An Adjustable Robust Optimization Approach for the Expansion Planning of a Virtual Power Plant: Electronic companion

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I. Solution of the Subproblem

Based on [1], the single-level approximation of the subproblem at iteration ν of the outer loop and iteration μ of the inner loop is formulated as follows:

$$\min_{\Psi^{\mathrm{S}}} \ \Pi \tag{1a}$$

subject to:

$$\text{Contraints of the Uncertainty Set } \Lambda \tag{1b}$$

$$\Bigg\{ \Pi \geq \sum_{t\in\mathcal{T}} \Bigg[\sum_{h\in\mathcal{H}} \Bigg(\underline{P}^{\mathrm{E}} \underline{\varphi}_{ht\mu'} - \overline{P}^{\mathrm{E}} \overline{\varphi}_{ht\mu'} - \sum_{c\in\mathcal{C}\setminus\tilde{\mathcal{C}}} \overline{P}_c^{\mathrm{C}} \overline{\rho}_{cht\mu'} - \sum_{c\in\tilde{\mathcal{C}}} \overline{P}_c^{\mathrm{C}} u_c^{\mathrm{C,I}(\nu)} \iota_{cht\mu'}^{\mathrm{C}} + \sum_{d\in\mathcal{D}} \Big(p_{dht}^{\mathrm{D,min}} \underline{\gamma}_{dht\mu'} - p_{dht}^{\mathrm{D,max}} \overline{\gamma}_{dht\mu'} \Big) - \sum_{r\in\mathcal{R}\setminus\tilde{\mathcal{R}}} K_{rht}^{\mathrm{R}} \overline{P}_r^{\mathrm{R}} \tau_{rht\mu'} - \sum_{r\in\tilde{\mathcal{R}}} K_{rht}^{\mathrm{R}} \overline{P}_r^{\mathrm{R}} u_r^{\mathrm{R,I}(\nu)} \iota_{rht\mu'}^{\mathrm{R}} - \sum_{s\in\mathcal{S}} \Bigg(\overline{P}_s^{\mathrm{S,D}} u_{sht}^{\mathrm{S}(\mu')} \overline{\kappa}_{sht\mu'} + \overline{P}_s^{\mathrm{S,C}} \Big(1 - u_{sht}^{\mathrm{S}(\mu')}\Big) \overline{\delta}_{sht\mu'} - \underline{S}_{sht}^{S} \underline{\psi}_{sht\mu'} + \overline{S}_{sht}^{S} \overline{\psi}_{sht\mu'} \Bigg) - \sum_{s\in\tilde{\mathcal{S}}} \Big(\overline{P}_s^{\mathrm{S,D}} u_s^{\mathrm{S,I}(\nu)} \iota_{sht\mu'}^{\mathrm{S,D}} + \overline{P}_s^{\mathrm{S,C}} u_s^{\mathrm{S,I}(\nu)} \iota_{sht\mu'}^{\mathrm{S,C}} \Big) \Bigg) + \sum_{d\in\mathcal{D}} d_d^{\mathrm{D}} \xi_{dt\mu'} + \sum_{s\in\mathcal{S}} S_{s0t}^{S} \phi_{s1t\mu'} \Bigg] \tag{1c}$$

$$-\alpha_{ht\mu'} + \underline{\varphi}_{ht\mu'} - \overline{\varphi}_{ht\mu'} = -N_t \lambda_{ht}^{\mathrm{E}}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1d}$$

$$\alpha_{ht\mu'} - \overline{\rho}_{cht\mu'} \leq c_c^{\mathrm{C}}, \forall c\in\mathcal{C}\setminus\tilde{\mathcal{C}}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1e}$$

$$\alpha_{ht\mu'} - \iota_{cht\mu'}^{\mathrm{C}} \leq c_c^{\mathrm{C}}, \forall c\in\tilde{\mathcal{C}}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1f}$$

$$-\alpha_{ht\mu'} + \underline{\gamma}_{dht\mu'} - \overline{\gamma}_{dht\mu'} + \xi_{dt\mu'} = 0, \forall d\in\mathcal{D}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1g}$$

$$\alpha_{ht\mu'} - \tau_{rht\mu'} \leq 0, \forall r\in\mathcal{R}\setminus\tilde{\mathcal{R}}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1h}$$

$$\alpha_{ht\mu'} - \iota_{rht\mu'}^{\mathrm{R}} \leq 0, \forall r\in\tilde{\mathcal{R}}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1i}$$

$$\alpha_{ht\mu'} - \overline{\kappa}_{sht\mu'} + \frac{1}{\eta_s^{\mathrm{S,D}}} \phi_{sht\mu'} \leq 0, \forall s\in\mathcal{S}\setminus\tilde{\mathcal{S}}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1j}$$

