INSTITUTO FEDERAL
Paraná

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Ministério da Educação

Curso: Engenharia de Software | **Turno:** Noturno

Disciplina: Matemática Discreta e Lógica | **Professor:** Azuaite A. Schneider

Lista 3 – Álgebra Booleana e circuitos lógicos

1. Na expressão $S = \overline{A} \cdot B \cdot C \cdot (\overline{A + D})$, substituindo-se as variáveis A, B, C e D, por 0, 1, 1 e 1, respectivamente, qual será o valor de S?

2. Na expressão $S = [D + (\overline{A + B}) \cdot C] \cdot E$, substituindo-se as variáveis A, B, C, D e E, por 0, 0, 1, 1 e 1, respectivamente, qual será o valor de S?

3. Escreva a tabela verdade de cada uma das expressões abaixo.

   (a) $A + \overline{B + C}$ (b) $A\left(\overline{B + C}\right) + AB$ (c) $\overline{\overline{A} + B + C}$

4. Construa o circuito para as expressões abaixo:

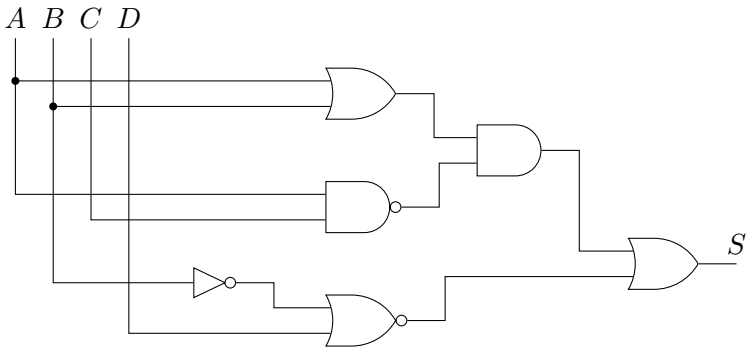
   (a) $S = AB + CDE$
   (b) $S = A + (B + CD) \cdot (B + A)$
   (c) $S = (A + B) \cdot (C + D) \cdot E$
   (d) $S = A \cdot B \cdot (C + D) + E$
   (e) $S = (A + B) \cdot (C + D) + E$
   (f) $S = A + (BC + DE) + FG + H$
   (g) $S = A(B + C)$
   (h) $S = (\overline{A + B})(C + (A + \overline{D}))$
   (i) $S = B\overline{C}A + \overline{(\overline{C} + D)}$
   (j) $S = ((A + \overline{\overline{B} + D}) \cdot (\overline{C} + A) + B) \cdot \overline{A + B}$
   (k) $S = A + B + \overline{C}B + \overline{A}$

5. Simplifique as expressões booleanas a seguir:

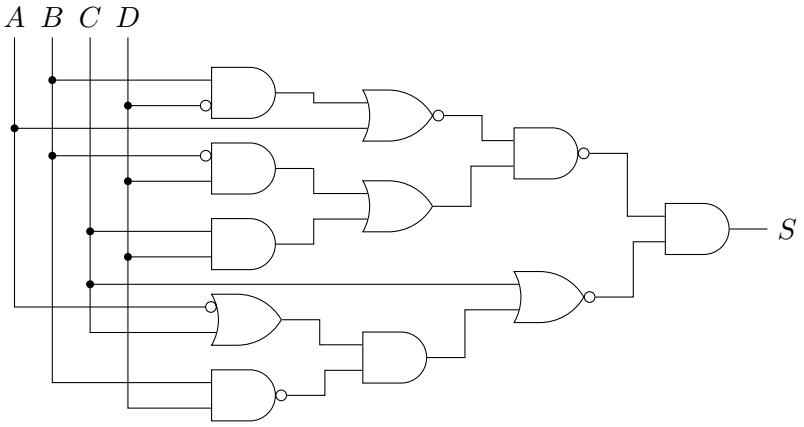
   (a) $A \cdot B + A \cdot \overline{B} + \overline{A} \cdot B$.
   (b) $(A + B) \cdot (A + C)$.
   (c) $\overline{A} \cdot \overline{B} + \overline{A} \cdot B + A \cdot \overline{B}$.
   (d) $(A + B) \cdot (A + \overline{B})$.
   (e) $A \cdot B + A \cdot \overline{B} + \overline{A} \cdot B + \overline{A} \cdot \overline{B}$.
   (f) $(A + B) \cdot (\overline{A} + B)$.
   (g) $A \cdot B + \overline{A} \cdot B$.
   (h) $(A + \overline{B}) \cdot (\overline{A} + \overline{B})$.
   (i) $\overline{A} \cdot \overline{B} + A \cdot B$.
   (j) $(A + B) \cdot (\overline{A} + \overline{B})$.
   (k) $S = AB + \overline{BC} + AC$
   (l) $S = \overline{ABD} + B\overline{CD} + \overline{BCD} + \overline{ABD}$

