An Adjustable Robust Optimization Approach for the Expansion Planning of a Virtual Power Plant: Electronic Companion

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I. Solution of the Subproblem

Based on [1], the single-level approximation of the subproblem at iteration ν of the outer loop and iteration μ of the inner loop is formulated as follows:

$$\min_{\Psi^{\mathrm{S}}} \; \Pi \tag{1a}$$

subject to:

$$\text{Constraints of the uncertainty set } \Lambda \tag{1b}$$

$$\Bigg\{ \Pi \geq \sum_{t\in\mathcal{T}} \Bigg[\sum_{h\in\mathcal{H}} \Bigg(\underline{P}^{\mathrm{E}} \underline{\varphi}_{ht\mu'} - \overline{P}^{\mathrm{E}} \overline{\varphi}_{ht\mu'} - \sum_{c\in\mathcal{C}\setminus\tilde{\mathcal{C}}} \overline{P}^{\mathrm{C}}_c \overline{\rho}_{cht\mu'} - \sum_{c\in\tilde{\mathcal{C}}} \overline{P}^{\mathrm{C}}_c u_c^{\mathrm{C,I}(\nu)} \iota^{\mathrm{C}}_{cht\mu'} + \sum_{d\in\mathcal{D}} \Big(p^{\mathrm{D,min}}_{dht} \underline{\gamma}_{dht\mu'} - p^{\mathrm{D,max}}_{dht} \overline{\gamma}_{dht\mu'} \Big) - \sum_{r\in\mathcal{R}\setminus\tilde{\mathcal{R}}} K^{\mathrm{R}}_{rht} \overline{P}^{\mathrm{R}}_r \tau_{rht\mu'} - \sum_{r\in\tilde{\mathcal{R}}} K^{\mathrm{R}}_{rht} \overline{P}^{\mathrm{R}}_r u_r^{\mathrm{R,I}(\nu)} \iota^{\mathrm{R}}_{rht\mu'} - \sum_{s\in\mathcal{S}} \Big(\overline{P}^{\mathrm{S,DI}}_s u^{\mathrm{S}(\mu')}_{sht} \overline{\kappa}_{sht\mu'} + \overline{P}^{\mathrm{S,CH}}_s \Big(1 - u^{\mathrm{S}(\mu')}_{sht}\Big) \overline{\delta}_{sht\mu'} - \underline{S}^{\mathrm{S}}_{sht} \underline{\psi}_{sht\mu'} + \overline{S}^{\mathrm{S}}_{sht} \overline{\psi}_{sht\mu'} \Big) - \sum_{s\in\tilde{\mathcal{S}}} \Big(\overline{P}^{\mathrm{S,DI}}_s u_s^{\mathrm{S,I}(\nu)} \iota^{\mathrm{S,DI}}_{sht\mu'} + \overline{P}^{\mathrm{S,CH}}_s u_s^{\mathrm{S,I}(\nu)} \iota^{\mathrm{S,CH}}_{sht\mu'} \Big) \Bigg) + \sum_{d\in\mathcal{D}} d^{\mathrm{D}}_d \xi_{dt\mu'} + \sum_{s\in\mathcal{S}} S^{\mathrm{S}}_{s0t} \phi_{s1t\mu'} \Bigg] \tag{1c}$$

$$-\alpha_{ht\mu'} + \underline{\varphi}_{ht\mu'} - \overline{\varphi}_{ht\mu'} = -N_t \lambda^{\mathrm{E}}_{ht}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1d}$$

$$\alpha_{ht\mu'} - \overline{\rho}_{cht\mu'} \leq c^{\mathrm{C}}_c, \forall c\in\mathcal{C}\setminus\tilde{\mathcal{C}}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1e}$$

$$\alpha_{ht\mu'} - \iota^{\mathrm{C}}_{cht\mu'} \leq c^{\mathrm{C}}_c, \forall c\in\tilde{\mathcal{C}}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1f}$$

