Latent Semantic Analysis

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Recap: vector space model

- Represent both doc and query by <u>concept</u> vectors
  – Each concept defines one dimension
  – *K* concepts define a high-dimensional space
  – Element of vector corresponds to concept weight
    - E.g., $d=(x_1,\ldots,x_k)$, x_i is "importance" of concept i
- Measure relevance
  – Distance between the query vector and document vector in this concept space

Recap: what is a good “basic concept”?

- Orthogonal
  - Linearly independent basis vectors
    - “Non-overlapping” in meaning
    - No ambiguity
- Weights can be assigned automatically and accurately
- Existing solutions
  - Terms or N-grams, i.e., bag-of-words
  - Topics, i.e., topic model ← We will come back to this later

Recap: TF weighting

- Two views of document length
  - A doc is long because it is verbose
  - A doc is long because it has more content
- Raw TF is inaccurate
  - Document length variation
  - “Repeated occurrences” are less informative than the “first occurrence”
  - Relevance does not increase proportionally with number of term occurrence
- Generally penalize long doc, but avoid over-penalizing
  - Pivoted length normalization

Recap: IDF weighting

- Solution
  - Assign higher weights to the rare terms
  - Formula
    - $IDF(t) = 1 + \log(\frac{N}{df(t)})$

Non-linear scaling

Total number of docs in collection

Number of docs containing term t

  - A corpus-specific property
    - Independent of a single document

Recap: TF-IDF weighting

- Combining TF and IDF
  - Common in doc → high tf → high weight
  - Rare in collection→ high idf→ high weight
  - $w(t,d) = TF(t,d) \times IDF(t)$
- Most well-known document representation schema in IR! (G Salton et al. 1983)

"Salton was perhaps the leading computer scientist working in the field of information retrieval during his time." - wikipedia

[Gerard Salton Award](#) – highest achievement award in IR

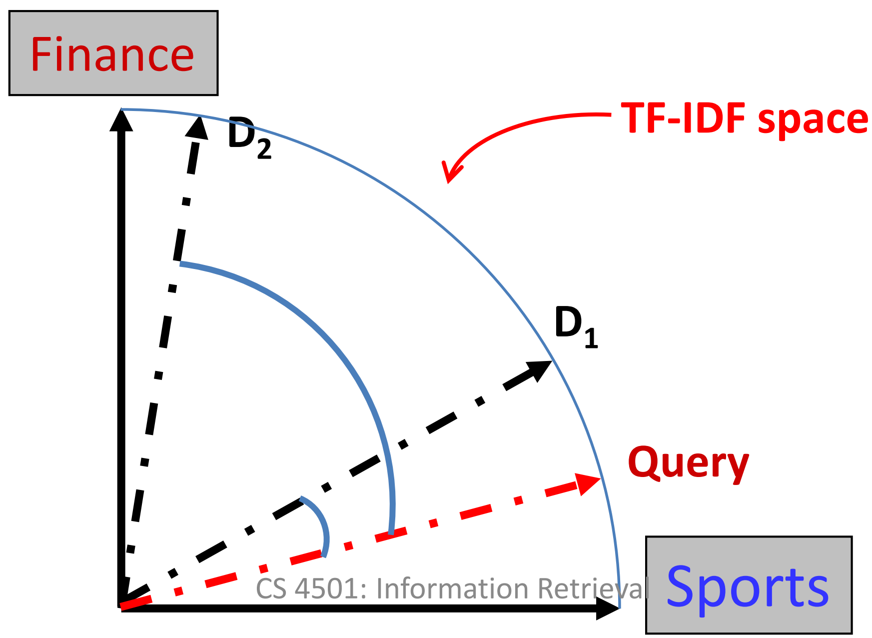
Recap: cosine similarity

- Angle between two vectors

  TF-IDF vector

  - $cosine(V_q, V_d) = \frac{V_q \times V_d}{|V_q|_2 \times |V_d|_2} = \boxed{\frac{V_q}{|V_q|_2}} \times \frac{V_d}{|V_d|_2}$
  - Document length normalized

  Unit vector



Recap: disadvantages of VS Model

- Assume term independence
- Assume query and document to be the same
- Lack of “predictive adequacy”
  - Arbitrary term weighting
  - Arbitrary similarity measure
- Lots of parameter tuning!