6. Determine as expressões das funções lógicas dos circuitos abaixo:

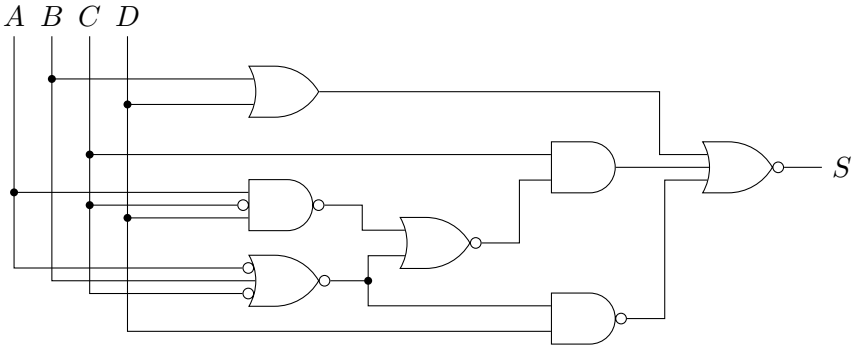
(a) Circuito 1



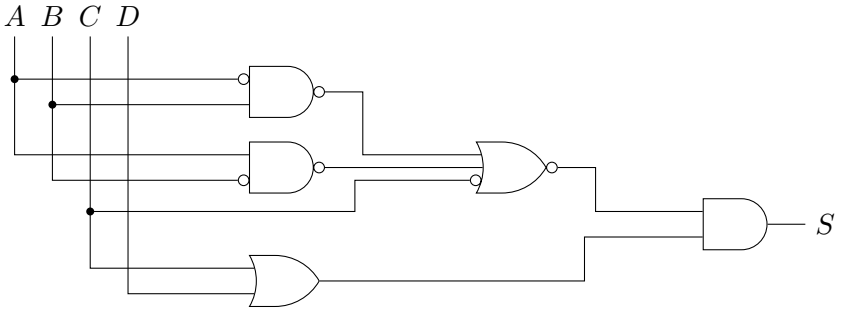
(b) Circuito 2



(c) Circuito 3

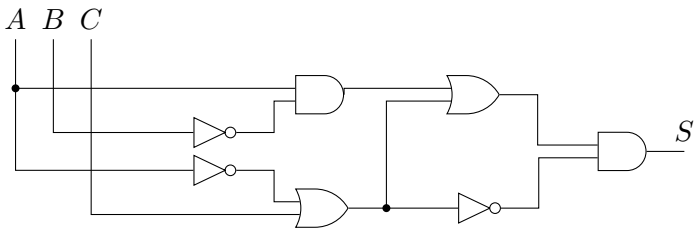


(d) Circuito 4

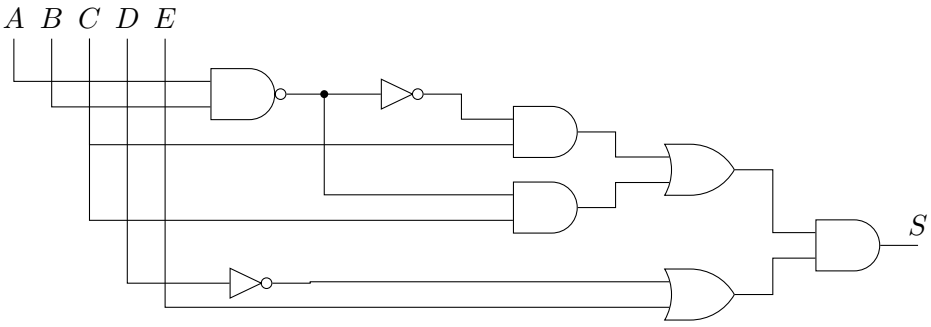


7. Para cada um dos circuitos abaixo:

Circuito 1



Circuito 2



(a) Determine uma expressão lógica a partir do circuito lógico.