$$-\alpha_{ht\mu'} + \underline{\gamma}_{dht\mu'} - \overline{\gamma}_{dht\mu'} + \xi_{dt\mu'} = 0, \forall d\in\mathcal{D}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1g}$$

$$\alpha_{ht\mu'} - \tau_{rht\mu'} \leq 0, \forall r\in\mathcal{R}\setminus\tilde{\mathcal{R}}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1h}$$

$$\alpha_{ht\mu'} - \iota^{\mathrm{R}}_{rht\mu'} \leq 0, \forall r\in\tilde{\mathcal{R}}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1i}$$

$$\alpha_{ht\mu'} - \overline{\kappa}_{sht\mu'} + \frac{1}{\eta^{\mathrm{S,DI}}_s} \phi_{sht\mu'} \leq 0, \forall s\in\mathcal{S}\setminus\tilde{\mathcal{S}}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1j}$$

$$\alpha_{ht\mu'} - \overline{\kappa}_{sht\mu'} + \frac{1}{\eta^{\mathrm{S,DI}}_s} \phi_{sht\mu'} - \iota^{\mathrm{S,DI}}_{sht\mu'} \leq 0, \forall s\in\tilde{\mathcal{S}}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1k}$$

$$-\alpha_{ht\mu'} - \overline{\delta}_{sht\mu'} - \eta^{\mathrm{S,CH}}_s \phi_{sht\mu'} \leq 0, \forall s\in\mathcal{S}\setminus\tilde{\mathcal{S}}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1l}$$

$$-\alpha_{ht\mu'} - \overline{\delta}_{sht\mu'} - \eta^{\mathrm{S,CH}}_s \phi_{sht\mu'} - \iota^{\mathrm{S,CH}}_{sht\mu'} \leq 0, \forall s\in\tilde{\mathcal{S}}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1m}$$

$$\phi_{sht\mu'} - \phi_{sh+1t\mu'} + \underline{\psi}_{sht\mu'} - \overline{\psi}_{sht\mu'} = 0, \forall s\in\mathcal{S}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1n}$$

$$\underline{\varphi}_{ht\mu'}, \overline{\varphi}_{ht\mu'} \geq 0, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1o}$$

$$\overline{\rho}_{cht\mu'} \geq 0, \forall c\in\mathcal{C}\setminus\tilde{\mathcal{C}}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1p}$$

$$\iota^{\mathrm{C}}_{cht\mu'} \geq 0, \forall c\in\tilde{\mathcal{C}}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1q}$$

$$\underline{\gamma}_{dht\mu'}, \overline{\gamma}_{dht\mu'} \geq 0, \forall d\in\mathcal{D}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1r}$$

$$\xi_{dt\mu'} \geq 0, \forall d\in\mathcal{D}, \forall t\in\mathcal{T} \tag{1s}$$

$$\tau_{rht\mu'} \geq 0, \forall r\in\mathcal{R}\setminus\tilde{\mathcal{R}}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1t}$$

$$\iota^{\mathrm{R}}_{rht\mu'} \geq 0, \forall r\in\tilde{\mathcal{R}}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1u}$$

$$\overline{\delta}_{sht\mu'}, \overline{\kappa}_{sht\mu'}, \underline{\psi}_{sht\mu'}, \overline{\psi}_{sht\mu'} \geq 0, \forall s\in\mathcal{S}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1v}$$

$$\iota^{\mathrm{S,CH}}_{sht\mu'}, \iota^{\mathrm{S,DI}}_{sht\mu'} \geq 0, \forall s\in\tilde{\mathcal{S}}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \Bigg\}, \tag{1w}$$

