we can write

$$a_\ell = \frac{e^{i\delta_\ell}\sin\delta_\ell(k)}{k} = \frac{\sin\delta_\ell(k)}{k\,e^{-i\delta_\ell}}$$

$$= \frac{\sin\delta_\ell(k)}{k(\cos\delta_\ell - i\sin\delta_\ell)} = \frac{1}{k\frac{\cos\delta_\ell}{\sin\delta_\ell} - ik}$$

$$\therefore a_\ell = \frac{1}{k\cot(\delta_\ell) - ik}$$

we can write the scattering cross-section as:

$$\boxed{\sigma = \sum_\ell \frac{4\pi(2\ell+1)}{k^2 + k^2\cot^2\delta_\ell(k)}}$$