VS model in practice

- Document and query are represented by <u>term</u> vectors
  - Terms are not necessarily <u>orthogonal</u> to each other
    - Synonymy: car v.s. automobile
    - Polysemy: fly (action v.s. insect)

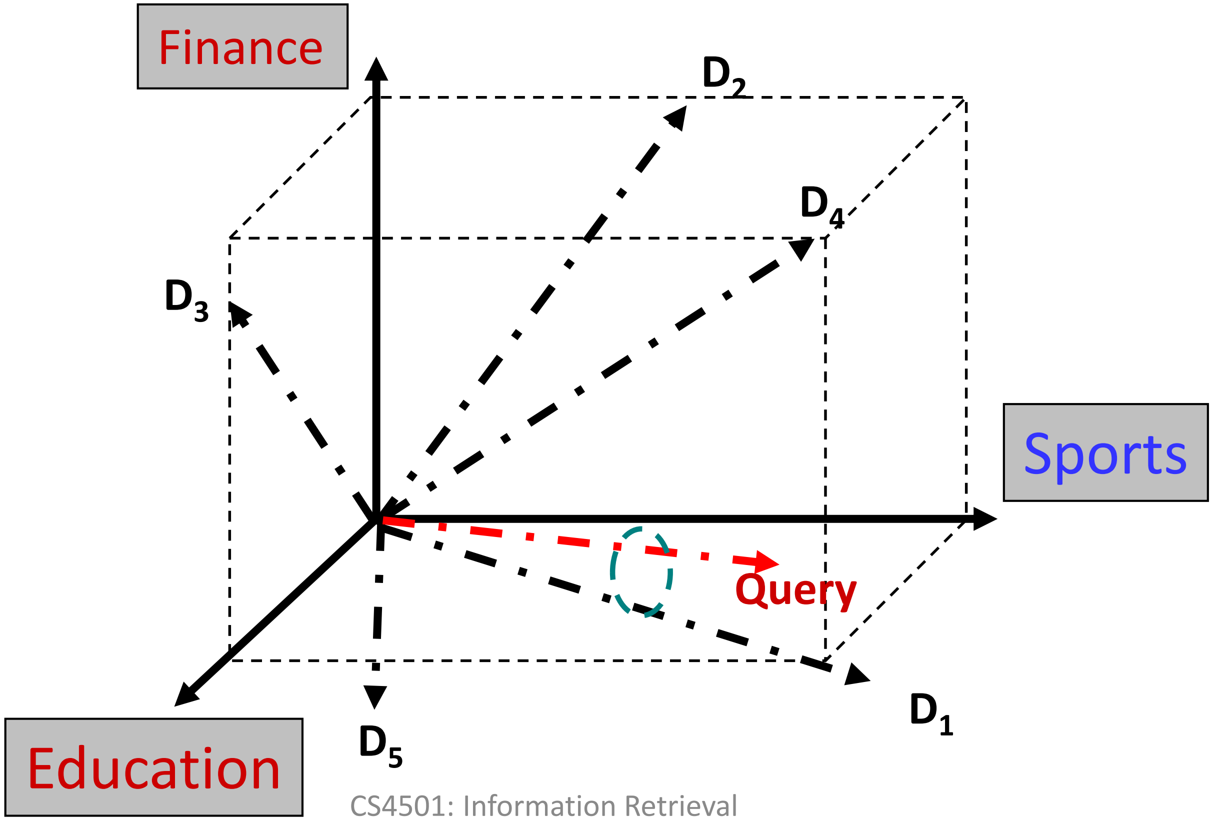
TABLE 1. Sample term by document matrix.[a]

	Access	Document	Retrieval	Information	Theory	Database	Indexing	Computer	REL	MATCH
Doc 1	x	x	x			x	x		R	
Doc 2				x*	x			x*		M
Doc 3			x	x*				x*	R	M

[a]Query: "IDF in *computer-based information look-up*"