$$\mu' = 1, \ldots, \mu,$$

where S^{S}_{s0t} is the initial level of stored energy of storage unit s in representative day t [MWh] and set $\Psi^{\mathrm{S}}=\Big\{ \Pi, \Upsilon^{\mathrm{ML}}, \Big\{ \big\{ \{ \overline{\rho}_{cht\mu'} \}_{\forall c\in\mathcal{C}\setminus\tilde{\mathcal{C}}}, \{ \iota^{\mathrm{C}}_{cht\mu'} \}_{\forall c\in\tilde{\mathcal{C}}}, \{ \underline{\gamma}_{dht\mu'}, \overline{\gamma}_{dht\mu'} \}_{\forall d\in\mathcal{D}}, \{ \tau_{rht\mu'} \}_{\forall r\in\mathcal{R}\setminus\tilde{\mathcal{R}}}, \{ \iota^{\mathrm{R}}_{rht\mu'} \}_{\forall r\in\tilde{\mathcal{R}}}, \{ \overline{\delta}_{sht\mu'}, \overline{\kappa}_{sht\mu'}, \underline{\psi}_{sht\mu'}, \overline{\psi}_{sht\mu'}, \phi_{sht\mu'} \}_{\forall s\in\mathcal{S}}, \{ \iota^{\mathrm{S,CH}}_{sht\mu'}, \iota^{\mathrm{S,DI}}_{sht\mu'} \}_{\forall s\in\tilde{\mathcal{S}}}, \alpha_{ht\mu'}, \underline{\varphi}_{ht\mu'}, \overline{\varphi}_{ht\mu'} \big\}_{\forall h\in\mathcal{H}}, \{ \xi_{dt\mu'} \}_{\forall d\in\mathcal{D}} \Big\}_{\forall t\in\mathcal{T}, \mu'=1,\ldots,\mu} \Big\}$ includes new variables indexed by subscript μ'. Note that parameters $u^{\mathrm{S}(\mu')}_{sht}$ denote the optimal values for u^{S}_{sht} resulting from the lower-level optimization of the subproblem at inner-loop iteration μ'.

The goal of the single-level approximation of the subproblem is the minimization of Π (1a). Constraints (1b) define the uncertainty set. In (1c), the value of the lower-level dual

TABLE I
NUMBERS OF DAYS GROUPED IN REPRESENTATIVE DAYS

t	1	2	3	4	5
N_t [days]	140	158	28	34	5

TABLE II
DATA FOR CONVENTIONAL GENERATING UNITS

Unit	$\hat{C}_c^{\mathrm{C}}$ [\$/MWh]	$\underline{C}_c^{\mathrm{C}}$ [\$/MWh]	$\overline{C}_c^{\mathrm{C}}$ [\$/MWh]	$C_c^{\mathrm{C,I}}$ [10^6 \$]	$\overline{P}_c^{\mathrm{C}}$ [MW]
Existing unit	40	8	16	–	60
Candidate 1	25	5	10	80	55
Candidate 2	30	6	12	60	45
Candidate 3	35	7	14	40	40
Candidate 4	10	2	4	120	30
Candidate 5	15	3	6	100	20

objective function for given $u_{sht}^{\mathrm{S}(\mu')}$ represents a lower bound for Π. Finally, constraints (1d)–(1w) correspond to the lower-level dual constraints for given $u_{sht}^{\mathrm{S}(\mu')}$.

Problem (1) includes bilinear terms in (1c) in the form of products of middle-level variables and lower-level dual (continuous) variables. Particularly, these bilinear terms are $p_{dht}^{\mathrm{D,min}}\underline{\gamma}_{dht\mu'}$, $p_{dht}^{\mathrm{D,max}}\overline{\gamma}_{dht\mu'}$, and $d_d^{\mathrm{D}}\xi_{dt\mu'}$. Nonetheless, such bilinear terms can be replaced with equivalent mixed-integer linear expressions as explained in [2]. Hence, problem (1) is reformulated as a mixed-integer linear program (MILP).

The optimal values for middle-level variables c_c^{C}, d_d^{D}, $p_{dht}^{\mathrm{D,max}}$, $p_{dht}^{\mathrm{D,min}}$, and $\lambda_{ht}^{\mathrm{E}}$ are fed into the lower-level problem of the subproblem. The solution of this MILP provides new values for lower-level binary variables u_{sht}^{S}.