(b) Simplifique a expressão lógica e construa um circuito equivalente a partir da expressão.

8. Através de manipulações algébricas e usando os postulados e os teoremas da Álgebra de Boole, verifique as seguintes igualdades:

(a) $(A + \overline{B} + A \cdot B) \cdot (A + \overline{B}) \cdot \overline{A} \cdot B = 0$

(b) $(A + \overline{B} + A \cdot \overline{B}) \cdot (A \cdot B + \overline{A} \cdot C + B \cdot C) = A \cdot B + \overline{A} \cdot \overline{B} \cdot C$

(c) $\overline{A} \cdot B \cdot (\overline{D} + D \cdot \overline{C}) + (A + D \cdot \overline{A} \cdot C) \cdot B = B$

Postulados da Álgebra Booleana

	Regra/Postulado	**Regra/Postulado**	**Sigla**
1	$A + 0 = A$	$A \cdot 0 = 0$	IDENT
2	$A + 1 = 1$	$A \cdot 1 = A$	IDENT
3	$A + \overline{A} = 1$	$A \cdot \overline{A} = 0$	IDENT
4	$A + A = A$	$A \cdot A = A$	IDENT
5	$\overline{\overline{A}} = A$	$(A + B) \cdot (A + C) = A + B \cdot C$	DN/AUX
6	$A + B = B + A$	$A \cdot B = B \cdot A$	COMUT
7	$(A + B) + C = A + (B + C)$	$(A \cdot B) \cdot C = A \cdot (B \cdot C)$	ASSOC
8	$A \cdot (B + C) = A \cdot B + A \cdot C$	$A + (B \cdot C) = A + B \cdot A + C$	DIST
9	$\overline{A + B} = \overline{A} \cdot \overline{B}$	$\overline{A \cdot B} = \overline{A} + \overline{B}$	DM
10	$A + A \cdot B = A$	$A + \overline{A} \cdot B = A + B$	AUX

Gabarito

1. $S = \overline{A} \cdot B \cdot C \cdot \left(\overline{A + D}\right) = \overline{0} \cdot 1 \cdot 1 \cdot \left(\overline{0 + 1}\right) = 1 \cdot 1 \cdot 1 \cdot \left(\overline{1}\right) = 1 \cdot 0 = 0$

2. $S = \left[D + \overline{(A + B) \cdot C}\right] \cdot E = \left[1 + \overline{(0 + 0) \cdot 1}\right] \cdot 1 = \left[1 + \overline{0 \cdot 1}\right] \cdot 1 = \left[1 + \overline{0}\right] \cdot 1 = [1 + 1] \cdot 1 = 1 \cdot 1 = 1$

3.

a)

A	B	C	$B + C$	$\overline{B + C}$	$A + \overline{B + C}$
0	0	0	0	1	1
0	0	1	1	0	0
0	1	0	1	0	0
0	1	1	1	0	0
1	0	0	0	1	1
1	0	1	1	0	1
1	1	0	1	0	1
1	1	1	1	0	1

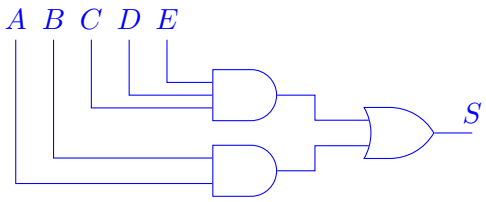
b)

A	B	C	$\overline{B + C}$	$A \cdot (\overline{B + C})$	AB	$A \cdot (\overline{B + C}) + AB$
0	0	0	1	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	0	0	0
1	0	0	1	1	0	1
1	0	1	0	0	0	0
1	1	0	0	0	1	1
1	1	1	0	0	1	1