Choosing basis for VS model

- A concept space is preferred
  - Semantic gap will be bridged



How to build such a space

- Automatic term expansion
  - Construction of thesaurus
    - WordNet
  - Clustering of words
- Word sense disambiguation
  - Dictionary-based
    - Relation between a pair of words should be similar as in text and dictionary's description
  - Explore word usage context

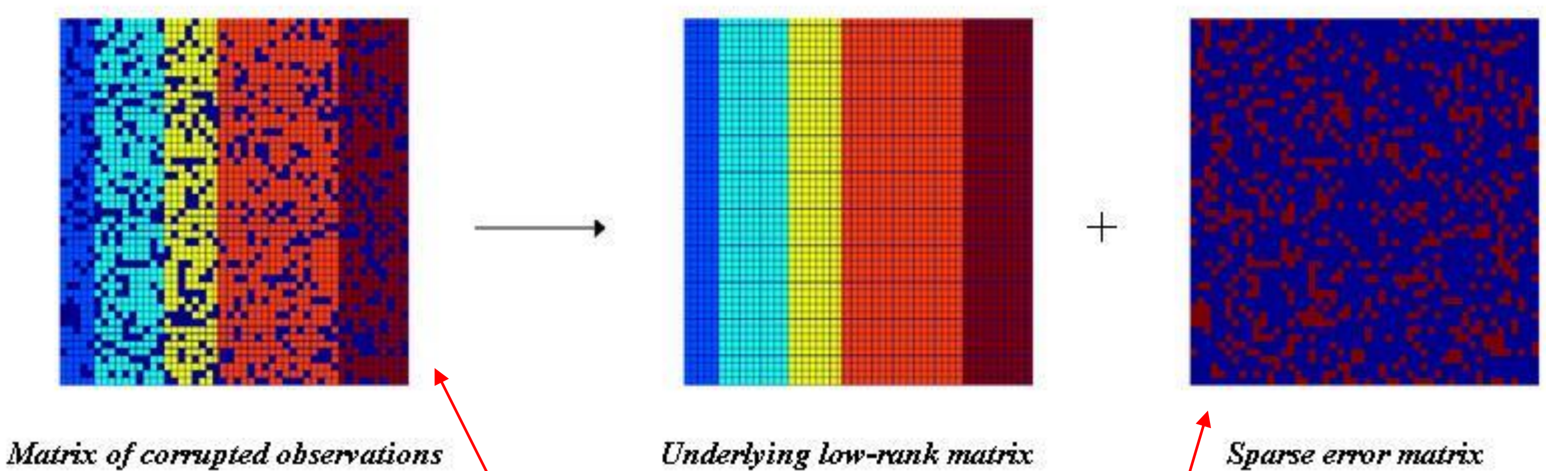
How to build such a space

- Latent Semantic Analysis
  - Assumption: there is some underlying latent semantic structure in the data that is partially obscured by the randomness of word choice with respect to retrieval
  - It means: the observed term-document association data is contaminated by random noise

How to build such a space

- Solution
  - Low rank matrix approximation

*Imagine this is *true* concept-document matrix*



Imagine this is our observed term-document matrix

Random noise over the word selection in each document

Latent Semantic Analysis (LSA)

- Low rank approximation of term-document matrix $C_{M \times N}$
  - Goal: remove noise in the observed term-document association data
  - Solution: find a matrix with rank k which is closest to the original matrix in terms of Frobenius norm

$$\hat{Z} = \underset{Z|rank(Z)=k}{\operatorname{argmin}} \|C - Z\|_F$$
$$= \underset{Z|rank(Z)=k}{\operatorname{argmin}} \sqrt{\sum_{i=1}^{M} \sum_{j=1}^{N} \left(C_{ij} - Z_{ij}\right)^2}$$