II. DATA FOR THE CASE STUDY

Based on historical data, we group the 365 days of the year into five representative days using a modified version of the K-means clustering technique applied in [3]. The numbers of days grouped in representative days are shown in Table I.

Data for existing and candidate conventional generating units are displayed in Table II. We consider two generation technologies, namely, a peak and a base technology. The peak technology features higher forecast production cost coefficients but lower investment cost coefficients. The existing conventional unit and candidate units 1, 2, and 3 correspond to the peak technology. On the other hand, the base technology is characterized by lower forecast production cost coefficients but higher investment cost coefficients. Candidate units 4 and 5 correspond to the base technology.

Capacity factor data of photovoltaic solar units for each representative day are obtained from version 2017.1.17 of the System Advisor Model [4], based on weather data from the National Solar Radiation Data Base [5]. Photovoltaic power production data represent the average production over 12 different locations, thereby avoiding the effect of extremely favorable/disadvantageous solar conditions. For the sake of simplicity, we assume that all existing and prospective photovoltaic solar units have the same capacity factors. Fig. 1 shows the hourly capacity factors for every representative day. Capacities and investment cost coefficients for photovoltaic solar units are listed in Table III.

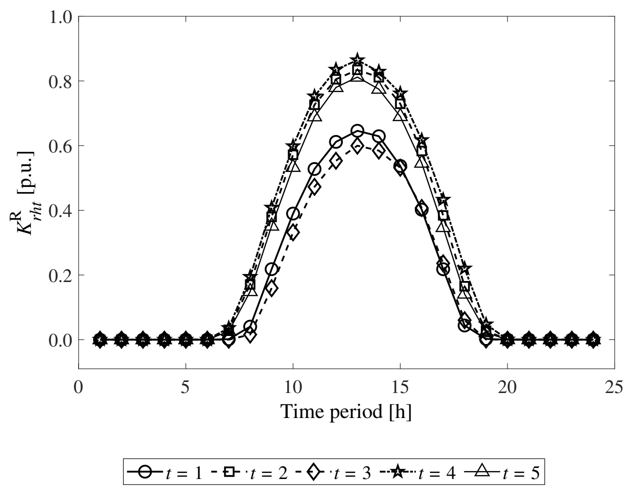

Fig. 1. Hourly capacity factors of photovoltaic solar units.

TABLE III
DATA FOR PHOTOVOLTAIC SOLAR UNITS

Unit	$\overline{P}_r^{\mathrm{R}}$ [MW]	$C_r^{\mathrm{R,I}}$ [10^6 \$]
Existing unit	80	–
Candidate 1	70	105.00
Candidate 2	65	78.00
Candidate 3	50	50.00
Candidate 4	35	34.30
Candidate 5	27	25.92

The initial level of stored energy of the existing storage unit is 80 MWh, whereas candidate storage units are initially discharged. The lower bounds for the levels of stored energy of the existing and candidate storage units are 0 MWh across the time span except for the last hourly period of each representative day for the existing storage unit. In this case, for $h = |\mathcal{H}|$, the lower bounds are set equal to the corresponding initial level. Furthermore, the charging and discharging efficiency rates of all storage units are both equal to 0.9 p.u. The remaining data of the storage units, i.e., the rated charging and discharging power capacities, the upper bounds for the levels of stored energy, and the investment cost coefficients, are provided in Table IV.

Regarding the flexible demands, the forecast minimum and maximum consumption levels are shown in Figs. 2 and 3, respectively, whereas the forecast values of the minimum daily energy consumption are presented in Table V. Upper and lower bounds result from considering a $\pm 20\%$ variation about the corresponding forecast values.