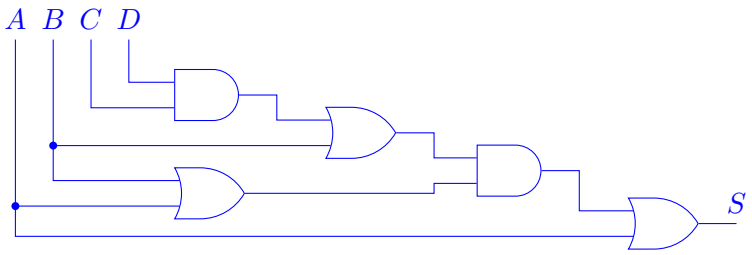
c)

A	B	C	$\overline{A}$	$\overline{A}+B+C$	$\overline{\overline{A}+B+C}$
0	0	0	1	1	0
0	0	1	1	1	0
0	1	0	1	1	0
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	0	1	0
1	1	0	0	1	0
1	1	1	0	1	0

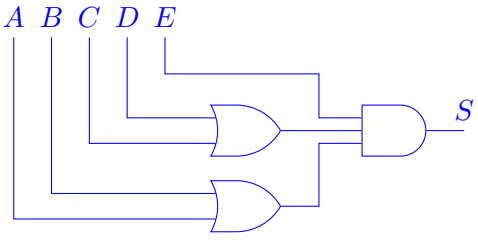
4. a)



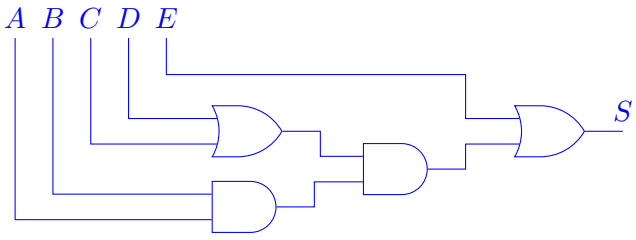
b)



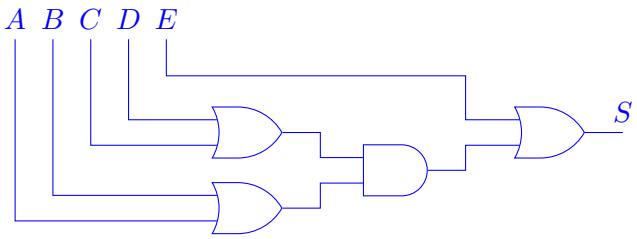
c)



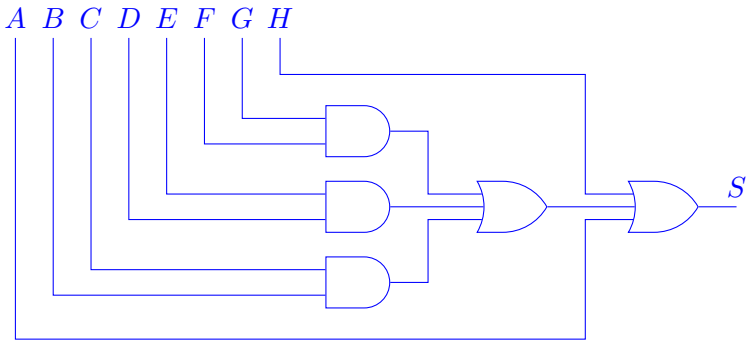
d)



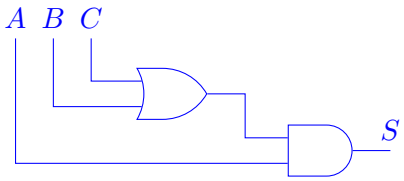
e)



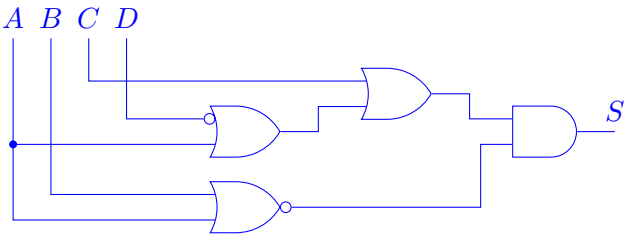
f)



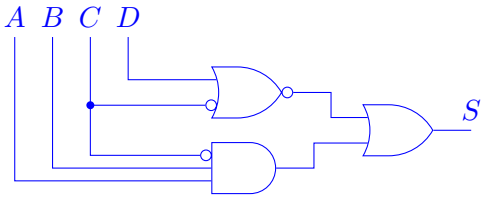
g)