Basic concepts in linear algebra

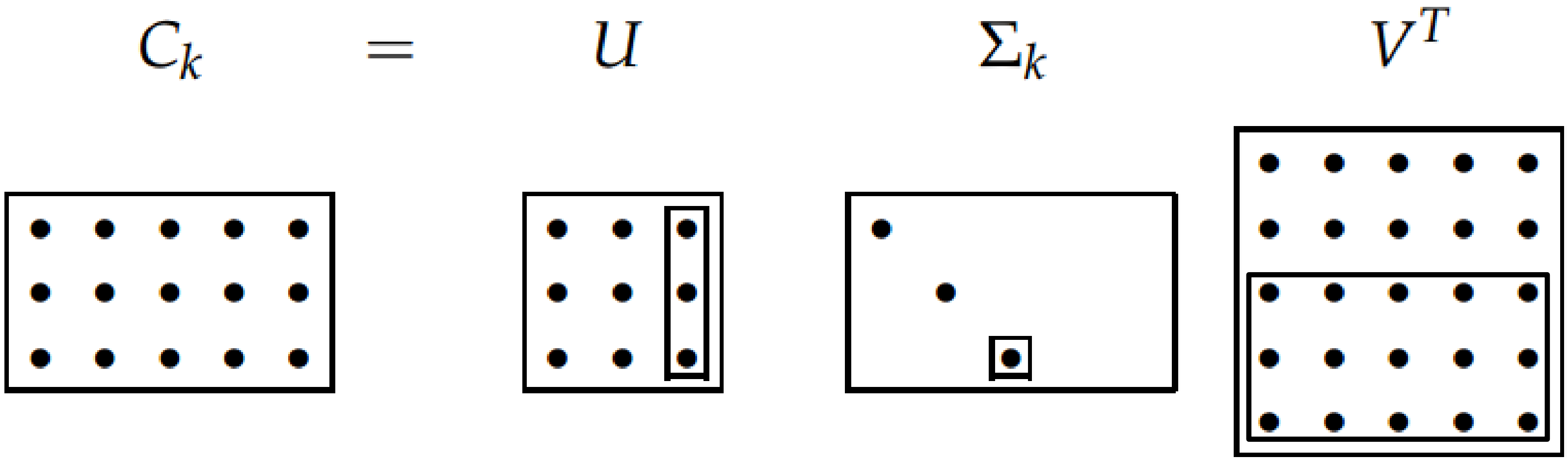
- Symmetric matrix
  - $C = C^T$
- Rank of a matrix
  - Number of linearly independent rows (columns) in a matrix $C_{M \times N}$
  - $rank(C_{M \times N}) \leq \min(M, N)$

Basic concepts in linear algebra

- Eigen system
  - For a square matrix $C_{M\times M}$
  - If $Cx = \lambda x$, x is called the right eigenvector of C and λ is the corresponding eigenvalue
- For a symmetric full-rank matrix $C_{M\times M}$
  - We have its eigen-decomposition as
    - $C = Q\Lambda Q^T$
    - where the columns of Q are the orthogonal and normalized eigenvectors of C and Λ is a diagonal matrix whose entries are the eigenvalues of C

Basic concepts in linear algebra

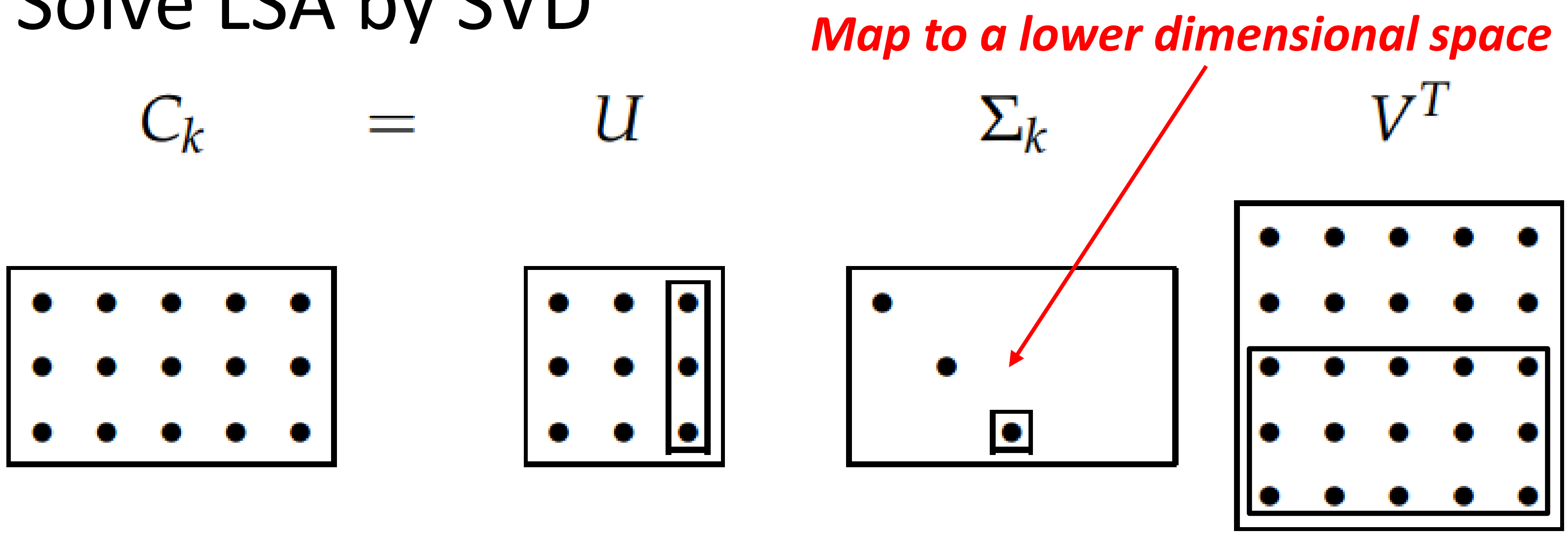
- Singular value decomposition (SVD)



