\ cntrctn$.
Uniqueness of fixed point: $(1 - \gamma)d(x^*, x^{**}) \le 0$.
remember: cntrctns have domain = co-domain.

if $[a, b]$ is a clsd int then any cont f
$f : [a, b] \to [a, b]$ that has at least one fixed point, is cont by IVT. (evaluate int to show)
Then the SCC $\Rightarrow f(x_n) \to f(x^*)$ as $n \to \infty$. ∴The lim $f(x_n) = (x_{n+1})$ and so, $f(x^*) = x^*$.

EVT Let K be a non-empty and cmpct set of ℝ and $f : K \to \mathbb{R}$ be cont then f has a max (M) and a min (m) on K.
$\exists\ x_0$ and $x_1 \in K$: $\forall x \in K, f(x_0) \le f(x) \le f(x_1)$. by PCS $f(K)$ is cmpct and non-empty.
$\forall x \in K, f(x) \in f(K)$ so $m \le f(x) \le M$.

IVT Let $f : [a, b] \to \mathbb{R}$ be cont and L be a real between $f(a)$ and $f(b)$. $\Rightarrow \exists c \in [a, b] : f(c) =$ L.
assume $f(a) \le L \le f(b)$, let $c = sup_{x\in[a,b]}|f(x) \le L\}$.
If $\exists(x_n) : f(x_n) \le L$ and $x_n \to c$ as $n \to \infty$. by OLT, assume that $c < b$ (treat $c = b$ separately), and argue that $f(c + 1/n) > L$ and so $f(c) \ge L$.
note: Some fs are not cont (see Brouwer). but satisfy the IVT: eg.$f(x) = \sin(1/x)$,with $f(0) = 0$.

Preservation of ints
Let I be an int and $f : I \to \mathbb{R}$ be cont. Then $f(I)$ is an int.
Take $y < y' \in f(I)$ and L between y and y', $\exists x, x' \in I$: $f(x) = y, f(x') = y'$, apply the IVT between x and x' to L.

Brouwer
Let $f : [a, b] \to [a, b]$ then f has at least one fixed point. Let I be an op int of ℝ.(unless specified)
remark:derivatives always satisfy IVT... even when they are not cont

I is op unless specified
Deriv are not nec cont

5.3 1st order expansion
$g : I \to \mathbb{R}$ and $c \in I$. Then g is diff at c ⇔ $\exists L \in \mathbb{R}$ and a f defined on an op int around 0 : $lim_{h\to 0} r(h) = 0$ and $\forall h$ in this int, $g(c+h) = g(c)+Lh+hr(h)$.
In this case, we have $g'(c) = L$ $\therefore g(c + h) = g(c) + g'(c)h + hr(h)$.

diff⇒ cont $g : I \to \mathbb{R}$ and $c \in I$ if g is diff at c the g is cont at c.

ADT$f, g \in I \to \mathbb{R}$ and $c \in I$, assume that f and g are diff at c.
(i) $\forall k \in \mathbb{R}, kf$ is diff at c and $(kf)'(c) = kf'(c)$.
(ii) f + g is diff at c and $(f + g)'(c) = f'(c) + g'(c)$.
(iii) fg is diff at c and $(fg)'(c) = f'(c)g(c) + f(c)g'(c)$.
(iv) if denom does not vanish...The quot rule.
def of diff and ALTfL.

Chain Rule Let I and J be op, $f : I \to J$ and $g : J \to \mathbb{R}$ and $c \in I$. if f is diff at c and g is diff at f(c) then $g \circ f$ is diff at c and $(g \circ f)'(c) = g'(f(c))f'(c)$ $h = (x - c)$ then $f(c+h) = f(c)+f'(c)h+hr(h)$ and $g(f(c) + k) = g(f(c)) + g'(f(c))k+kq(k)$ where $r(h) \to 0$ as $h \to 0$. let $k = f(c + h)f(c) = f'(c)h + hr(h) \to 0$ as $h \to 0$, we get $g(f(c+h)) = \dots$