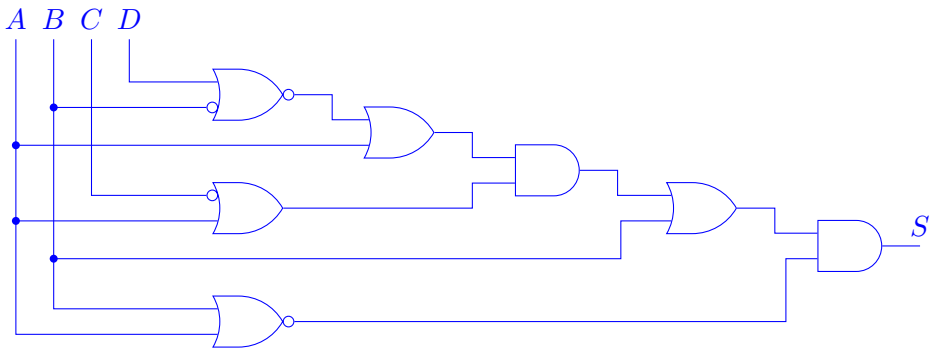
h)



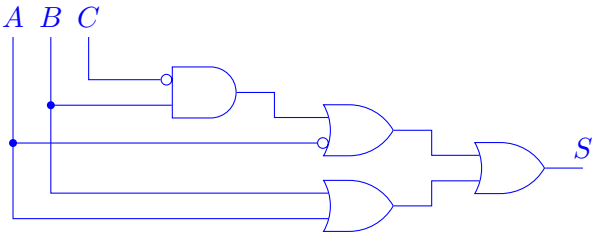
i)



j)



k)



5. a)

$$\begin{aligned} A \cdot B + A \cdot \overline{B} + \overline{A} \cdot B &= A \cdot \left(B + \overline{B}\right) + \overline{A} \cdot B && \text{DIST} \\ &= A \cdot 1 + \overline{A} \cdot B && \text{IDENT} \\ &= A + \overline{A} \cdot B && \text{IDENT} \\ &= A + B && \text{AUX} \end{aligned}$$

b)

$$(A + B) \cdot (A + C) = A + B \cdot C \qquad \text{DIST}$$

c)

$$\begin{aligned} \overline{A} \cdot \overline{B} + \overline{A} \cdot B + A \cdot \overline{B} &= \overline{A}\left(\overline{B} + B\right) + A\overline{B} && \text{DIST} \\ &= \overline{A} + A\overline{B} && \text{IDENT} \\ &= \overline{A} + \overline{B} && \text{AUX} \end{aligned}$$

d)

$$\begin{aligned} (A + B) \cdot \left(A + \overline{B}\right) &= A + B \cdot \overline{B} && \text{DIST} \\ &= A + 0 && \text{IDENT} \\ &= A && \text{IDENT} \end{aligned}$$

e)

$$\begin{aligned} AB + A\overline{B} + \overline{A}B + \overline{A} \cdot \overline{B} &= A \cdot \left(B + \overline{B}\right) + \overline{A}\left(B + \overline{B}\right) && \text{DIST} \\ &= A \cdot 1 + \overline{A} \cdot 1 && \text{IDENT} \\ &= A + \overline{A} && \text{IDENT} \\ &= 1 && \text{IDENT} \end{aligned}$$

f)

$$\begin{aligned}(A+B)\cdot(\overline{A}+B) &= (B+A)\cdot(B+\overline{A}) && \text{COMUT}\\ &= B+(A\cdot\overline{A}) && \text{DIST}\\ &= B+0 && \text{IDENT}\\ &= B && \text{IDENT}\end{aligned}$$

g)

$$\begin{aligned}A\cdot B+\overline{A}\cdot B &= B\cdot A+B\cdot\overline{A} && \text{COMUT}\\ &= B\cdot(A+\overline{A}) && \text{DIST}\\ &= B\cdot 1 && \text{IDENT}\\ &= B && \text{IDENT}\end{aligned}$$

h)

$$\begin{aligned}(A+\overline{B})\cdot(\overline{A}+\overline{B}) &= (A\cdot\overline{A})+\overline{B} && \text{DIST}\\ &= 0+\overline{B} && \text{IDENT}\\ &= \overline{B} && \text{IDENT}\end{aligned}$$