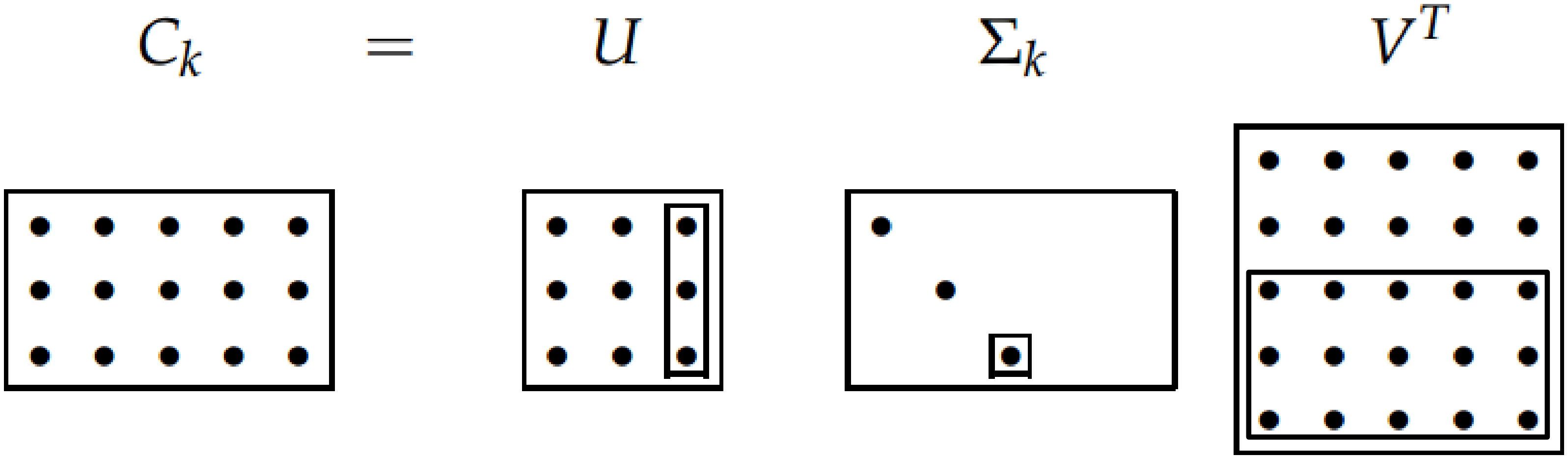
  - We define $C^k_{M \times N} = U_{M \times k} \Sigma_{k \times k} V^T_{N \times k}$
    - where we place Σ_{ii} in a descending order and set $\Sigma_{ii} = \sqrt{\lambda_i}$ for $i \leq k$, and $\Sigma_{ii} = 0$ for $i > k$

Latent Semantic Analysis (LSA)

- Solve LSA by SVD



1. Perform SVD on document-term adjacency matrix
2. Construct $C_{M\times N}^{k}$ by only keeping the largest k singular values in Σ non-zero