IET $g : I \to \mathbb{R}$ and $c \in I$ if c is an extremum of f and f is diff at c then $f'(c) = 0$.
max and mins occur when derivative = 0
Assume that c is a max of f. $f(c + h) \le f(c)$. Using (5.3) shows that $f'(c)h + hr(h) \le 0$. Taking $h > 0$ gives $f'(c) + r(h) \le 0$. Use OLT (or fal version) to deduce that $f'(c) \le 0$. Taking $h < 0$ in (5.3) gives $f'(c) + r(h)0$ and thus $f'(c) \ge 0$.

5.12 Location of extrema Let $f : [a, b] \to \mathbb{R}$ be cont on $[a, b]$ and diff on (a, b). Then f has minima and maxima, and these extrema are either a, b or $c \in (a, b)$: $f'(c) = 0$. try $f = x^3$! EVT, f has at least one min and max. If one of these extrema is not a or b, then it lies in (a, b) and it is an extremum of f on the op int (a, b). Thus, the IET can be applied and f′ vanishes at this extremum.

Rolle's Let $a < b$. Let $f : [a, b] \to \mathbb{R}$ be cont on $[a, b]$ and diff on (a, b). If $f(a) = f(b)$ then $\exists c \in (a, b) : f'(c) = 0$. by EVT, f has a min and a max on the end points then f is constant on [a, b] f′ = 0 on (a,b),else max/min in (a,b) then by IET f′ = 0.

MVT Let $a < b$ and $f : [a, b] \to \mathbb{R}$ be cont on $[a, b]$ and diff on (a, b). $\Rightarrow c \in (a, b) : f'(c) = \frac{f(b)-f(a)}{b-a}$
*Rolles to the difference between f and the straight line going through (a, f (a)) and (b, f (b)), i.e. to the f g defined by $g(x) = f(x) - [f(a) + \frac{f(b)-f(a)}{b-a}(x - a)]$.

Lipschitz estimate Let $a < b$. Let f be cont on [a, b] and diff on (a, b). Then $|f(b) - f(a)| \le |b - a|sup|f'(x)|_x \in (a, b)$. max speed = dist*max accel. MVT

Monotony and sign of derivative If f is diff on an op int I and if f′= 0 (resp. $f' \ge 0$,or $f' \le 0$) on I then f is constant (resp. increasing, or decreasing) on I. MVT

5.19 If $f : I \to \mathbb{R}$ is cont and $a \in I$ then $F(x) = \int_a^x f(s)ds$ on I, and F′ = f on I.

FTCLet I be an op int, and $f : I \to \mathbb{R}$ be diff. Assume that f′ is cont on I. Then, $\forall a, x \in I$, $f(x) = f(a) + \int_a^x f'(s)ds$. 5.19... F′ = f′

Taylor exp Let I be an op int that contains 0 and $n \in \mathbb{N} \cup \{0\}$. Let $f : I \to \mathbb{R}$ be $(n + 1)$ times continuously diff Then, $\forall x \in I$, $f(x) = f(0) + f'(0)x + \dots + \frac{f^{(n)}(0)}{n!}x^n + O(x^{n+1})$

O notation
1.if $k \ge m$, $O(x^k) + O(x^m) = O(x^m)$, (smallest remains)
2.$O(x^{n+1}) = x^n O(x)$,
3. $\frac{O(x^{n+1})}{x^n} = O(x) \to 0$ as $x \to 0$.

common Taylor exp.
$*e^x = 1 + x + \dots + \frac{x^n}{n!} + O(x^{n+1})$.
$*\sin(x) = x - \frac{x^3}{3!} + \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + O(x^{2n+3})$.
$*\cos(x) = 1 - \frac{x^2}{2!} + \dots + (-1)^n \frac{x^{2n}}{(2n)!} + O(x^{2n+2})$.
$*\frac{1}{1-x} = 1 + x + \dots + x^n + O(x^{n+1})$.
$*\log(1 + x) = x - \frac{x^2}{2} + \dots + (-1)^{n-1}\frac{x^n}{n} + O(x^{n+1})$.