i)

$$\begin{aligned}\overline{A}\cdot\overline{B}+A\cdot B &= (\overline{A}\cdot\overline{B}+A)\cdot(\overline{A}\cdot\overline{B}+B) && \text{DIST}\\ &= (A+\overline{A}\cdot\overline{B})\cdot(B+\overline{B}\cdot\overline{A}) && \text{COMUT}\\ &= (A+\overline{B})\cdot(\overline{B}+\overline{A}) && \text{AUX}\end{aligned}$$

j)

$$\begin{aligned}(A+B)\cdot(\overline{A}+\overline{B}) &= ((A+B)\cdot\overline{A})+((A+B)\cdot\overline{B}) && \text{DIST}\\ &= A\overline{A}+B\overline{A}+A\overline{B}+B\cdot\overline{B} && \text{DIST}\\ &= 0+B\overline{A}+A\overline{B}+0 && \text{IDENT}\\ &= A\overline{B}+B\overline{A} && \text{COMUT}\end{aligned}$$

k)

$$\begin{aligned}AB+\overline{BC}+AC &= AB+\overline{B}+\overline{C}+AC && \text{DM}\\ &= \overline{B}+BA+\overline{C}+CA && \text{ASSOC/COMUT}\\ &= \overline{B}+A+\overline{C}+A && \text{AUX}\\ &= \overline{B}+\overline{C}+A && \text{IDENT}\end{aligned}$$

l)

$$
\begin{aligned}
S &= \overline{ABD} + B\overline{CD} + \overline{BCD} + \overline{ABD} \\
&= \overline{A} + \overline{B} + \overline{D} + B(\overline{C} + \overline{D}) + \overline{B} + \overline{C} + \overline{D} + \overline{A} + \overline{B} + \overline{D} && \text{DM} \\
&= \overline{A} + \overline{B} + \overline{C} + \overline{D} + B(\overline{C} + \overline{D}) && \text{IDENT} \\
&= \overline{A} + \overline{C} + \overline{D} + \overline{B} + B(\overline{C} + \overline{D}) && \text{ASSOC} \\
&= \overline{A} + \overline{C} + \overline{D} + \overline{B} + \overline{C} + \overline{D} && \text{AUX} \\
&= \overline{A} + \overline{B} + \overline{C} + \overline{D} && \text{IDENT}
\end{aligned}
$$

6.

a) $S = \left((A + B) \cdot \overline{A \cdot C}\right) + \left(\overline{\overline{B} + D}\right)$

b) $S = \overline{\left(\overline{B \cdot \overline{D} + A}\right) \cdot (\overline{B} \cdot D + C \cdot D)} \cdot \left(\left(\overline{(\overline{A} + C) \cdot \overline{B} \cdot D}\right) + C\right)$

c) $S = (B + D) + \overline{\left(\overline{A\overline{C}D} + \overline{\overline{A} + B + \overline{C}}\right) \cdot C + \overline{(\overline{A} + B + \overline{C}) \cdot D}}$

d) $S = \overline{\left(\overline{\overline{A} \cdot B} + \overline{A \cdot \overline{B}} + \overline{C}\right)} \cdot (C + D)$

7. Circuito 1

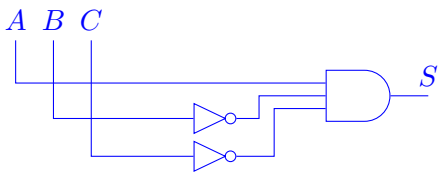
a) Expressão lógica (booleana):

$$(A \cdot \overline{B} + \overline{A} + C) \cdot \overline{(\overline{A} + C)}$$

b) Simplificação booleana:

$$
\begin{aligned}
(A \cdot \overline{B} + \overline{A} + C) \cdot \overline{(\overline{A} + C)} &= (\overline{A} + A\overline{B} + C) \cdot \overline{A} \cdot \overline{C} && \text{ASSOC e DM} \\
&= (\overline{A} + \overline{B} + C) \cdot A \cdot \overline{C} && \text{AUX e DN} \\
&= \overline{A}A\overline{C} + A\overline{C}\overline{B} + CA\overline{C} && \text{DIST} \\
&= 0 \cdot \overline{C} + A\overline{C}\overline{B} + 0 \cdot A && \text{IDENT} \\
&= 0 + A\overline{C}\overline{B} + 0 && \text{IDENT} \\
&= A\overline{C}\overline{B} && \text{IDENT}
\end{aligned}
$$