- $D_{M\times M} = C_{M\times N} \times C_{M\times N}^T$
  - D_{ij}: document-document similarity by counting how many terms co-occur in d_i and d_j
  - $D = (U\Sigma V^T) \times (U\Sigma V^T)^T = U\Sigma^2 U^T$
    - Eigen-decomposition of document-document similarity matrix
    - d_i's new representation is then $\left(U\Sigma^{\frac{1}{2}}\right)_i$ in this system(space)
    - In the lower dimensional space, we will only use the first k elements in $\left(U\Sigma^{\frac{1}{2}}\right)_i$ to represent d_i
- The same analysis applies to $T_{N\times N} = C_{M\times N}^T \times C_{M\times N}$

Geometric interpretation of LSA

- $C^k_{M\times N}(\mathrm{i},\mathrm{j})$ measures the relatedness between d_i and w_j in the k-dimensional space
- Therefore
  - As $C^k_{M\times N} = U_{M\times k}\Sigma_{k\times k}V^T_{N\times k}$
  - d_i is represented as $\left(U_{M\times k}\Sigma^{\frac{1}{2}}_{k\times k}\right)_i$
  - w_j is represented as $\left(V_{N\times k}\Sigma^{\frac{1}{2}}_{k\times k}\right)_j$

Latent Semantic Analysis (LSA)