$$\alpha_{ht\mu'} - \overline{\kappa}_{sht\mu'} + \frac{1}{\eta_s^{\mathrm{S,D}}} \phi_{sht\mu'} - \iota_{sht\mu'}^{\mathrm{S,D}} \leq 0, \forall s\in\tilde{\mathcal{S}}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1k}$$

$$-\alpha_{ht\mu'} - \overline{\delta}_{sht\mu'} - \eta_s^{\mathrm{S,C}} \phi_{sht\mu'} \leq 0, \forall s\in\mathcal{S}\setminus\tilde{\mathcal{S}}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1l}$$

$$-\alpha_{ht\mu'} - \overline{\delta}_{sht\mu'} - \eta_s^{\mathrm{S,C}} \phi_{sht\mu'} - \iota_{sht\mu'}^{\mathrm{S,C}} \leq 0, \forall s\in\tilde{\mathcal{S}}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1m}$$

$$\phi_{sht\mu'} - \phi_{sh+1t\mu'} + \underline{\psi}_{sht\mu'} - \overline{\psi}_{sht\mu'} = 0, \forall s\in\mathcal{S}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1n}$$

$$\underline{\varphi}_{ht\mu'}, \overline{\varphi}_{ht\mu'} \geq 0, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1o}$$

$$\overline{\rho}_{cht\mu'} \geq 0, \forall c\in\mathcal{C}\setminus\tilde{\mathcal{C}}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1p}$$

$$\iota_{cht\mu'}^{\mathrm{C}} \geq 0, \forall c\in\tilde{\mathcal{C}}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1q}$$

$$\underline{\gamma}_{dht\mu'}, \overline{\gamma}_{dht\mu'} \geq 0, \forall d\in\mathcal{D}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1r}$$

$$\xi_{dt\mu'} \geq 0, \forall d\in\mathcal{D}, \forall t\in\mathcal{T} \tag{1s}$$

$$\tau_{rht\mu'} \geq 0, \forall r\in\mathcal{R}\setminus\tilde{\mathcal{R}}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1t}$$

$$\iota_{rht\mu'}^{\mathrm{R}} \geq 0, \forall r\in\tilde{\mathcal{R}}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1u}$$

$$\overline{\delta}_{sht\mu'}, \overline{\kappa}_{sht\mu'}, \underline{\psi}_{sht\mu'}, \overline{\psi}_{sht\mu'} \geq 0, \forall s\in\mathcal{S}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \tag{1v}$$

$$\iota_{sht\mu'}^{S,C}, \iota_{sht\mu'}^{S,D} \geq 0, \forall s\in\tilde{\mathcal{S}}, \forall h\in\mathcal{H}, \forall t\in\mathcal{T} \Bigg\}, \tag{1w}$$

$$\mu' = 1, \ldots, \mu,$$

where S_{s0t}^{S} is the initial level of stored energy of storage unit s in representative day t [MWh] and set $\Psi^{\mathrm{S}} = \Big\{ \Pi, \Upsilon^{\mathrm{ML}}, \Big\{ \{ \{ \overline{\rho}_{cht\mu'} \}_{\forall c\in\mathcal{C}\setminus\tilde{\mathcal{C}}}, \{ \iota_{cht\mu'}^{\mathrm{C}} \}_{\forall c\in\tilde{\mathcal{C}}}, \{ \underline{\gamma}_{dht\mu'}, \overline{\gamma}_{dht\mu'} \}_{\forall d\in\mathcal{D}}, \{ \tau_{rht\mu'} \}_{\forall r\in\mathcal{R}\setminus\tilde{\mathcal{R}}}, \{ \iota_{rht\mu'}^{\mathrm{R}} \}_{\forall r\in\tilde{\mathcal{R}}}, \{ \overline{\delta}_{sht\mu'}, \overline{\kappa}_{sht\mu'}, \underline{\psi}_{sht\mu'}, \overline{\psi}_{sht\mu'}, \phi_{sht\mu'} \}_{\forall s\in\mathcal{S}}, \{ \iota_{sht\mu'}^{\mathrm{S,C}}, \iota_{sht\mu'}^{\mathrm{S,D}} \}_{\forall s\in\tilde{\mathcal{S}}}, \alpha_{ht\mu'}, \underline{\varphi}_{ht\mu'}, \overline{\varphi}_{ht\mu'} \}_{\forall h\in\mathcal{H}}, \{ \xi_{dt\mu'} \}_{\forall d\in\mathcal{D}} \Big\}_{\forall t\in\mathcal{T}, \mu'=1,\ldots,\mu} \Big\}$ includes new variables indexed by subscript μ'. Note that parameters $u_{sht}^{\mathrm{S}(\mu')}$ denote the optimal values for u_{sht}^{S} resulting from the lower-level optimization of the subproblem at inner-loop iteration μ'.