Finally, forecast energy market prices are obtained from historical energy market prices for the Electric Reliability Council

TABLE IV
DATA FOR STORAGE UNITS

Unit	$\overline{P}_s^{\mathrm{S,CH}}$ [MW]	$\overline{P}_s^{\mathrm{S,DI}}$ [MW]	$\overline{S}_{sht}^{\mathrm{S}}$ [MWh]	$C_s^{\mathrm{S,I}}$ [10^6 \$]
Existing unit	80	80	160	–
Candidate 1	70	70	140	24.50
Candidate 2	60	60	120	19.20
Candidate 3	55	55	110	16.50
Candidate 4	40	40	80	10.80
Candidate 5	25	25	50	6.25

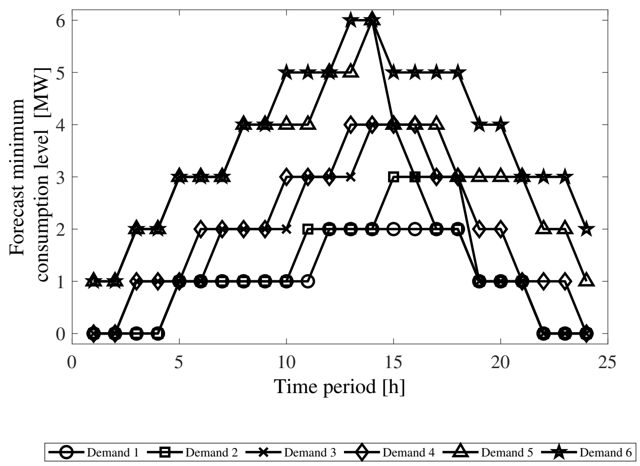

Fig. 2. Forecast minimum power consumption levels of the flexible demands.

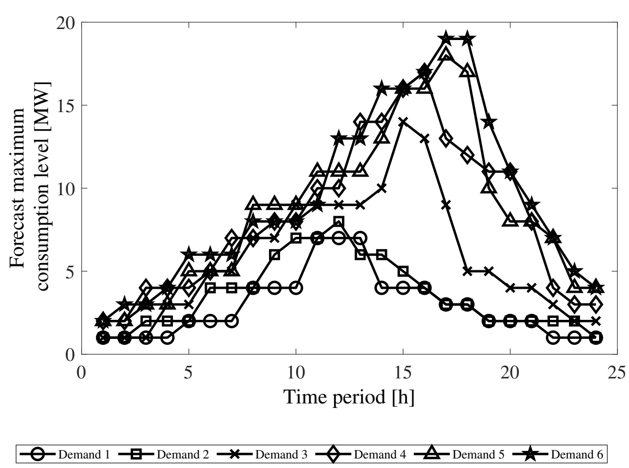

Fig. 3. Forecast maximum power consumption levels of the flexible demands.

of Texas North Central zone [6]. Fig. 4 shows the forecast hourly energy market prices of every representative day. Upper and lower bounds respectively result from considering a 40% increase and a 20% decrease over the corresponding forecast values.

TABLE V
FORECAST VALUES FOR THE MINIMUM DAILY ENERGY CONSUMPTION OF THE FLEXIBLE DEMANDS

Demand	$\hat{D}_d^{\mathrm{D}}$ [MWh]
1	52.0
2	65.0
3	104.0
4	130.0
5	156.0
6	182.0

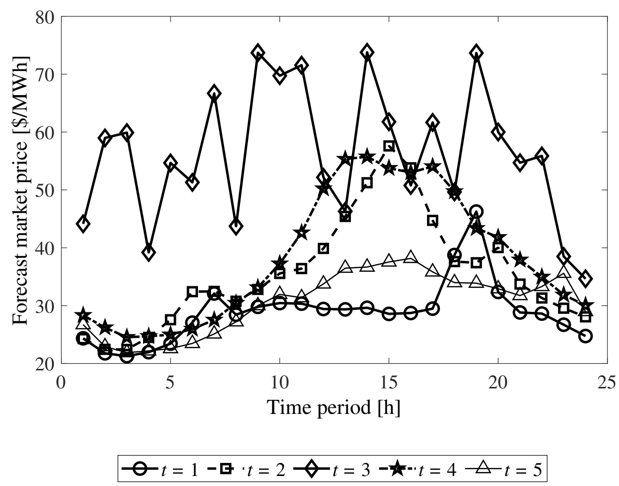

Fig. 4. Forecast hourly energy market prices.

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