pwcv relies on $N_{x,\epsilon}$
let$(f_n(x)) \in A$ and use seq cv or ALT iff $f_n(x)$ cv $\forall x \in A \Rightarrow (f_n(x)) \to f$(if dv for any x then it doesn't cv pw.
unif cv relies on N_ϵ
$sup(|f_n(x) - f(x)|) \to 0$ as $n \to \infty$:
(a) Write for a generic x, $|f_n(x) - f(x)|$.
(b) find small ω_n, not depending on x:$|f_n(x) - f(x)| \le \omega_n$.
(c) If $(\omega_n)n \to 0$, then $(f_n) \to f$ unif
unif⇒ p.w.
converse is false.

f_n cv behaviour
-pw:[0,1] unif:[0,a] :$x^n \to f = 0$
-pw:[0,1] unif:[a,1] :hat $\to f = 0$
-pw:[0,1] : $\frac{nx}{1+nx}$
$\to f = 1$ or 0 $(x = 0)$
-unif:[-1,1] f_n, pw$f'_n \to f'$: $\sqrt{x^2 + \frac{1}{n}} \to f = |x|$(¬diff).
-unif:[0,1], $\frac{nx}{n+x} \to f = x$
-unif:[ℝ] $f'_n \nrightarrow$ pw(ℝ)
$\frac{\sin(nx)}{n} \to f = 0$
-pw cont $f_n \to f$ discont: $Arctan(x)$.
-pw:[0,1] bd $f_n \to f$ unbd: $\frac{nx}{(1+nx^2)}$
-unif:[0,1] ¬ diff $f_n \to f$ diff at $x_0 \in [0, 1]$: $g_n = \frac{1}{n}, x \le x_0$ and $g_n = 0, x > x_0$.

f behaviour
-cont $f : \mathbb{R} \to \mathbb{R}$ op I: $f(I)$ is ¬op : $f = x^2$.
-cont $f : \mathbb{R} \to \mathbb{R}$ clsd I: $f(I)$ is op$[0, \infty)$: $f = Arctan(x)$.
-cont bdf : $(0, 1) \to \mathbb{R}$ max/¬min: $f = -(x - \frac{1}{2})^2$.
-cont $f : (0, 1) \to \mathbb{R}$, cauchy$(x_n) \in (0, 1)$: $f(x_n)$ is cauchy: $x_n = \frac{1}{(n+1)}, f(x) = 1/x$.
-cont bd $f : A \to \mathbb{R}$,$f(A)$¬ bd$A = (0, 1]$, $f = Log(x)$.
-cont $f_n \to f$ cont pw[0,1] $(x_n) \to x$ but $f(x_n) \nrightarrow f(x)$: $f = hat$.
-nowhere cont Dirichlet (if $\in Q$: 1, else: 0)
-cont at 0 only mod.Dirichlet ($f = n*$ Dirichlet).
-cont on Irrationals: Thomae (if $\in Q$: 0, else: $\frac{1}{n}$)

CUL Let (f_n) and $c \in A$. Assume that each f_n is cont at c. If (f_n) cv unif to f on A then f is also cont at c⇒ if each (f_n) is cont on A, then f is cont on A. USE if each f_n is cont but f is not cont (contrapos) ⇒ cv is not unif.

Integration and unif Lim If $(f_n)$is cont on [a, b] and cv unif. $\to f$ on $[a, b] \Rightarrow \int_a^b f_n(x)dx \to \int_a^b f(x)dx$

unifcauchy Let (f_n) be defined on A. (f_n) cv unif on A ⇔ $\forall\epsilon > 0, \exists N \in \mathbb{N} : \forall n, p \ge N$ and $\forall x \in A, |f_n(x) - f_p(x)| \le \epsilon$. cauchy ∴ cv pwto f(x), by OLT $p \to \infty$ this shows the def.

diff of the lim Let (f_n) be defined on an int [a, b], f_n are diff on (a, b). Assume that (f_n) cv unif to f on [a, b] and that (f'_n) cv unif to g on (a, b). Then f is diff on (a, b) and f′ = g. ergo: the lim of the derivative is the derivative of the lim.