The goal of the single-level approximation of the subproblem is the minimization of Π (1a). Constraints (1b) define the uncertainty set. In (1c), the value of the lower-level dual objective function for given $u_{sht}^{\mathrm{S}(\mu')}$ represents a lower bound for Π. Finally, constraints (1d)–(1w) correspond to the lower-level dual constraints for given $u_{sht}^{\mathrm{S}(\mu')}$.

Problem (1) includes bilinear terms in (1c) in the form of products of middle-level variables and lower-level dual (continuous) variables. Particularly, these bilinear terms are

TABLE I
WEIGHTS ASSIGNED TO REPRESENTATIVE DAYS

t	1	2	3	4	5
N_t [days]	140	158	28	34	5

TABLE II
DATA FOR CONVENTIONAL GENERATING UNITS I

Unit	$\hat{C}_c^{\mathrm{C,F}}$	$\hat{C}_c^{\mathrm{C,SD}}$	$\hat{C}_c^{\mathrm{C,SU}}$	$\hat{C}_c^{\mathrm{C,V}}$	$C_c^{\mathrm{C,I}}$
	[\$]	[\$]	[\$]	[\$/MWh]	[10^6 \$]
Existing unit	50	100	100	40	–
Candidate 1	35	70	70	25	80
Candidate 2	40	80	80	30	60
Candidate 3	45	90	90	35	40
Candidate 4	20	40	40	10	120
Candidate 5	25	50	50	15	100

TABLE III
DATA FOR CONVENTIONAL GENERATING UNITS II

Unit	$\underline{P}_c^{\mathrm{C}}$	$\overline{P}_c^{\mathrm{C}}$
	[MW]	[MW]
Existing unit	15	60
Candidate 1	15	55
Candidate 2	10	45
Candidate 3	10	40
Candidate 4	8	30
Candidate 5	5	20

$p_{dht}^{\mathrm{D,min}}\underline{\gamma}_{dht\mu'}$, $p_{dht}^{\mathrm{D,max}}\overline{\gamma}_{dht\mu'}$, and $d_d^{\mathrm{D}}\xi_{dt\mu'}$. Nonetheless, such bilinear terms can be replaced with equivalent mixed-integer linear expressions as explained in [2]. Hence, problem (1) is reformulated as a mixed-integer linear program (MILP).

The optimal values for middle-level variables c_c^{C}, d_d^{D}, $p_{dht}^{\mathrm{D,max}}$, $p_{dht}^{\mathrm{D,min}}$, and $\lambda_{ht}^{\mathrm{E}}$ are fed into the lower-level problem of the subproblem. The solution of this MILP provides new values for lower-level binary variables u_{sht}^{S}.

II. DATA FOR THE CASE STUDY

Based on historical data, we group the 365 days of the year into 5 representative days using a modified version of the K-means clustering technique proposed in [3]. The weight assigned to each representative day is shown in Table I.

Data for existing and candidate conventional generating units are displayed in Tables II and III. We consider two generation technologies, namely, a peak and a base technology. The peak technology features higher forecast production cost coefficients but lower investment cost coefficients. The existing conventional unit and candidate units 1, 2, and 3 correspond to the peak technology. On the other hand, the base technology is characterized by lower forecast production cost coefficients but higher investment cost coefficients. Candidate units 4 and 5 correspond to the base technology.

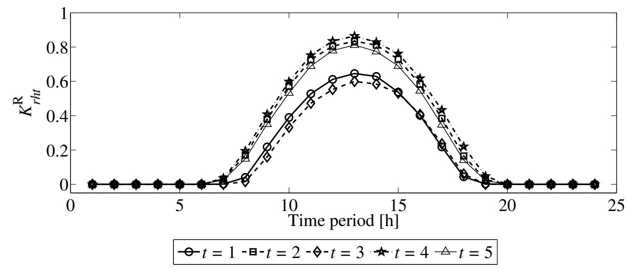

Fig. 1. Hourly capacity factors of photovoltaic solar units for each representative day.

TABLE IV
DATA FOR PHOTOVOLTAIC SOLAR UNITS

Unit	$\overline{P}_r^{\mathrm{R}}$ [MW]	$C_r^{\mathrm{R,I}}$ [\$$10^6$/MW]
Existing unit	80	–
Candidate 1	70	1.50
Candidate 2	65	1.20
Candidate 3	50	1.00
Candidate 4	35	0.98
Candidate 5	27	0.96

Confidence bounds for production cost coefficients of conventional generating units are obtained from the third column of Table II, which shows their forecast values. Lower bounds are a 20% decrease with respect to the forecast values whereas upper bounds are a 40% increase with respect to the forecast values for all production cost coefficients of conventional generating units.