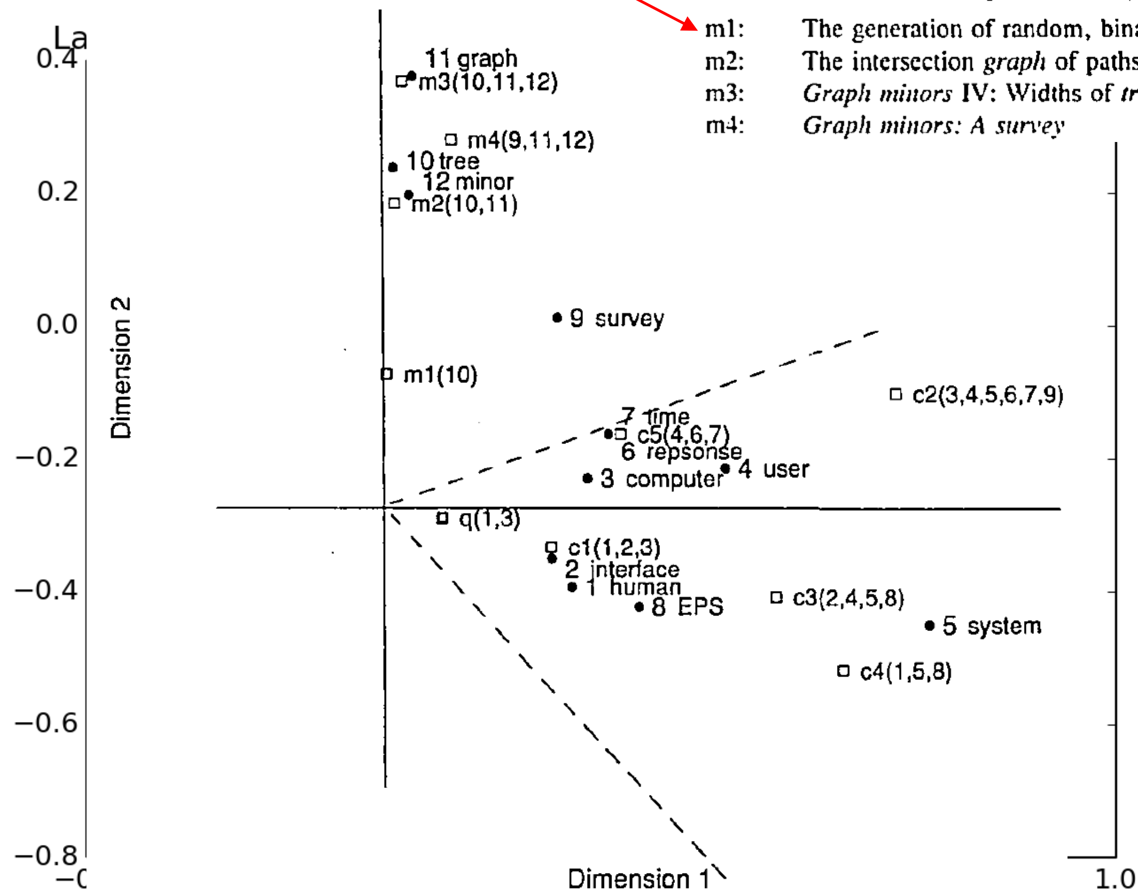
- Visualization

HCI

Graph theory

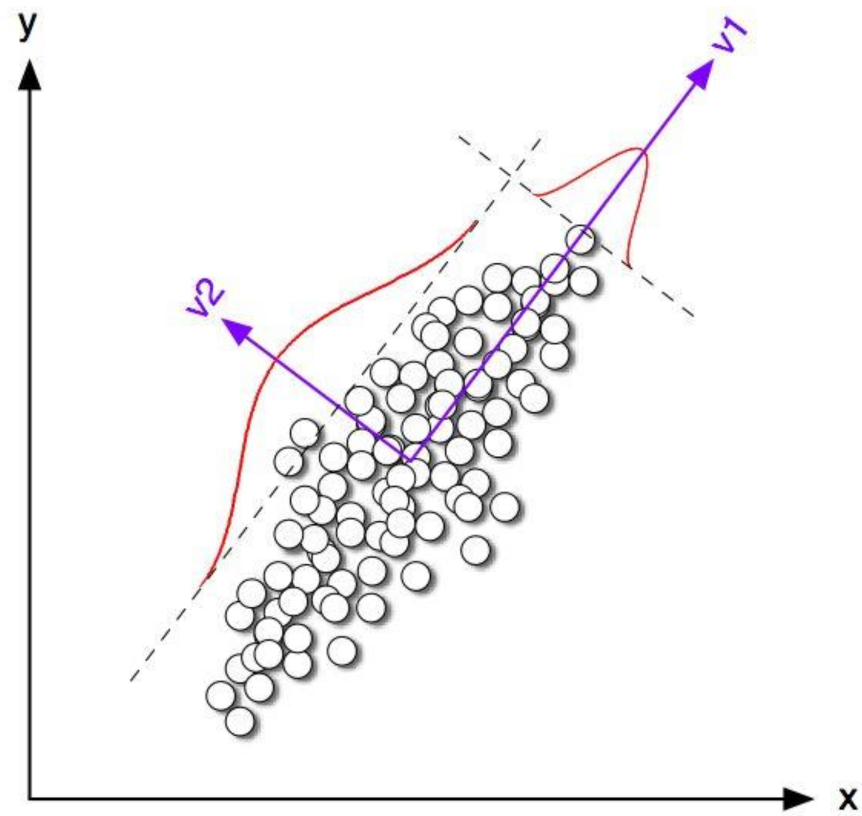
Titles

c1:	*Human* machine *interface* for Lab ABC *computer* applications
c2:	A *survey of user* opinion of *computer system response time*
c3:	The *EPS user interface* management *system*
c4:	*System* and *human system* engineering testing of *EPS*
c5:	Relation of *user*-perceived *response time* to error measurement
m1:	The generation of random, binary, unordered *trees*
m2:	The intersection *graph* of paths in *trees*
m3:	*Graph minors* IV: Widths of *trees* and well-quasi-ordering
m4:	*Graph minors: A survey*



What are those dimensions in LSA

- Principle component analysis



Latent Semantic Analysis (LSA)

- What we have achieved via LSA
  – Terms/documents that are closely associated are placed near one another in this new space
  – Terms that do not occur in a document may still close to it, if that is consistent with the major patterns of association in the data
  – A good choice of concept space for VS model!