Circuito 1 simplificado



Circuito 2

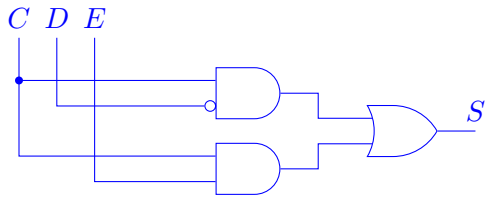
a) Expressão lógica (booleana):

$$\left(\overline{(A \cdot B)} \cdot C + \overline{AB} \cdot C\right) \cdot (\overline{D} + E)$$

b) Simplificação booleana:

$$\begin{aligned}
\left(\overline{(A \cdot B)} \cdot C + \overline{AB} \cdot C\right) \cdot (\overline{D} + E) &= (AB \cdot C + \overline{AB} \cdot C) \cdot (\overline{D} + E) && \text{DN} \\
&= (C \cdot (AB + \overline{AB})) \cdot (\overline{D} + E) && \text{DIST} \\
&= (C \cdot 1) \cdot (\overline{D} + E) && \text{IDENT} \\
&= C \cdot (\overline{D} + E) && \text{IDENT} \\
&= C\overline{D} + CE && \text{DIST}
\end{aligned}$$

Circuito 2 simplificado



8. a)

$$\begin{aligned}
(A + \overline{B} + A \cdot B) \cdot (A + \overline{B}) \cdot \overline{A} \cdot B &= (A + \overline{B}) \cdot (A + \overline{B}) \cdot \overline{A} \cdot B && \text{AUX} \\
&= (A + \overline{B}) \cdot \overline{A} \cdot B && \text{IDENT} \\
&= A\overline{A}B + \overline{B} \cdot \overline{A} \cdot B && \text{DIST} \\
&= 0 \cdot B + 0 \cdot \overline{A} && \text{IDENT} \\
&= 0 + 0 && \text{IDENT} \\
&= 0 && \text{IDENT}
\end{aligned}$$

b)

$$\begin{aligned}
&(A + \overline{B} + A \cdot \overline{B}) \cdot (A \cdot B + \overline{A} \cdot C + BC) \\
&= (A + \overline{B}) \cdot (AB + C(\overline{A} + B)) && \text{AUX e DIST} \\
&= AAB + AC(\overline{A} + B) + \overline{B}AB + \overline{B}C(\overline{A} + B) && \text{DIST} \\
&= AB + AC\overline{A} + ACB + 0 \cdot A + \overline{B}C\overline{A} + \overline{B}CB && \text{DIST e IDENT} \\
&= AB + ABC + C \cdot 0 + 0 + \overline{AB}C + C \cdot 0 && \text{IDENT e ASSOC} \\
&= AB + \overline{AB}C && \text{AUX}
\end{aligned}$$

c)

$$\begin{aligned}
\overline{A} \cdot B(\overline{D} + D \cdot \overline{C}) + (A + D \cdot \overline{A} \cdot C) \cdot B &= \overline{A}B(\overline{D} + \overline{C}) + (A + DC) \cdot B && \text{AUX} \\
&= \overline{A}B\overline{D} + \overline{A}B\overline{C} + AB + DCB && \text{DIST} \\
&= B(A + \overline{AD}) + \overline{A}B\overline{C} + BCD && \text{DIST} \\
&= B(A + \overline{D}) + \overline{A}B\overline{C} + BCD && \text{AUX} \\
&= BA + B\overline{D} + \overline{A}B\overline{C} + BCD && \text{DIST} \\
&= B(A + \overline{AC}) + B(D + CD) && \text{DIST} \\
&= B(A + \overline{C}) + B(\overline{D} + C) && \text{AUX} \\
&= BA + B\overline{C} + B\overline{D} + BC && \text{DIST} \\
&= BA + B\overline{D} + B(\overline{C} + C) && \text{DIST} \\
&= BA + B\overline{D} + B && \text{IDENT} \\
&= B + B\overline{D} && \text{AUX} \\
&= B && \text{AUX}
\end{aligned}$$