Capacity factor data of photovoltaic solar units for each representative day are obtained from version 2017.1.17 of the System Advisor Model [4], based on weather data from the National Solar Radiation Data Base [5]. Photovoltaic power production data represent the average production over 12 different locations, thereby avoiding the effect of extremely favorable/disadvantageous solar conditions. For the sake of simplicity, we assume that all existing and prospective photovoltaic solar units have the same capacity factors. Fig. 1 shows the hourly capacity factors for each representative day. Capacities and investment cost coefficients for photovoltaic solar units are listed in Table IV.

The initial level of stored energy of the existing storage unit is 80 MWh, whereas candidate storage units are initially discharged. The lower bound for the level of stored energy of the existing and candidate storage units is 0 MWh across the time span except for the last hourly period of each representative day for the existing storage unit. In this case, for $h = |\mathcal{H}|$, the lower bound is set equal to the initial level. Furthermore, the charging and discharging efficiencies of all storage units are both equal to 0.9 p.u. The remaining data of the storage units, i.e., the maximum charging and discharging power levels, the upper bounds for the level of stored energy, and the investment cost coefficients are provided in Table V.

Regarding the flexible demands, the forecast minimum and

TABLE V
DATA FOR STORAGE UNITS

Unit	$\overline{P}_s^{\mathrm{S,C}}$ [MW]	$\overline{P}_s^{\mathrm{S,D}}$ [MW]	$\overline{S}_{sht}^{\mathrm{S}}$ [MWh]	$C_s^{\mathrm{S,I}}$ [$\$10^3$/MW]
Existing unit	80	80	160	–
Candidate 1	70	70	140	350
Candidate 2	60	60	120	320
Candidate 3	55	55	110	300
Candidate 4	40	40	80	270
Candidate 5	25	25	50	250

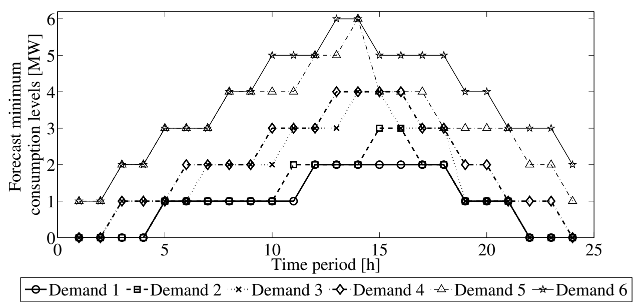

Fig. 2. Forecast minimum power consumption levels of the flexible demands

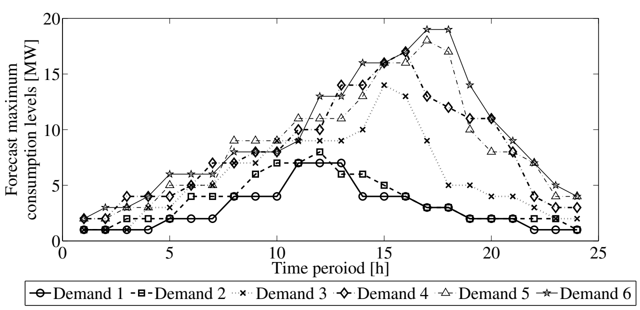

Fig. 3. Forecast maximum power consumption levels of the flexible demands

maximum consumption levels are shown in Figs. 2 and 3, respectively, whereas the forecast values of minimum daily energy consumption are displayed in Table VI. Lower bounds for all these date are a 20% decrease with respect to their forecast values while upper bounds are a 20% increase with respect to their forecast values.

Finally, forecast energy market prices are obtained from historical energy market prices for the Electric Reliability Council of Texas North Central zone [6]. Fig. 4 shows the forecast hourly energy market prices of each representative day. Lower bounds are a 20% decrease with respect to the forecast values whereas upper bounds are a 40% increase with respect to the forecast values for energy market prices.

TABLE VI
FORECAST VALUES FOR THE MINIMUM DAILY ENERGY CONSUMPTION OF THE FLEXIBLE DEMANDS

Demand	$\hat{D}_d^{\mathrm{D}}$ [MWh]
1	52.0
2	65.0
3	104.0
4	130.0
5	156.0
6	182.0

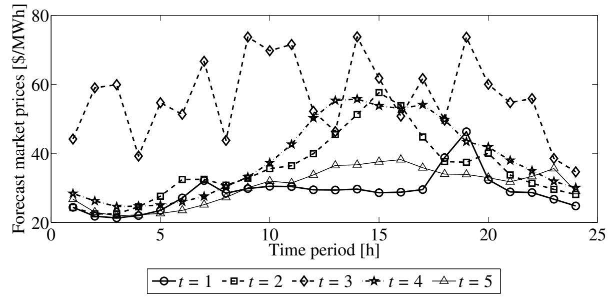

Fig. 4. Forecast hourly energy market prices for each representative day.

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