Darboux f′ always satisfies IVT: ie. if $f : I \to \mathbb{R}$ is diff on op int I and a,b lie in I ⇒ $\forall f'(a) \le L \le f'(b) \exists c \in (a, b) : f'(c) = L$.

6.15
1.If $\sum(f_n)$ cv unif, then the lim of $\sum(f_n)$ is cont.
2.If $\sum(f_n)$ and $\sum(f'_n)$ cv unif on (a,b), then the lim of $\sum(f_n)$ is diff and the lim of $(\sum(f_n))' = \sum(f'_n)$. CUL and diff of the lim, remember to use the ACT or the ADT when needed.

unif cauchy for series $\sum(f_n)$ cv unif on A ⇔ $\forall\epsilon > 0, \exists N \in \mathbb{N} : \forall n, p \ge N$ and $\forall x \in A, \Rightarrow |f_p(x) + \dots + f_n(x)| < \epsilon$. Cauchy for seq of reals.

M-test Let (f_n) be defined on A and $(M_n) \in \mathbb{R} : \forall x \in A, |f_n(x)| \le M_n$. If the positive series of M_n cv then $\sum(f_n)$ cv unif on A. cauchy for series and triang ineq

bestbuddy M-test find an UB M_n of $|f_n|$ for each n. To show that M_n cv use any of the tests for series of reals.

prove lim of $\sum f_n$ is cont or diff
(1)Establish unif cv of $\sum f_n$ (probably by M-test)
(2)If each f_n is cont then 6.15 (1) $\Rightarrow f = \sum f_n$ is cont. (finished for cont). (3)Establish unif cv of $\sum f'_n$
(4) by 6.15 (2) f is diff with $f' = \sum f'_n$. (M-test, ∴find UB$(|f'_n|)$ $= R_n$: $\sum R_n$ cv.
Note that the unif cv do not have to be on the entire domain over which we want to establish the cont or diff of f. If cv are unif on any cmpctset in this domain, then the cont or diff of f is valid on the entire domain.

cv of power series
Assume $\sum a_n x^n$ cv at some $c \ne 0$.
1. $\forall x \in \mathbb{R} : |x| < |c|, \sum a_n x^n$ cv abs.
2. for any $0 \le r < |c|, \sum a_n x^n$ cv unif on [r, r].
$|x| < R$, it cv abs. $|x| = R$, no idea. $|x| > R$, it dv.

6.22 (Radius of cv)
(a_n) of non zero reals: $lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = \ell$. Then $\sum a_n x^n$ has $R = \frac{1}{\ell}$ if $\ell > 0$ or $R \not< \infty$ if $\ell = 0$.

find R and int of $\sum a_n x^n$
if $\frac{a_{n+1}}{a_n}$ cv $R = \frac{1}{\ell}$
else: $R = sup\{x \ge 0; \sum a_n x^n$ cv}via ratio test.
int: if the $\sum a_n x^n$ cv at $x = -R$ and/or $x = R$, then $I = [R, R]$, $I = (-R, R]$, $I = [R, R)$ or $I = (R, R)$.
Int of cv $[-1, 1] : \sum \frac{x^n}{n^2}$
$(-1, 1) : \sum x^n$
$(-1, 1] : \sum \frac{-x^n}{n}$
$[-1, 1) : \sum \frac{x^n}{n}$

diff power series $\sum a_n x^n$ and $\sum na_n x^{n-1}$ have same R. As a consequence,$\sum a_n x^n$ is diff on (R, R) to recover f, integrate! and to find integration const C find the value of f(0)