LSA for retrieval

- Project queries into the new document space
  - $\tilde{q} = qV_{N\times k}\Sigma_{k\times k}^{-1}$
    - Treat query as a pseudo document of term vector
    - Cosine similarity between query and documents in this lower-dimensional space

LSA for retrieval

HCI

Graph theory

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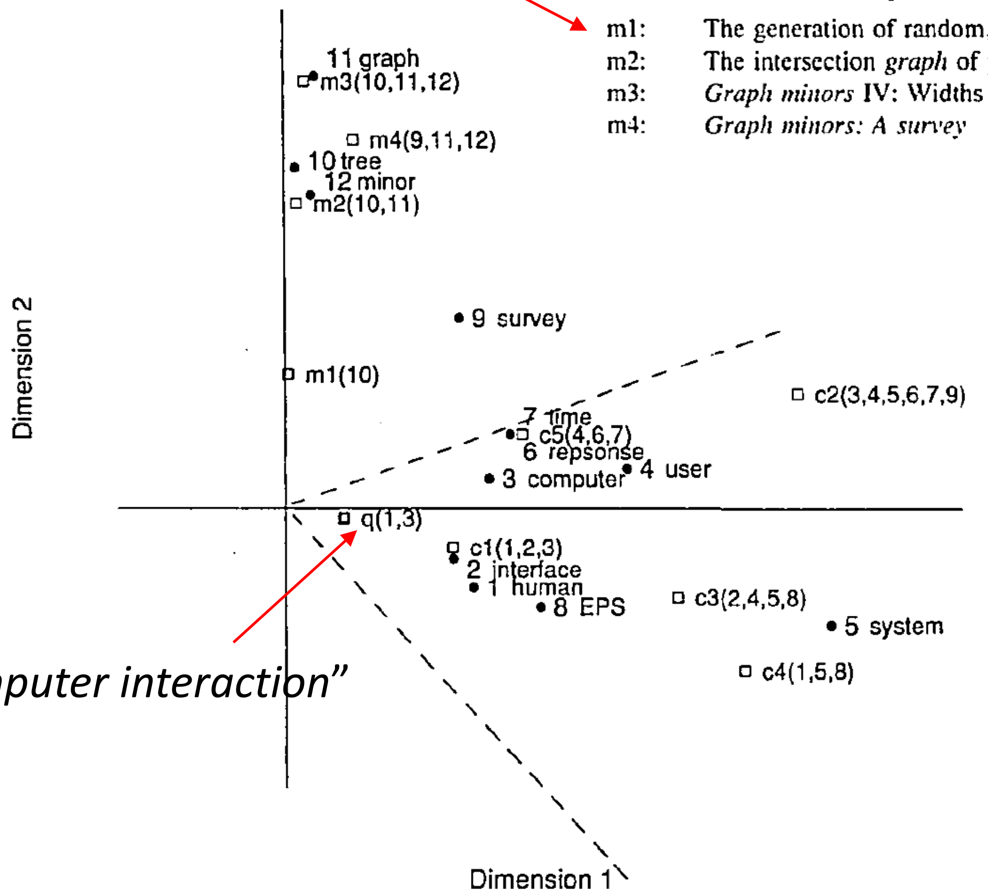

q: “*human computer interaction*”

Discussions

- Computationally expensive
  - Time complexity $O(MN^2)$
- Empirically helpful for recall but not for precision
  - Recall increases as k decreases
- Optimal choice of k
- Difficult to handle dynamic corpus
- Difficult to interpret the decomposition results

We will come back to this later!

LSA beyond text

- Collaborative filtering
  - User item matrix stores for each user the rating for the items

	i_1	i_2	i_3	i_4	i_5	...	i_M
u_1	2	0	3	2	5	...	1
u_2	0	4	0	0	0	...	5
u_3	0	2	0	0	0	...	4
u_4	1	0	4	2	4	...	2
...	...	...	...	...	...	...	...
u_K	2	...	4	...	4	...	1

Predicting unknown ratings

LSA beyond text

- Eigen face

LSA beyond text

- Cat from deep neuron network

One of the neurons in the artificial neural network, trained from still frames from unlabeled YouTube videos, learned to detect cats.

What you should know

- Assumption in LSA
- Interpretation of LSA
  - Low rank matrix approximation
  - Eigen-decomposition of co-occurrence matrix for documents and terms
- LSA for IR

Today's reading

- Chapter 13: Matrix decompositions and latent semantic indexing
  - All the chapters!
- Deerwester, Scott C., et al. "Indexing by latent semantic analysis." *JAsIs* 41.6 (1990): 391-407.